The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1666 - 29

SCHOOL ON QUANTUM PHASE TRANSITIONS
AND
NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Ultracold Atoms in optical lattice potentials

Presented by:

Markus Greiner

University of Colorado at Boulder, USA

ICTP SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES 2005

Ultracold Atoms in optical lattice potentials

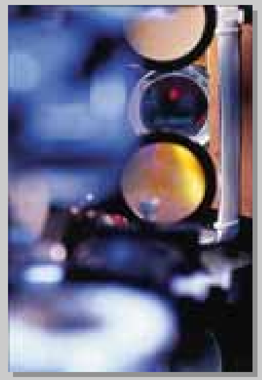
Experiments at the interface between atomic physics and condensed matter physics, quantum optics, molecular physics and quantum information

Markus Greiner

markus.greiner@colorado.edu

most experiments discussed in this lecture have been performed in the group of Ted Hänsch and I. Bloch at the

Ludwig-Maximilians-Universität, München and Max-Planck-Institut für Quantenoptik, Garching.



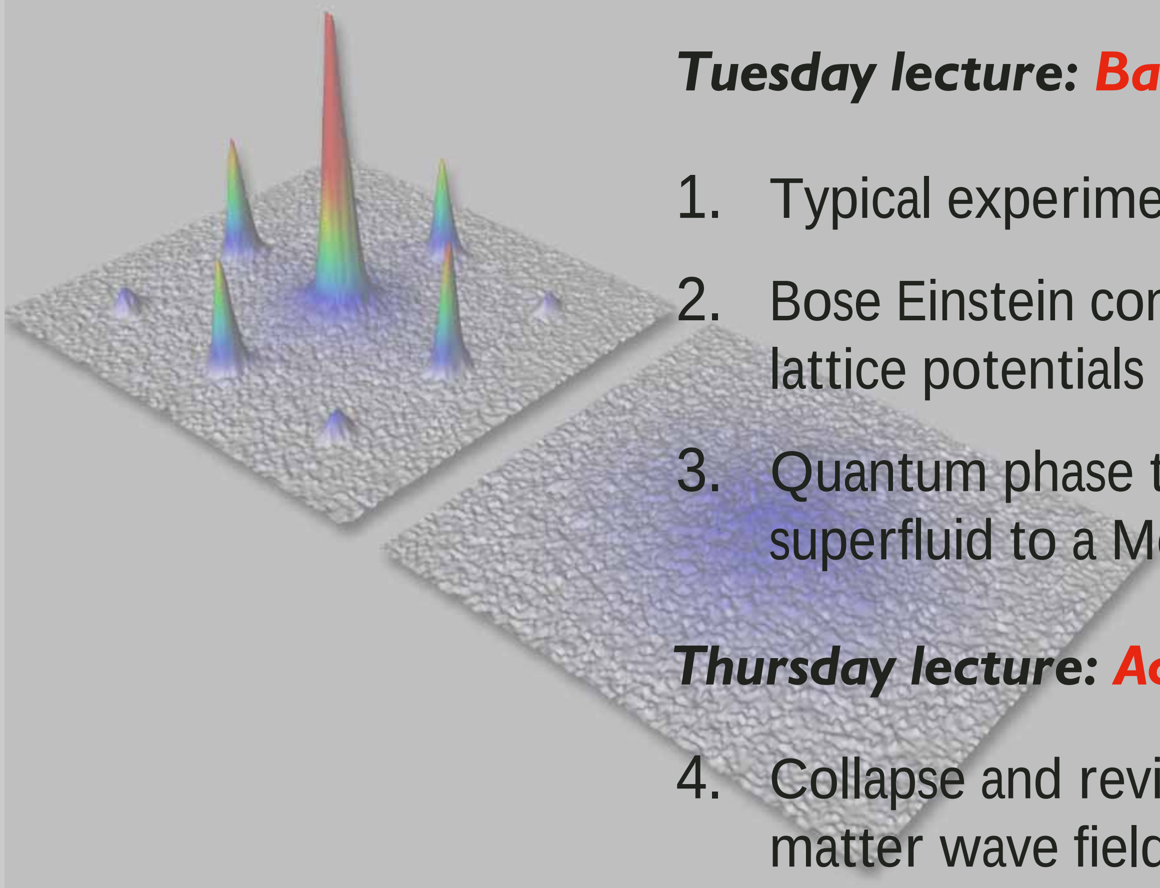
I am presently at JILA, Boulder, Co, in the group of D. Jin, working with fermionic condensates.

LMU

MPQ

Max-Planck-Institut für Quantenoptik

Ultracold Atoms in optical lattice potentials



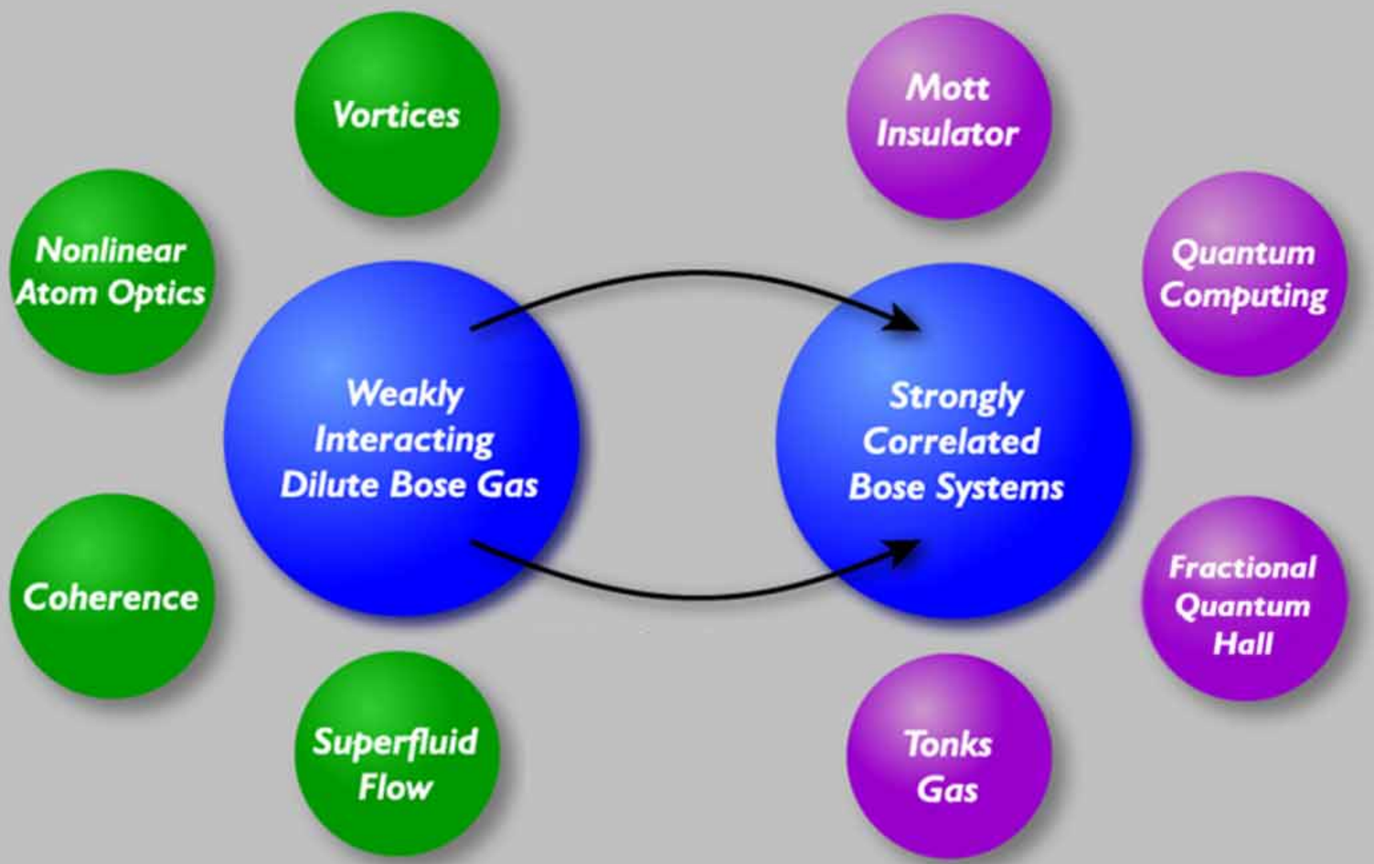
Tuesday lecture: Basics

1. Typical experimental setup
2. Bose Einstein condensates in optical lattice potentials
3. Quantum phase transition from a superfluid to a Mott insulator

Thursday lecture: Advanced topics

4. Collapse and revival of a macroscopic matter wave field due to cold collisions
5. Quantum gates with neutral atoms
6. Low dimensional systems

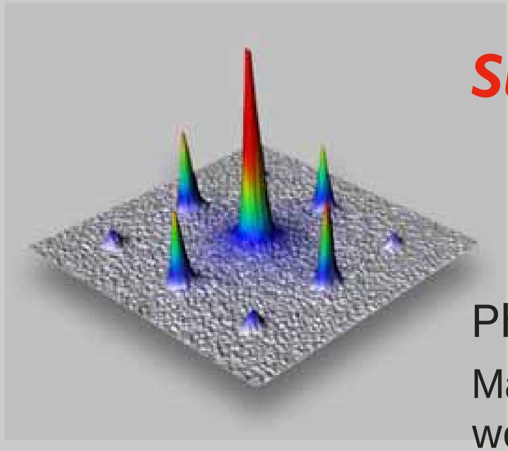
Introduction

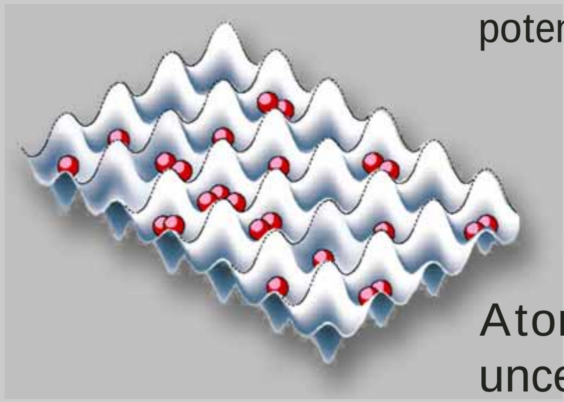


LMU

MPQ
Max-Planck-Institut für Quantenoptik

Superfluid – Mott Insulator Transition





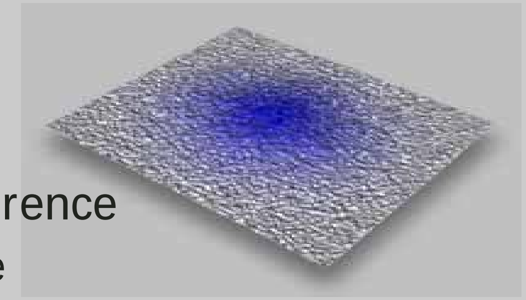
Superfluid

Phase coherence

Macroscopic phase well defined in each potential well

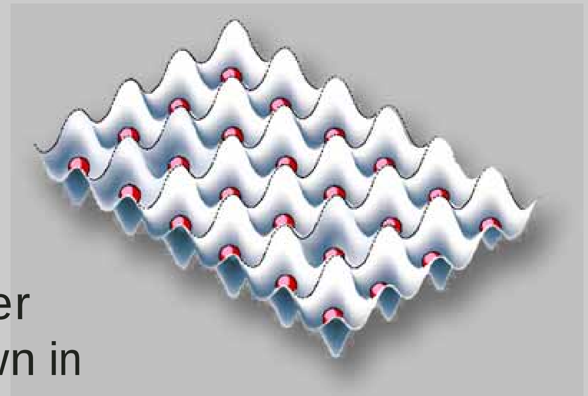
Atom number uncertain in each potential well

Mott Insulator



No Phase coherence

Macroscopic phase uncertain in each potential well

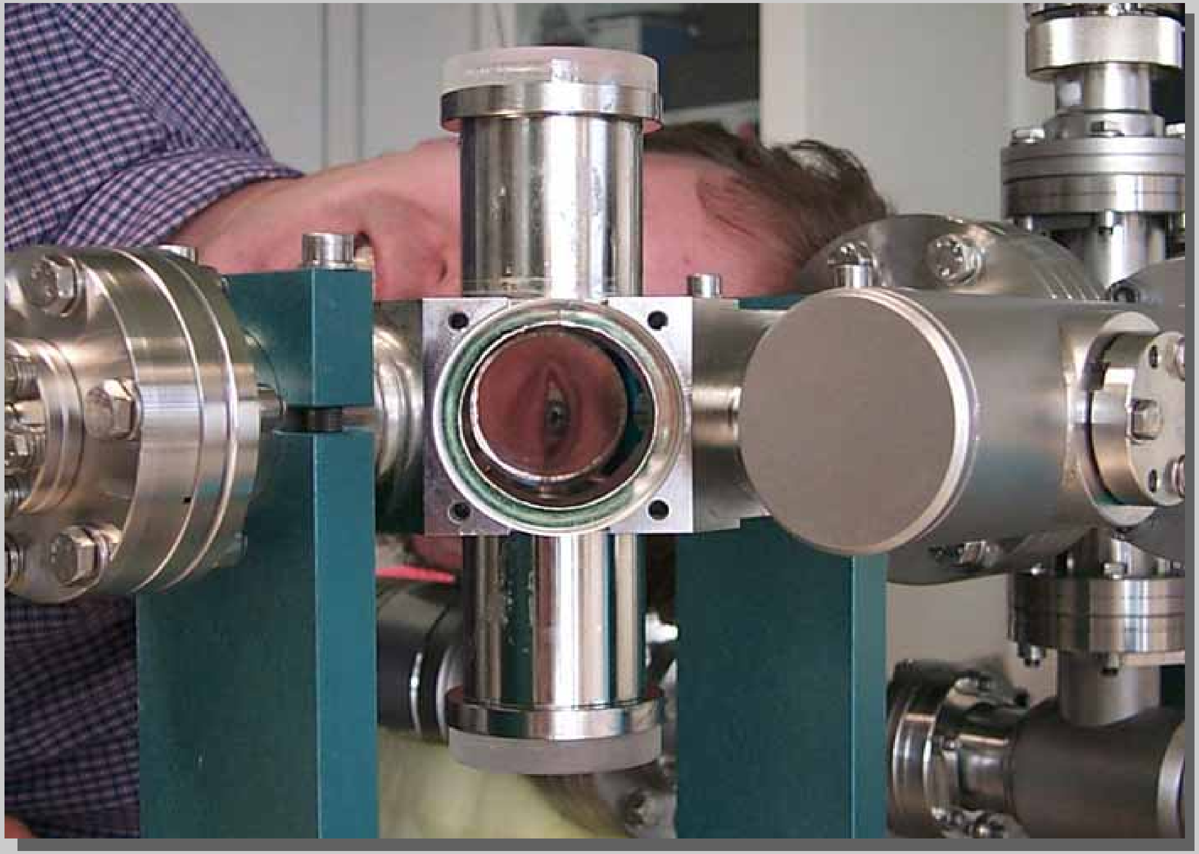


Atom number exactly known in each potential well

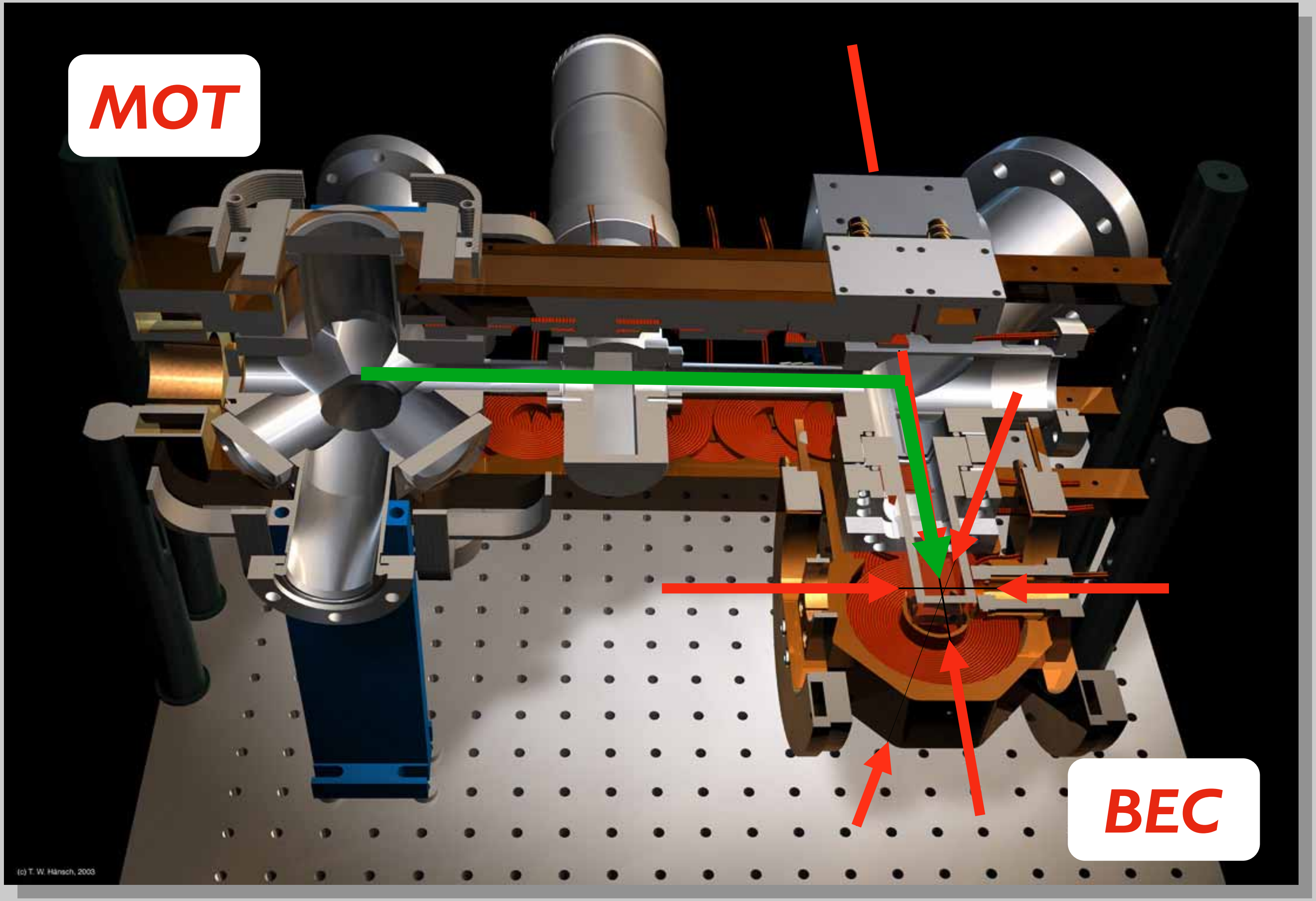
→ atom number correlations

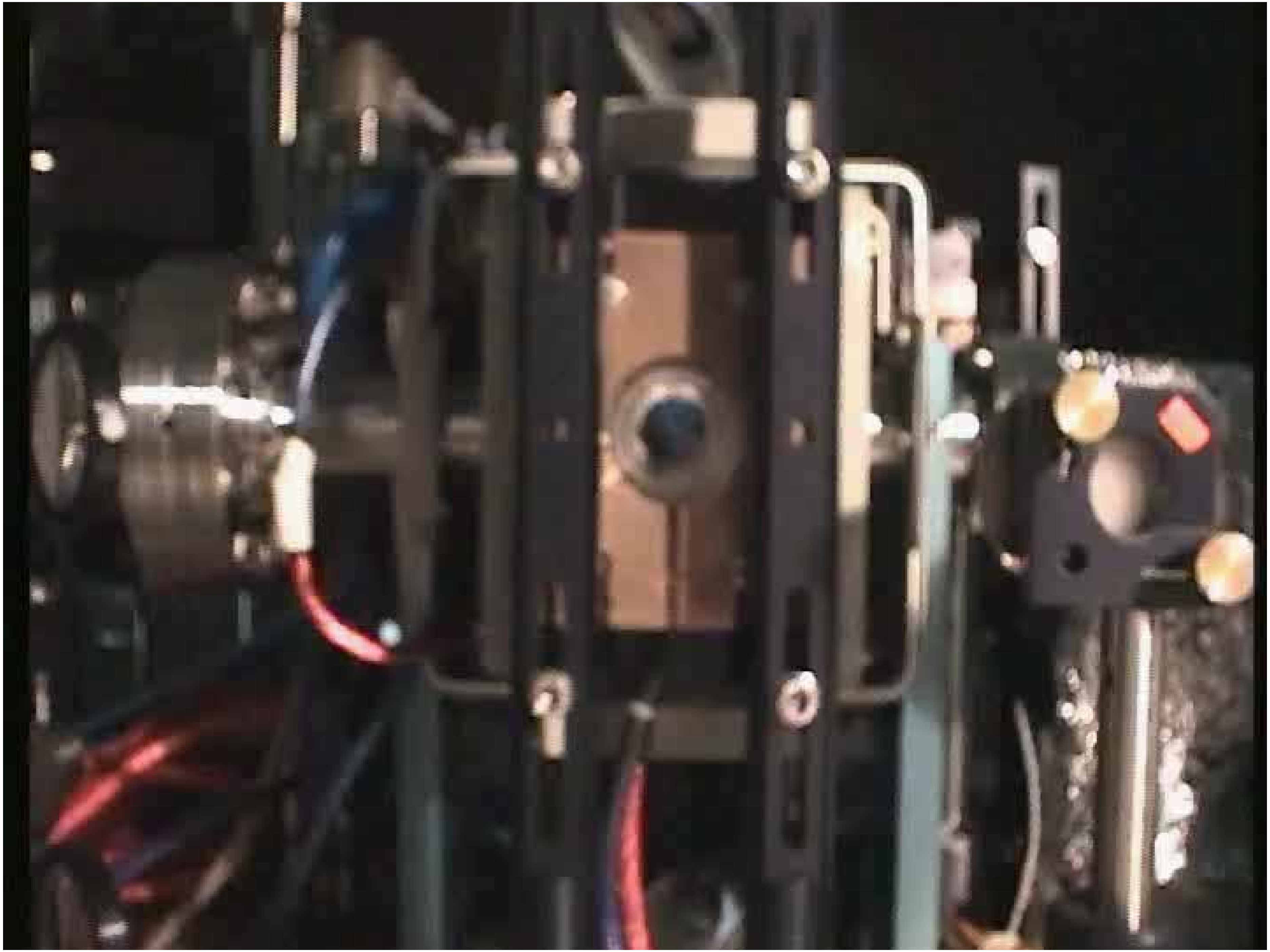
Theory: M.P.A. Fisher et al, Proposal: D. Jaksch et al.

1. Experimental setup for lattice experiments



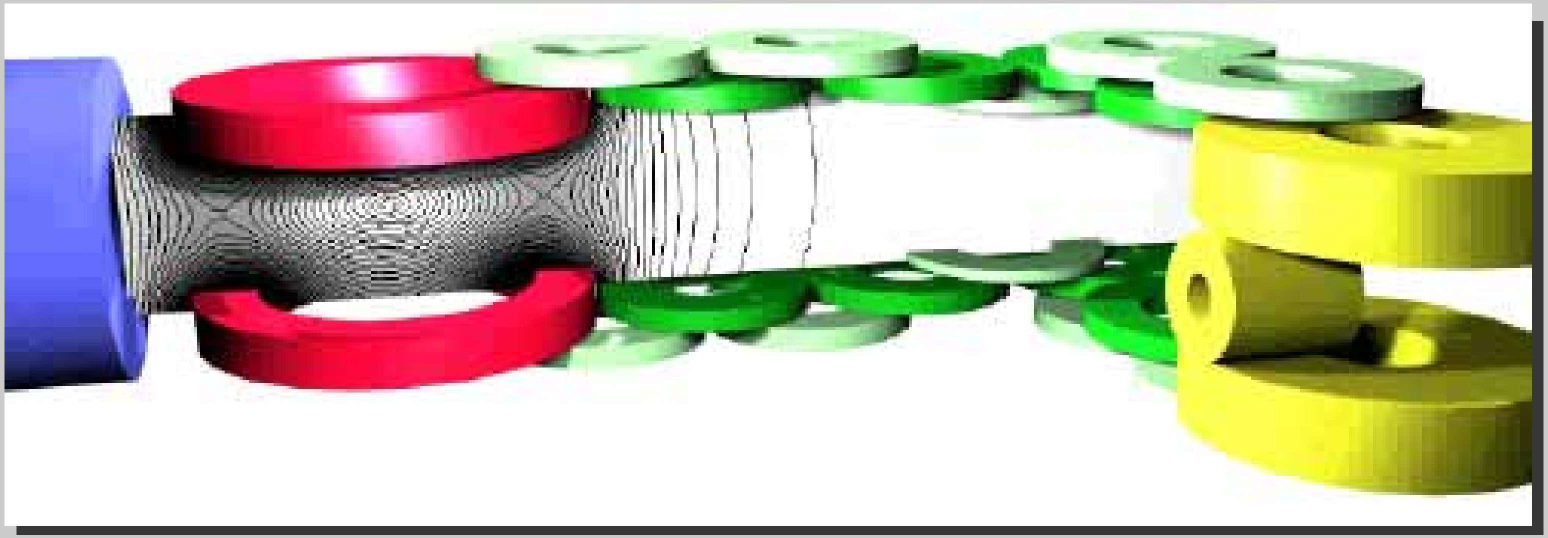
Magnetic Transport of Cold Atoms





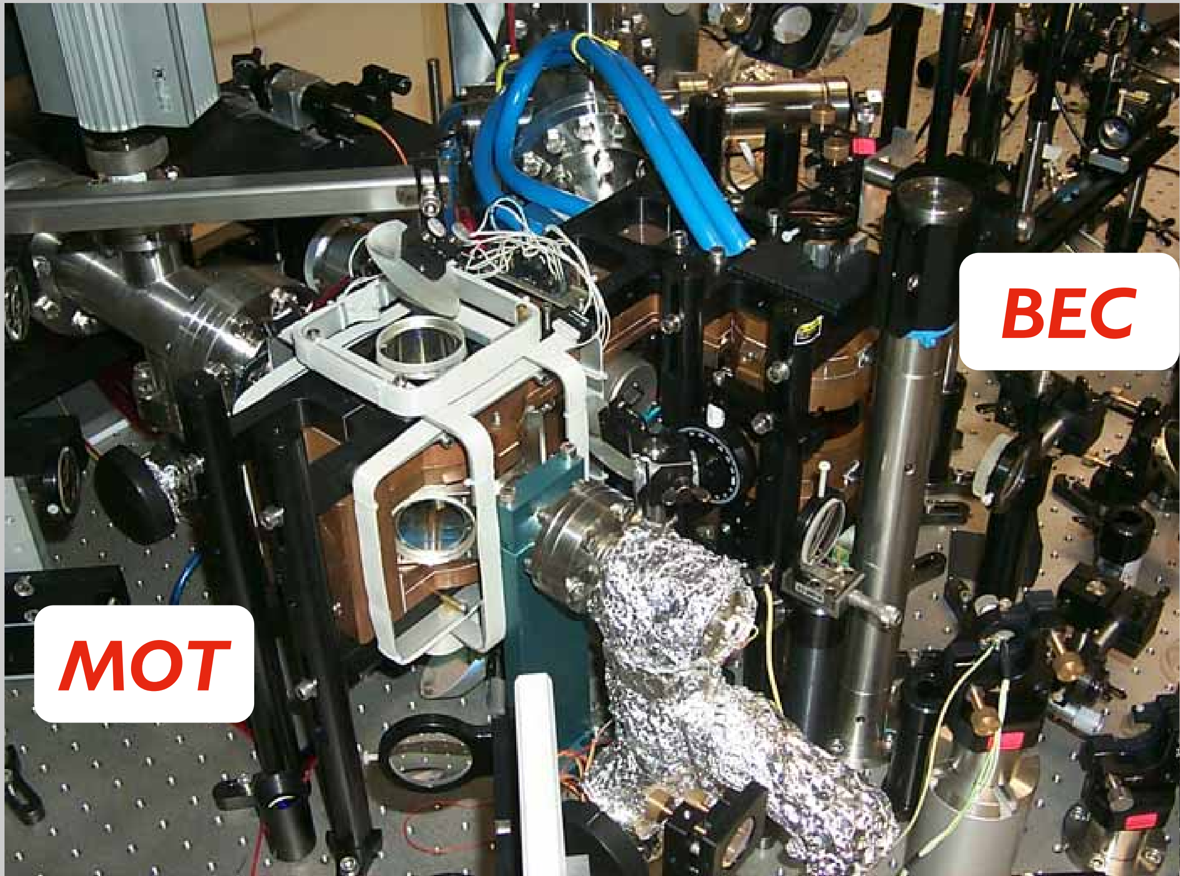
Magnetic Transport of Cold Atoms

Magnetic transport of atoms



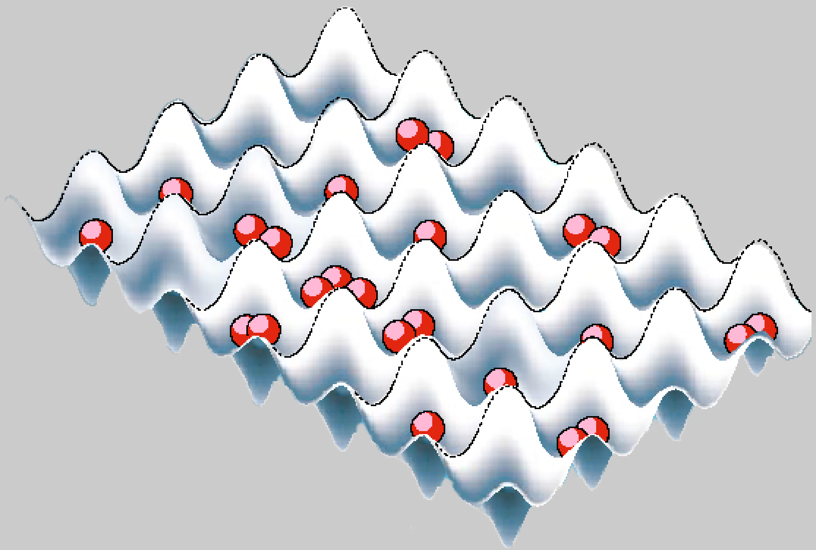
M. Greiner et al., PRA 63, 031401

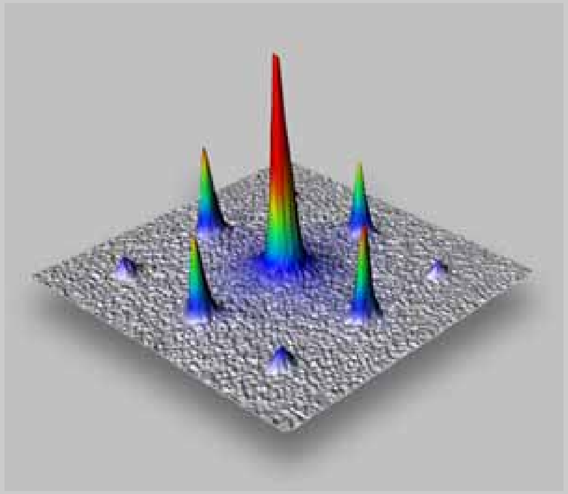
Experimental setup

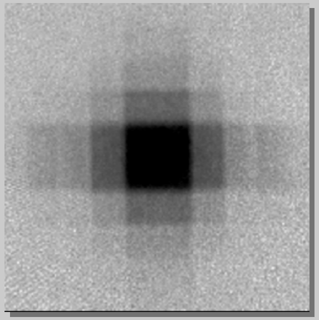


BEC apparatus

2. Bose einstein condensates in optical lattice potentials







Trapping Atoms in Light Field - Optical Dipole Potentials

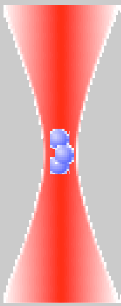
An electric field induces a dipole moment: $\vec{d} = \alpha \vec{E}$

Energy of a dipole in an electric field: $U_{dip} = -\vec{d} \cdot \vec{E}$

$$U_{dip} \propto -\alpha(\omega)\, I(\vec{r})$$

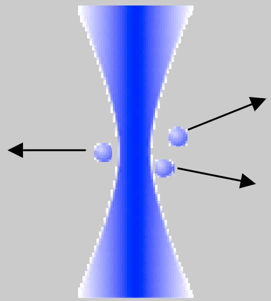
Red detuning:

Atoms are trapped in the intensity maxima



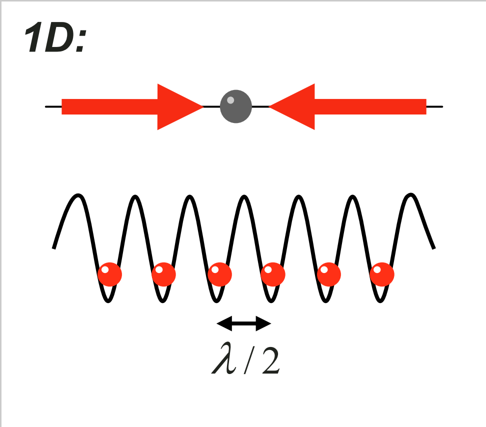
Blue detuning:

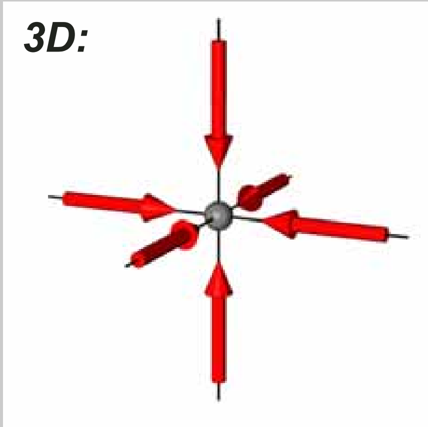
Atoms are repelled from the intensity maxima

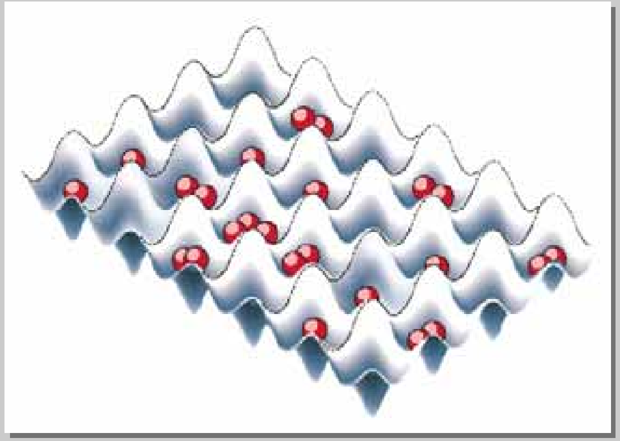


See R. Grimm et al., Adv. At. Mol. Opt. Phys. 42, 95-170 (2000).

3D periodic optical dipole potential







$$V(x) \propto \sin^2(kx) + \sin^2(ky) + \sin^2(kz)$$ *+ harmonic confinement*

- *Resulting potential consists of a simple cubic lattice*
- *BEC coherently populates more than 100,000 lattice sites*

V_0 *up to 40 E_{recoil}*

ω_r *up to $2\pi \times 50$ kHz*

n ≈ *1-3 atoms on average per site*

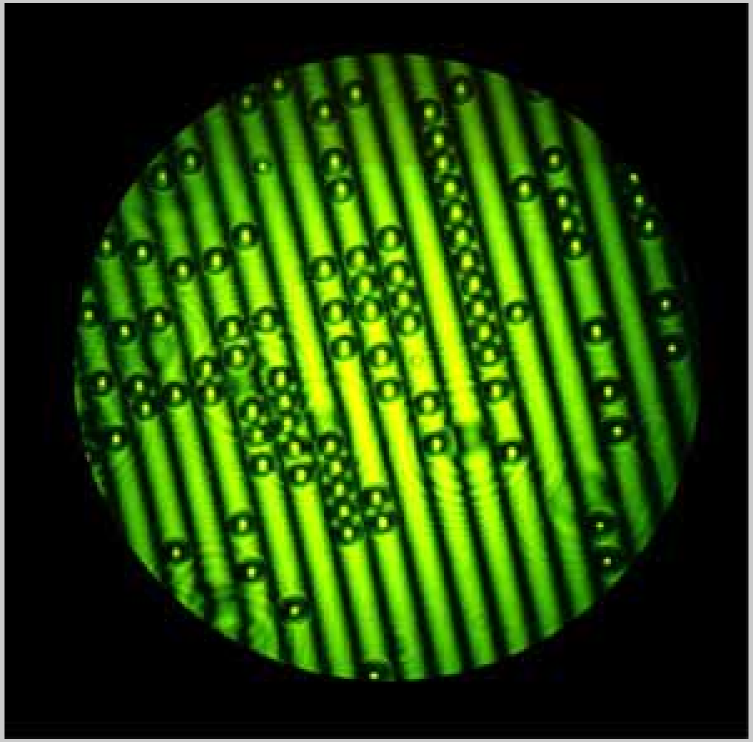
See eg. Jessen and Deutsch, Adv. At. Mol Opt. Phys. 37, (1996)
R. Grimm et al., Adv. At. Mol. Opt. Phys. 42, 95-170 (2000).

Optical dipole trap also possible with classical particles

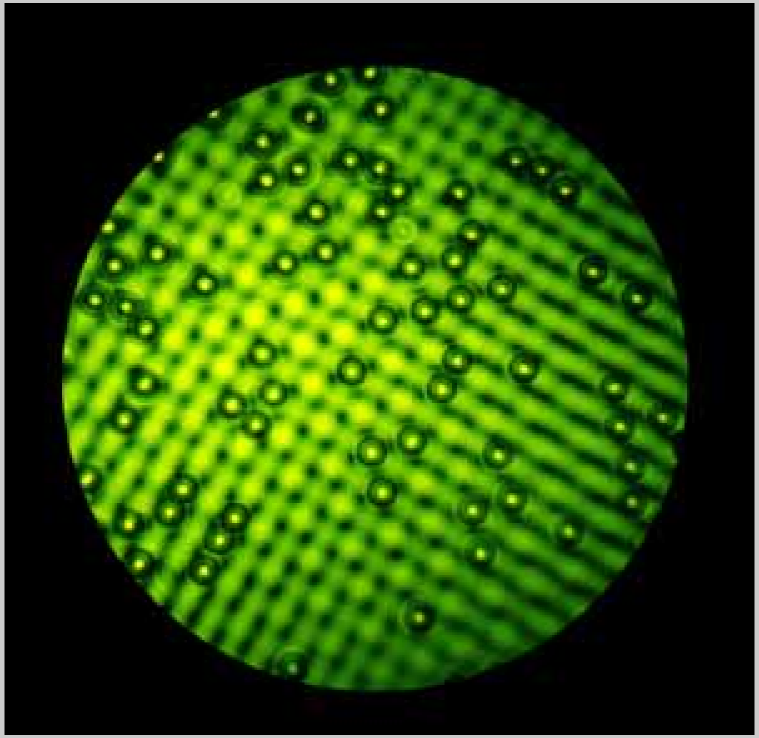
"Optical lattice" : 4 μm polystyrol particles in water

conservative light force for macroscipic particles → optical tweezers

2 beam lattice:

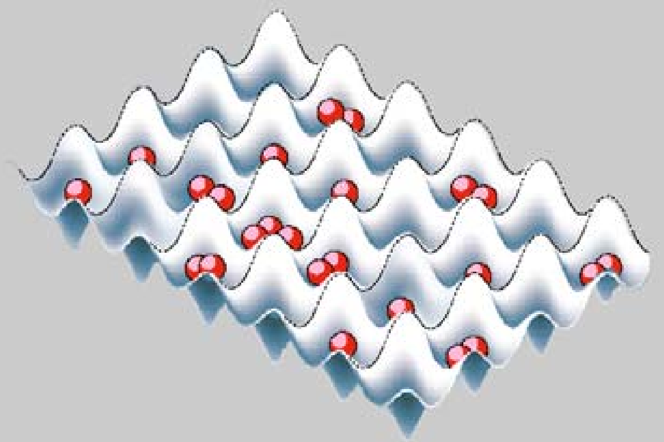


4 beam lattice:



Typical lattice parameters for a 3D lattice

Atomic Species	^{87}Rb
Wavelength	830-850 nm
Waist ($1/e^2$)	125 µm
Polarization	Orthogonal between standing wave pairs
Intensity control	All beams intensity stabilized
Lattice geometry	Simple cubic
Lattice spacing	425 nm



Start with a pure condensate in a magnetic trap

Turn on lattice potential adiabatically, so that the wave function remains in the many body ground state of the system !

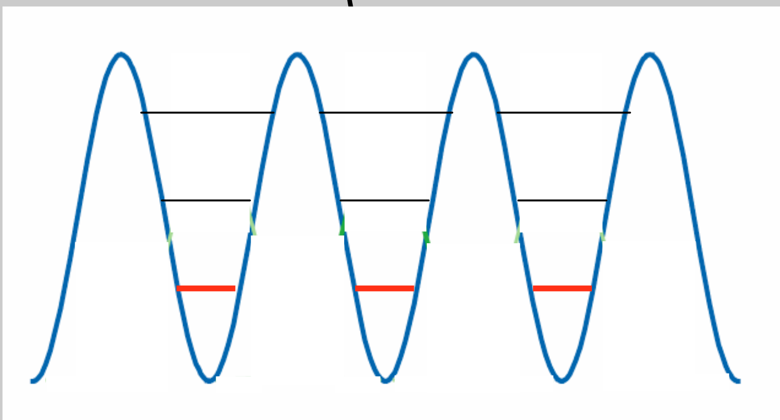
Macroscopic Wave Function of a BEC in an Optical Lattice

$$\Psi(x) = \sum_j A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$

Number of atoms on j^{th} lattice site

Phase of wave function on j^{th} lattice site

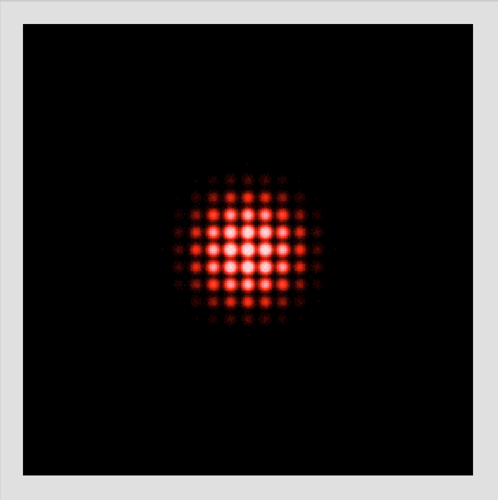
Localized wave function on J^{th} lattice site



Lattice potential

Detecting the Atoms in the Lattice

Spacing between neighboring lattice sites (≈425 nm) is too small to be detectable by optical means !



(simulation)

Switch off the lattice light

↓

Localized wavefunctions expand and interfere with each other

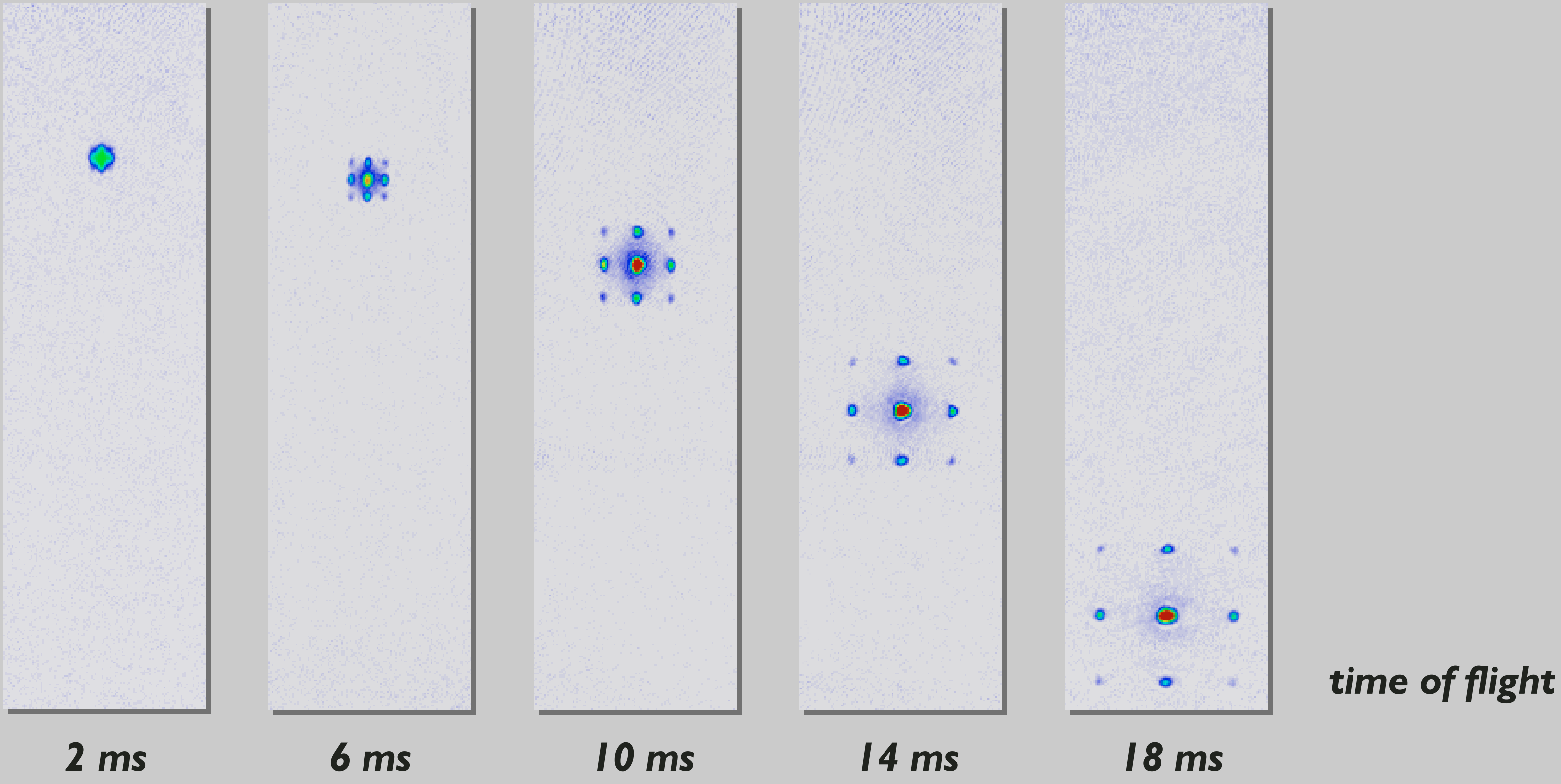
↓

Observe the multiple matter wave interference pattern !

→ Momentum distribution

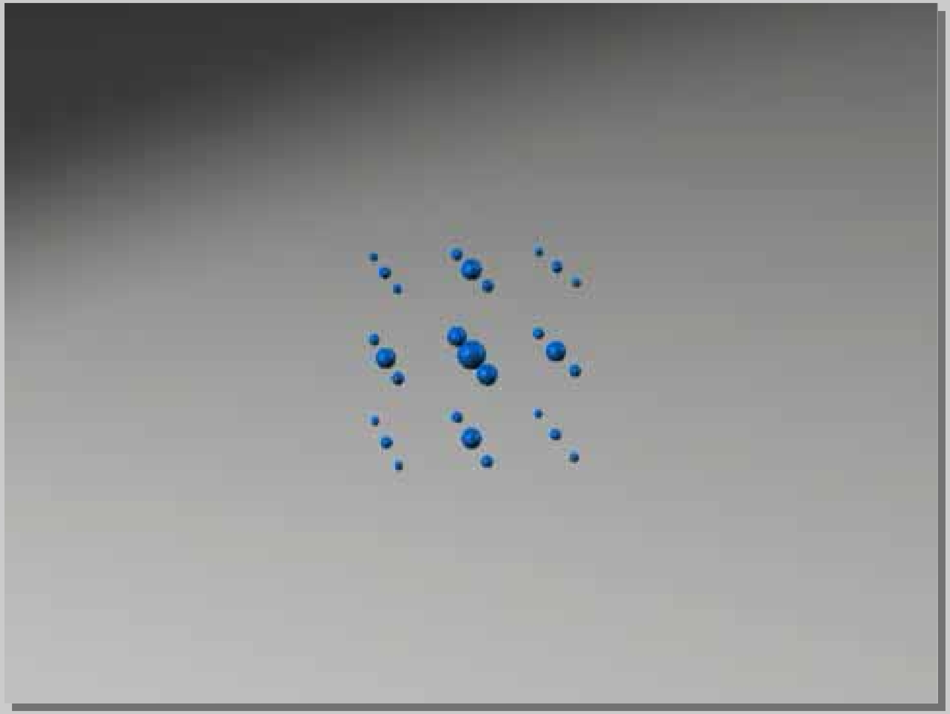
Matter Wave Interference Pattern of a BEC in an Optical Lattice

Time of flight measurement



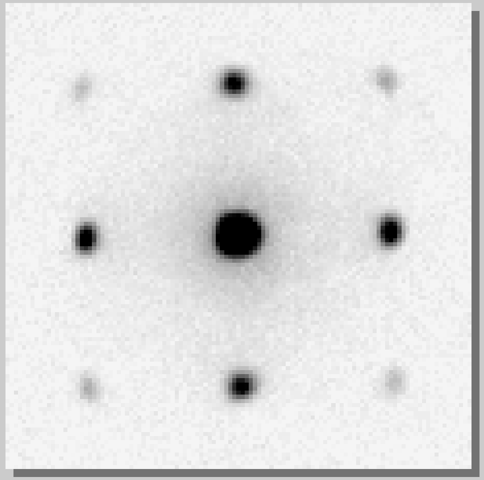
Individual condensates in the lattice expand and interfere with each other, revealing the momentum distribution of the atoms in the lattice.

Interference Pattern of a 3D Lattice



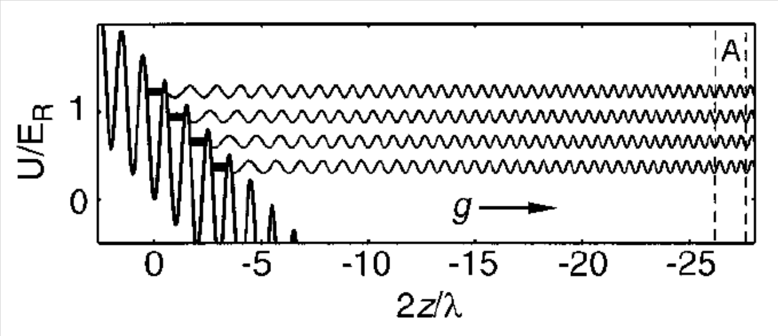
Time of flight images

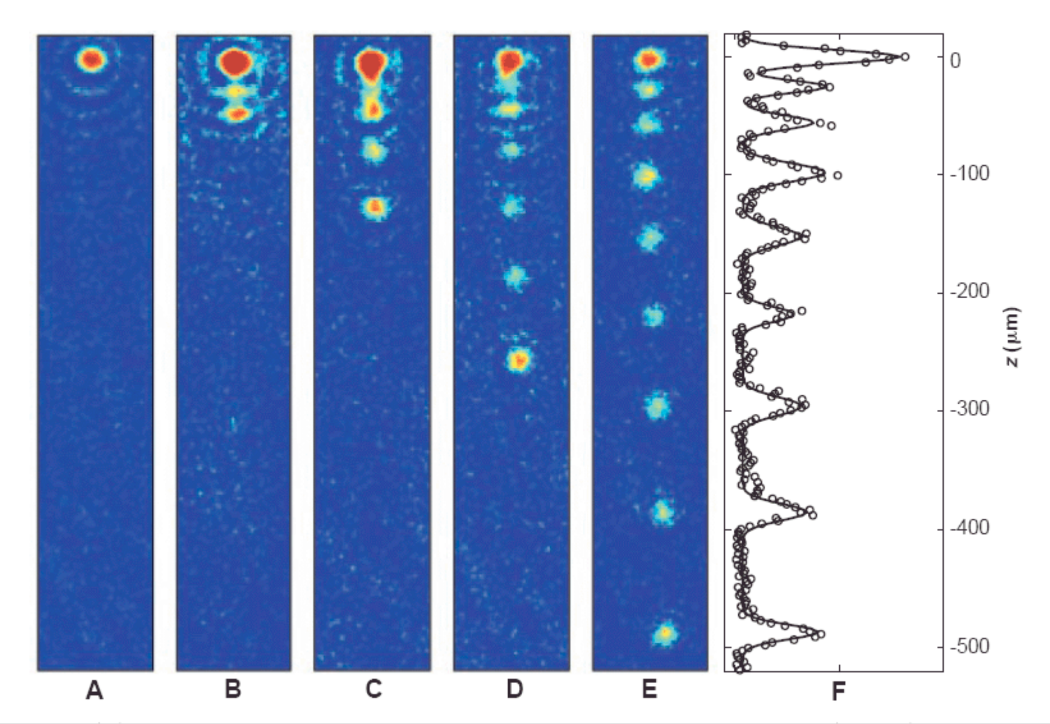
➔ *Momentum distribution*



First BEC lattice experiments

Kasevich (Yale)
BEC in a vertically oriented lattice
→ coherent matter waves tunnel out of each lattice site, interfere, and form **"pulsed atom laser"**





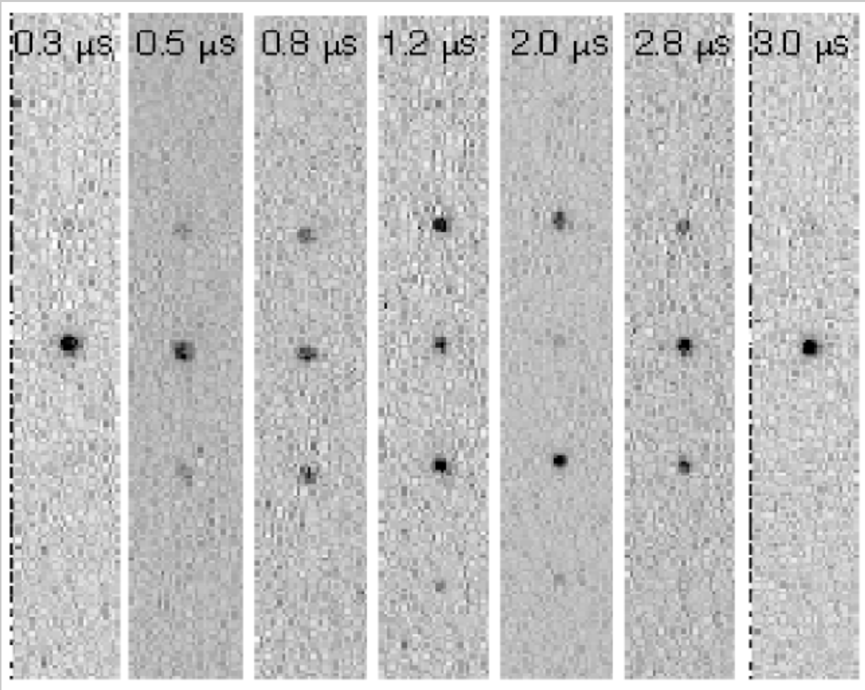
Anderson *et al.*, Science 282,1686 (1998)

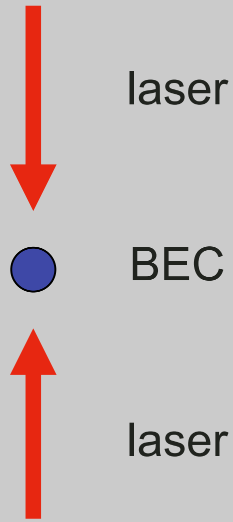
First BEC lattice experiments

Bragg type lattices:

lattice light is pulsed on for a short moment
e.g. Bill Phillips group, NIST

Ovchinnikov *et al.*, PRL 83, 284 (1999)





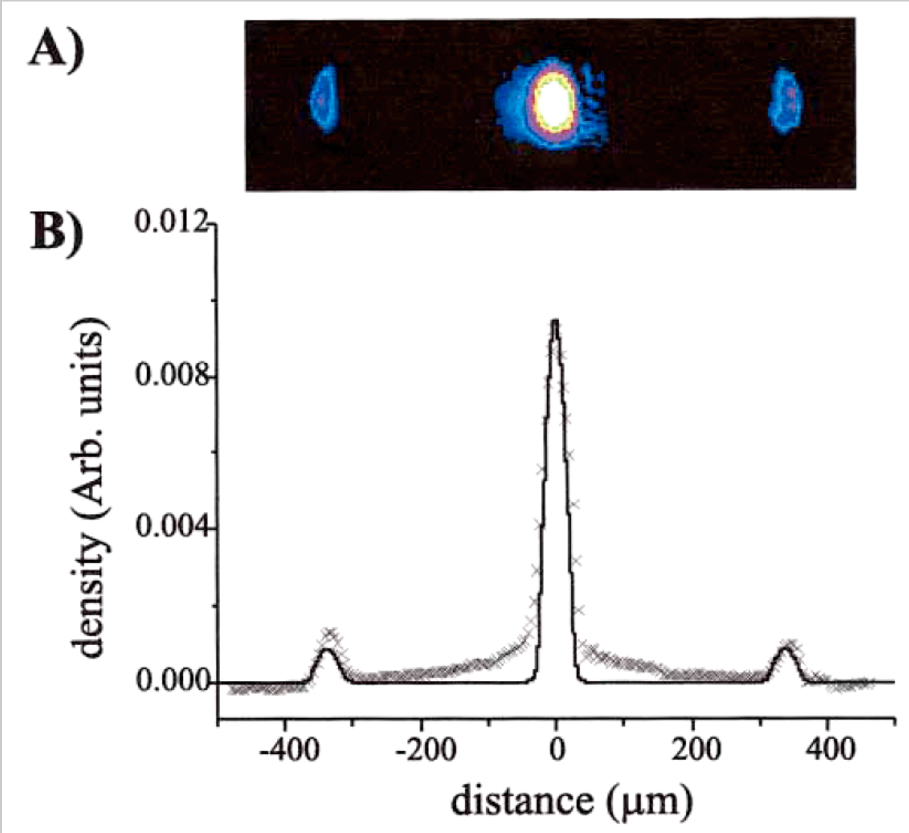
First BEC lattice experiments

1D lattice

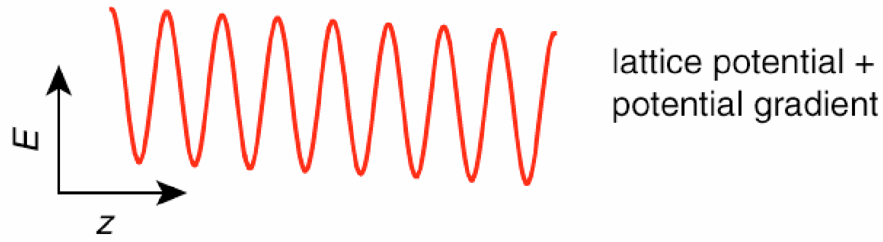
BEC is adiabatically loaded into 1D standing wave, e.g. in Inguscio's group (Florence)

→ Studying josephson junction arrays (tunneling, dynamical instabilities …)

e.g.
Pedri et al., PRL 87, 220401 (2001)
Cataliotti et al., Science 293, 843 (2001)



Preparing Arbitrary Phase Differences Between Neighbouring Lattice Sites

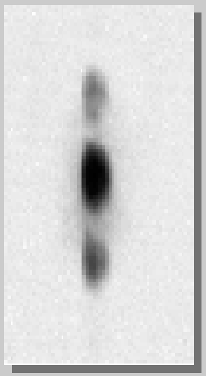


$$\phi_j = E_j \cdot t / \hbar$$

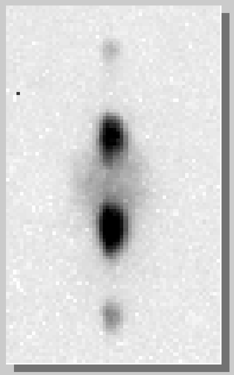
Phase difference between neighboring lattice sites

$$\Delta\phi_j = (V'\lambda/2) \cdot t/\hbar$$

(cp. Bloch-Oscillations)

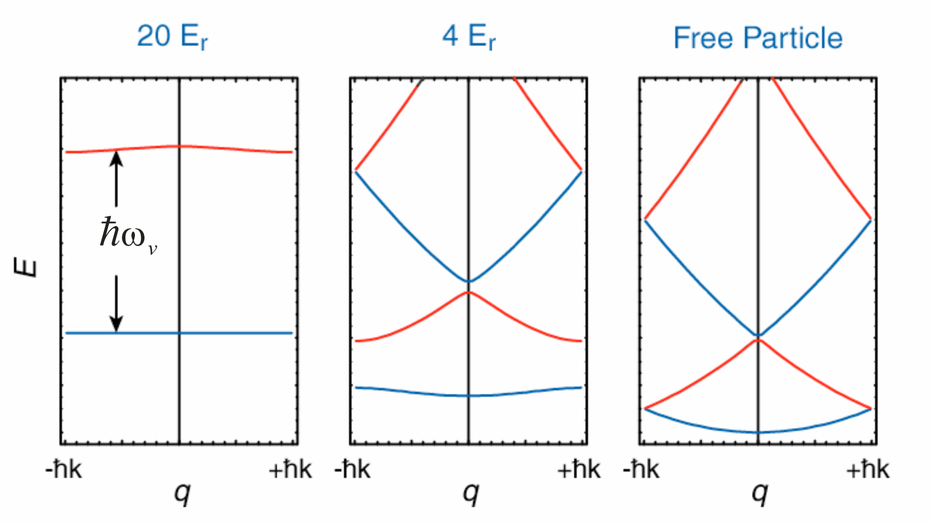


$$\Delta\phi = 0$$



$$\Delta\phi = \pi$$

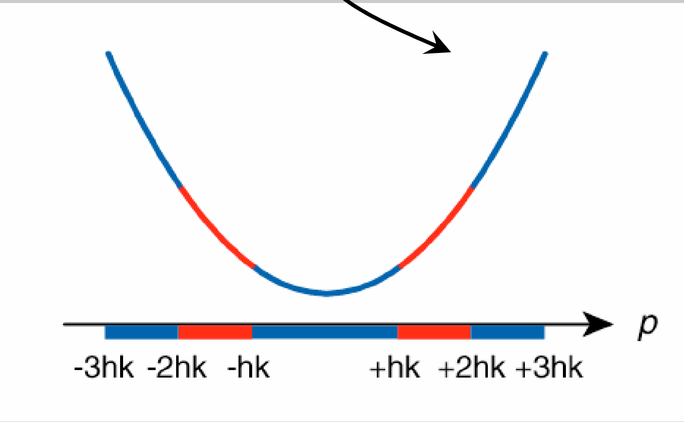
Mapping the Population of the Energy Bands onto the Brillouin Zones



Crystal momentum

Crytsal momentum is conserved while lowering the lattice depth adiabatically !

Population of nth band is mapped onto nth Brillouin zone !

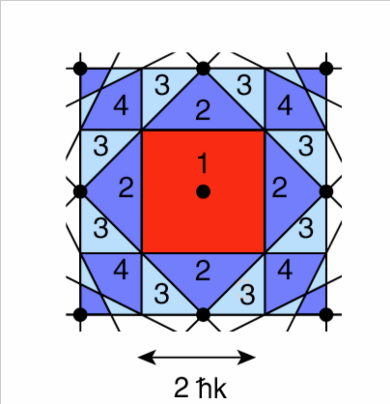


Free particle momentum

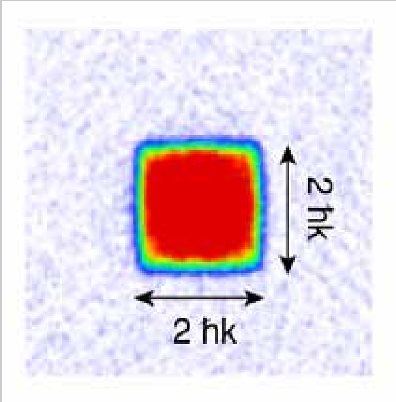
A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

Imaging the Brillouin zones

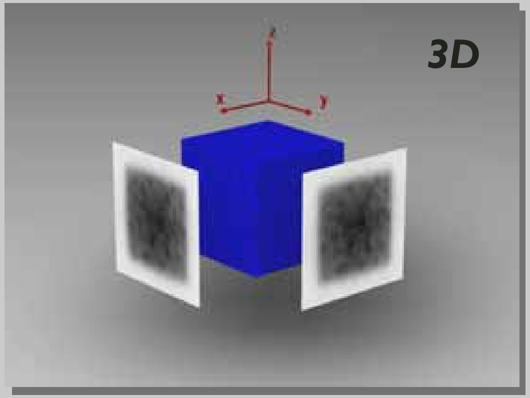
Brillouin Zones in 2D

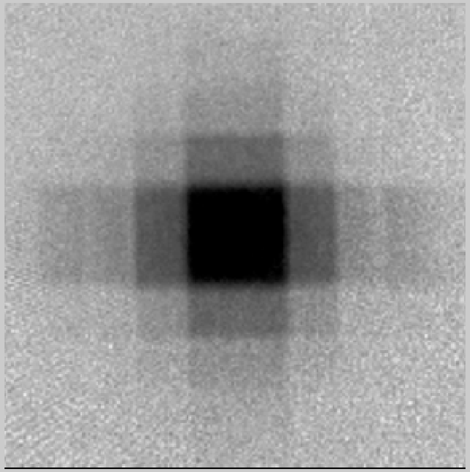


Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



2D





Populating higher energy bands by raman transitions

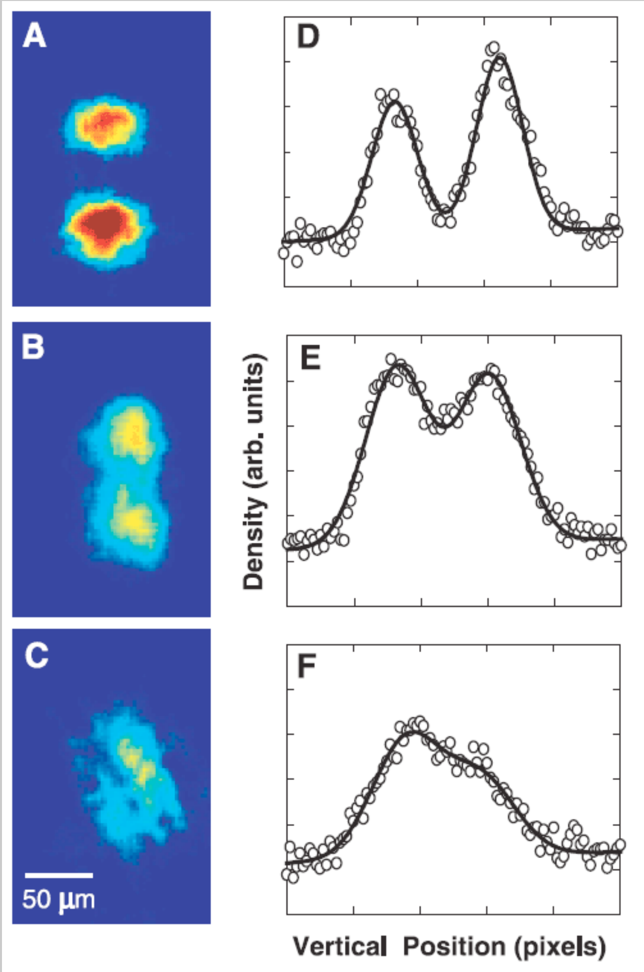
M. Greiner et al. PRL 87, 160405 (2001)

Squeezed states in a Bose-Einstein condensate

In deep optical lattices, repulsive interaction between atoms can cause number squeezing:

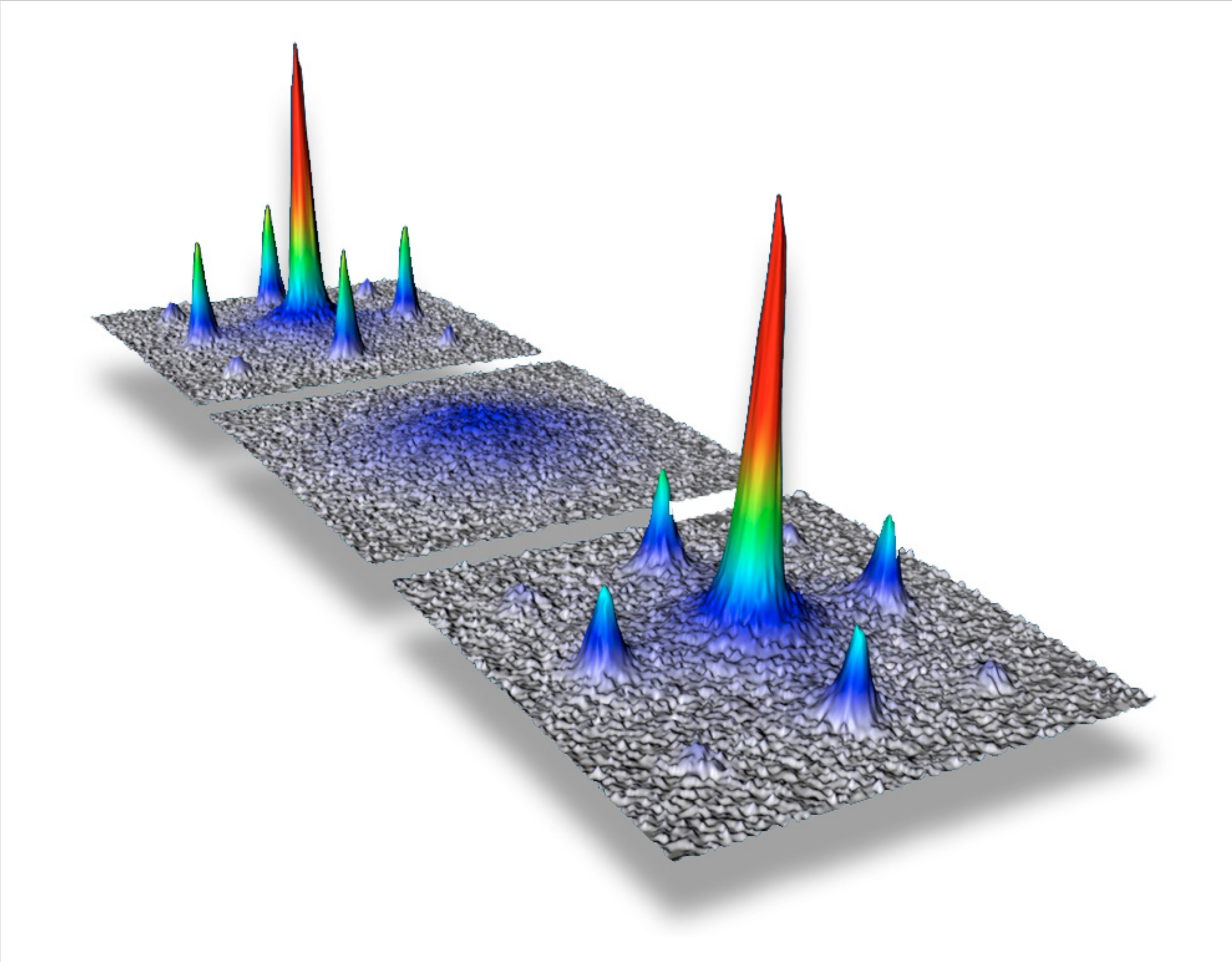
- *Atom number fluctuations on each lattice site get reduced*
- *Therefore the macroscopic phase, as a conjugate variable, becomes more uncertain*

This number squeezing has been observed in the experiment of Mark Kasevich for a 1D lattice:

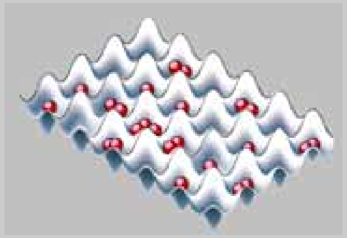


Orzel et al., Science 291, 2386 (2001)

3. Quantum phase transition from a superfluid to a Mott insulator



Bose-Hubbard Hamiltonian

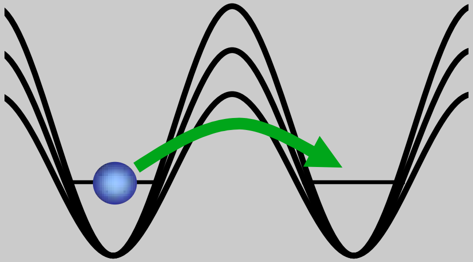


$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Tunneling term:

J: **tunneling matrix element**

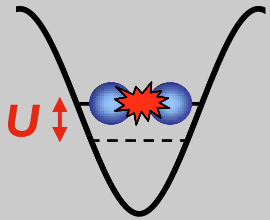
$\hat{a}_i^\dagger \hat{a}_j$ **: tunneling from site j to site i**



Interaction term:

U **: on-site interaction matrix element**

$\hat{n}_i(\hat{n}_i - 1)$ **: n atoms collide with n-1 atoms on same site**



Ratio between tunneling J and interaction U can be widely varied by changing depth of 3D lattice potential!

MI in opt. latt.: proposed by Dieter Jaksch et al. in the group of Peter Zoller, Innsbruck

M.P.A. Fisher et al, PRB 40, 546 (1989), D. Jaksch et al., PRL 81, 3108 (1998)

Superfluid Limit

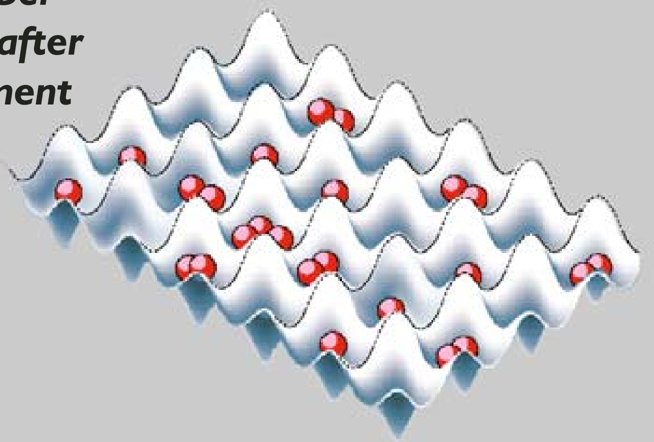
$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

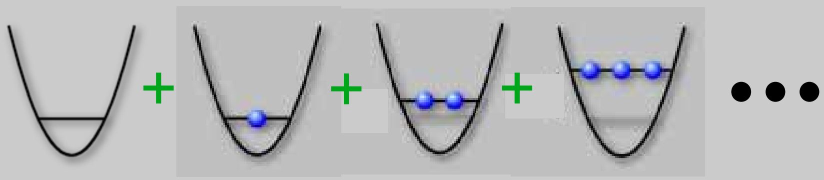
Atoms are delocalized over the entire lattice !
Macroscopic wave function φ_i describes this state very well.

$$|\Psi_{SF}\rangle \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger \right)^N |0\rangle$$

$$\varphi_i = \langle \hat{a}_i \rangle; \quad |\Psi\rangle_i = e^{-|\varphi_i|^2/2} \sum_n \frac{\varphi_i^n}{\sqrt{n!}} |n\rangle$$

Atom number distribution after a measurement





Coherent state with well defined macroscopic phase φ_i and poissonian atom number distribution at each lattice site

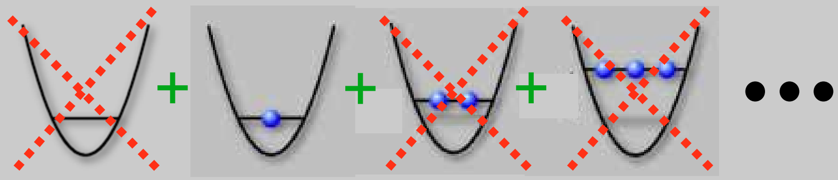
Mott-Insulator ground state in the "Atomic Limit"

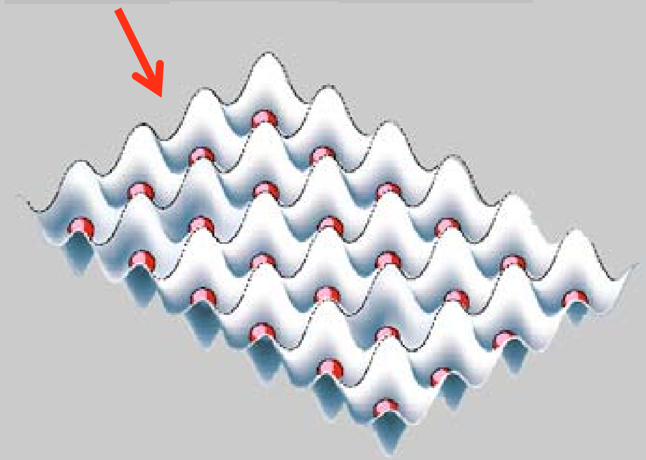
$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

MI ground state: Atoms are completely localized to lattice sites !

$$|\Psi_{Mott}\rangle \propto \prod_{i=1}^{M} \left(a_i^\dagger\right)^n |0\rangle$$

Fock states with vanishing atom-number fluctuation are formed.





→ no macroscopic phase

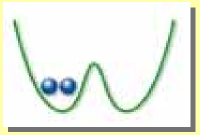
$$\langle a_i \rangle = 0$$

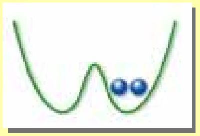
Proposal: Mott with BEC in 3D lattice:
D. Jaksch et al., PRL 81, 3108 (1998)

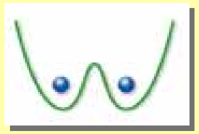
The Simplest Possible "Lattice": 2 Atoms in a Double Well

Superfluid State

$$\frac{1}{\sqrt{2}}(\phi_l + \phi_r) \otimes \frac{1}{\sqrt{2}}(\phi_l + \phi_r)$$

0.25 x  +

0.25 x  +

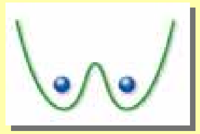
0.5 x 

$\langle n \rangle = 1$

$\langle E_{int} \rangle = \frac{1}{2} U$

MI State

$$\frac{1}{\sqrt{2}}\phi_l \otimes \phi_r + \frac{1}{\sqrt{2}}\phi_r \otimes \phi_l$$



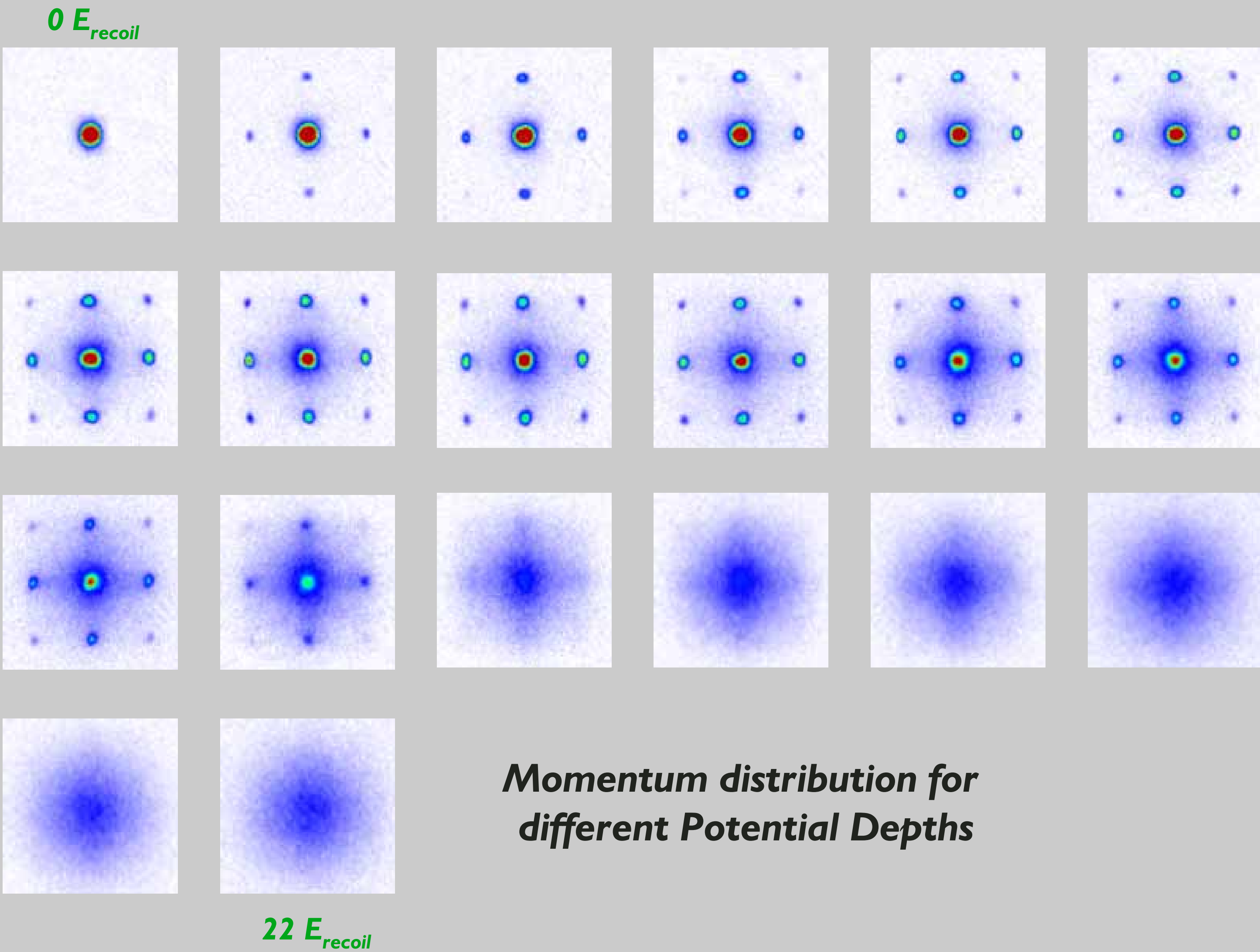
$\langle n \rangle = 1$

$\langle E_{int} \rangle = 0$

Average atom number per site: $\langle n \rangle = 1$ (Superfluid State), $\langle n \rangle = 1$ (MI State)

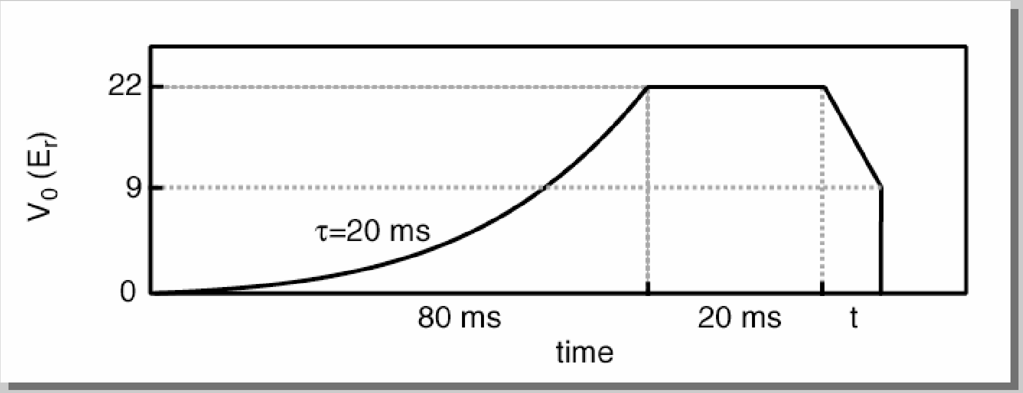
Average onsite Interaction per site: $\langle E_{int} \rangle = \frac{1}{2} U$ (Superfluid State), $\langle E_{int} \rangle = 0$ (MI State)

Entering the Mott Insulator Regime

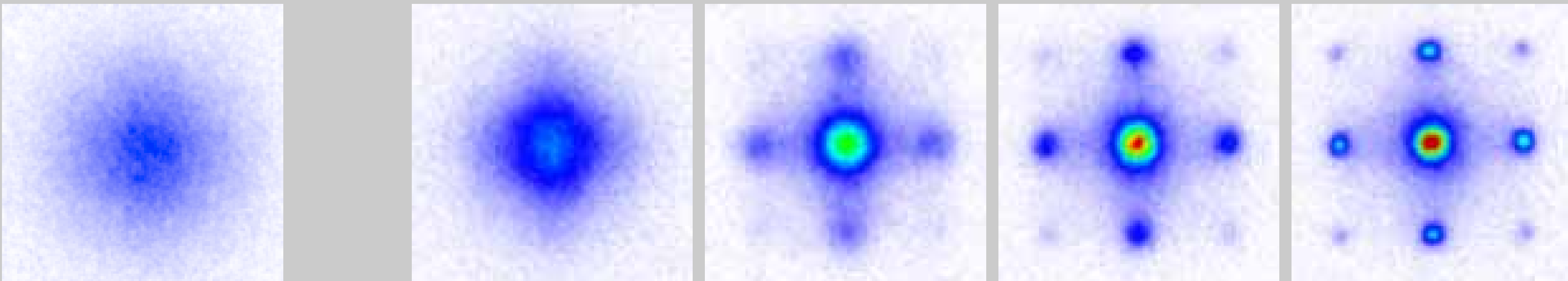


Momentum distribution for different Potential Depths

Can We Restore Coherence ?

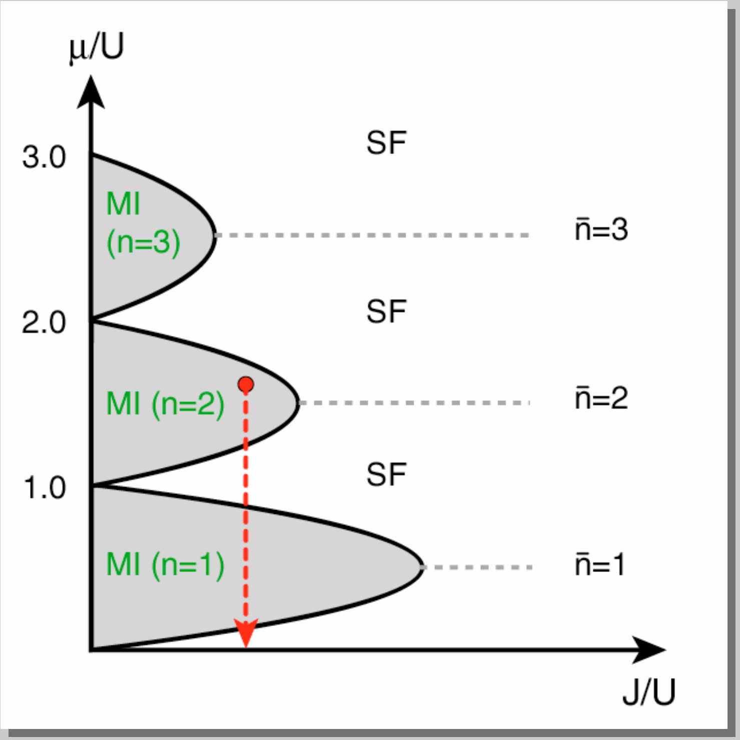


Ramp down for different times t and monitor momemtum distribution !

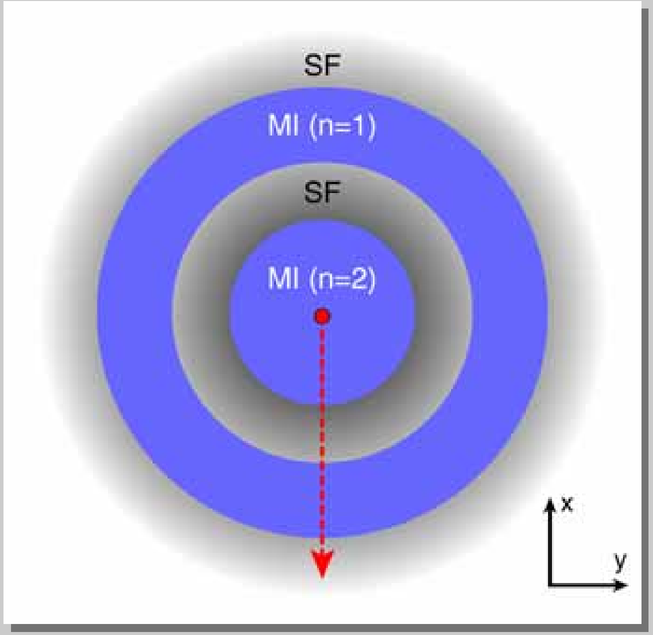


Before ramping down ***0.1 ms*** ***1.4 ms*** ***4 ms*** ***14 ms***

Mott insulator in an inhomogeneous system



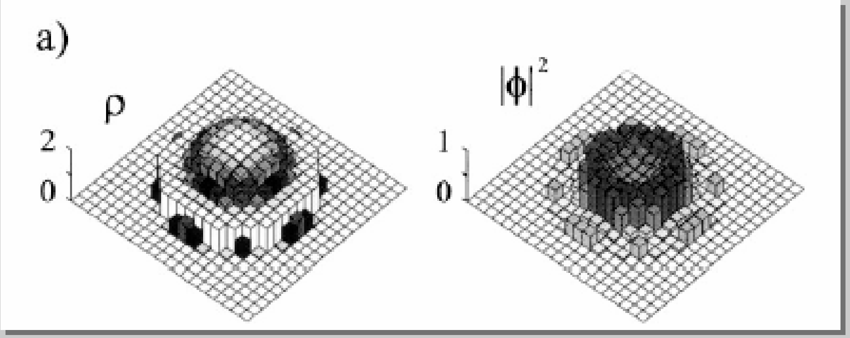
Jaksch et al. PRL 81, 3108 (1998)



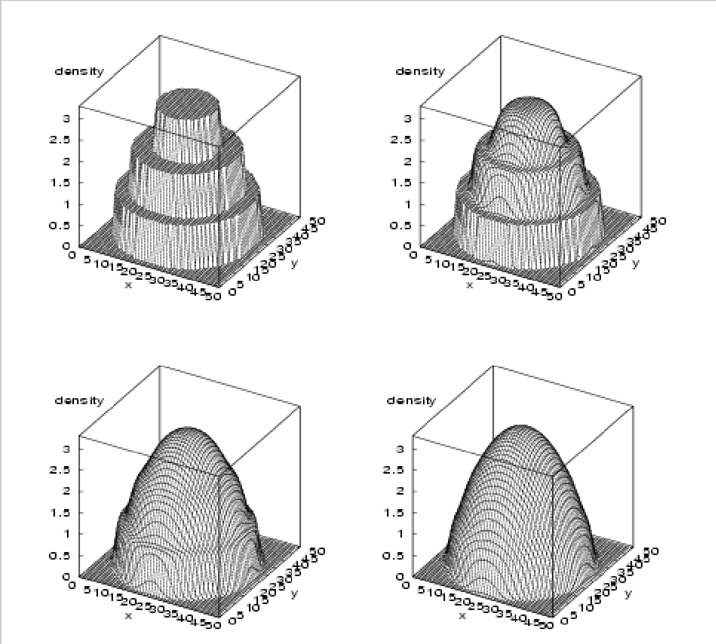
For an inhomogeneous system an effective local chemical potential can be introduced

$$\mu_{loc} = \mu - \varepsilon_i$$

Ground State of an Inhomogeneous System

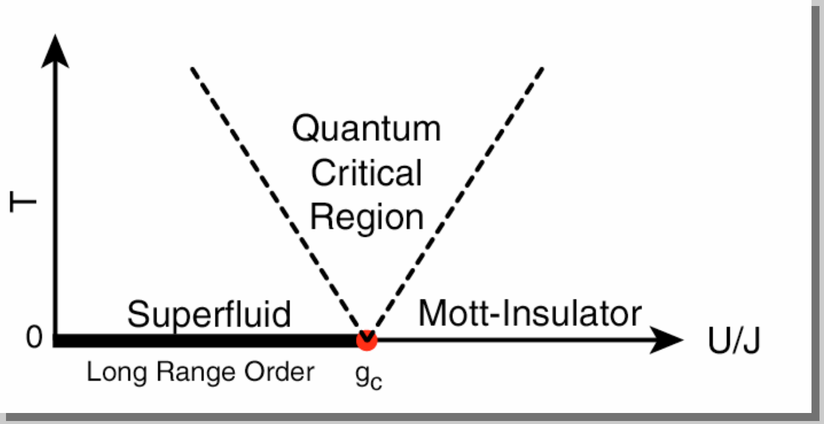


From Jaksch et al. PRL 81, 3108 (1998)



From M. Niemeyer and H. Monien (private communication)

Quantum Phase Transition (QPT) from a Superfluid to a Mott-Insulator



At the critical point g_c the system will undergo a phase transition from a superfluid to an insulator !

This phase transition occurs even at T=0 and is driven by quantum fluctuations !

Characteristic for a QPT

- *Excitation spectrum is dramatically modified at the critical point.*
- *$U/J < g_c$ (Superfluid regime)*
  Excitation spectrum is gapless
- *$U/J > g_c$ (Mott-Insulator regime)*
  Excitation spectrum is gapped

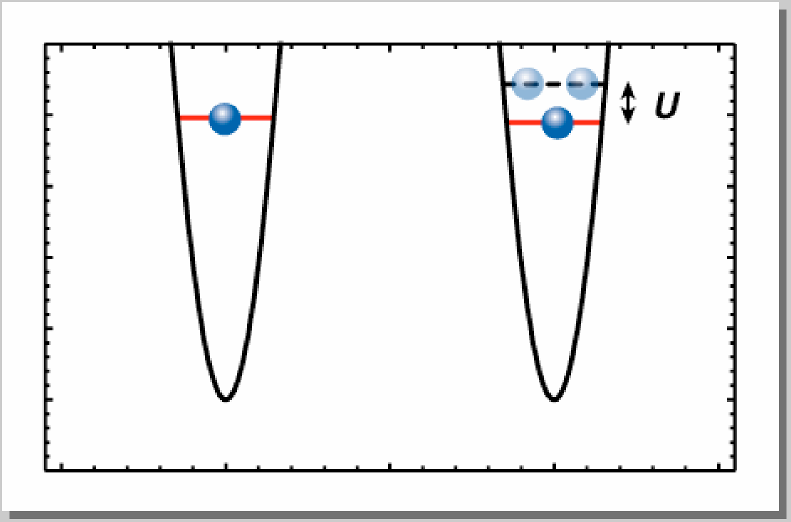
Critical ratio for:

$U/J = z\ 5.8$

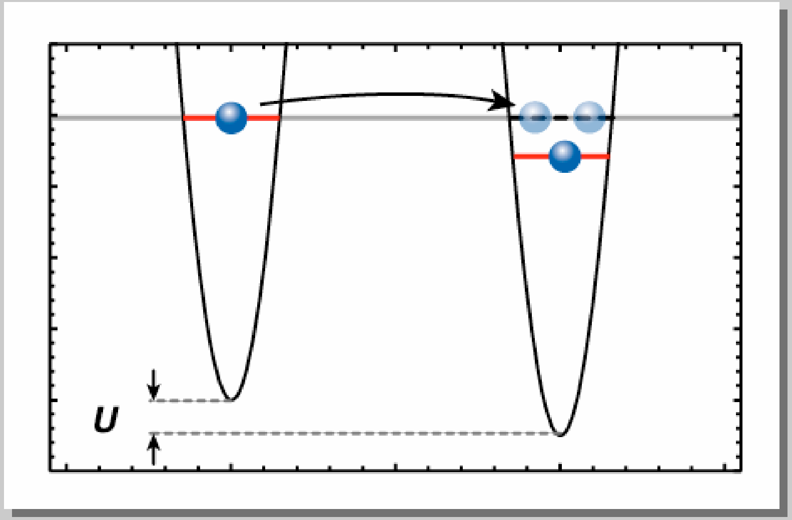
see Subir Sachdev, Quantum Phase Transitions, Cambridge University Press

Creating Excitations in the MI Phase

Mott-Insulator with $n_i = 1$ atom per lattice site



Without gradient potential



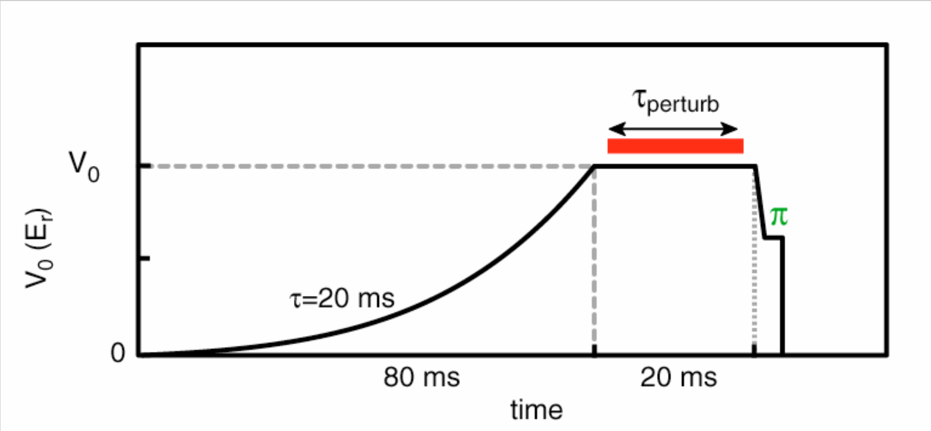
With gradient potential

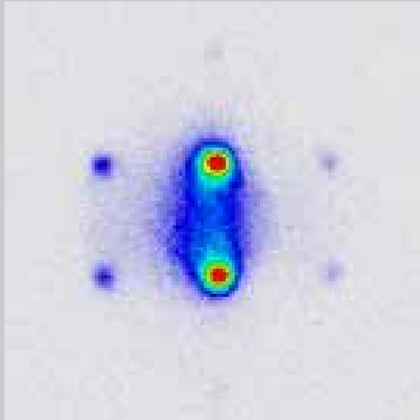
Special case: $\Delta E_{ij} = U$

Energy Scales:

$$\hbar\omega_{\mathrm{n}} \approx 20\ U$$
$$U \approx 10\text{–}300\ J$$

Measuring Excitation Probability vs. Pertubation Gradient

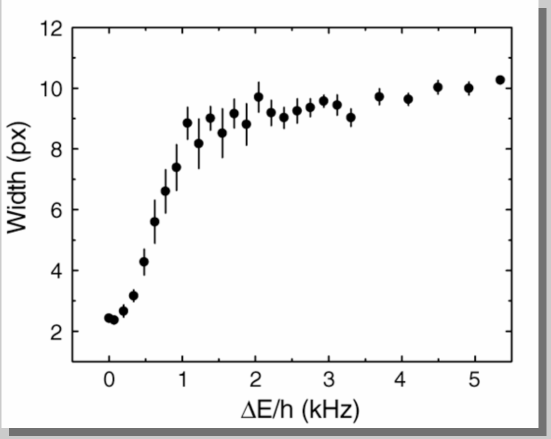




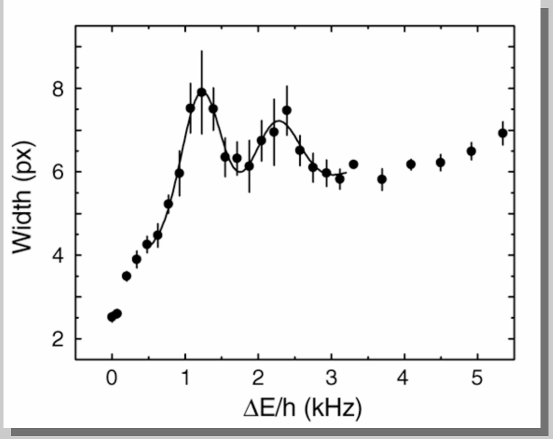
1. *Ramp up to a fixed lattice depth V0*
2. *Apply a gradient for time* $t_{perturb}$
3. *Ramp down to a potential depth of 10* E_{recoil}
4. *Apply a p-pulse*
5. *Measure width of interference peaks*

If excitations are created, the width of the detected interference peaks will broaden !

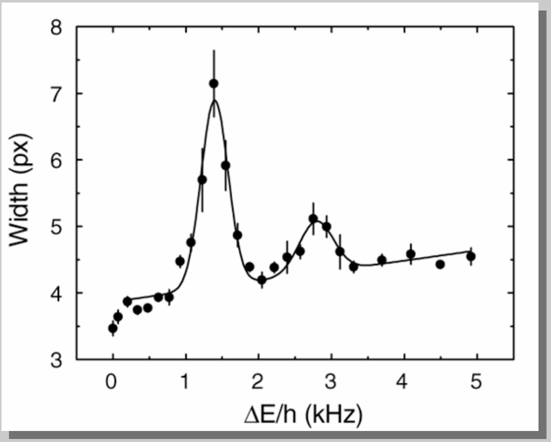
Excitation Probability vs. Gradient



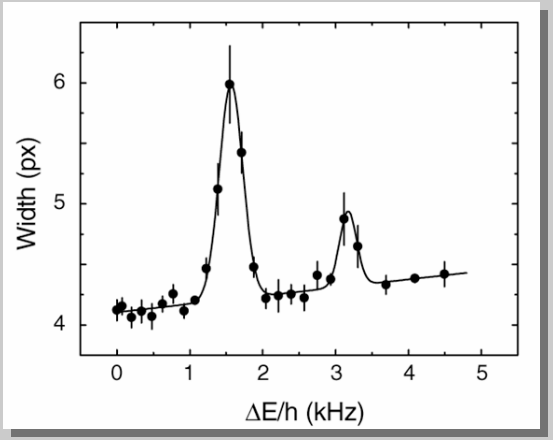
$10\ E_{recoil}$ $t_{perturb} = 2\ ms$



$13\ E_{recoil}$ $t_{perturb} = 4\ ms$

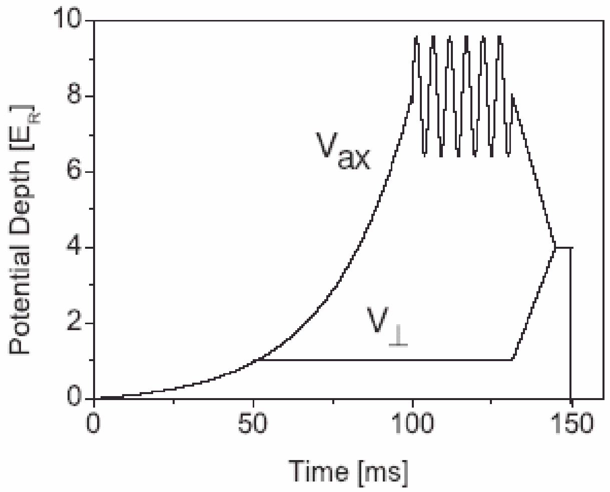


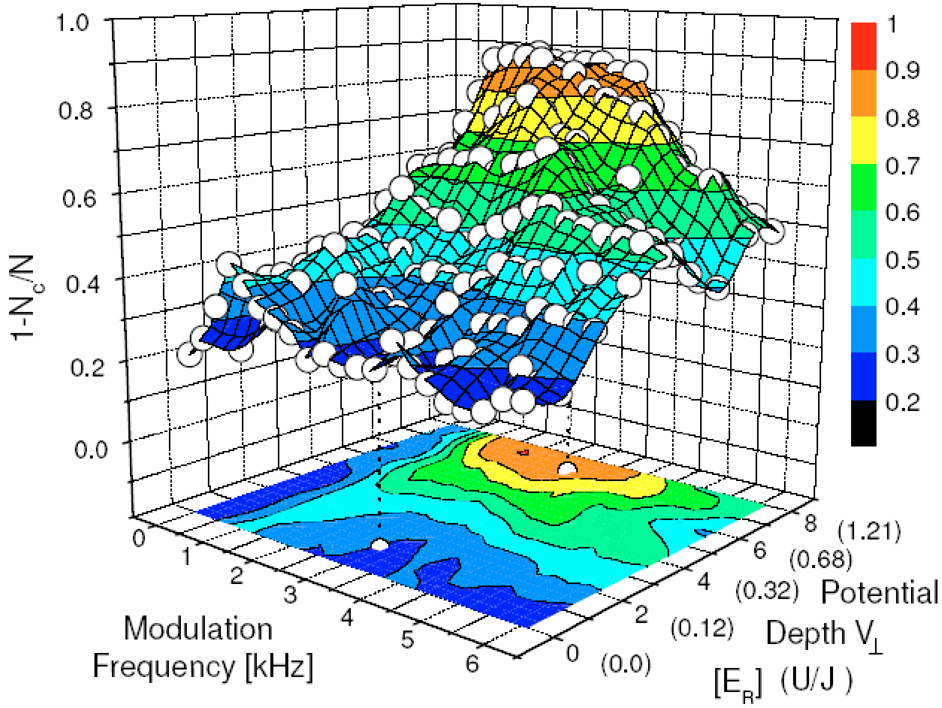
$16\ E_{recoil}$ $t_{perturb} = 9\ ms$



$20\ E_{recoil}$ $t_{perturb} = 20\ ms$

Excitations of bosons in an optical lattice

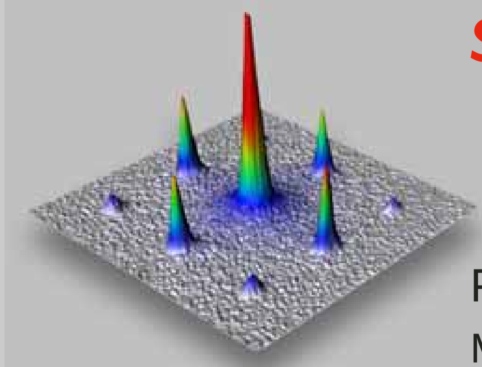




Schori et al.,
PRL 93:240402 (2004)

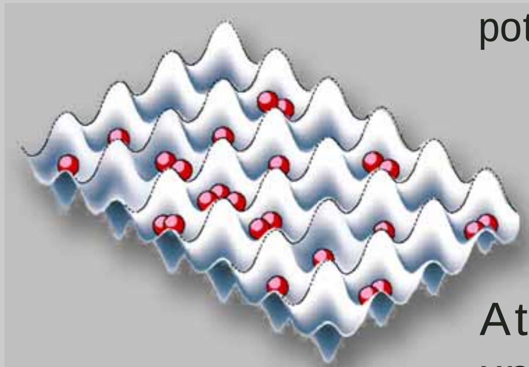
Conclusion Lecture 1

Superfluid



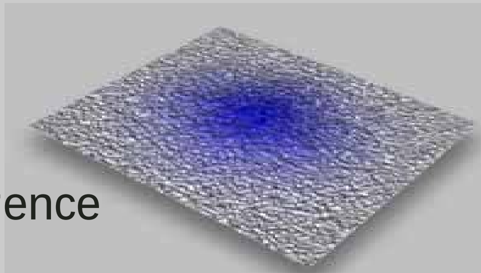
Phase coherence

Macroscopic phase well defined in each potential well



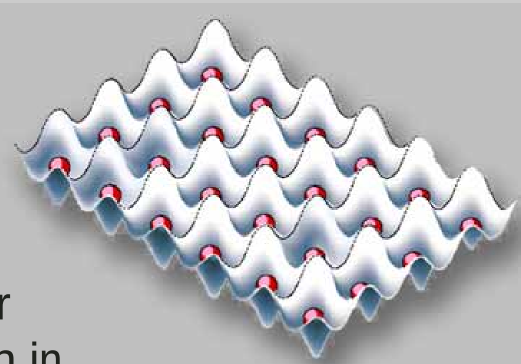
Atom number uncertain in each potential well

Mott Insulator



No Phase coherence

Macroscopic phase uncertain in each potential well



Atom number exactly known in each potential well

→ atom number correlations

M.P.A. Fisher et al, D. Jaksch et al.

Condensed matter physics with ultracold atoms

Real materials
complicated:
- various interactions
- disorder

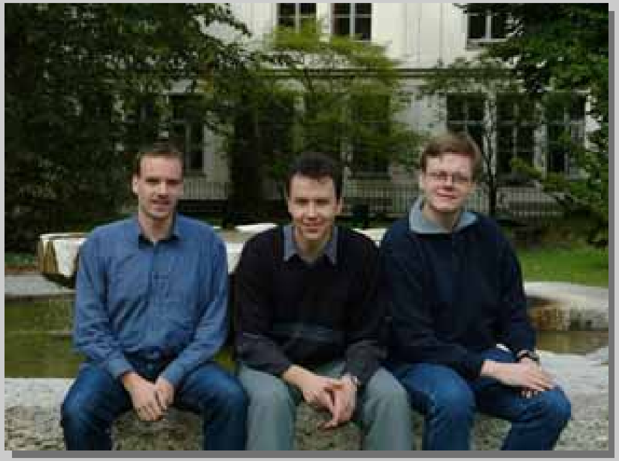
?

Direct experimental test of condensed matter models:

Condensed matter models
difficult to calculate, especially for fermions

Ultracold atoms in optical lattices
clean realization of condensed matter models

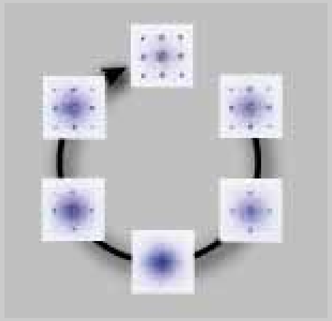
Thanks

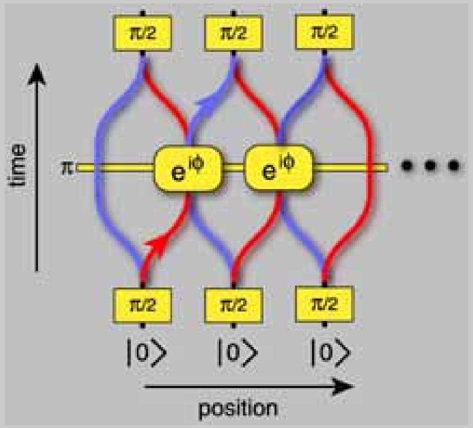


Thanks to my Munich collegues:
Olaf Mandel, Immanuel Bloch
(now at Mainz, Germany)
Ted Hänsch (MPQ Munich)

Second lecture:

- Collapse and revival of a macroscopic matter wave
- Quantum gates with neutral atoms
- Low dimensional systems





ICTP SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES 2005

Ultracold Atoms in optical lattice potentials

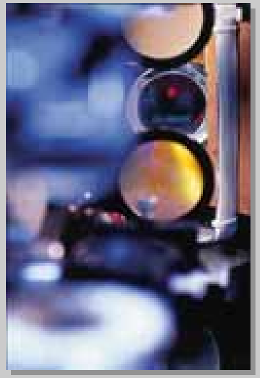
Experiments at the interface between atomic physics and condensed matter physics, quantum optics, molecular physics and quantum information

Markus Greiner

markus.greiner@colorado.edu

most experiments discussed in this lecture have been performed in the group of Ted Hänsch and I. Bloch at the

Ludwig-Maximilians-Universität, München and Max-Planck-Institut für Quantenoptik, Garching.



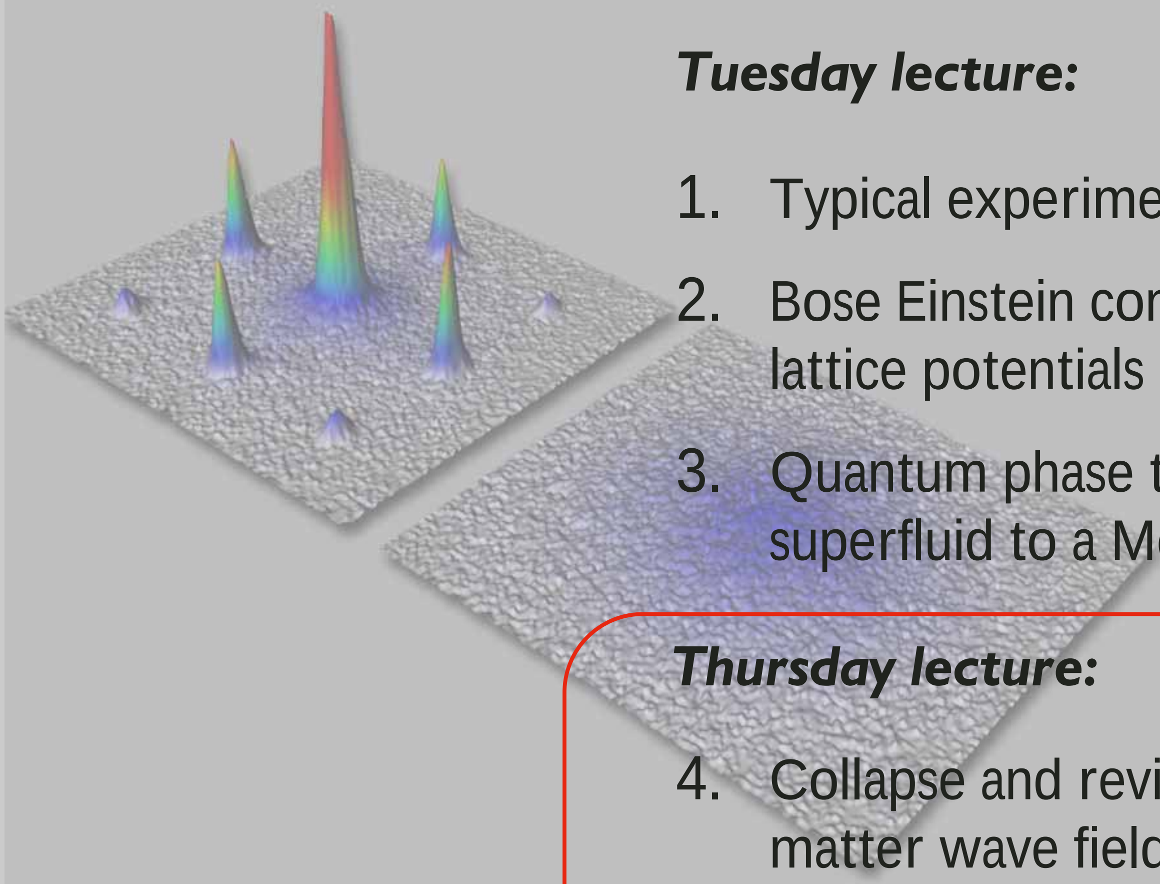
I am presently at JILA, Boulder, Co, in the group of D. Jin, working with fermionic condensates.

LMU

MPQ

Max-Planck-Institut für Quantenoptik

Ultracold Atoms in optical lattice potentials



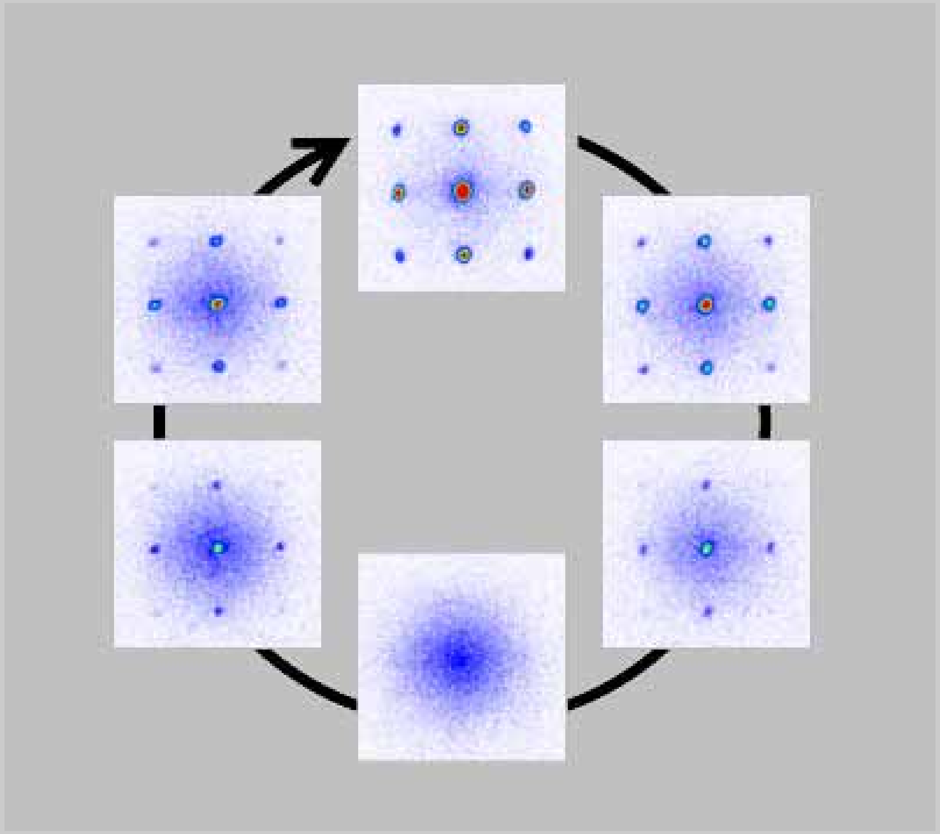
Tuesday lecture:

1. Typical experimental setup
2. Bose Einstein condensates in optical lattice potentials
3. Quantum phase transition from a superfluid to a Mott insulator

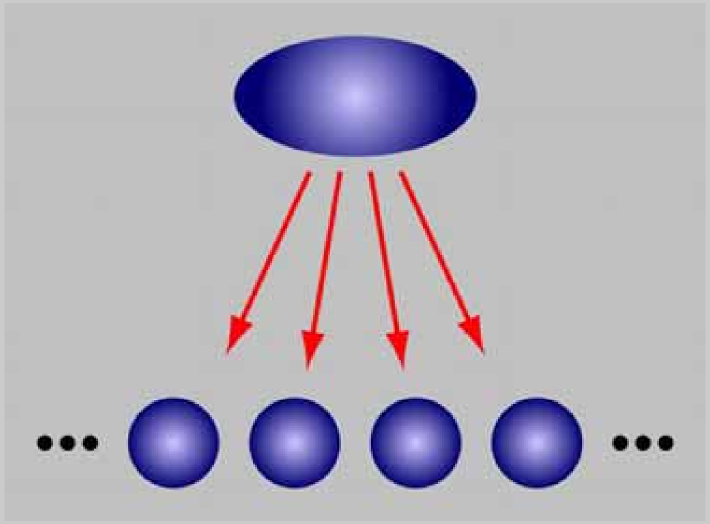
Thursday lecture:

4. Collapse and revival of a macroscopic matter wave field due to cold collisions
5. Quantum gates with neutral atoms

4. Collapse and revival of a macroscopic matter wave



Collapse and Revival of a Macroscopic Matter Wave Field



Splitting a condensate:

→ ***Well defined relative phase***

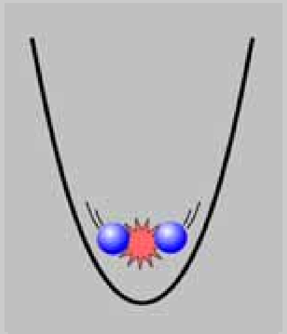
- ***How does the phase correlation evolve in time?***
- ***What happens to the individual matter wave fields?***

Non equilibrium experiment:

Rapidly increase lattice potential to isolate potential wells
→ Superfluid state is ***projected*** into Mott insulator regime

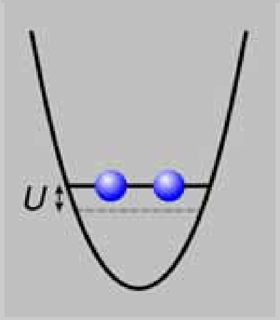
Dynamical Evolution of a Many Atom State due to Cold Collision

How do collisions affect the many body state in time ?



$$\hat{H} = \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Phase evolution of the quantum state of two interacting atoms:



Collisional phase

$$|2\rangle(t) = |2\rangle \cdot e^{-iUt/\hbar}$$

- *Phase shift is coherent !*
- *Leads to dramatic effects beyond mean-field theories !*
- *Can be used for quantum computation (see Jaksch, Briegel, Cirac, Zoller schemes)*

Collisional phase of n-atoms in a trap:

$$E_n t / \hbar = \frac{1}{2} U n (n-1) t / \hbar$$

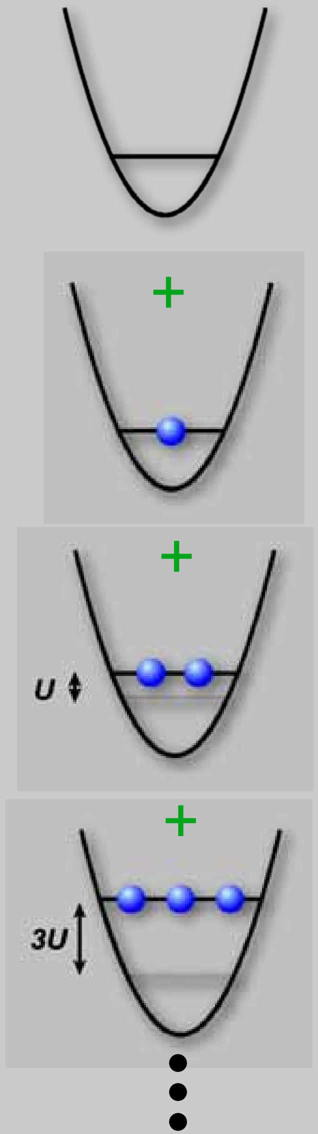
Time Evolution of a Coherent State due to Cold Collisions

$$\hat{H} = \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Coherent state in each lattice site no eigenstate !

$$|\Psi\rangle_i = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

=



1

1

$e^{-iUt/\hbar}$

$e^{-i3Ut/\hbar}$

1. ***Here*** α ***= amplitude of the coherent state***

2. ***Here*** $|\alpha|^2$ ***= average number of atoms per lattice site***

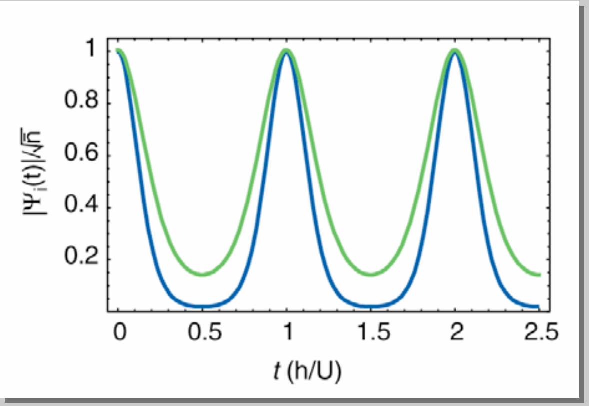
Collapse and Revival of the Macroscopic Wave Function due to Cold Collisions

Quantum state in each lattice site (e.g. for a coherent state)

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\frac{1}{2}U\,n(n-1)t/\hbar} \left|n\right\rangle$$

Macroscopic wave function in i[th] lattice site

$$\Psi_i(t) = {}_i\left\langle \Psi(t) \right| \hat{a}_i \left| \Psi(t) \right\rangle_i$$



1. *Macroscopic wave function **collapses** but **revives** after times multiple times of **h/U** !*
2. *Collapse time depends on the **variance** s_N of the atom number distribution !*

Wright et al. 1997; Imamoglu, Lewenstein & You et al. 1997, Castin & Dalibard 1997

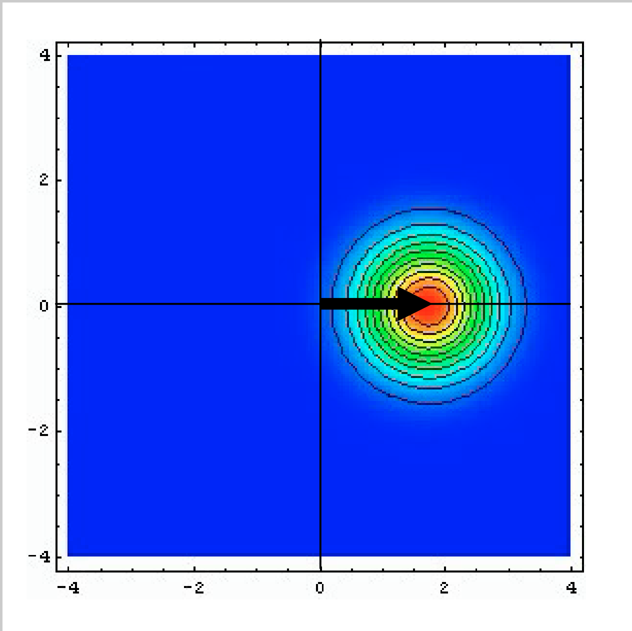
Dynamical Evolution of a Coherent State due to Cold Collisions

The dynamical evolution can be visualized through the Q-function

$$Q = \frac{\left|\langle \alpha | \psi_i(t) \rangle\right|^2}{\pi}$$

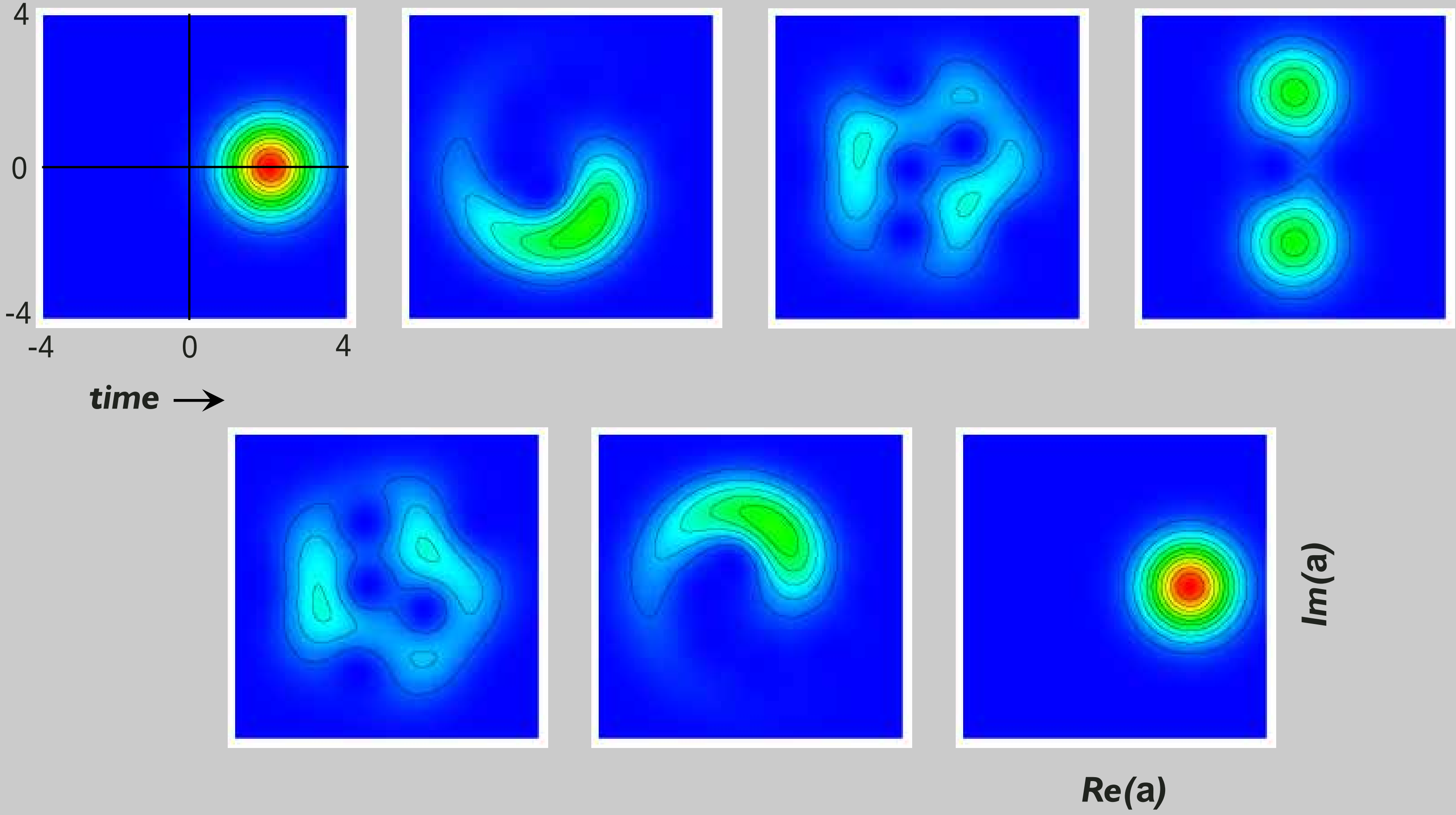
Characterizes overlap of our input state with an aribtrary coherent state $|\alpha\rangle$

Im(α)

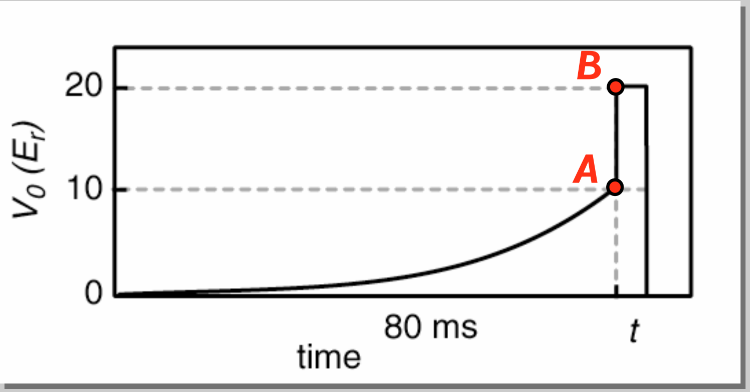


Re(α)

Dynamical Evolution of a Coherent State due to Cold Collisions



Freezing Out Atom Number Fluctuations



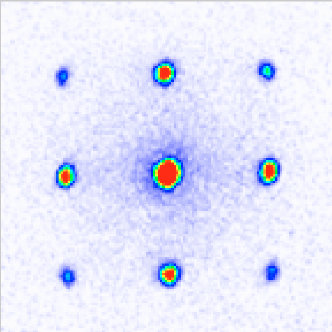
Ramp up lattice fast from the superfluid regime (A) to the MI regime (B), such that atoms do not have time to tunnel !

Atom number fluctuations at (A) are "frozen" !

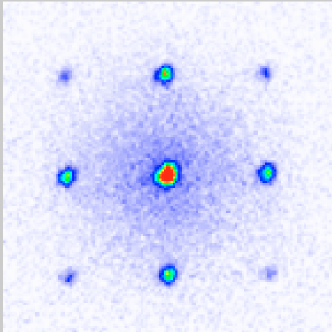
Coherent state in each lattice site with well defined macroscopic phase:

$$\left|\Psi(0)\right\rangle_i = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} \left|n\right\rangle$$

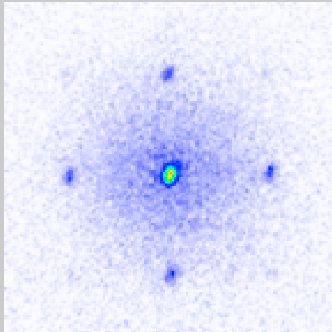
Dynamical Evolution of the Interference Pattern



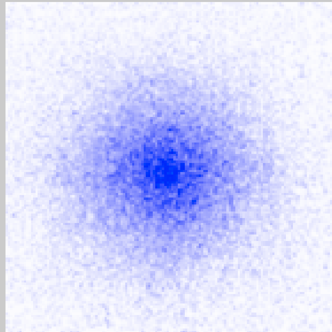
t=50 μs



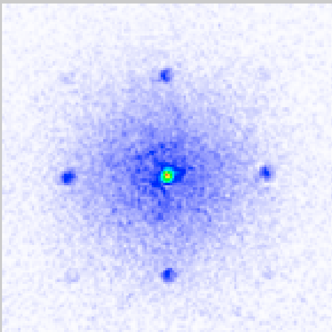
t=150 μs



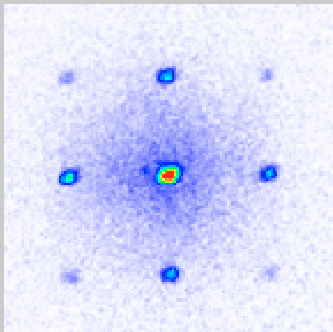
t=200 μs



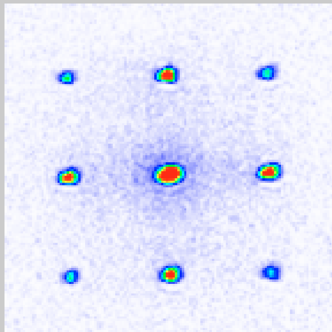
t=300 μs



t=400 μs



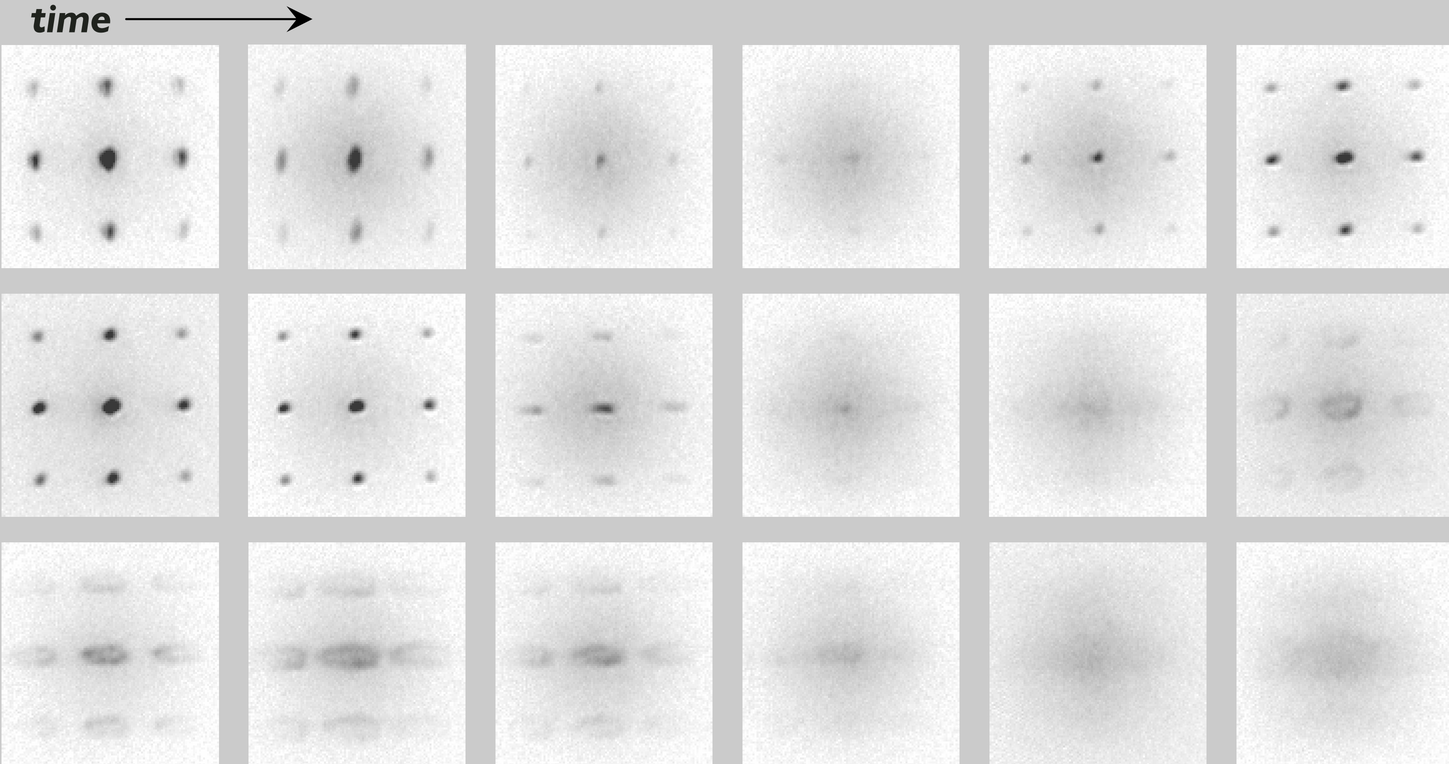
t=450 μs

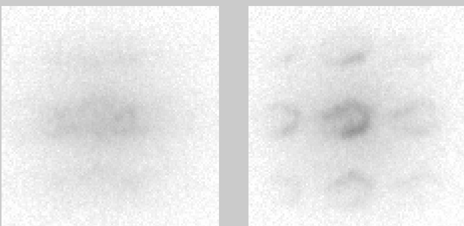


t=600 μs

After a potential jump from $V_A=8E_r$ to $V_B=22E_r$.

Collapse and Revival (Experiment at $V_0 = 20\ E_r$)

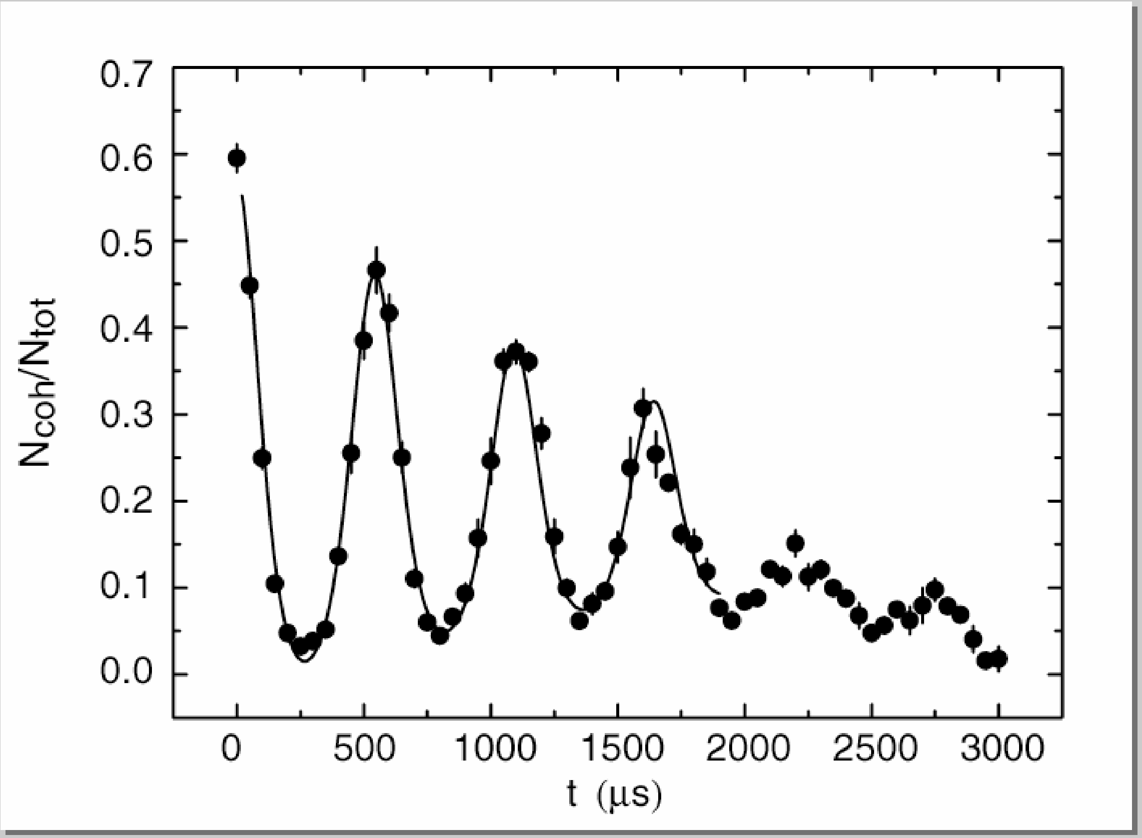




Frequency of Revival: 1.6(1) kHz
Resonance Gradient: 1.57 (15) kHz
Theory (Wannier functions) U/h: 1.67 kHz

Collapse and Revival N_{coh}/N_{tot}

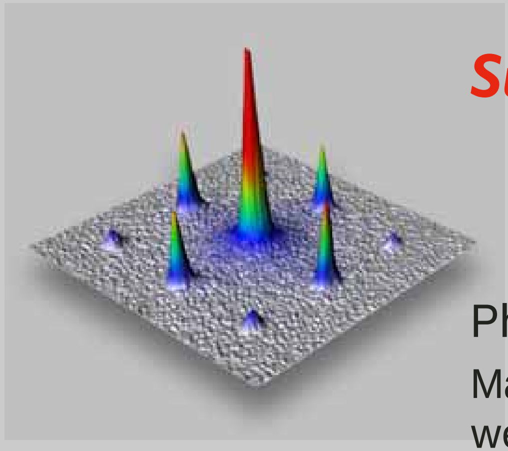
Oscillations after lattice potential jump from 8 E_{recoil} to 22 E_{recoil}



Up to 5 revivals are visible !

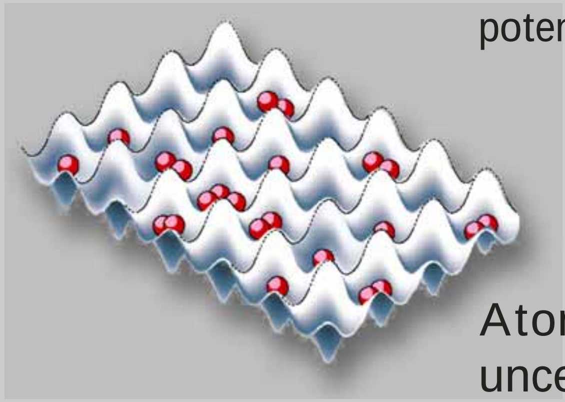
SF - MI

Superfluid



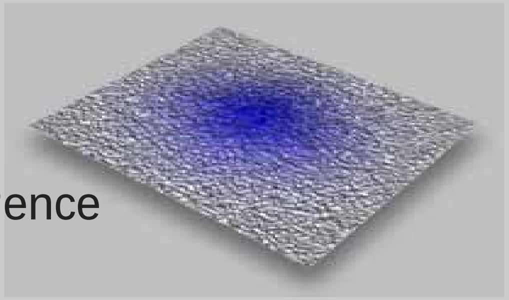
Phase coherence

Macroscopic phase well defined in each potential well



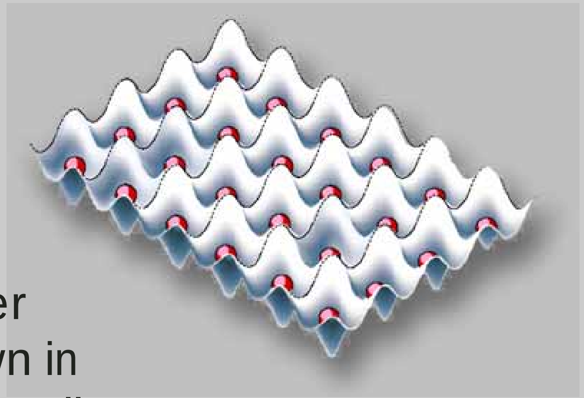
Atom number uncertain in each potential well

Mott Insulator



No Phase coherence

Macroscopic phase uncertain in each potential well



Atom number exactly known in each potential well

→ atom number correlations

M.P.A. Fisher et al, D. Jaksch et al.

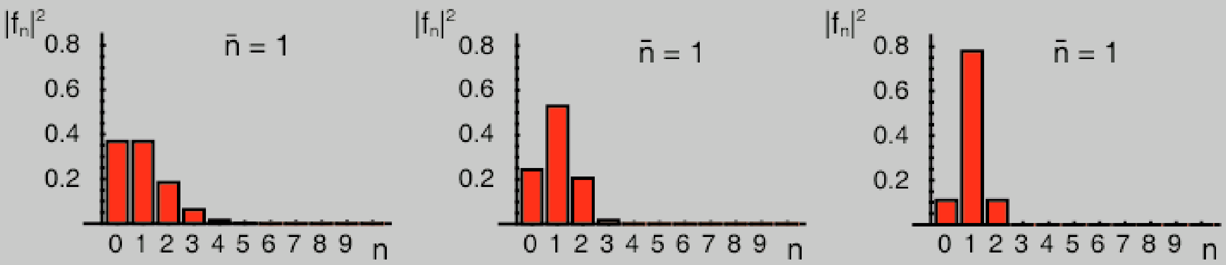
Gutzwiller approximation for finite onsite interaction U

$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Gutzwiller approximation: ***Many body state is approximated as a product of localized states*** $|\Phi_i\rangle$ ***on each lattice site***

$$|\Psi_{GW}\rangle = \prod_M |\Phi_i\rangle \qquad |\Phi_i\rangle = \sum_{n=0}^{\infty} f_n^{(i)} |n\rangle$$

Number squeezing for finite U/J



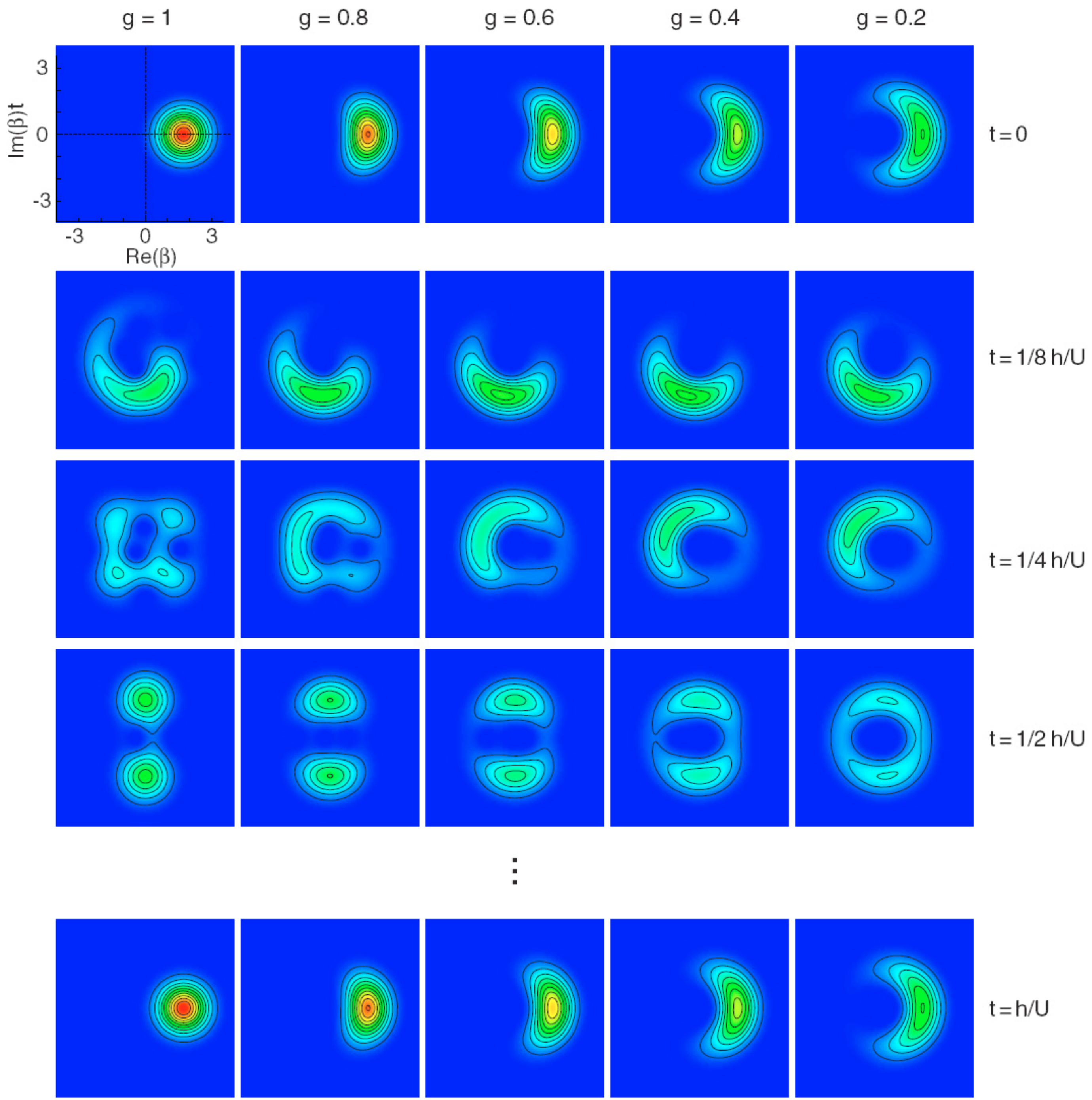
U/J=0 ***U/J=7*** ***U/J=22***

In the limit of large Boson number:
Orzel et al.: Squeezed states in a Bose-Einstein condensate, Science, 291, 2001

Different number squeezing

Time evolution

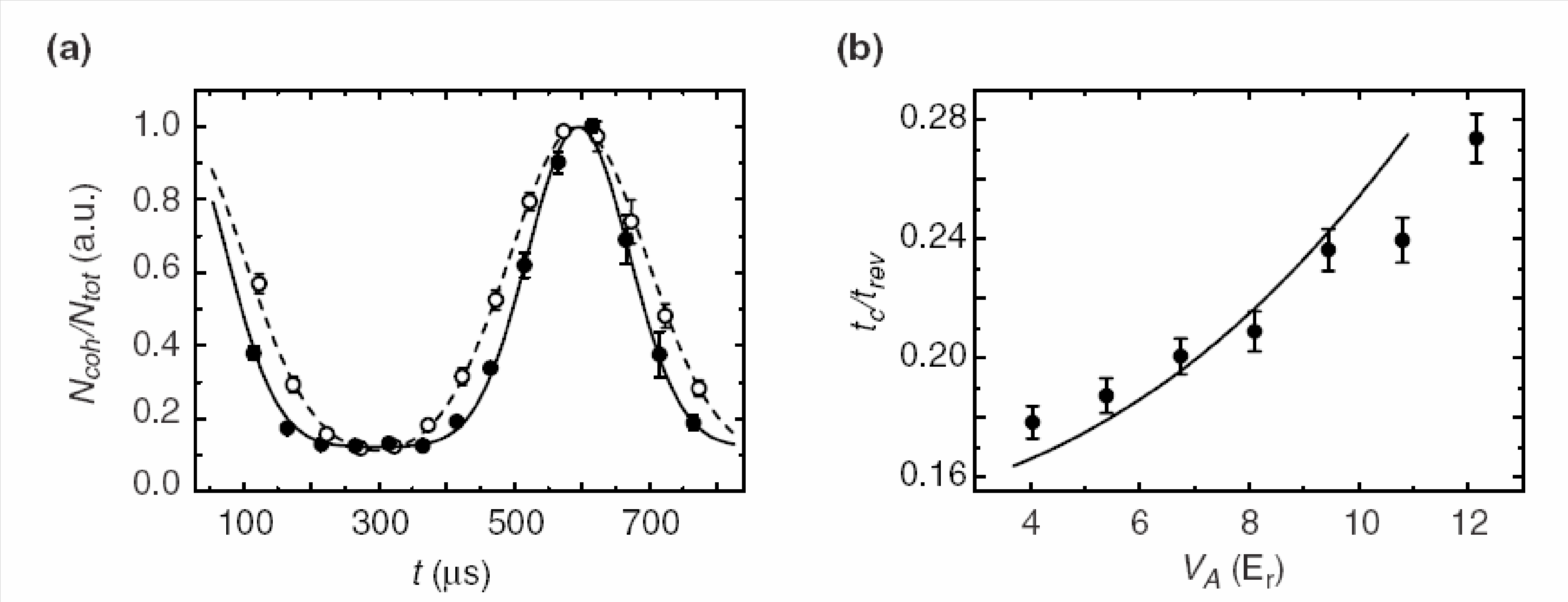
Full revival:



Measurement of number squeezing

Collapse time depends on variance σ_n^2 of atom number statistics:

$$t_c = t_{rev} / (2\pi\sigma_n)$$

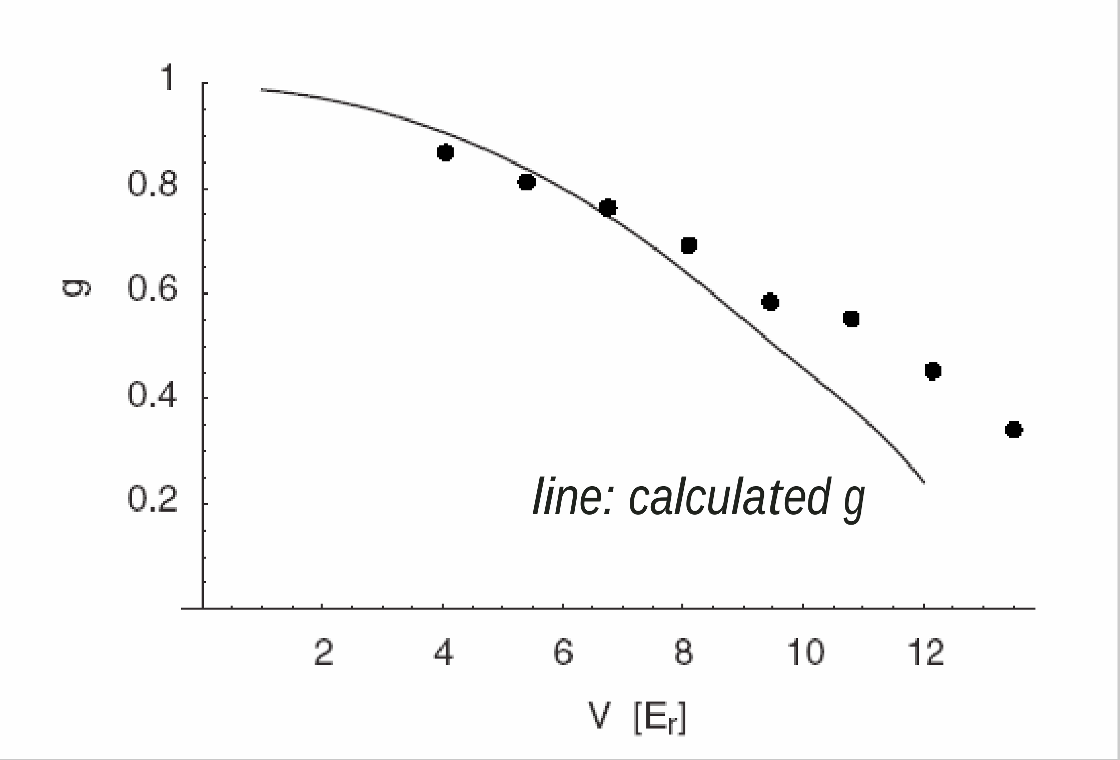


Measured atom number squeezing

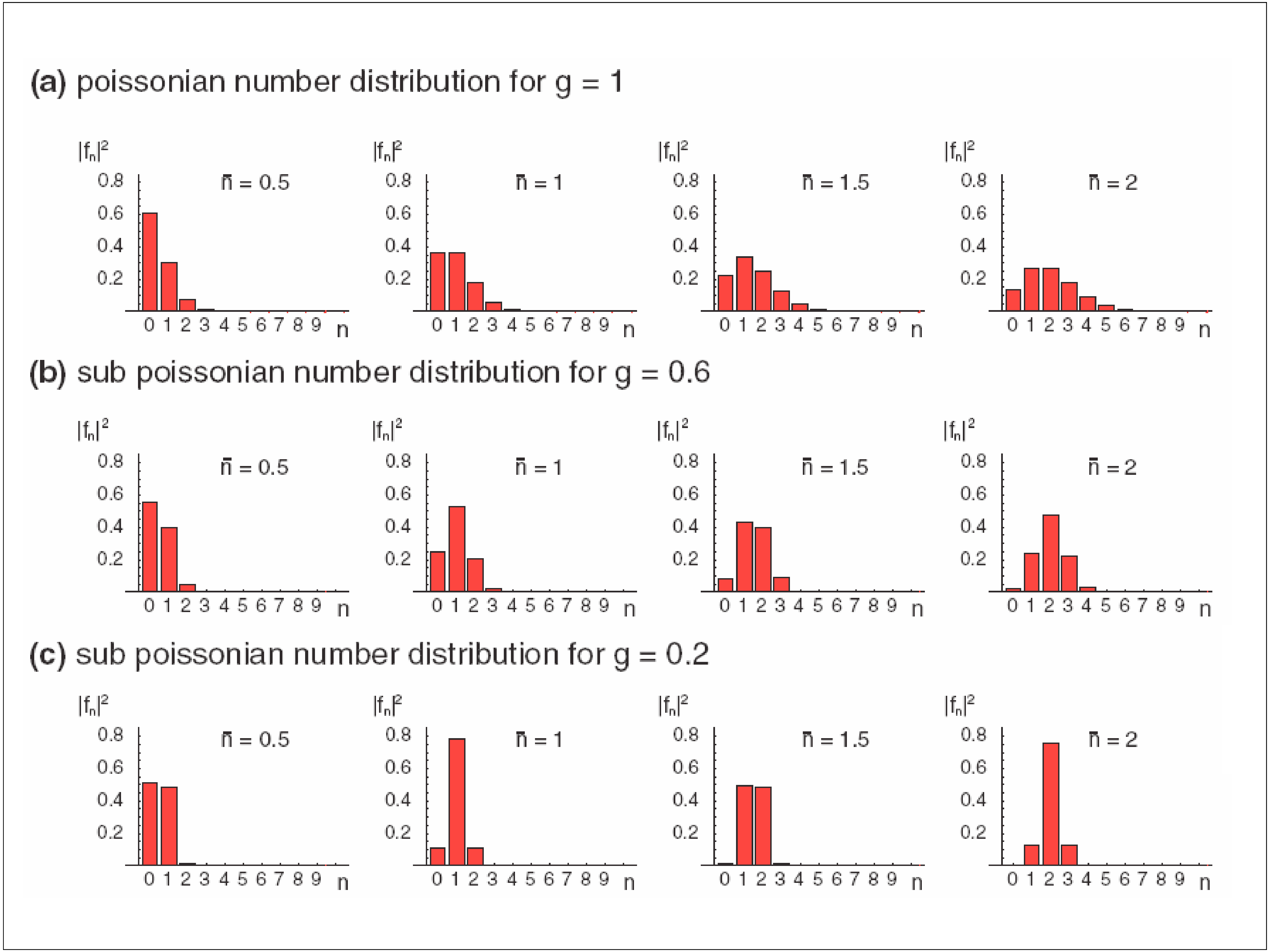
squeezing parameter g:

$$\left|\Phi_i\right\rangle = \sum_{n=0}^{\infty} g^{\frac{n(n-1)}{2}} \frac{\lambda^{n/2}}{\sqrt{n}} \left|n\right\rangle$$

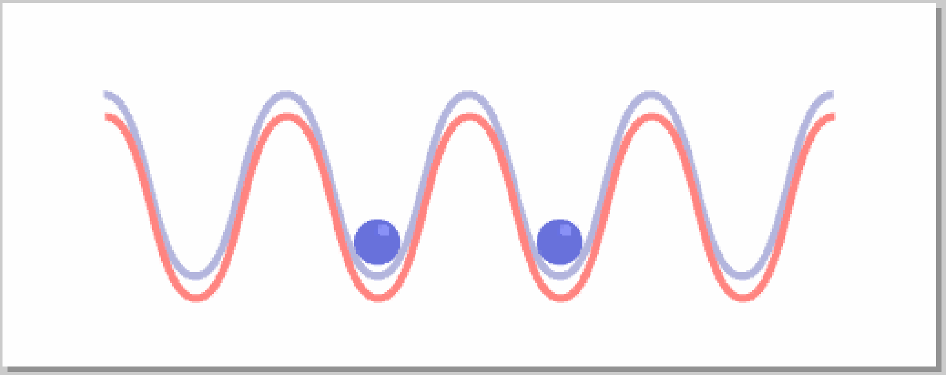
Rokhsar et al.,
PRB 44,10328 (1991)



potential depth

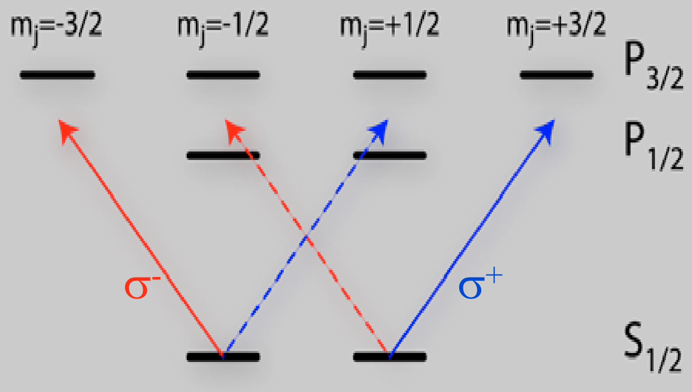

(a) poissonian number distribution for g = 1
(b) sub poissonian number distribution for g = 0.6
(c) sub poissonian number distribution for g = 0.2
$|f_n|^2$
0.8
0.6
0.4
0.2
$\bar{n}$ = 0.5
$\bar{n}$ = 1
$\bar{n}$ = 1.5
$\bar{n}$ = 2
0 1 2 3 4 5 6 7 8 9 n

5. Universal quantum gates with ultracold atoms

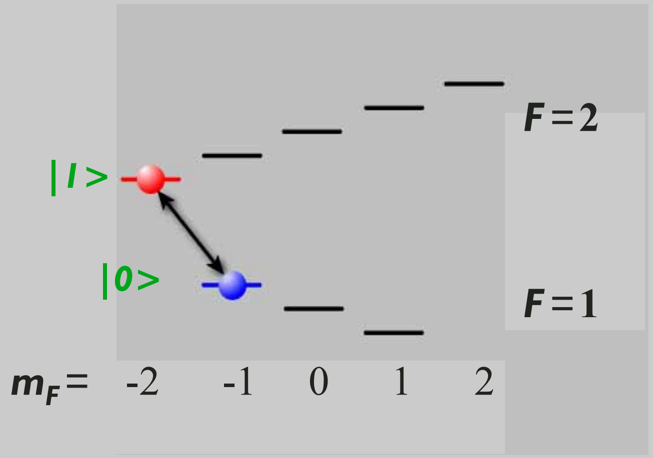


State Selective Lattice Potentials

^{87}Rb Fine-structure



Hyperfine structure



State selective lattice potential

$$|1>: \quad V_1(x,\theta) = V_-(x,\theta)$$

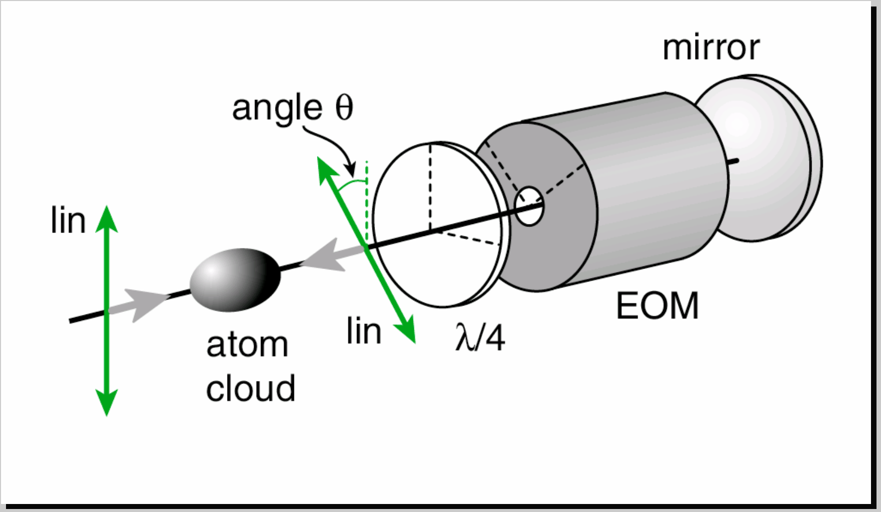
$$|0>: \quad V_0(x,\theta) = \frac{1}{4}V_-(x,\theta) + \frac{3}{4}V_+(x,\theta)$$

$V_-(x,\theta)$ formed by σ_- polarized Light

$V_+(x,\theta)$ formed by σ_+ polarized light

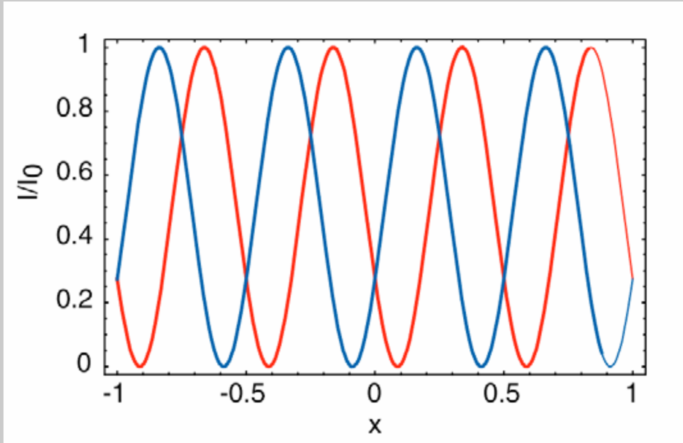
D. Jaksch et al., PRL 82,1975(1999), G. Brennen et al., PRL 82, 1060 (1999)
Overview: I. Deutsch & P. Jessen, Optical Lattices, Adv. At. Mol. Phys. 36, 91 (1996).

Moving the Lattice Potential



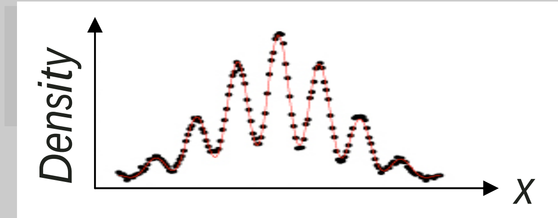
Lin angle Lin standing wave configuration can be decomposed into a σ^+ and σ- standing wave !

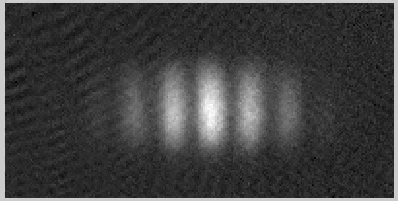
$$I_+ = I_0 \sin^2(kx + \theta/2)$$
$$I_- = I_0 \sin^2(kx - \theta/2)$$



D. Jaksch et al., PRL 82,1975(1999), G. Brennen et al., PRL 82, 1060 (1999)
Overview: I. Deutsch & P. Jessen, Optical Lattices, Adv. At. Mol. Phys. 36, 91 (1996).

Delocalization "by Hand": Trapped Atom Interferometer





Measured time of flight picture

TOF

↑

$\pi/2$ microwave pulse

↑

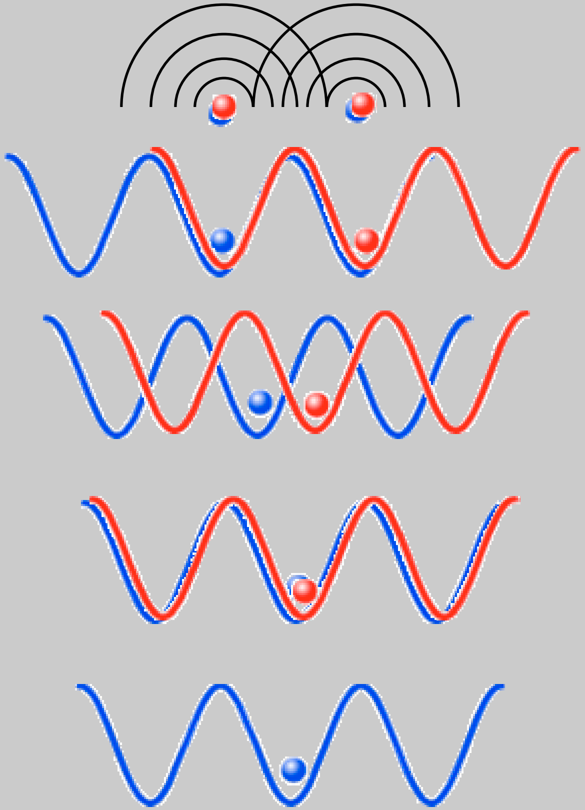
Shift

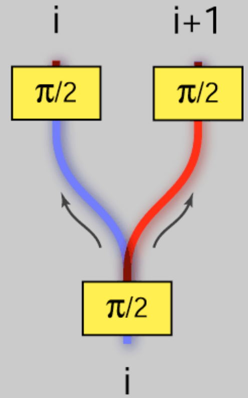
↑

$\pi/2$ microwave pulse

↑

Initial state |0>





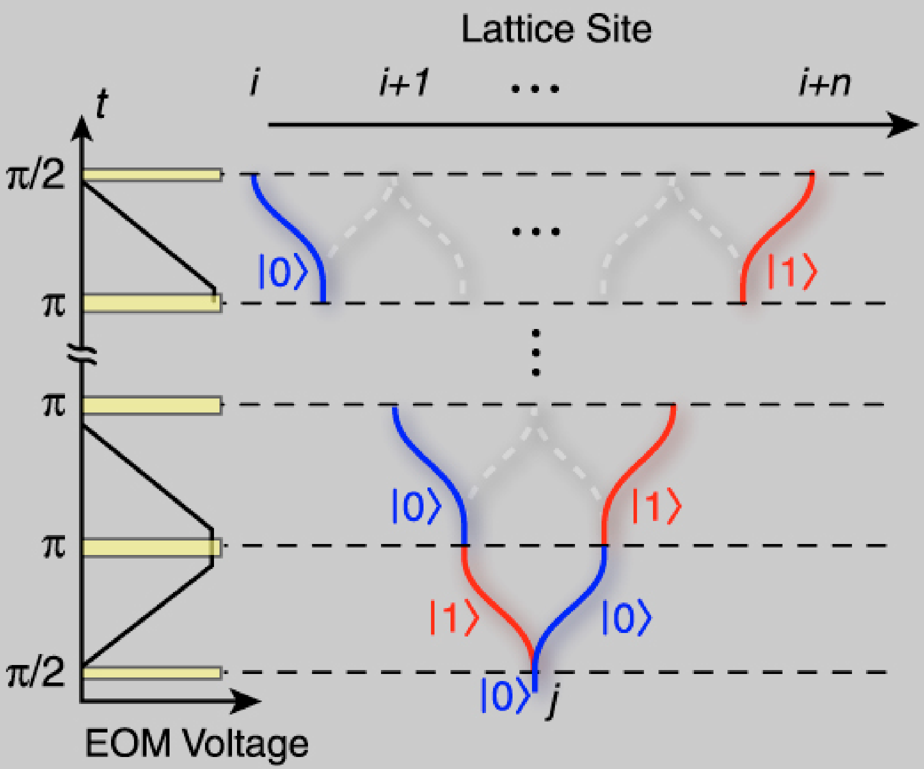
Theory :

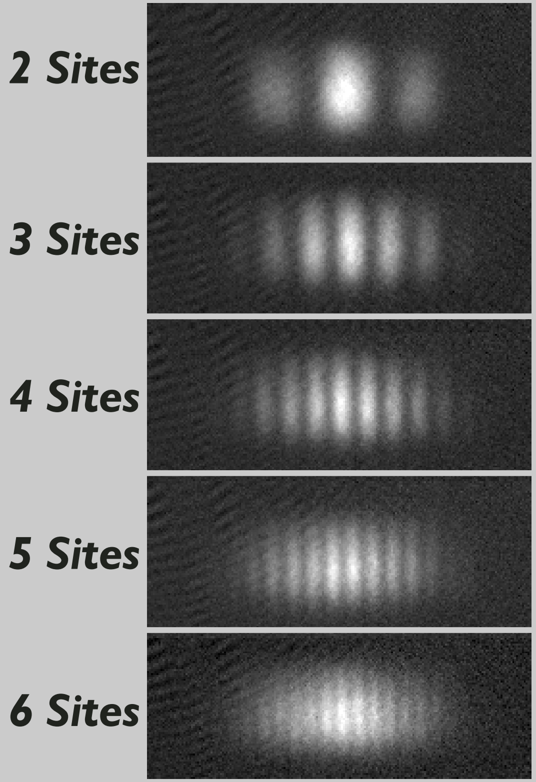
D. Jaksch et al., PRL 82,1975(1999)

G. Brennen et al., PRL 82, 1060 (1999)

A. Sorensen et al., PRL 83, 2274 (1999)

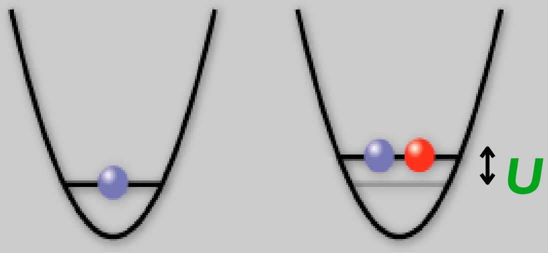
Complete Delocalization Sequence





O. Mandel, M. Greiner et al., preprint arXiv: cond-mat/0301169 (2003)

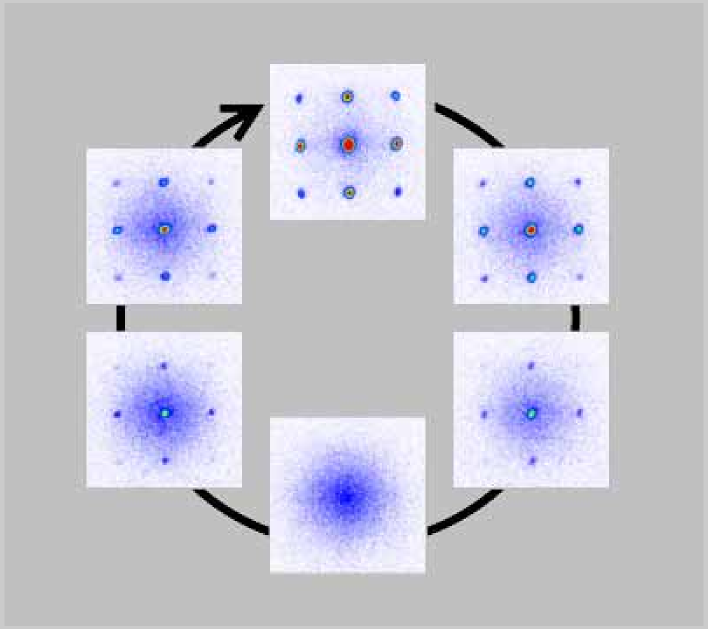
Quantum gates with neutral atoms



2 atoms at same site: collisional phase shift

$$e^{i\phi} = e^{iUt_{hold}/\hbar}$$

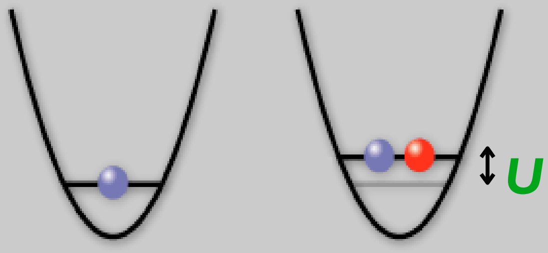
→*Collisions between atoms just lead to a coherent collsional phase* ϕ



Demonstrated in Collapse and Revival experiment,

M. Greiner, O. Mandel et al., Nature 419, 51 (2002)

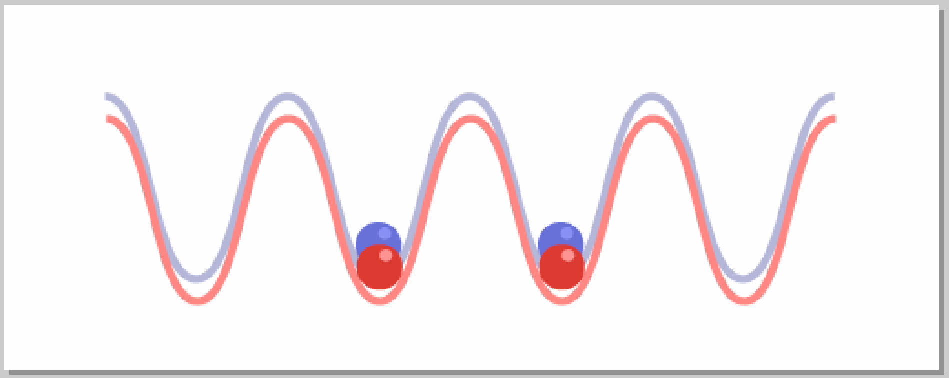
Quantum gates with neutral atoms



2 atoms at same site: collisional phase shift

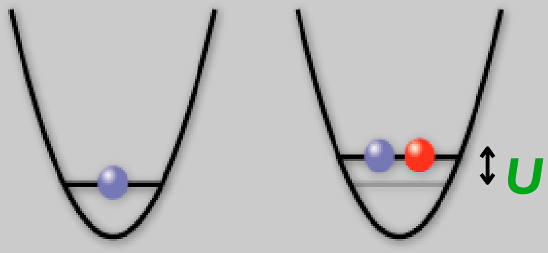
$$e^{i\phi} = e^{iUt_{hold}/\hbar}$$

Fundamental quantum gate:



D. Jaksch et al., PRL 82,1975(1999)
G. Brennen et al., PRL 82, 1060 (1999)
A. Sorensen et al., PRL 83, 2274 (1999)

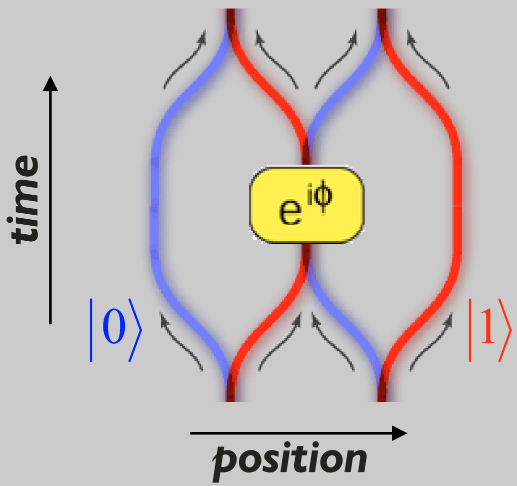
Quantum gates with neutral atoms



2 atoms at same site: *collisional phase shift*

$$e^{i\phi} = e^{iUt_{hold}/\hbar}$$

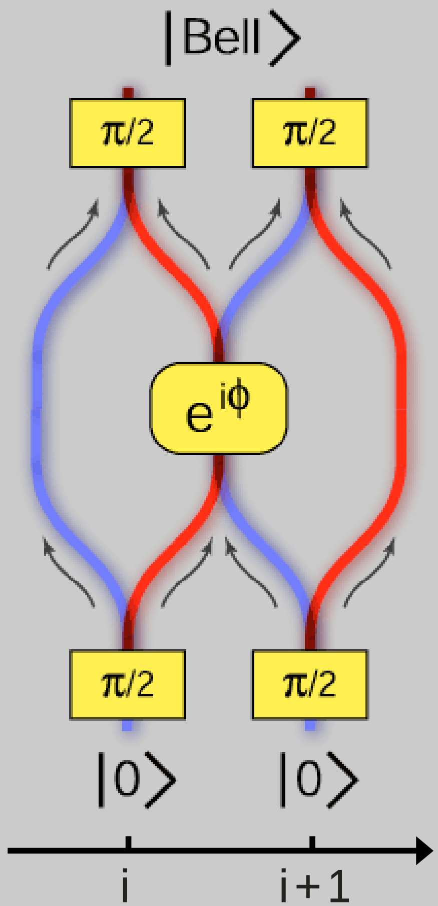
Fundamental** quantum **gate:



Input state	*Final state*
$\|0\rangle\|0\rangle$	$\|0\rangle\|0\rangle$
$\|0\rangle\|1\rangle$	$\|0\rangle\|1\rangle$
$\|1\rangle\|0\rangle$	$\|1\rangle\|0\rangle \cdot e^{i\phi}$
$\|1\rangle\|1\rangle$	$\|1\rangle\|1\rangle$

D. Jaksch et al., PRL 82,1975(1999)
G. Brennen et al., PRL 82, 1060 (1999)
A. Sorensen et al., PRL 83, 2274 (1999)

Engineering a Cluster-state

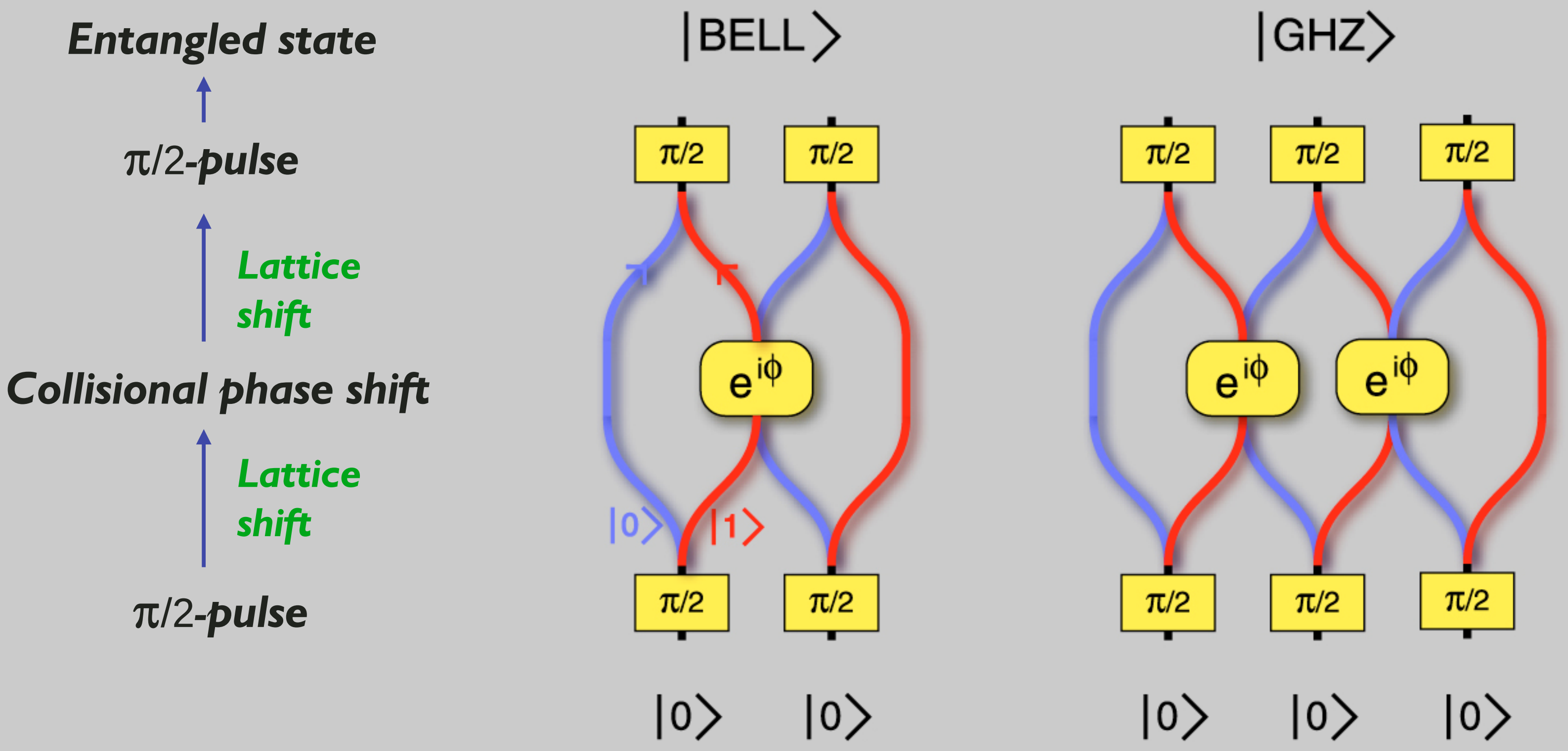


$$\tfrac{1}{2}(1-e^{i\varphi})|Bell\rangle + \tfrac{1}{2}(1+e^{i\varphi})|1\rangle_i|1\rangle_{i+1}$$

$$\tfrac{1}{2}\left(|0\rangle_i|0\rangle_{i+1} + |0\rangle_i|1\rangle_{i+2} + e^{i\varphi}|1\rangle_{i+1}|0\rangle_{i+1} + |1\rangle_{i+1}|1\rangle_{i+2}\right)$$

$$\tfrac{1}{2}\left(|0\rangle_i + |1\rangle_i\right)\left(|0\rangle_{i+1} + |1\rangle_{i+1}\right)$$

Entanglement due to Controlled Cold Collisions



- *With N atoms one obtains maximally entangled "cluster states" !*
- *Cluster state has maximally "connectedness" and "persistency"*

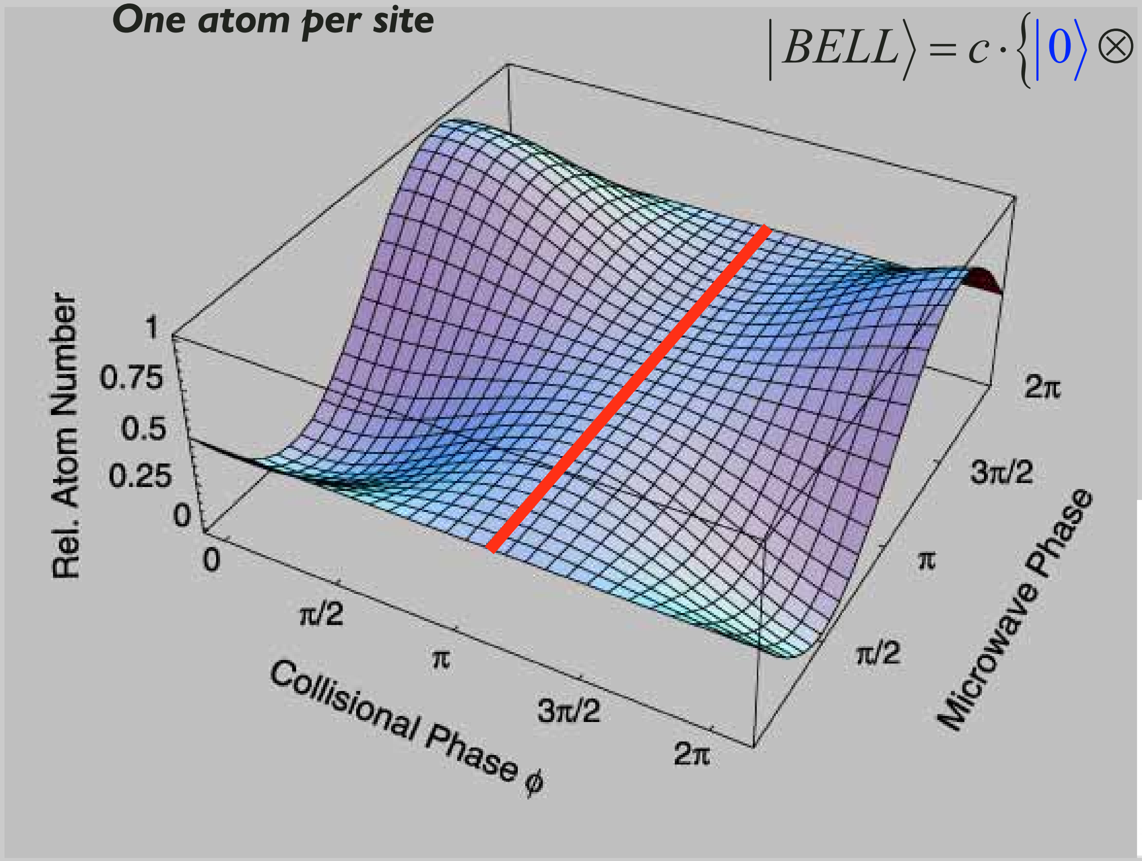
D. Jaksch et al., PRL 82,1975 (1999), H.-J. Briegel et al., J.Mod.Opt.

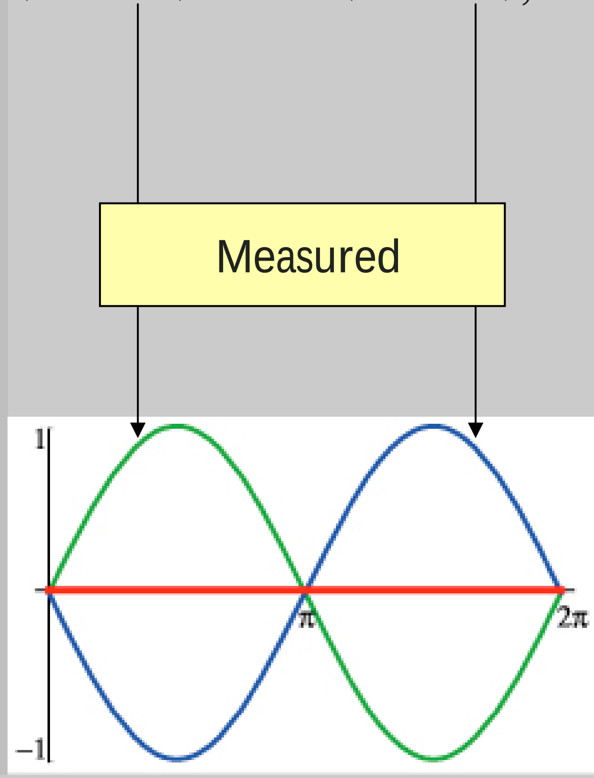
H.-J. Briegel & R. Raussendorf PRL 86, 910 (2001) & PRL 86, 5188 (2001).

Collapse and Revival of the Ramsey fringe

$$\phi = \pi \Rightarrow \psi = |BELL\rangle$$

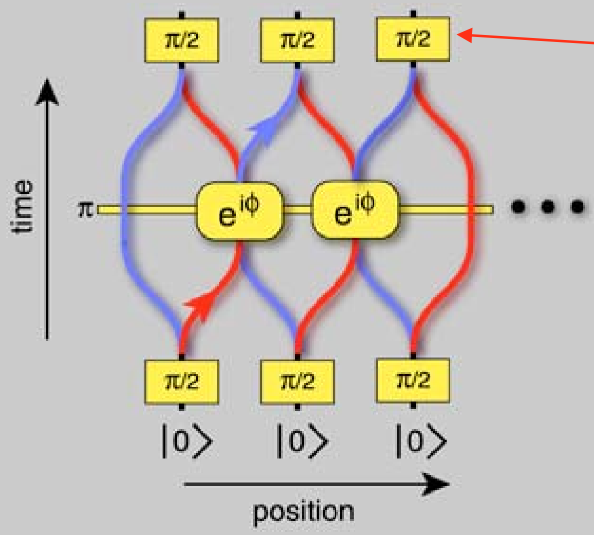
$$|BELL\rangle = c\cdot\left\{|0\rangle\otimes\left(|0\rangle-|1\rangle\right)+|1\rangle\otimes\left(|0\rangle+|1\rangle\right)\right\}$$





D. Jaksch, PhD-Thesis, Innsbruck

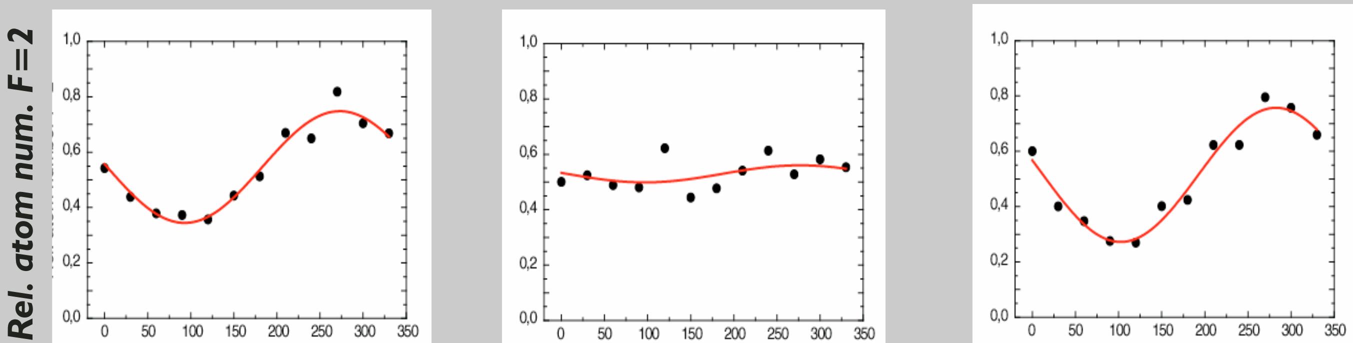
Ramsey Fringe vs. Collisional Phase



Variation of the phase of 2^{nd} $\pi/2$ pulse:

- *Observation of Ramsey fringes*
- *Collapse of Ramsey fringe for **entangled** state for a Collisional phase $\phi = \pi$*
- *Revival of Ramsey fringe for **disentangled** state for a Collisional phase $\phi = 2\pi$*

(for details see: D. Jaksch, PhD-Thesis)



phase of 2^{nd} $\pi/2$ pulse — $\phi \approx 0$

entangled — $\phi \approx \pi$

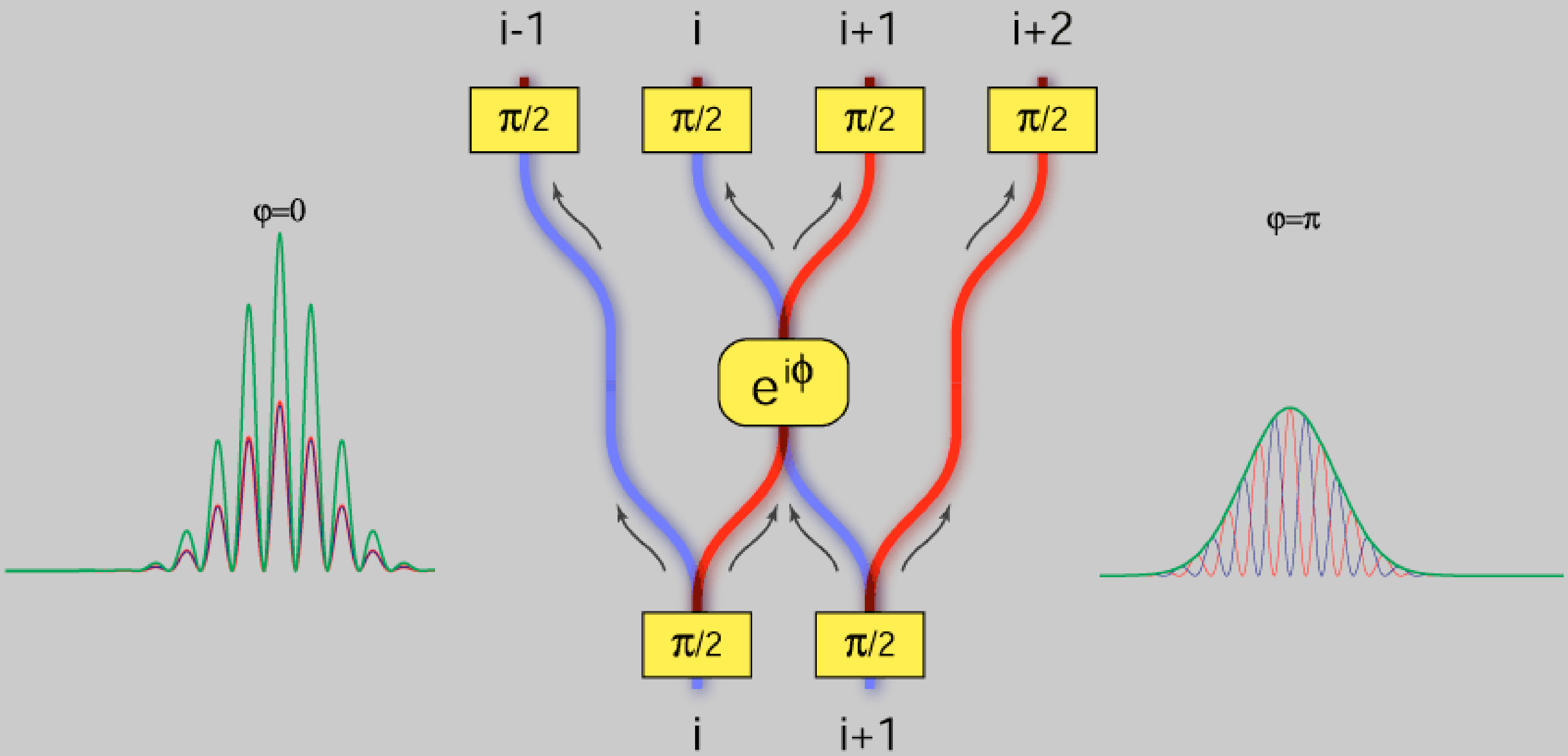
disentangled — $\phi \approx 2\pi$

Collisional phase ϕ

Conditional Double-Slit

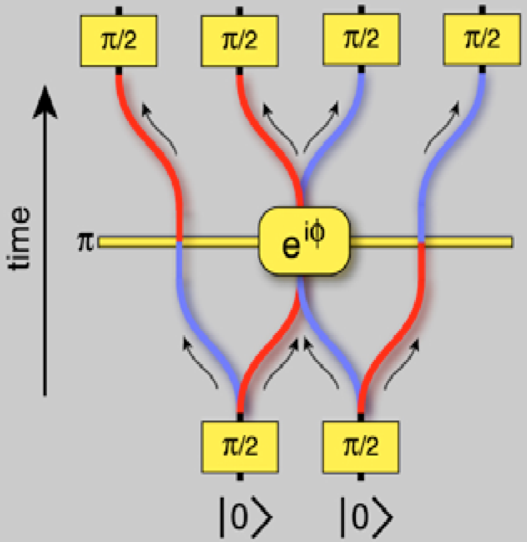
After $\pi/2$-Puls,
detection in $|0\rangle$:

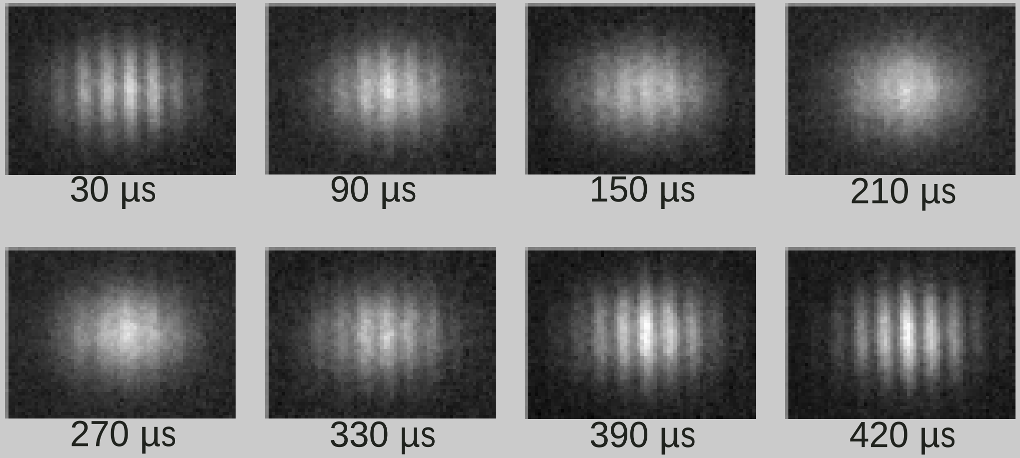
$$\frac{1}{4}\left\{|0\rangle_{i-1}\otimes\left(|0\rangle_{i}+|0\rangle_{i+2}\right)+|0\rangle_{i+1}\otimes\left(e^{i\varphi}|0\rangle_{i}+|0\rangle_{i+2}\right)\right\}$$

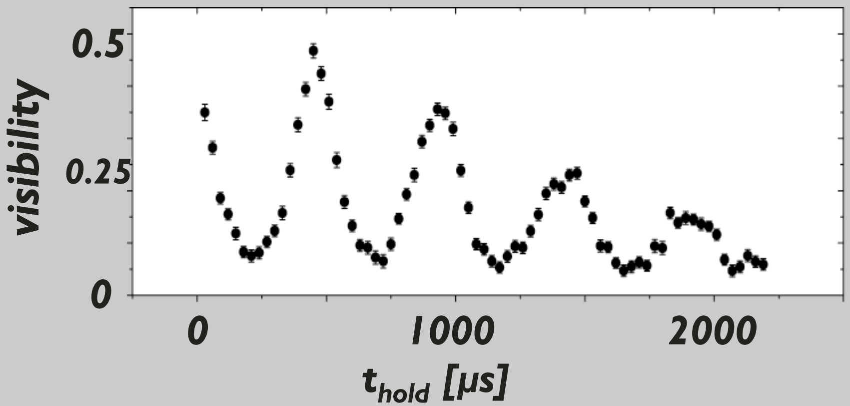


Entanglement Dynamics Sequence

Alternative sequence to measure whole Ramsey fringe in single experimental run



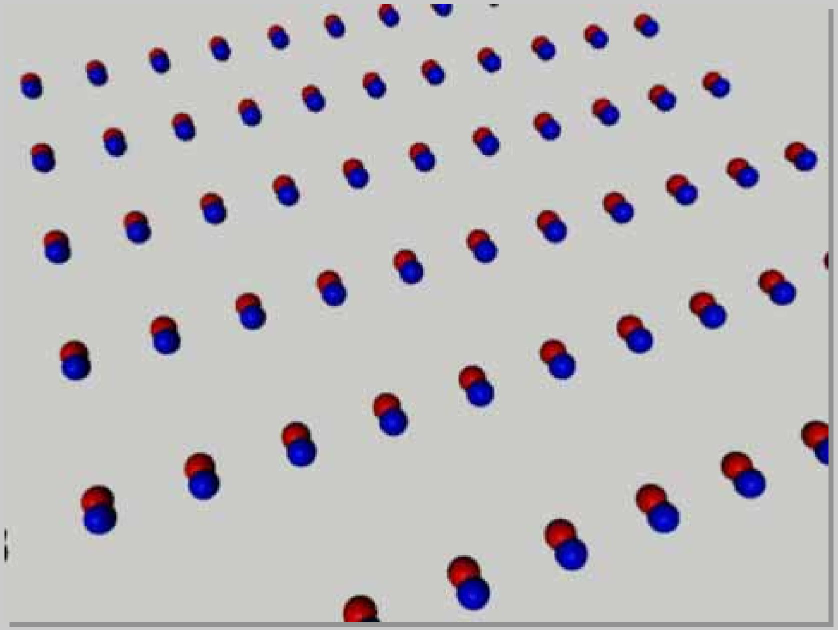




Olaf Mandel, Markus Greiner, Artur Widera, Tim Rom, Theodor W. Hänsch, Immanuel Bloch
Nature 425, 937 (2003)

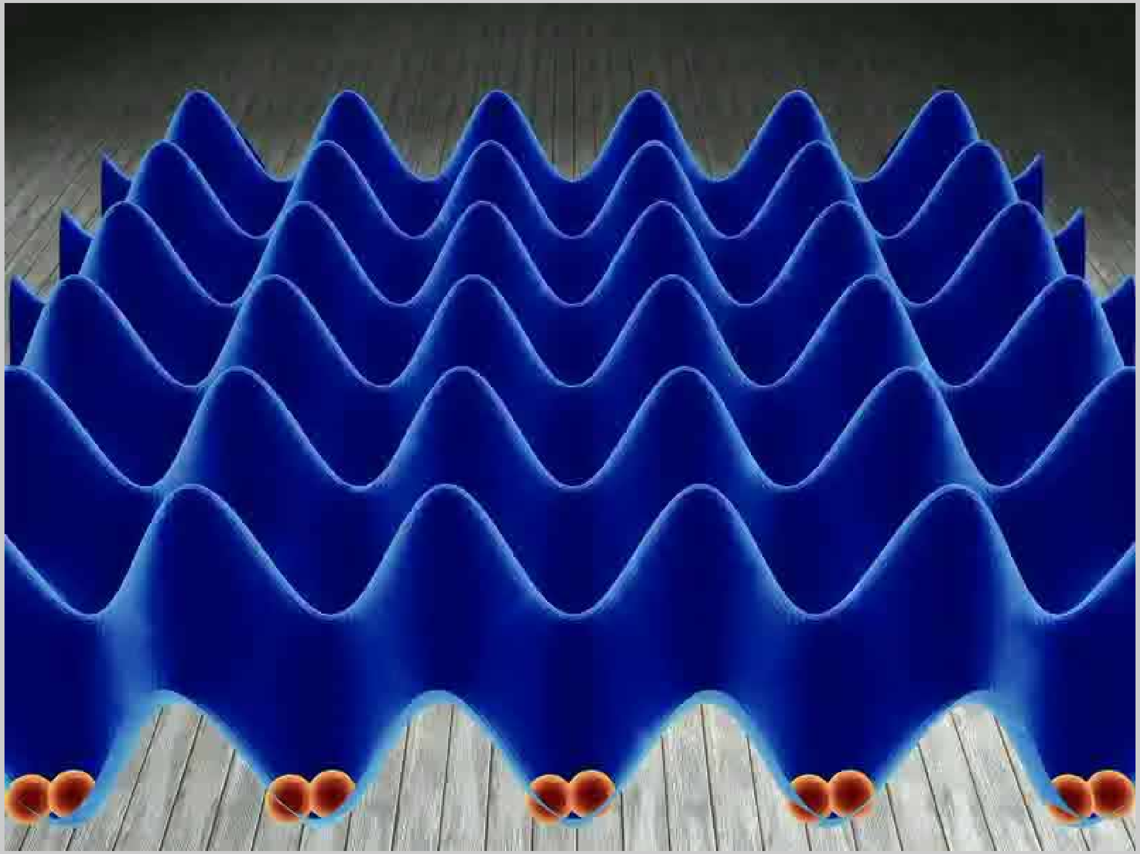
Applications

- *Simulation of solid state hamiltonians (Ising, Heisenberg)*
- *Quantum Random Walks in optical lattices*
- *Adding addressability of single lattice sites*
- *Resource for quantum computing*



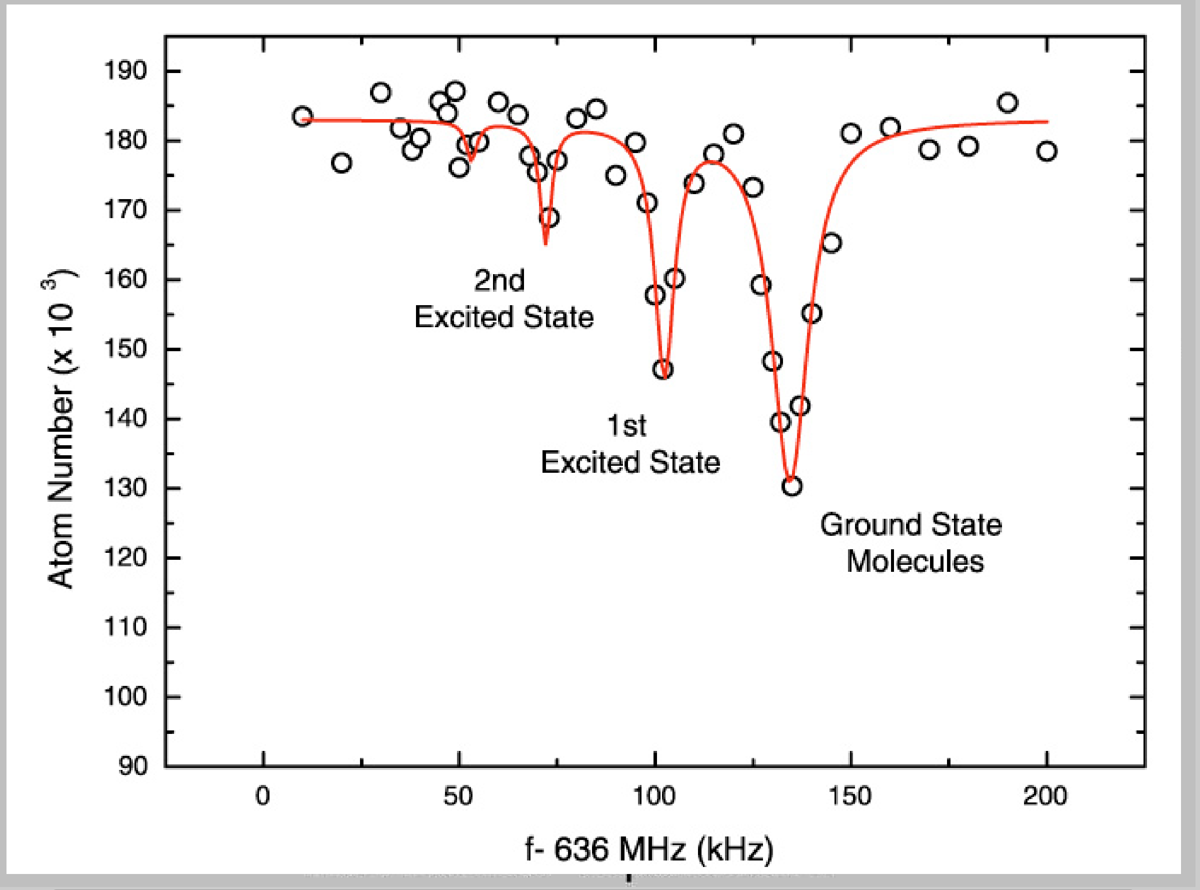
H.-J. Briegel & R. Raussendorf PRL 86, 910 (2001) & PRL 86, 5188 (2001).
W. Dür & H.-J. Briegel, PRL 90, 067901 (2003)

Application of MI: Molecule formation by photo association



Mott insulator state with two atoms per lattice site:
Great environment to form molecules !

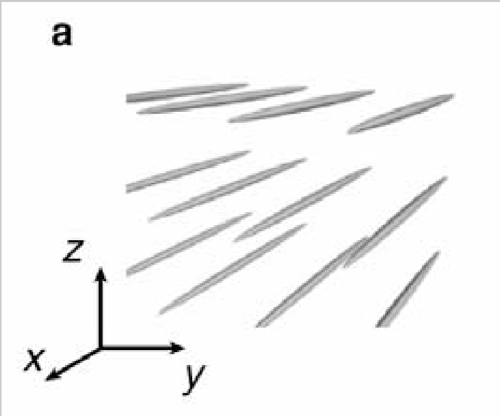
Application of MI: Molecule formation by photo association

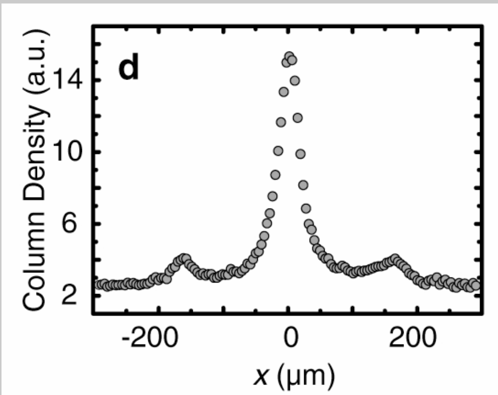


Mott insulator state with two atoms per lattice site: Great environment to form molecules !

T. Rom, et al., PRL 93, 073002 (2004)

Tonks-Girardeau Gas





Strongly interacting 1D gas:

Fermionization of bosonic particles

Munich:

- *Tubes: red detuned 2D lattice*
- *Additional lattice along tubes to increase effective mass*
- *Detection via momentum distribution*

B. Paredes et al., Nature 429, 277-281 (2004)

Penn State (D. Weiss):

- *Tubes: blue detuned 2D lattice*

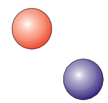
T. Kinoshita, Science 305, 1125 – 1128 (2004)

Zurich: *1D regime (see lectures Michael Koehl)*

Condensed matter physics with ultracold atoms

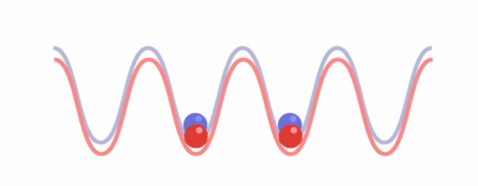
A large variety of complex condensed matter and many-body Hamiltonians can be realized in a controlled way:

- combining different spin states of atoms

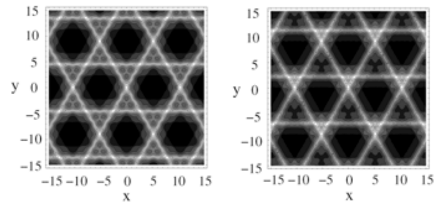


- using both fermionic and bosonic atoms

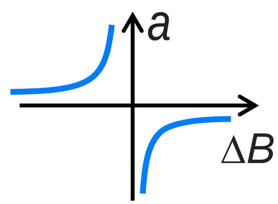
  → boson mediated fermion-fermion interaction



- spin selective potentials
- varying lattice geometries, e.g. Kagome



- Feshbach resonances



- add disorder
- …

Research possibilities

This allows to realize exciting quantum phases:

- magnetic order, e.g. Antiferromagnetic phases
- Frustrated phases in Kagome lattices
- High Tc
- spin waves in lattices
- (fractional) Quantum Hall physics with Bosons
- disorder: Bose-glass phase, Anderson localization
- Quantum information: detection of highly entangled state, using them for "teleportation"
- …

→ "Quantum simulator" in the sense of Feynman

Condensed matter physics with ultracold atoms

Real materials

complicated:

- various interactions
- disorder

?

Direct experimental test of condensed matter models:

Condensed matter models

difficult to calculate, especially for fermions

Ultracold atoms in optical lattices

clean realization of condensed matter models