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Grand Unified Models from Strings on Orbifolds

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GRAND UNIFICATION

IN FOUR AND MORE DIMENSIONS

- GUTs in $D=4$

  motivation, symmetries, proton decay

- GUTs in $D>4$

  orbifolds, symmetry breaking, phenomenology

- Strings on orbifolds

  geometry of symm. breaking and zero modes

(1) GUTs in D = 4 *

Motivation: the structure of the SM, its gauge group

$$G_{SM} = SU(3) \times SU(2) \times U(1) ,$$

with the SU(2) Higgs doublets H_1, H_2, and the quarks and leptons

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; u_R^c , \; d_R^c \; ; \; l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \; e_R^c , \; (\nu_R^c) ,$$

which come in 3 generations, suggests the embedding in a unified group:

$$\begin{aligned} G_{SM} &\subset SU(5) \quad \text{or} \quad SU(4) \times SU(2) \times SU(2) \\ &\subset SO(10) \\ &\subset E_6 , \quad \ldots \end{aligned}$$

* cf. S. Raby, PDG, http://pdg.lbl.gov/

Remarkably, matter fields are in complete GUT representations, i.e., for SU(5),

$$\underline{10} = (q_L, u_R^c, e_R^c)\,, \quad \underline{5}^* = (d_R^c, l_L)\,, \quad (\underline{1} = \nu_R^c)\,,$$

or for the Pati-Salam group $SU(4) \times SU(2) \times SU(2)$,

$$(4,2,1) = (q_L, l_L)\,, \quad (4^*,1,2) = (u_R^c, d_R^c, \underline{\nu_R^c}, e_R^c)\,;$$

all quarks and leptons of one generation are unified in a single multiplet for SO(10):

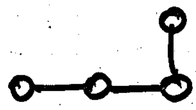
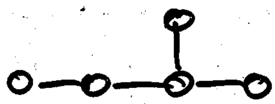
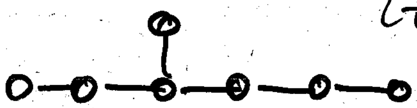
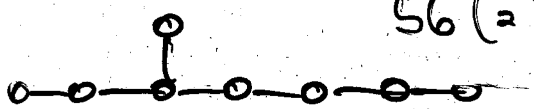
$$16 = 10 + 5^* + 1 = (4,2,1) + (4^*,2,1)$$

The GUT group SO(10) is singled out; for larger groups, unknown 'matter' is needed, e.g., $\underline{27}$ of E_6

NOTE: Higgs and gauge fields are split multiplets (unknown fields have to be added to form GUT reps; just bosons for non-SUSY)

'Exceptional sequence' of groups

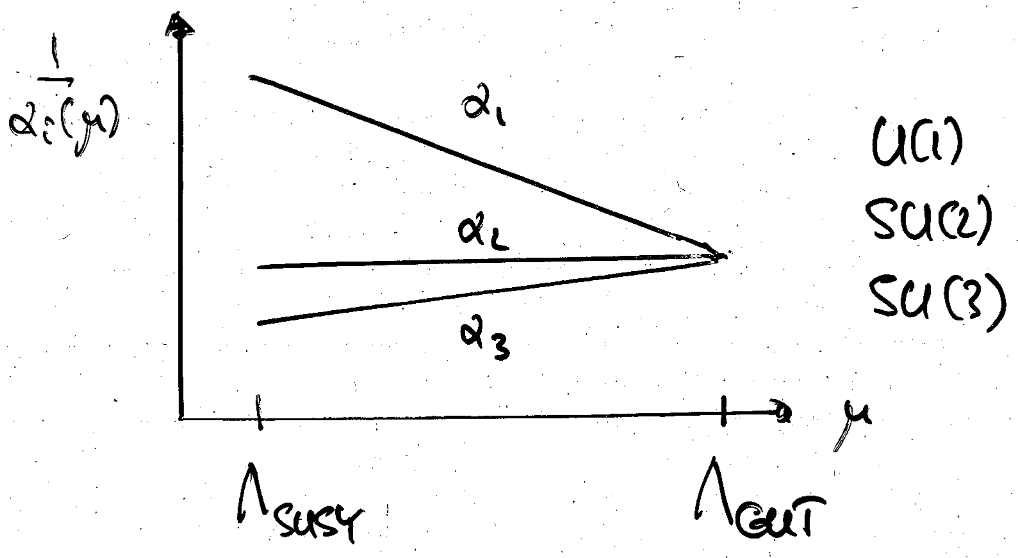
cf. D. Olive '81, ...

Group E_n		Coset space $E_{n+1}/E_n \times U(1)$
$E_3 = SU_3 \times SU(2)$		$(3,2)$ + c.c.
$E_4 = SU_5$		10 + c.c.
$E_5 = SO_{10}$		$16\ (= \bar{5} + 10 + 1)$ + c.c.
E_6		$27\ (= 16 + 10 + 1)$ + c.c.
E_7		$56\ (= 27 + \overline{27} + 1 + 1) + 1$ + c.c.
E_8		

→ natural embedding of SM and GUT groups in E_8 (cf heterotic string)

– Hints for the unification scale

• unification of couplings → S. Martin, J. Ellis



'requires' low energy SUSY, $\Lambda_{SUSY} \sim 1$ TeV, implies $\Lambda_{GUT} \sim 10^{16}$ GeV

→ standard SUSY GUT picture

• small neutrino masses / seesaw → J. Beacom

$$m_{\nu_3} \sim \frac{m_{D_3}^2}{M_3} \sim \frac{v^2}{\Lambda_{GUT}} \sim 0.004 \text{ eV}$$

cf. $(\Delta m^2_{sol})^{1/2} \simeq 0.05$ eV,

$(\Delta m^2_{atm})^{1/2} \simeq 0.009$ eV; agreement accidental?

– Proton Decay

SU(5) GUT ; Higgs fields :

$$\underbrace{H_2(5) \;,\; H_1(5^*)}_{SU(2)\times U(1)-} \;,\quad \underset{SU(5)-\text{breaking}}{\Sigma(24)}$$

superpotential for mass generation :

$$W = \frac{1}{2} m \,\mathrm{tr}\,\Sigma^2 + \frac{1}{3} a \,\mathrm{tr}\,\Sigma^3 + \lambda H_1 (\Sigma + 3\sigma) H_2$$

$$+ \frac{1}{4} Y_2^{ij} 10_i 10_j H_2 + \sqrt{2}\, Y_1^{ij} 10_i 5^*_j H_1$$

$$\langle\Sigma\rangle = \sigma \begin{pmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{pmatrix}$$

breaks $SU(5) \to SU(3) \times SU(2) \times U(1)$;

$$H_2 = (\underset{SU(2)}{H_2}, \underset{SU(3)}{T_2}) \;,\quad H_1 = (H_1, T_1^c)$$

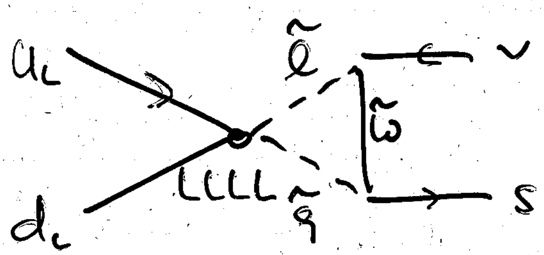
$T_1^c T_2$ mass term : $m_T = 5\lambda\sigma$

$H_1 H_2$: $m = 0$ (i.e. $O(v)$)

mass parameters have to be adjusted to order $\frac{v}{\sigma} \sim 10^{-13}$, **doublet - triplet splitting problem !**

Proton decay from dim-5 operators :

$$W_5 = \frac{1}{m_T}\left(\frac{1}{2}(q Y_2 q)(q Y_1 \ell) + (u^c Y_2 e^c)(u^c Y_1 d^c)\right)$$



,

$\Gamma(p \to K^+ \bar{\nu})$ typically too large !

minimal SU(5) excluded (Murayama, Pierce)?

NO ! Yukawa couplings have to be corrected, e.g.

$$\Delta W_Y = f_1\, 10\, 10\, \frac{\Sigma}{M_{P\ell}}\, H_2 + f_2\, 10\, 10\, H_2 \frac{\Sigma}{M_{P\ell}} + \ldots$$

In consistent SU(5), proton decay can be suppressed (Bajc, Perez, Senjanović ; Dorsner ; Costa, Wiesenfeldt), but no 'natural' model

(2) GUTs in D > 4

Motivation: simplicity of symmetry breaking, solution of doublet-triplet splitting problem; no dim 5 p-decay; (naturally embedded in string theory)
Kawamura '01; Altarelli, Feruglio '01; Hall, Nomura, hep-ph/0103125; Hebecker, March-Russell, hep-ph/0107039

→ **review:** M. Quiros, hep-ph/0302189

SU(5) model, D = 5, SUSY
(8 supercharges, N = 2)

gauge fields: V_M, Σ, λ^i $(i = 1,2)$

symplectic Majorana spinors: $\lambda^i = \epsilon^{ij} C \bar{\lambda}_j^T$

matter fields: H^i, $i = 1,2$, $\psi = \psi_L^1 + \psi_R^2$

(hypermultiplet)

action :

$$S = \int d^5x \left\{ \frac{1}{g^2} \left(-\frac{1}{2} \mathrm{tr}\, V_{MN}^2 - \mathrm{tr}(D_M \Sigma)^2 + \mathrm{tr}(X^I)^2 \right.\right.$$

$$- \mathrm{tr}(\bar{\lambda}_i i\gamma^M D_M \lambda^i) + \mathrm{tr}(\bar{\lambda}_i [\Sigma, \lambda^i])$$

$$- (D_M H)_i^\dagger (D^M H^i) - i\bar{\Psi}\gamma^M D_M \Psi + F_i^\dagger F_i$$

$$- \bar{\Psi}\Sigma\Psi + H_i^\dagger \sigma^{I\,i}{}_j X^I H^j$$

$$\left. + H_i^\dagger \Sigma^2 H^i + i\sqrt{2}\,\bar{\Psi}\lambda^i \epsilon_{ij} H^j + \mathrm{h.c.} \right\}$$

N=2 SUSY !

auxiliary fields : X^I , F_i , $SU(2)_R$!

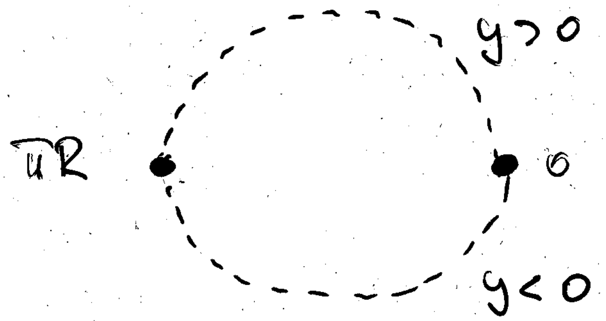
N=1 field content (4D) :

$$V = (V_\mu, \lambda_L) \quad , \quad \Phi = \left(\frac{\Sigma + i V_5}{\sqrt{2}}, \lambda_R^c \right)$$

vector , chiral multiplet

$$H = (H^1, \Psi^1_L) \quad , \quad H^c = (H_2^\dagger, \Psi_R^{2c})$$

compactification on orbifold S^1/Z_2 :



$y = x^5$

$y \in [-\pi R, +\pi R]$

S^1 : $y \in [-\pi R, +\pi R]$

S^1/Z_2 : identify $y \sim -y$,

πR •- - - - - - - - - - - -• 0 , $y \in [0, \pi R]$

Field theory on orbifold : boundary conditions!

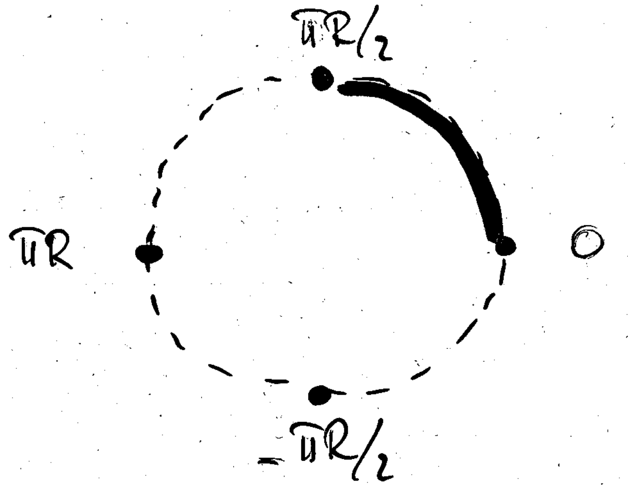
restrict FI to fields with ($P \in SU(5)$) :

$$P V(x,-y) P^{-1} = + V(x,y) \,, \quad P \Phi(x,-y) P^{-1} = - \Phi(x,y)$$

$$P H(x,-y) P^{-1} = + H(x,y) \,, \quad P H^c(x,-y) P^{-1} = - H^c(x,y)$$

action invariant ; away from fixpoints $y=0, \pi R$ mapping on manifold and field space ; <u>at fixpoints</u> <u>breaking</u> of <u>SUSY</u> from $N=2$ to $N=1$

Breaking of SU(5) to SU(3) x SU(2) x U(1) :



$$S^1/Z_2 \times Z_2' : \quad y \sim -y \quad , \quad \frac{\pi R}{2} + y \sim \frac{\pi R}{2} - y$$

$$y \in [0, \frac{\pi R}{2}]$$

$$P = 1 \quad , \quad P' = \mathrm{diag}(-1, -1, -1, 1, 1)$$

zero modes (N=1 multiplets) :

V : $(8,1)_0$, $(1,3)_0$, $(1,1)_0$

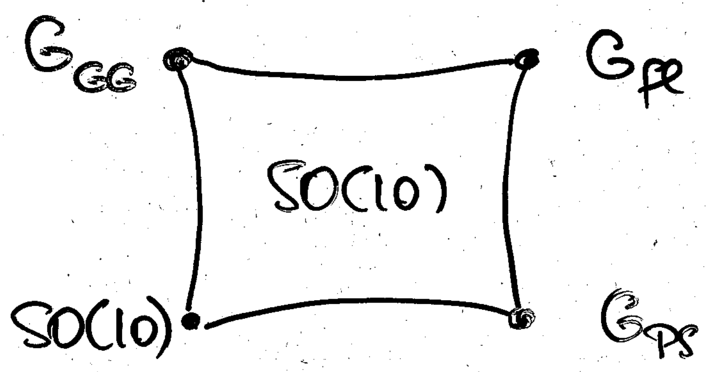
H_2 : $(1,2)_2$ (H_{++} ...)

H_1 : $(1,2)_{-2}$ (H^c_{++} ...)

GUT breaking implies doublet-triplett splitting !
no dim 5 proton decay ; many models with interesting flavour structure

Extension to $SO(10) \to SU(3) \times SU(2) \times U(1)^2$:

$SO(10)$ gauge theory in 6D, $N=2$ SUSY, compactified on $T^2/_{Z_2 \times Z_2' \times Z_2''}$:



SM gauge group from intersection of Pati-Salam and Georgi-Glashow :

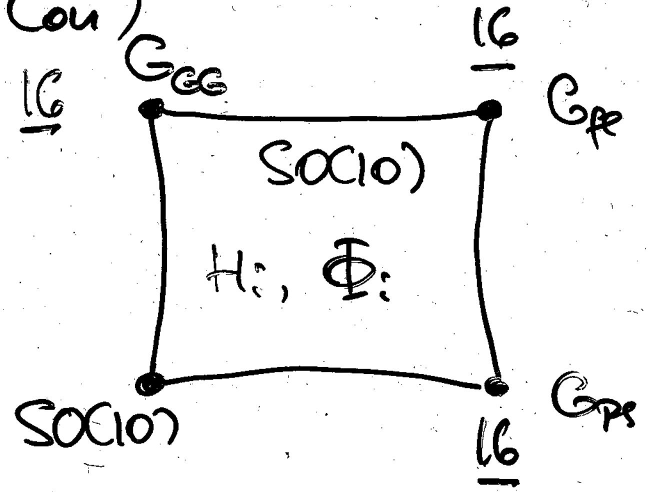
$$G_{PS} = SU(4) \times SU(2) \times SU(2) \ , \quad G_{GG} = SU(5) \times U(1)_X \ ,$$
$$G_{fl} = SU(5)' \times U(1)_{X'} \ ,$$

$$G_{SM'} \equiv G_{SM} \times U(1)_X$$
$$= G_{PS} \cap G_{GG} = G_{GG} \cap G_{fl}$$

symmetry breaking is <u>local</u> ; one expects also localized 'matter' field ('twisted sector') ; doublet triplet splitting OK

– Phenomenology

interplay of brane fields and split multiplets from bulk fields leads to interesting flavour physics ; example hep-ph/0304142 (Asaka, WB, Covi)



$H_i = 10\ ,\quad \Phi_i = 16$

split bulk zero modes :

$$L = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix}\ ,\quad L^c = \begin{pmatrix} \nu_4^c \\ e_4^c \end{pmatrix}\ ,\quad G_5^c = d_4^c\ ,\quad G_6 = d_4$$

mass matrices :

$$W = d_\alpha m^d_{\alpha\beta} d^c_\beta + e^c_\alpha m^e_{\alpha\beta} e_\beta + n^c_\alpha m^D_{\alpha\beta} \nu_\beta$$
$$+ u^c_i m_{ij} u_j + \frac{1}{2} n_i^c M_{ij} n_j^c$$

$\underline{i,j} = 1 \dots 3$; $\underline{\alpha,\beta} = 1 \dots 4$

pattern of mass matrices for universal Yukawa couplings at each fixpoint:

$$\frac{1}{\tan\beta}\, m^u \sim \frac{v_1 M_*}{v_N^2}\, m^N \sim \begin{pmatrix} \mu_1 & & 0 \\ & \mu_2 & \\ 0 & & \mu_3 \end{pmatrix}$$

$$m^d \sim m^e \sim m^D \sim \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 & \tilde{M}_4 \end{pmatrix}$$

$\mu_i, \tilde{\mu}_i = O(v_1)$, hierarchical ; $\tilde{M}_i = O(\Lambda_{GUT})$;
integrate out heavy states: quark and lepton mass matrices in terms of 6 parameters

INPUT : $\mu_3 \simeq m_t$, $\mu_2 \simeq m_c$, $\mu_1 \simeq m_u$

$\tilde{\mu}_3 \simeq m_b$, $\tilde{\mu}_2 \simeq m_s$, $\frac{\tilde{\mu}_1}{\tilde{\mu}_2} = V_{us} = \Theta_c$

OUTPUT : $V_{cb} \sim \frac{m_s}{m_b}$, $V_{ub} \sim \Theta_c \frac{m_s}{m_b}$,

$\frac{m_d}{m_s} \sim \gamma\, \Theta_c$, $\gamma \equiv \frac{\mu_2}{\tilde{\mu}_2} \sim \frac{m_c m_b}{m_t m_s} \sim 0.1$

$M_3 : M_2 : M_1 \sim m_t : m_c : m_u$

$m_{\nu_3} : m_{\nu_2} : m_{\nu_1} \sim 1 : \gamma : \gamma$

Θ_{23}, Θ_{12} large , $\Theta_{13} \sim \gamma$ (LG o.k.)

reason for difference between quark and neutrino mass hierarchy:

$$m_\nu = -\bar{m}^{DT}\frac{1}{\bar{M}}\bar{m}^D \sim \bar{m}^{dT}\frac{1}{m^u}\bar{m}^d$$

$$\frac{m_{\nu_1}}{m_{\nu_3}} \sim \left(\frac{m_d}{m_b}\right)^2\left(\frac{m^u}{m_t}\right)^{-1} \sim 0.1$$

i.e., missmatch between up- and down-quark mass hierarchies + seesaw causes small neutrino mass hierarchy!

proton decay (with Costa, Wiesenfeldt):

		BR[%]				
		$e^+\pi^0$	$\mu^+\pi^0$	$\bar{\nu}\pi^+$	e^+K^0	$\boxed{\mu^+K^0}$
6D SO(10)	I	75	4	19	1	<1
	II	71	5	23	1	<1
4D SU(5)		54	<1	27	<1	18

reason for difference: localization of 1st family in 6D model (different results for 5D model)

$$\Gamma^{-1}(p\to e^+\pi^0) \simeq \left(\frac{M_c}{9\times10^{15}\,\mathrm{GeV}}\right)^4 5.3\times10^{33}\,\mathrm{yr}$$

$M_c = \Lambda_{GUT} \to 1\times10^{35}$ yr **OK**

(3) Strings on orbifolds

basic idea: Dixon, Harvey, Vafa, Witten '86;
early semi-realistic models (G_{SM}, 3 generations
+ extra stuff): Ibáñez, Nilles, Quevedo, Kim '87; ...
recent interest: embedding of orbifold GUTs,
Kobayashi, Raby, Zhang $T^6/\mathbb{Z}_6$ hep-ph/0403065,
hep-ph/0409098; Förste, Nilles, Vaudrevange,
Wingerter $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ hep-ph/0406208; WB,
Hamaguchi, Lebedev, Ratz $T^6/\mathbb{Z}_6$ hep-ph/0412318

starting point: heterotic string, $E_8 \times E_8'$,
with one or more of the compact dimensions
'large', i.e. $O(\frac{1}{\Lambda_{GUT}})$ ('anisotropic orbifold':
Hebecker, Trapletti hep-th/0411131)
Symmetry breaking:

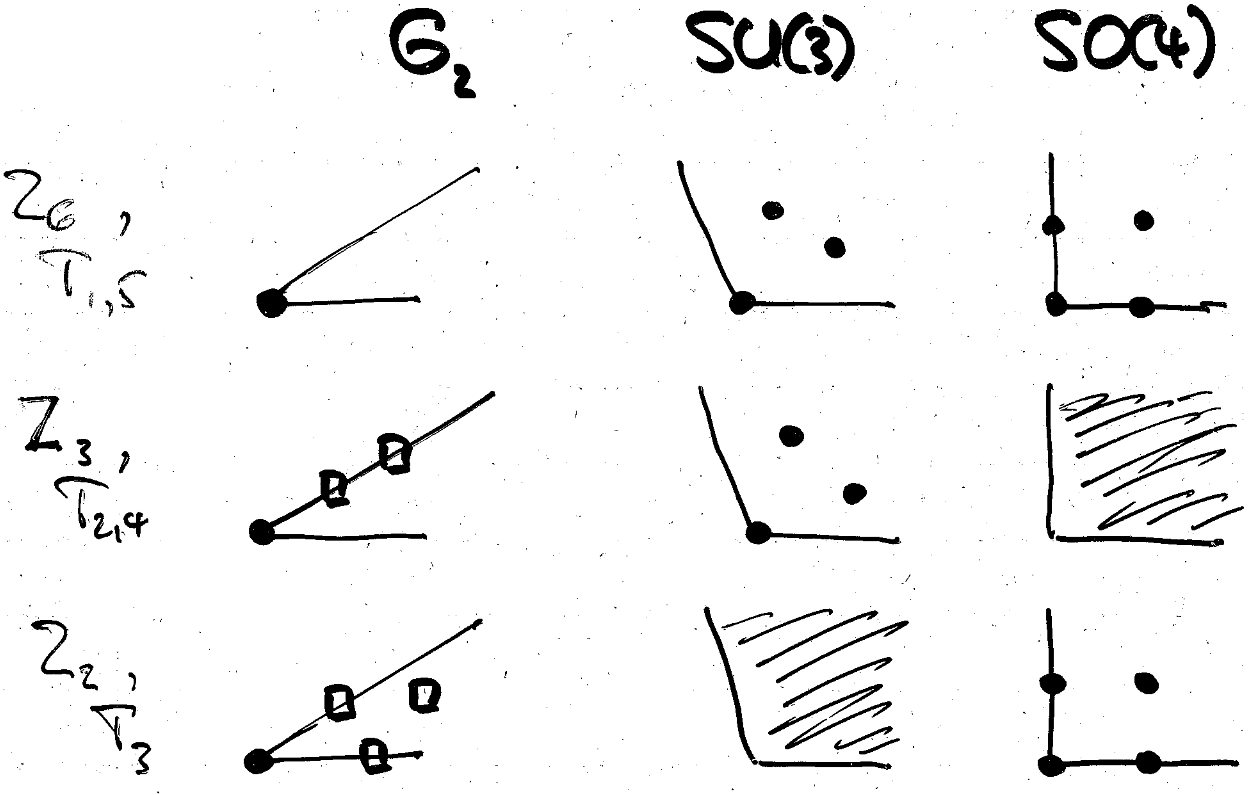
$$SU(3)\times SU(2)\times U(1) \subset SO(10)$$
$$\subset E_6 \dots \subset E_8$$

– Geometry of symmetry breaking

with K. Hamaguchi, O. Lebedev, M. Ratz

search: compactification which has fixed points with $E_8 \supset G \supset SO(10)$, $(\underline{16},\ldots)$; three generations, G_{SM} in $D=4$

$$O = T^6_{G_2 \times SU(3) \times SO(4)} \Big/ Z_6$$



characteristic feature: invariant planes w.r.t. subtwists; additional fixed points in G_2 plane an example:

$\mathcal{O}$: $v_6 = \frac{1}{6}(0;-1,-2,3)$

$T_{E_8\times E_8'}/\mathbb{Z}_6$:

$$V_6 = \left(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{3}\ 0\,0\,0\,0\,0\right)\left(-\tfrac{1}{3}\ 0^7\right)$$

$$W_2 = \left(-\tfrac{1}{2}\ \tfrac{1}{2}\ 0\ \tfrac{1}{2}\ \tfrac{1}{2}\ 0\,0\,0\right)\left(1\ 0^7\right)\ , \quad W_2' = 0$$

$$W_3 = \left(\tfrac{1}{3}\ 0\ 0\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\right)\left(0^8\right)$$

unbroken gauge group (up to U(1) factors):

$$G = SU(3)\times SU(2)\times [SO(14)]$$

hidden sector

<u>massless spectrum</u>:

$$2\times(16;1) + \Big((3,2;1)_{\frac{1}{6}} + (\bar{3},1;1)_{-\frac{2}{3}} + (\bar{3},1;1)_{\frac{1}{3}}$$
$$(1,2;1)_{-\frac{1}{2}} + (1,1;1)_{1} + (1,1;1)_{0}\Big)$$

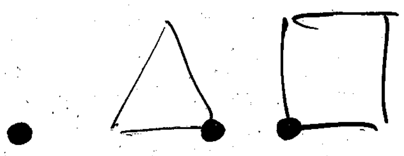
3 gen.; 2 sequ.

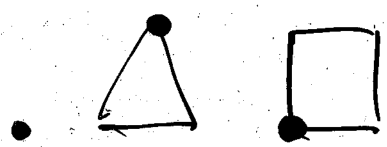
$$+\ 16\big((\bar{3},1;1)+(3,1;1)\big) + 34\,(1,2;1)$$
$$+\ 7\,(1,1;14) + 86\,(1,1;1)$$

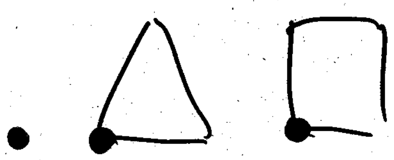
vector-like

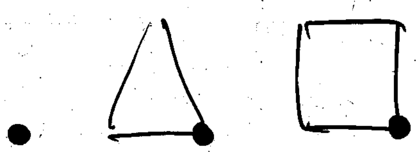
Geometry of local symmetries (V_6)

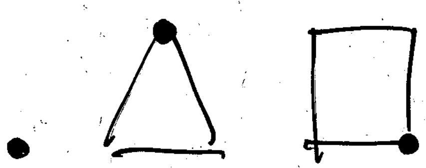
always: $SO(14)$ in hidden sector

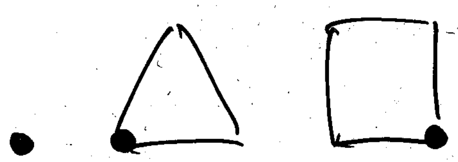

$SO(10)\times SO(4)'$ $\supset SU(5)$


$SO(12)$ "


$SU(7)$ "


$SO(10)'\times SO(4)$ $\supset SU(4)_{PS}$


$SU(7)'$ "


$SU(7)''$ "

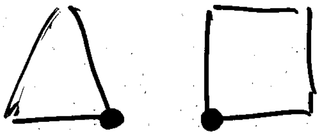
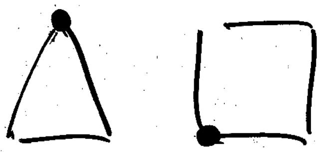
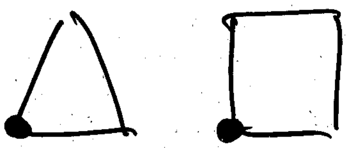
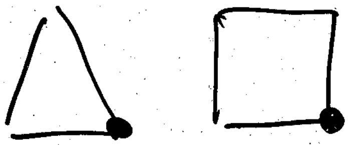
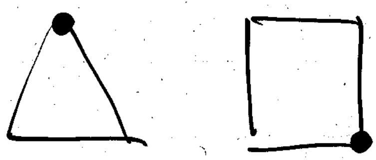
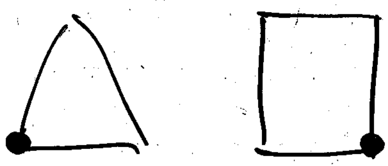
multiplicity: 2 ($n_2' = 0, 1$) ;

all groups different, intersection yields G_{SM}

'hybrid' unification

Localization of zero modes (T_1)

all singlets w.r.t. SO(14)

-  $\{(16,1,1)\} + 2\,(1,2,1) + (1,1,2)$
-  $12 + 5 \times 1$
  $\supset 5 + \bar{5}$ [SU(5)]
-  $7 + \bar{7} + 3 \times 1$
  $\supset 5 + \bar{5}$
-  $(1,2,1) + (1,1,2)$
  $\supset (1,2)_0$ [G_{SM}]
-  $7' + 1$
  $\supset (\bar{3},1)_{-\frac{1}{6}} + (1,2)_0$
-  $7'' + 1$
  $\supset (3,1)_{\frac{1}{6}} + (1,2)_0$

normal   exotic

multiplicity : 2   ($n_2' = 0, 1$)

$\rightarrow \quad 2 \times 16 + 6\,((3,1) + (\bar{3},1)) + 14\,(1,2)$ vector-like

$T_{2,4}, T_3$: more complicated

untwisted sector :

U_1 : $(\bar{3},1;1)_{-\frac{2}{3}}$, $(1,1;1)_1$, $(1,1;1)_0$

$\qquad\quad U^c_3 \qquad\qquad E^c_3 \qquad\qquad N^c_0$

U_2 : $(3,2;1)_{\frac{1}{6}}$, $(1,1;14)$

$\qquad\quad Q_3$

U_3 : $(1,2;1)_{-\frac{1}{2}}$ $\quad$ $(1,2;1)_{\frac{1}{2}}$

$\qquad\quad H_d \qquad\qquad H_u$

$(*, *; *)$ $\leftarrow$ $SO(14)$

$SU(3)_c \quad SU(2)_L$

<u>puzzling</u> <u>feature</u> of SM : Higgs + gauge fields in split multiplets , matter in GUT reps.

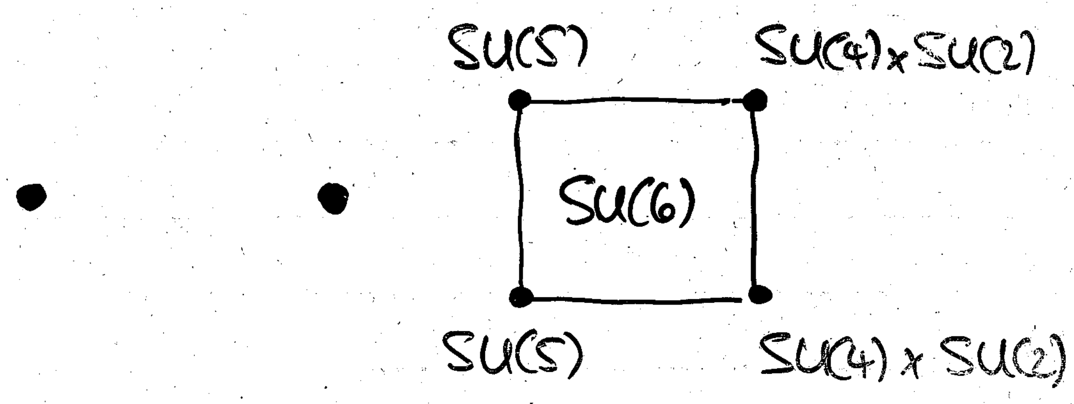
reason (mostly) : localization at fixed points with enhanced (GUT) symmetry

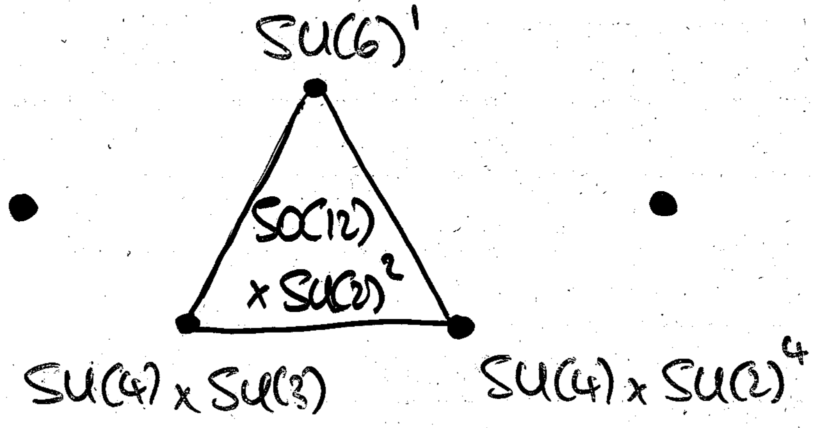
Intermediate GUTs

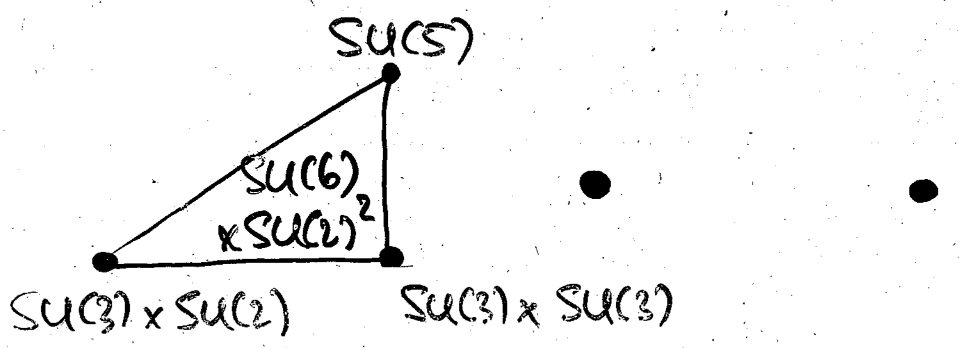
2 planes: $R_s \sim \frac{1}{M_{string}}$, 1 plane: $R_u \sim \frac{1}{M_{GUT}} \gg R_s$

$\rightarrow$ 6D effective field theory (unification of couplings, dynamically possible ?)

3 possibilities:







same zero modes, but <u>different</u> <u>localization</u>

$\rightarrow$ different Yukawa couplings

— Yukawa couplings etc.

Breaking of $U(1)$ factors? mass generation? decoupling of vector-like zero modes? requires Yukawa couplings:

$$\langle V_{-\frac{1}{2}} \, V_{-\frac{1}{2}} \, V_{-1} \, V_0 \ldots V_0 \rangle$$

→ <u>selection</u> <u>rules</u>; only one renormalizable quark-lepton coupling:

$U_1 U_2 U_3$: $\quad U_3^c Q_3 H_u \quad$ top-quark

Flat directions of superpotential? large Majorana ν-masses from breaking of B-L? quark-lepton mass hierarchy from higher-dimensional operators? electroweak symmetry breaking?

......

supersymmetry breaking?

... **geometrical understanding of quark-lepton quantum numbers intriguing ...**

HOPES FOR THE (NEAR) FUTURE

EXP

- SUSY at LHC
- proton decay , $\mu \to e\gamma$
- absolute neutrino mass scale (cosmology) , Θ_{13}

TH

- realistic orbifold extension of SM
- relation to other compactifications (Calabi Yau, D-branes)
- SUSY breaking and moduli stabilization