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Coulomb and Magnetic Field Effects in Nanoscale SN and SNS junctions

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Coulomb and Magnetic Field Effects in Nanoscale SN and SNS Junctions

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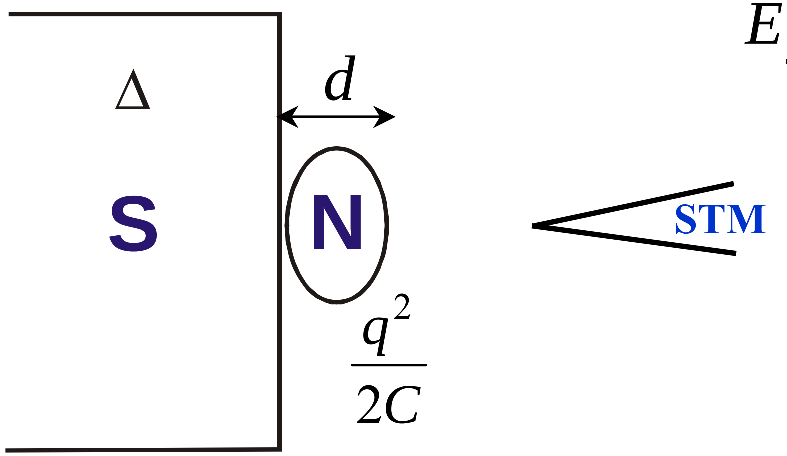
Formulation of problem

$t << 1$ - SN interface transparency

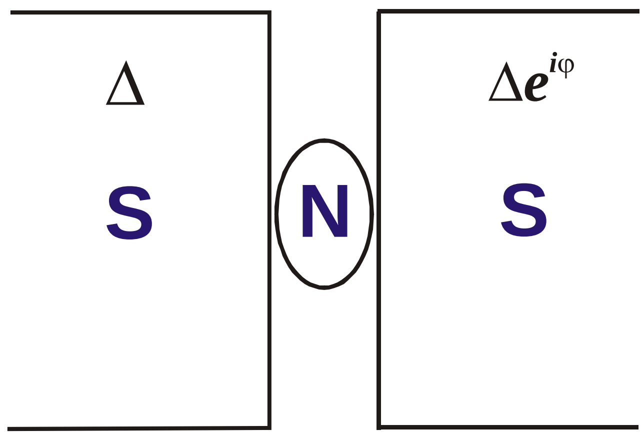
$G \sim N_{\mathrm{ch}}\, t >> 1$ - SN interface conductance in units of $e^2/\hbar$



$E_g = \dfrac{G\delta}{4}$ - minigap in the absence of Coulomb interaction

1) $\widetilde{E}_g(H) = ?$

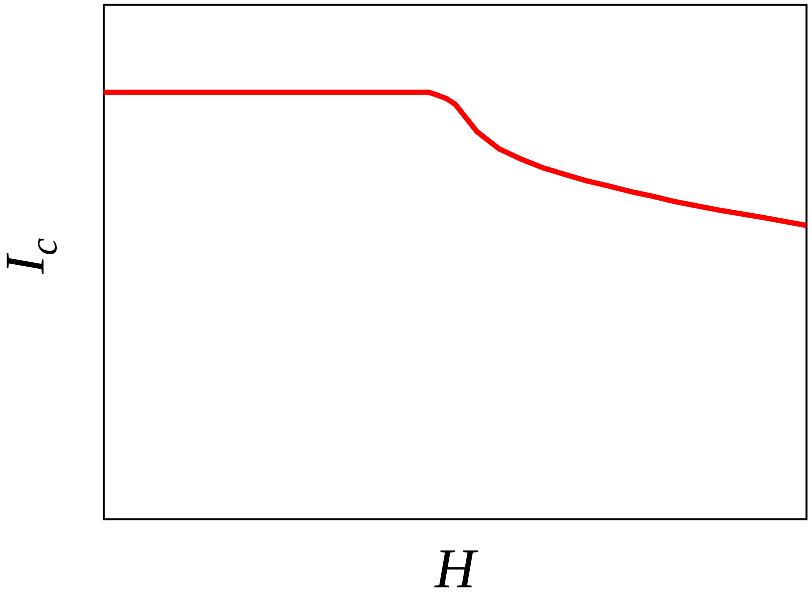
2) $E_g^{\mathrm{tun}}(H) = ?$



3) $I(\varphi)\big|_H = ?$

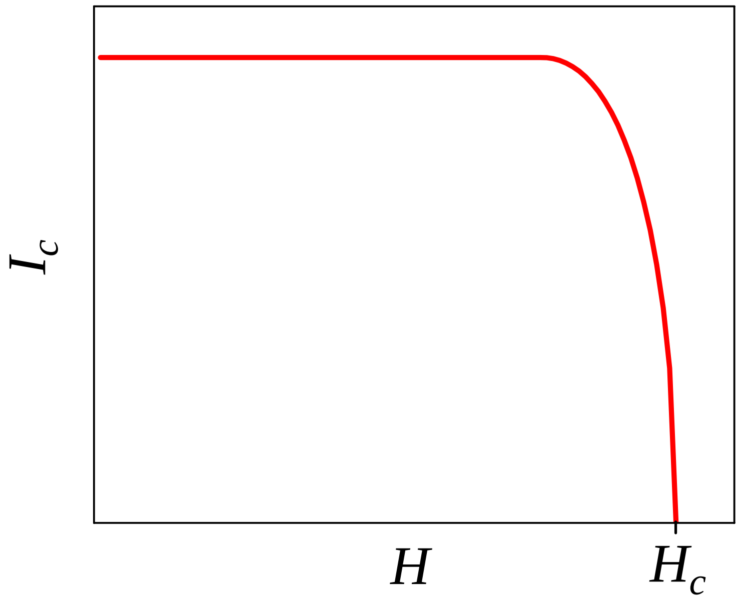
$I_c(H) = ?$

Effects of magnetic field (an example)



Weak Coulomb interaction

Strong Coulomb interaction



Parameters

$$E_{\text{Th}} > \Delta >> (E_g, E_C, H) >> \delta$$

$$E_{\text{Th}} = \frac{D}{d^2}$$ - Thouless energy

$$E_g = \frac{G\delta}{4}$$ - minigap in the absence of Coulomb interaction

$$E_C = \frac{e^2}{2C}$$ - Coulomb energy

$$\delta = \frac{1}{N_0 V}$$ - mean level spacing

Possible experimental parameters:

size of the grain $d \sim 50$ nm

interface transparency $t \sim 10^{-5}$

→

$E_C, E_J \sim 0.1 \div 1$ K

$E_g \sim 10^{-2}$ K

$G \sim 50$

Technique

Zero-dimensional replica sigma-model in Matsubara representation:

$$S = -\frac{\pi}{\delta}\mathrm{Tr}\Big[(\varepsilon + iH)\hat{\tau}_3\widetilde{Q} + \hat{\tau}_1 e^{2i\hat{\tau}_3 K(\tau)} E_g \widetilde{Q}\Big] + \int d\tau \frac{\dot{K}^2}{4E_C}$$

$K(\tau) = \int^{\tau} \phi(t)dt,$ ϕ - fluctuating electric potential of the grain

$\widetilde{Q}$ - matrix in the Nambu, energy, and replica spaces

$\widetilde{Q}_{\tau\tau'} = e^{-i\hat{\tau}_3 K(\tau)} Q_{\tau\tau'} e^{i\hat{\tau}_3 K(\tau)}$ - takes into account the shift of the whole energy band by the electric potential (Kamenev, Andreev)

In the superconductor:

$$Q_S = 2\pi\delta(\varepsilon - \varepsilon')\frac{(\varepsilon + iH)\hat{\tau}_3 + \Delta\hat{\tau}_1}{\sqrt{(\varepsilon + iH)^2 + \Delta^2}} \approx 2\pi\delta(\varepsilon - \varepsilon')\hat{\tau}_1 \qquad \text{at} \quad \varepsilon, H << \Delta$$

Physical quantities

$$\rho_{\uparrow}(E) = \frac{1}{2\delta}\,\mathrm{tr}\left(\hat{\tau}_3 \widetilde{Q}_{\varepsilon\varepsilon}\right)\Big|_{\varepsilon\to -iE}$$

- thermodynamic density of states

$$\rho_{\uparrow}^{\mathrm{tun}}(E) = \frac{1}{2\delta}\,\mathrm{tr}\left(\hat{\tau}_3 Q_{\varepsilon\varepsilon}\right)\Big|_{\varepsilon\to -iE}$$

- tunneling density of states

$$I = \frac{2e}{\hbar}\frac{dF}{d\varphi}$$

- supercurrent in SNS junction

Solution

K - fast variable

$\widetilde{Q}$ - slow variable

The dynamics of K is determined by a simple Hamiltonian:

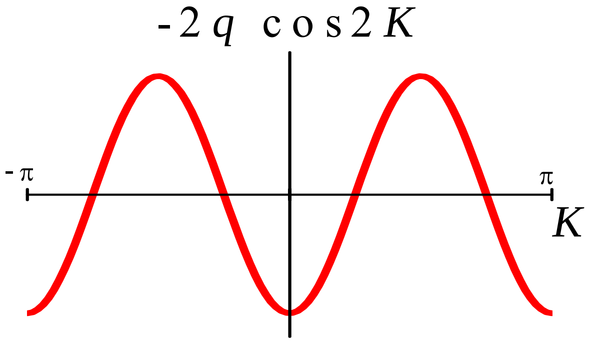
$$\hat{H} = E_C \left(-\frac{\partial^2}{\partial K^2} - 2q \cos 2K \right)$$

We need $E_0(q)$ - the ground state

$$\begin{cases} \dfrac{\widetilde{E}_g}{E_g} = -\dfrac{1}{2E_C} \dfrac{\partial E_0}{\partial q} \\ q = \dfrac{E_g \widetilde{E}_g}{E_C \delta} \log \dfrac{2\Delta}{\max(\widetilde{E}_g, H) + \sqrt{\max^2(\widetilde{E}_g, H) - \widetilde{E}_g^2}} \end{cases}$$

- self-consistent system of equations that determines the minigap $\widetilde{E}_g$

Limiting cases



$$\psi(-\pi) = \psi(\pi)$$

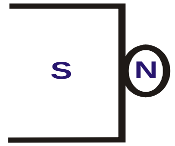
$q >> 1: \quad E_C << E_J$ - weak Coulomb interaction, $E_0(q) = -2E_C\left(q - \sqrt{q}\right)$

$q << 1: \quad E_C >> E_J$ - strong Coulomb interaction, $E_0(q) = -E_C \dfrac{q^2}{2}$

$$E_C = \frac{e^2}{2C}$$ - Coulomb energy

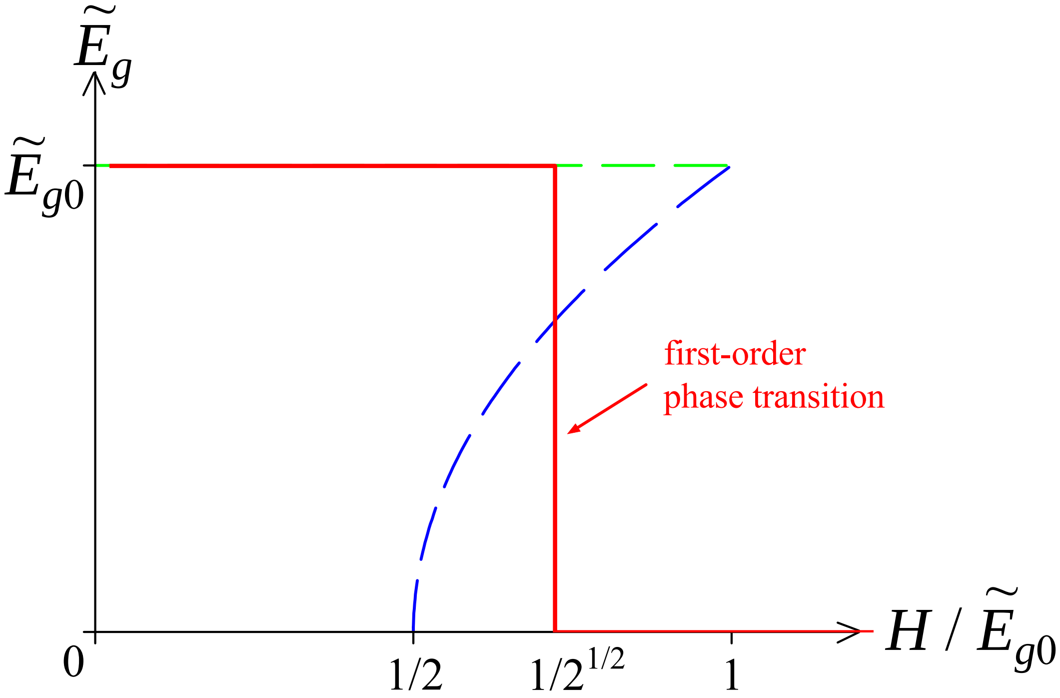
$$E_J = \frac{E_g^2}{\delta} \log \frac{2\Delta}{E_g}$$ - Josephson energy between superconductors with gaps Δ and E_g ($\Delta \gg E_g$)

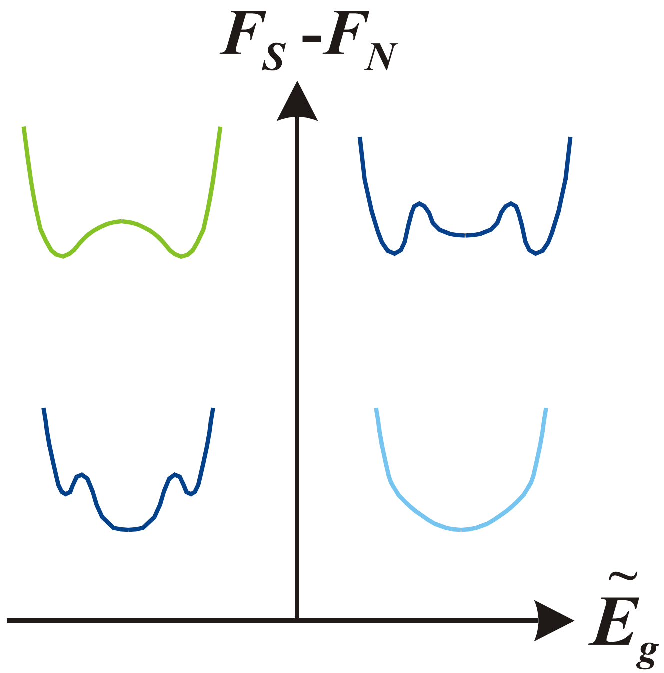
Minigap.
Strong Coulomb interaction



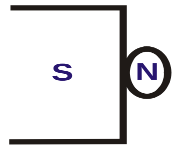
$$\tilde{E}_g \equiv \tilde{E}_{g0} = 2\Delta \exp\left(-\frac{2E_C\delta}{E_g^2} \right) \qquad \text{at} \quad H < \tilde{E}_{g0} / \sqrt{2}$$

$$\tilde{E}_g = 0 \qquad \text{at} \quad H > \tilde{E}_{g0} / \sqrt{2}$$





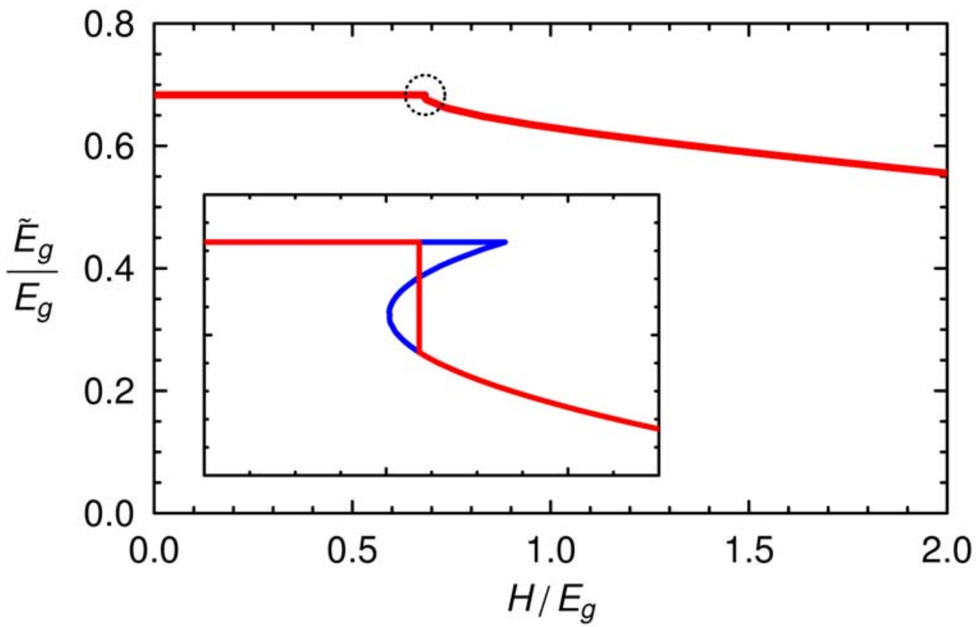
Minigap.
Weak Coulomb interaction



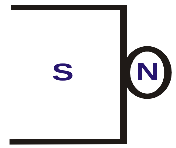
$$\tilde{E}_g = E_g - \frac{1}{2}\sqrt{\frac{E_C\delta}{\log\frac{2\Delta}{E_g}}} \qquad \text{at } H < E_g$$

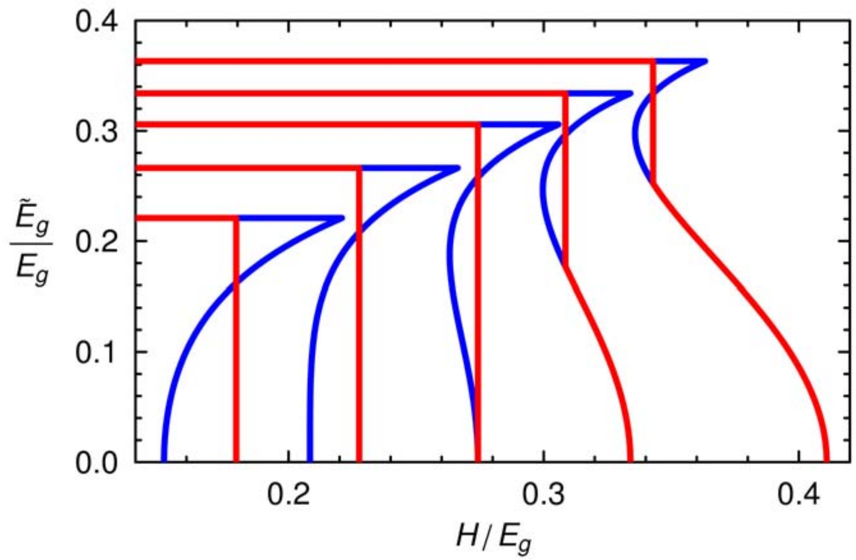
$$\tilde{E}_g = E_g - \frac{1}{2}\sqrt{\frac{E_C\delta}{\log\frac{2\Delta}{H+\sqrt{H^2-E_g^2}}}} \qquad \text{at } H > E_g \quad \text{except narrow vicinity of } H = E_g$$

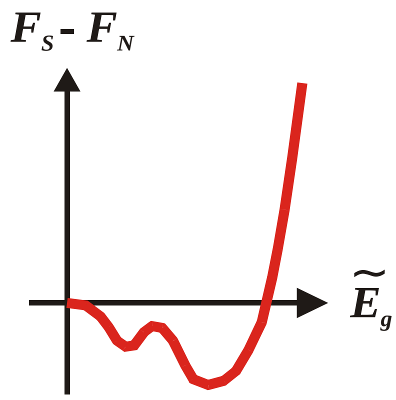
- threshold dependence on H



Minigap.
Moderate Coulomb interaction

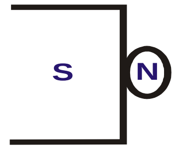


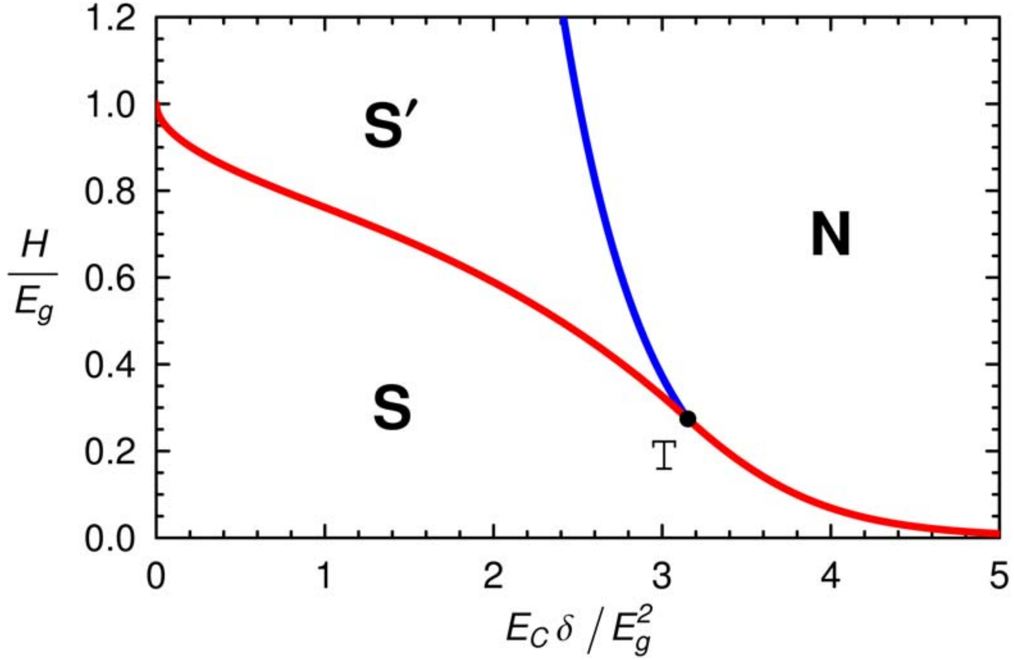




- two superconducting solutions at $\frac{7E_C\delta\Delta^2}{E_g^4}\exp\left(-\frac{4E_C\delta}{E_g^2}\right) > 1$
- first-order phase transition between them as H varies

Phase diagram of the grain



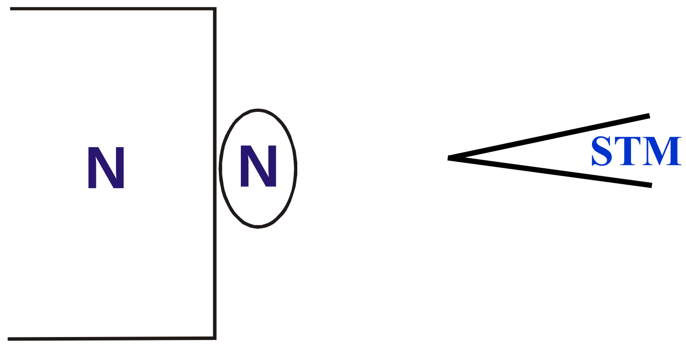


- **S** – strong gapped state (E_g does not depend on H)
- **S'** – weak gapped state (E_g suppressed by H)
- **N** – gapless state

- red line – I order phase transition
- blue line – II order phase transition

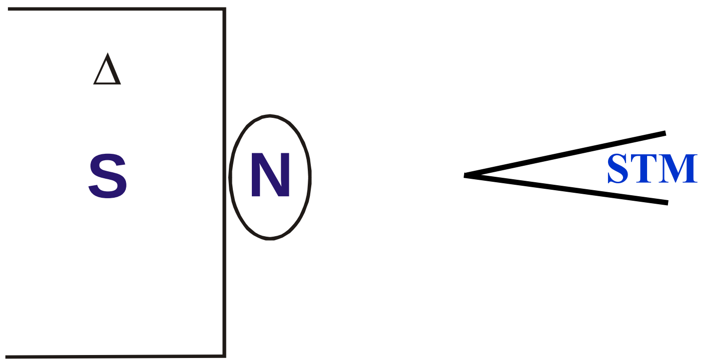
Coulomb blockade of tunneling

$$\rho^{\text{tun}}(E) \propto \frac{dI}{dV}$$ - tunneling density of states



Normal reservoir:

- Coulomb blockade at $G << 1$
- lifted at $G >> 1$

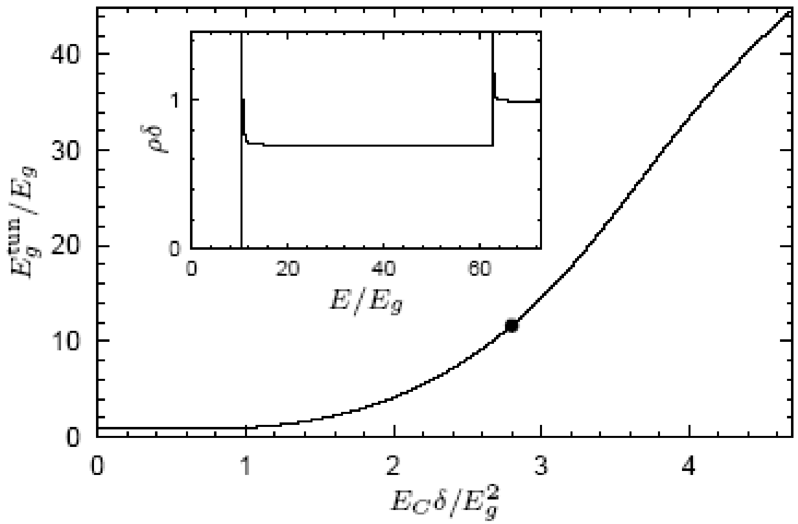


Superconducting reservoir:

- Coulomb blockade persists even at $G >> 1$

Minigap in the tunneling density of states

Without magnetic field:



$$E_g^{\rm tun} = \widetilde{E}_g + E_1 - E_0$$

Coulomb gap

With magnetic field:

$$\rho_\uparrow^{\rm tun}(E) = \frac{1}{\delta}\sum_n P_n\left[\mathrm{Re}\frac{|E - E_{n0} - H|}{\sqrt{(E - E_{n0} - H)^2 - \widetilde{E}_g^2}}\theta(E - E_{n0}) + \mathrm{Re}\frac{|E + E_{n0} - H|}{\sqrt{(E + E_{n0} - H)^2 - \widetilde{E}_g^2}}\theta(-E - E_{n0})\right]$$

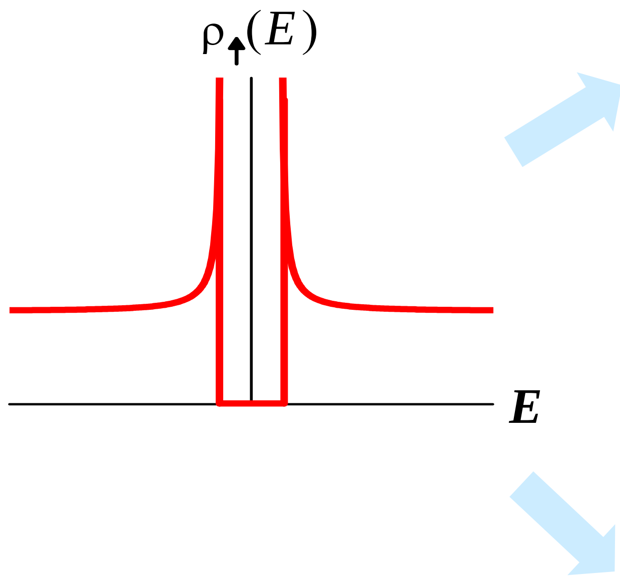
$$P_n = \left|\langle 0|\cos K|n\rangle\right|^2 + \left|\langle 0|\sin K|n\rangle\right|^2, \quad E_{n0} = E_n - E_0$$

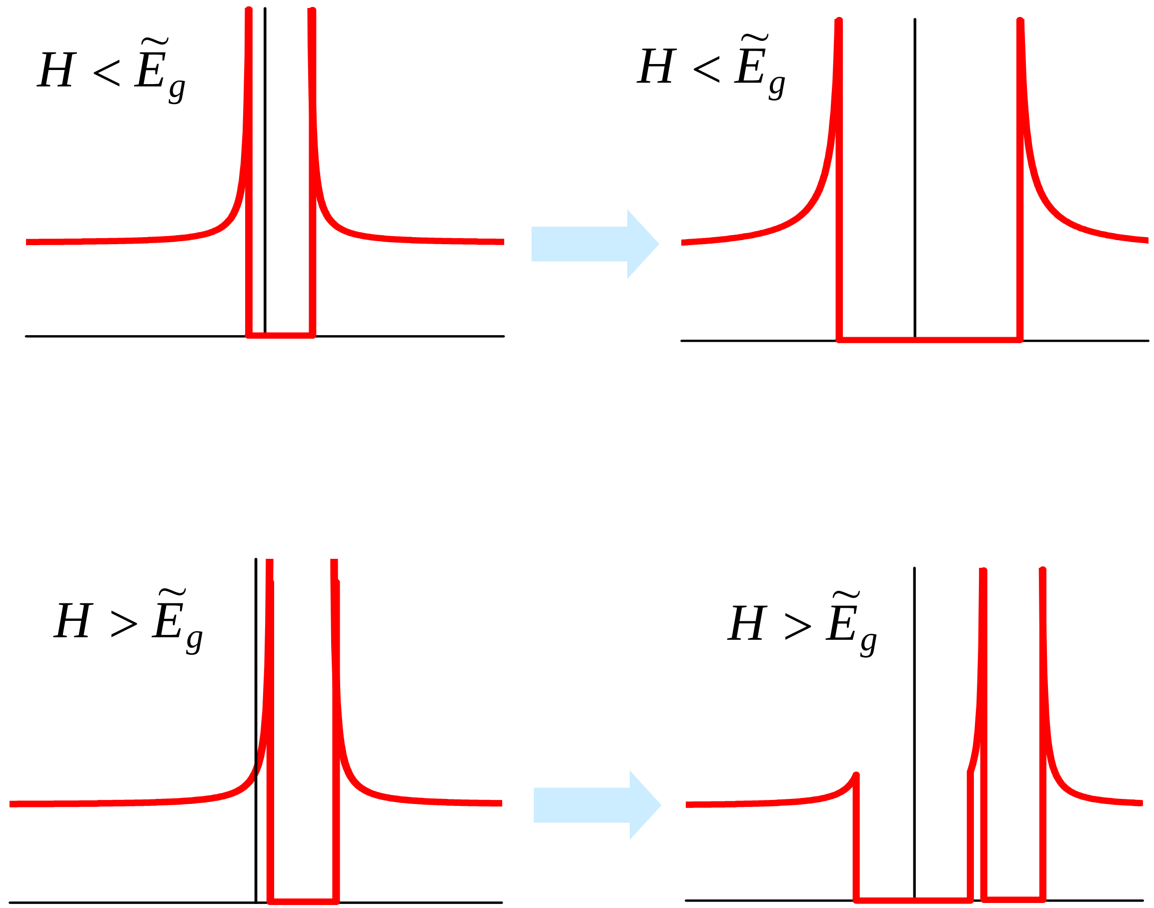
Tunneling DoS in the presence of magnetic field *H*

2) Shift by H:

3) Coulomb gap around $E = 0$:

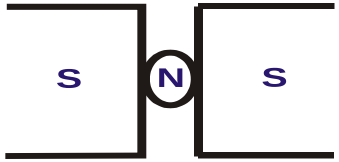
1) Thermodynamic minigap $\widetilde{E}_g$:





$$\rho_{\text{total}}(E) = \rho_{\uparrow}(E) + \rho_{\downarrow}(E), \qquad \rho_{\downarrow}(E) = \rho_{\uparrow}(-E)$$

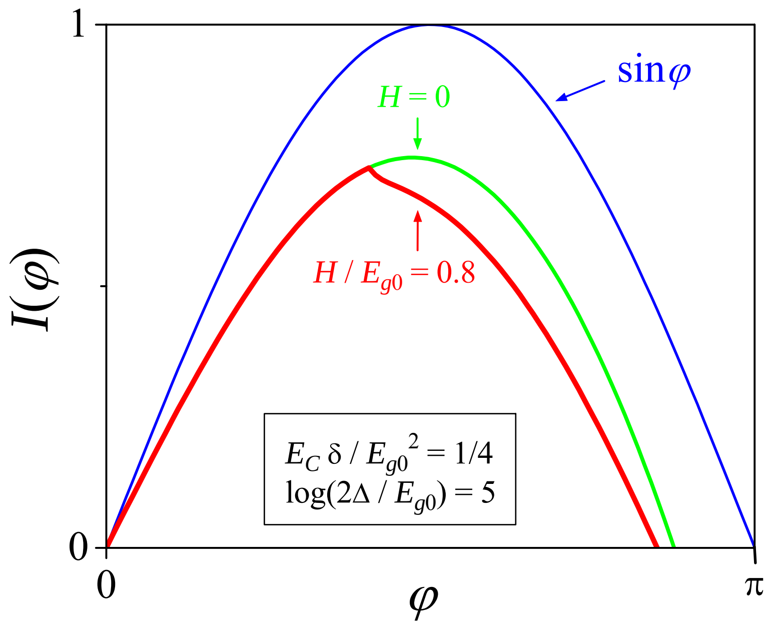
Josephson effect.
Weak Coulomb interaction

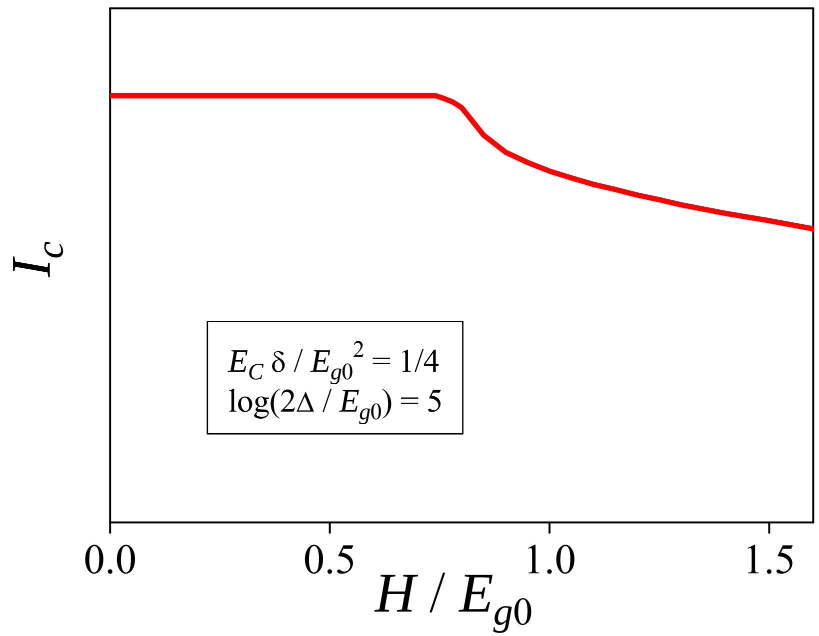


$$E_g(\varphi) = E_{g0}\cos(\varphi/2)$$

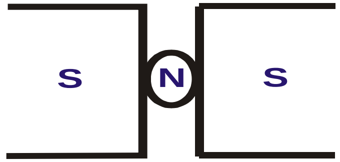
$$I(\varphi) = \frac{e}{\hbar}\frac{E_{g0}^2}{\delta}\log\frac{2\Delta}{E_g(\varphi)}\left(1 - \frac{1}{\cos\frac{\varphi}{2}}\sqrt{\frac{E_C\delta}{E_{g0}^2\log\frac{2\Delta}{E_g(\varphi)}}}\right)\sin\varphi \qquad \text{at } H < E_g(\varphi)$$

$$I(\varphi) = \frac{e}{\hbar}\frac{E_{g0}^2}{\delta}\log\frac{2\Delta}{H+\sqrt{H^2-E_g^2(\varphi)}}\left(1 - \frac{1}{\cos\frac{\varphi}{2}}\sqrt{\frac{E_C\delta}{E_{g0}^2\log\frac{2\Delta}{H+\sqrt{H^2-E_g^2(\varphi)}}}}\right)\sin\varphi \qquad \text{at } H > E_g(\varphi)$$



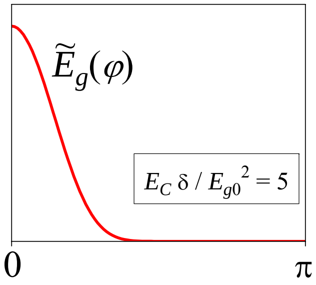


Josephson effect.
Strong Coulomb interaction



$$\tilde{E}_g(\varphi) = 2\Delta \exp\left(-\frac{2E_C\delta}{E_{g0}^2\cos^2(\varphi/2)}\right)$$

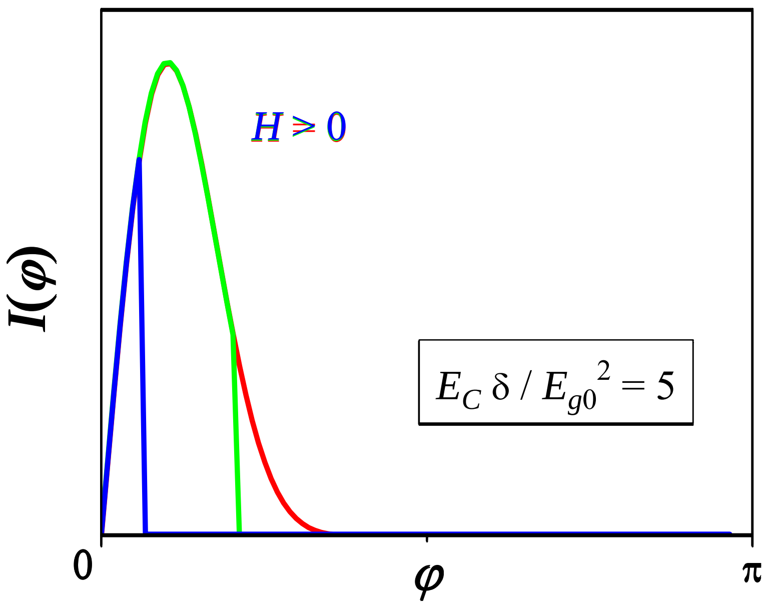
- minigap depends on the phase difference

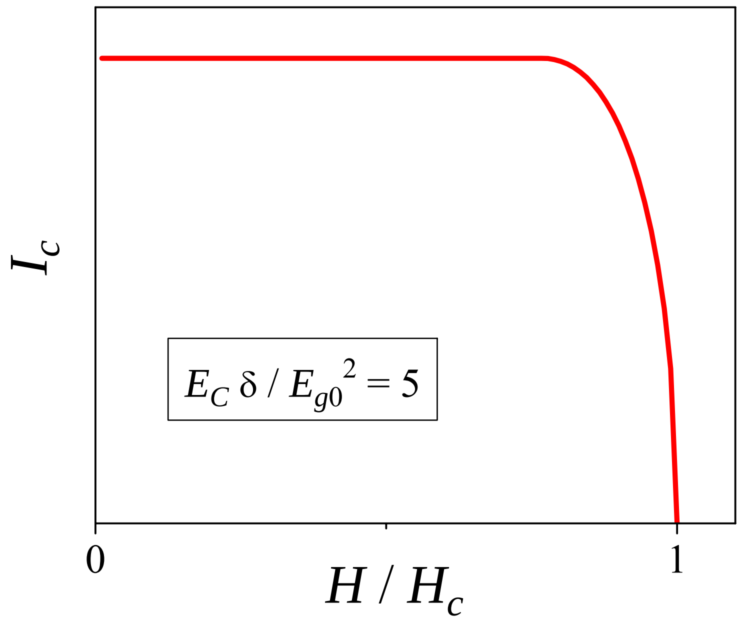


$$I(\varphi) = \frac{e}{\hbar}\frac{8\Delta^2 E_C}{E_{g0}^2}\frac{\sin\varphi}{\cos^4(\varphi/2)}\exp\left(-\frac{4E_C\delta}{E_{g0}^2\cos^2(\varphi/2)}\right) \qquad \text{at } H < \frac{\tilde{E}_g(\varphi)}{\sqrt{2}}$$

$$I(\varphi) = 0 \qquad \text{at } H > \frac{\tilde{E}_g(\varphi)}{\sqrt{2}}$$

I_c is reached at $\varphi_c = \sqrt{\dfrac{E_{g0}^2}{2E_C\delta}} << \pi$





Conclusions

Proximity and Josephson effects in the presence of Coulomb interaction and magnetic field

- **Weak Coulomb interaction:**
  - **Weak dependence of the minigap and the critical current on the magnetic field**
  - **Two superconducting states; first-order phase transition between them**
- **Strong Coulomb interaction:**
  - **Suppression of the induced superconductivity in the grain by weak magnetic field ($H << \Delta$); first-order phase transition into the normal state**
  - **Minigap vanishes abruptly**
  - **Steep drop of $I_c(H)$**
- **Moderate Coulomb interaction:**
  - **Two superconducting states; first-order phase transition between them**

The form of the $E_g(H)$ or $I_c(H)$ dependence demonstrates the strength of the Coulomb interaction