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Supersymmetry: Status and New Developments

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Supersymmetry: Status and New Developments

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The Plan:

Lecture 1: Motivation and Introduction to Supersymmetry

Lecture 2: Supersymmetric Interactions and the Minimal Supersymmetric Standard Model

Lecture 3: Models of Supersymmetry Breaking

Lecture 4: Signals for Supersymmetry

Based in part on "A supersymmetry primer", `hep-ph/9709356`

A .pdf file of these lectures can be found at:
`http://zippy.physics.niu.edu/ICTP05/`

Lecture 1: Motivation and Introduction to Supersymmetry

There are many reasons to believe that the next discoveries beyond the presently known Standard Model will involve **supersymmetry (SUSY)**.

Some of them are:

- A good cold dark matter particle
- A light Higgs boson, in agreement with precision electroweak constraints
- Unification of gauge couplings
- Mathematical beauty
- Indirect effects (that come and go!) on observables like the anomalous magnetic moment of the muon and $Z \longrightarrow b\bar{b}$.

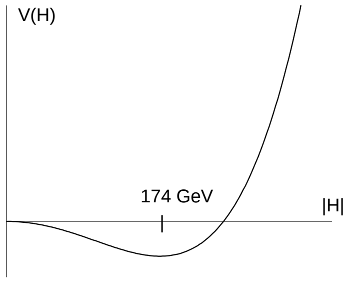
However, in my opinion, they are all insignificant compared to the one really good reason to suspect that supersymmetry is real.

That is. . .

The Hierarchy Problem

Consider the potential for H, the complex scalar field that is the electrically neutral part of the Standard Model Higgs field:

$$V(H) = m_H^2 |H|^2 + \frac{\lambda}{2}|H|^4$$



For electroweak symmetry breaking to agree with the experimental m_Z, we need:

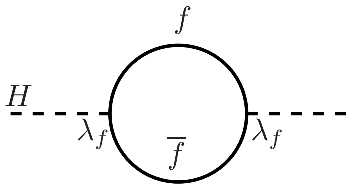
$$\langle H \rangle = \sqrt{-m_H^2/\lambda} \approx 175 \text{ GeV}$$

The requirement of unitarity in the scattering of Higgs bosons and longitudinal W bosons tells us that λ is not much larger than 1. Therefore,

$$-(\text{few hundred GeV})^2 \lesssim m_H^2 < 0.$$

However, this appears fine-tuned (in other words, incredibly and mysteriously lucky!) when we consider the likely size of quantum corrections to m_H^2.

Contributions to m_H^2 from a Dirac fermion loop:



The correction to the Higgs squared mass parameter from this loop diagram is:

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\text{UV}}^2 + 6m_f^2 \ln\left(M_{\text{UV}}/m_f\right) + \ldots\right]$$

where λ_f is the coupling of the fermion to the Higgs field H.

M_{UV} should be interpreted as the ultraviolet cutoff scale(s) at which new physics enters to cut off the loop integrations.

So m_H^2 is sensitive to the **largest** mass scales in the theory.

For example, some people believe that String Theory is responsible for modifying the high energy behavior of physics, making the theory finite. Going from string theory to field theory, integrations over Euclidean momenta are modified according to:

$$\int d^4p\,[\ldots] \;\rightarrow\; \int d^4p\, e^{-p^2/M^2_{\rm string}}\,[\ldots]$$

Using this, one obtains from each Dirac fermion one-loop diagram:

$$\Delta m_H^2 \;\sim\; -\frac{\lambda_f^2}{8\pi^2} M^2_{\rm string} + \ldots$$

A typical guess is that $M_{\rm string}$ is comparable to $M_{\rm Planck} \approx 2.4 \times 10^{18}$ GeV.

This makes it difficult to explain how m_H^2 could be so small, after incorporating these relatively huge corrections.

The Hierarchy Problem is that we already know:

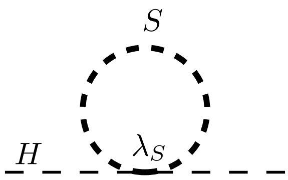
$$\frac{|m_H^2|}{M^2_{\rm Planck}} \;\lesssim\; 10^{-32}$$

Why should this number be so small, if individual radiative corrections Δm_H^2 can be of order $M^2_{\rm Planck}$ or $M^2_{\rm string}$, multiplied by loop factors?

This applies even if you don't trust String Theory (and why should you?), and some other unspecified quantum gravitation effects at $M_{\rm Planck}$, or any other very large mass scale, make the loop integrals converge.

An incredible coincidence seems to be required to make the corrections to the Higgs squared mass cancel to give a much smaller number.

Scalar loops give a "quadratically divergent" contribution to the Higgs squared mass also. Suppose S is some heavy complex scalar particle that couples to the Higgs.



$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[M_{\rm UV}^2 - 2m_S^2 \ln\left(M_{\rm UV}/m_S\right) + \ldots\right]$$

(Note that the coefficient of the $M_{\rm UV}^2$ term from a scalar loop has the **opposite sign** of the fermion loop.)

In dimensional regularization, the terms proportional to $M_{\rm UV}^2$ do not occur. One could adopt dimensional regularization (although it seems unphysical for this purpose), and also assume that the Higgs does not couple directly to *any* heavy particles. But there is still a problem...

Indirect couplings of the Higgs to heavy particles still give a problem:

(a) (b)

Here F is any heavy fermion that shares gauge quantum numbers with the Higgs boson. Its mass, m_F does not come from the Higgs boson and can be arbitrarily large. From these diagrams one finds (x is a group-theory factor):

$$\Delta m_H^2 = x\left(\frac{g^2}{16\pi^2}\right)^2\left[kM_{\rm UV}^2 + 48m_F^2 \ln(M_{\rm UV}/m_F) + \ldots\right]$$

Here k depends on the choice of cutoff procedure (and is 0 in dimensional regularization). However, the contribution proportional to m_F^2 is always present.

More generally, *any* indirect communication between the Higgs boson and very heavy particles, or very high-mass phenomena in general, can give an unreasonably large contribution to m_H^2.

The systematic cancellation of loop corrections to the Higgs mass squared requires the type of conspiracy that is better known to physicists as a **symmetry**.

Fermion loops and boson loops gave contributions with opposite signs:

$$\Delta m_H^2 = -\frac{\lambda_f^2}{16\pi^2}(2M_{\rm UV}^2) + \ldots \qquad \text{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\rm UV}^2 + \ldots \qquad \text{(complex scalar)}$$

So we need a **SUPERSYMMETRY** = a symmetry between fermions and bosons.

It turns out that this makes the cancellation not only possible, but automatic.

More on this later, but first, an historical analogy. . .

An analogy: Coulomb self-energy correction to the electron's mass

H. Murayama, `hep-ph/0002232`

If the electron is really pointlike, the classical electrostatic contribution to its energy is infinite.

Model the electron as a solid sphere of uniform charge density and radius R:

$$\Delta E_{\rm Coulomb} = \frac{3e^2}{20\pi\epsilon_0 R}$$

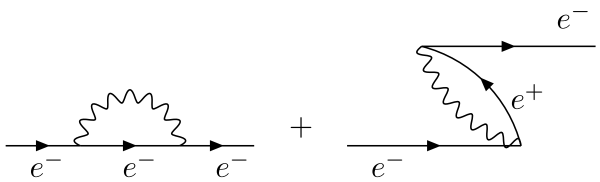
Interpreting this as a correction $\Delta m_e = \Delta E_{\rm Coulomb}/c^2$ to the electron mass:

$$m_{e,\text{physical}} = m_{e,\text{bare}} + (1\,\text{MeV}/c^2)\left(\frac{0.9\times 10^{-17}\,\text{meters}}{R}\right).$$

A divergence arises if we try to take $R \to 0$. Naively, we might expect $R \gtrsim 10^{-17}$ meters, to avoid having to tune the bare electron mass to better than 1%, for example:

$$0.511\,\text{MeV}/c^2 = -100.000\,\text{MeV}/c^2 + 100.511\,\text{MeV}/c^2.$$

However, there is another important quantum mechanical contribution:



The virtual positron effect cancels most of the Coulomb contribution, leaving:

$$m_{e,\text{physical}} = m_{e,\text{bare}} \left[1 + \frac{3\alpha}{4\pi} \ln\left(\frac{\hbar/m_e c}{R}\right) + \ldots \right]$$

with $\hbar/m_e c = 3.9 \times 10^{-13}$ meters. Even if R is as small as the Planck length 1.6×10^{-35} meters, where quantum gravity effects become dominant, this is only a 9% correction.

The existence of a "partner" particle for the electron, the positron, is responsible for eliminating the dangerously huge contribution to its mass.

If we did not yet know about the positron, we would have had three options:

- **Assume that the electron is not point-like, and has structure at a measurable size R.**
- **Assume that the electron is (nearly?) pointlike, and there is a mysterious fine-tuning between the bare mass and the Coulomb correction to it.**
- **Predict that the electron's "partner", the positron, must exist.**

Today we know that the last option is the correct one.

The "reason" for the positron's existence can be understood from a **symmetry**, namely the Poincaré invariance of Einstein's relativity when applied to the quantum theory of electrons and photons, QED.

The reason for the cancellation of the Coulomb contribution is the approximate chiral symmetry of the QED Lagrangian.

Supersymmetry

A SUSY transformation turns a boson state into a fermion state, and vice versa. So the operator Q that generates such transformations acts, schematically, like:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

This means that Q must be an anticommuting spinor. This is an intrinsically complex object, so $Q^\dagger$ is also a distinct symmetry generator:

$$Q^\dagger|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad Q^\dagger|\text{Fermion}\rangle = |\text{Boson}\rangle$$

The possible forms for such theories are highly restricted by the Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula Theorem.
In a 4-dimensional theory with chiral fermions (like the Standard Model) and non-trivial scattering, then Q carries spin-$1/2$ with L helicity, and $Q^\dagger$ has spin-$1/2$ with R helicity, and they must satisfy. . .

The Supersymmetry Algebra

$$\begin{aligned} \{Q, Q^\dagger\} &= P^\mu \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0 \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0 \\ [T^a, Q] &= [T^a, Q^\dagger] = 0 \end{aligned}$$

Here $P^\mu = (H, \vec{\mathbf{p}})$ is the generator of spacetime translations, and T^a are the gauge generators. (This is schematic, with spinor indices suppressed for now. We will restore them later.)

The single-particle states of the theory fall into irreducible representations of this algebra, called **supermultiplets**. Fermion and boson members of a given supermultiplet are **superpartners** of each other. By definition, if $|\Omega\rangle$ and $|\Omega'\rangle$ are superpartners, then $|\Omega'\rangle$ is equal to some combination of $Q, Q^\dagger$ acting on $|\Omega\rangle$.

Therefore, since P^2 and T^a commute with $Q, Q^\dagger$, all members of a given supermultiplet must have the same (mass)^2 and gauge quantum numbers.

Each supermultiplet contains equal numbers of fermions and bosons

Proof: Consider the operator $(-1)^{2S}$ where S is spin angular momentum. Then

$$(-1)^{2S} = \begin{cases} -1 & \text{acting on fermions} \\ +1 & \text{acting on bosons} \end{cases}$$

So, $(-1)^{2S}$ must anticommute with Q and $Q^\dagger$. Now consider all states $|i\rangle$ in a given supermultiplet with the same momentum eigenvalue $p^\mu \neq 0$. These form a complete set of states, so $\sum_j |j\rangle\langle j| = 1$. Now do a little calculation:

$$\begin{aligned} p^\mu \mathrm{Tr}[(-1)^{2S}] = \sum_i \langle i|(-1)^{2S} P^\mu |i\rangle &= \sum_i \langle i|(-1)^{2S} Q Q^\dagger |i\rangle + \sum_i \langle i|(-1)^{2S} Q^\dagger Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2S} Q Q^\dagger |i\rangle + \sum_i \sum_j \langle i|(-1)^{2s} Q^\dagger |j\rangle\langle j|Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2S} Q Q^\dagger |i\rangle + \sum_j \langle j|Q(-1)^{2S} Q^\dagger |j\rangle \\ &= \sum_i \langle i|(-1)^{2S} Q Q^\dagger |i\rangle - \sum_j \langle j|(-1)^{2S} Q Q^\dagger |j\rangle \\ &= 0. \end{aligned}$$

The trace just counts the number of boson minus the number of fermion degrees of freedom in the supermultiplet. Therefore, $p^\mu(n_B - n_F) = 0$.

Types of supermultiplets

Chiral (or "Scalar" or "Matter" or "Wess-Zumino") supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

2 real spin-0 scalars = 1 complex scalar. ($n_B = 2$)

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or "Vector") supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

1 real spin-1 massless gauge vector boson. ($n_B = 2$)

The Standard Model $\gamma, Z, W^\pm, g$ must fit into these.

Gravitational supermultiplet:

1 two-component Weyl fermion gravitino, helicity $\pm\frac{3}{2}$. ($n_F = 2$)

1 real spin-2 massless graviton. ($n_B = 2$)

How do the Standard Model quarks and leptons fit in?

Each quark or charged lepton is 1 Dirac = 2 Weyl fermions

Electron: $\Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$ $\begin{array}{l} \longleftarrow \text{ two-component Weyl LH fermion} \\ \longleftarrow \text{ two-component Weyl RH fermion} \end{array}$

Each of e_L and e_R is part of a chiral supermultiplet, so each has a complex, spin-0 superpartner, called $\widetilde{e}_L$ and $\widetilde{e}_R$ respectively. They are called the "left-handed selectron" and "right-handed selectron", although they carry no spin.

The conjugate of a right-handed Weyl spinor is a left-handed Weyl spinor. Define two-component left-handed Weyl fields: $e \equiv e_L$ and $\bar{e} \equiv e_R^\dagger$. So, there are two left-handed chiral supermultiplets for the electron:

$$(e,\, \widetilde{e}_L) \quad \text{and} \quad (\bar{e},\, \widetilde{e}_R^*).$$

The other charged leptons and quarks are similar. We do not need ν_R in the Standard Model, so there is only one neutrino chiral supermultiplet for each family:

$$(\nu_e,\, \widetilde{\nu}_e).$$

Chiral supermultiplets of the Minimal Supersymmetric Standard Model (MSSM):

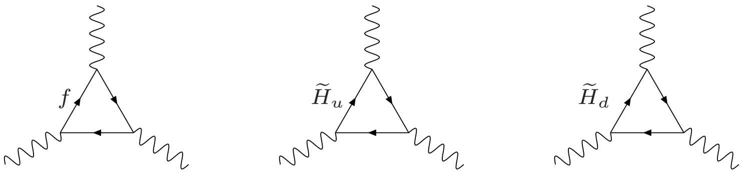
Names		spin 0	spin 1/2	$SU(3)_C,\, SU(2)_L,\, U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
($\times 3$ families)	$\bar{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
($\times 3$ families)	$\bar{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$

The superpartners of the Standard Model particles are written with a $\widetilde{\;}$. The scalar names are obtained by putting an "s" in front, so they are generically called **squarks** and **sleptons**, short for "scalar quark" and "scalar lepton".

The Standard Model Higgs boson requires two different chiral supermultiplets, H_u and H_d. The fermionic partners of the Higgs scalar fields are called **higgsinos**. There are two charged and two neutral Weyl fermion higgsino degrees of freedom.

Why do we need two Higgs supermultiplets? Two reasons:

1) Anomaly Cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \quad = \; 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present. Otherwise, the electroweak gauge symmetry would not be allowed!

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge $+2/3$ quarks (top).
Only the H_d Higgs scalar can give masses to charge $-1/3$ quarks (bottom) and the charged leptons. We will show this later.

The vector bosons of the Standard Model live in gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\mathbf{1},\ \mathbf{1},\ 0)$

The spin-$1/2$ **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\widetilde{\ }$. The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

However, the winos and the bino are not mass eigenstate particles; they mix with each other and with the higgsinos of the same charge.

Recall that if supersymmetry were an exact symmetry, then superpartners would have to be exactly degenerate with each other. For example,

$$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511 \text{ GeV}$$
$$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$$
$$m_{\tilde{g}} = m_{\text{gluon}} = 0 + \text{QCD-scale effects}$$

etc.

But new particles with these properties have been ruled out long ago, so:
Supersymmetry must be broken in the vacuum state chosen by Nature.

Supersymmetry is thought to be spontaneously broken and therefore hidden, the same way that the electroweak symmetry is hidden from very low-energy experiments.

For a clue as to the nature of SUSY breaking, return to our motivation in the Hierarchy Problem. The Higgs mass parameter gets corrections from each chiral supermultiplet:

$$\Delta m_H^2 = \frac{1}{16\pi^2}(\lambda_S - \lambda_F^2)M_{\text{UV}}^2 + \ldots$$

The corresponding formula for Higgsinos has no term proportional to M_{UV}^2; fermion masses always diverge at worst like $\ln(M_{\text{UV}})$. Therefore, if supersymmetry were exact and unbroken, it must be that:

$$\lambda_S = \lambda_F^2,$$

in other words, the dimensionless (scalar)^4 couplings are the squares of the (scalar)-(fermion)-(antifermion) couplings.

If we want SUSY to be a solution to the hierarchy problem, we must demand that this is still true even after SUSY is broken:
The breaking of supersymmetry must be “soft”. This means that it does not change the dimensionless terms in the Lagrangian.

The effective Lagrangian of the MSSM can therefore be written in the form:

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$$

- $\mathcal{L}_{\rm SUSY}$ contains all of the gauge interactions and Yukawa interactions dimensionless scalar couplings, and preserves exact supersymmetry
- $\mathcal{L}_{\rm soft}$ violates supersymmetry, and contains only mass terms and couplings with *positive* mass dimension.

If $m_{\rm soft}$ is the largest mass scale in $\mathcal{L}_{\rm soft}$, then by dimensional analysis,

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(M_{\rm UV}/m_{\rm soft}) + \ldots \right],$$

where λ stands for dimensionless couplings. This is because Δm_H^2 must vanish in the limit $m_{\rm soft} \to 0$, in which SUSY is restored. Therefore, we expect that $m_{\rm soft}$ should not be much larger than roughly 1000 GeV.

This is the best reason to be optimistic that SUSY will be discovered at the Fermilab Tevatron or the CERN Large Hadron Collider in this decade.

Without further justification, soft SUSY breaking might seem like a rather arbitrary requirement. Fortunately, we will see later that it arises naturally from the spontaneous breaking of theories with exact SUSY.

One might also ask if there is any good reason why the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far. There is!

- All of the particles in the MSSM that have been discovered as of 1995 (quarks, leptons, gauge bosons) would be exactly massless if the electroweak symmetry were not broken. So their masses are expected to be at most of order $v = 175$ GeV, the electroweak breaking scale. **In other words, they are required to be light.**
- All of the particles in the MSSM that have not yet been discovered as of 2005 (squarks, sleptons, gauginos, Higgsinos, Higgs scalars) can get a mass even without electroweak symmetry breaking. **They are not required to be light.**

Notations for two-component (Weyl) fermions

Metric tensor: $\eta^{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$
Position, momentum four-vectors: $x^\mu = (t, \vec{x});\qquad p^\mu = (E, \vec{p})$

Left-handed (LH) two-component Weyl spinor: $\psi_\alpha \quad \alpha = 1, 2$
Right-handed (RH) two-component Weyl spinor: $\psi^\dagger_{\dot\alpha} \quad \dot\alpha = 1, 2$

The Hermitian conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor, and vice versa:

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha} \equiv \psi^\dagger_{\dot\alpha}$$

(Some other people call this $\overline{\psi}_{\dot\alpha}$)

Therefore, **all** spin-$1/2$ fermionic degrees of freedom in any theory can be defined in terms of a list of left-handed Weyl spinors, $\psi_{i\alpha}$ where i is a flavor index. With this convention, right-handed Weyl spinors always carry a dagger: $\psi^{\dagger i}_{\dot\alpha}$.

Products of spinors are defined as:

$$\psi\xi \equiv \psi_\alpha \xi_\beta \epsilon^{\beta\alpha} \qquad \text{and} \qquad \psi^\dagger \xi^\dagger \equiv \psi^\dagger_{\dot\alpha} \xi^\dagger_{\dot\beta} \epsilon^{\dot\alpha\dot\beta}$$

Since ψ and ξ are anti-commuting fields, the antisymmetry of $\epsilon^{\alpha\beta}$ implies:

$$\psi\xi = \xi\psi = (\psi^\dagger\xi^\dagger)^* = (\xi^\dagger\psi^\dagger)^*.$$

To make Lorentz-covariant quantities, define matrices $(\overline{\sigma}_\mu)^{\dot\alpha\beta}$ and $(\sigma_\mu)_{\alpha\dot\beta}$ with:

$$\overline{\sigma}_0 = \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad \overline{\sigma}_n = -\sigma_n = (\vec{\sigma})_n \quad \text{(for } n = 1, 2, 3).$$

Then the Lagrangian for an arbitrary collection of LH Weyl fermions ψ_i is:

$$\mathcal{L} = -i\psi^{\dagger i}\overline{\sigma}^\mu D_\mu \psi_i - \tfrac{1}{2} M^{ij}\psi_i\psi_j - \tfrac{1}{2} M_{ij}\psi^{\dagger i}\psi^{\dagger j}$$

where D_μ = covariant derivative, and the mass matrix M^{ij} is symmetric, with $M_{ij} \equiv (M^{ij})^*$.

Two LH Weyl spinors ξ, χ can form a 4-component Dirac or Majorana spinor:

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$$

In the 4-component formalism, the Dirac Lagrangian is:

$$\mathcal{L} = -i\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m\overline{\Psi}\Psi, \quad \text{where} \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix},$$

In the two-component fermion language, with spinor indices suppressed:

$$\mathcal{L} = -i\xi^\dagger \overline{\sigma}^\mu \partial_\mu \xi - i\chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi - m(\xi\chi + \xi^\dagger \chi^\dagger),$$

up to a total derivative. This follows from using the identity:

$$-i\chi\sigma^\mu \partial_\mu \chi^\dagger = -i\partial_\mu \chi^\dagger \overline{\sigma}^\mu \chi.$$

A Majorana fermion can be described in 4-component language in the same way by identifying $\chi = \xi$, and multiplying the Lagrangian by a factor of $\frac{1}{2}$ to compensate for the redundancy.

For example, to describe the Standard Model fermions in 2-component notation:

$$\begin{aligned} \mathcal{L} \;=\; & -iQ^{\dagger i}\overline{\sigma}^\mu D_\mu Q_i - i\bar{u}^{\dagger i}\overline{\sigma}^\mu D_\mu \bar{u}_i - i\bar{d}^{\dagger i}\overline{\sigma}^\mu D_\mu \bar{d}_i \\ & -iL^{\dagger i}\overline{\sigma}^\mu D_\mu L_i - i\bar{e}^{\dagger i}\overline{\sigma}^\mu D_\mu \bar{e}_i \end{aligned}$$

with the family index $i = 1, 2, 3$ summed over, color and weak isospin and spinor indices suppressed, and D_μ the appropriate Standard Model covariant derivative, for example,

$$\begin{aligned} D_\mu L &= \left[\partial_\mu + i\frac{g}{2}W_\mu^a \tau^a - i\frac{g'}{2}B_\mu\right] \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ D_\mu \bar{e} &= \left[\partial_\mu + ig' B_\mu\right] \bar{e} \end{aligned}$$

with τ^a ($a = 1, 2, 3$) equal to the Pauli matrices, and the gauge eigenstate weak bosons are related to the mass eigenstates by

$$\begin{aligned} W_\mu^\pm &= (W_\mu^1 \mp W_\mu^2)/\sqrt{2}, \\ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} &= \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \end{aligned}$$

Two-component spinor language is much more natural and convenient for SUSY, because the supermultiplets are in one-to-one correspondence with the LH Weyl fermions.

More generally, two-component spinor language is more natural for any theory of physics beyond the Standard Model, because it is an Essential Truth that parity is violated. Nature does not treat left-handed and right-handed fermions the same, and the higher we go in energy, the more essential this becomes.

The simplest SUSY model: a free chiral supermultiplet

The minimum particle content for a SUSY theory is a complex scalar ϕ and its superpartner fermion ψ. We must at least have kinetic terms for each, so:

$$S = \int d^4x \, (\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion})$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad\qquad \mathcal{L}_{\rm fermion} = -i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

A SUSY transformation should turn ϕ into ψ, so try:

$$\delta\phi = \epsilon\psi; \qquad\qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

where $\epsilon =$ infinitesimal, anticommuting, constant spinor, with dimension [mass]$^{-1/2}$, that parameterizes the SUSY transformation. Then we find:

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi.$$

We would like for this to be canceled by an appropriate SUSY transformation of the fermion field. . .

To have any chance, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and must contain one spacetime derivative. There is only one possibility, up to a multiplicative constant:

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi; \qquad \delta\psi^\dagger_{\dot\alpha} = -i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*$$

With this guess, one obtains:

$$\delta\mathcal{L}_{\text{fermion}} = -\delta\mathcal{L}_{\text{scalar}} + (\text{total derivative})$$

so the action S is indeed invariant under the SUSY transformation, justifying the guess of the multiplicative factor. This is called the free Wess-Zumino model.

Furthermore, if we take the commutator of two SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) = i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\phi$$

Since ∂_μ corresponds to the spacetime 4-momentum P_μ, this has exactly the form demanded by the SUSY algebra discussed earlier. (More on this soon.)

The fact that two SUSY transformations give back another symmetry (namely a spacetime translation) means that the SUSY algebra "closes".

If we do the same check for the fermion ψ:

$$\begin{aligned}\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) &= i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\psi_\alpha \\ &\quad -i\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) + i\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)\end{aligned}$$

The first line is expected, but the second line only vanishes on-shell (when the classical equations of motion are satisfied). This seems like a problem, since we want SUSY to be a valid symmetry of the quantum theory (off-shell)!

To show that there is no problem, we introduce another bosonic spin-0 field, F, called an auxiliary field. Its Lagrangian density is:

$$\mathcal{L}_{\text{aux}} = F^*F$$

Note that F has no kinetic term, and has dimensions [mass]^2, unlike an ordinary scalar field. It has the not-very-exciting equations of motion $F = F^* = 0$.

The auxiliary field F does not affect the dynamics, classically or in the quantum theory. But it does appear in modified SUSY transformation laws:

$$\begin{aligned} \delta\phi &= \epsilon\psi \\ \delta\psi_\alpha &= i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F \\ \delta F &= i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi \end{aligned}$$

Now the total Lagrangian

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi - i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F$$

is still invariant, and also one can now check:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}X) - \delta_{\epsilon_1}(\delta_{\epsilon_2}X) \;=\; i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu X$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$, without using equations of motion.

So in the “modified” theory, SUSY does close off-shell as well as on-shell.

The auxiliary field F is really just a book-keeping device to make this simple.
In retrospect, we can see why we needed it by considering the number of degrees of freedom on-shell (classically) and off-shell (quantum mechanically):

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

(Going on-shell eliminates half of the propagating degrees of freedom of the fermion, because the Lagrangian density is linear in time derivatives, so that the fermionic canonical momenta are not independent phase-space variables.)

The auxiliary field will also plays an important role when we add interactions to the theory, and in gaining a simple understanding of SUSY breaking.

Noether's Theorem tells us that for every symmetry, there is a conserved current, and SUSY is not an exception. The **supercurrent** J^μ_α is an anti-commuting 4-vector that also carries a spinor index.

By the usual Noether procedure, one finds for the supercurrent (and its conjugate $J^\dagger$), in terms of the variations of the fields δX for $X = (\phi, \phi^*, \psi, \psi^\dagger, F, F^*)$:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \quad \equiv \quad \sum_X \delta X \, \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu,$$

where K^μ satisfies $\delta\mathcal{L} = \partial_\mu K^\mu$. One finds:

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*; \qquad\qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi.$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0; \qquad\qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0,$$

as can be verified by use of the equations of motion.

From the conserved supercurrents one can construct the conserved charges:

$$Q_\alpha = \sqrt{2} \int d^3x \; J^0_\alpha; \qquad\qquad Q^\dagger_{\dot\alpha} = \sqrt{2} \int d^3x \; J^{\dagger 0}_{\dot\alpha},$$

As quantum mechanical operators, they satisfy:

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\,\delta X$$

for any field X. Let us also introduce the 4-momentum operator $P^\mu = (H, \vec{P})$, which satisfies:

$$[P_\mu, X] = i\partial_\mu X.$$

Now by using the canonical commutation relations of the fields, one finds:

$$\begin{aligned} \left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right] &= 2(\epsilon_2 \sigma_\mu \epsilon_1^\dagger - \epsilon_1 \sigma_\mu \epsilon_2^\dagger)\, P^\mu \\ \left[\epsilon Q + \epsilon^\dagger Q^\dagger,\, P\right] &= 0 \end{aligned}$$

This implies…

The SUSY Algebra

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu,$$
$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$$
$$[Q_\alpha, P^\mu] = [Q^\dagger_{\dot\alpha}, P^\mu] = 0$$

This time in non-schematic form, with the spinor indices and the factors of 2 in their proper places.

(Note that the commutators turned into anti-commutators in the first two, when we extracted the anti-commutating spinors ϵ_1, ϵ_2.)

Covered in Lecture 1:

- The Hierarchy Problem, $m_Z \ll m_{\rm Planck}$, is a strong motivation for supersymmetry (SUSY)
- In SUSY, all particles fall into:
  - Chiral supermultiplet $=$ complex scalar boson and fermion partner
  - Gauge supermultiplet $=$ vector boson and gaugino fermion partner
  - Gravitational supermultiplet $=$ graviton and gravitino fermion partner
- The Minimal Supersymmetric Standard Model (MSSM) introduces squarks, sleptons, Higgsinos, gauginos as the superpartners of Standard Model states
- Two-component fermion notation: ψ_α $=$ LH fermion, $\psi^\dagger_{\dot\alpha}$ $=$ RH fermion
- The Wess-Zumino Model Lagrangian describes a single chiral supermultiplet
- The Supersymmetry Algebra

Lecture 2: Supersymmetric interactions and the Minimal Supersymmetric Standard Model

Masses and Interactions for Chiral Supermultiplets

In Lecture 1, we found the Lagrangian describing a collection of free, massless chiral supermultiplets

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i$$

is invariant under the transformations parameterized by a constant spinor ϵ_α:

$$\begin{aligned} \delta\phi_i &= \epsilon\psi_i, \\ \delta(\psi_i)_\alpha &= -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi_i + \epsilon_\alpha F_i \\ \delta F_i &= -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi_i \end{aligned}$$

Now we try to add to this a Lagrangian describing interactions:

$$\mathcal{L}_{\text{int}} = \left(-\tfrac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + x^{ij} F_i F_j\right) + \text{c.c.} + U$$

where, to be renormalizable, W^{ij}, W^i, x^{ij}, and U are polynomials in ϕ_i, ϕ^{*i} with degrees $1, 2, 0$, and 4, respectively.

Now one can compute $\delta\mathcal{L}$ under the SUSY transformation, and require that it be a total derivative, so that the action $S = \int d^4x\mathcal{L}$ is invariant.

This turns out to work if and only if $x^{ij} = 0$ and $U = 0$, and:

$$\begin{aligned} W^{ij} &= \frac{\partial^2 W}{\partial\phi_i\partial\phi_j} = M^{ij} + y^{ijk}\phi_k \\ W^i &= \frac{\partial W}{\partial\phi_i} = M^{ij}\phi_j + \tfrac{1}{2}y^{ijk}\phi_j\phi_k \end{aligned}$$

where we have defined a useful function:

$$W = \tfrac{1}{2}M^{ij}\phi_i\phi_j + \tfrac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$$

called the **superpotential**. Note that it does not depend on ϕ^{*i}, only the ϕ_i. It is an analytic function of the scalar fields treated as complex variables.

The superpotential W contains masses M^{ij} and couplings y^{ijk}, which are each automatically symmetric under interchange of i, j, k.

Supersymmetry is very restrictive; you cannot just do anything you want!

The Lagrangian terms involving auxiliary fields F_i, F^{*i} are:

$$\mathcal{L} = F^{*i}F_i + W^iF_i + W_i^*F^{*i}$$

So the equations of motion are now:

$$F^{*i} = -W^i = M^{ij}\phi_j + \tfrac{1}{2}y^{ijk}\phi_j\phi_k,$$

This is still algebraic; no spacetime derivatives. By eliminating the auxiliary fields, we get the complete Lagrangian:

$$\begin{aligned} \mathcal{L} = &-\partial^\mu\phi^{*i}\partial_\mu\phi_i - V(\phi_i, \phi^{*i}) \\ &-i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i - \tfrac{1}{2}\left(M^{ij}\psi_i\psi_j + y^{ijk}\phi_i\psi_j\psi_k + \text{c.c.}\right) \end{aligned}$$

where the scalar potential is:

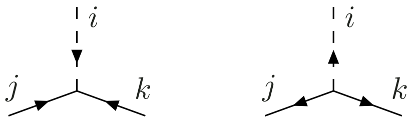
$$\begin{aligned} V(\phi_i, \phi^{*i}) = F_iF^{*i} = W^iW_i^* = M_{ik}M^{kj}\phi^{*i}\phi_j + \frac{1}{2}M^{in}y_{jkn}\phi_i\phi^{*j}\phi^{*k} \\ +\frac{1}{2}M_{in}y^{jkn}\phi^{*i}\phi_j\phi_k + \frac{1}{4}y^{ijn}y_{kln}\phi_i\phi_j\phi^{*k}\phi^{*l} \end{aligned}$$

The Feynman rules for our interacting chiral supermultiplets are:

Propagators: $\frac{-i}{p^2+M^2}$ $\quad$ $\frac{-ip\cdot\overline{\sigma}}{p^2+M^2}$ $\quad$ $\frac{-iM^{ij}}{p^2+M^2}$ $\quad$ $\frac{-iM_{ij}}{p^2+M^2}$

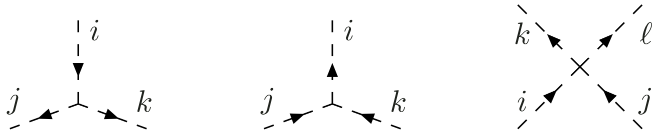
Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$. $\surd$

Yukawa interactions:



$-iy^{ijk}$ $\qquad$ $-iy_{ijk}$

Scalar interactions:



$-iM^{in}y_{njk}$ $\qquad$ $-iM_{in}y^{njk}$ $\qquad$ $-iy^{ijn}y_{k\ell n}$

The superpotential parameters M^{ij}, y^{ijk} determine all non-gauge interactions.

Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains a gauge boson A^a_μ and a gaugino λ^a_α. The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$; $1, 2, 3$ for $SU(2)_L$; 1 for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} - i\lambda^{\dagger a}\overline{\sigma}^\mu \nabla_\mu \lambda^a + \tfrac{1}{2}D^a D^a$$

where D^a is a real spin-0 auxiliary field with no kinetic term, and

$$\nabla_\mu \lambda^a = (\partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c)$$

The action is invariant under the SUSY transformation:

$$\begin{aligned}
\delta A^a_\mu &= -\tfrac{1}{\sqrt{2}}\left(\epsilon^\dagger \overline{\sigma}_\mu \lambda^a + \lambda^{\dagger a}\overline{\sigma}_\mu \epsilon\right), \\
\delta \lambda^a_\alpha &= -\tfrac{i}{2\sqrt{2}}(\sigma^\mu \overline{\sigma}^\nu \epsilon)_\alpha\, F^a_{\mu\nu} + \tfrac{1}{\sqrt{2}}\epsilon_\alpha\, D^a, \\
\delta D^a &= \tfrac{i}{\sqrt{2}}\left(\epsilon^\dagger \overline{\sigma}^\mu \nabla_\mu \lambda^a - \nabla_\mu \lambda^{\dagger a}\overline{\sigma}^\mu \epsilon\right).
\end{aligned}$$

The auxiliary field D^a is again needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell $(n_B = n_F = 2)$	2	2	0
off-shell $(n_B = n_F = 4)$	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, one must turn the ordinary derivatives into covariant ones:

$$\begin{aligned} \partial_\mu \phi_i \quad &\rightarrow \quad \nabla_\mu \phi_i = \partial_\mu \phi_i + igA^a_\mu (T^a \phi)_i \\ \partial_\mu \psi_i \quad &\rightarrow \quad \nabla_\mu \psi_i = \partial_\mu \psi_i + igA^a_\mu (T^a \psi)_i \end{aligned}$$

One must also add three new terms to the Lagrangian:

$$\begin{aligned} \mathcal{L} \quad = \quad & \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) \\ & + g(\phi^* T^a \phi) D^a . \end{aligned}$$

You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY transformations and gauge transformations.

The part of the Lagrangian involving the auxiliary fields D^a is:

$$\mathcal{L} = \tfrac{1}{2} D^a D^a + g(\phi^* T^a \phi)$$

So the D^a obey purely algebraic equations of motion $D^a = -g(\phi^* T^a \phi)$, and so can be eliminated from the theory. The resulting scalar potential is:

$$\begin{aligned} V(\phi_i, \phi^{*i}) \quad &= \quad F^{*i} F_i + \tfrac{1}{2} D^a D^a \\ &= \quad W^*_i W^i + \tfrac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2 \end{aligned}$$

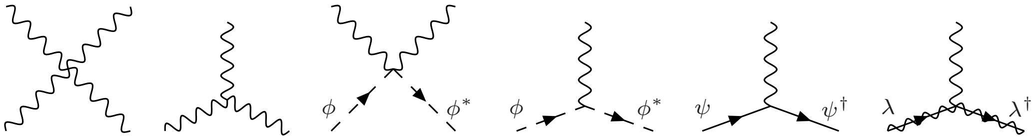
The two types of contributions to the scalar potential are called “F-term” and “D-term”. Note:

- Since V is a sum of squares, it is automatically ≥ 0.
- The scalar potential in SUSY theories is completely determined by the fermion masses, Yukawa couplings, and gauge couplings.

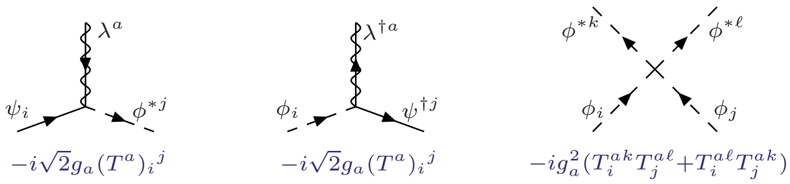
But both of these statements will be modified when we break SUSY.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



There are also interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



These interactions have the greatest impact in producing SUSY events at colliders. Experimental measurements of the magnitudes of these couplings will provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown rigorously that the quadratic sensitivity to $M_{\rm UV}$ does not arise in SUSY theories with these terms added in:

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j\phi^{*j}\phi_i \\ &\quad -\left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right),\end{aligned}$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

As long as the theory does not have gauge-singlet chiral supermultiplets, one can also include:

$$\mathcal{L}_{\text{maybe soft}} = -\tfrac{1}{2} c_i^{jk} \phi^{*i} \phi_j \phi_k + \text{c.c.}$$

Technically, the MSSM allows such terms as soft; however, they turn out to be completely negligible in known models of spontaneous SUSY breaking.

One might also wonder why we have not included possible soft mass terms for the chiral supermultiplet fermions, like $\mathcal{L} = \frac{1}{2} m^{ij} \psi_i \psi_j + \text{c.c.}$ They would be redundant; they can always be absorbed into a redefinition of the superpotential and the terms $(m^2)_i^j$ and c_i^{jk}.

How to make a realistic SUSY Model:

- Choose a gauge symmetry group.
  (In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.)
- Choose a superpotential W; must be invariant under the gauge symmetry.
  (In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses.)
- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.
  (This is where almost all of the arbitrariness in the MSSM is.)

Let's do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$\boxed{W_{\rm MSSM} = \tilde{\bar u}\mathbf{y_u}\tilde Q H_u - \tilde{\bar d}\mathbf{y_d}\tilde Q H_d - \tilde{\bar e}\mathbf{y_e}\tilde L H_d + \mu H_u H_d}$$

The objects H_u, H_d, $\tilde Q$, $\tilde L$, $\tilde{\bar u}$, $\tilde{\bar d}$, $\tilde{\bar e}$ appearing here are the scalar fields appearing in the left-handed chiral supermultiplets. Recall that $\bar u$, $\bar d$, $\bar e$ are the conjugates of the right-handed parts of the quark and lepton fields.

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, and higher-order quantum corrections, they are the same as in the Standard Model. (All gauge and family indices are suppressed.)

Note that, as promised earlier, we need both H_u and H_d, because terms like $\tilde{\bar u}\mathbf{y_u}\tilde Q H_d^*$ and $\tilde{\bar d}\mathbf{y_d}\tilde Q H_u^*$ are not allowed in the superpotential, since they are not analytic.

In the approximation that only the t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar t t H_u^0 - \bar t b H_u^+) - y_b(\bar b t H_d^- - \bar b b H_d^0) \\ & - y_\tau(\bar\tau \nu_\tau H_d^- - \bar\tau\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned}$$

Here the $\tilde{}$ are omitted to reduce clutter, and $Q_3 = (t\, b)$; $L_3 = (\nu_\tau\, \tau)$; $H_u = (H_u^+\, H_u^0)$; $H_d = (H_d^0\, H_d^-)$ $\bar u_3 = \bar t$; $\bar d_3 = \bar b$; $\bar e_3 = \bar\tau$.

Note that the minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

$$m_t = y_t v_u; \qquad m_b = y_b v_d; \qquad m_\tau = y_\tau v_d.$$

The Soft SUSY-breaking Lagrangian for the MSSM

$$\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} \;=\;& -\tfrac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + \text{c.c.}\\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + \text{c.c.}\\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_{\widetilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\widetilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\widetilde{\overline{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\widetilde{\overline{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\widetilde{\overline{e}}}}\,\widetilde{\overline{e}}^\dagger\\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}$$

The first line gives masses to the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).
The second line consists of (scalar)^3 interactions.
The third line is (mass)^2 terms for the squarks and sleptons.
The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$\begin{aligned}
& M_1,\, M_2,\, M_3,\, \mathbf{a_u},\, \mathbf{a_d},\, \mathbf{a_e} \;\sim\; m_{\text{soft}};\\
& \mathbf{m^2_{\widetilde{Q}}},\, \mathbf{m^2_{\widetilde{L}}},\, \mathbf{m^2_{\widetilde{\overline{u}}}},\, \mathbf{m^2_{\widetilde{\overline{d}}}},\, \mathbf{m^2_{\widetilde{\overline{e}}}},\, m^2_{H_u},\, m^2_{H_d},\, b \;\sim\; m^2_{\text{soft}}
\end{aligned}$$

where $m_{\text{soft}} \lesssim 1$ TeV.

The squark and slepton squared masses and (scalar)^3 couplings are 3×3 matrices in family space. The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals exist. None are completely convincing.

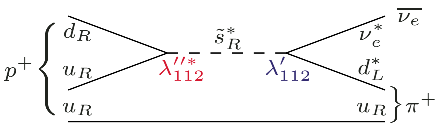
The question can be answered experimentally by discovering the pattern of Higgs and squark and slepton and gaugino masses, because they are the main terms in the SUSY-breaking Lagrangian.

Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk}L_iL_j\bar{e}_k + \lambda'_{ijk}L_iQ_j\bar{d}_k + \mu'_iL_iH_u$$
$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k$$

These violate lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot simply require B and L conservation, since they are already known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the superpotential, saving the proton.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

This is **exactly equivalent** to matter parity, because the product of $(-1)^{2S}$ is always $+1$ for any interaction vertex that conserves angular momentum.

However, particle within the same supermultiplet do not carry the same R-parity. You can check that all of the known Standard Model particles and the Higgs scalar bosons carry $P_R = +1$, while all of the squarks and sleptons and higgsinos and gauginos carry $P_R = -1$.

Consequences of R-parity

The particles with odd R-parity ($P_R = -1$) are the "supersymmetric particles" or "sparticles".

Every interaction vertex in the theory must contain an even number of $P_R = -1$ sparticles. Three extremely important consequences:

- The lightest sparticle with $P_R = -1$, called the "Lightest Supersymmetric Particle" or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so can make an attractive candidate for the non-baryonic dark matter required by cosmology.
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSPs (usually just one). The LSP escapes the detector, with a missing momentum signature.

Where does R-parity come from?

One way that matter parity could arise is as a surviving subgroup of a continuous gauge symmetry. For example, if $U(1)_{B-L}$ symmetry is gauged, and then broken at very high energy by a VEV of some field that carried an even integer value of $3(B-L)$, then matter parity will automatically be an exact symmetry of the MSSM.

For our purposes, the MSSM is **defined** to conserve R-parity.

(However, there are alternatives to R-parity, for example **baryon triality**, a Z_3 discrete symmetry:

$$Z_3^B = e^{2\pi i(B-2Y)/3}$$

If Z_3^B is multiplicatively conserved, then the proton is absolutely stable, but the LSP is not.)

Electroweak symmetry breaking and the Higgs bosons

There are two complex Higgs scalar doublets, (H_u^+, H_u^0) and (H_d^0, H_d^-), rather than one in the Standard Model. The classical scalar potential is:

$$\begin{aligned} V \;=\; & (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ & + b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.} \\ & + \tfrac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ & + \tfrac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2. \end{aligned}$$

The $|\mu|^2$ parts come from the F-terms. The g^2 and g'^2 parts come from the D-terms. The other terms come from the soft SUSY-breaking Lagrangian.

We must now minimize this potential, and show that it is compatible with the known electroweak symmetry breaking.

First, the freedom to do $SU(2)_L$ gauge transformations allows us to take $H_u^+ = 0$ at the minimum without loss of generality. Then one can show that $\partial V / \partial H_u^+ = 0$ also requires $H_d^- = 0$. So, at the minimum of the potential, $U(1)_{\rm EM}$ will be unbroken, as required. We are left with. . .

$$\begin{aligned} V \;=\; & (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\,H_u^0 H_d^0 + \text{c.c.}) \\ & + \tfrac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned}$$

A redefinition of the phase of H_u^0 can absorb any phase in b, so take it real and positive. This implies that at the minimum, $H_u^0 H_d^0$ is also real and positive, so H_u^0 and H_d^0 have opposite phases. Since they have opposite weak hypercharges ($\pm\frac{1}{2}$), a $U(1)_Y$ gauge rotation can make them both real and positive at the minimum, without loss of generality.

Must require that $H_u^0 = H_d^0 = 0$ is **not** the minimum. Then:

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2).$$

Also, we need the potential to be bounded from below. This requires:

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.$$

If these conditions are met, typically with $m_{H_u}^2 < 0$ in realistic models, then spontaneous electroweak breaking $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$ occurs.

The resulting Higgs VEVs can be parameterized:

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle, \qquad \text{where}$$
$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\ \text{GeV})^2$$
$$\tan\beta \equiv v_u/v_d.$$

The conditions for a minimum, from $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$, are:

$$|\mu|^2 + m_{H_u}^2 = b\cot\beta + (m_Z^2/2)\cos 2\beta$$
$$|\mu|^2 + m_{H_d}^2 = b\tan\beta - (m_Z^2/2)\cos 2\beta$$

These allow us to eliminate two parameters in favor of m_Z^2 and $\tan\beta$.

The quark and lepton masses are related to these VEVs by:

$$y_t = \frac{m_t}{v\sin\beta}, \qquad y_b = \frac{m_b}{v\cos\beta}, \qquad y_\tau = \frac{m_\tau}{v\cos\beta}, \qquad \text{etc.}$$

If we want the Yukawa couplings to avoid getting non-perturbatively large up to very high scales, we must have:

$$1.5 \lesssim \tan\beta \lesssim 55$$

“The μ Problem”

Solve for outputs m_Z and $\tan\beta$, using Lagrangian parameters as inputs:

$$\tan\beta = r + \sqrt{r^2 - 1}$$
$$m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2}{\sqrt{1 - 1/r^2}} - 2|\mu|^2 - m_{H_d}^2 - m_{H_u}^2$$

where

$$r = (2|\mu|^2 + m_{H_d}^2 + m_{H_u}^2)/2b.$$

Without miraculous cancellations, we expect that all of the (mass)^2 parameters appearing in these equations should be within an order of magnitude of m_Z^2. However, μ is a SUSY-**respecting** parameter appearing in the superpotential, while $m_{H_u}^2$, $m_{H_d}^2$ and b are SUSY-**breaking** parameters. Why should they be comparable in size?

Define mass-eigenstate Higgs bosons: h^0, H^0, A^0, G^0, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

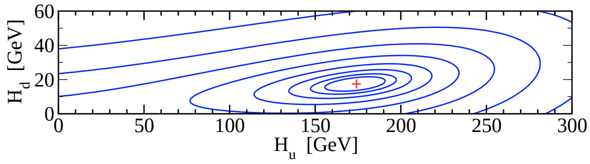
$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

Now, expand the potential to second order in these fields to obtain the masses:

$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{h^0,H^0}^2 &= \tfrac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\Big), \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \end{aligned}$$

The mixing angle α obeys $\tan 2\alpha = \left(\frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2}\right) \tan 2\beta$, and is traditionally chosen to be negative. The Goldstone bosons have $m_{G^0} = m_{G^\pm} = 0$; they are absorbed by the Z, $W^\pm$ bosons to give them masses, as in the Standard Model.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson h^0 corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level, it is easy to show from above that the lightest Higgs scalar obeys:

$$m_{h^0} < m_Z.$$

This has been ruled out by LEP2. However, taking into account loop effects, m_{h^0} can be considerably larger. Assuming that all superpartners are lighter than 1000 GeV, and that perturbation theory is valid up to $M_{\rm GUT}$, one finds:

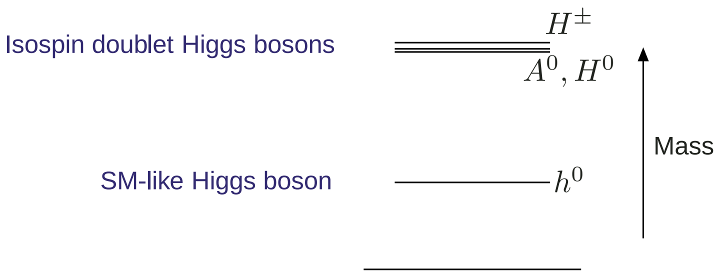
$$m_{h^0} \lesssim 130 \text{ GeV}$$

in the MSSM. By adding more supermultiplets, the bound increases to 150 GeV. By not requiring that the theory stays perturbative, one can get up to 200 GeV.

The decoupling limit for the Higgs bosons

If $m_{A^0} \gg m_Z$, then:

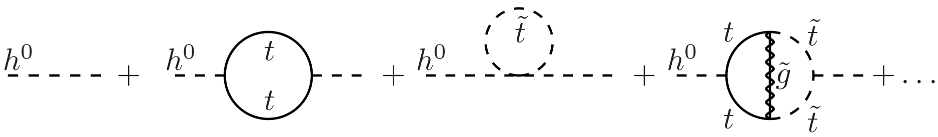
- h^0 has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A^0, H^0, H^\pm$ form an isospin doublet, and are much heavier than h^0



Many models of SUSY breaking approximate this decoupling limit.

Radiative corrections to the Higgs mass in SUSY:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



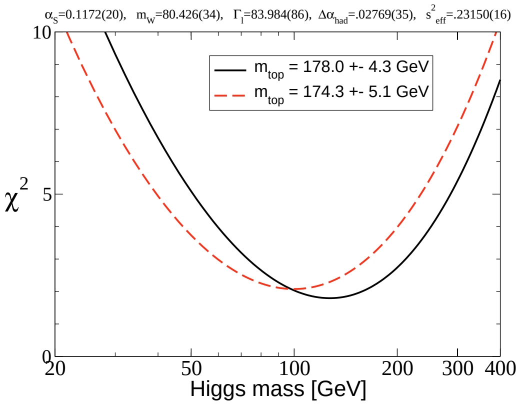
At tree-level:	m_Z^2	pure electroweak
At one-loop:	$y_t^2 m_t^2$	top Yukawa comes in
At two-loop:	$\alpha_S y_t^2 m_t^2$	SUSYQCD comes in
At three-loop:	$\alpha_S^2 y_t^2 m_t^2$	

At a future International Linear Collider, one might be able to measure m_{h^0} to within 50 MeV or better. This will require three-loop calculations to match theory to experiment!

Precision electroweak fits (slightly) favor a heavier top quark and a light Standard Model Higgs boson

(K. Tobe, J. Wells, SPM 2005)

From global fit to LEP1, SLAC, Tevatron measurements of m_W, m_Z, G_F, s^2_{eff}, Γ_{leptons}, $\Delta\alpha_{\text{hadrons}}$, α_S



Theoretical uncertainties, and LEP2 measurements above $\sqrt{s} = m_Z$, and SUSY loops can affect this weakly, but don't change the trend.

Neutralinos

The neutral higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$, $\tilde{W}^0$) mix with each other after electroweak symmetry breaking to form four **neutralino** fermion states. In the gauge eigenstate basis $\psi_i^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$ for $i = 1, 2, 3, 4$, the neutralino mass terms in the Lagrangian are

$$\mathcal{L}_{\text{neutralino mass}} = -\tfrac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g'v_d/\sqrt{2} & g'v_u/\sqrt{2} \\ 0 & M_2 & gv_d/\sqrt{2} & -gv_u/\sqrt{2} \\ -g'v_d/\sqrt{2} & gv_d/\sqrt{2} & 0 & -\mu \\ g'v_u/\sqrt{2} & -gv_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

The diagonal terms are just the gaugino masses in the soft SUSY-breaking Lagrangian. The $-\mu$ entries can be traced back to the superpotential. The off-diagonal terms come from the gaugino-Higgs-Higgsino interactions, and are always less than m_Z.

The physical neutralino mass eigenstates $\tilde{N}_i$ (another popular notation is $\tilde{\chi}_i^0$) are obtained by diagonalizing the mass matrix with a unitary matrix.

$$\tilde{N}_i = \mathbf{N}_{ij}\psi_j^0,$$

where

$$\text{diag}(m_{\tilde{N}_1}, m_{\tilde{N}_2}, m_{\tilde{N}_3}, m_{\tilde{N}_4}) = \mathbf{N}^*\mathbf{M}\mathbf{N}^{-1},$$

with $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$.

In many models of SUSY breaking, one finds:

$$M_1 \approx 0.5 M_2 < |\mu| \qquad \text{and} \qquad m_Z \ll |\mu|$$

where the "0.5" is really $\frac{5}{3}\tan^2\theta_W$. In that case, the lightest neutralino state $\tilde{N}_1$ is mostly bino, with mass nearly equal to M_1.

The lightest neutralino fermion, $\tilde{N}_1$, is a likely candidate for the cold dark matter that seems to be required by cosmology.

Charginos

Similarly, the charged higgsinos H_u^+, H_d^- and the charged winos W^+, W^- mix to form **chargino** fermion mass eigenstates.

$$\mathcal{L}_{\text{chargino mass}} = -\tfrac{1}{2}(\psi^\pm)^T \mathbf{M}_{\tilde{C}} \psi^\pm + \text{c.c.}$$

where, in 2×2 block form,

$$\mathbf{M}_{\tilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix} \qquad \text{with} \qquad \mathbf{X} = \begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix}$$

The mass eigenstates $\widetilde{C}^\pm_{1,2}$ (many other sources use $\tilde{\chi}^\pm_{1,2}$) are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V}\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U}\begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}.$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions.

The chargino mixing matrices are chosen so that

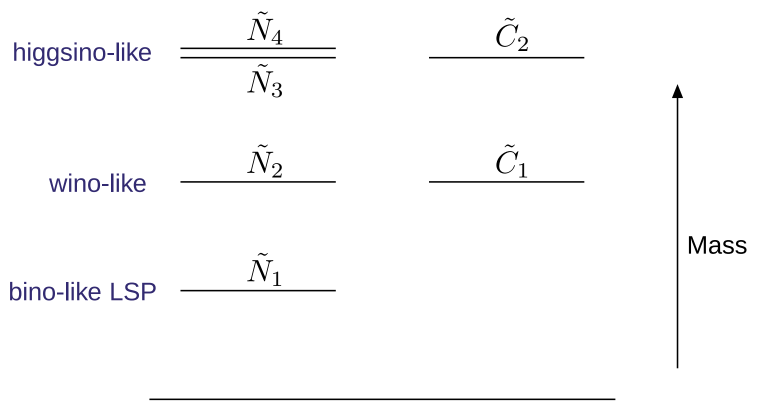
$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with positive real entries $m_{\widetilde{C}_i}$. In this case, one can solve for the tree-level mass^2 eigenvalues in simple closed form:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} &= \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \\ &\mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \end{aligned}$$

In many models of SUSY breaking, one finds that $M_2 \ll |\mu|$, so the lighter chargino is mostly wino with mass close to M_2, and the heavier is mostly higgsino with mass close to $|\mu|$.

A typical mass hierarchy for the neutralinos and charginos, assuming $m_Z \ll |\mu|$ and $M_1 \approx 0.5 M_2 < |\mu|$.



Although this is a very popular scenario, it is NOT guaranteed.

The Gluino

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. Therefore, at tree-level, its mass is the same as the corresponding parameter in the soft SUSY-breaking Lagrangian:

$$M_{\tilde{g}} = M_3$$

However, the quantum corrections to this are quite large (again, because this is a color octet!). If one calculates the one-loop pole mass of the gluino, one finds:

$$M_{\tilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_s}{4\pi}\big[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\big]\Big)$$

where Q is the renormalization scale, the sum is over all 12 squark multiplets, and

$$A_{\tilde{q}} = \int_0^1 dx\, x \ln\big[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x) - i\epsilon\big].$$

This correction can be of order 5% to 25%, depending on the squark masses!
It tends to **increase** the gluino mass, compared to the tree-level prediction.

Squarks and Sleptons

To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R$),
- a 3×3 matrix for sneutrinos ($\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau$)

Fortunately, the general hypothesis of flavor-blind soft parameters predicts that most of these mixing angles are very small.

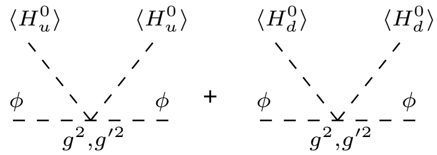
The first- and second-family squarks and sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs $(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{e}_L, \widetilde{\mu}_L)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$.

For detailed predictions, one must take into account “D-term” corrections to the mass^2 of each scalar ϕ:

$$\Delta_\phi = (T_{3\phi} g^2 - Y_\phi g'^2)(v_d^2 - v_u^2) = (T_{3\phi} - Q_\phi \sin^2\theta_W)\cos 2\beta\, m_Z^2,$$

where $T_{3\phi}$, Y_ϕ, and Q_ϕ are the third component of weak isospin, the weak hypercharge, and the electric charge of ϕ.

Diagrammatically, these come from:

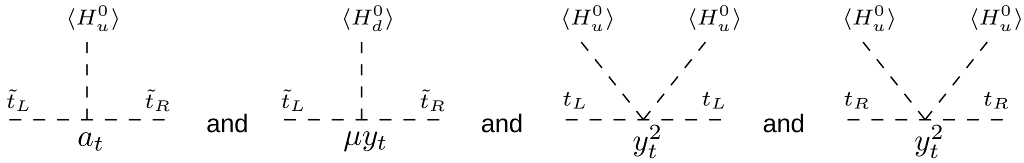


This leads to model-independent sum rules

$$m_{\tilde{e}_L}^2 - m_{\tilde{\nu}_e}^2 = m_{\tilde{d}_L}^2 - m_{\tilde{u}_L}^2 = g^2(v_u^2 - v_d^2)/2 = -\cos 2\beta\, m_W^2.$$

Since $\cos 2\beta < 0$ in the allowed range $\tan\beta > 1$, it follows that $m_{\tilde{e}_L} > m_{\tilde{\nu}_e}$ and $m_{\tilde{d}_L} > m_{\tilde{u}_L}$, with the magnitude of the splittings constrained by electroweak symmetry breaking.

For the third-family squarks and sleptons, there are additional effects proportional the large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings. For the top quark, we have corrections with the diagrammatic representations:



in addition to the D-term contributions. The first diagram comes directly from the soft SUSY-breaking Lagrangian, and the others from the F-term contribution to the scalar potential. So, in the $(\tilde{t}_L, \tilde{t}_R)$ basis, the top squark mass^2 matrix is:

$$\begin{pmatrix} m_{\tilde{Q}_3}^2 + m_t^2 + \Delta_{\tilde{t}_L} & a_t^* v_u - \mu y_t v_d \\ a_t v_u - \mu^* y_t v_d & m_{\bar{\tilde{u}}_3}^2 + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix}$$

Therefore, the top-squark system has a significant mixing, with the off-diagonal entries “repelling” the two mass^2 eigenvalues.

Diagonalizing the top squark mass^2 matrix, one finds mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s^*_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

where $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ by convention, and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$.

In a completely analogous way, there is a non-trivial mixing for the bottom squark and tau slepton states:

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{b}} & -s^*_{\tilde{b}} \\ s_{\tilde{b}} & c_{\tilde{b}} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix};$$

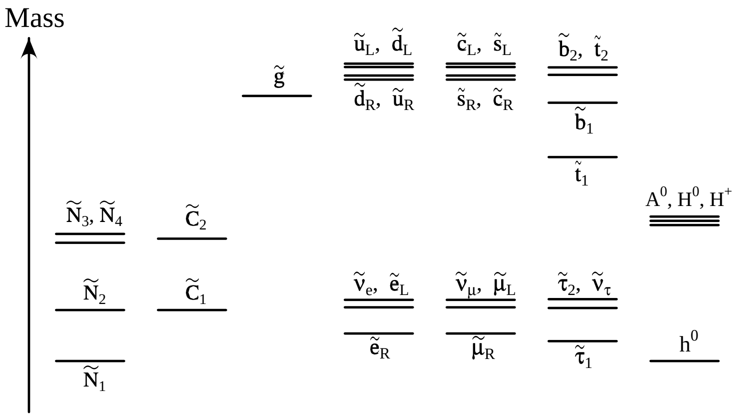
$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{\tau}} & -s^*_{\tilde{\tau}} \\ s_{\tilde{\tau}} & c_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

The same sort of mixing occurs for the first- and second-family squarks and sleptons, but is considered negligible because the Yukawa couplings are small, and by the assumption of flavor-blindness, the relevant a-terms are also.

The undiscovered particles in the MSSM:

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h^0\ H^0\ A^0\ H^\pm$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$
squarks	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$	" " " " $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$
charginos	$1/2$	-1	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "

A sample mass spectrum for the MSSM



This may or may not look anything like the Real World. It incorporates, qualitatively, some theoretical prejudices that are common to many different models.

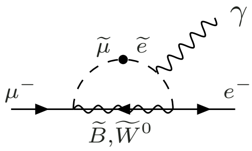
In the next lecture, we will explore various candidate ideas for how SUSY is broken, and relate them to features of the SUSY spectrum.

Lecture 3: Models of SUSY Breaking

Hints of an Organizing Principle

Fortunately, we already know that the MSSM soft terms cannot be arbitrary, because of experimental constraints on flavor violation.

For example, if there is a smuon-selectron mixing (mass)^2 term $\mathcal{L} = -m^2_{\tilde{\mu}_L \tilde{e}_L} \tilde{e}_L \tilde{\mu}^*_L$, and $\tilde{M} =$ Max$[m_{\tilde{e}_L}, m_{\tilde{e}_R}, M_2]$, then by calculating this one-loop diagram, one finds the decay width:



$\mu^- \rightarrow e^- \gamma$

$$\Gamma(\mu^- \rightarrow e^- \gamma) = 5 \times 10^{-21} \text{ MeV} \left(\frac{m^2_{\tilde{\mu}_L \tilde{e}_L}}{\tilde{M}^2} \right)^2 \left(\frac{100 \text{ GeV}}{\tilde{M}} \right)^4$$

For comparison, the experimental limit is (from MEGA at LAMPF):

$$\Gamma(\mu^- \rightarrow e^- \gamma) < 3.6 \times 10^{-27} \text{ MeV}.$$

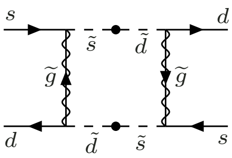
So the amount of smuon-selectron mixing in the soft Lagrangian is limited by:

$$\left(\frac{m^2_{\tilde{\mu}_L \tilde{e}_L}}{\tilde{M}^2} \right) < 10^{-3} \left(\frac{\tilde{M}}{100 \text{ GeV}} \right)^2$$

Another example: $K^0 \leftrightarrow \overline{K^0}$ mixing.

Let $\mathcal{L} = -m^2_{\tilde{d}_L \tilde{s}_L} \tilde{d}_L \tilde{s}^*_L$ be the flavor-violating term, and $\tilde{M} =$ Max$[m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{g}}]$.

Comparing this diagram with Δm_{K^0} gives:



$K^0 \leftrightarrow \overline{K^0}$

$$\frac{m^2_{\tilde{d}_L \tilde{s}_L}}{\tilde{M}^2} < 0.04 \left(\frac{\tilde{M}}{500 \text{ GeV}} \right)$$

The experimental values of ϵ and ϵ'/ϵ in the effective Hamiltonian for the $K^0, \overline{K^0}$ system also give strong constraints on the amount of $\tilde{d}_L, \tilde{s}_L$ and $\tilde{d}_R, \tilde{s}_R$ mixing and CP violation in the soft terms.

Similarly:

The $D^0, \overline{D^0}$ system constrains $\tilde{u}_L, \tilde{c}_L$ and $\tilde{u}_R, \tilde{c}_R$ soft SUSY-breaking mixing.

The $B^0_d, \overline{B^0_d}$ system constrains $\tilde{d}_L, \tilde{b}_L$ and $\tilde{d}_R, \tilde{b}_R$ soft SUSY-breaking mixing.

In general, the soft-SUSY breaking terms must be either very heavy, or nearly flavor-blind, to avoid flavor-changing violating experimental limits.

The Flavor-Preserving Minimal Supersymmetric Standard Model

Take an idealized limit in which in which the squark and slepton (mass)^2 matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space:

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}}\mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{u}}}} = m^2_{\tilde{\bar{u}}}\mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{d}}}} = m^2_{\tilde{\bar{d}}}\mathbf{1}; \quad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}}\mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{e}}}} = m^2_{\tilde{\bar{e}}}\mathbf{1}.$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will. Also assume:

$$\mathbf{a_u} = A_{u0}\,\mathbf{y_u}; \qquad \mathbf{a_d} = A_{d0}\,\mathbf{y_d}; \qquad \mathbf{a_e} = A_{e0}\,\mathbf{y_e}.$$

Also, assume no new CP-violating phases:

$$M_1,\ M_2,\ M_3,\ A_{u0},\ A_{d0},\ A_{e0} = \text{real}$$

The Higgs mass parameters $m^2_{H_u}$ and $m^2_{H_d}$ are real, and μ and b can be chosen real by convention.

The Flavor-Preserving Minimal Supersymmetric Standard Model (continued)

The new parameters, besides those already found in the Standard Model, are:

- M_1, M_2, M_3 (3 real gaugino masses and 3 signs)
- $m^2_{\tilde{Q}}, m^2_{\tilde{\bar{u}}}, m^2_{\tilde{\bar{d}}}, m^2_{\tilde{L}}, m^2_{\tilde{\bar{e}}}$ (5 squark and slepton mass^2 parameters)
- A_{u0}, A_{d0}, A_{e0} (3 real scalar^3 couplings and 3 signs)
- $m^2_{H_u}, m^2_{H_d}, b, \mu$ (4 real parameters)

So there are 15 real parameters and 6 signs in this model.
The parameters μ and $b \equiv B\mu$ are often traded for the known Higgs VEV $v = 175$ GeV, $\tan\beta$, and sign(μ).
Most viable SUSY breaking models are special cases of this.

However, these are Lagrangian parameters that run with the renormalization scale, Q. Therefore, one must also choose an "input scale" Q_0 where the flavor-independence holds.

What is the input scale Q_0 ?

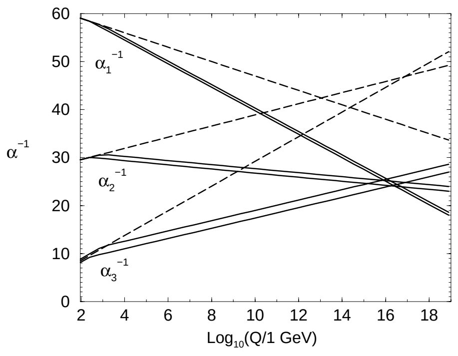
Perhaps:

- $Q_0 = M_{\rm Planck}$, or
- $Q_0 = M_{\rm string}$, or
- $Q_0 = M_{\rm GUT}$, or
- Q_0 is some other scale associated with the type of SUSY breaking.

In any case, one can pick the SUSY-breaking parameters at Q_0 as boundary conditions, then run them down to the weak scale using their renormalization group (RG) equations. Flavor violation will remain small, because the Yukawa couplings of the first two families are small.

At the weak scale, use the renormalized parameters to predict physical masses, decay rates, cross-sections, dark matter relic density, etc.

A reason to be optimistic that this program can succeed: the SUSY unification of gauge couplings. The measured α_1, α_2, α_3 are run up to high scales using the RG equations of the Standard Model (dashed lines) and the MSSM (solid lines).



At one-loop order, the RG equations are:

$$\frac{d}{d(\ln Q)} \alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3)$$

with $b_a^{\rm SM}$=$(41/10, -19/6, -7)$ in the Standard Model, and $b_a^{\rm MSSM}$=$(33/5, 1, -3)$ in the MSSM because of the extra particles in the loops. The results for the MSSM are in agreement with unification at $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV.

If this hint is real, we can reasonably hope that a similar extrapolation for the soft SUSY-breaking parameters can also work.

Origins of SUSY breaking

Up to now, we have simply put SUSY breaking into the MSSM explicitly.

To gain deeper understanding, let us consider how SUSY could be spontaneously broken. This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad Q^\dagger_{\dot\alpha}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\left(||Q_1^\dagger|0\rangle||^2 + ||Q_1|0\rangle||^2 + ||Q_2^\dagger|0\rangle||^2 + ||Q_2|0\rangle||^2\right) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

Recall that in SUSY, the potential energy

$$V = \sum_i F^{*i}F_i + \tfrac{1}{2}\sum_a D^aD^a$$

is a sum of squared of auxiliary fields. So, for spontaneous SUSY breaking, one must arrange that **no** state (or field configuration, classically) has all $F_i = 0$ and all $D^a = 0$.

Models of SUSY breaking where

- $F_i \neq 0$ are called “O’Raifeartaigh models” or “F-term Breaking models”
- $D^a \neq 0$ are called “Fayet-Iliopoulis models” or “D-term breaking models”

Let us do a simple example of each.

D-term breaking: the Fayet-Iliopoulis model

Suppose a $U(1)$ gauge symmetry is present, with some scalar supermultiplets carrying its charges. There is a supersymmetric and gauge-invariant term:

$$\mathcal{L} = -\kappa D$$

where κ is called the Fayet-Iliopoulis constant, and D is the auxiliary field for the $U(1)$ gauge supermultiplet. The part of the potential involving D is:

$$V = \kappa D - \tfrac{1}{2}D^2 - gD\sum_i q_i|\phi_i|^2.$$

The q_i are the $U(1)$ charges of scalar fields ϕ_i. The equation of motion for D is:

$$D = \kappa - g\sum_i q_i|\phi_i|^2.$$

Now suppose the ϕ_i have superpotential masses M_i. (Gauge invariance requires that they come in pairs with opposite charges.) Then the potential will be:

$$V = \sum_i |M_i|^2|\phi_i|^2 + \tfrac{1}{2}(\kappa - g\sum_i q_i|\phi_i|^2)^2.$$

Note that $V = 0$ is not possible for any ϕ_i. So SUSY must break...

D-term breaking (continued)

$$V = \sum_i |M_i|^2|\phi_i|^2 + \tfrac{1}{2}(\kappa - g\sum_i q_i|\phi_i|^2)^2.$$

If the superpotential masses are large enough ($M_i^2 > gq_i\kappa$ for each i), then the minimum of the potential is at:

$$\phi_i = 0, \qquad D = \kappa, \qquad V = \tfrac{1}{2}\kappa^2$$

The scalar and fermion masses are not degenerate:

$$\begin{aligned} m^2_{\phi_i} &= M_i^2 - gq_i\kappa \\ m^2_{\psi_i} &= M_i^2 \end{aligned}$$

This is a clear sign that SUSY has indeed been broken.

One might hope that the $U(1)_Y$ of the MSSM could get a D-term VEV to break SUSY. Unfortunately, the MSSM squarks and sleptons do not have superpotential masses, so they will just get VEVs in such a way as to make the $D_Y = 0$. This would be horrible: $SU(3)_C$ and $U(1)_{\rm EM}$ would be broken completely, but SUSY would **not** be broken!

More generally, D-term breaking for any $U(1)$ turns out to have great difficulty in giving acceptably large masses to gauginos. So F-term breaking is usually considered more promising as the main source of SUSY breaking...

An important note about F-term breaking in general:

The idea: pick a set of n supermultiplets (ϕ_i, ψ_i, F_i) with $i = 1, \ldots, n$, and a superpotential W, in such a way that $F^{*i} = -\partial W/\partial\phi_i = 0$ have no simultaneous solution. Then $V = \sum_i |F_i|^2 > 0$.

If the superpotential is

$$W = L^i\phi_i + \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k,$$

then

$$F^{*i} = -L^i - M^{ij}\phi_j - \frac{1}{2}y^{ijk}\phi_j\phi_k.$$

So at least one of the L^i must be non-zero to break SUSY; otherwise we could easily arrange $V = 0$ just by choosing all $\phi_i = 0$. This W can only be gauge-invariant if the corresponding ϕ_i is a gauge singlet.

Therefore, F-term breaking of SUSY requires a gauge-singlet chiral supermultiplet as a necessary, but not sufficient, condition.

However, the gauge-singlet may consist of composite fields.

F-term breaking: the O'Raifeartaigh Model

The simplest example has $n = 3$ chiral supermultiplets, with ϕ_1 the required singlet, and:

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

Then the auxiliary fields are:

$$F_1 = k - \frac{y}{2}\phi_3^{*2}, \qquad F_2 = -m\phi_3^*, \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*.$$

The reason SUSY must be broken is that $F_1 = 0$ and $F_2 = 0$ are not compatible. The minimum of this potential is at $\phi_2 = \phi_3 = 0$, with ϕ_1 not determined (classically). Quantum corrections fix the true minimum to be at $\phi_1 = 0$. At the minimum:

$$F_1 = k, \qquad V = k^2 > 0.$$

F-term breaking (continued)

If you assume $m^2 > yk$ and expand the scalar fields around the minimum at $\phi_1 = \phi_2 = \phi_3 = 0$, you will find 6 real scalars with tree-level squared masses:

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk,\ m^2 + yk.$$

Meanwhile, there are 3 Weyl fermions with squared masses

$$0,\ m^2,\ m^2.$$

The fact that the fermions and scalars aren't degenerate is a clear sign that SUSY has indeed been spontaneously broken.

The 0 mass^2 eigenvalues belong to the complex scalar ϕ_1 and its superpartner ψ_1. The masslessness of ϕ_1 corresponds to the flat direction of the classical potential. It is lifted by quantum corrections at one loop, resulting in:

$$m^2_{\phi_1} = \frac{y^4k^2}{48\pi^2 m^2}.$$

However, ψ_1 remains **exactly** massless, even including loop effects. **Why?**

The Goldstino ($\tilde{G}$)

In general, the spontaneous breaking of a global symmetry gives rise to a massless Nambu-Goldstone mode with the same quantum numbers as the broken symmetry generator. Here, the broken generator is the fermionic charge Q_α, so the Nambu-Goldstone particle must be a massless, neutral, Weyl fermion, called the **Goldstino**. It is always the fermion that lives in the same supermultiplet with the auxiliary field that got a VEV to break SUSY.

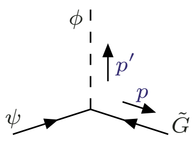
After SUSY breaking, you can show using Noether's Theorem that the Goldstino has an effective Lagrangian of the form (assuming F-term breaking for simplicity):

$$\mathcal{L}_{\text{Goldstino}} = -i\widetilde{G}^\dagger \overline{\sigma}^\mu \partial_\mu \widetilde{G} - \frac{1}{\langle F \rangle}(\widetilde{G}\partial_\mu J^\mu + \text{c.c.})$$

where J^μ is the fermionic supercurrent, and contains products of all of the fields and their superpartners.

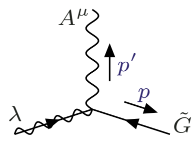
The Goldstino therefore has derivatively coupled interactions with every particle-sparticle pair:

Fermion-Scalar-Goldstino couplings



$$-\frac{i}{\langle F \rangle}(p' \cdot \sigma)(p \cdot \overline{\sigma})$$

Vector-Gaugino-Goldstino couplings



$$-\frac{i}{2\sqrt{2}\langle F \rangle}(p' \cdot \sigma\overline{\sigma}^\mu - \sigma^\mu p' \cdot \overline{\sigma})(p \cdot \sigma)$$

Note these both grow with $\frac{1}{\langle F \rangle}$, so they are more important if the mass scale of SUSY breaking is smaller. (More on this later.)

The interactions are well-defined in the $\langle F \rangle \to 0$ limit, because the derivatives in the numerator combine to give factors like $m_\phi^2 - m_\psi^2$ and $m_A^2 - m_\lambda^2$ on-shell, and these also vanish in the limit that there is no SUSY breaking.

The Goldstino is a consequence of spontaneously breaking **global** SUSY. Including gravity, SUSY becomes a local symmetry. The spinor ϵ_α used to define the SUSY transformations is no longer constant.

The resulting locally supersymmetric theory is **supergravity**. In unbroken supergravity, the graviton has a massless spin-$\frac{3}{2}$ partner (with only helicities $\pm\frac{3}{2}$) called the **gravitino**, with odd R-parity ($P_R = -1$).

When local SUSY is spontaneously broken, the gravitino absorbs the would-be massless Goldstino as its helicity $\pm\frac{1}{2}$ components, and acquires a mass:

$$m_{3/2} \sim \frac{\langle F \rangle}{M_{\rm Planck}}$$

This follows by dimensional analysis, since $m_{3/2}$ must vanish if SUSY-breaking is turned off $(\langle F \rangle \to 0)$ or gravity is turned off $(M_{\rm Planck} \to \infty)$. The gravitino inherits the couplings of the Goldstino it has eaten.

Spontaneous Breaking of SUSY requires us to extend the MSSM

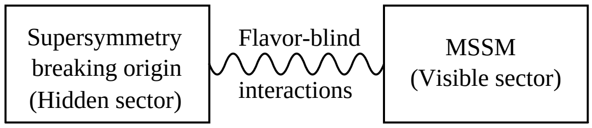
- D-term breaking using $U(1)_Y$ can't work
- There is no gauge-singlet chiral supermultiplet in the MSSM that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any (gaugino)-(gaugino)-(scalar) coupling that could turn into a gaugino mass term when the scalar gets a VEV.

We also have the clue that SUSY breaking must be essentially flavor-blind in order to not conflict with experiment.

This leads to the following general schematic picture of SUSY breaking. . .

The MSSM soft SUSY-breaking terms arise indirectly or radiatively, not from tree-level renormalizable couplings directly to the SUSY-breaking sector.



Spontaneous SUSY breaking occurs in a "hidden sector" of particles with no (or tiny) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly. As a bonus, if the mediating interactions are flavor-blind, then the soft SUSY-breaking terms of the MSSM will be also.

There are two obvious guesses for the flavor-blind interactions: gravitational and the ordinary gauge interactions.

Planck-scale Mediated SUSY Breaking (also known as "gravity mediation")

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since m_{soft} must vanish in the limit that SUSY breaking is turned off $(\langle F \rangle \to 0)$ and in the limit that gravity becomes irrelevant $(M_P \to \infty)$.

Since we know $m_{\text{soft}} \sim$ few hundred GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ or } 10^{12} \text{ GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\begin{aligned}\mathcal{L}_{\text{PMSB}} \;=\; & -\Big(\frac{f^a}{2M_P}F\lambda^a\lambda^a + \text{c.c.}\Big) - \frac{k_i^j}{M_P^2}FF^*\phi_i\phi^{*j} \\ & -\Big(\frac{\alpha^{ijk}}{6M_P}F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P}F\phi_i\phi_j + \text{c.c.}\Big)\end{aligned}$$

This is (part of) a fully supersymmetric Lagrangian that arises in supergravity, but it could have other origins too. When we replace F by its VEV $\langle F\rangle$, we get exactly the soft SUSY-breaking Lagrangian of lecture 2, with:

- Gaugino masses: $M_a = f^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious that these are flavor-blind!

A dramatic simplification occurs if one assumes a “minimal” form for the kinetic terms and gauge interactions in the underlying supergravity theory. (Whether this assumption is reasonable or not remains controversial.)

This means $f^a = f$ for all gauge interactions, $k_i^j = k\delta_i^j$ for all scalar fields, and $\alpha^{ijk} = \alpha y^{ijk}$ and $\beta^{ij} = \beta M^{ij}$. Then all of the MSSM soft terms can be written in terms of just four parameters:

- A common gaugino mass: $m_{1/2} = f\frac{\langle F\rangle}{M_P}$
- A common scalar squared mass: $m_0^2 = k\frac{|\langle F\rangle|^2}{M_P^2}$
- A scalar^3 coupling prefactor: $A_0 = \alpha\frac{\langle F\rangle}{M_P}$
- A scalar mass^2 prefactor $B_0 = \beta\frac{\langle F\rangle}{M_P}$

This simplified parameter space is often called “Minimal Supergravity” or “mSUGRA”.

The "mSUGRA" parameter space

In terms of the four parameters $m_{1/2}$, m_0^2, A_0, and B_0:

$$M_3 = M_2 = M_1 = m_{1/2}$$
$$\mathbf{m^2_{\tilde{Q}}} = \mathbf{m^2_{\tilde{\bar{u}}}} = \mathbf{m^2_{\tilde{\bar{d}}}} = \mathbf{m^2_{\tilde{L}}} = \mathbf{m^2_{\tilde{\bar{e}}}} = m_0^2\,\mathbf{1}$$
$$m^2_{H_u} = m^2_{H_d} = m_0^2$$
$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e}$$
$$b = B_0\mu.$$

These values of the soft terms should probably be taken at the renormalization scale $Q_0 = M_P$, and then run down to the weak scale. However, it is traditional to use $Q_0 = M_{\rm GUT}$ instead, because nobody has any idea how to extrapolate above $M_{\rm GUT}$! Part, but not all, of the error incurred in doing so can be reabsorbed into the definitions of $m_{1/2}$, m_0^2, A_0, and B_0.

Some particular models can be even more predictive, in principle:

- Dilaton-dominated: $m_0^2 = m_{3/2}^2$, $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$
- Polonyi: $m_0^2 = m_{3/2}^2$, $A_0 = (3-\sqrt{3})m_{3/2}$
- "No-scale" or "Gaugino mass dominated": $m_{1/2} \gg m_0, A_0$

However, there is no clear theoretical reason why things should be so simple.

The modern viewpoint is to take $m_{1/2}$, m_0^2, A_0, and B_0 as crude, but convenient, parameterizations of our ignorance of SUSY breaking.

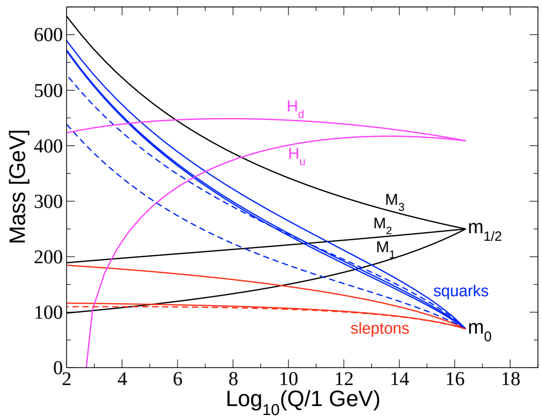
Renormalization Group Running for mSUGRA with $m_{1/2} = 250$ GeV, $m_0 = 70$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, and sign(μ) = +1

Gaugino masses M_1, M_2, M_3

Slepton masses (dashed=stau)

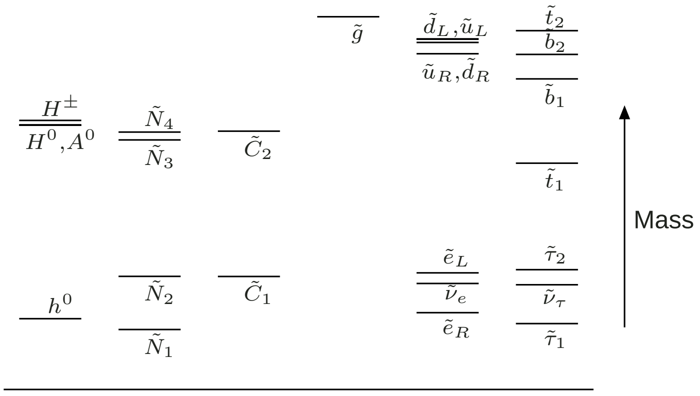
Squark masses (dashed=stop)

Higgs: $(m_{H_u}^2 + \mu^2)^{1/2}$, $(m_{H_d}^2 + \mu^2)^{1/2}$



Electroweak symmetry breaking occurs because $m_{H_u}^2 + \mu^2$ runs negative near the electroweak scale. This is due directly to the large top quark Yukawa coupling.

Here is the resulting sparticle mass spectrum:



This is typical, qualitatively, of mSUGRA models with relatively large $m_{1/2}$.

Notes: The Higgs sector is in the decoupling limit, with h^0 near the LEP2 limit. A neutralino is the LSP. The gluino is the heaviest sparticle. The lightest squark is the top squark. The lightest slepton is the tau slepton.

Gauge-Mediated SUSY Breaking (GMSB) models

The idea: SUSY breaking is transmitted from a hidden sector by the ordinary $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions. This makes them automatically flavor-blind!

To do this, introduce new, heavy, chiral supermultiplets, called **messengers**, which couple to $\langle F \rangle$ and also to the MSSM gauge bosons and gauginos.

If the typical messenger particle masses are $M_{\rm mess}$, the MSSM soft terms are:

$$m_{\rm soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\rm mess}}$$

The $\alpha_a/4\pi$ is a one-loop factor for diagrams involving gauge interactions. This follows by dimensional analysis, since $m_{\rm soft}$ must vanish as $\langle F \rangle \rightarrow 0$, or as the messengers become very heavy.

Note that $\sqrt{\langle F \rangle}$ can be as low as 10^4 GeV, if $M_{\rm mess}$ is comparable.

This is much lower than in Planck-scale Mediated SUSY Breaking. Therefore, these are also sometimes called “low-scale SUSY breaking” models.

GMSB models typically predict that the gravitino (which has absorbed the Goldstino) is the LSP. This is because, provided that $M_{\rm mess} \ll M_P$,

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \quad \ll \quad m_{\rm soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\rm mess}}$$

In fact, $m_{3/2}$ can be as low as 0.1 eV, for $\sqrt{\langle F \rangle} \sim 10^4$ GeV.

The lightest of the MSSM superpartner states is often called the Next-to-Lightest Supersymmetric Particle (NLSP).

The NLSP need not be neutral, since it can decay into its Standard Model partner and the Goldstino/gravitino.

Minimal Gauge-Mediated SUSY Breaking model

For a minimal model, take a set of new chiral supermultiplets q, $\overline{q}$, ℓ, $\overline{\ell}$ that transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3}); \quad \overline{q} \sim (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}); \quad \ell \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}); \quad \overline{\ell} \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2}).$$

These supermultiplets contain messenger quarks $\psi_q, \psi_{\overline{q}}$ and scalar quarks $q, \overline{q}$ and messenger leptons $\psi_\ell, \psi_{\overline{\ell}}$ and scalar leptons $\ell, \overline{\ell}$. These particles must get very large masses so as not to have been discovered already. They do so by coupling to a gauge-singlet chiral supermultiplet S through a superpotential:

$$W_{\rm mess} = y_2 S\ell\overline{\ell} + y_3 Sq\overline{q}.$$

The scalar component of S and its auxiliary field are both assumed to acquire VEVs, denoted $\langle S\rangle$ and $\langle F_S\rangle$ respectively.

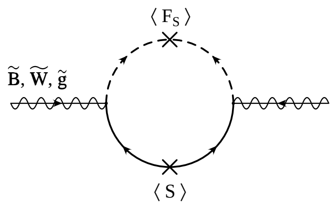
Note that the chiral supermultiplet S might be composite, and $\langle F_S\rangle \neq 0$ might come from an O'Raifeartaigh model, or from some more complicated dynamical mechanism. (We don't need to know!)

Minimal Gauge-Mediated SUSY Breaking model (continued)

The effect of SUSY breaking is to split the messenger masses:

$$\begin{array}{llll} \ell, \overline{\ell}: & m^2_{\text{fermions}} = |y_2\langle S\rangle|^2\,, & & m^2_{\text{scalars}} = |y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|\,; \\ q, \overline{q}: & m^2_{\text{fermions}} = |y_3\langle S\rangle|^2\,, & & m^2_{\text{scalars}} = |y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|\,. \end{array}$$

The SUSY-breaking apparent here is transmitted to the MSSM gauginos through one-loop graphs: The results are

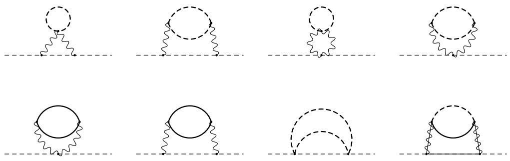


$$M_a = \frac{\alpha_a}{4\pi}\Lambda, \qquad \text{where} \qquad \Lambda \equiv \frac{\langle F_S\rangle}{\langle S\rangle}.$$

The MSSM gauge bosons do not get such a mass shift, since they are protected by gauge invariance. So SUSY breaking has been successfully communicated to the MSSM.

Minimal Gauge-Mediated SUSY Breaking model (continued)

The MSSM scalars do not get any masses at 1-loop order, but do at 2-loops from these Feynman diagrams:



The result for each MSSM scalar ϕ can be written:

$$m_\phi^2 = 2\Lambda^2\Big[\Big(\frac{\alpha_3}{4\pi}\Big)^2 C_3^\phi + \Big(\frac{\alpha_2}{4\pi}\Big)^2 C_2^\phi + \Big(\frac{\alpha_1}{4\pi}\Big)^2 C_1^\phi\Big], \qquad \text{where}$$

$$C_3^\phi = \begin{cases} 4/3 & \text{for } \phi = \widetilde{Q}_i, \widetilde{\bar{u}}_i, \widetilde{\bar{d}}_i; \\ 0 & \text{for } \phi = \widetilde{L}_i, \widetilde{\bar{e}}_i, H_u, H_d \end{cases}$$

$$C_2^\phi = \begin{cases} 3/4 & \text{for } \phi = \widetilde{Q}_i, \widetilde{L}_i, H_u, H_d; \\ 0 & \text{for } \phi = \widetilde{\bar{u}}_i, \widetilde{\bar{d}}_i, \widetilde{\bar{e}}_i \end{cases}$$

$$C_1^\phi = 3Y_\phi^2/5 \quad \text{for each } \phi \text{ with weak hypercharge } Y_\phi.$$

These squared masses are positive (fortunately!).

The Minimal GMSB model can be generalized by putting N copies of the messenger sector. The results then become:

$$M_a = \frac{\alpha_a}{4\pi} N\Lambda, \qquad \text{(gauginos)}$$

$$m_\phi^2 = 2N\Lambda^2\Big[\Big(\frac{\alpha_3}{4\pi}\Big)^2 C_3^\phi + \Big(\frac{\alpha_2}{4\pi}\Big)^2 C_2^\phi + \Big(\frac{\alpha_1}{4\pi}\Big)^2 C_1^\phi\Big], \qquad \text{(scalars)}$$

The parameters of this model framework are just:

- $N =$ number of messengers,
- $M_{\rm mess} =$ typical messenger mass scale,
- $\Lambda =$ effective SUSY-breaking order parameter
- μ, or equivalently $\tan\beta$ and sign(μ)

These models can be further generalized by including more exotic messengers, perhaps with widely varying masses.

GMSB model predictions

The scalar^3 terms $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$, arise first at two-loop order, and are suppressed by an additional factor of $\alpha_a/4\pi$ compared to gaugino masses. So it is an excellent approximation to set them $= 0$.

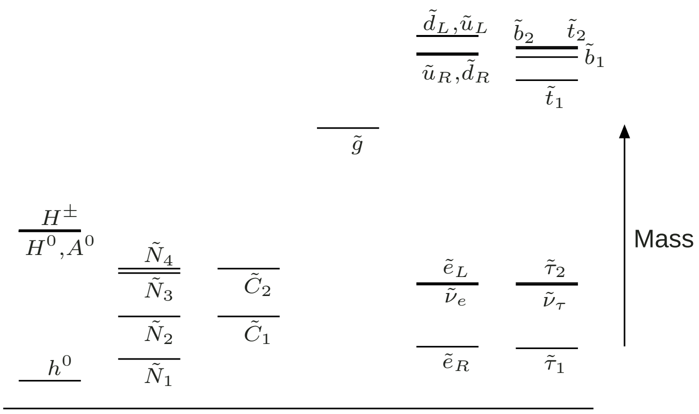
Because gaugino masses arise at one loop, and scalar squared masses arise at two loops, they are roughly comparable:

$$M_a, m_\phi \sim \frac{\alpha}{4\pi}\Lambda.$$

However, note that the gaugino masses scale like N, while the scalar masses scale like $\sqrt{N}$.
For $N = 1$, a bino-like neutralino will be the NLSP.
For $N \geq 2$, a stau will be the NLSP.

The above predictions for gaugino and scalar masses hold at the renormalization scale $Q_0 = M_{\rm mess}$. They must be run down to the electroweak scale. (This generates non-zero $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$, and modifies the other predictions.)

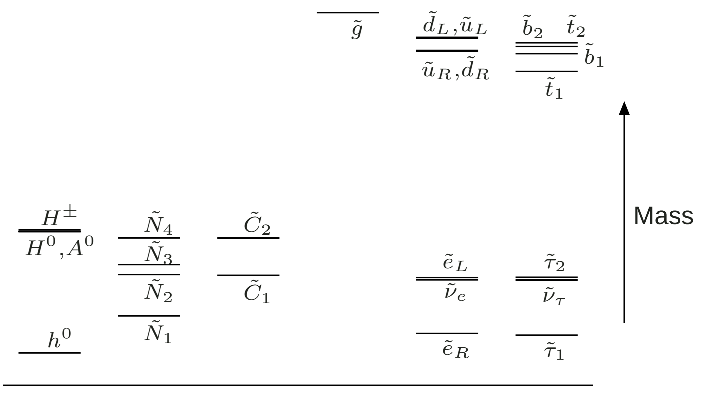
A sample sparticle mass spectrum for Minimal GMSB
with $N = 1$, $\Lambda = 150$ TeV, $M_{\rm mess} = 300$ TeV, $\tan\beta = 15$, sign(μ) = +1



The NLSP is a neutralino, which can decay to the nearly massless Goldstino/gravitino by: $\tilde{N}_1 \to \gamma\tilde{G}$. This decay can be prompt, or with a macroscopic decay length.

A sample sparticle mass spectrum for non-minimal GMSB

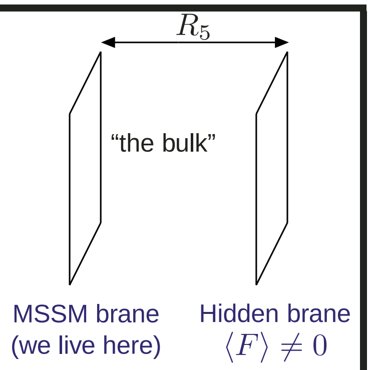
with $N = 3$, $\Lambda = 60$ TeV, $M_{\rm mess} = 120$ TeV, $\tan\beta = 15$, sign(μ) = +1



The NLSP is a stau ($\tilde{\mu}_R$ and $\tilde{e}_R$ are not much heavier). It can decay to the nearly massless Goldstino/gravitino by: $\tilde{\tau}_1 \rightarrow \tau\tilde{G}$. The decay could be prompt, or with a macroscopic length.

Extra-Dimensional Mediated SUSY Breaking

The Idea: Make the separation between hidden sector and visible sector a physical distance, for example along a hidden 5th dimension. The MSSM field theory is confined to a 4d "brane", and SUSY is spontaneously broken on another, parallel, 4d brane.



Provided that the distance between the branes is large, $R_5 \gg 1/M_P$, then dangerous flavor-violating effects in the MSSM soft terms can be suppressed by factors of $e^{-R_5 M_P}$.

There are various sub-ideas to play with here, including:

- Only gravity propagates in the bulk (Anomaly-Mediated SUSY Breaking) Randall, Sundrum; Giudice, Luty, Murayama, Ratazzi
- Gauge supermultiplets propagate in the bulk (Mirabelli,Peskin; Kaplan, Kribs, Schmaltz; Chacko, Luty, Nelson, Ponton; . . .)
- SUSY broken by 5d boundary conditions (Scherk-Schwarz SUSY Breaking)

Perhaps the most predictive scenario is:

Anomaly Mediated SUSY Breaking

The idea: assume that only gravity propagates in the bulk. Then all SUSY-breaking effects are suppressed, except a contribution from the conformal anomaly which is always present.

One can show that the resulting soft terms are given in terms of the renormalization group quantities (beta functions and anomalous dimensions) as:

$$\begin{aligned} M_a &= (\beta_{g_a}/g_a)m_{3/2} && \text{(gaugino masses)} \\ (m^2)_i^j &= -\tfrac{1}{2}\frac{d\gamma_i^j}{d(\ln Q)}m_{3/2}^2 && \text{(scalar masses)} \end{aligned}$$

These are flavor-blind, to a good approximation, because they are dominated by gauge couplings. Unfortunately, the MSSM sleptons are predicted to have negative squared mass!

Anomaly Mediated SUSY Breaking (continued)

Many fixes have been suggested to add positive contributions to the slepton squared masses. Perhaps the simplest is to simply add a common m_0^2 to each of the scalar squared masses.

Then the parameters of the model are just

- $m_{3/2}$ (AMSB SUSY breaking scale)
- m_0 (ad hoc scalar squared mass)
- μ or equivalently, $\tan\beta$ and sign(μ)

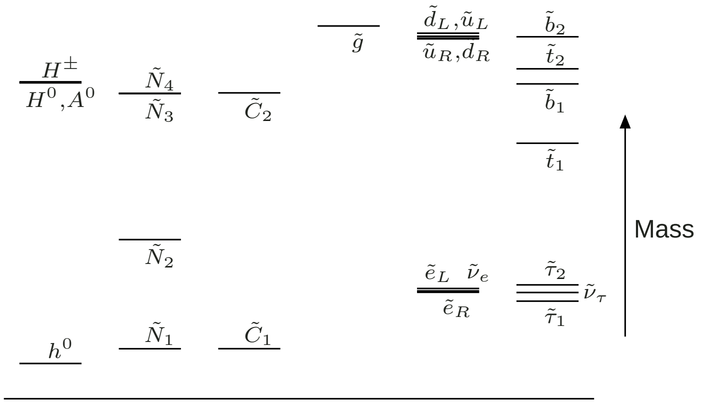
The most striking feature of the model is that the LSP is a wino-like neutralino. Typically, the charged wino can just barely decay into it:

$$\tilde{C}_1^+ \to \pi^+ \tilde{N}_1$$

where the pion is extremely soft, and therefore difficult to detect.

A sample sparticle mass spectrum for AMSB

(with $m_{3/2} = 60$ TeV, $m_0 = 450$ GeV, $\tan\beta = 10$, sign$(\mu) = +1$):



The LSP is $\tilde{N}_1$, but with $\tilde{C}_1$ only 160 MeV heavier. The sleptons are tightly packed. Other sparticles are much heavier.

Lecture 4: Signals for Supersymmetry

- Dark Matter
- Sparticle Decays
- LEP2 limits
- Tevatron $p\overline{p}$ collider
- LHC pp collider
- ILC e^-e^+ collider
- Special features of Gauge-Mediated SUSY Breaking

The Lightest SUSY Particle as Cold Dark Matter

Recent results in experimental cosmology suggest the existence of cold dark matter with a density:

$$\Omega_{\text{CDM}} h^2 = 0.113 \pm 0.009 \pm 0.018 \qquad \text{(WMAP 2003)}$$

where h = Hubble constant in units of 100 km/(sec Mpc).

A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.113$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

As a crude estimate, a weakly interacting particle that annihilates in collisions with a characteristic mass scale M will have

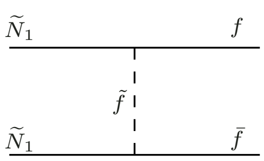
$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \left(\frac{150 \text{ GeV}}{M} \right)^2$$

So, a stable, weakly interacting particle with mass of order 100 GeV is a likely candidate. In particular, a neutralino LSP ($\widetilde{N}_1$) may do it, if R-parity is conserved.

Contributions to the annihilation cross section $\langle \sigma v \rangle$ for a neutralino LSP

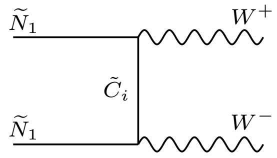
To have a viable SUSY model with a $\widetilde{N}_1$ LSP, it must not have too large a relic density Ωh^2, so $\langle \sigma v \rangle$ must not be too small. Let us examine the main contributions to the annihilation:

1) Annihilation through t-channel slepton and squark exchange. (If sleptons are lighter, they contribute more.)



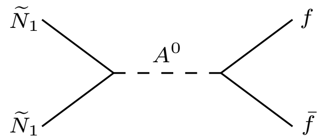
When $\widetilde{N}_1$ is mostly bino, as in many mSUGRA models, the interaction vertices are both $\sqrt{2} g' Y_f$, and this is the dominant process. To be efficient enough, the slepton masses ($m_{\tilde{e}_R}$, $m_{\tilde{\mu}_R}$, and $m_{\tilde{\tau}_1}$) must not be too large. In fact, for slepton masses > 100 GeV that are not ruled out by LEP2, the cross-section is too small, and so Ωh^2 comes out much too large in most of parameter space.

2) Annihilation through t-channel chargino exchange. This relies on a non-zero higgsino or wino content in $\widetilde{N}_1$.



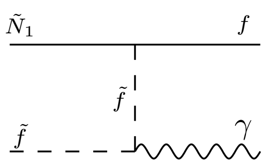
This diagram can provide sufficient annihilation of dark matter LSPs even if all squarks and sleptons are heavy, but only if μ is small (called the "focus point" region) or M_2 is not much larger than M_1.

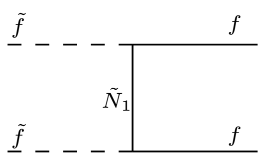
3) Resonant annihilation through an s-channel neutral Higgs boson. This process is s-wave for the pseudo-scalar Higgs A^0, and is p-wave suppressed for h^0 and H^0.



This process can be very efficient near resonance, and is mainly important for large $\tan\beta \gtrsim 40$, because the $A^0 b\bar{b}$ coupling is proportional to $m_b \tan\beta$.

4) Co-annihilation of neutralinos with sleptons (or top squarks).
If one or more sleptons or squarks is only slightly heavier that $\widetilde{N}_1$, then they will coexist in thermal equilibrium in the early universe. This can increase the efficiency of annihilation in Standard Model states, for example through $\tilde{N}_1\tilde{f} \to f\gamma$ and $\tilde{f}\tilde{f} \to ff$:

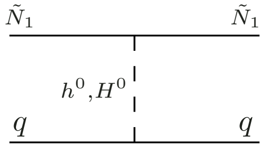


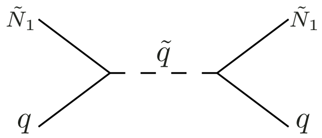


This co-annihilation effect is usually only important if $(m_{\tilde{f}} - m_{\tilde{N}_1})/m_{\tilde{N}_1} \lesssim 1/20$, so that significant numbers of $\tilde{f}$ can be in thermal equilibrium with $\tilde{N}_1$.

Direct Detection of $\tilde{N}_1$ Dark Matter LSPs

Neutralinos moving through a detector can recoil from nucleons:





The suppression due to small quark Yukawa couplings in the first diagram can be overcome by the coherent effect of many nucleons.

Typical recoil energies are only $E \sim 100$ keV. The predicted event rates are very low (a few per kilogram of detector per day, or less). This depends on the mixing matrix of $\tilde{N}_1$, and also on the local density and velocity distribution of dark matter.

Several present and future experiments using germanium detectors and scintillators will probe much, but not all, of the mSUGRA parameter space.

Indirect detection of Dark Matter LSPs

Neutralino LSPs in the centers of the Sun and the Galaxy can annihilate to Standard Model particles with high energies, which can then be seen directly or indirectly.

$$\begin{aligned}
\tilde{N}_1\tilde{N}_1 &\rightarrow e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-,\ \nu\overline{\nu},\ q\overline{q} \\
\tilde{N}_1\tilde{N}_1 &\rightarrow ZZ \rightarrow f\overline{f}f'\overline{f}' \\
\tilde{N}_1\tilde{N}_1 &\rightarrow W^+W^- \rightarrow \ell^+\nu\ell^-\overline{\nu}
\end{aligned}$$

For one promising example, ν_μ produced (either directly or indirectly) in $\tilde{N}_1\tilde{N}_1$ annihilation can travel to Earth and then undergo a charged current interaction leading to detection of upward-going muons.

Present and future neutrino telescopes are indirect dark matter detectors.

SUSY signatures at colliders

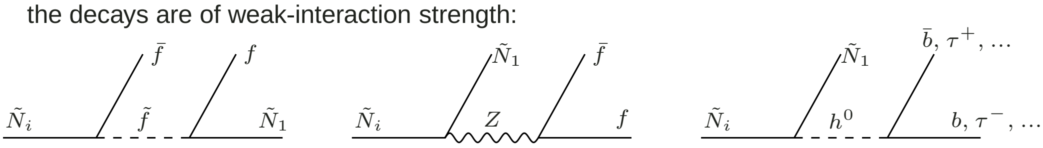
We will concentrate mostly on models with conserved R-parity and a neutralino LSP dark matter candidate $(\tilde{N}_1)$. Recall:

- The most important interactions for producing sparticles are gauge interactions, and interactions related to gauge interactions by SUSY. Their strength is known, up to mixing of sparticles.
- Two sparticles produced in each event, with opposite momenta.
- The LSPs are neutral and extremely weakly interacting, so they carry away energy and momentum.
  - At e^+e^- colliders, the total energy can be accounted for, so one sees missing energy, $\not{E}$.
  - At hadron colliders, the component of the momentum along the beam is unknown on an event-by-event basis, so only the energy component in particles transverse to the beam is observable. So one sees "missing transverse energy", $\not{E}_T$.

Sparticle Decays

1) Neutralino Decays

If R-parity is conserved and $\tilde{N}_1$ is the LSP, then it cannot decay. For the others, the decays are of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, Z boson, or Higgs boson h^0) might be on-shell, if that two-body decay is kinematically allowed.

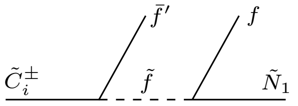
In general, the visible decays are either:

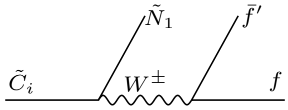
$$\tilde{N}_i \to q\bar{q}\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$
$$\tilde{N}_i \to \ell^+\ell^-\tilde{N}_1 \qquad \text{(seen in detector as } \ell^+\ell^- + \not{E}\text{)}$$

Some SUSY signals rely on leptons in the final state. This is more likely if sleptons are relatively light. If $\tilde{N}_i \longrightarrow \tilde{N}_1 h^0$ is kinematically open, then it often dominates. This is called the "spoiler mode", because leptonic final states are rare.

2) Chargino Decays

Charginos $\tilde{C}_i$ have decays of weak-interaction strength:





In each case, the intermediate boson (squark or slepton $\tilde{f}$, or W boson) might be on-shell, if that two-body decay is kinematically allowed.

In general, the decays are either:

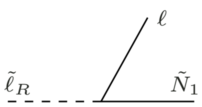
$$\tilde{C}_i^{\pm} \to q\bar{q}'\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$
$$\tilde{C}_i^{\pm} \to \ell^{\pm}\nu\tilde{N}_1 \qquad \text{(seen in detector as } \ell^{\pm} + \not{E}\text{)}$$

Again, leptons in final state are more likely if sleptons are relatively light.

For both neutralinos and charginos, a relatively light, mixed $\tilde{\tau}_1$ can lead to enhanced τ's in the final state. This is increasingly important for larger $\tan\beta$. Tau identification may be a crucial limiting factor for experimental SUSY.

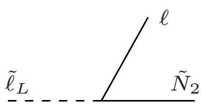
3) Slepton Decays

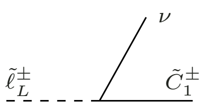
When $\tilde{N}_1$ is the LSP and has a large bino content, the sleptons $\tilde{e}_R$, $\tilde{\mu}_R$ (and often $\tilde{\tau}_1$ and $\tilde{\tau}_2$) prefer the direct two-body decays with strength proportional to g'^2:

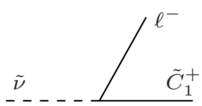


(seen in detector as $\ell^{\pm} + \not{E}$)

However, the left-handed sleptons $\tilde{e}_L$, $\tilde{\mu}_L$, $\tilde{\nu}$ have no coupling to the bino component of $\tilde{N}_1$, so they often decay preferentially through $\tilde{N}_2$ or $\tilde{C}_1$, which have a large wino content, with strength proportional to g^2:



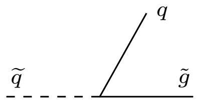




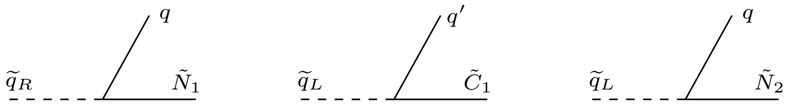
with $\tilde{N}_2$ and $\tilde{C}_1$ decaying as before.

4) Squark Decays

If the decay $\tilde{q} \longrightarrow q\tilde{g}$ is kinematically allowed, it will always dominate, because the squark-quark-gluino vertex has QCD strength:



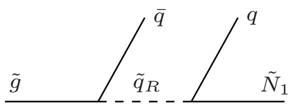
Otherwise, right-handed squarks prefer to decay directly to a bino-like LSP, while left-handed squarks prefer to decay to a wino-like $\tilde{C}_1$ or $\tilde{N}_2$:



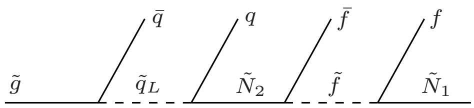
If a top squark is light, then the decays $\tilde{t}_1 \longrightarrow t\tilde{g}$ and $\tilde{t}_1 \longrightarrow t\tilde{N}_1$ may not be kinematically allowed, and it may decay only into charginos: $\tilde{t}_1 \longrightarrow b\tilde{C}_1$. If even that is not allowed, it has only a suppressed flavor-changing decay: $\tilde{t}_1 \longrightarrow c\tilde{N}_1$.

5) Gluino Decays

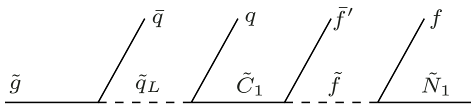
The gluino can only decay through squarks, either on-shell (if allowed) or virtual. For example:



$jj + \not{E}$ or $t\bar{t} + \not{E}$



$jjjj + \not{E}$ or $t\bar{t}jj + \not{E}$ or $jj\ell^+\ell^- + \not{E}$



$jjjj + \not{E}$ or $t\bar{t}jj + \not{E}$ or $jj\ell^\pm + \not{E}$

Because $m_{\tilde{t}_1} \ll$ other squark masses, top quarks can appear in these decays.

The possible signatures of gluinos and squarks are typically numerous and complicated because of these and other **cascade decays**.

An important feature of gluino decays with one lepton:

In each case, $\tilde{g} \rightarrow jj\ell^{\pm} + \not{E}$, and the lepton has either charge with equal probability. (The gluino does not "know" about electric charge.)

So, events with at least one gluino, and exactly one charged lepton in the final state from each sparticle that was produced, will have probability 0.5 to have **same-charge leptons**, and probability 0.5 to have opposite-charge leptons.

This is important at hadron collider, where Standard Model backgrounds with same-charge leptons are much smaller.

$$\text{(SUSY)} \rightarrow \ell^{+}\ell'^{+} + \text{ jets } + \not{E}_T$$

SUSY Limits from LEP2 e^+e^- collisions up to $\sqrt{s} = 208$ GeV

The CERN LEP2 collider had the capability of producing all sparticle-antisparticle pairs, except for the gluino:

$$e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-,\ \tilde{C}_1^+\tilde{C}_1^-,\ \tilde{N}_1\tilde{N}_2,\ \tilde{N}_2\tilde{N}_2,\ \gamma\tilde{N}_1\tilde{N}_1,\ \tilde{q}\tilde{q}^*$$

Exclusions for charged sparticles are typically close to the kinematic limit, except when mass difference are small. For example, at 95% CL:

$$m_{\tilde{C}_1} > 103 \text{ GeV} \qquad (m_{\tilde{C}_1} - m_{\tilde{N}_1} > 3 \text{ GeV or} < 100 \text{ MeV})$$

$$m_{\tilde{C}_1} > 92 \text{ GeV} \qquad (\text{any heavier than } \tilde{N}_1)$$

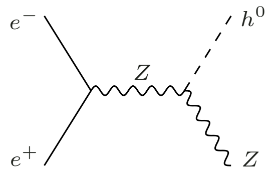
and

$$m_{\tilde{e}_R} > 100 \text{ GeV} \qquad (m_{\tilde{e}_R} - m_{\tilde{N}_1} > 5 \text{ GeV})$$

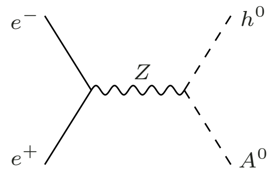
See `http://lepsusy.web.cern.ch/lepsusy/`
for detailed results.

LEP2 Searches for Higgs bosons

The most important constraints on SUSY parameter space come from searches for the MSSM Higgs bosons at LEP2. The relevant processes include:



$\propto \sin^2(\beta - \alpha)$



$\propto \cos^2(\beta - \alpha)$

The first diagram is the same as for the Standard Model Higgs search in the decoupling limit, where $\sin^2(\beta - \alpha) \approx 1$. Many SUSY models fall into this category, and the LEP2 bound (nearly) applies:

$$m_{h^0} > 114.4\,\mathrm{GeV} \qquad (95\%\ \mathrm{CL})$$

General bounds in SUSY are much weaker, but "most" of parameter space in the MSSM yields a Standard-Model-like lightest Higgs boson.

Impact of the LEP2 bound on m_{h^0}

Recall that in the decoupling limit:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$

For $\cos(2\beta) \approx 1$, we therefore need, roughly:

$$\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 600\,\mathrm{GeV}.$$

This suggests a pessimistic attitude toward discovering squarks at the Tevatron.

However, there are many ways out. Enlargement of the Higgs sector, for example by adding a singlet Higgs supermultiplet, can give positive contributions to m_h^0.

Tevatron Signals for SUSY in $p\overline{p}$ collisions at $\sqrt{s} = 1.96$ TeV

Trilepton + $\not\!\!E_T$ Signal at the Tevatron

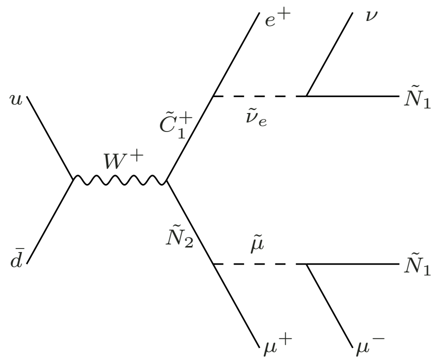
This signal arises if one can produce a pair of wino-like sparticles

$$p\overline{p} \longrightarrow \tilde{C}_1^{\pm}\tilde{N}_2,$$

which then each decay leptonically with a significant branching fraction,

$$\tilde{N}_2 \rightarrow \ell^+\ell^-\tilde{N}_1, \qquad \tilde{C}_1^{\pm} \rightarrow \ell^{\pm}\nu\tilde{N}_1$$

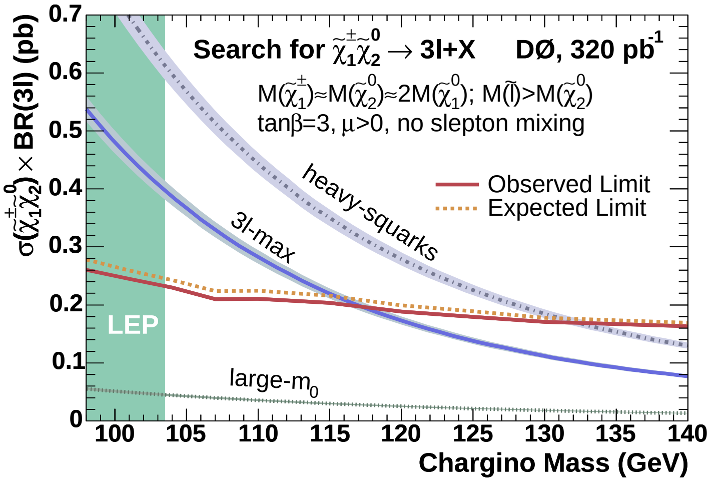
With no hard jets in the event, and three identified leptons, the Standard Model backgrounds are small. Here is a typical Feynman diagram for the whole event:



$$p\bar{p} \rightarrow \ell^+\ell^-\ell'^{\pm} + \not\!\!E_T$$

Decays of $\tilde{C}_1^{\pm}$ and $\tilde{N}_2$ through virtual squarks and/or virtual h^0 kill the signal. Decays through Z, W hurt the signal. Decays through sleptons, as shown, help the signal.

Latest Tevatron trilepton search results (from DØ `hep-ex/0504032`; CDF preliminary similar)



This is for an mSUGRA-like model scenario. The greatest sensitivity comes when $0 < m_{\tilde{\ell}} - m_{\tilde{N}_2} \lesssim 20$ GeV, with squarks much heavier. More data, better τ ID will result in interesting explorations of SUSY parameter space.

Multi-Jets + $\not{E}_T$ at Tevatron

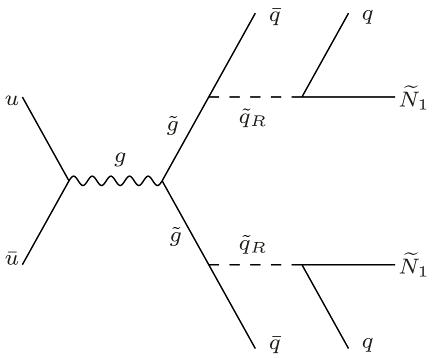
Another strategy: look for events with gluino-gluino, gluino-squark, and squark-squark pair production:

$$p\overline{p} \to \tilde{g}\tilde{g},\ \tilde{g}\tilde{q},\ \tilde{q}\tilde{q}$$

followed by decays **without** leptons:

$$\tilde{g} \to q\bar{q}\tilde{N}_1, \qquad \tilde{q} \to q\tilde{N}_1$$

A typical Feynman diagram for the whole event:



By vetoing isolated, energetic leptons, the Standard Model backgrounds with $\not{E}_T$ from $W \to \ell\nu$ are reduced.

This is most likely to be a viable signal for models that don't fall into the mSUGRA category, with a relatively lighter gluino.

Like-Charge Dileptons + $\not\!\!E_T$ at Tevatron

Exploit the fact mentioned earlier that gluinos decay into leptons of either charge with equal probability:

$$p\overline{p} \longrightarrow \tilde{g}\tilde{g} \longrightarrow \text{(jets)} + \ell^{\pm}\ell^{\pm} + \not\!\!E_T .$$

Multi-b-jets + $\not\!\!E_T$ at Tevatron

Produce gluons that decay into bottom quark and bottom squark:

$$p\overline{p} \longrightarrow \tilde{g}\tilde{g} \longrightarrow (b\tilde{b}_1)(\bar{b}\tilde{b}_1^*) \longrightarrow (b\bar{b}\tilde{N}_1)(b\bar{b}\tilde{N}_1) \longrightarrow (b\bar{b})(b\bar{b}) + \not\!\!E_T .$$

Light Top Squarks at Tevatron

Top squarks with $m_{\tilde{t}_1} < \text{Min}[m_{\tilde{N}_1} + m_b + m_W, m_{\tilde{C}_1} + m_b, m_b + m_{\tilde{\nu}}]$ have only suppressed flavor-violating decays to charm:

$$p\overline{p} \rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow (c\tilde{N}_1)(\bar{c}\tilde{N}_1) \rightarrow jj + \not\!\!E_T$$

These searches are ongoing. . .

LHC Signals for SUSY in pp collisions at $\sqrt{s} = 14$ TeV

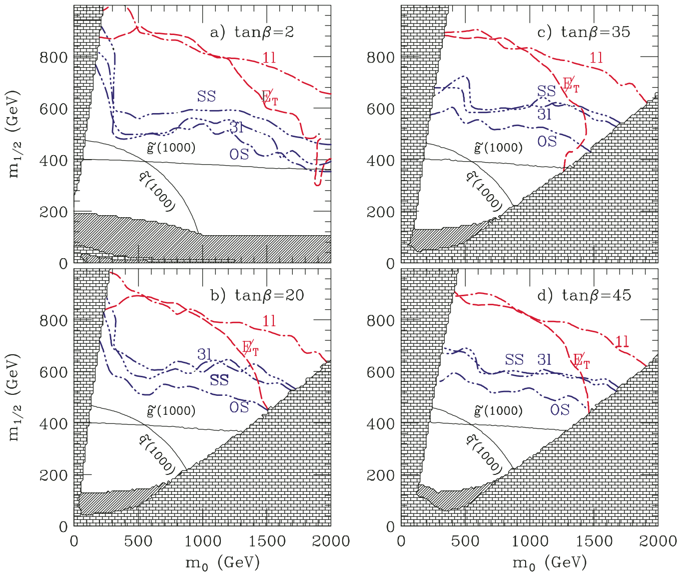
The LHC is a gluon-gluon collider, to first approximation

The dominant production cross-sections are:

$$pp \to \tilde{g}\tilde{g},\ \tilde{g}\tilde{q},\ \tilde{q}\tilde{q}$$

Event rates are very high. Discovery signals can be classified by the number of leptons in the event.

Discovery reach extends well beyond 1 TeV in both gluino and squark masses.

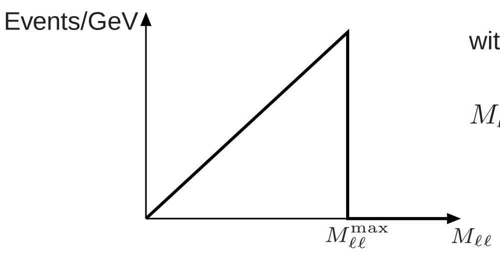


Baer, Chen, Drees, Paige, Tata 1999

A post-discovery measurement at the LHC: kinematic edge in $\tilde{N}_2$ decay

Combinations of sparticle masses can be measured at the LHC from observing edges in kinematic distributions.

As one of many examples, consider all events leading to $\tilde{N}_2 \to \ell\tilde{\ell} \to \ell^+\ell^-\tilde{N}_1$, from all SUSY production sources. Theoretically, the distribution of the invariant mass for the lepton pair should have this shape (see e.g. Atlas TDR):



with

$$M_{\ell\ell}^{\max} = m_{\tilde{N}_2}\left(1 - \frac{m_{\tilde{\ell}}^2}{m_{\tilde{N}_2}^2}\right)^{1/2}\left(1 - \frac{m_{\tilde{N}_1}^2}{m_{\tilde{\ell}}^2}\right)^{1/2}$$

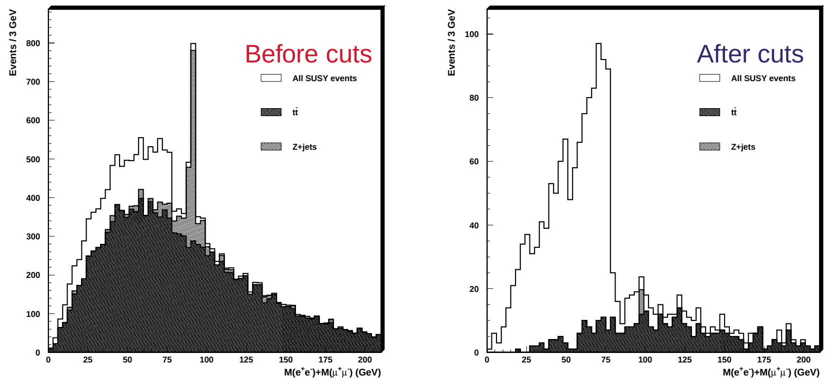
To remove backgrounds, require events to have very large $\not{E}_T$, several hard jets, and two energetic leptons.

Also, the signal from $\tilde{N}_2$ decay has only same-flavor leptons. Therefore, one can enhance the edge shape by subtracting events that pass the cuts, but with opposite lepton flavors:

$$[e^+e^-] + [\mu^+\mu^-] - [e^+\mu^-] - [\mu^+e^-]$$

Also cuts $\not{E}_T > 150$ GeV and $E_{\ell\ell} > 100$ GeV eliminate the $t\bar{t}$ and Z-peak backgrounds.

CMS study, Chiorboli and Tricomi



Gluino mass reconstruction at the LHC

The gluino mass can be reconstructed once the $\tilde{N}_2$ mass has been found from the dilepton mass edge. For a particularly favorable case, consider decays through $\tilde{b}_1$, so that the gluino decay results in

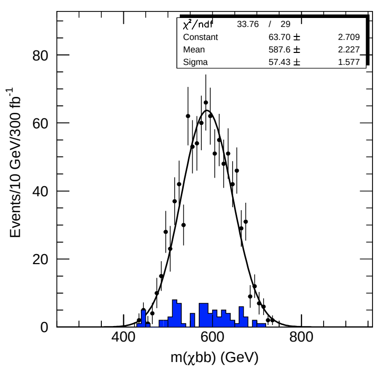
$$\tilde{g} \to \bar{b}\tilde{b}_1 \to b\bar{b}\tilde{N}_2 \to b\bar{b}\ell^+\ell^-\tilde{N}_1$$

By selecting events close to the (now) known dilepton edge, the impact of the unknown $\tilde{N}_1$ momentum can be minimized. Then one reconstructs the invariant mass

$$M(b\bar{b}\tilde{N}_2),$$

and fits the resulting peak to a Monte Carlo generated Gaussian.

(Here $m_{\tilde{g}} = 595$ GeV.)

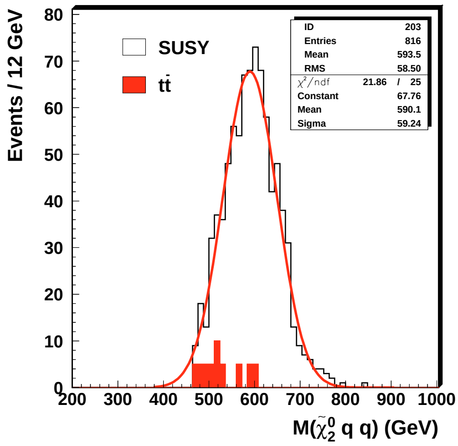


ATLAS study: Gjelsten, Hisano, Kawagoe, Lytken, Miller, Nojiri, Osland, Polesello

Here is another study, this time based on the decay (with no b-tagging):

$$\tilde{g} \to \bar{q}\tilde{q} \to q\bar{q}\tilde{N}_2 \to q\bar{q}\ell^+\ell^-\tilde{N}_1$$

Again, a peak is reconstructed based on the invariant mass $M(jj\tilde{N}_2)$, using the known position of the dilepton edge to minimize the effects of the unknown $\tilde{N}_1$ momentum.



CMS study, Chiorboli and Tricomi

A measurement of the gluino mass is crucial to a quantitative understanding of SUSY breaking. This can only be obtained at the LHC.

However, kinematic measurements at the LHC are essentially determinations of mass **differences**, with the unknown $\tilde{N}_1$ LSP mass used as an input to the fits.

The most accurate determination of absolute masses can be obtained at an International Linear Collider.

International Linear Collider Signals for SUSY in e^+e^- collisions

Essential features of the e^+e^- collider environment for SUSY:

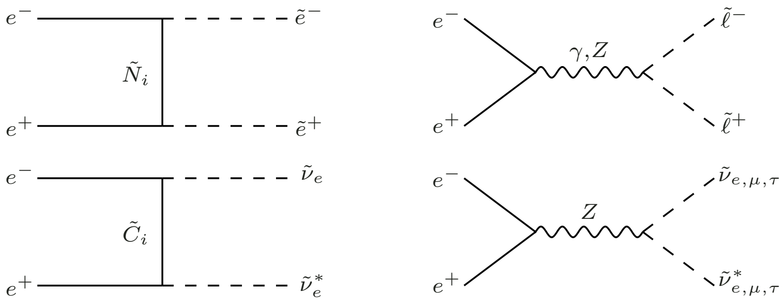
- All sparticles, except the gluino, can be produced in pairs up to nearly the kinematic limits.
- Clean environment allows reconstruction of events, elimination of backgrounds.
- Polarized beam allow signals and backgrounds to be turned on and off.
- Essentially all events can be recorded; no need for hadron-collider-style triggers that might throw away unexpected interesting events.
- Possible site conveniently located near Northern Illinois University in beautiful DeKalb IL, USA

Slepton pair production in e^-e^+ collisions

The processes

$$
\begin{aligned}
e^-e^+ &\longrightarrow \tilde{e}_R^-\tilde{e}_R^+,\ \tilde{e}_R^-\tilde{e}_L^+,\ \tilde{e}_L^-\tilde{e}_R^+,\ \tilde{e}_L^-\tilde{e}_L^+,\ \tilde{\mu}_R^-\tilde{\mu}_R^+,\ \tilde{\mu}_R^-\tilde{\mu}_L^+,\ \tilde{\mu}_L^-\tilde{\mu}_R^+,\ \tilde{\mu}_L^-\tilde{\mu}_L^+,\\
&\qquad \tilde{\tau}_1^-\tilde{\tau}_1^+,\ \tilde{\tau}_1^-\tilde{\tau}_2^+,\ \tilde{\tau}_2^-\tilde{\tau}_1^+,\ \tilde{\tau}_2^-\tilde{\tau}_2^+\\
e^-e^+ &\longrightarrow \tilde{\nu}_e\tilde{\nu}_e^*,\ \tilde{\nu}_\mu\tilde{\nu}_\mu^*,\ \tilde{\nu}_\tau\tilde{\nu}_\tau^*,
\end{aligned}
$$

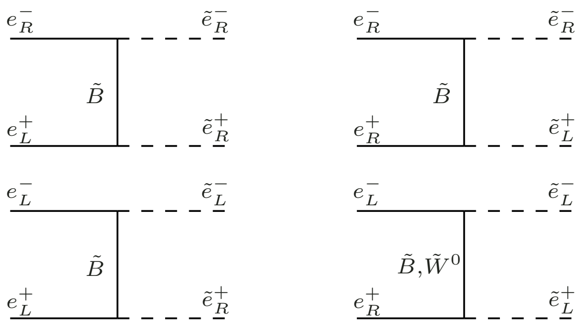
proceed by the following Feynman diagrams:



Only sleptons of the first family can be produced by the t-channel diagrams.

Selectron separation using polarized beams

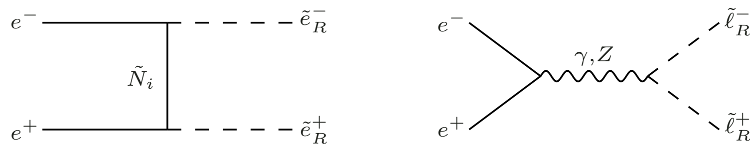
The t-channel neutralino exchange diagrams are dominant near threshold. Separated into e^- and e^+ helicities, using the interactions that are dictated by the SUSY Lagrangian:



This allows a clean separation of the selectron masses, by using e^- (and e^+) beams with known helicity. In the cases of pure $e_R^-e_R^+$ and $e_L^-e_L^+$, the s-channel diagrams do not contribute at all.

Kinematic endpoints in slepton pair production

Right-handed sleptons can be produced in e^-e^+ collisions by the following Feynman diagrams, followed by $\tilde{\ell}_R^{\pm} \to \ell^{\pm}\tilde{N}_1$:

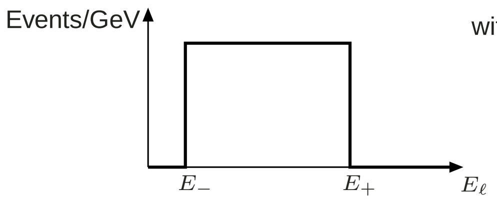


Note that the first diagram is possible only for selectrons. Because the $e^-\tilde{N}_1\tilde{e}_R^-$ coupling is the "supersymmetrization" of the gauge interaction, it comes entirely from the R helicity of the initial-state electron.

There is a background from $e^-e^+ \to W^-W^+$, which can be reduced by using polarized electron (and perhaps positron) beams: $e_R^-e_L^+ \to \ell_R^-\ell_R^+$. (This is not so nice for $\tilde{e}_L\tilde{e}_L$ production.) There is also a SUSY background, from $e^-e^+ \to \tilde{N}_2\tilde{N}_1 \to (\ell^+\ell^-\tilde{N}_1)\tilde{N}_1$.

Plotting the lepton energies allows masses to be determined:

In theory, the lepton energy distribution should look like:

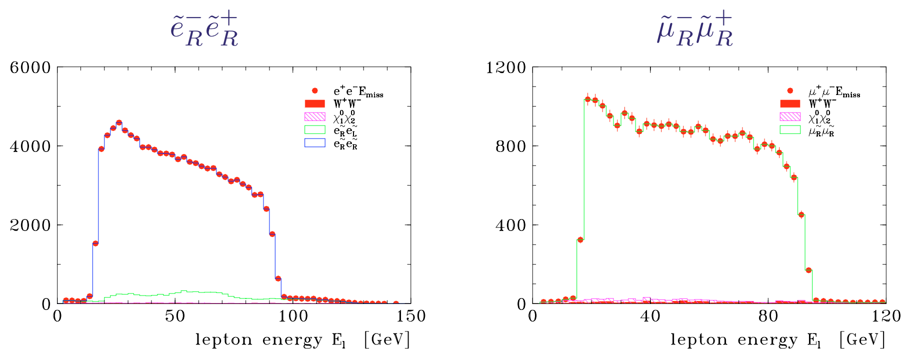


with

$$E_{\pm} = \frac{1}{2}\left(1 - \frac{m_{\tilde{N}_1}^2}{m_{\tilde{\ell}_R}^2}\right)\left(\frac{s}{4} \pm m_{\tilde{\ell}_R}^2\right)^{1/2}$$

After including cuts and beam and detector effects

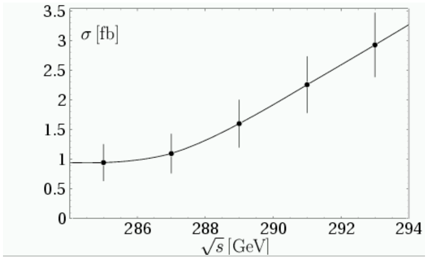
(H.-U. Martyn, LC note, $\sqrt{s} = 400$ GeV and 200 fb^{-1})



By measuring the two kinematic endpoints, the masses of the two particles involved in the decay can be determined with very high accuracy.

One can also do threshold scans by tuning $\sqrt{s}$, to observe the turn-on of production: $\tilde{\mu}_R^- \tilde{\mu}_R^+$, 10 fb^{-1} per point

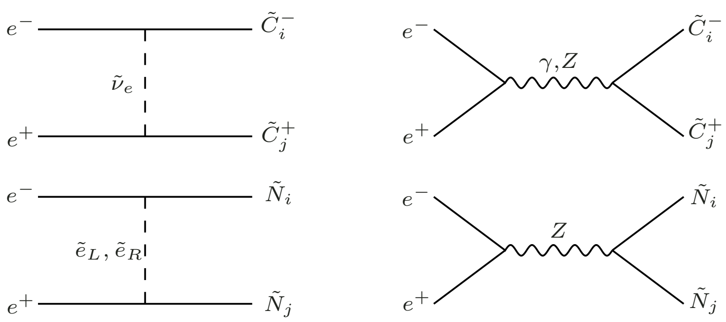
(Freitas, Manteuffel, Zerwas 2004)



Determination of $\tilde{\tau}$ masses is more difficult.

Chargino and Neutralino Production at an $e^- e^+$ collider

The processes $e^- e^+ \to \tilde{C}_i^+ \tilde{C}_j^-$ and $e^- e^+ \to \tilde{N}_i \tilde{N}_j$ proceed through these Feynman diagrams:

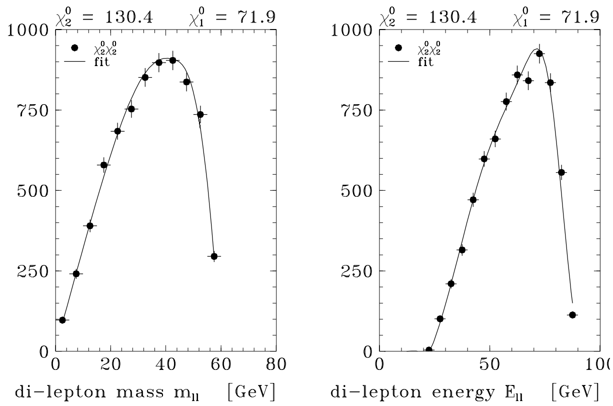


Again, the relative strengths of these contributions can be dialed by changing the polarization of the beams, taking advantage of the different couplings of $\tilde{\nu}_e$, $\tilde{e}_L$ and $\tilde{e}_R$ to e_L^-, e_R^-, e_R^+, and e_L^+ and the gaugino components of $\tilde{C}_i$, $\tilde{N}_i$. This is dictated by the dimensionless part of the SUSY Lagrangian.

Measurement of $\tilde{N}_2$ mass

Look for events in which both neutralinos decay leptonically:

$$e^-_L e^+_R \to \tilde{N}_2 \tilde{N}_2 \to (\ell^+ \ell^- \tilde{N}_1)(\ell'^+ \ell'^- \tilde{N}_1) \to \ell^+ \ell^- \ell'^+ \ell'^- + \not{E}$$



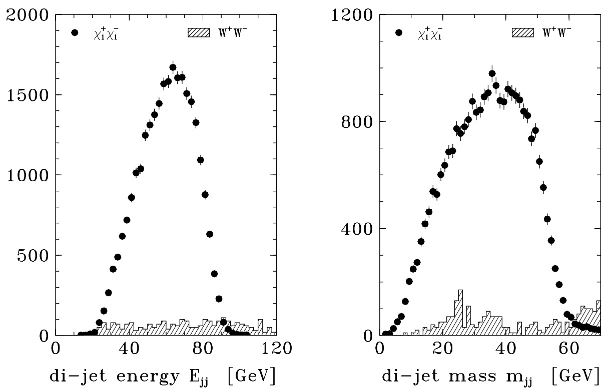
$m_{\tilde{N}_2} = 130.4$ GeV,
$m_{\tilde{N}_1} = 71.9$ GeV,
$\sqrt{s} = 320$ GeV,
160 fb $^{-1}$,
TESLA TDR

Fit this to Monte Carlo, using the known $\tilde{N}_1$ mass.
A scan of $\sqrt{s}$ near threshold can make the measurement more precise.

Measurement of $\tilde{C}_1$ mass

Look for events in which one chargino decays hadronically, the other leptonically:

$$e^-_L e^+_R \to \tilde{C}^-_1 \tilde{C}^+_1 \to (q\bar{q}' \tilde{N}_1)(\ell^\pm \nu \tilde{N}_1) \to jj\ell^\pm + \not{E}$$

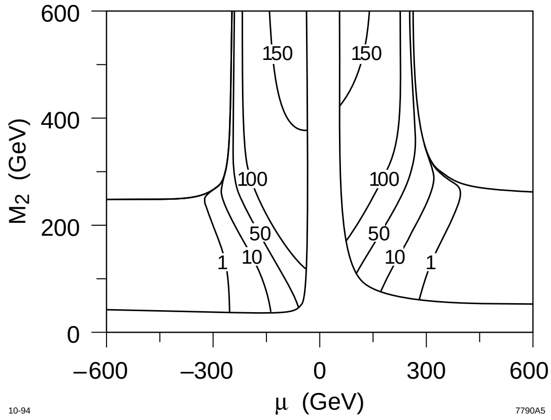


$m_{\tilde{C}_1} = 127.7$ GeV,
$m_{\tilde{N}_1} = 71.9$ GeV,
$\sqrt{s} = 320$ GeV,
160 fb $^{-1}$,
TESLA TDR

Fit this to Monte Carlo, using the known $\tilde{N}_1$ mass.
A scan of $\sqrt{s}$ near threshold can make the measurement more precise.

Cross-sections can be very sensitive probes of SUSY parameters

Contours of $\sigma(e^-_R e^+ \to \tilde{C}^-_1 \tilde{C}^+_1)$, in fb, as a function of M_2 and μ, assuming mSUGRA model with $\tan\beta = 4$:



Feng, Peskin, Murayama, Tata, 1995

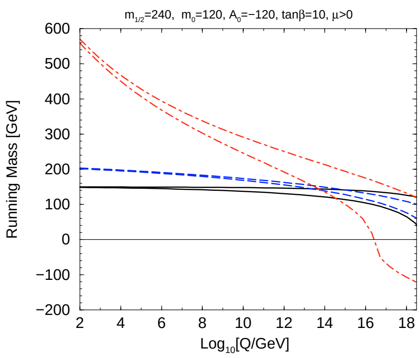
$\sqrt{s} = 500$ GeV

What can we do with the mass precision obtained at LHC and ILC?

Test possible organizing principles of SUSY breaking!

A case study (Snowmass 2001 P3 working group):

See also Allanach, Blair, Kraml, Martyn, Polesello, Porod, Zerwas, hep-ph/0403133, and earlier references therein, for similar work.



LH squarks, LH sleptons, RH sleptons

Optimistic Assumptions:

2% uncertainty in $M_{\rm gluino}$

1% uncertainty in $M_{\rm squarks}$

0.5% uncertainty in $M_{\rm sleptons}$

1% uncertainty in α_s

0% theoretical uncertainty (!)

Goal: improve this, both on theory side and experiment side

NLSP decay in Gauge-Mediated SUSY Breaking

Recall that GMSB models have the special property that the LSP is a very light Goldstino/gravitino $(\tilde{G})$. The Next-Lightest SUSY Particle (NLSP) can decay into its Standard Model partner and $\tilde{G}$.

This can completely change the SUSY signals at colliders!

In general, the NSLP can have a decay length that is microscopic, comparable to detector elements, or macroscopic:

$$\Gamma(\mathrm{NLSP} \rightarrow \mathrm{SM\,particle} + \tilde{\mathrm{G}}) \;=\; 2 \times 10^{-3}\,\kappa \left(\frac{\mathrm{M_{NLSP}}}{100\ \mathrm{GeV}}\right)^5 \left(\frac{\sqrt{\langle \mathrm{F} \rangle}}{100\ \mathrm{TeV}}\right)^{-4} \mathrm{eV}$$

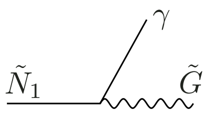
where κ is a mixing matrix factor. If the NLSP has energy E in the lab frame, its decay length will be:

$$d = 9.9 \times 10^{-3}\,\frac{1}{\kappa}\left(\frac{E^2}{M^2_{\mathrm{NLSP}}} - 1\right)^{1/2} \left(\frac{M_{\mathrm{NLSP}}}{100\ \mathrm{GeV}}\right)^{-5} \left(\frac{\sqrt{\langle F \rangle}}{100\ \mathrm{TeV}}\right)^{4} \mathrm{cm}$$

which can be anywhere from sub-micron to kilometers, depending on $\langle F \rangle$.

Neutralino NLSP in Gauge-Mediated SUSY Breaking

If the NLSP is $\tilde{N}_1$, it can have decays:



There are three general possibilities:

1) If the $\tilde{N}_1$ decays are prompt, then every SUSY event will be tagged by two additional **energetic, isolated photons**. There is still missing energy carried away by the $\tilde{G}$. Standard Model backgrounds are very small, so it is relatively easy to discover SUSY with the inclusive signal (X means “anything”)

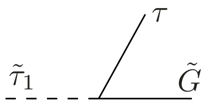
$$X + \gamma\gamma + \not{E}.$$

2) If the $\tilde{N}_1$ decays are delayed, but still occur within the detector, then one can look for photons that do not point back to the interaction vertex. This can be a striking signal, depending on the experimental environment.

3) If the $\tilde{N}_1$ decays occur outside of the detector, then the signals are the same as discussed earlier.

Stau NLSP in Gauge-Mediated SUSY Breaking

If the NLSP is the lightest stau, $\tilde{\tau}_1$, then it can have decays:

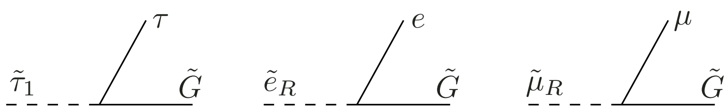


1) If the $\tilde{\tau}_1$ decays are prompt, then every SUSY event will be tagged by two **energetic, isolated τ's**.

2) If the $\tilde{\tau}_1$ decays occur outside of the detector, then one can look for slow, highly ionizing tracks as they move through the detector. These may appear to be slow "muons", or they may be missed if the timing gates do not accommodate them. They can be identified by their anomalously high ionization rate in the detector, or by their long time-of-flight.

Slepton co-NLSP in Gauge-Mediated SUSY Breaking

In GMSB models, $\tilde{\tau}_1$, $\tilde{e}_R$, $\tilde{\mu}_R$ can be nearly degenerate (to within less that $m_\tau = 1.8$ GeV. In that case, SUSY particles will decay to final states involving one of them, and they each act independently as the NLSP, with decays to $\tilde{G}$:



1) If the NLSP decays are prompt, then every SUSY event will be tagged by two **energetic, isolated leptons** (e, μ, τ) with uncorrelated flavors, and often uncorrelated charges.

2) If the NLSP decays occur outside of the detector, then one can look for slow, highly ionizing tracks, just as for the stau NLSP case.

A bold prediction: supersymmetry will be discovered in this decade.

Perhaps this is not so bold. After all, almost half of the particles have been found already!

A slightly bolder prediction: some feature of it will be completely unexpected by (nearly) everyone.

Expect the unexpected!