The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organizatio

International Atomic Energy Agency

Summer School and Conference on Poisson Geometry

Trieste - Italy, 4 - 22 July 2005

ABSTRACTS

A. Alekseev
University of Geneva
Title: Ginzburg-Weinstein via Gelfand-Zeitlin

Abstract: Let U(n) be the unitary group, and $u(n)^*$ the dual of its Lie algebra, equipped with the Kirillov Poisson structure. In their 1983 paper, Guillemin-Sternberg introduced a densely defined Hamiltonian action of a torus of dimension $(n-1)n/2$ on $u(n)^*$, with moment map given by the Gelfand-Zeitlin coordinates. A few years later, Flaschka-Ratiu described a similar, `multiplicative' Gelfand-Zeitlin system for the Poisson Lie group $U(n)^*$. By the Ginzburg-Weinstein theorem, $U(n)^*$ is isomorphic to $u(n)^*$ as a Poisson manifold. Flaschka-Ratiu conjectured that one can choose the Ginzburg-Weinstein diffeomorphism in such a way that it intertwines the linear and nonlinear Gelfand-Zeitlin systems. Our main result gives a proof of this conjecture, and produces a canonical Ginzburg-Weinstein diffeomorphism.

The talk is based on the joint work with E. Meinrenken, math.DG/0506112.

M. Boucetta
Title: On the Riemann-Poisson manifolds

Abstract: A Riemann-Poisson manifold is a Riemannian manifold endowed with a Poisson tensor parallel with respect to the contravariant Levi-Civita connection associated with the metric and the Poisson tensor. The aim of this talk is to introduce the notion of Riemann-Poisson manifold, to give some general properties of such structures. Many examples will be given in order to show the interest of this notion.

Rui L. Fernandes
Title: Stability of symplectic leaves

Abstract: Let M be manifold and S a compact symplectic leaf of some Poisson structure π on M. We give a criterion to decide if S is a stable leaf, i.e., if every nearby Poisson structure has a nearby symplectic leaf diffeomorphic to S. A similar result holds for Lie algebroids and we will explain the relationship between these two results.
This is joint work with Marius Crainic.

Vladimir Fock

Title: Poisson structure and quantization of cluster varieties.

Abstract: Cluster varieties is a class of varieties admitting certain discrete set of birational parameterizations by algebraic tori with prescribed transition functions. Among such varieties there are Lie groups, grassmanians, character varieties of 2D Riemann surfaces, Teichmueller spaces, certain configuration spaces and many others. Cluster approach to these varieties allows to describe explicitly and in a unified way such structures as the canonical Poisson structure as well as its quantization, canonical discrete group action respecting all these structure, tropicalisation, canonical bases of functions and many others.
The talk is based on joint work with A.B.Goncharov.

Victor Guillemin

Title: The notion of moment map for families of symplectomorphisms.

Abstract: As Alan Weinstein observed in his article "Symplectic geometry" (Bull. AMS 1981, 1-13), the moment map associated with the Hamiltonian action of a Lie group defines a "moment Lagrangian" whose properties encode many of the main features of moment geometry. In my talk I'll describe an analogous object for families of symplectomorphisms (i.e. no groups.)

ZJ Liu

Title: On (co-)morphisms of Lie pseudoalgebras and groupoids
(joint work with Z. Chen)

Abstract: A unified description of morphisms and comorphisms of Lie pseudoalgebras is given by showing that both of their graphs can be realized as Lie subpseudoalgebras of a Lie pseudoalgebra, which is named the ψ-sum defined in this paper. We also provide similar descriptions for the morphisms and the comorphisms of Lie algebroids and groupoids. It will be seen that the three versions of our main theorems express actually a same fact from different points of view (algebraic, geometric and global versions respectively).

Jiang-Hua Lu

Title: Examples of Poisson varieties associated to semi-simple groups

Abstract: We will describe a class of Poisson varieties with the actions by two groups G_1 and G_2 such that all non-empty intersections of G_1 and G_2 orbits are regular Poisson subvarieties. This is joint work with M. Yakimov.

Yoshiaki Maeda

Title: Deformation quantizations and gerbes

Abstract: We will present how an example of gerbes appears naturally from the deformation quantization. We will also give some basic examples of gerbes which seems toy models of our construction of the gerbes of star exponential functions of quadratics in the Weyl algebra. We also will propose a new geometric object, which we would like to call "pile" , by introducing flat connections on the gerbes.

A. Odzijewicz

Title: Noncommutative Kahler-like structures in quantization

Abstract: There are complementary methods of mathematical description od quantum physical systems. These are the algebraic methods based on the theory of C^*-algebras and the geometric methods that find an elegant presentation as Kostant-Souriau quantization and $*-$product quantization.

In our approach we take an effort to unify both these methods by use of the notion of coherent states map. The coherent states map $\K$ means a symplectic map of the classical phase space M into quantum phase space, i.e. complex projective Hilbert space $\CP(\M)$. It appears that using the coherent states map one can unify, in some sense, the classical and quantum description of the considered physical system.

Bearing in mind the above fact we introduce the notion of a polarized algebra of observables $\A$, which is univocally determined by coherent states map $\K$. In the case when $\K$ is Gaussian coherent states map of linear phase space $\R^{2N}$ into $\CP(\M)$, the algebra $\A$ is Heisenberg-Weyl algebra. So the C^*-algebra $\A$ is a natural generalization of the latter one to the case of a general phase space M. We prove some important properties of $\A$ and explain the relation of the structure of $\A$ to the structures such as e.g. prequantum line bundle and polarization which play a crucial role in Kostant-Souriau quantization.

As a result one can distinguish additional structures on C^*-algebra $\A$ that are responsible for the symplectic (K\"ahler in the special case) structures of classical phase space M. This structure denotes the existence of a commutative Banach subalgebra $\overline\P$ of $\A$ which has physical interpretation as the algebra of annihilation operators. Its classical counterpart is the polarization in the sense of Kostant-Souriau quantization. So, it is natural to understand $\overline\P$ as the quantum polarization and call $(\A,\overline\P)$ the polarized C^*-algebras or quantum K\"ahler manifold.

We introduce the notion of an abstract coherent state on $(\A,\overline\P)$. The coherent states in this sense generalize the notion of vacuum to the case of a general phase space. On the another hand using the coherent states one can study the algebra $\A$ by reducing many problems to the investigation of its polarization $\overline\P$, which is more handy because of commutativity.

Also we show the fundamental properties of coherent states: on $\A$ they can be considered as classical states of some classical phase space being subspace of space of multiplicative functional on the polarization $\overline\P$. Additionally, when we apply the GNS construction to the coherent states we obtain Hilbert space which is a generalization of Hardy space, which is exactly obtained when $M=\D$ is a unit disc in $\C$ and $\A$ is Toeplitz algebra.

We show how to reconstruct from $(\A,\overline\P)$ the classical phase space M and coherent states map $\K$. This gives rise to the method of reconstruction of the classical mechanics picture for the quantum one.

Norbert Poncin

Title: Automorphisms and derivations of quantum and classical Poisson algebras

Abstract: Algebraic characterizations of topological spaces and manifolds go back to the Gel'fand-Neumark theory. The classical result of Pursell and Shanks, which states that the Lie algebra of smooth vector fields over a smooth manifold characterizes the smooth structure of the variety, is the starting point of a multitude of papers. Many types of Lie algebras of vector fields have been considered. Let us mention the Lie algebra of vector fields preserving a given generalized foliation, the Lie algebras of Hamiltonian vector fields or Poisson brackets of functions, of Jacobi brackets in general, Lie algebras of vector fields on orbit spaces and G-manifolds, Lie algebras of vector fields on affine and toric varieties, Lie algebroids, ...

The first objective of our joint works with J. Grabowski is to prove that the Lie algebra ${\cal D}(M)$ of all linear differential operators, its Lie-subalgebra ${\cal D}^1(M)$ of all first order differential operators, and the Poisson-Lie algebra ${\cal S}(M)$ of all symmetric contravariant tensors (the symbols of the preceding operators) over a smooth, a real-analytic or holomorphic manifold M, characterize the structure of M.

Our algebraic approach to these problems leads to the definition of ‘quantum Poisson algebras’ and ‘classical Poisson algebras’. We show that if two (quantum or classical) Poisson algebras are isomorphic as Lie algebras, their "basic algebras of functions" are isomorphic as associative algebras---an algebraic Shanks-Pursell type result, which implies the aforementioned initial goal.

We also provide an explicit description of all automorphisms and derivations of the infinite-dimensional algebras ${\cal D}^1(M)$, ${\cal S}(M)$, and ${\cal D}(M)$. The problem of distinguishing those derivations that generate one-parameter groups of automorphisms and describing these groups will also be solved.

Parts of our proofs are based upon canonical and equivariant quantization. We will briefly present these techniques and compare their efficiency as computing devices.

Martin Schlichenmaier, University of Luxembourg
Berezin-Toeplitz quantization of the moduli space of flat SU(N) connections

Abstract: As was shown by Bordemann, Meinrenken, and Schlichenmaier the Berezin-Toeplitz operator quantization and its associated star product give a unique natural quantization for a quantisable compact K¨ahler manifold. This procedure is applied for the moduli space of gauge equivalence classes of SU(N) connections on a fixed Riemann surface. In this context the Verlinde spaces and the Verlinde bundle over Teichm¨uller space show up. As it is well-known these moduli spaces can also be described as the moduli spaces of stable rang N algebraic bundles. Recent results of J. Andersen on the asymptotic faithfulness of the representation of the mapping class group on the space of covariantly constant sections of the Verlinde bundle are presented.

Yuri Vorobiev
University of Sonora &
Moscow Institute of Electronics and Mathematics
Title: Symplectic Leaf Neighborhood Theorems

Abstract: We discuss some criteria of Poisson equivalence over a symplectic leaf which are based on a homotopy argument for coupling Poisson structures. We also derive necessary and sufficient conditions for the linearizability of a Poisson structure at a (singular) symplectic leaf in the case when the transverse Lie algebra of the leaf is semisimple of compact type. This semilocal generalization of Conn's Linearization Theorem shows that, in general, there is a cohomological obstruction to the Poisson linearizability at a symplectic leaf of nonzero dimension.

Alan Weinstein
Title: Poisson geometry and deformation quantization near a pseudoconvex boundary
(Joint work with Eric Leichtnam and Xiang Tang)

Abstract: Let X be a complex manifold with strongly pseudoconvex boundary M. If ψ is a defining function for M, then $-\log\psi$ is plurisubharmonic in a neighborhood of X, and the 2-form $\sigma = i \del \delbar(-\log \psi)$ is the symplectic form associated to a K\"ahler structure on the complement of M in a neighborhood in X of M; it blows up along M.

We show that the Poisson structure obtained by inverting σ extends smoothly across M and that, up to isomorphism near M, it is completely determined by the contact structure on M when M is compact. We also study the boundary behavior of the ``Berezin-Toeplitz" deformation quantization attached to the K\"ahler structure. The proofs use a complex Lie algebroid determined by the CR structure on M, along with some ideas of Epstein, Melrose, and Mendoza concerning manifolds with contact boundary.

Henrique Bursztyn

Title: Quasi-Poisson theory via Dirac geometry

Abstract: I will discuss various aspects of the (twisted) Dirac geometry underlying the theory of quasi-Poisson actions and D/G-valued moment maps.

Ping Xu

Title: Some remarks on generalized complex structures

Abstract: We discuss two aspects of generalized complex structures from the viewpoint of Poisson geometry.

(1) We study the reduction of generalized complex structures generalizing the Marsden-Weinstein reduction in symplectic case and the complex structure reduction of a Kahler manifold as studied by Guillemin-Sternberg, and Hitchin et. al.

(2) We study the relation between generalized complex structures and Poisson-Nijenhuis structures.

This is a joint work with Mathieu Stienon.
