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### Exotic Properties of Superconductor Ferromagnet Structures

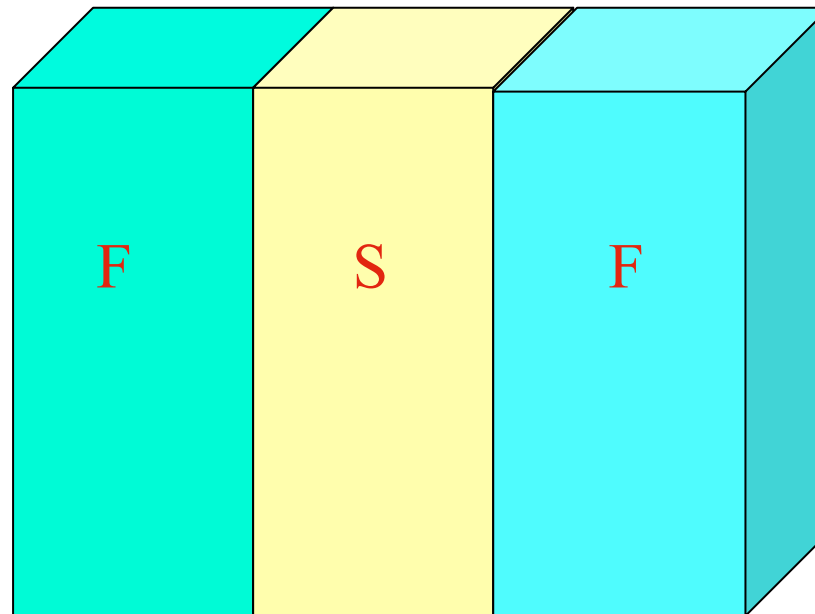
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These are preliminary lecture notes, intended only for distribution to participants

# Exotic Properties of Superconductor-Ferromagnet Structures.

F.S. Bergeret, A.F. Volkov, K.B. Efetov



S-superconductor, F-ferromagnet

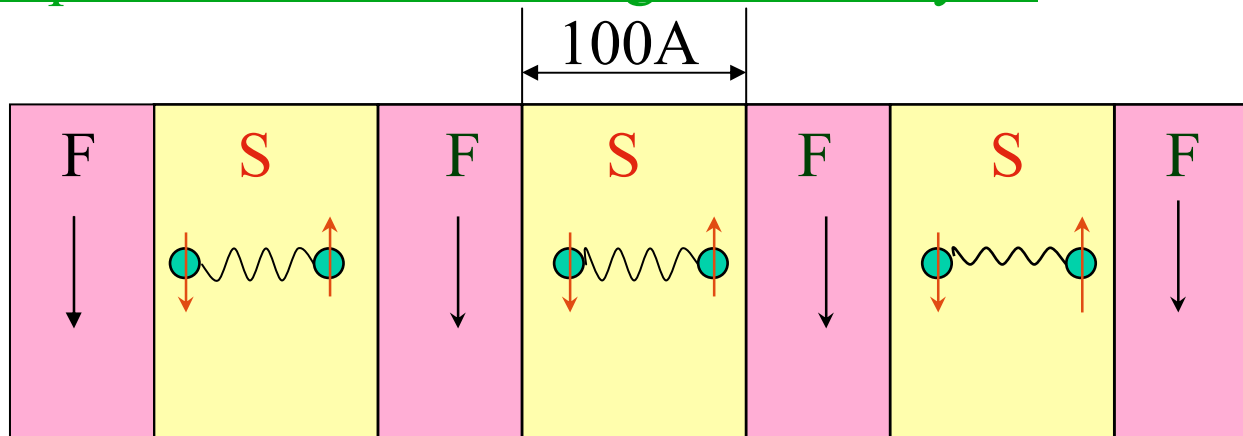
Ferromagnetism destroys superconductivity:

(Ginzburg 1956, orbital effect)

BCS Theory:

Singlet pairing is destroyed by the exchange field.

Superconductor-Ferromagnet multilayers



Due to proximity effects the superconductor and the ferromagnet act on each other!

$$T_K = 1000K$$

$$T_C = 1 - 10K$$

Zeeman splitting is most important. Effect of the magnetic field on the orbital motion is neglected.

General picture originating from several decades of the study of mutual interaction between ferromagnets and superconductors:

The ferromagnetism destroys superconductivity.

(Exception: magnetic superconductors with a weak magnetic order (Anderson and Suhl (1959), Bulaevskii, Buzdin, Kulic, Panyukov (1979))

However,

in certain situations (inhomogeneous magnetization) the conventional superconductivity is not destroyed by the strong ferromagnetism and can even be enhanced by it.

Moreover, a magnetic moment can be generated in the superconductor over long distances (inverse proximity effect).



Exotic properties!

Three different effects:

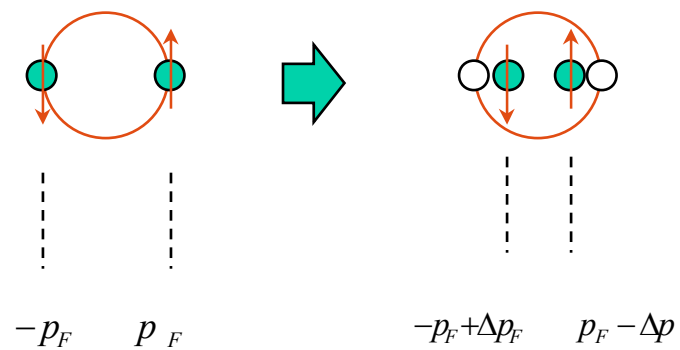
1. Generation of an odd triplet superconductivity penetrating in the ferromagnets over long distances.
2. Enhancement of the Josephson current by an exchange field.
3. Spin screening of magnetic moments in superconductors.

F.S. Bergeret, A.F. Volkov and K.B. Efetov,  
published in a number of Phys. Rev. Lett. and Phys. Rev. B,  
(2001-2004) and Rev. Mod. Phys. (accepted)

Odd triplet superconductivity

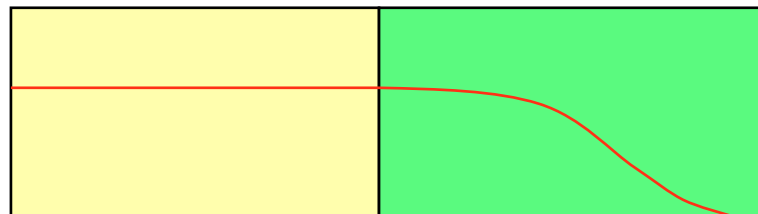
Q. What is the difference between the  
proximity effects in S/N and S/F contacts?

## Superconductor/Normal Metal



S

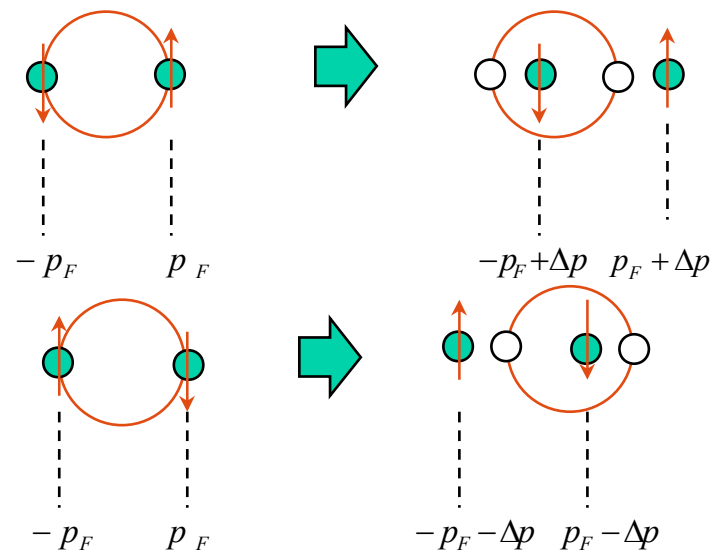
N



$$\xi_T = \sqrt{D/2\pi T}$$

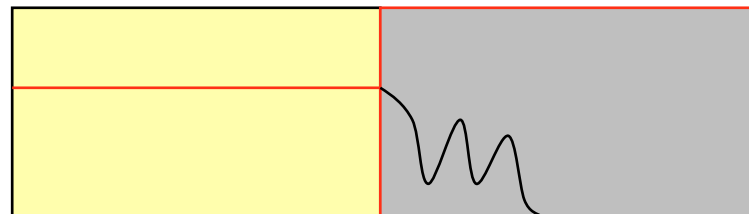
T-Temperature

## Superconductor/Ferromagnet



S

F



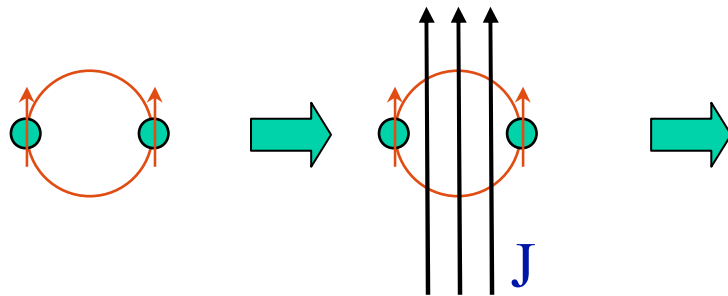
$$\xi_T \geq \xi_J$$

$$\xi_J = \sqrt{D/J}$$

J-Exchange energy

Q. Can the superconducting condensate penetrate the ferromagnet over distances exceeding  $\xi_J$  or it is absolutely impossible?

A. It can if it is a triplet one.



No destruction of Cooper pairs:  
everything is as in a normal  
metal!

How to get a triplet condensate?

It can be generated “by hand” making the magnetization of different layers non-collinear to each other!

## Known types of superconductivity in nature:

1. Singlet **s**-wave pairing (conventional, observed in traditional superconductors).
2. Triplet **p**-pairing (superfluid  $He^3$ ,  $Sr_2RuO_4$ )
3. Singlet **d**-pairing (high  $T_c$  cuprates)

Triplet pairing has been possible because the condensate function  $F$  is odd in momentum  $\longrightarrow$  no contradiction with Pauli principle.

$$F(r, t; r', t') = \langle \Psi_{\uparrow}(r, t) \Psi_{\uparrow}(r', t') \rangle$$
$$\Delta(r, r'; t) = V(r - r') F(r, t; r', t)$$

The function  $\Delta(r, r', t)$  vanishes at coinciding coordinates and times.

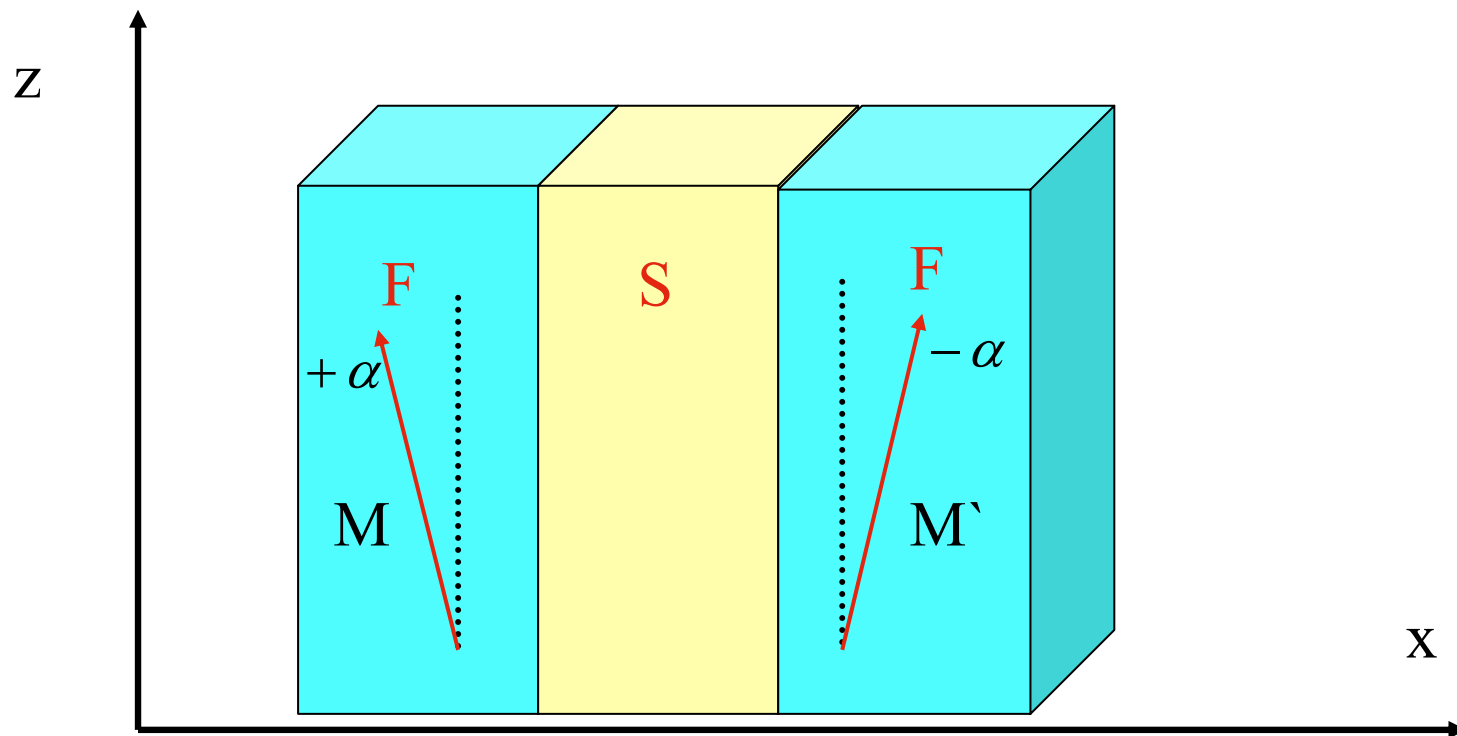
However, another proposal:

$V$  depends on time,  $\Delta(r, r', t, t')$  is odd with respect to  $t \leftrightarrow t'$  (s-wave).  
Berezinskii (1975), Balatskii, Abrahams (1992) (odd gap superconductivity).



Now: odd triplet condensate can be created in the ferromagnet (s-wave pairing, not sensitive to potential impurities). No gap, but superconductivity is possible!

Triplet condensate (the simplest structure)



$\mathbf{M}$  and  $\mathbf{M}'$  are not collinear ( $+\alpha$  and  $-\alpha$ )

## Model:

$$H = H_{BSC} + H_Z$$

$$H_{BCS} = \int \left( \sum_{\alpha=\uparrow,\downarrow} \psi_{\alpha}^{\dagger}(\mathbf{r}) (\varepsilon(-i\nabla) - \varepsilon_F) \psi_{\alpha}(\mathbf{r}) - g \psi_{\uparrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r}) \right) d\mathbf{r}$$

$$H_Z = -J \sum_{\alpha=\uparrow,\downarrow} \int \psi_{\alpha}^{\dagger}(\mathbf{r}) \mathbf{m} \cdot \boldsymbol{\sigma}_{\alpha\beta} \psi_{\beta}(\mathbf{r}) d\mathbf{r}$$

It is assumed that:

$g > 0, J = 0$  in the superconductor,  $g = 0, J > 0$  in the ferromagnet,  $\mathbf{m}$  is a unit vector directed along the magnetization.

In the main approximation one has the standard singlet coupling in the superconductor and no condensate in the ferromagnet.

However, proximity effects!  Triplet component appears.

Method of quasiclassical (4x4) Green functions:  
in the limit  $J\tau \leq 1$  Usadel equation.

$$-D\nabla_{\mathbf{R}}(\hat{g}_0\nabla_{\mathbf{R}}\hat{g}_0)+[(\omega\hat{\rho}_3-i\hat{\Delta}(\mathbf{R})+i\hat{V}_0(\mathbf{R})),\hat{g}_0(\mathbf{R},\omega)]=0$$

$$\hat{g}_0^2 = 1$$

$D$  is the classical diffusion coefficient

Normal  $g$  and anomalous  $f$  2x2 Green functions:  
equation in the ferromagnets

$$D\partial_x^2\hat{f}-2|\omega|\hat{f}+i\text{sgn}(\omega)(\hat{f}\hat{V}^*-\hat{V}\hat{f})=0$$

$$\hat{V}=J\begin{pmatrix}\cos\alpha & \pm i\sin\alpha \\ \mp i\sin\alpha & -\cos\alpha\end{pmatrix}$$

Structure of the functions  $f$ :

$$f = i\hat{\tau}_2(f_3(x)\hat{\sigma}_3 + f_0(x)) + i\hat{\tau}_1\hat{\sigma}_1f_1(x)$$

$\sigma, \tau$  -Pauli matrices  
(spin, Nambu)

$$f_3 \propto \langle \psi_{\uparrow}\psi_{\downarrow} \rangle - \langle \psi_{\downarrow}\psi_{\uparrow} \rangle$$

-Singlet condensate

$$f_0 \propto \langle \psi_{\uparrow}\psi_{\downarrow} \rangle + \langle \psi_{\downarrow}\psi_{\uparrow} \rangle$$

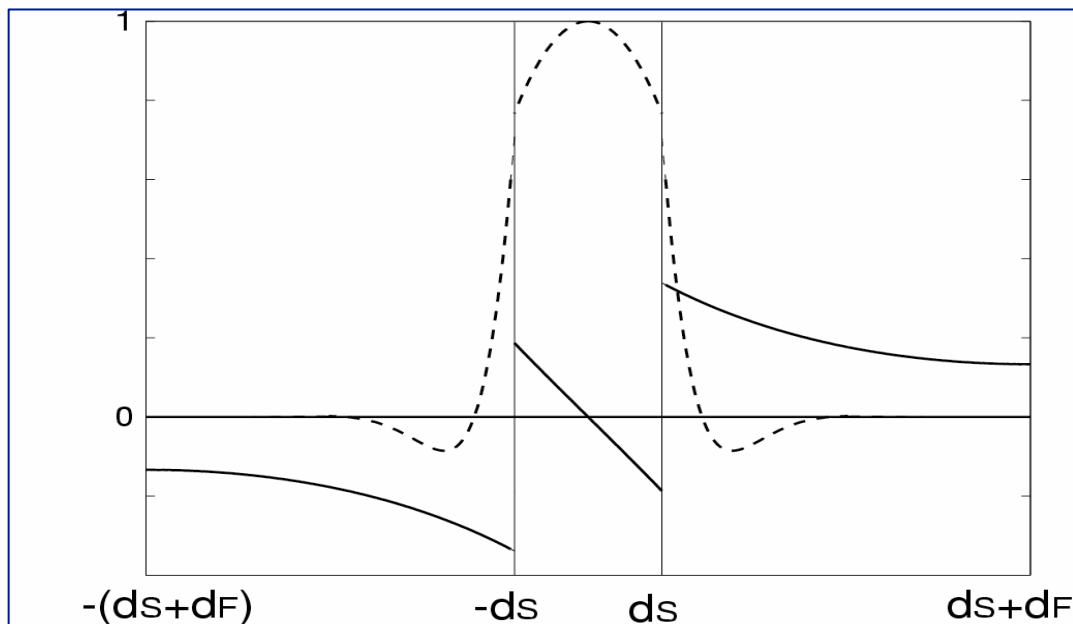
-Triplet condensate (with projection 0 on z-axis)

$$f_1 \propto \langle \psi_{\uparrow}\psi_{\uparrow} \rangle \propto \langle \psi_{\downarrow}\psi_{\downarrow} \rangle$$

-Triplet condensate (with projection +1,-1)

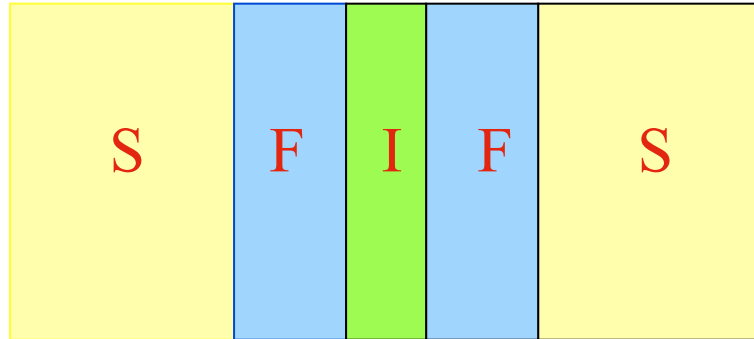
## Properties of the triplet component:

- 1) The singlet component  $f_3$  penetrates the ferromagnetic region over a short distance  $\xi_J = \sqrt{D_F/J}$  (even function in  $\omega$ , symmetric in momentum).
- 2)  $f_0$  and  $f_1$  are odd functions of  $\omega$  (odd condensate!) and are symmetric functions in the momentum space. They penetrate the ferromagnetic region over a long distance  $\xi_T = \sqrt{D_F/2\pi T}$ . At  $J \gg T$  long range penetration.
- 3) The maximum is achieved at  $\alpha = \pi/4$ . No contribution at  $\alpha = 0, \pi$ .



Spatial dependence of Im(SC) (dashed line) and Re(TC) (solid line). Only the long range part of TC is represented (which is the reason for the discontinuity).

## Enhancement of the Josephson current by an exchange field.



**Q.** When is the Josephson current maximal (the magnetic moments in F-layers are parallel, antiparallel or absent)?

**A.** The antiparallel configuration is most favorable.

$$I_J^{(p)} = \frac{\Delta^2(T)4\pi T}{eR} \times \sum_{\varepsilon} \frac{\varepsilon_n^2 + \Delta^2(T, h) - h^2}{[\varepsilon_n^2 + \Delta^2(T, h) - h^2]^2 + 4\varepsilon_n^2 h^2},$$

$$I_J^{(a)} = \frac{\Delta^2(T)4\pi T}{eR} \times \sum_{\varepsilon} \frac{1}{\sqrt{[\varepsilon_n^2 + \Delta^2(T, h) - h^2]^2 + 4\varepsilon_n^2 h^2}}$$

$$1 = \lambda\pi T \sum_{\varepsilon} \operatorname{Re} \frac{1}{\sqrt{(\varepsilon_n + ih)^2 + \Delta^2(T, h)}}.$$

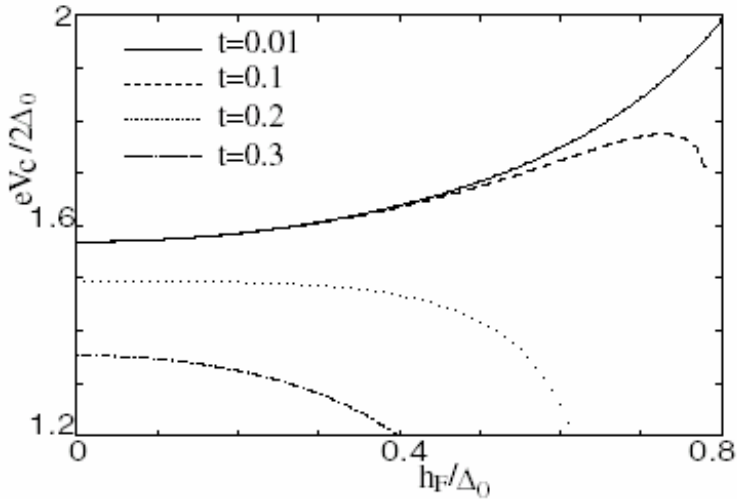


FIG. 2. Dependence of the normalized critical current on  $h$  for different temperatures in the case of an antiparallel orientation. Here  $eV_c = eRI_c$ ,  $h_F$  is the effective exchange field,  $t = T/\Delta_0$ , and  $\Delta_0$  is the superconducting order parameter at  $T = 0$  and  $h = 0$ .

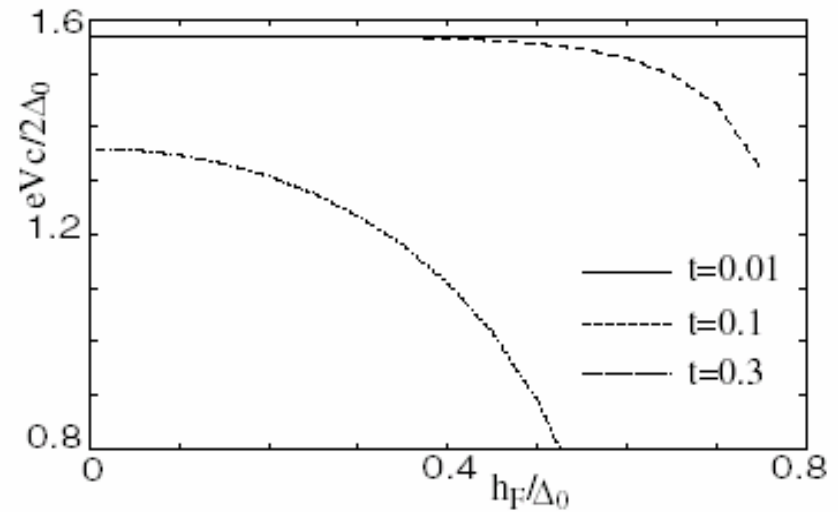
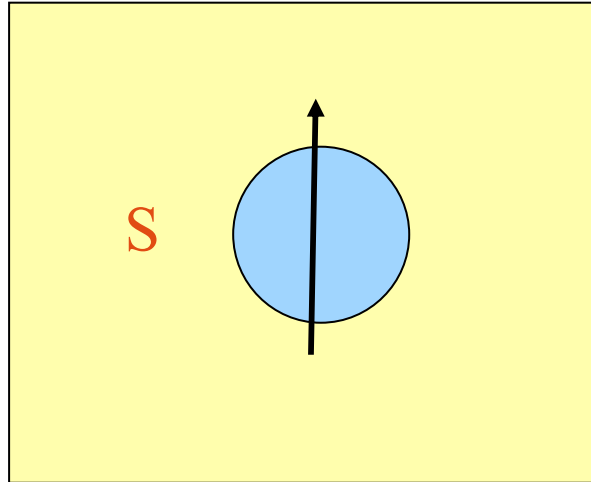


FIG. 3. The same dependence as in Fig. 2 in the case of a parallel orientation.

## Spin screening of magnetic moments in superconductors.

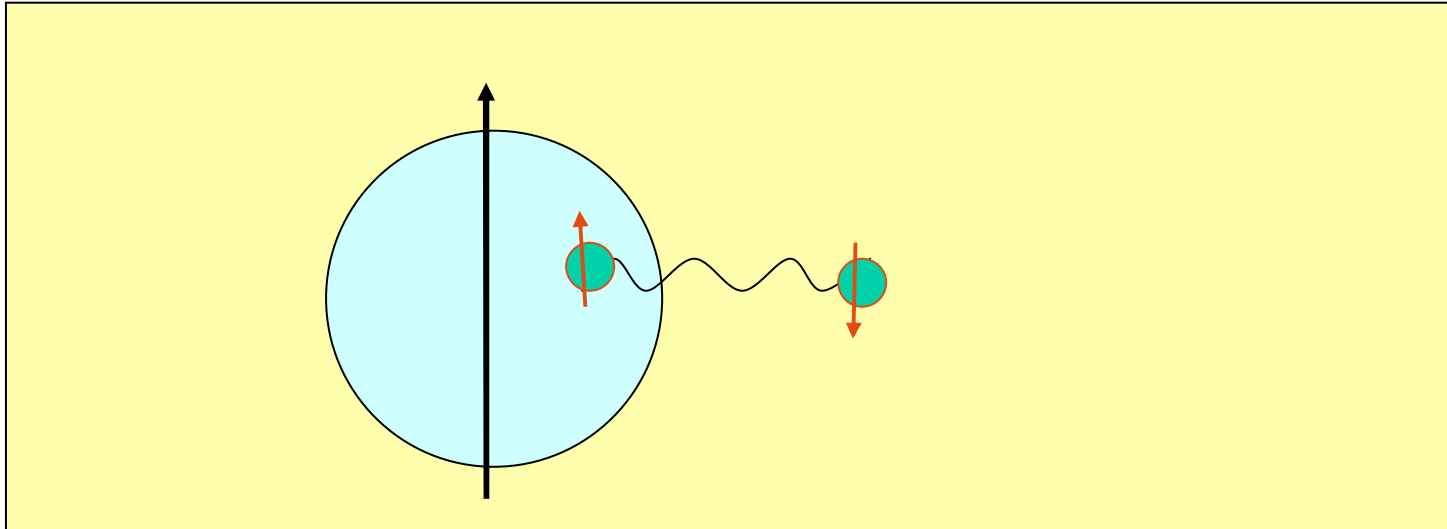


Conventional screening by the orbital electron motion in the superconductor (Meissner Effect) is well known.

**Q.** Is the screening possible if the superconductor is thin and the Meissner effect is suppressed?

Yes, it is possible due to spins of the electrons.

Qualitative picture.



The magnetic moment is induced in the superconductor over the distance of the order of the size of the Cooper pairs (inverse proximity effect).

A full screening is possible.



## Usadel Equations

$$D\nabla(\check{g}\nabla\check{g}) - \omega[\hat{\tau}_3\hat{\sigma}_0, \check{g}] + iJ[\hat{\tau}_3\hat{\sigma}_3, \check{g}] = -i[\check{\Delta}, \check{g}].$$

and boundary conditions

$$\gamma_F(\check{g}\mathbf{n}\nabla\check{g})_F = \gamma_S(\check{g}\mathbf{n}\nabla\check{g})_S; \quad \gamma_F(\check{g}\mathbf{n}\nabla\check{g})_F = -[\check{g}_S, \check{g}_F],$$

The induced magnetic moment takes the form

$$\delta M = \mu\delta \sum_p \left( \langle c_{p\uparrow}^\dagger c_{p\uparrow} - c_{p\downarrow}^\dagger c_{p\downarrow} \rangle \right) = -\mu i\pi\nu T \sum_{\omega=-\infty}^{\omega=+\infty} \text{Tr}(\hat{\sigma}_3\hat{g})/2,$$

Full screening if the transparency of the interface is high!

## Conclusions

Simple theory but interesting effects:

Conventional superconductivity is not necessarily destroyed by strong ferromagnets and non-trivial proximity effects may occur.

There are many experimental indications in favor of the considered effects.

Experimental indications for the inverse proximity effect.

I.A. Garifullin et al., Appl. Magn. Reson., 22, 439 (2002)

J. Stahn et al., Phys. Rev. B 71, 140509 (2005)