The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR 1666 - 2

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Quantum Phase Transitions, Strongly Interacting Systems, and Cold Atoms

Presented by:

Eugene Demler

Physics Department, Harvard University

Quantum Phase Transitions, Strongly Interacting Systems, and Cold Atoms

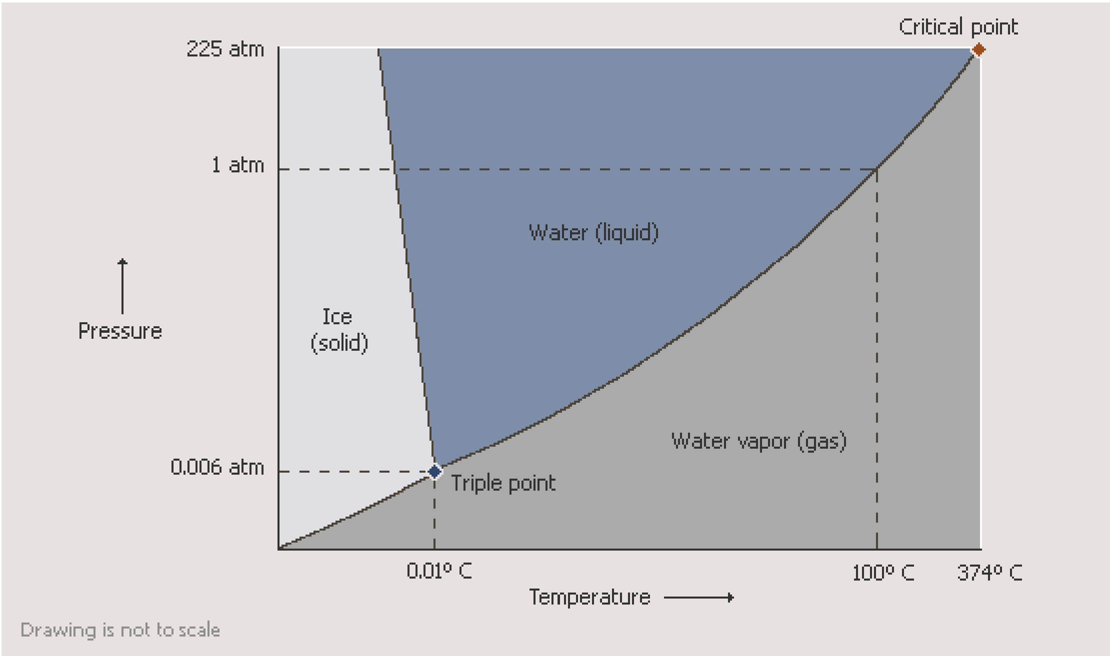
Eugene Demler

Physics Department, Harvard University

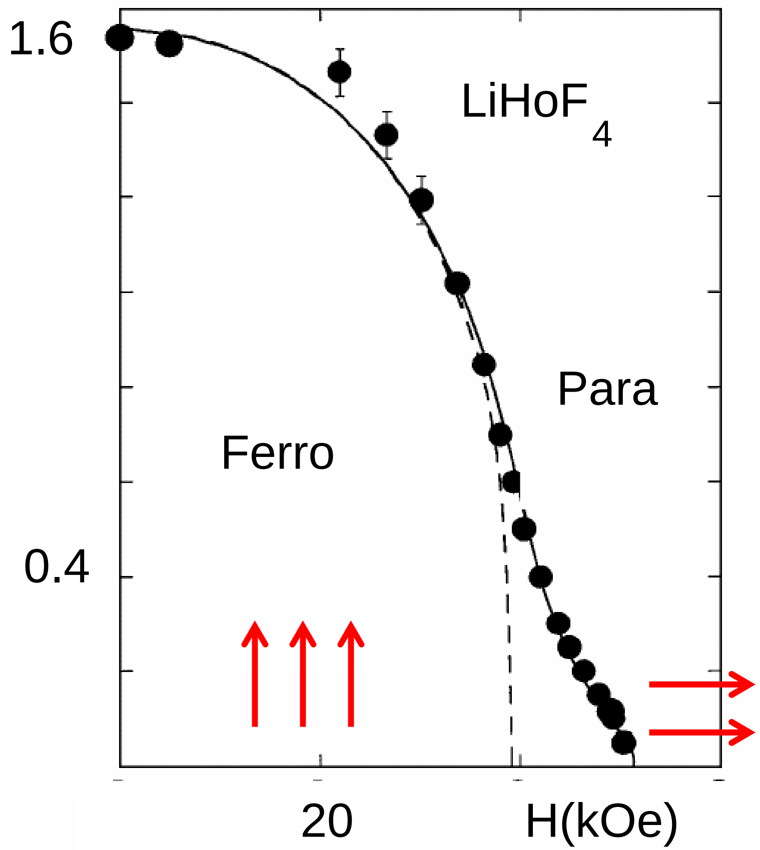
Collaborators:

Ehud Altman, Ignacio Cirac, Bert Halperin, Walter Hofstetter, Adilet Imambekov, Ludwig Mathey, Mikhail Lukin, Anatoli Polkovnikov, Anders Sorensen, Charles Wang, Fei Zhou, Peter Zoller

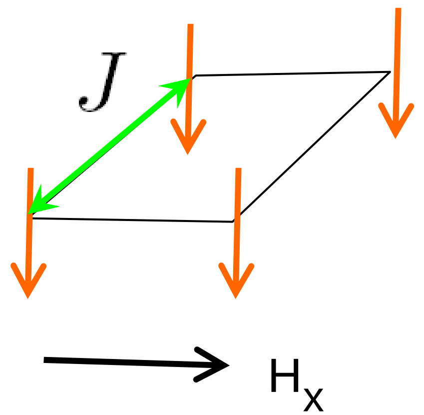
Classical Phase transitions: Phase Diagram for Water



Ising Model in Transverse Field

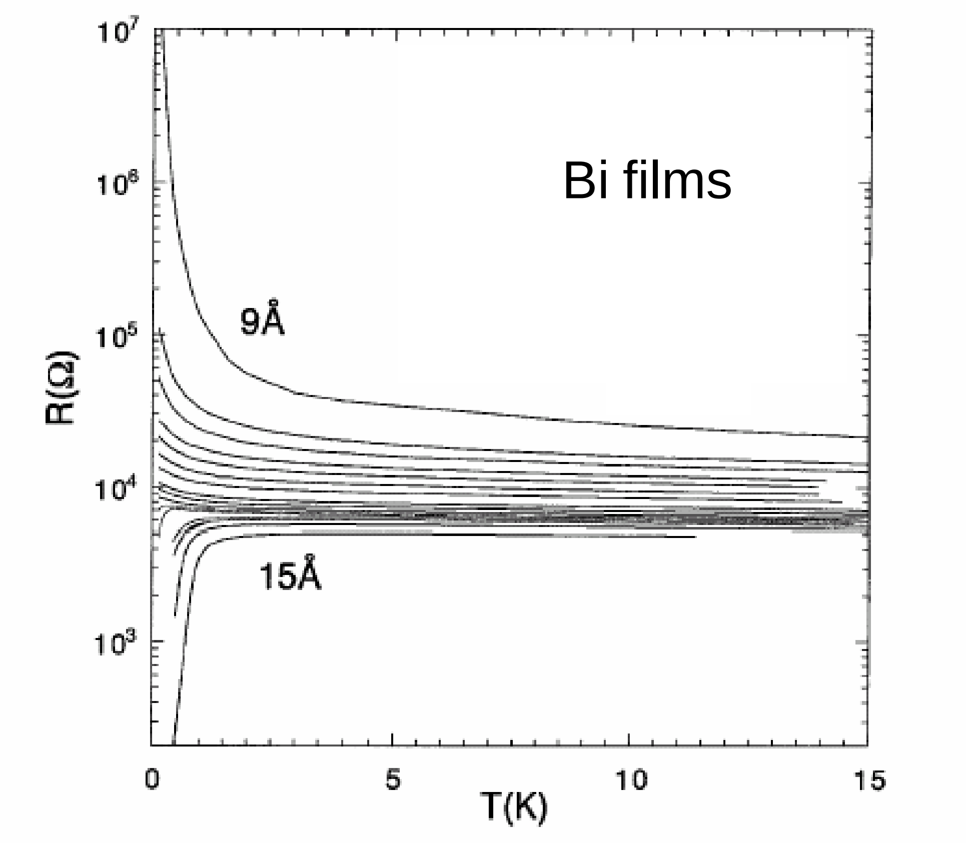


Bitko et al., PRL 77:940 (1996)



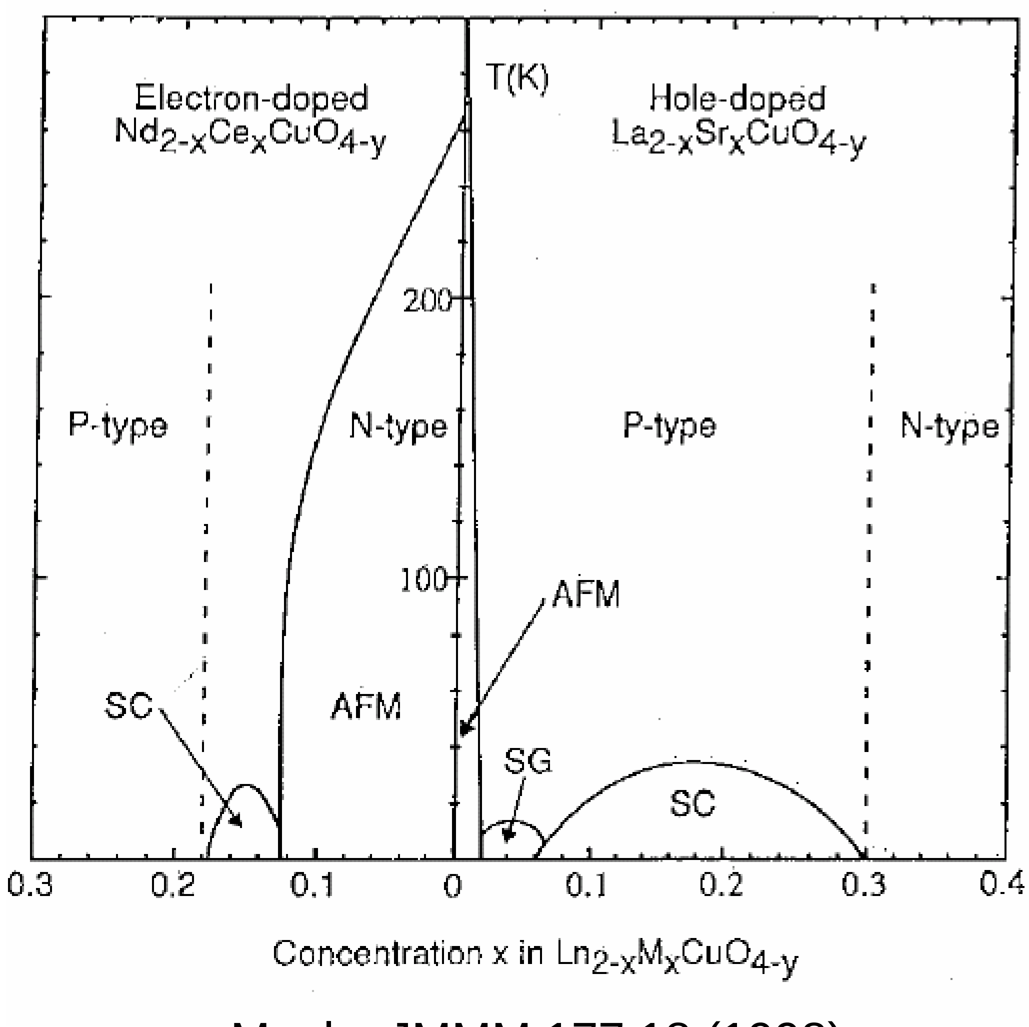
$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i^z S_j^z - H_x \sum_i S_i^x$$

Superconductor to Insulator Transition in Thin Films



Marcovic et al., PRL 81:5217 (1998)

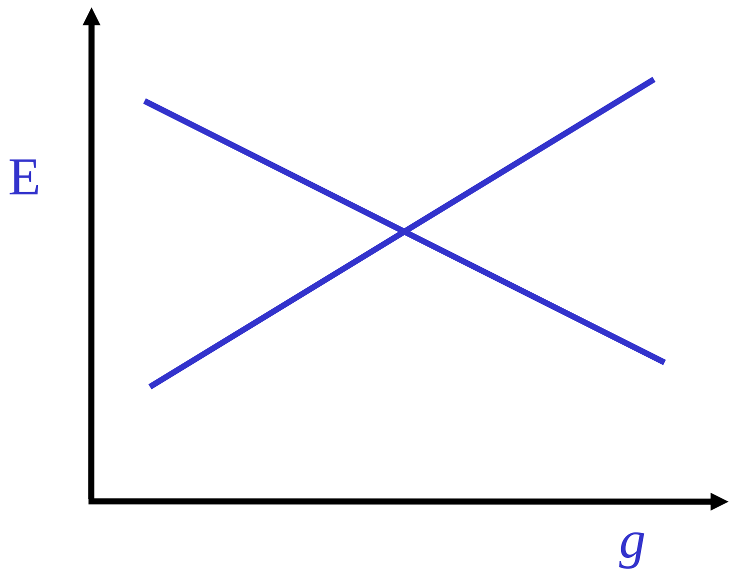
High Temperature Superconductors



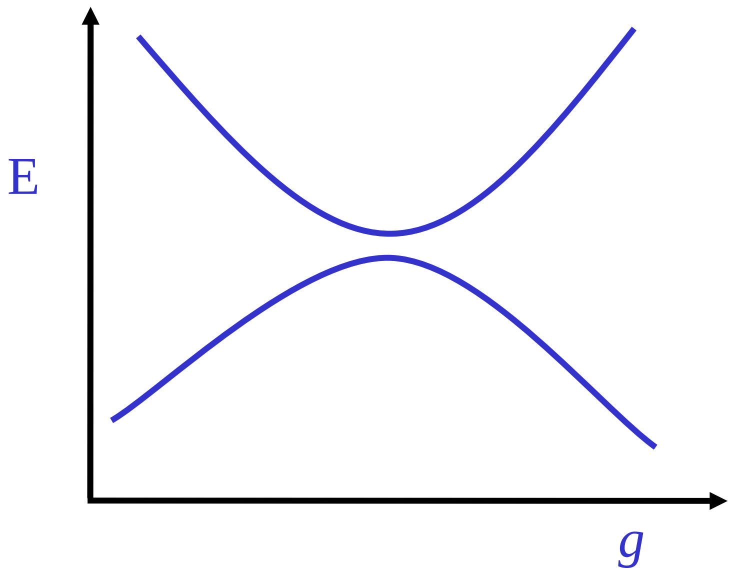
Maple, JMMM 177:18 (1998)

Quantum Phase Transition

Level crossing at T=0

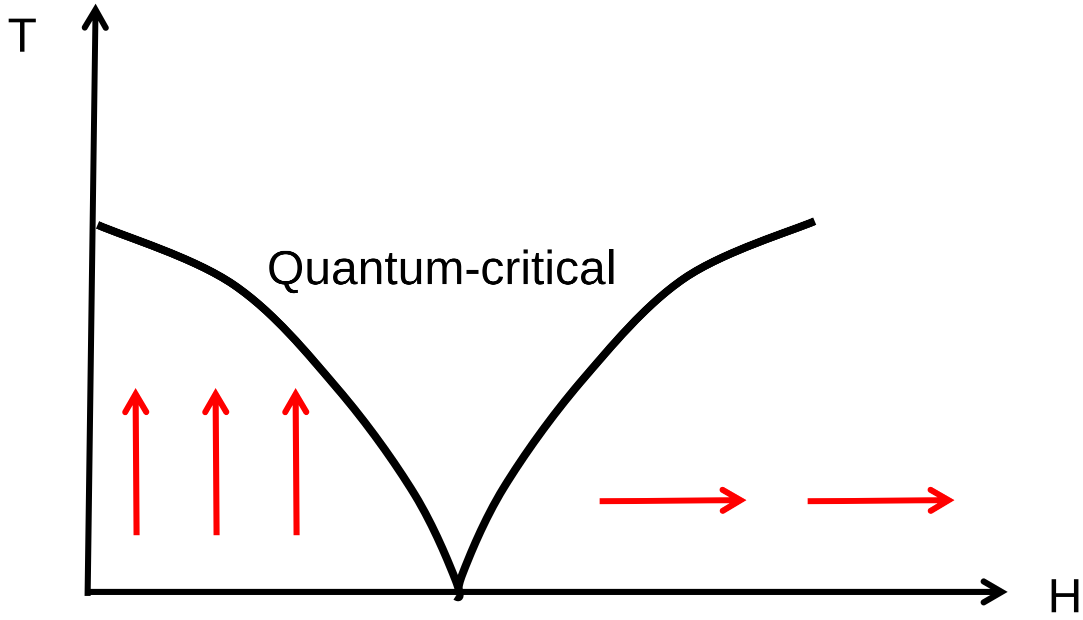


True level crossing.
First order phase transition



Avoided level crossing.
Second order phase transition

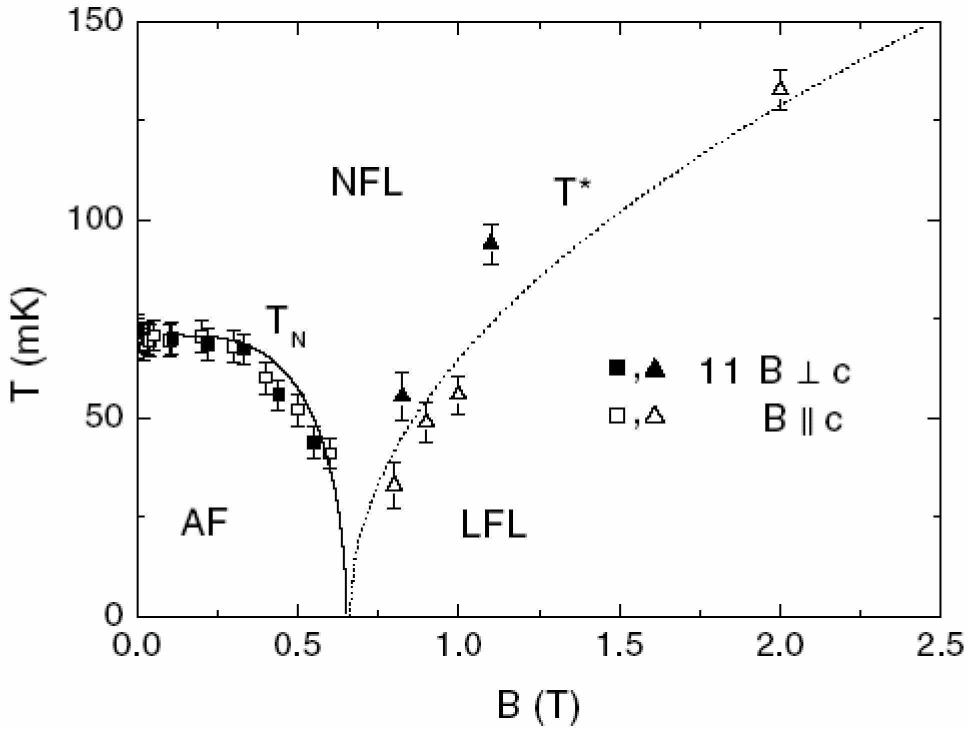
Quantum Critical Region



Quantum critical point controls a wide quantum critical region

Quantum critical region does not have well defined quasiparticles

Quantum Critical Point in $YbRh_2Si_2$



Gegenwart et al., PRL 89:56402(2002)

NFL – non Fermi liquid

AF – antiferromagnetic

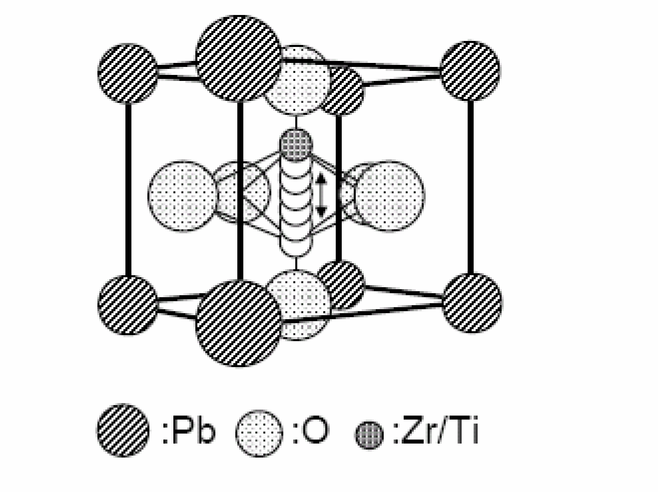
LFL – Landau Fermi liquid

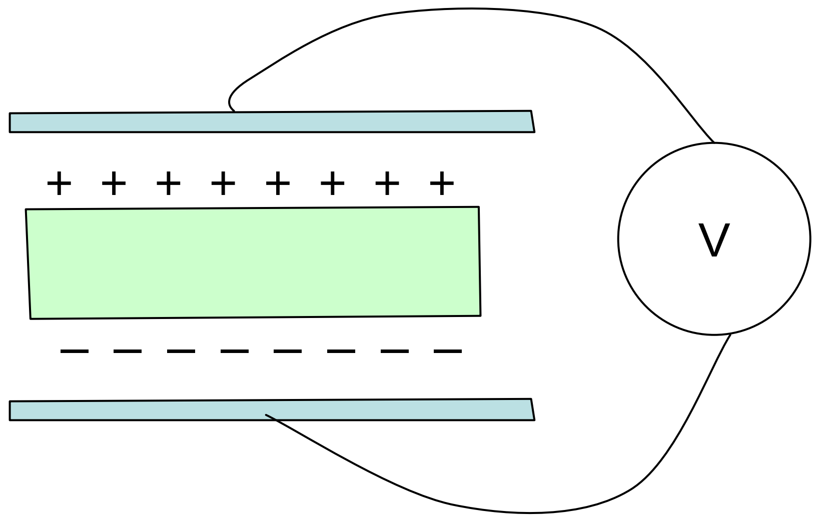
Quantum States of Matter.
Why are they interesting?

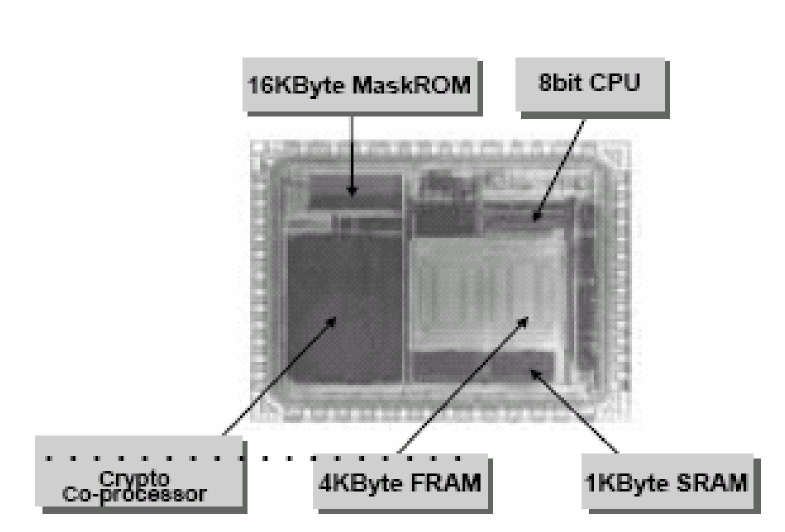
- **Understanding fundamental properties of complex quantum systems**

- **Technological applications**

Applications of Quantum Materials: Ferroelectric RAM





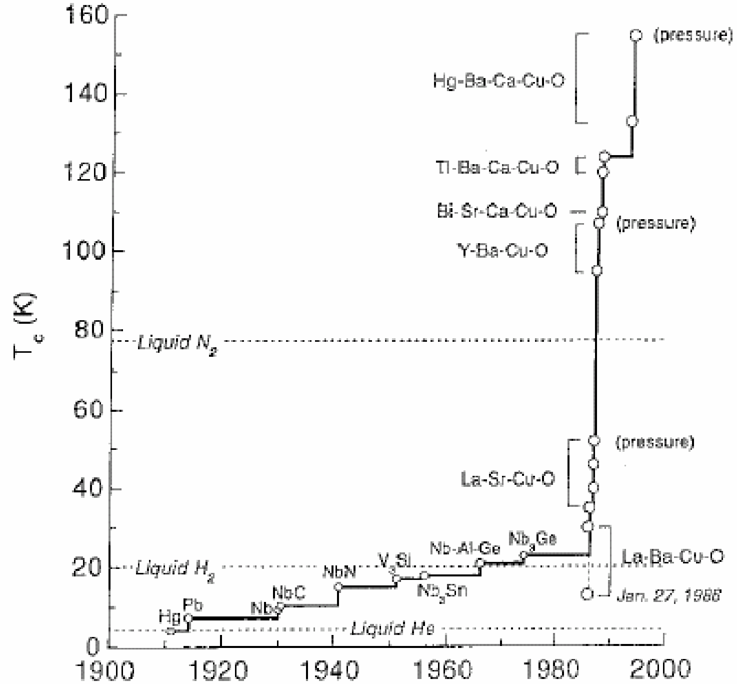


FeRAM in Smart Cards

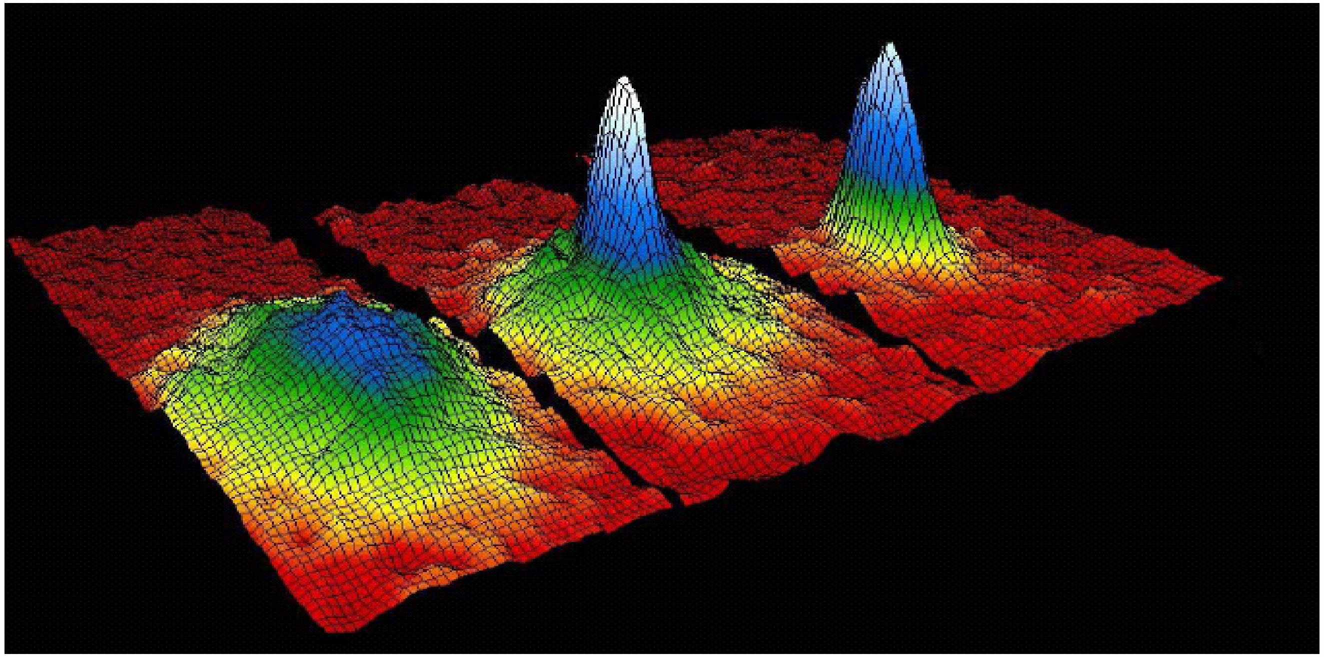
Non-Volatile Memory

High Speed Processing

Applications of Quantum Materials: High Tc Superconductors



Bose-Einstein Condensation



Cornell et al., Science 269, 198 (1995)

$$n \sim 10^{14}\mathrm{cm}^3 \quad T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$$

Ultralow density condensed matter system

Interactions are weak and can be described theoretically from first principles

New Era in Cold Atoms Research

Focus on systems with strong interactions

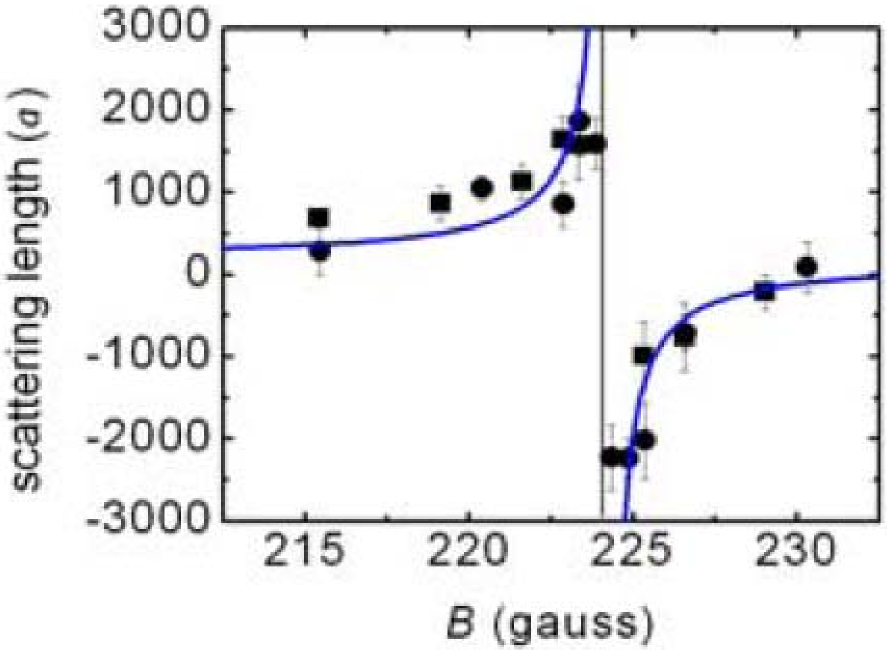
- Optical lattices
- Feshbach resonances
- Rotating condensates
- One dimensional systems
- Systems with long range dipolar interactions

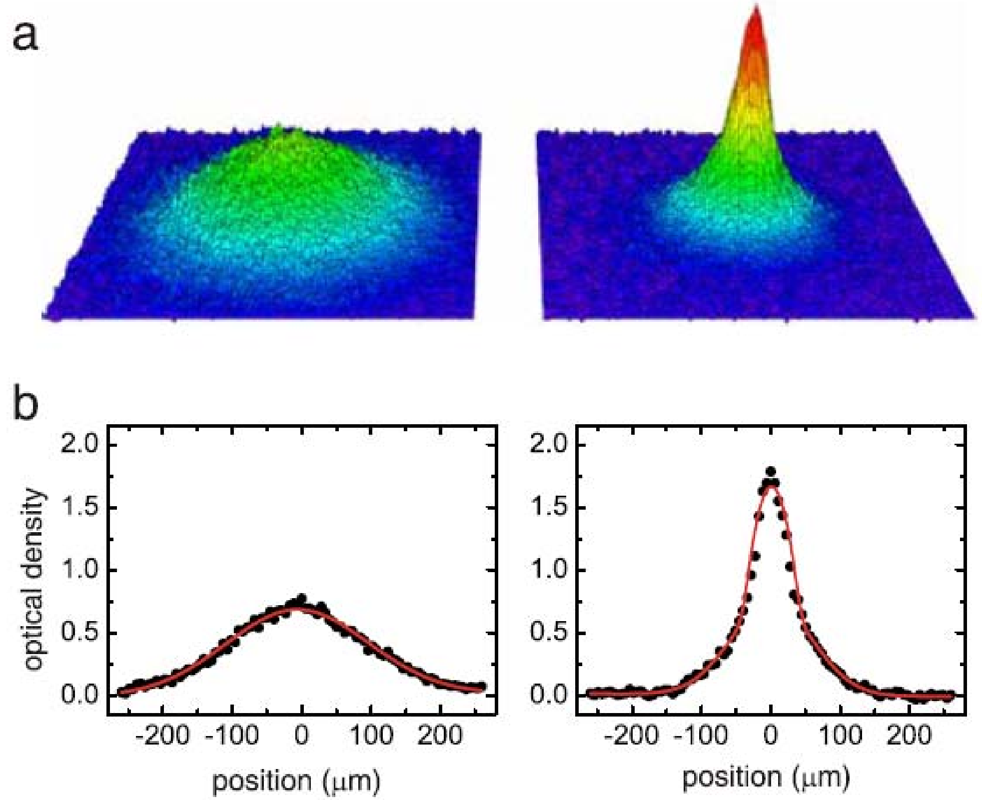
Feshbach Resonance and Fermionic Condensates

Greiner et al., Nature 426:537 (2003)

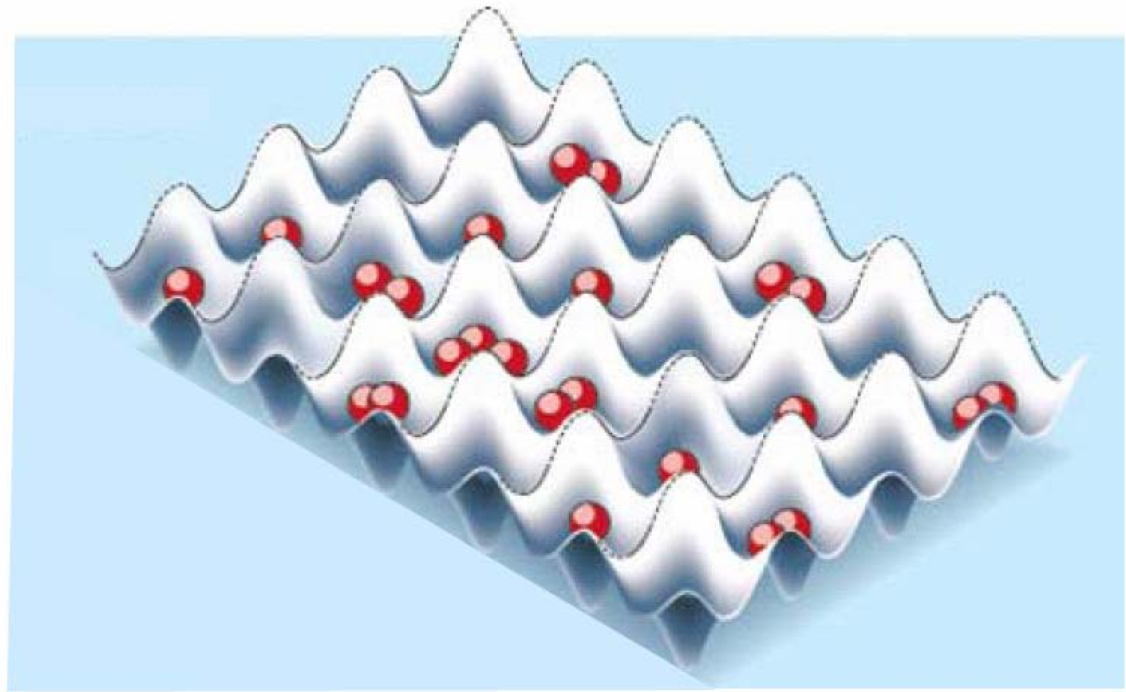
See also Jochim et al., Science 302:2101 (2003)

Zwierlein et al., PRL 91:250401 (2003)





Atoms in Optical Lattices



Theory: Jaksch et al. PRL 81:3108(1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);

Strongly Correlated Systems

Electrons in Solids

$$E_{\text{int}} \sim 1 \div 4 \text{ eV} \sim 10^4 \text{ K}$$

$$E_{\text{kin}} \sim 1 \div 10 \text{ eV} \sim 10^5 \text{ K}$$

Atoms in optical lattices

$$E_{\text{int}} \sim E_{\text{kin}} \sim 10 \text{ kHz} \sim 10^{-6} \text{ K}$$

Simple metals. $E_{\text{int}} < E_{\text{kin}}$

Perturbation theory in Coulomb interaction applies.
Band structure methods wotk

Strongly Correlated Electron Systems. $E_{\text{int}} \geq E_{\text{kin}}$

Band structure methods fail.

Novel phenomena in strongly correlated electron systems:

Quantum magnetism, phase separation, unconventional superconductivity, high temperature superconductivity, fractionalization of electrons …

Cold Atoms with Strong Interactions

Goals

- Resolve long standing questions in condensed matter physics (e.g. the origin of high Tc superconductivity)
- Resolve matter of principle questions (e.g. spin liquids in two and three dimensions)
- Find new exciting physics

Outline

- Cold atoms in optical lattices. Hubbard model
- Two component Bose mixture

  Quantum magnetism. Competing orders. Fractionalized phases
- Spin one bosons

  Spin exchange interactions. Exotic spin order (nematic)
- Fermions

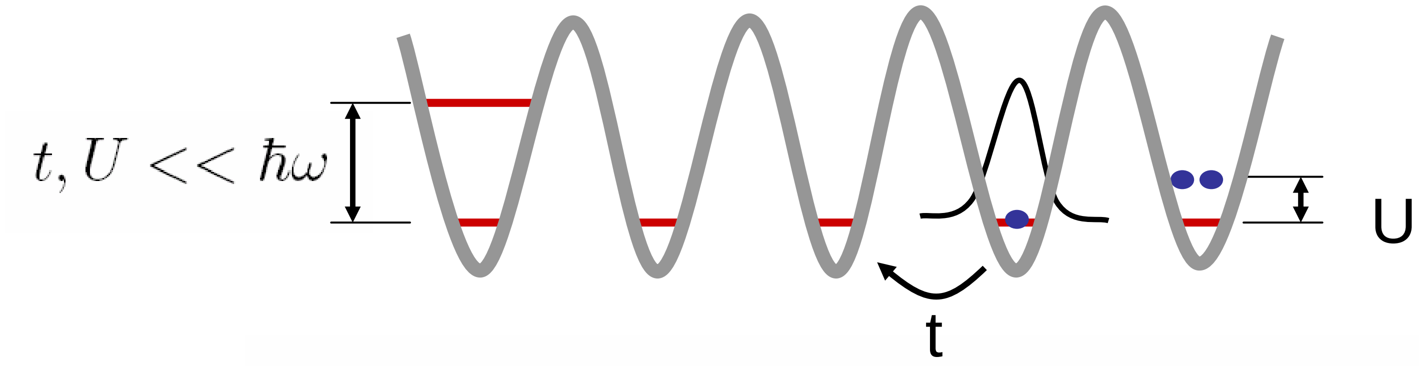
  Pairing in systems with repulsive interactions. Unconventional pairing. High Tc mechanism
- Boson-Fermion mixtures

  Polarons. Competing orders
- BEC on chips

  Interplay of disorder and interactions. Bose glass phase

Atoms in optical lattice.
Bose Hubbard Model

Bose Hubbard Model

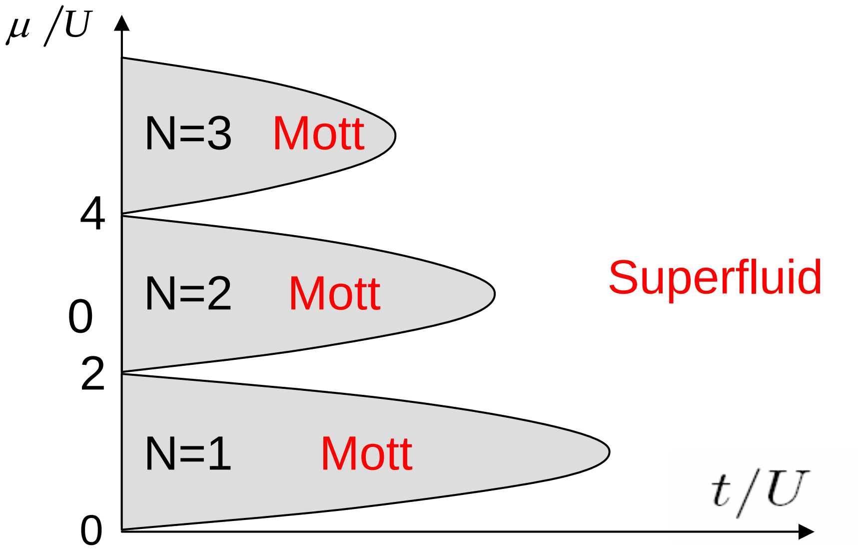


$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i^2 - \mu \sum_i n_i$$

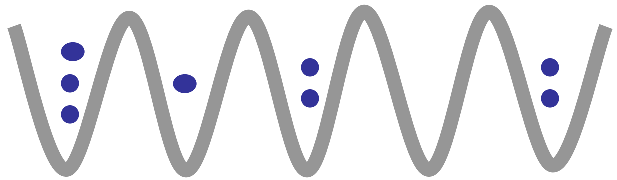
t – tunneling of atoms between neighboring wells

U – repulsion of atoms sitting in the same well

Bose Hubbard Model. Mean-field Phase Diagram.

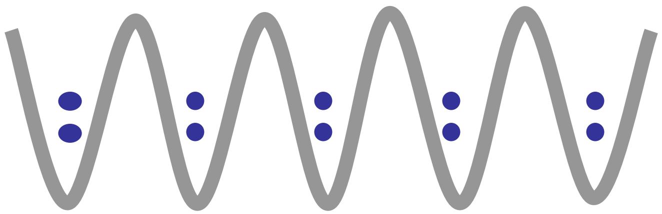


M.P.A. Fisher et al., PRB40:546 (1989)



$$U << Nt$$

Superfluid phase
Weak interactions



$$U >> Nt$$

Mott insulator phase
Strong interactions

Bose Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i^2 - \mu \sum_i n_i$$

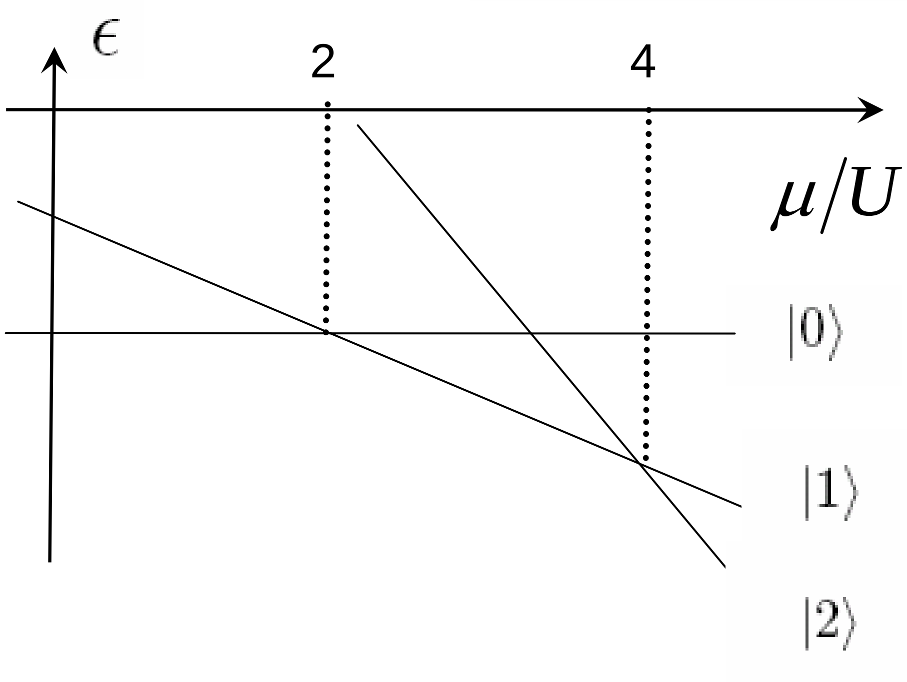
Set $t = 0$ Hamiltonian eigenstates are Fock states $|n\rangle = \frac{1}{\sqrt{n!}} (b_i^\dagger)^n \; |0\rangle$

$|0\rangle$ $\epsilon = 0$

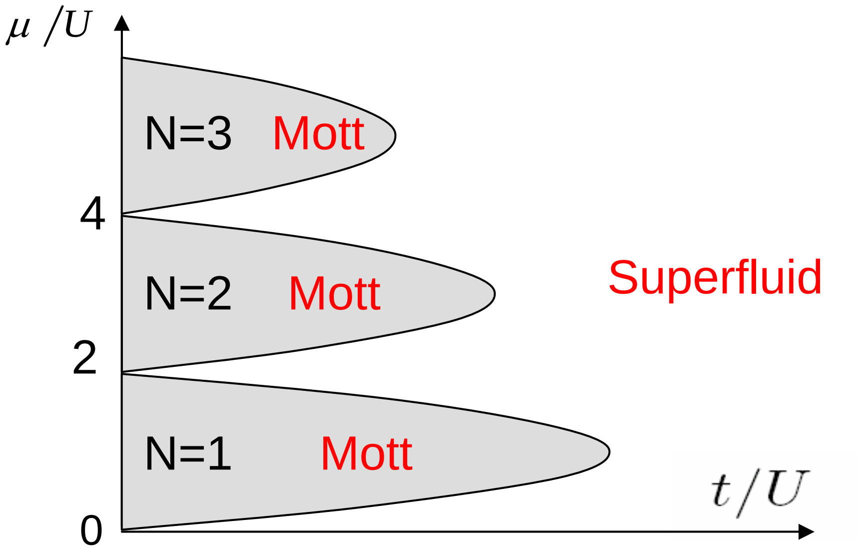
$|1\rangle$ $\epsilon = -\mu$

$|2\rangle$ $\epsilon = 2U - 2\mu$

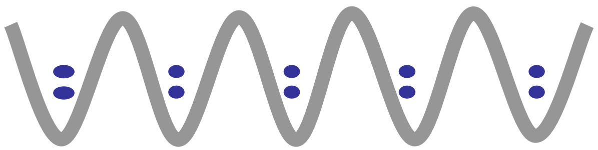
$|3\rangle$ $\epsilon = 6U - 3\mu$



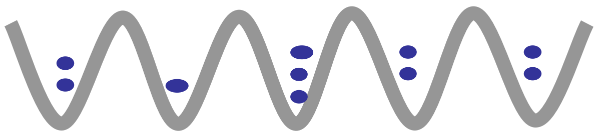
Bose Hubbard Model. Mean-field Phase Diagram.



Superfluid



Mott insulator phase



Particle-hole excitation $\Delta E \sim U - N\,t$

Tips of the Mott lobes $U \sim N\,t$

Gutzwiller variational wavefunction

$$|\Psi\rangle = \prod_i (f_0 |0\rangle + f_1 |1\rangle + f_2 |2\rangle + \dots)_i$$
$$= \prod_i (f_0 + f_1 b_i^\dagger + \frac{f_2}{\sqrt{2}} (b_i^\dagger)^2 + \dots) |0\rangle_i$$

Normalization $|f_0|^2 + |f_1|^2 + |f_2|^2 + \cdots = 1$

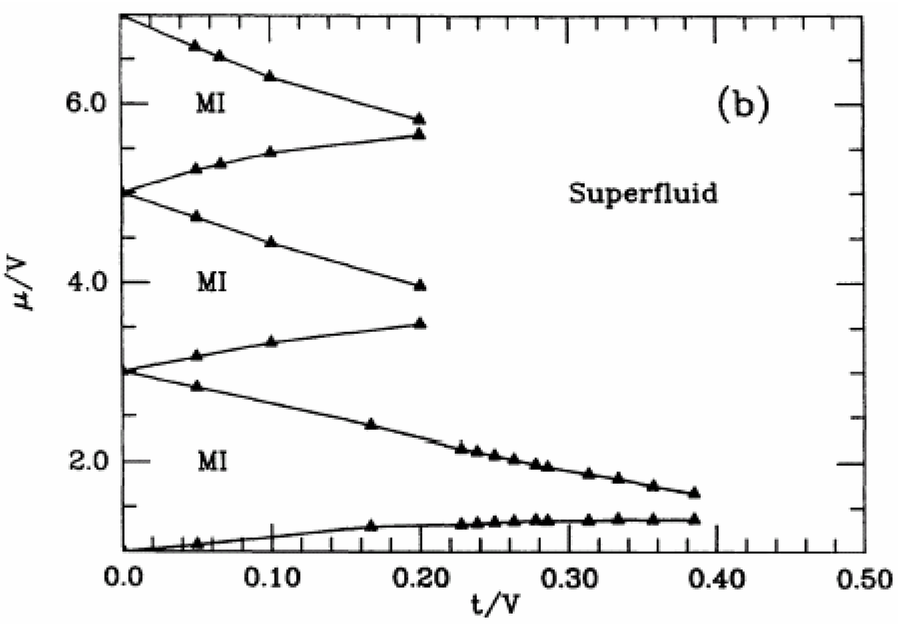
Interaction energy $\epsilon_{\mathrm{U}} = 2\ U\ |f_2|^2 + 6\ U\ |f_3|^2 + \dots$

Kinetic energy $\epsilon_{\mathrm{t}} = -zt \left| f_0^* f_1 + \sqrt{2}\ f_1^* f_2 + \sqrt{3}\ f_2^* f_3 + \dots \right|^2$

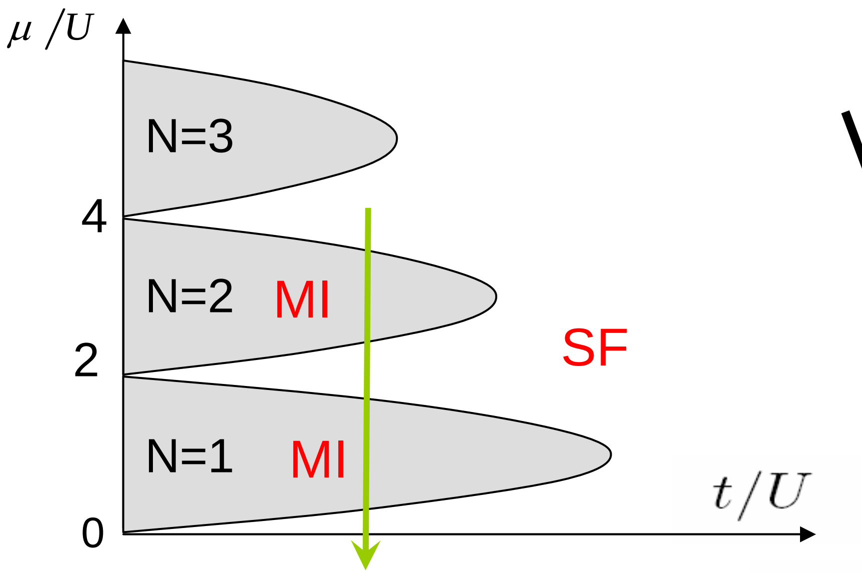
z – number of nearest neighbors

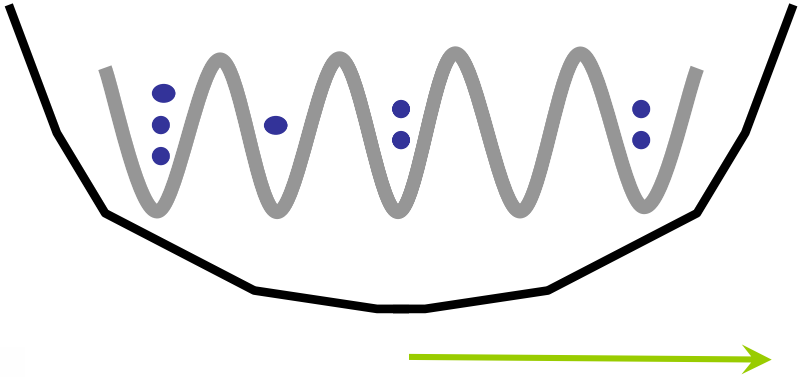
Phase Diagram of the 1D Bose Hubbard Model. Quantum Monte-Carlo Study

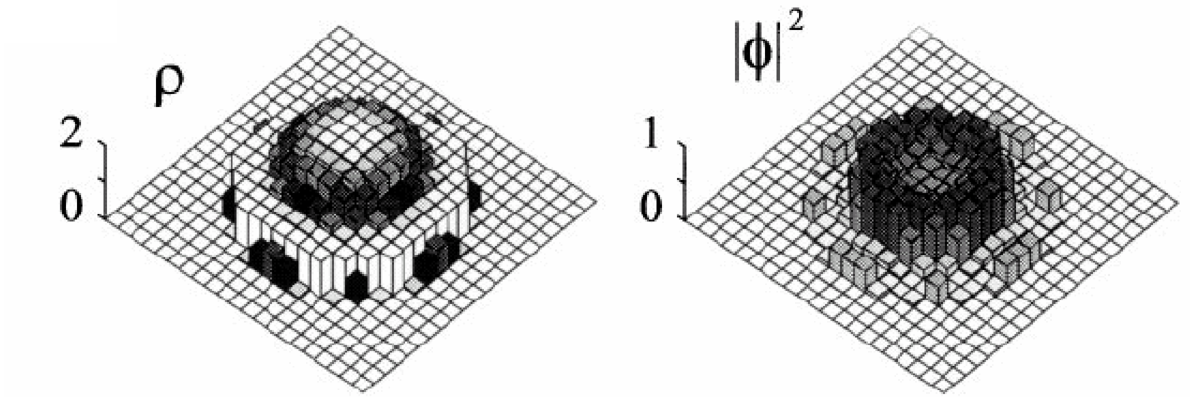
Batrouni and Scaletter, PRB 46:9051 (1992)



Optical Lattice and Parabolic Potential



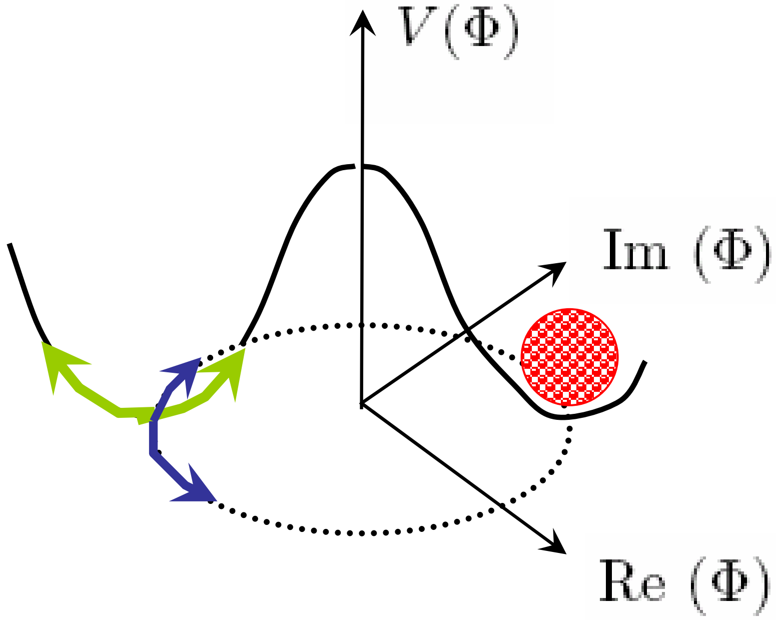




Jaksch et al.,
PRL 81:3108 (1998)

Superfluid Phase

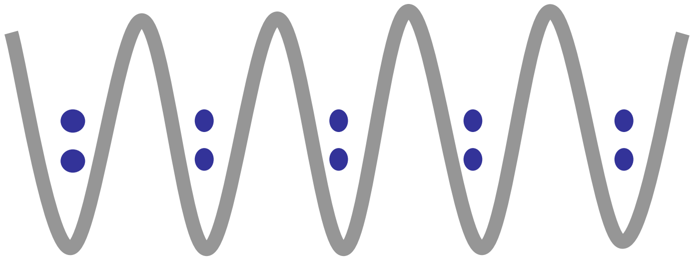
Order parameter $\langle a_i \rangle = \Phi = |\Phi| \, e^{i\theta}$ Breaks U(1) symmetry



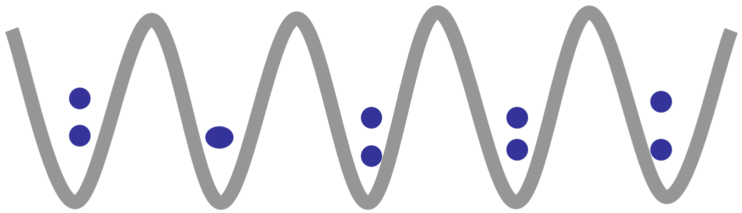
Phase (Bogoliubov) mode. Gapless Goldstone mode. $\omega = c \mid \vec{q} \mid$

Gapped amplitude mode. $\omega = \sqrt{\Delta^2 + c^2 \, q^2}$

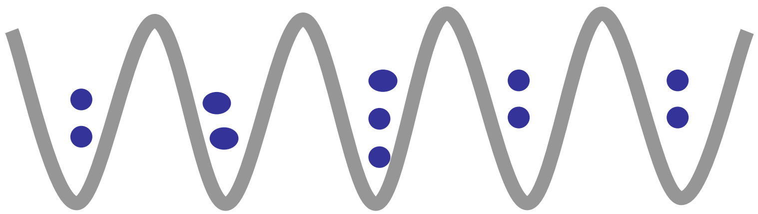
Mott Insulating Phase



Ground state

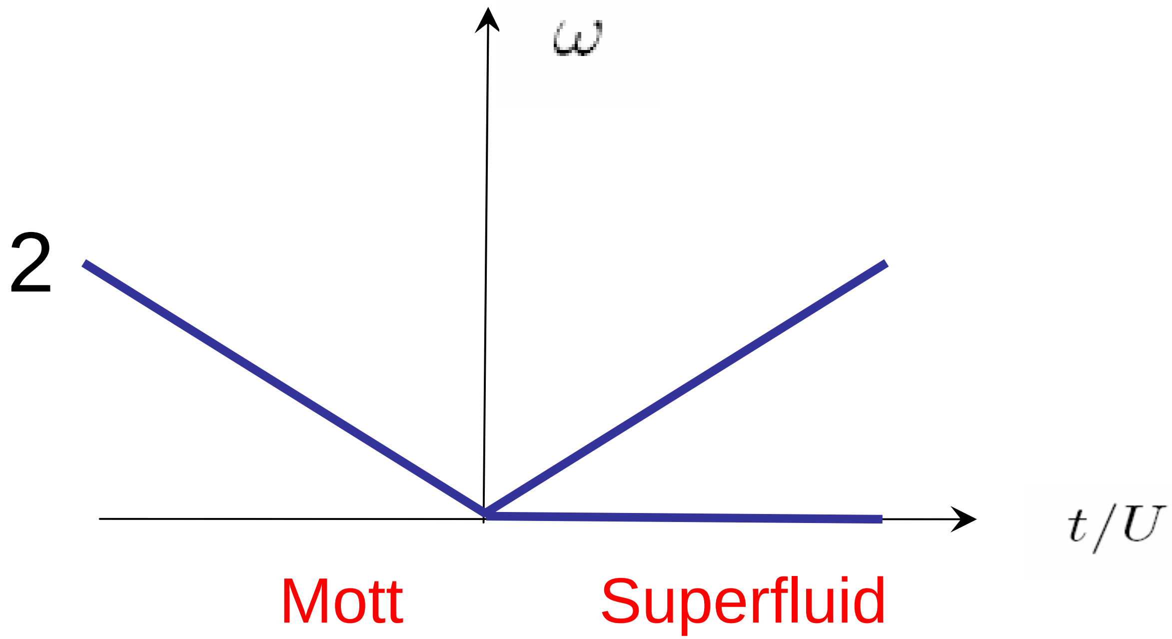


Hole excitation (gapped)



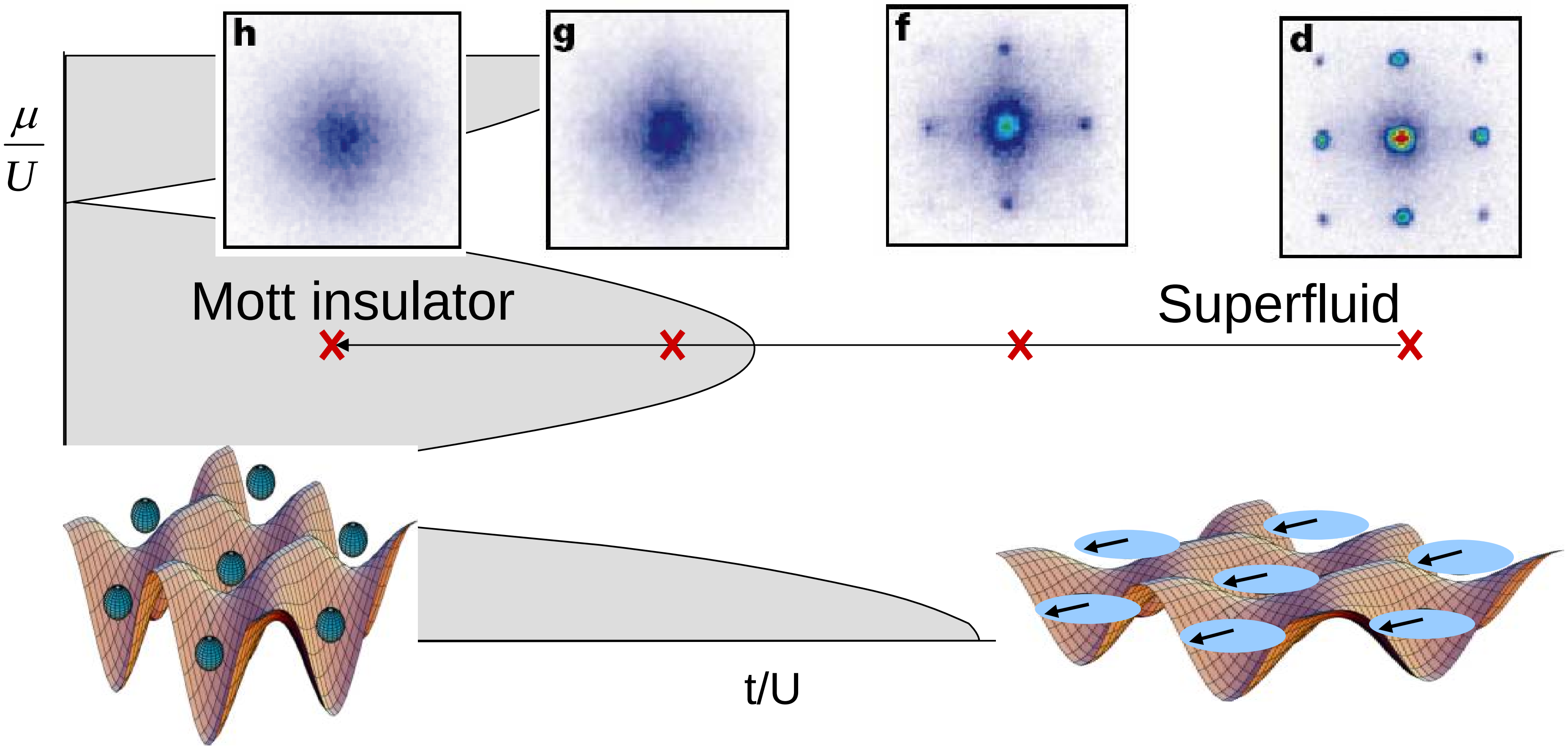
Particle excitation (gapped)

Excitations of the Bose Hubbard Model



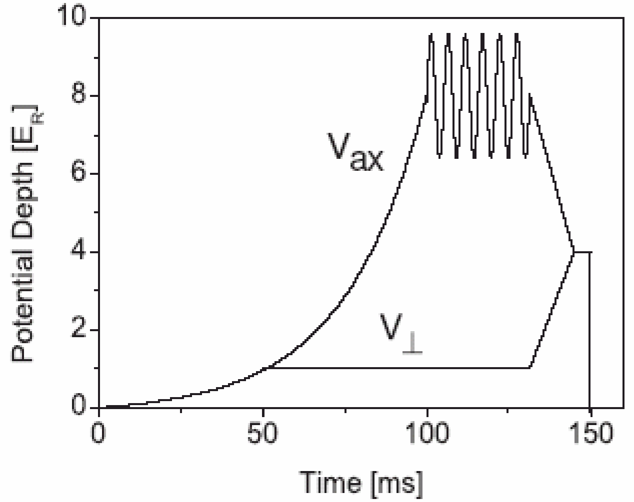
Superfluid to Insulator Transition

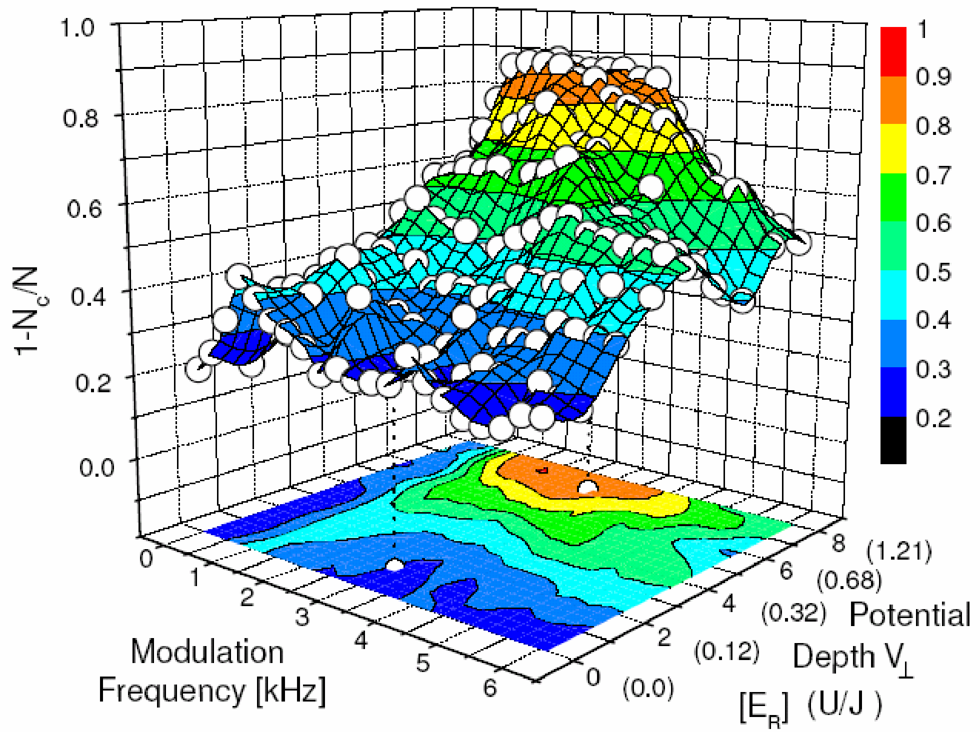
Greiner et al., Nature 415 (02)



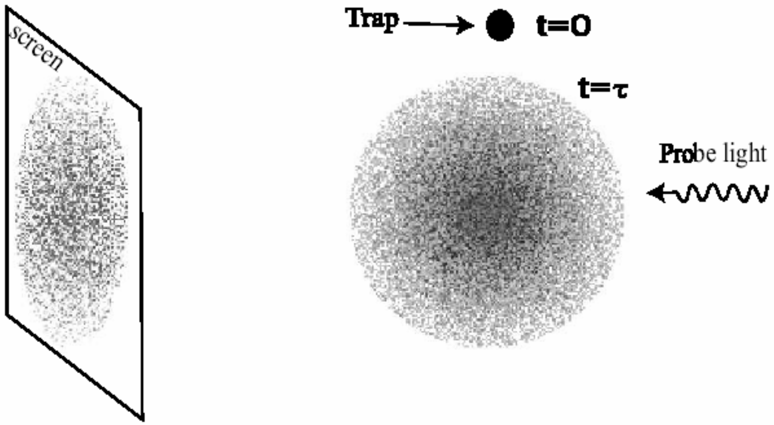
Excitations of Bosons in the Optical Lattice

Schori et al., PRL 93:240402 (2004)

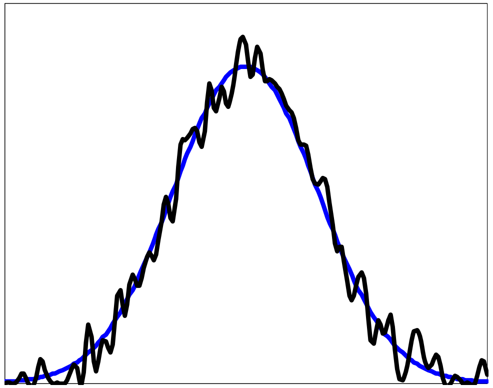


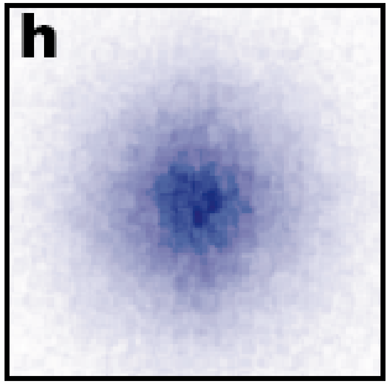


Time of Flight Experiments.



Quantum Noise Interferometry of Atoms in Optical Lattices.



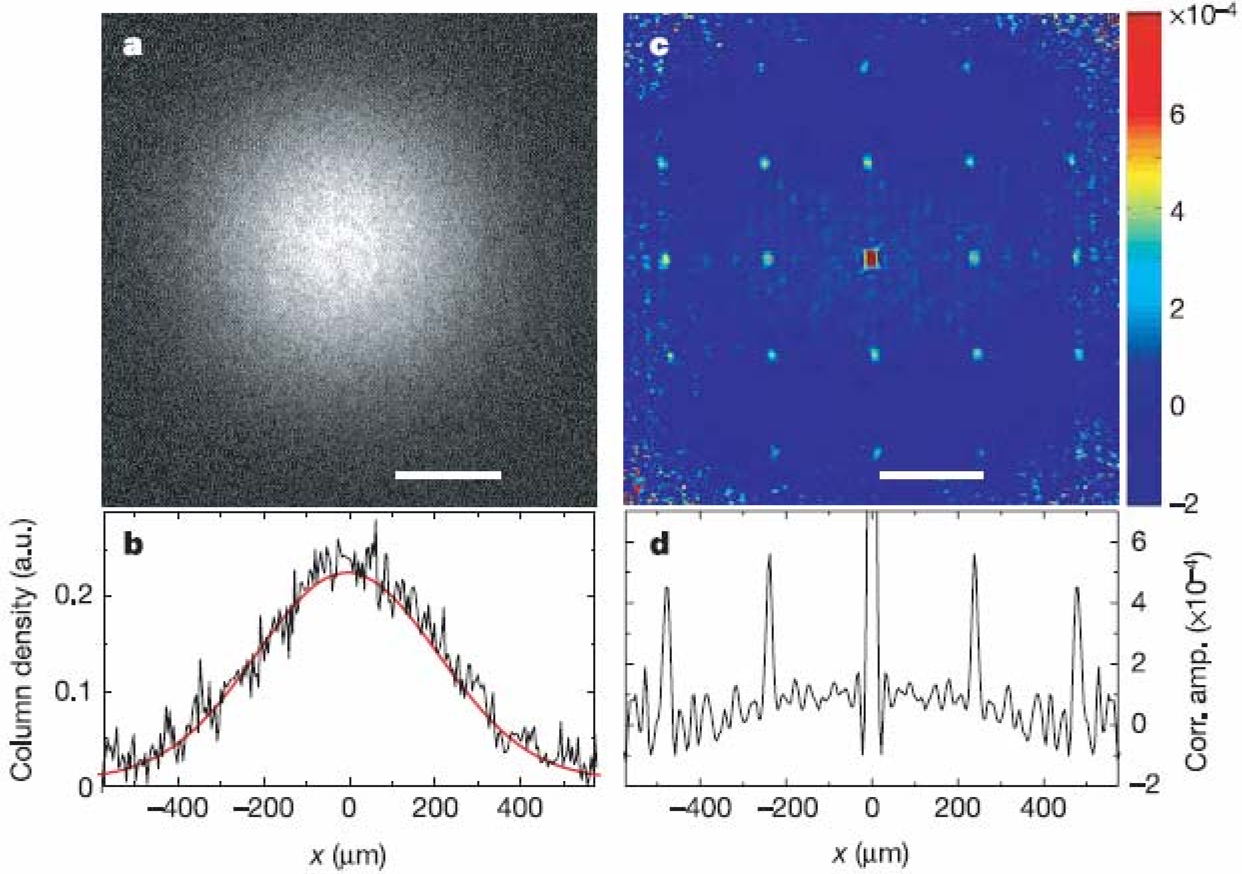


Second order coherence $G(r_1, r_2) = \langle n(r_1)n(r_2)\rangle - \langle n(r_1)\rangle\langle n(r_2)\rangle$

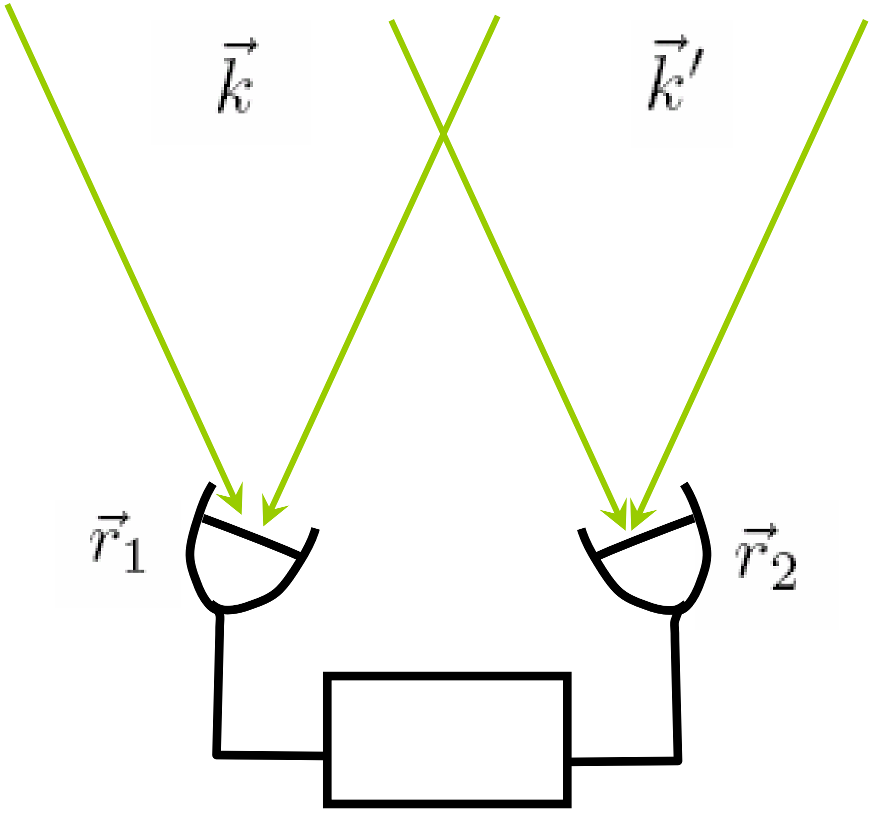
Second Order Coherence in the Insulating State of Bosons. Hanburry-Brown-Twiss experiment

Theory: Altman et al., PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005)

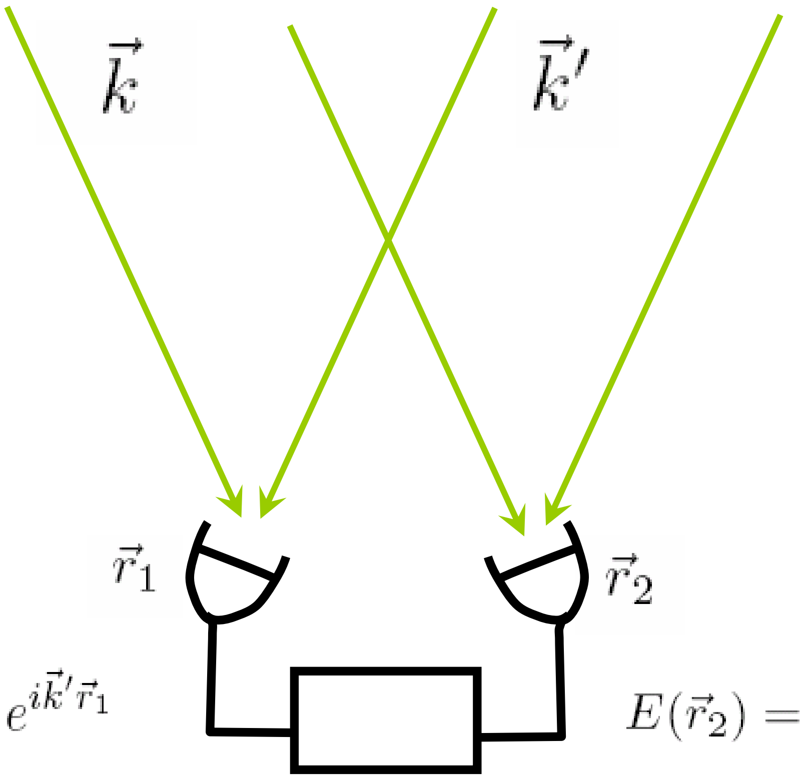


Hanburry-Brown-Twiss Stellar Interferometer



$$\langle\, I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k} - \vec{k}')\ (\vec{r}_1 - \vec{r}_2)\Big)$$

Hanburry-Brown-Twiss Interferometer

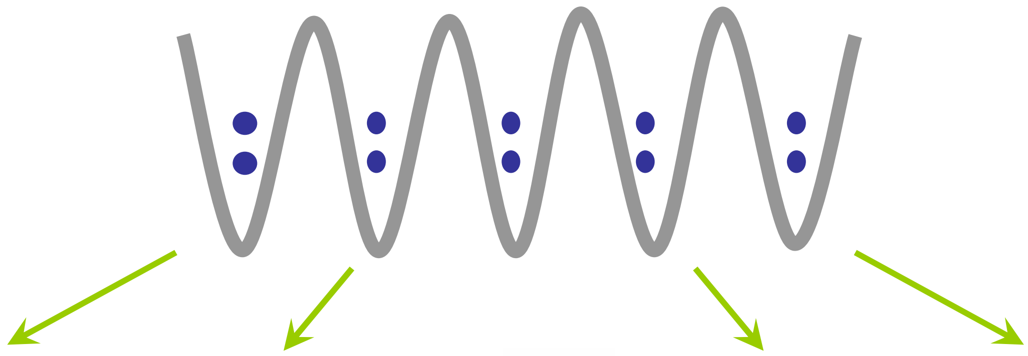


$$E(\vec{r}_1) = E_k e^{i\vec{k}\vec{r}_1} + E_{k'} e^{i\vec{k}'\vec{r}_1}$$

$$E(\vec{r}_2) = E_k e^{i\vec{k}\vec{r}_2} + E_{k'} e^{i\vec{k}'\vec{r}_2}$$

$$\begin{aligned} \langle I(r_1)I(r_2)\rangle &= \langle |E(r_1)|^2 |E(r_2)|^2 \rangle \\ &= \langle \left\{ |E_k|^2 + |E_{k'}|^2 + (E_k E^*_{k'} e^{i(\vec{k}-\vec{k}')\vec{r}_1} + \text{c.c.}) \right\} \left\{ |E_k|^2 + |E_{k'}|^2 + (E_k E^*_{k'} e^{i(\vec{k}-\vec{k}')\vec{r}_2} + \text{c.c.}) \right\} \rangle \\ &= \langle (|E_k|^2 + |E_{k'}|^2)^2 \rangle + \langle |E_k|^2 |E_{k'}|^2 \rangle [e^{i(\vec{k}-\vec{k}')\vec{r}_1} + \text{c.c.}] \end{aligned}$$

Second Order Coherence in the Insulating State of Bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k},\ \vec{k}+\vec{G}_1,\ \vec{k}+\vec{G}_2$

First order coherence: $\langle\ \rho(\vec{r})\ \rangle$

Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle$

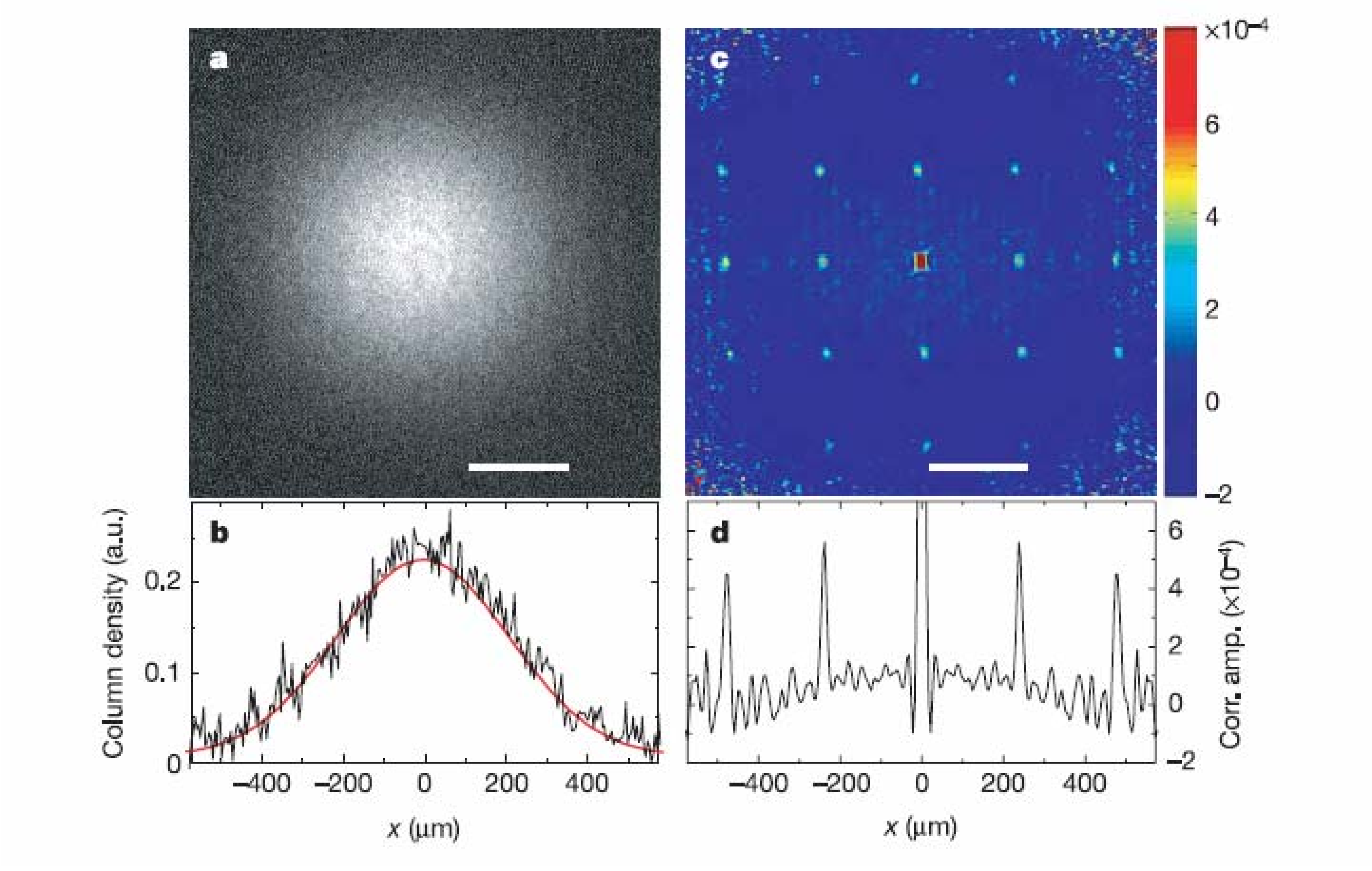
Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle = A_0 + A_1\ \cos\Big(\ \vec{G}_1(\vec{r}_1-\vec{r}_2)\ \Big) + A_2\ \cos\Big(\ \vec{G}_2(\vec{r}_1-\vec{r}_2)\ \Big) + \dots$$

Second Order Coherence in the Insulating State of Bosons. Hanburry-Brown-Twiss experiment

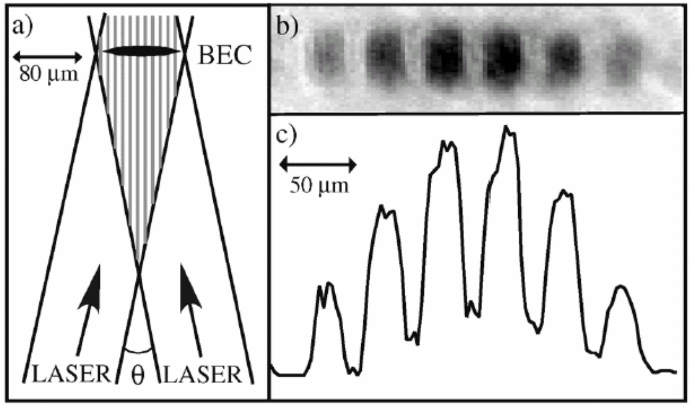
Theory: Altman et al., PRA 70:13603 (2004)

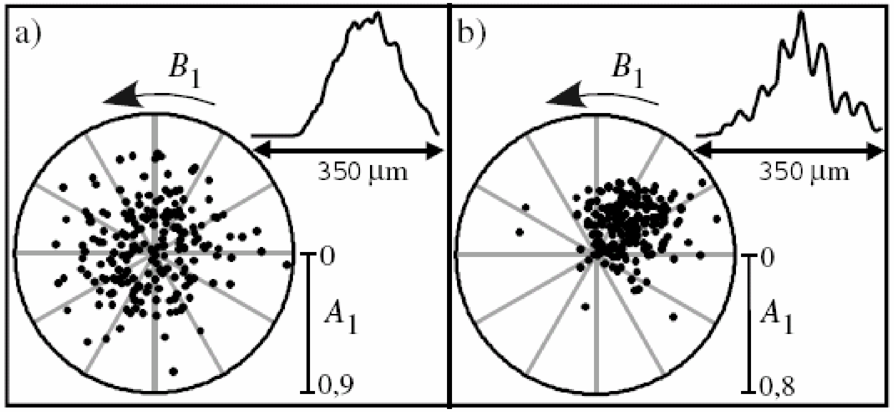
Experiment: Folling et al., Nature 434:481 (2005)



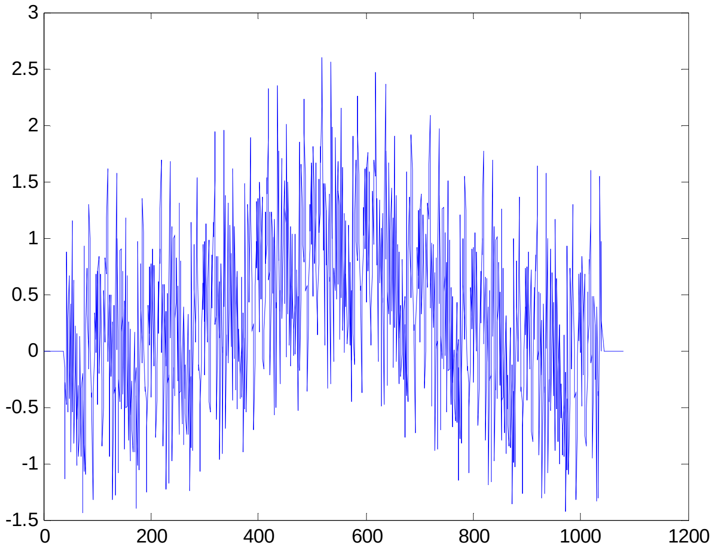
Interference of an Array of Independent Condensates

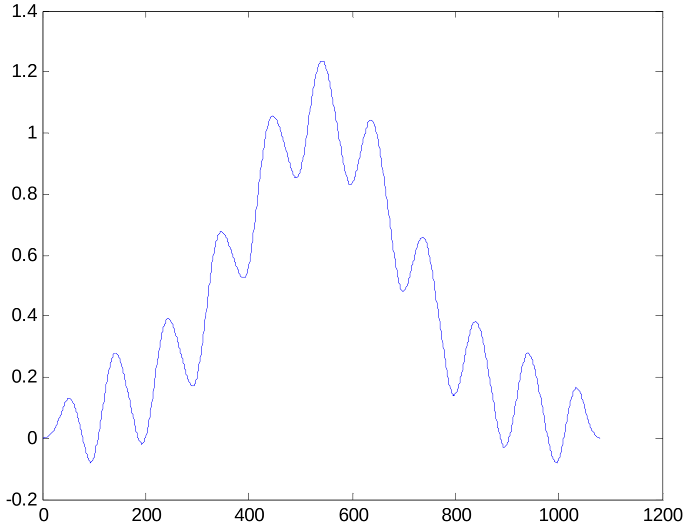
Hadzibabic et al., PRL 93:180403 (2004)





Smooth structure is a result of finite experimental resolution (filtering)



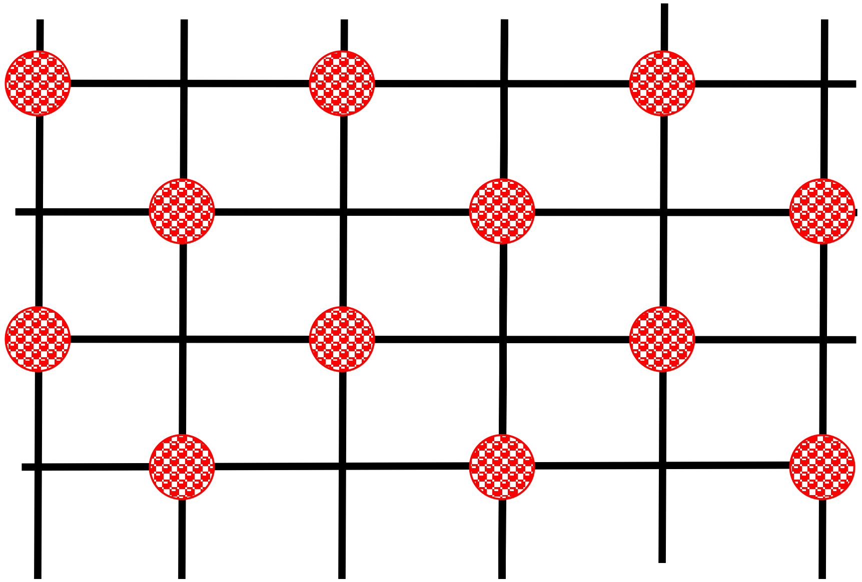


Extended Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_{ij} n_i U_{ij} n_j - \mu \sum_i n_i$$

U_0 - on site repulsion

U_1 - nearest neighbor repulsion

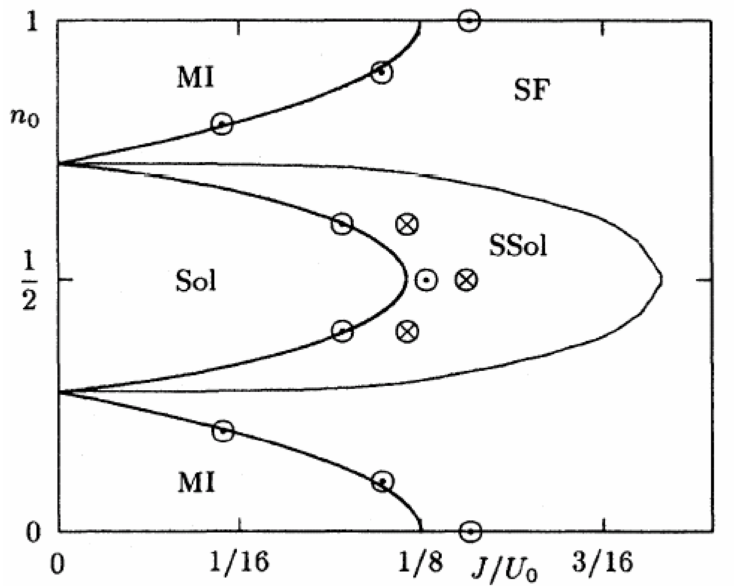


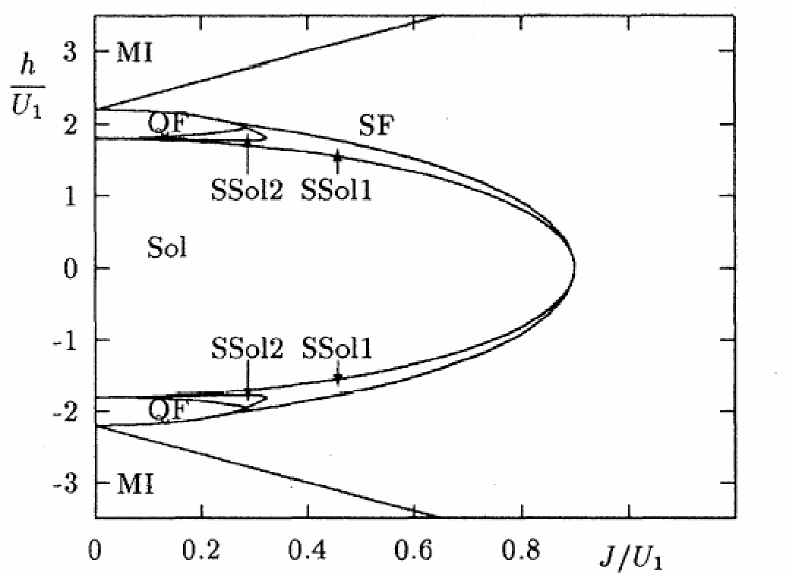
Checkerboard Phase:

Crystal phase of bosons.
Breaks translational symmetry

Extended Hubbard Model. Mean Field Phase diagram

van Otterlo et al., PRB 52:16176 (1995)



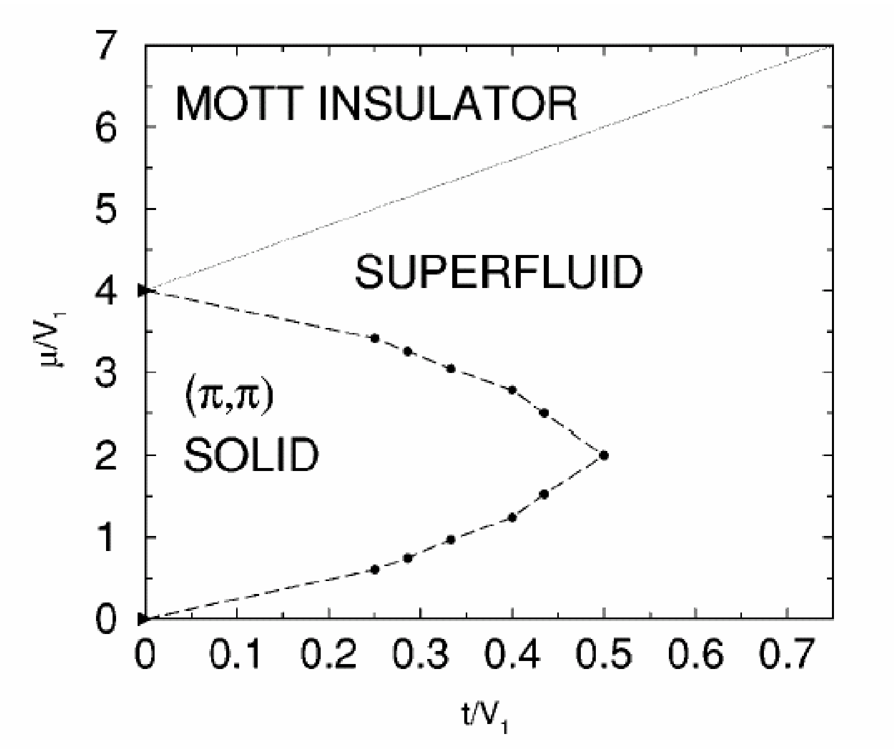


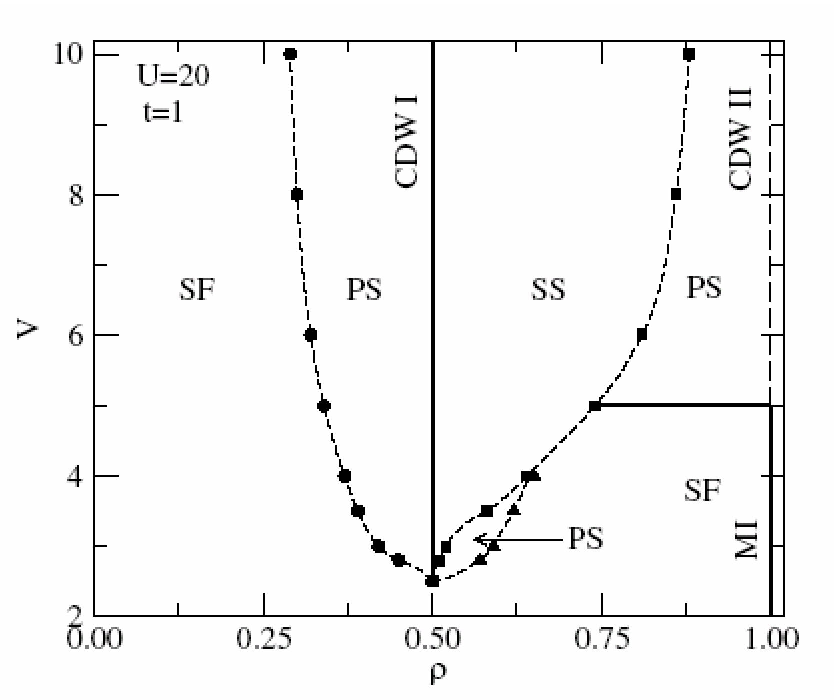
$$\frac{U_1}{U_0} = \frac{1}{5}$$

Hard core bosons. $\frac{U_2}{U_1} = \frac{1}{10}$

Supersolid – superfluid phase with broken translational symmetry

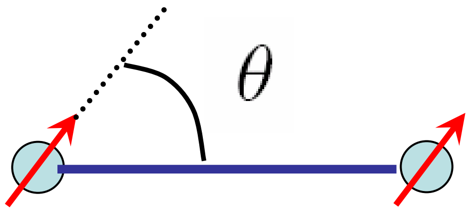
Extended Hubbard Model.
Quantum Monte Carlo study



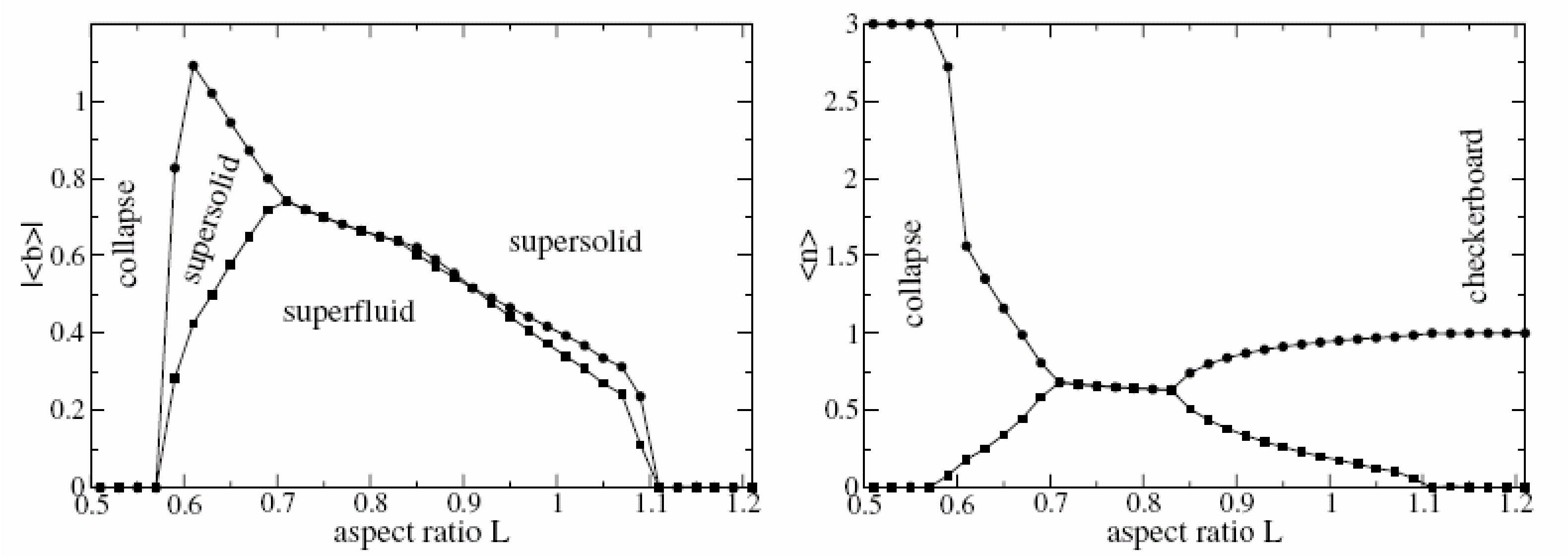


Hebert et al., PRB 65:14513 (2002)   Sengupta et al., PRL 94:207202 (2005)

Dipolar Bosons in Optical Lattices

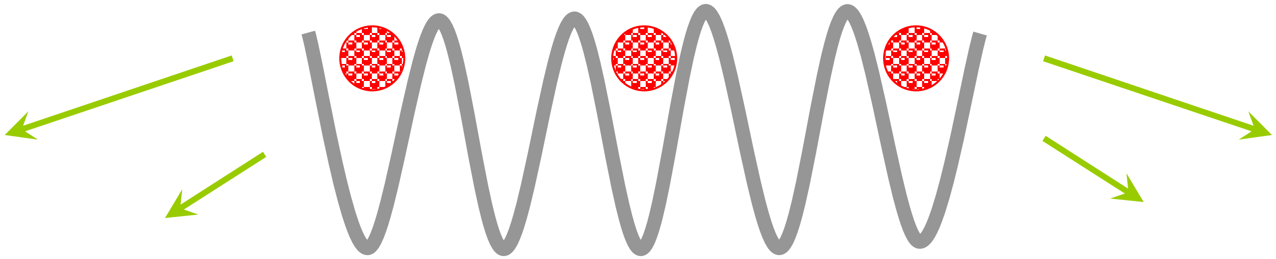


$$V_{\mathrm{int}} = d^2 \frac{1 - 3\cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} + \frac{4\pi\hbar^2 a}{m}\,\delta(\mathbf{r} - \mathbf{r}')$$



Goral et al., PRL88:170406 (2002)

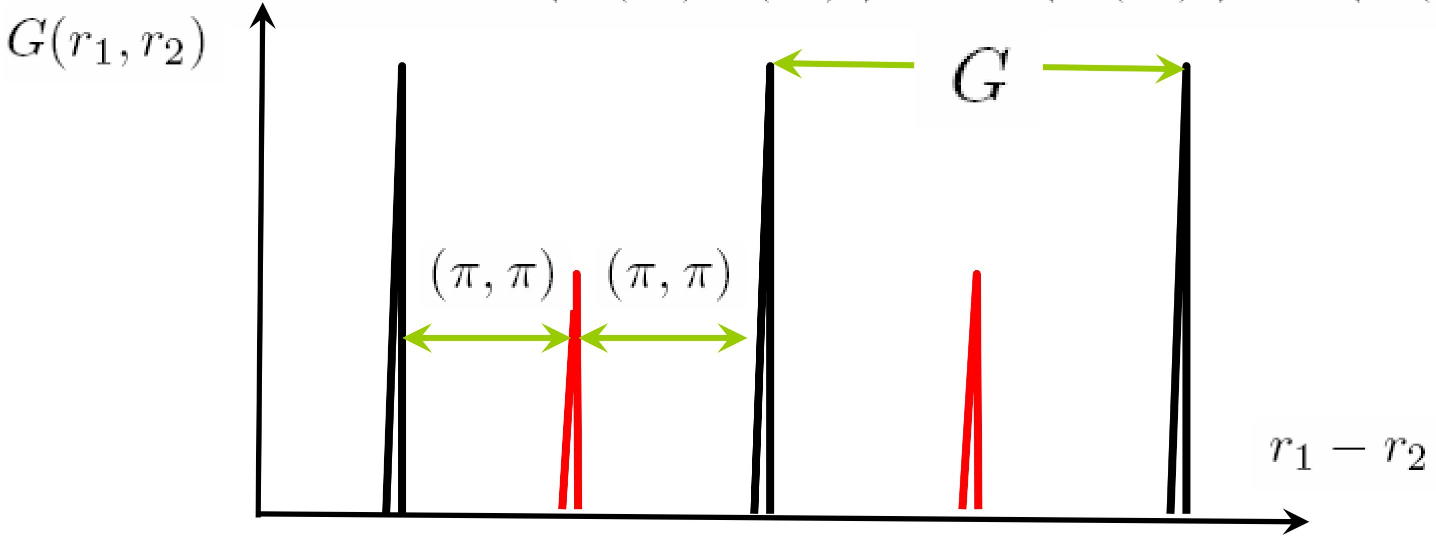
How to Detect a Checkerboard Phase?



$$r = \frac{\hbar\, k\, t}{m}$$

Correlation Function Measurements

$$\begin{aligned} G(r_1, r_2) &= \langle\, n(r_1)\; n(r_2)\,\rangle_{\mathrm{TOF}} - \langle\, n(r_1)\,\rangle_{\mathrm{TOF}}\,\langle\, n(r_2)\,\rangle_{\mathrm{TOF}} \\ &\sim \langle\, n(k_1)\; n(k_2)\,\rangle_{\mathrm{LAT}} - \langle\, n(k_1)\,\rangle_{\mathrm{LAT}}\,\langle\, n(k_2)\,\rangle_{\mathrm{LAT}} \end{aligned}$$

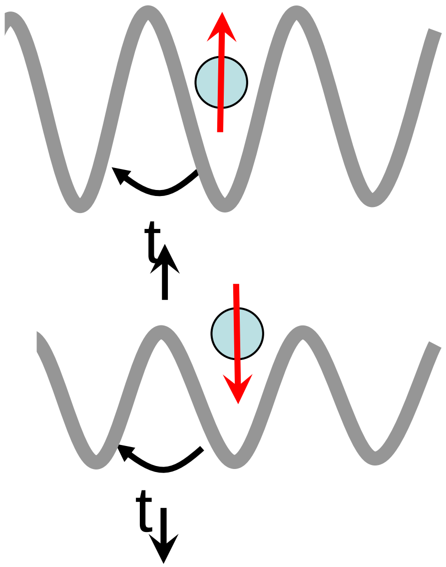


Two Component Bose Mixture in Optical Lattice.

Quantum magnetism. Competing orders.
Fractionalized phases

Two Component Bose Mixture in Optical Lattice

Example: $^{87}\mathrm{Rb}$. Mandel et al., Nature 425:937 (2003)



$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

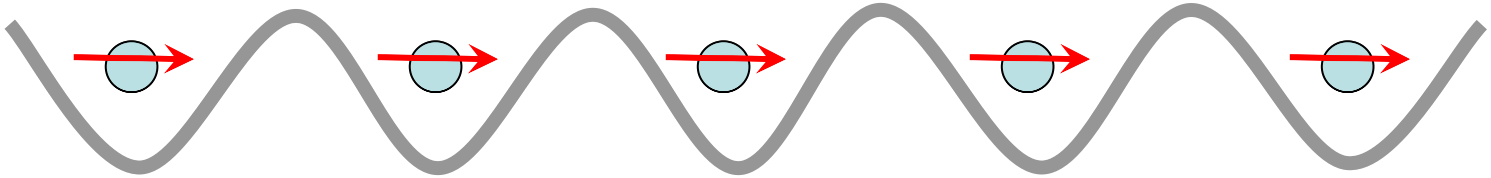
Two component Bose Hubbard Model

$$\mathcal{H} = -\ t_{\uparrow}\sum_{\langle ij\rangle} b^{\dagger}_{i\uparrow} b_{j\uparrow} - t_{\downarrow}\sum_{\langle ij\rangle} b^{\dagger}_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow}\sum_{i} n_{i\uparrow}(n_{\uparrow}-1)$$
$$+\ U_{\downarrow\downarrow}\sum_{i} n_{i\downarrow}(n_{\downarrow}-1) + U_{\uparrow\downarrow}\sum_{i} n_{i\uparrow}n_{\downarrow}$$

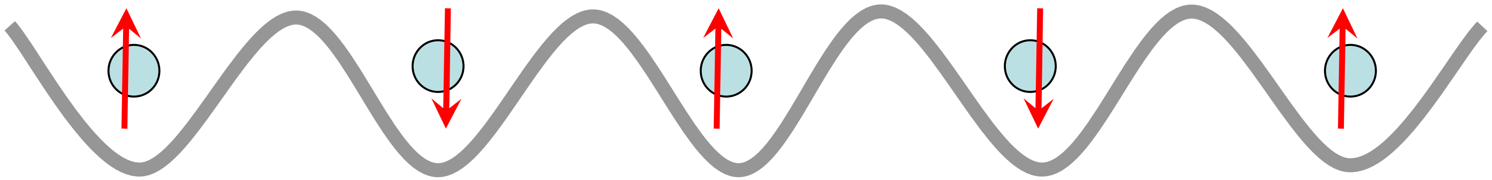
Two Component Bose Mixture in Optical Lattice. Magnetic order in an insulating phase

Insulating phases with N=1 atom per site. Average densities $n_{\uparrow} = n_{\downarrow} = \frac{1}{2}$

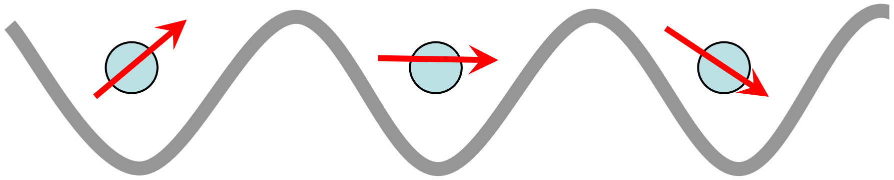
Easy plane ferromagnet $| \Psi \rangle = \prod_i \left(b_{i\uparrow}^{\dagger} + e^{i\phi} b_{i\downarrow}^{\dagger} \right) | 0 \rangle$



Easy axis antiferromagnet $| \Psi \rangle = \prod_{i \in A} b_{i\uparrow}^{\dagger} \prod_{i \in B} b_{i\downarrow}^{\dagger}$



Quantum Magnetism of Bosons in Optical Lattices



Kuklov and Svistunov, PRL (2003)

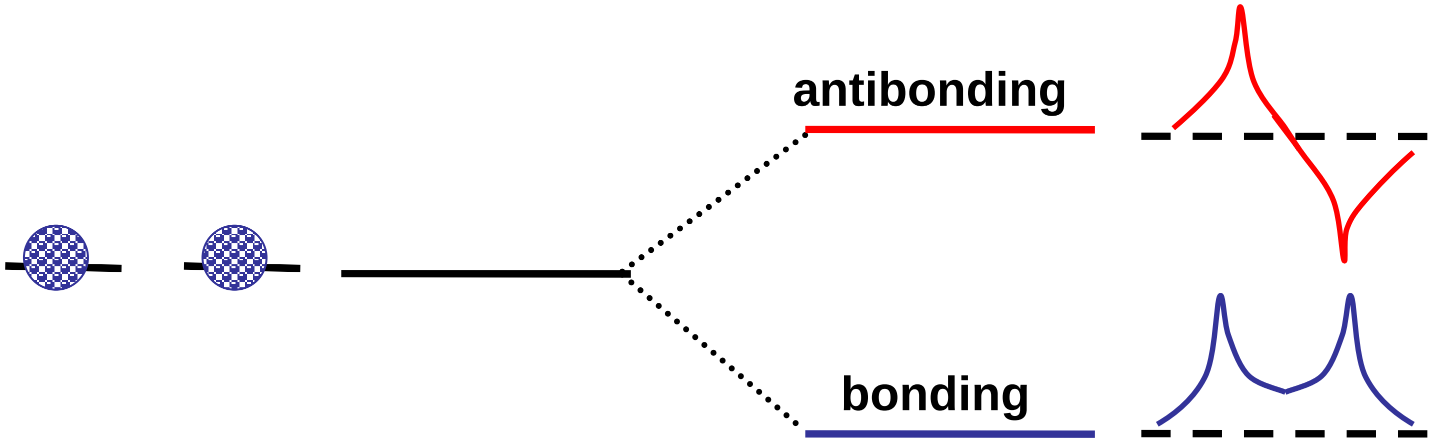
Duan et al., PRL (2003)

$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

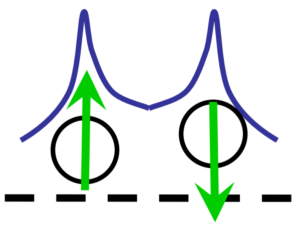
$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\, U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\, U_{\downarrow\downarrow}$

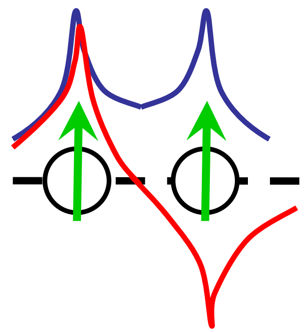
Exchange Interactions in Solids



Kinetic energy dominates: antiferromagnetic state

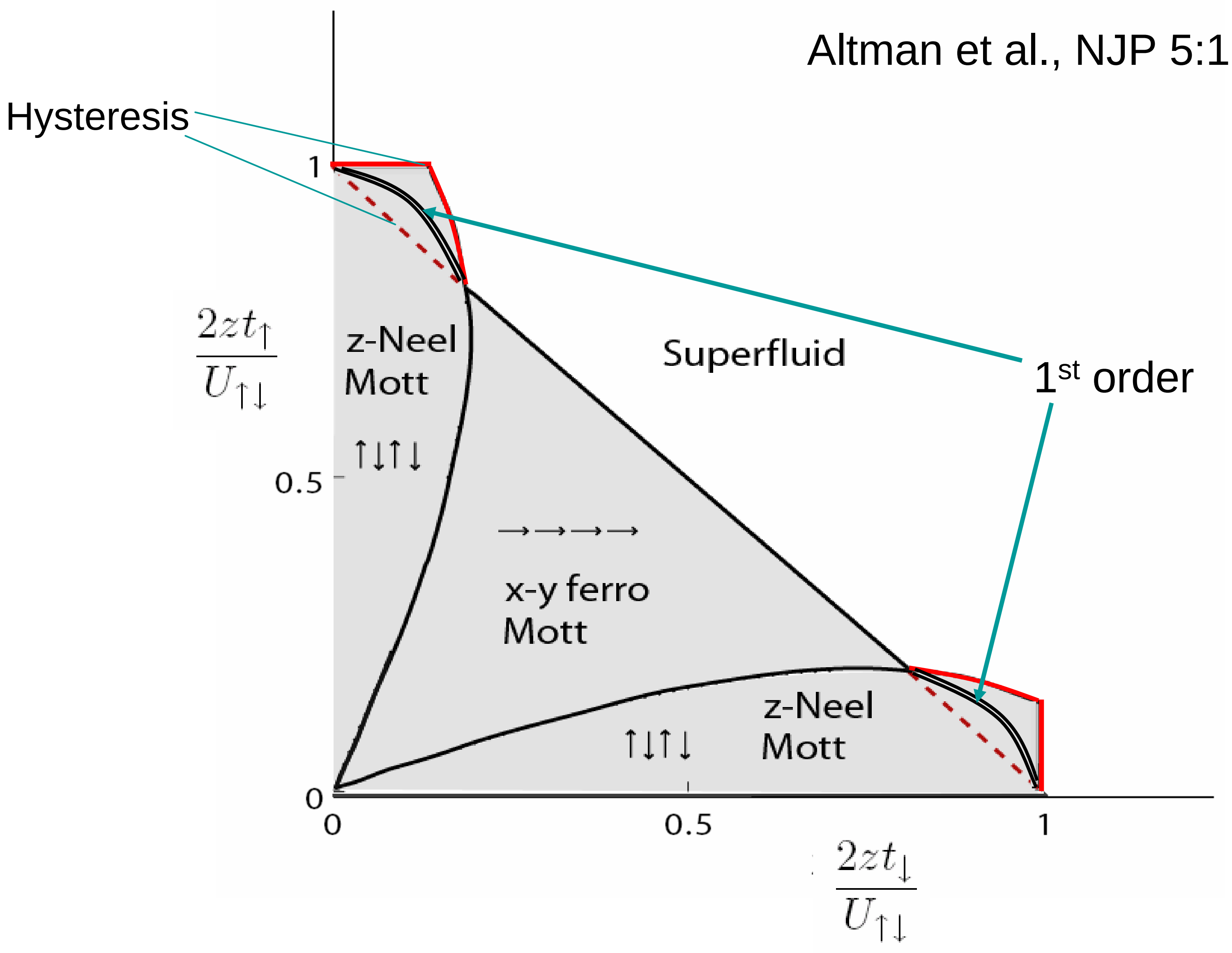


Coulomb energy dominates: ferromagnetic state

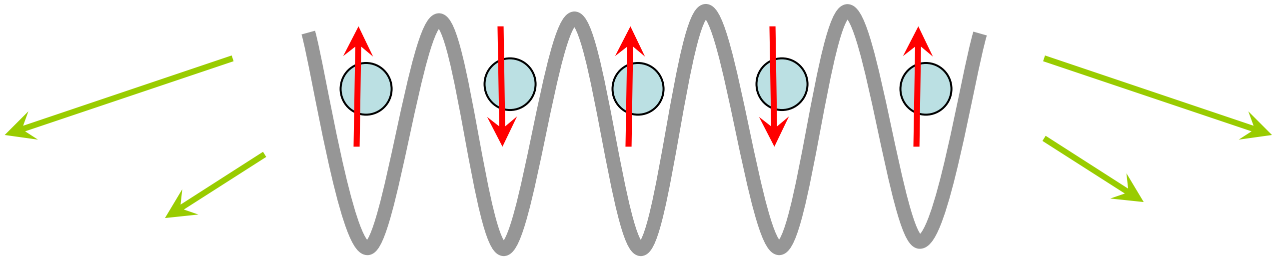


Two Component Bose Mixture in Optical Lattice.
Mean Field Theory + Quantum Fluctuations

Altman et al., NJP 5:113 (2003)



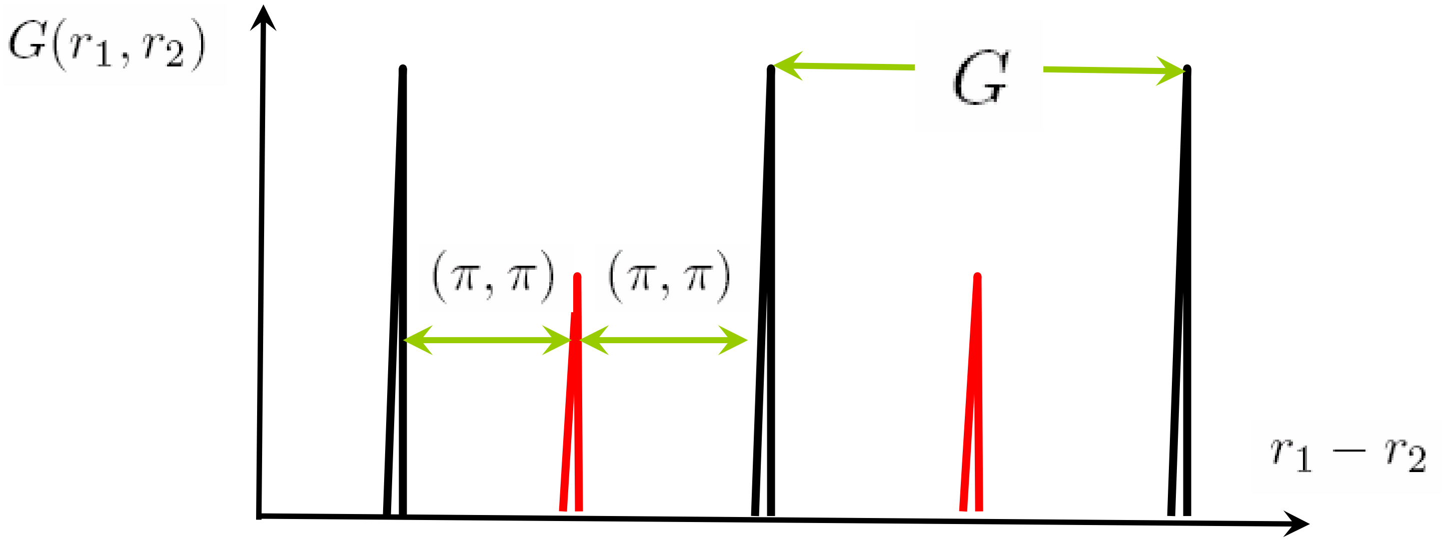
Probing Spin Order of Bosons



$$r = \frac{\hbar\ k\ t}{m}$$

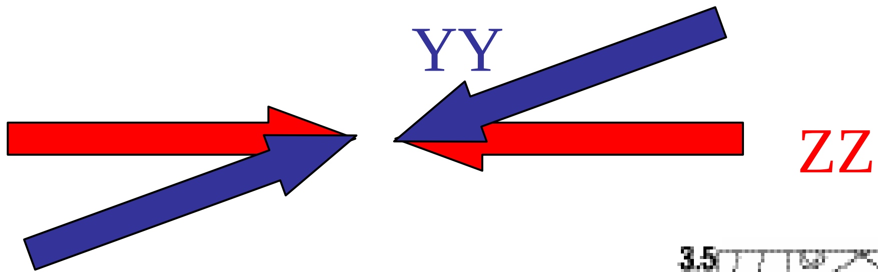
Correlation Function Measurements

$$\begin{aligned} G(r_1, r_2) &= \langle\ n(r_1)\ n(r_2)\ \rangle_{\mathrm{TOF}} - \langle\ n(r_1)\ \rangle_{\mathrm{TOF}}\ \langle\ n(r_2)\ \rangle_{\mathrm{TOF}} \\ &\sim \langle\ n(k_1)\ n(k_2)\ \rangle_{\mathrm{LAT}} - \langle\ n(k_1)\ \rangle_{\mathrm{LAT}}\ \langle\ n(k_2)\ \rangle_{\mathrm{LAT}} \end{aligned}$$



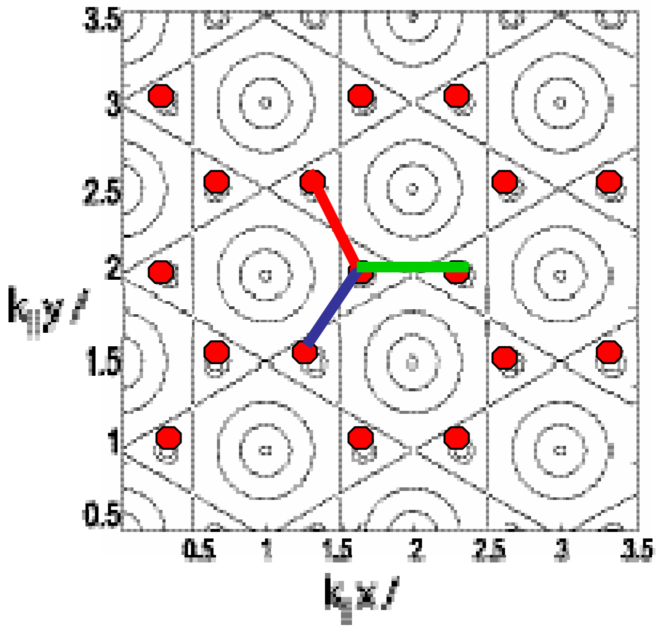
Engineering exotic phases

- Optical lattice in 2 or 3 dimensions: polarizations & frequencies of standing waves can be different for different directions



- Example: exactly solvable model Kitaev (2002), honeycomb lattice with

$$H = J_x \sum_{<i,j>\in x} \sigma_i^x \sigma_j^x + J_y \sum_{<i,j>\in y} \sigma_i^y \sigma_j^y + J_z \sum_{<i,j>\in z} \sigma_i^z \sigma_j^z$$



- Can be created with 3 sets of standing wave light beams !
- Non-trivial topological order, “spin liquid” + non-abelian anyons

…those has not been seen in controlled experiments

Spin F=1 Bosons in Optical Lattices

Spin exchange interactions. Exotic spin order (nematic)

Spinor Condensates in Optical Traps

Spin symmetric interaction of F=1 atoms

$$U(r_1 - r_2) = \delta(r_1 - r_2) \; (\; W_0 + W_2 \; \vec{S}_1 \vec{S}_2 \;)$$

$$W_2 = \frac{4\pi\hbar^2}{3m} \; (\; a_2 - a_0 \;)$$

Ferromagnetic Interactions for $W_2 < 0$

$^{87}\mathrm{Rb}$

$$a_0 = 110 \pm 4 \; a_\mathrm{B}$$

$$a_2 = 107 \pm 4 \; a_\mathrm{B}$$

Antiferromagnetic Interactions for $W_2 > 0$

$^{23}\mathrm{Na}$

$$a_0 = 46 \pm 5 a_\mathrm{B}$$

$$a_2 = 52 \pm 5 a_\mathrm{B}$$

Antiferromagnetic F=1 Condensates

Three species of atoms

$$\hat{a}_x \pm i\ \hat{a}_y = \hat{a}_{\pm 1} \qquad \hat{a}_z = \hat{a}_0$$

Mean Field

Ho, PRL 81:742 (1998) **Ohmi, Machida, JPSJ 67:1822 (1998)**

$$|\Psi\rangle = \left(n_x a_x^\dagger + n_y a_y^\dagger + n_z a_z^\dagger \right)^N |0\rangle$$

Beyond Mean Field. Spin Singlet Ground State

Law et al., PRL 81:5257 (1998); Ho, Yip, PRL 84:4031 (2000)

$$|\Psi\rangle = \left(a_x^{\dagger\,2} + a_y^{\dagger\,2} + a_z^{\dagger\,2} \right)^{N/2} |0\rangle$$

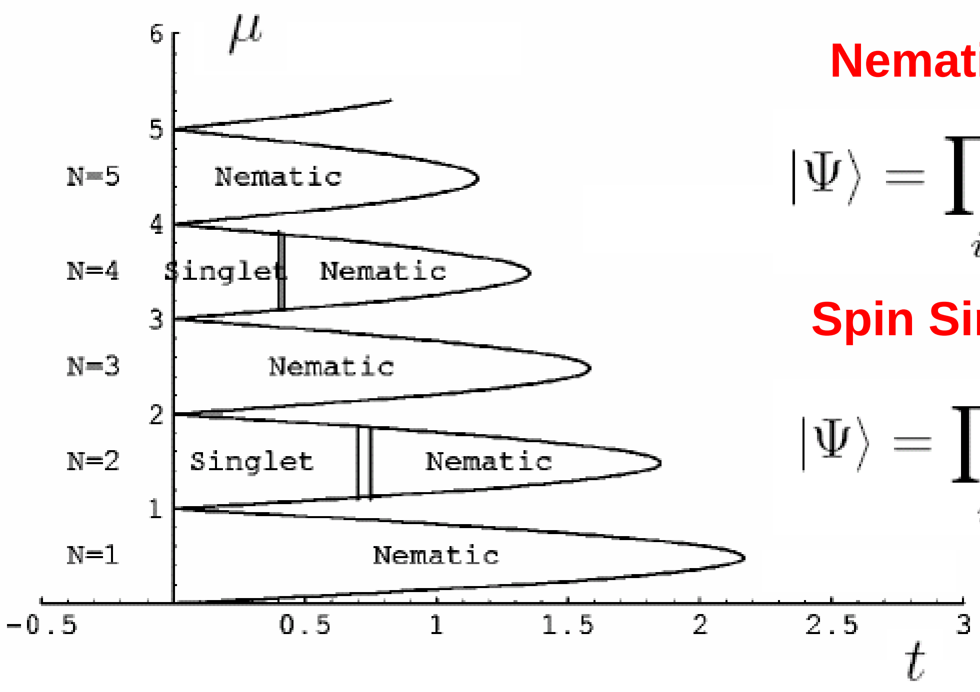
Experiments: Review in Ketterle's Les Houches notes

Antiferromagnetic Spin F=1 Atoms in Optical Lattices

Hubbard Hamiltonian **Demler, Zhou, PRL (2003)**

$$\mathcal{H} = -t \sum_{\langle ij \rangle} a^\dagger_{im} a_{jm} + U_0 \sum_i n_i^2 + U_2 \sum_i \vec{S}_i^2 - \mu \sum_i n_i$$

Symmetry constraints $n_i + S_i = \text{even}$



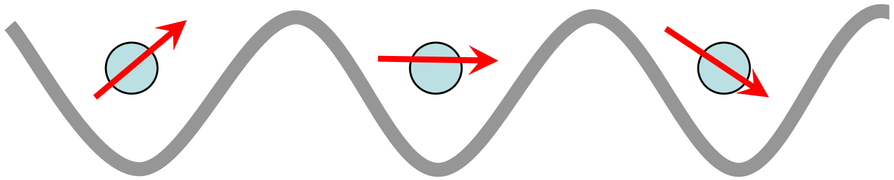
Nematic Mott Insulator

$$|\Psi\rangle = \prod_i (n_x a^\dagger_{ix} + n_y a^\dagger_{iy} + n_z a^\dagger_{iz})^N |0\rangle$$

Spin Singlet Mott Insulator

$$|\Psi\rangle = \prod_i (a^{\dagger 2}_{ix} + a^{\dagger 2}_{iy} + a^{\dagger 2}_{iz})^{N/2} |0\rangle$$

Nematic Insulating Phase for N=1



Effective S=1 spin model **Imambekov et al., PRA 68:63602 (2003)**

$$\mathcal{H} = -J_1 \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j - J_2 \sum_{\langle ij \rangle} \left(\vec{S}_i \vec{S}_j \right)^2$$

$$J_1 = \frac{2t^2}{U_0 + U_2} \qquad J_2 = \frac{2t^2}{3(U_0 + U_2)} + \frac{4t^2}{3(U_0 - U_2)}$$

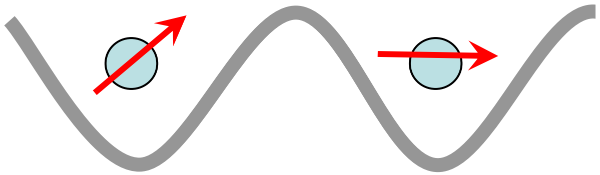
When $J_2 > J_1$ **the ground state is nematic in d=2,3.**

$$\langle S_a \rangle = 0 \qquad \langle S_a S_b \rangle \neq 0$$

One dimensional systems are dimerized: Rizzi et al., cond-mat/0506098

Nematic Insulating Phase for N=1.

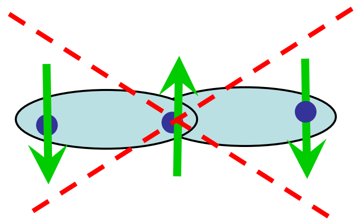
Two Site Problem



$$\mathcal{H} = -J_1 \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j - J_2 \sum_{\langle ij \rangle} \left(\vec{S}_i \vec{S}_j \right)^2$$

S_{tot}	$\vec{S}_1 \vec{S}_2$	$\left(\vec{S}_1 \vec{S}_2\right)^2$
2	**1**	**1**
0	**-2**	**4**

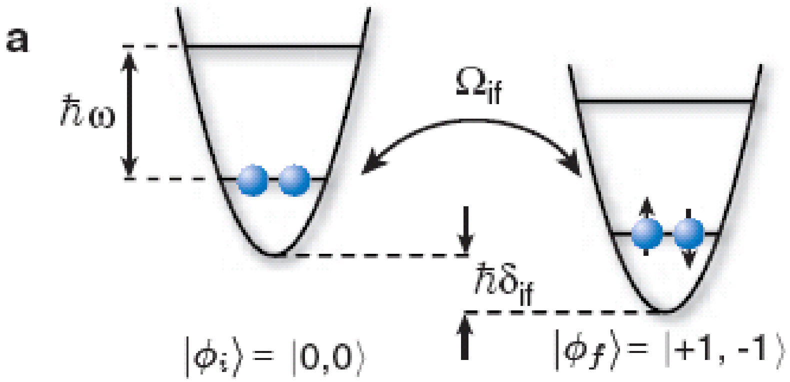
Singlet state is favored when $J_2 > J_1$



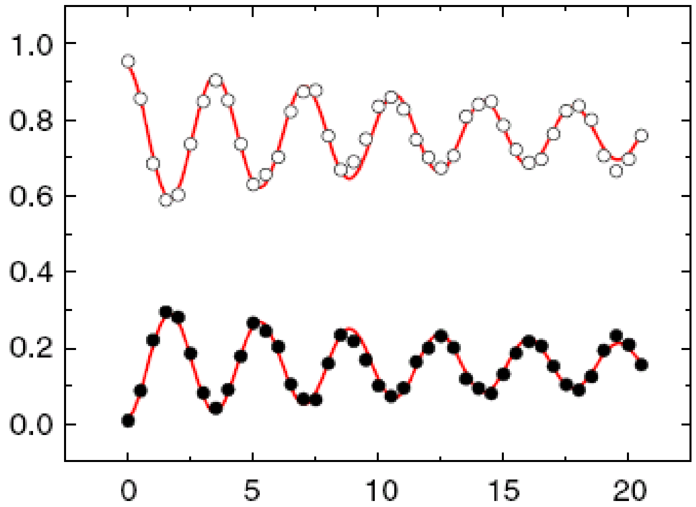
One can not have singlets on neighboring bonds. Nematic state is a compromise. It corresponds to a superposition of $S_{\text{tot}} = 0$ **and** $S_{\text{tot}} = 2$ **on each bond**

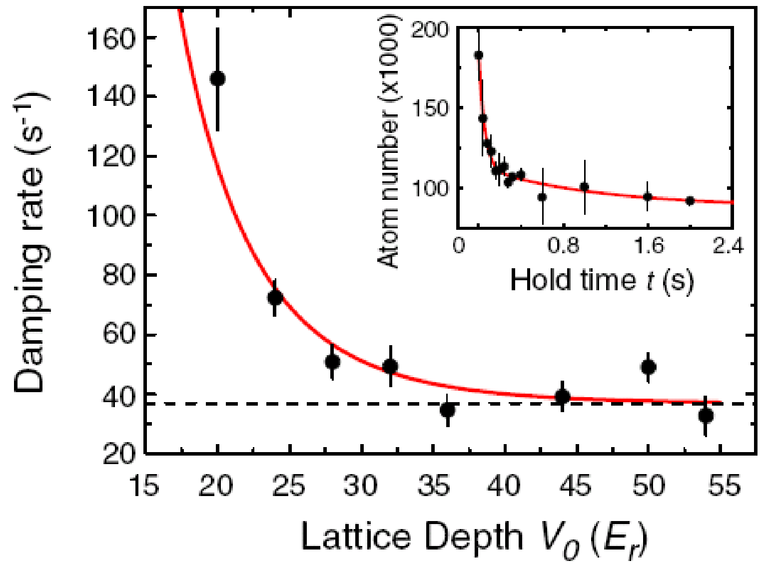
Coherent Spin Dynamics in Optical Lattices

Widera et al., cond-mat/0505492



$^{87}\mathrm{Rb}$ **atoms in the F=2 state**



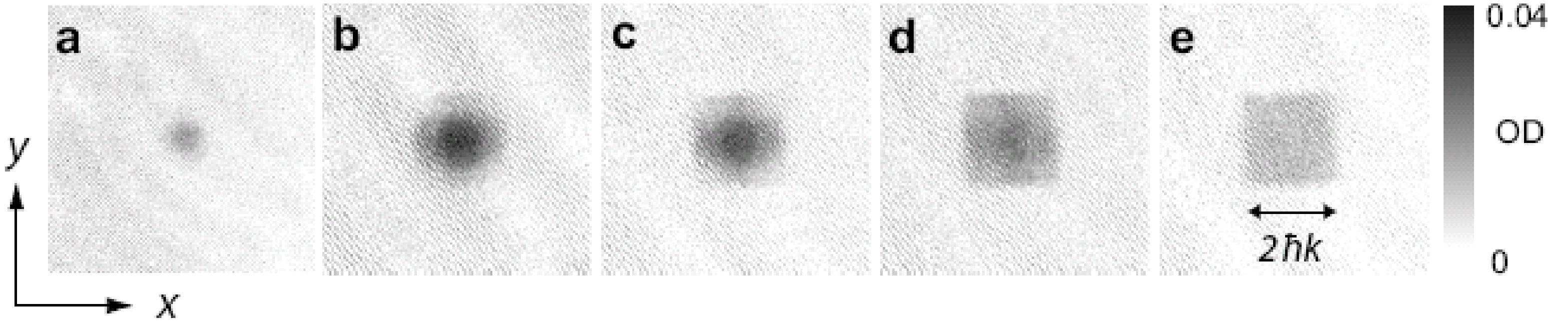


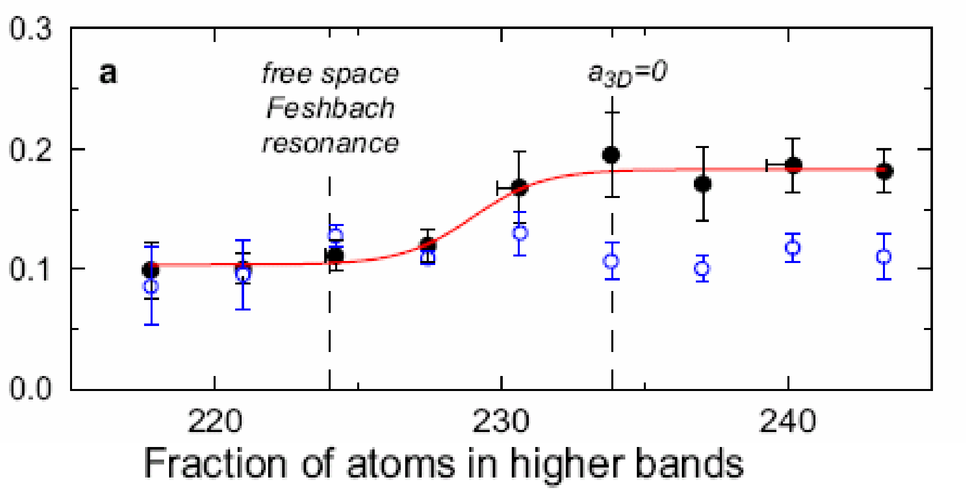
Fermionic Atoms in Optical Lattices

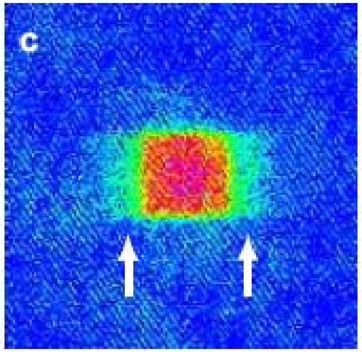
Pairing in systems with repulsive interactions.
Unconventional pairing. High Tc mechanism

Fermionic Atoms in a Three Dimensional Optical Lattice

Kohl et al., PRL 94:80403 (2005)



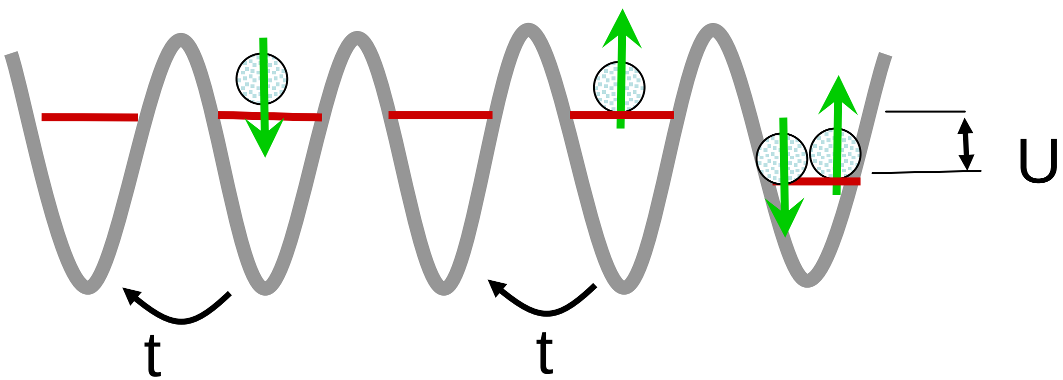




Fermions with Attractive Interaction

Hofstetter et al., PRL 89:220407 (2002)

$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$



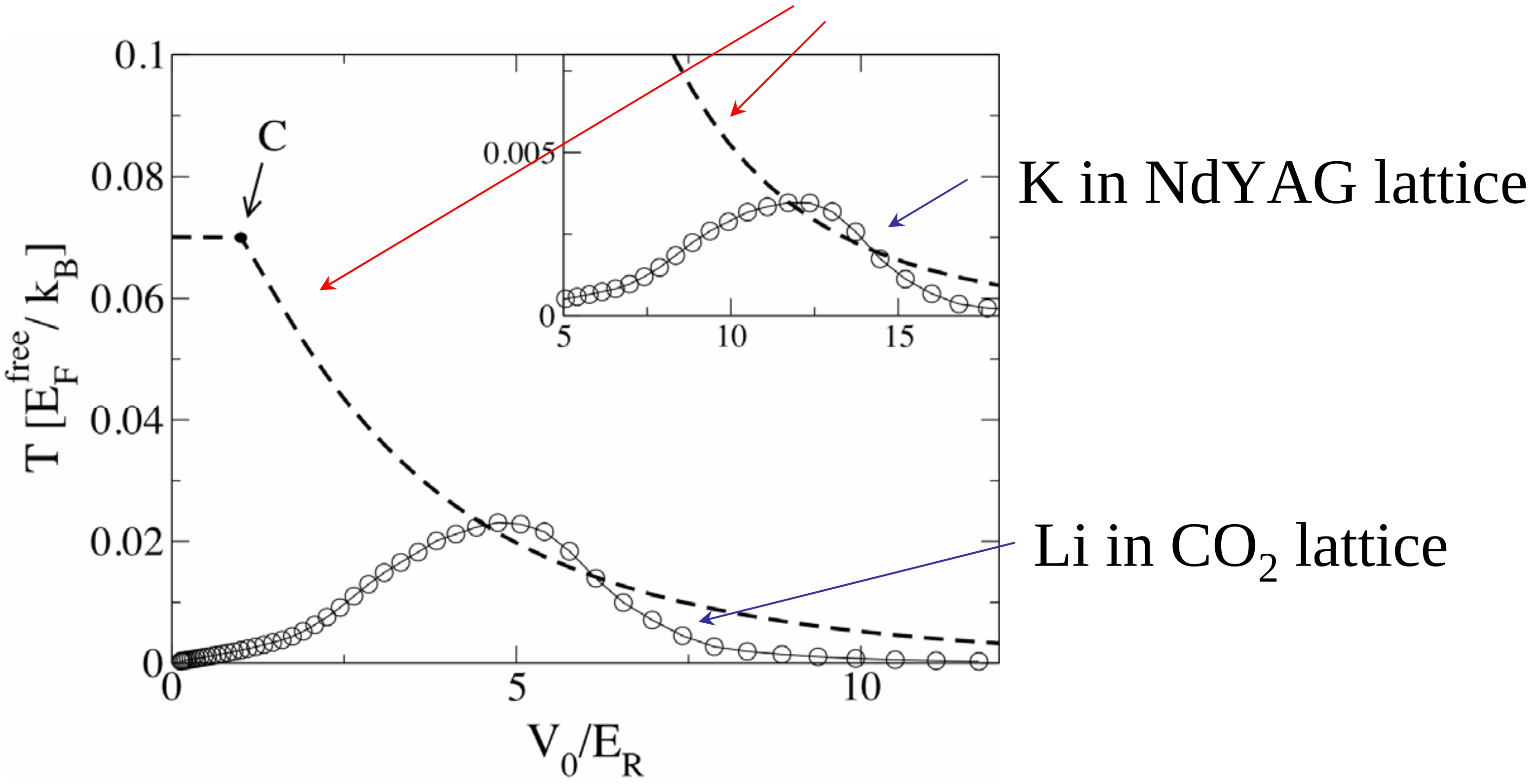
Highest transition temperature for $t \sim |U|$

$$T_\mathrm{c} \sim T_\mathrm{F}^\mathrm{free}\, n^{\frac{1}{3}}\, |a_\mathrm{s}|$$

Compare to the exponential suppresion of Tc w/o a lattice

Reaching BCS Superfluidity in a Lattice

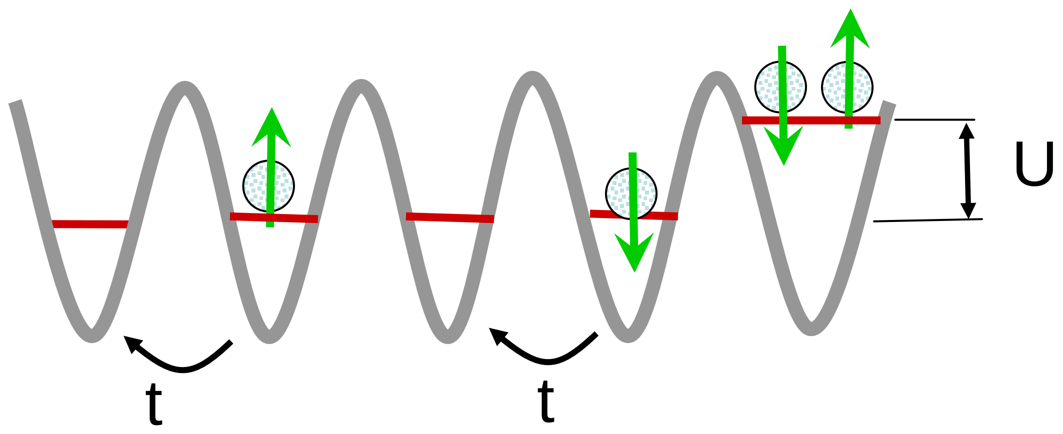
Turning on the lattice reduces the effective atomic temperature



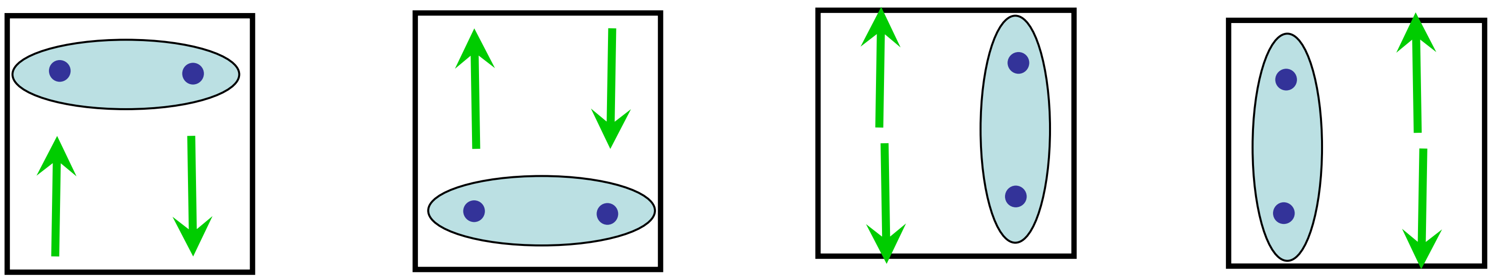
Superfluidity can be achived even with a modest scattering length

Fermions with Repulsive Interactions

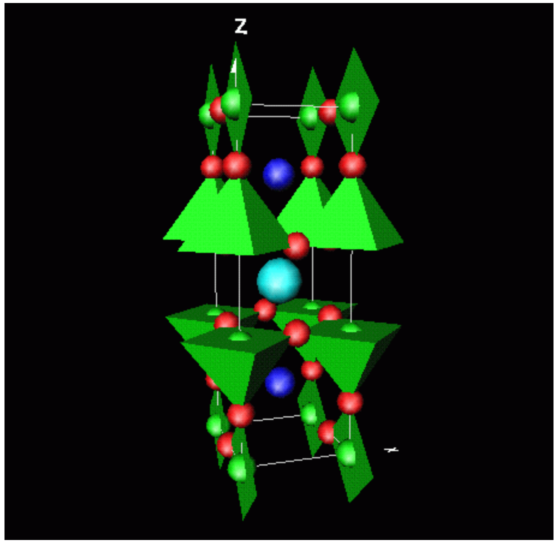
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$



Possible d-wave pairing of fermions

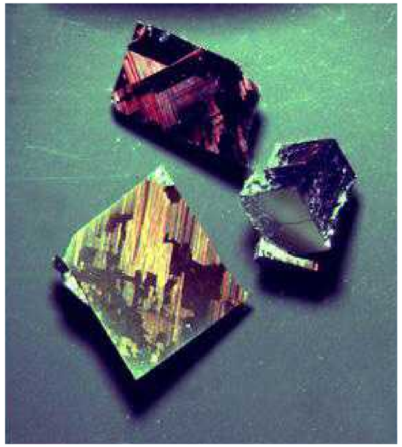


High Temperature Superconductors



$YBa_2Cu_3O_7$

Superconducting
Tc 93 K



Picture courtesy of UBC Superconductivity group

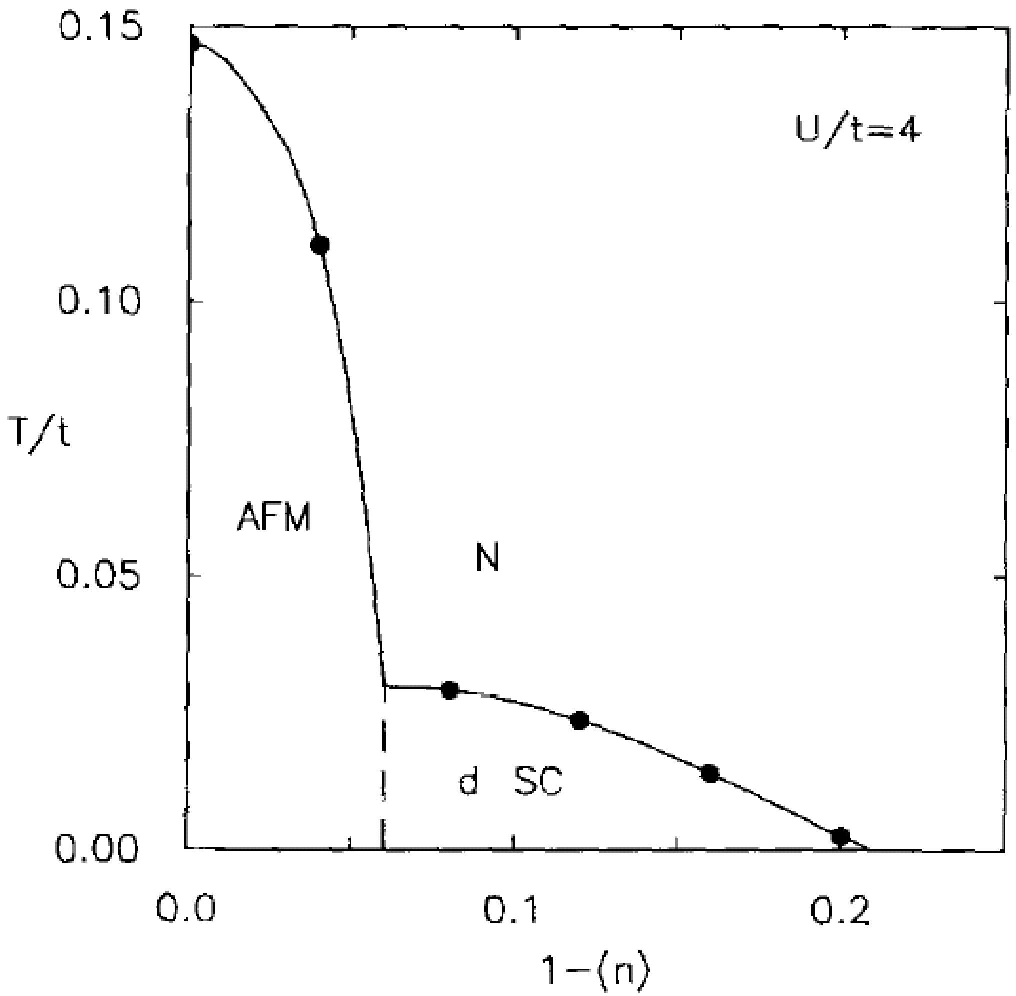
Hubbard model – minimal model for cuprate superconductors

P.W. Anderson, cond-mat/0201429

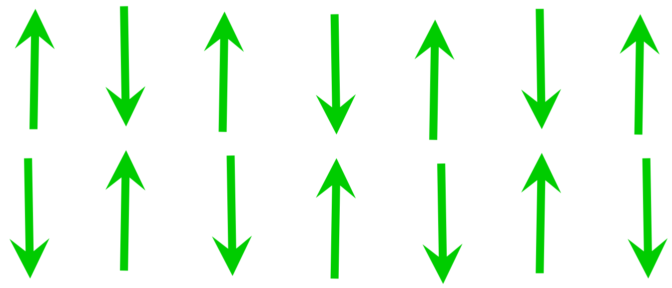
After many years of work we still do not understand the fermionic Hubbard model

Positive U Hubbard Model

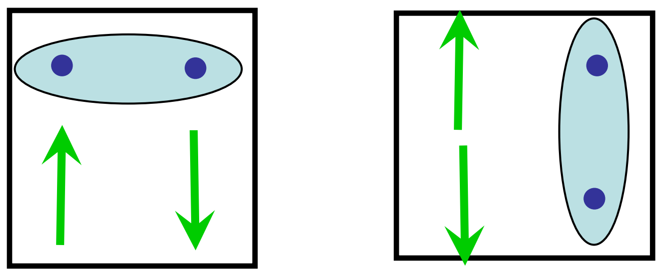
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)



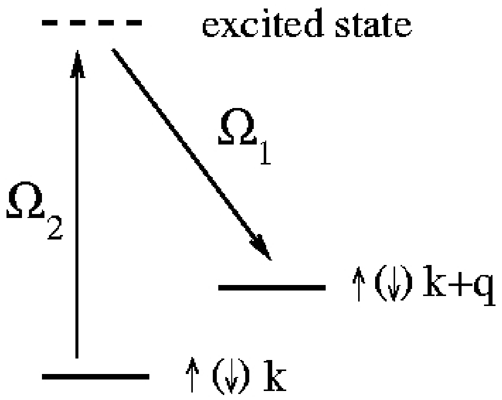
Antiferromagnetic insulator



D-wave superconductor



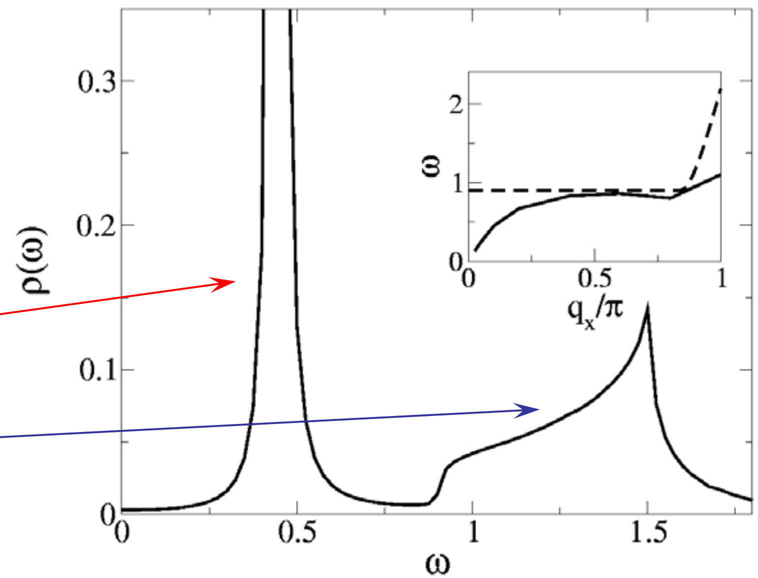
Superfluidity of Fermions in Optical Lattices. Probing excitation spectrum: Bragg scattering



- Pair of non-collinear laser beams create atomic excitation with given frequency and momentum
- Number of excited atoms:

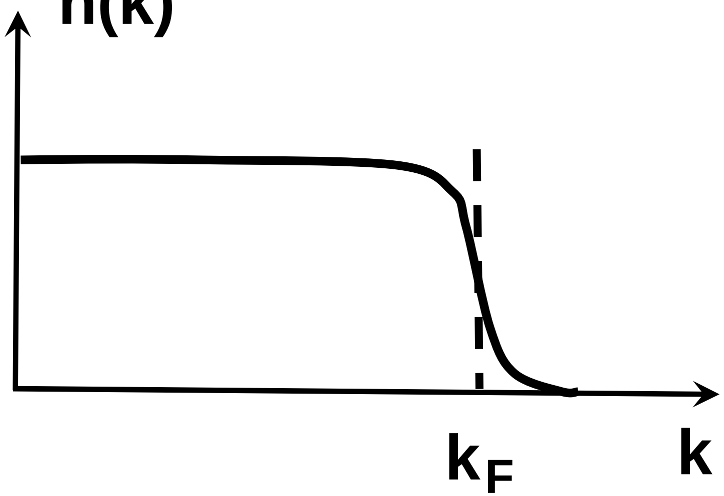
In superfluid phase:

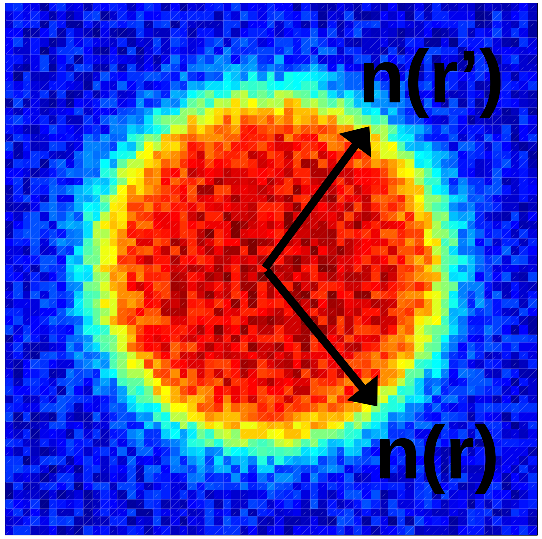
- energy gap
- sharp collective mode
- broad quasiparticle "continuum"

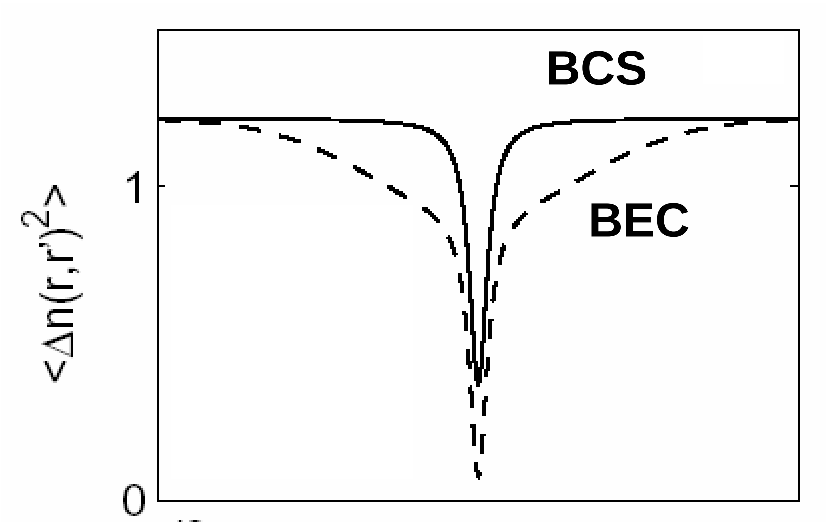


$$q_x = q_y = 0.1\pi, \quad n = 0.6, \quad U/t = -2.5$$

Second Order Interference from the BCS Superfluid





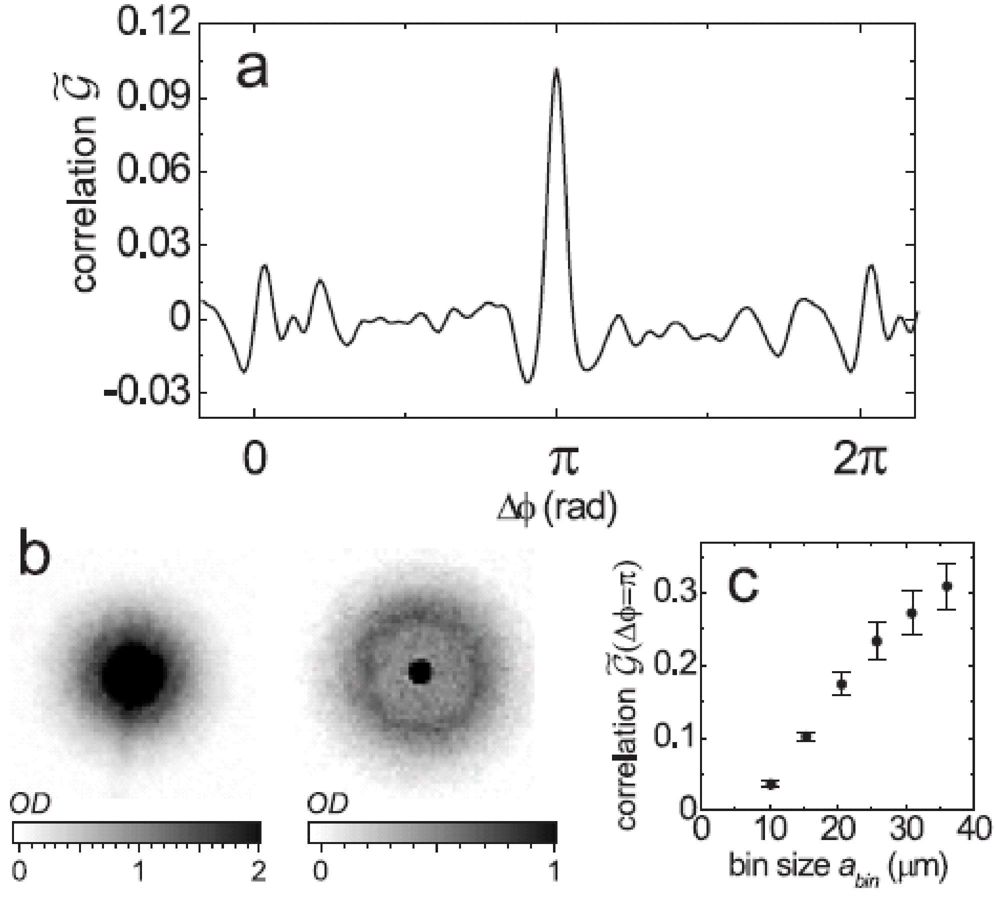


$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

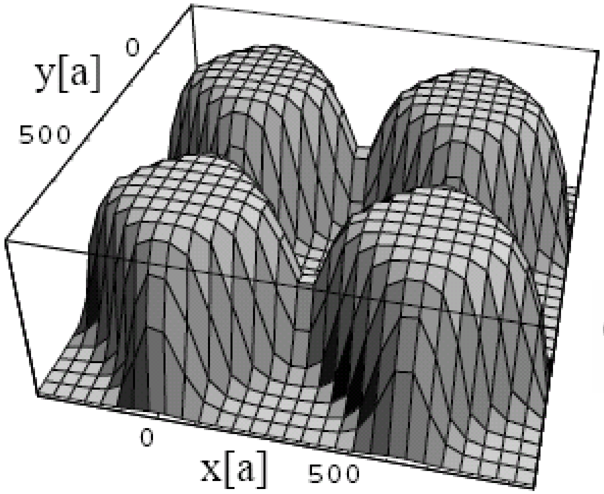
$$\Delta n(\mathbf{r},-\mathbf{r}) \left| \Psi_{BCS} \right\rangle = 0$$

Momentum Correlations in Paired Fermions

Greiner et al., PRL 94:110401 (2005)



Fermion Pairing in an Optical Lattice



Second Order Interference
In the TOF images

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

Normal State

$$G_{\mathrm{N}}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

Superfluid State

$$G_{\mathrm{S}}(r_1, r_2) = G_{\mathrm{N}}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

$$Q(r) = \frac{mr}{\hbar t}$$

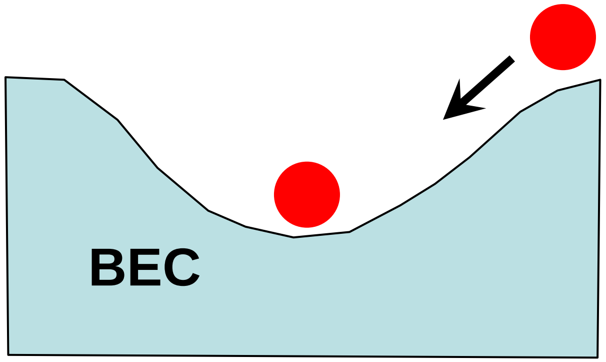
One can identify unconventional pairing

Boson Fermion Mixtures

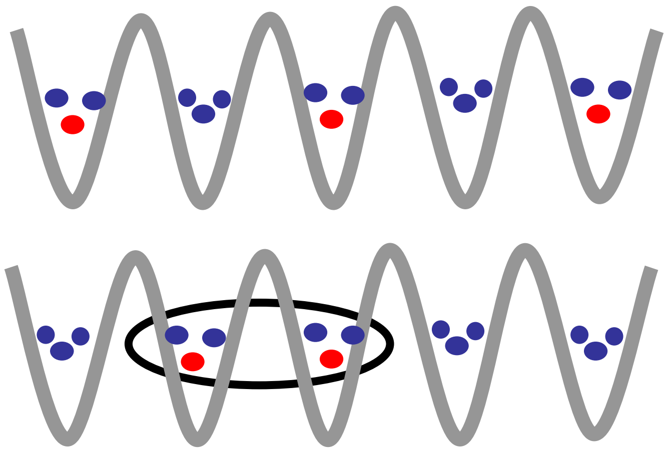
Polarons. Competing orders

Boson Fermion Mixtures

Experiments: ENS, Florence, JILA, MIT, Rice, …



Bosons provide cooling for fermions and mediate interactions. They create non-local attraction between fermions



Charge Density Wave Phase

Periodic arrangement of atoms

Non-local Fermion Pairing

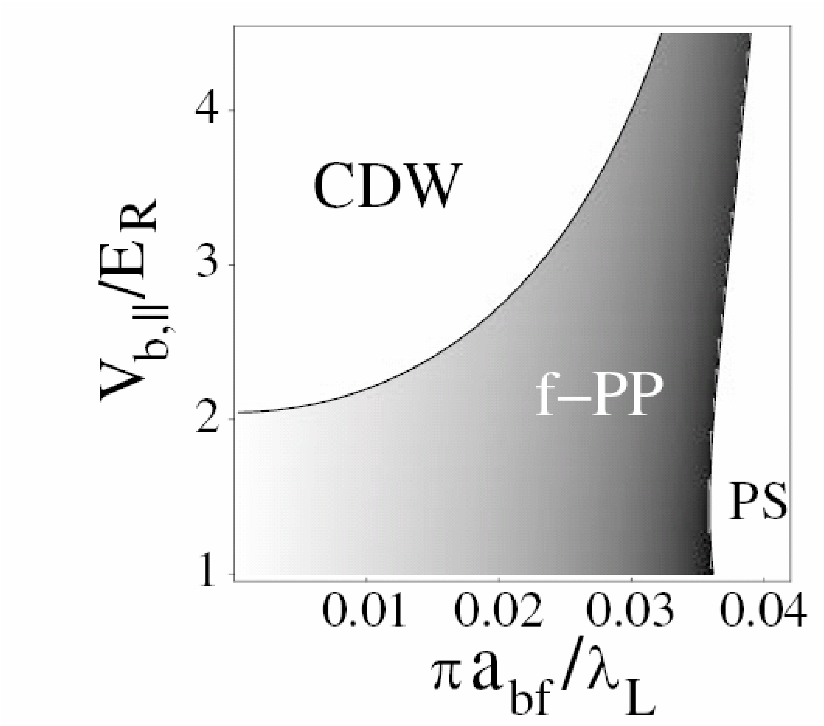
P-wave, D-wave, …

BF Mixtures in 1d Optical Lattices

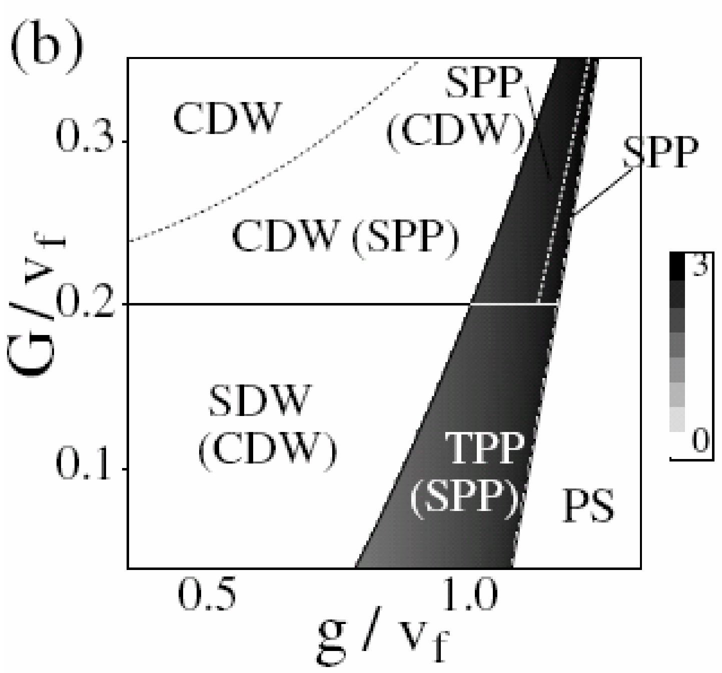
Cazalila et al., PRL (2003); Mathey et al., PRL (2004)

$$\mathcal{H} = -t_b \sum_{\langle ij \rangle} b_i^\dagger b_j - t_f \sum_{\langle ij \rangle} f_i^\dagger f_j - \mu_b \sum_i n_{bi} - \mu_f \sum_i n_{fi} + U_{bb} \sum_i n_{bi}^2 + U_{bf} \sum_i n_{bi} n_{fi}$$

Spinless fermions



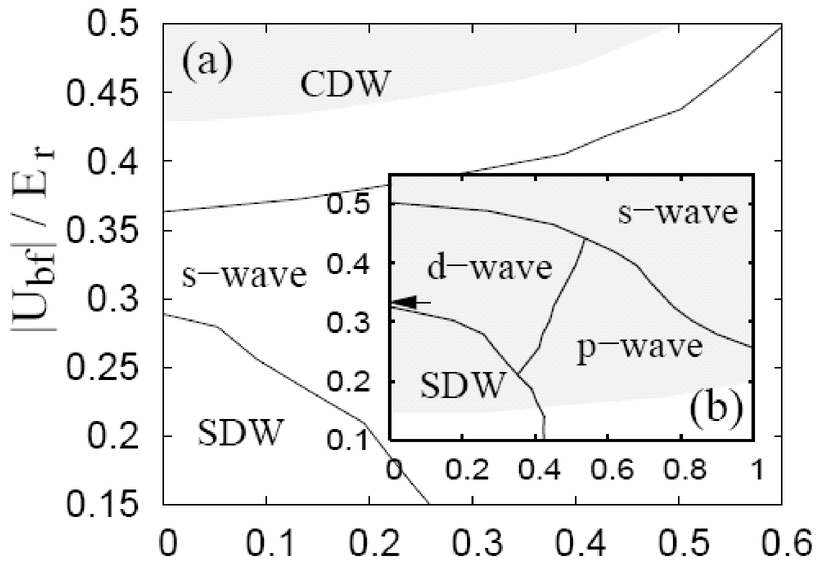
Spin ½ fermions



BF Mixtures in 2d Optical Lattices

Wang et al., cond-mat/0410492

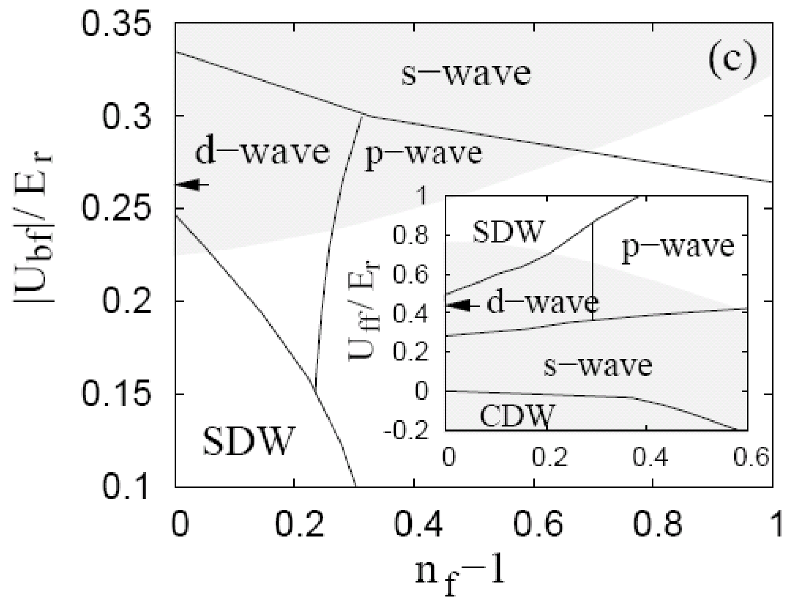
40K -- 87Rb



(a) λ**=1060nm**

(b) λ **=765.5nm**

40K -- 23Na



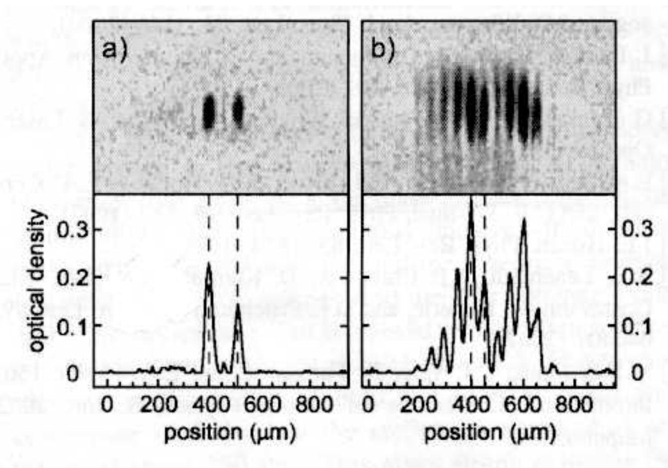
λ **=1060 nm**

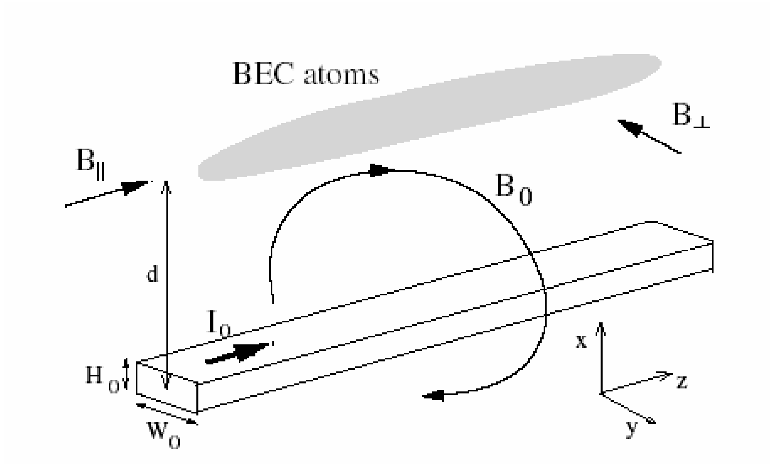
BEC on microchips.

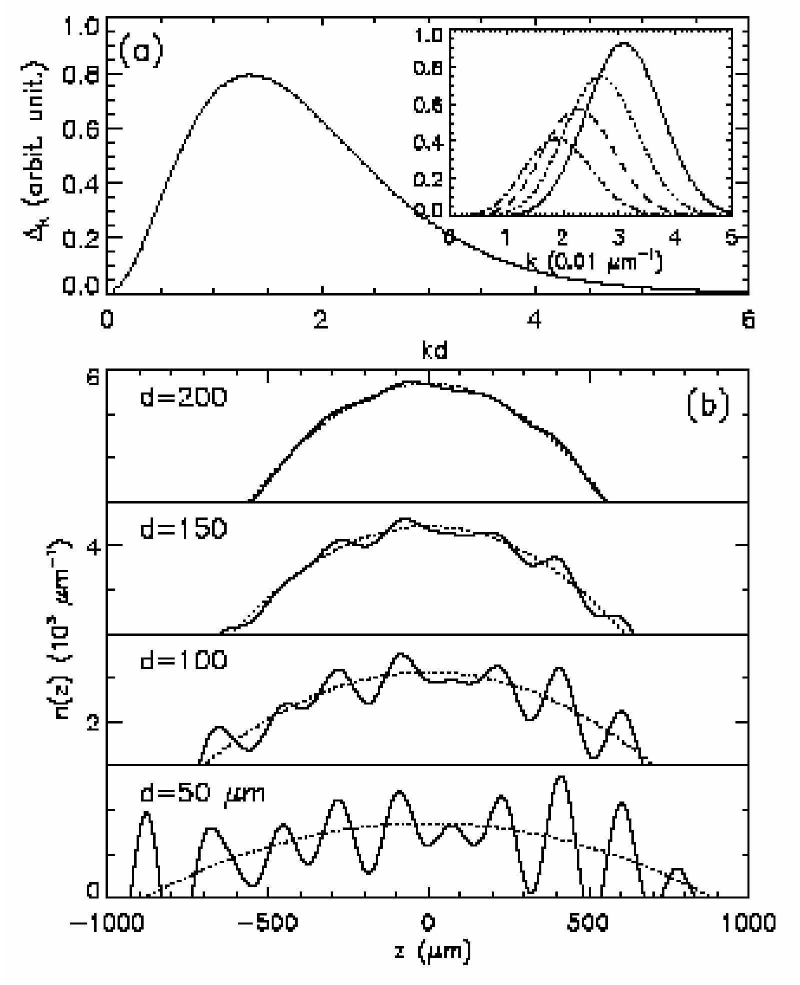
Interplay of disorder and interactions. Bose glass phase

Fragmented BEC in magnetic microtraps

Thywissen et al., EPJD (1999); Kraft et al., JPB (2002);
Leanhardt et al., PRL (2002); Fortagh et al., PRA (2002); …





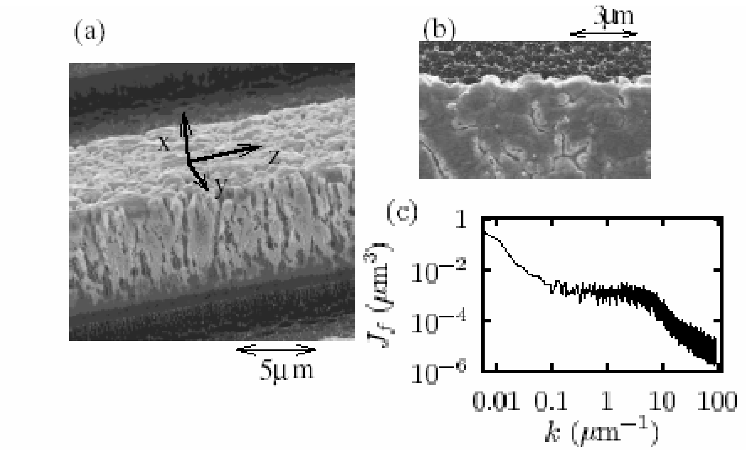


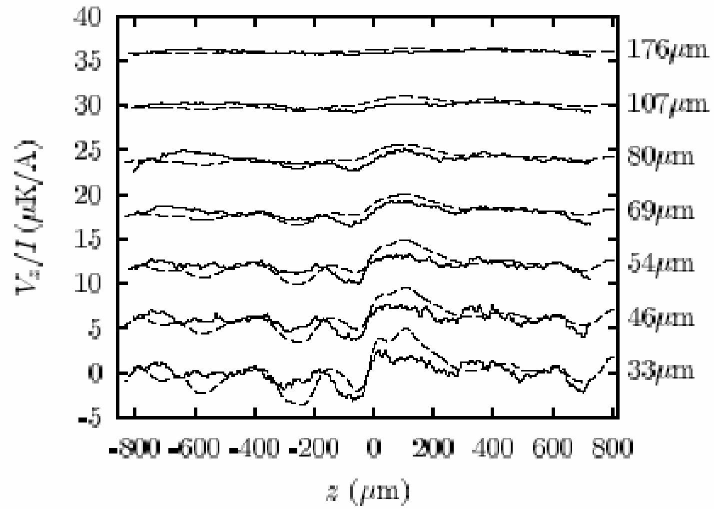
Theory: Wang et.al., PRL 92:076802 (2004)

BEC on atom chips.

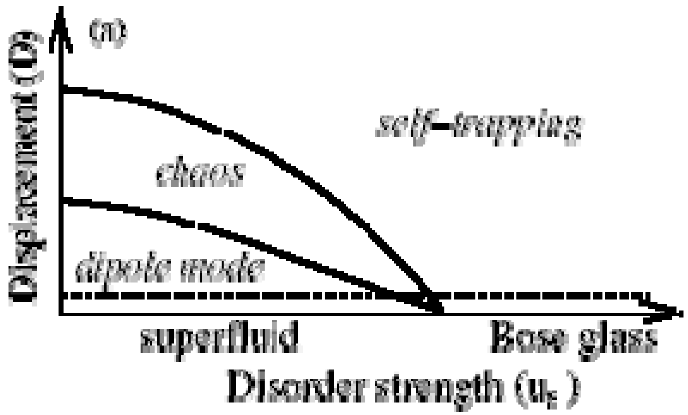
Esteve et al., PRA 70:43629 (2004)

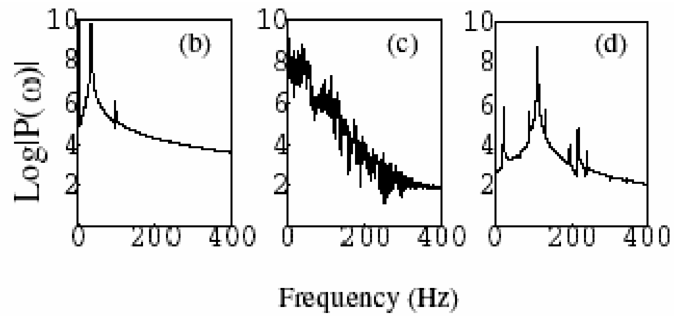
SEM image of wire





- Outlook: interplay of interactions and disorder: probing Bose glass phase





Conclusions:

Systems of cold atoms and molecules can be used to create several types of strongly correlated many-body systems. This opens interesting possibilities for

- **Simulating fundamental models in CM physics (e.g. Hubbard model)**
- **Understanding quantum magnetism**
- **Studying systems with unconventional fermion pairing**
- **Creating systems with topological order**
- **Understanding the interplay of disorder and interactions**
- **Engineering "quantum states"**