The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
Energy Agency

SMR 1666 - 10

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Feshbach resonances and Feshbach molecules

Presented by:

Chin Cheng

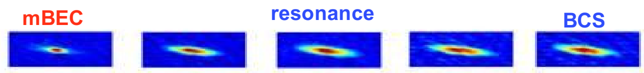
University of Chicago

School on Quantum Phase Transitions and

Non-Equilibrium Phenomena in Cold Atomic Gases *Trieste, 14 July 2005*

Ultracold atoms quantum gases

Feshbach Resonances and Feshbach Molecules



Cheng Chin

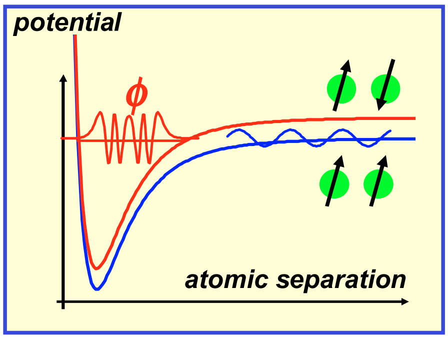
Experimentalphysik, Innsbruck University, Austria
James Franck Institute, Physics Dept., Univ. of Chicago

A Magnetic handle in ultracold gases

Ultracold atoms quantum gases

- **What is Feshbach resonance**
  - Single channel scattering
  - Two channel scattering
- **Li_2 molecules (open channel dominated)**
  - Atom-molecule thermalization
  - Collision properties of molecules
  - Precision molecular spectroscopy
  - Pairing gaps and collective modes (TOMORROW)
- **Cs_2 molecules (closed channel dominated)**
  - Highly rotation molecules (G- and L-wave)
  - Observation of Cs_4 quantum states
  - Molecule interferometry

Typical Feshbach resonances in cold atoms



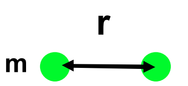
Open channel
Singlet potential with a foreigh bound statae

Closed channel
Triplet potential

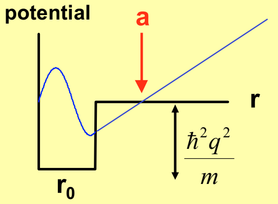
Feshbach coupling
Hyperfine interaction
Feshbach tuning
Zeeman effect

Experimental observables
1. *Inelastic and inelastic cross sections*
2. *Mean-field shifts and many-body effects*
3. *Pairing of atoms*

Resonant scattering (single channel)

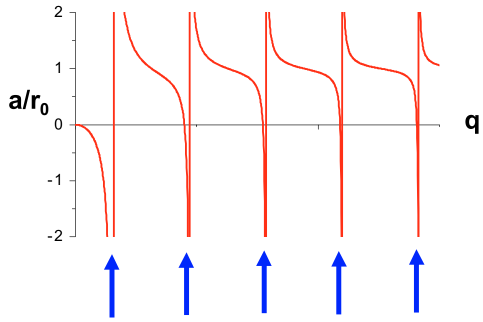


Zero scattering energy



$\Psi(r<r_0) \sim \frac{\sin q\, r}{r}$ $\Psi(r>r_0) \sim \frac{r-a}{r}$

B.C.: $\frac{1}{r-a} = \frac{q}{\tan q\, r_0}$



Formation of new bound state

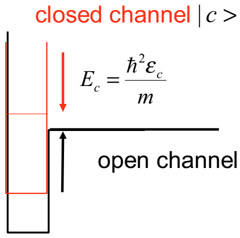
Near the resonance, binding energy is

$$E_m = \frac{\hbar^2}{m(a_{bg}-r_0)^2}$$

Gribakin and Flambaum PRA 48 563

Two-channel box model

closed channel $|c>$



$E_c = \frac{\hbar^2 \varepsilon_c}{m}$

open channel $|o>$

$$H = \frac{\hbar^2}{m}\begin{pmatrix} -q_c^2 & \Omega \\ \Omega & -q_o^2 \end{pmatrix} \quad \text{for } r < r_0$$

$$= \begin{pmatrix} \infty & 0 \\ 0 & 0 \end{pmatrix} \quad \text{for } r > r_0$$

Exact Solution for E=0

$$\psi \sim \frac{\sin q_+ r}{r}|+> + \frac{A \sin q_- r}{r}|-> \quad \text{for } r < r_0$$

$$\sim \frac{r-a}{r}|o> \quad \text{for } r > r_0$$

$$\frac{1}{r_0 - a} = \frac{q_+}{\tan q_+ r_0}\cos^2\theta + \frac{q_-}{\tan q_- r_0}\sin^2\theta$$

$|+>, |->$: eigenstates for $r < r_0$

$$\tan 2\theta = \frac{2\Omega}{q_o^2 - q_c^2}$$

Approx.: $\Omega << q_c^2, q_o^2$ and $\varepsilon_c << q_c / r_0$

$$\Rightarrow \frac{1}{a - r_0} = C + \frac{\gamma}{\varepsilon_c}$$

Background properties : $C = \frac{1}{a_{bg} - r_0}$

Feshbach coupling : $\gamma = 2 q_c^2 \theta^2 r_0^{-1}$

Magnetic dependence: $E_c + \mu B$

$$\Rightarrow a = a_{bg}\left(1 + \frac{\Delta B}{B - B_{res}}\right)$$

Feshbach bound state

Eigenstate for $E = -E_m$

$$\psi_m \sim \frac{\sin q_+ r}{r}|+> + \frac{A_m \sin q_- r}{r}|-> \quad \text{for } r < r_0$$

$$\sim \frac{e^{-\sqrt{\varepsilon_m} a}}{r}|o> \quad \text{for } r > r_0$$

Eigen energy $E_m = \frac{\hbar^2}{m}\varepsilon_m$

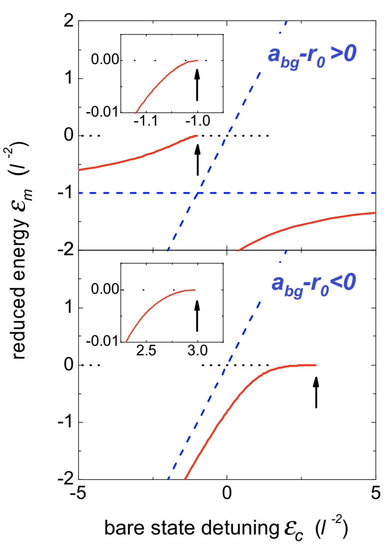
$$(\varepsilon_m + \varepsilon_c)\left(\sqrt{\varepsilon_m} - \frac{1}{a_{bg} - r_0}\right) = \gamma$$

Mixing fraction

$$\psi_m = \sin\phi\,|c> + \cos\phi\,|o>$$

$$\sin^2\phi = \int |\psi_c(r)|^2\, dv = \frac{2\sqrt{\varepsilon_m}\gamma}{(\varepsilon_m + \varepsilon_c)^2 + 2\sqrt{\varepsilon_m}\gamma}$$

generic energy structure

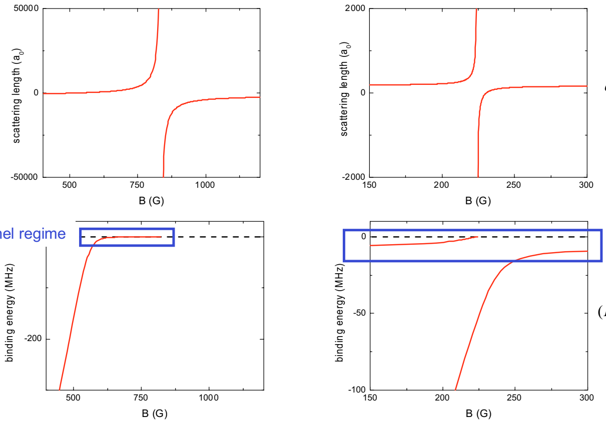


Scattering length and bound state

^{6}Li a_{bg}=-1405a_0 	^{40}K a_{bg}=+174a_0

scattering length

bound state energy



$$a = a_{bg}\left(1+\frac{\Delta B}{B-B_{res}}\right)$$

$$(E_m+E_c)\left(\sqrt{E_m}-\frac{\sqrt{\hbar^2/m}}{a_{bg}-r_0}\right)=\Gamma$$

binding energy = $\frac{\hbar^2}{m(a-r_0)^2}-\frac{2\hbar^5(a_{bg}-a)^2}{m^{5/2}(a-r_0)^5(a_{bg}-r_0)^2\Gamma}+...$

C. Chin, cond-mat/0506313

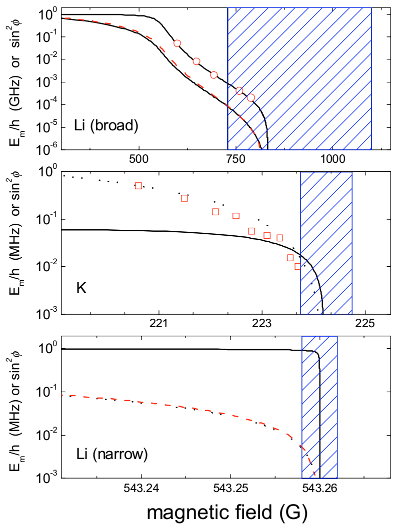
Example: Li_2 and K_2 molecules

Excellent agreement with all calculations and measurements!!

- **Small binding open channel**

$$E_m=\frac{\hbar^2}{m(a_{bg}-r_0)^2}$$

Li (broad)
mixing frac. < 10^{-3}
K40
mixing frac. < 10^{-1}
Li (narrow)
mixing frac. ~ 1



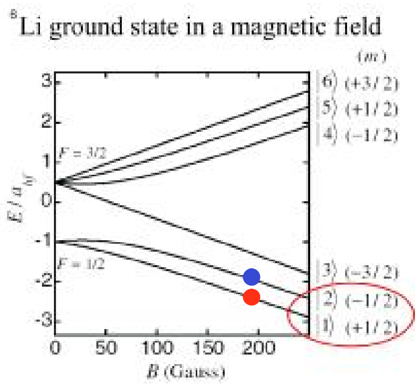
Comparison:

Binding energy (Innsbruck)
Q scattering calc. (NIST)
Mixing fraction (Rice Univ.)

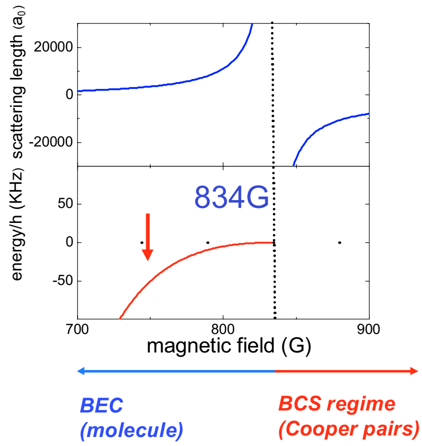
Binding energy (JILA)
Q scattering calc. (NIST)

Position & width (Rice Univ.)
Binding energy (NIST)

^{6}Li experiment



scattering between |1> and |2>



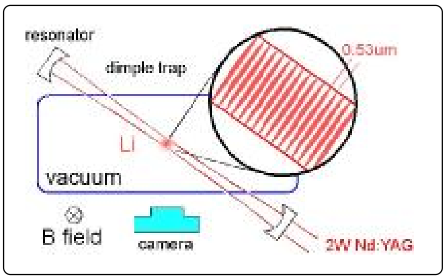
At high fields,

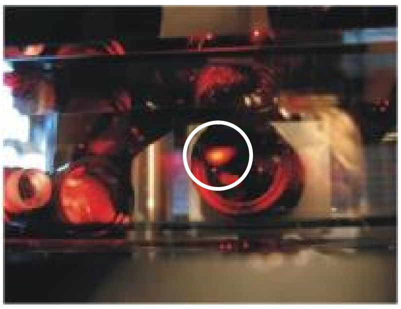
|2> = |2s, m_s=-1/2, m_i=0>

|1> = |2s, m_s=-1/2, m_i=-1>

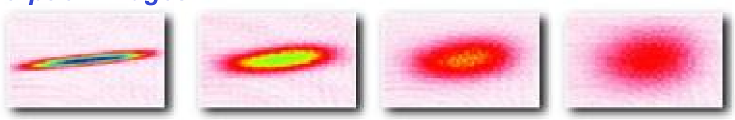
Li Experimental Setup

- Resonator set up at Brewster's angle. Opt. Lett 26, 1837 (2001)
- Efficient loading into the optical trap: ~ 10^7 thermal atoms





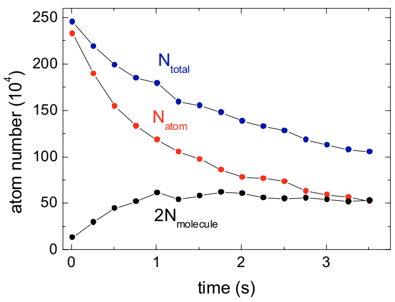
Absorption images



Time Evolution

Atoms ↔ molecules: thermalization

***B* = 690 G**:
mol. bind. energy $E_m = k_B$ 8μK
>> therm. energy $k_B T = k_B$ 2.5μK

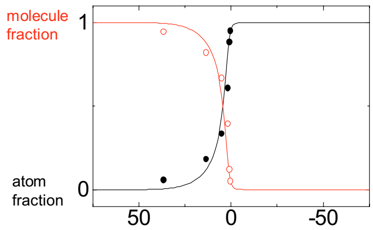


Exp: S. Jochim et al.PRL 91, 240402 (2003)

Atom→molecule: recombination
Li+Li+Li → Li_2+Li

Molecule→ Atom: coll. dissociation
Li_2+Li_2 → Li_2+Li+Li

Thermal equilibrium
$N_M / N_A = \phi_A e^{Em/kT}$
ϕ: phase space density



Theo: C. Chin and R. Grimm, PRA (2004)

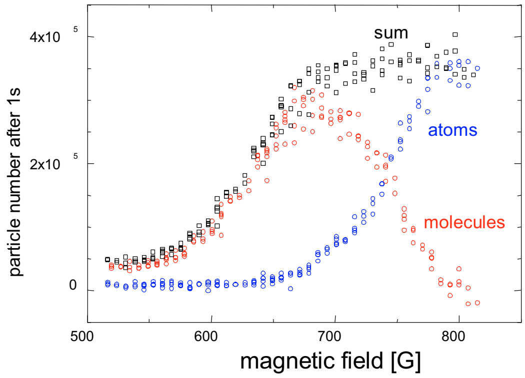
Collision properties of molecules

Small **a**
Large binding energy

molecule decay

atom-molecule conversion

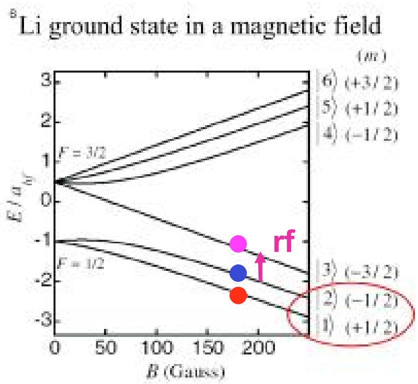
Large **a**
Small binidng energy



Experiment:
Long lifetimes near the resonance!!
> 40s at 764G
< 100ms below 500G

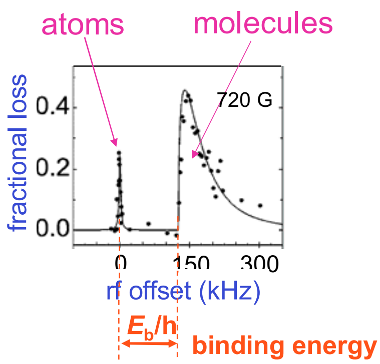
Theory:
decay rate: C $r_e (r_e/a)^{2.55}$
Mol. scattering length a_m=0.6a
Petrov et al ´04

Molecular binding energy: RF spectroscopy



$f_{atom} = E_{23}$

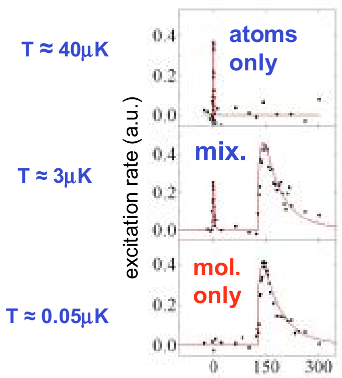
$f_{molecule} = E_{23} + E_b + 2E_K$



State 3 is initially unpopulated

Applications of RF spectroscopy

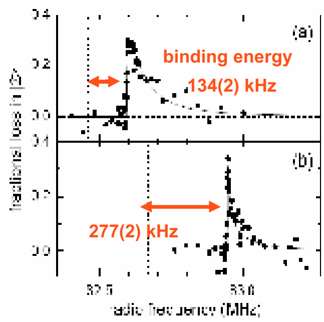
1. Distinguish atoms and weakly bound molecules



Also by Jin group at JILA

2. Precision molecular spectroscopy

Cooperation: A. Simoni, E. Tiesinga and P. Julienne, NIST



Experiment:
M. Bartenstein et al, PRL **94**, 103201 (2005)

Lineshape theory
C. Chin and P. Julienne PRA **71**, 012713 (2005)

3. Observation of Fermionic pairing gap

Outside the scope of this talk.

Please see *C. Chin et al., Science 04*

Bound-free and bound-boud transitions

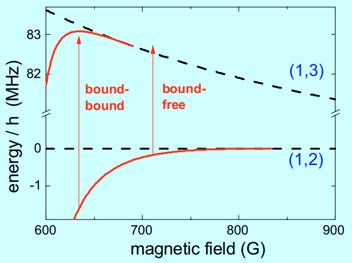
Bound-free transitions

$$\Gamma_f = \frac{h\Omega}{2}\left|\int \psi_K^*(r)\phi_m(r)dr\right|^2$$

$$= \frac{4}{\hbar}(1-\frac{a'}{a})^2 \frac{K^{1/2}E_b^{1/2}E_b^*}{(K+E_b)^2(K+E_b^*)}$$

Bound-free transitions

$$\Gamma_b = \frac{h\Omega}{2}\left|\int \phi_{m'}^*(r)\phi_m(r)dr\right|^2 = (\frac{a-a'}{a+a'})^2$$



Precision determination of scattering properties without collisions!!

EXPERIMENT:

Bound-free transitions

720.13(4)G	RF:82.593(2)MHz
694.83(4)G	RF:82.944(2)MHz

Bound-bound transitions

676.09(3)G	RF:83.2966(5)MHz
661.44(2)G	RF:83.6645(3)MHz

⇒

THEORY (Simoni, Tiesinga, Julienne)

a_s=45.167(8)a0, a_T=-2140(18)a0

Feshbach resonance positions

|1>+|2>: 834.1±1.5 G

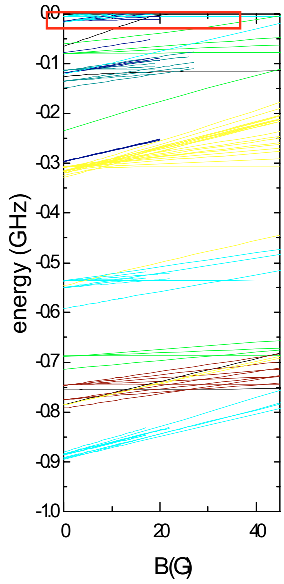
|1>+|3>: 690.4±0.5 G

|2>+|3>: 811.2±1.0 G

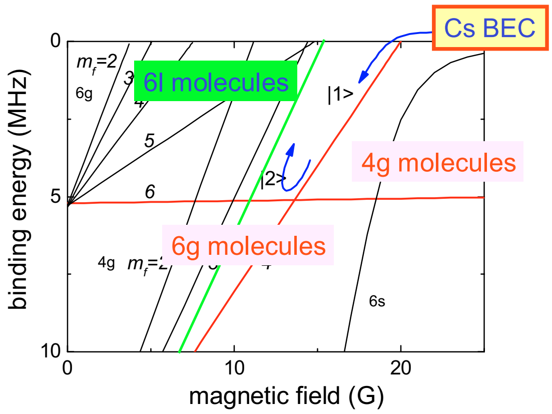
Breif summary on Li2 moleculese

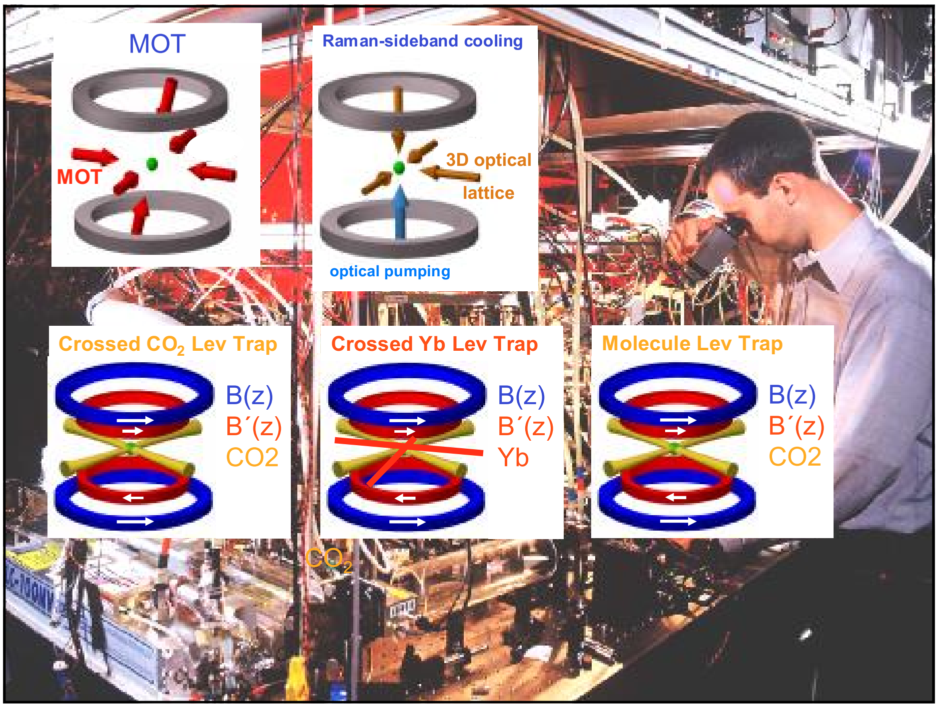
- **Molecule formation by recombination and thermal equilibrium**
  - Efficient atom-molecule conversion when kT<<Em
- **Collisional stability with positive scattering length**
  - Evaporation to BEC and BEC-BCS crossover (tomorrow)
- **Precision molecular RF spectroscopy**
  - Precise determination of the molecular energy structure
  - Pairing gap (tomorrow)
- **Collective modes and Fermionic hydrodynamics (tomorrow)**

Experiments on Cs_2 molecules



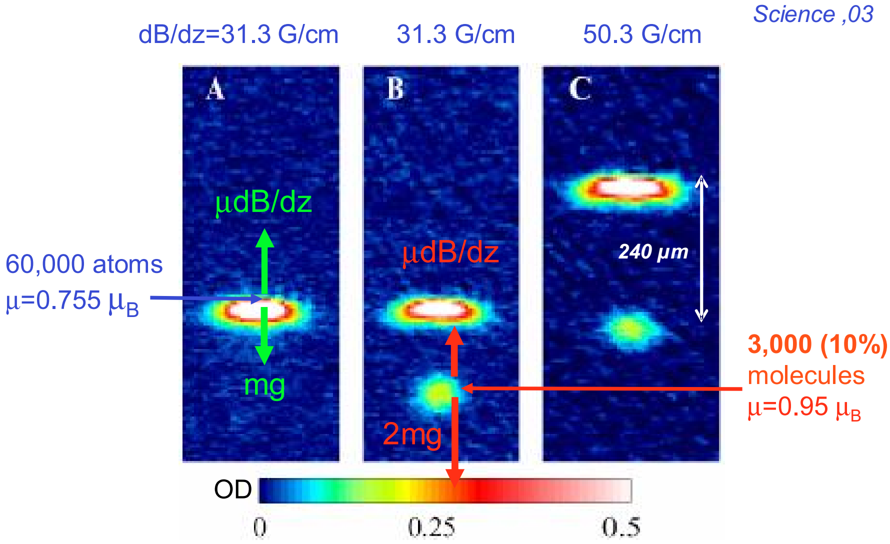
G-wave molecules: orbital angular momentum L = 4 hbar
G-wave resonance: coupling which changes L by 4hbar
due to high order spin-orbit interaction





Stern-Gerlach separation (5/2003)

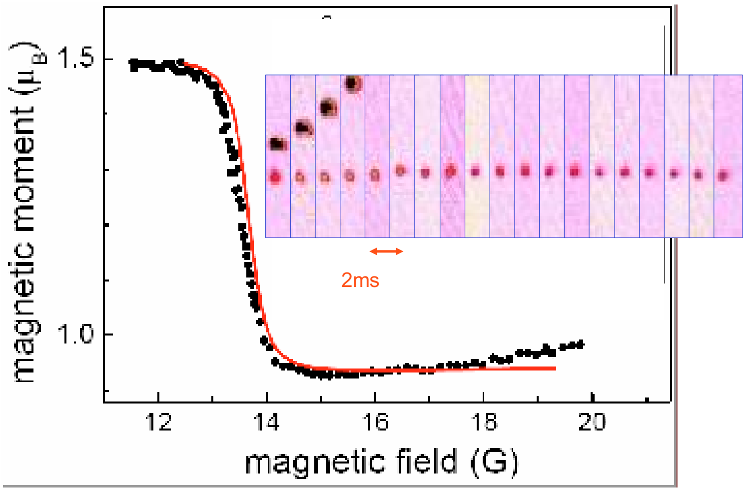
Science ,03



Also from Rb_2 7% (MPQ), Na_2 5% (MIT).

mapping out the avoided crossing

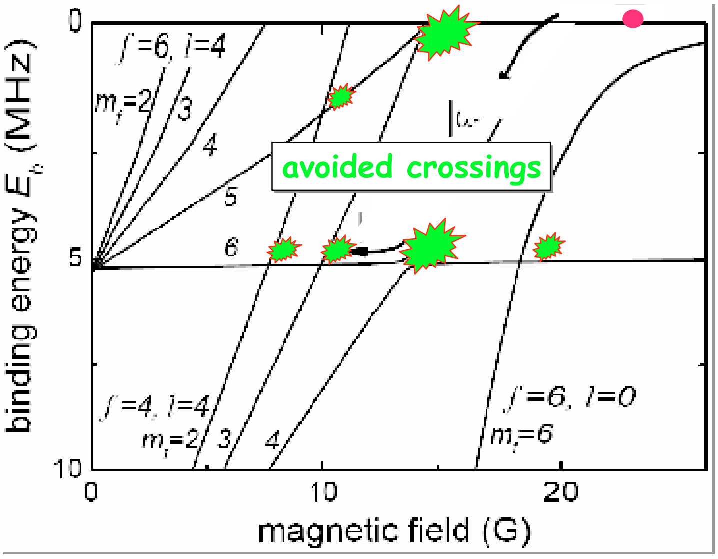
levitation condition ⇒ magnetic moment (precision 1%)



theory by E. Tiesinga et al., NIST

Convert 4g moleules to 6g molecules

Ultracold atoms quantum gases



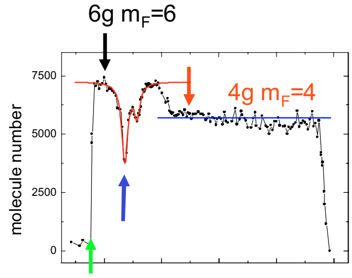
data from NIST group

Feshbach resonances of Cs_4 molecules

Ultracold atoms quantum gases

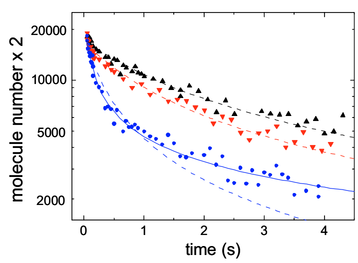
- Leave the molecules in the trap for 300ms.

6g m_F=6



What happened here?

Non-exponential decay

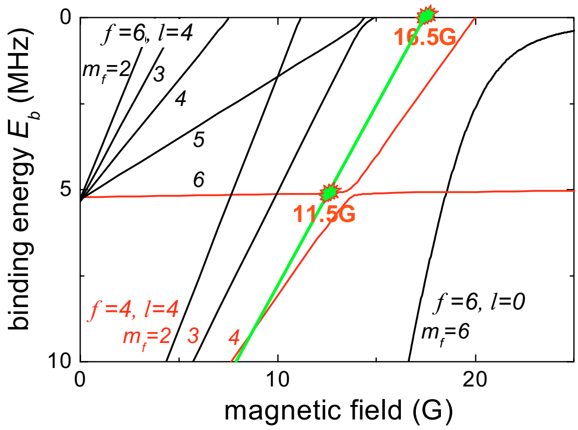


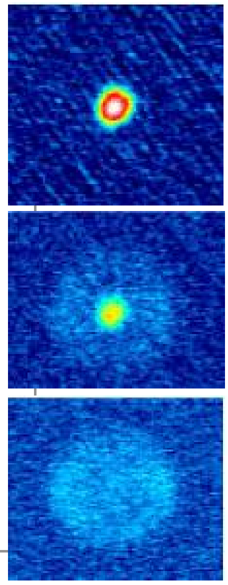
Binary decay constant:	
6g m_F=6	$3\times10^{-11}cm^3/s$
4g m_F=4	$5\times10^{-11}cm^3/s$
on resonance	$2\times10^{-10}cm^3/s$
(unitarity limit)	$4\times10^{-10}cm^3/s$

density dependence
resonance structure
⇒
molecular Feshbach resonances induced by Cs_4 states

puzzling observations

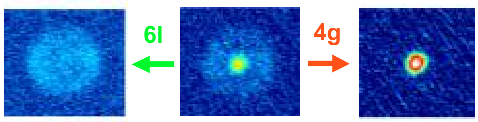
The mysterious state dissociates at 16.5G!!

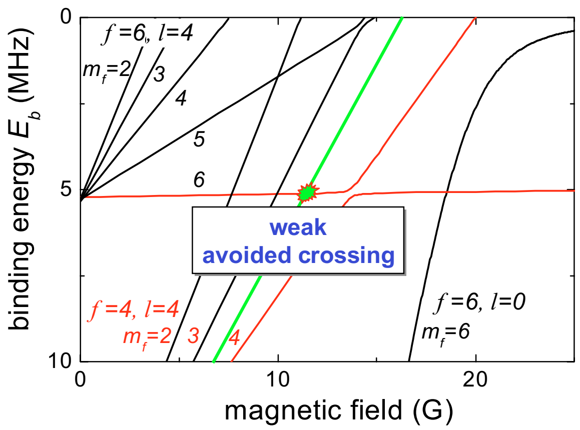




interesting experimental tools

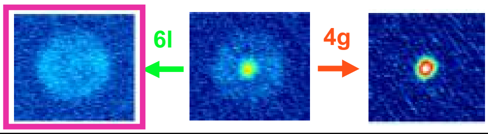
detection method for dissociation channel

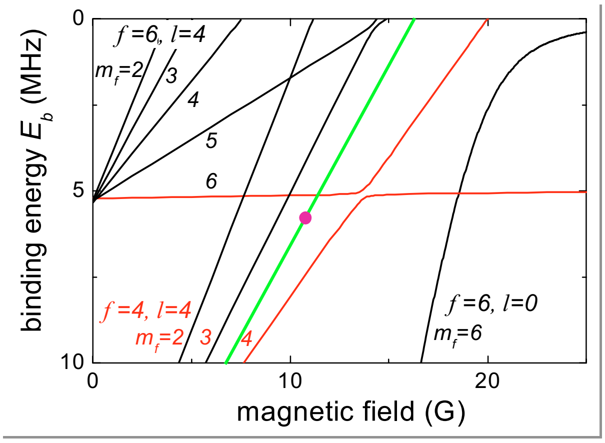




first experiment

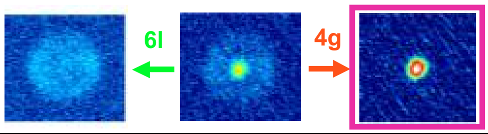
detection method for dissociation channel

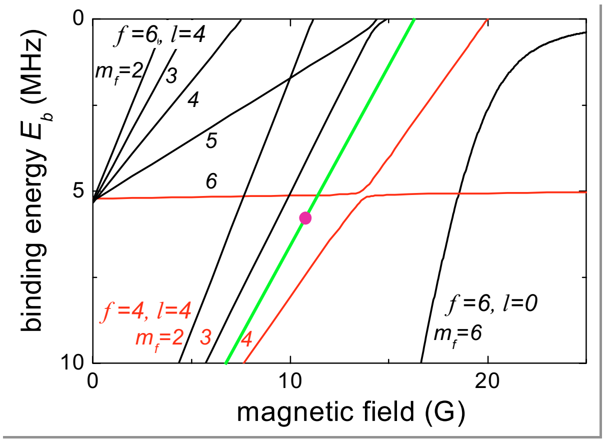




first experiment

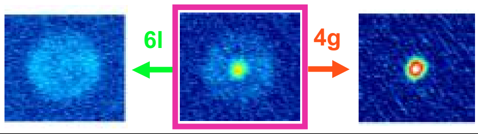
detection method for dissociation channel

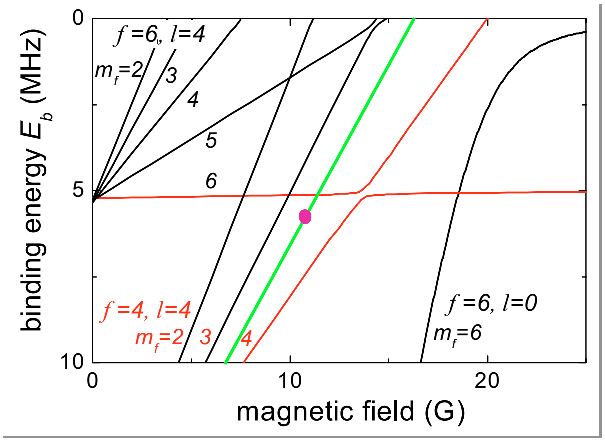




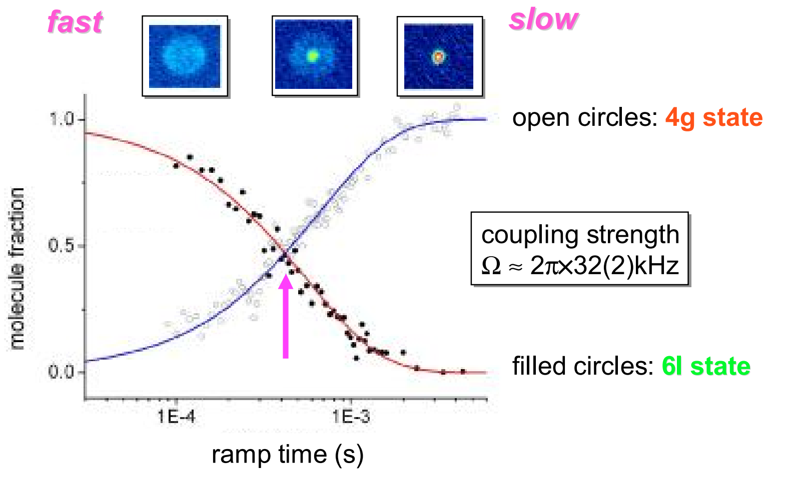
first experiment

detection method for dissociation channel



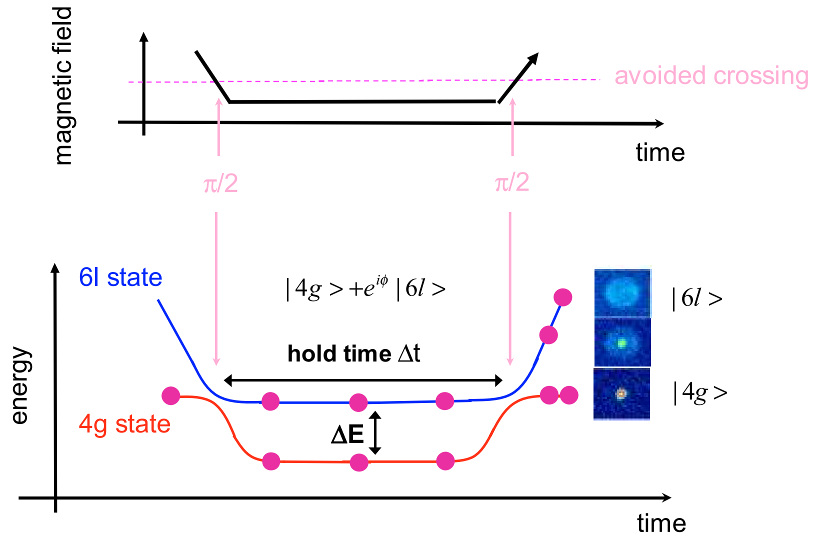


Landau-Zener → coupling strength

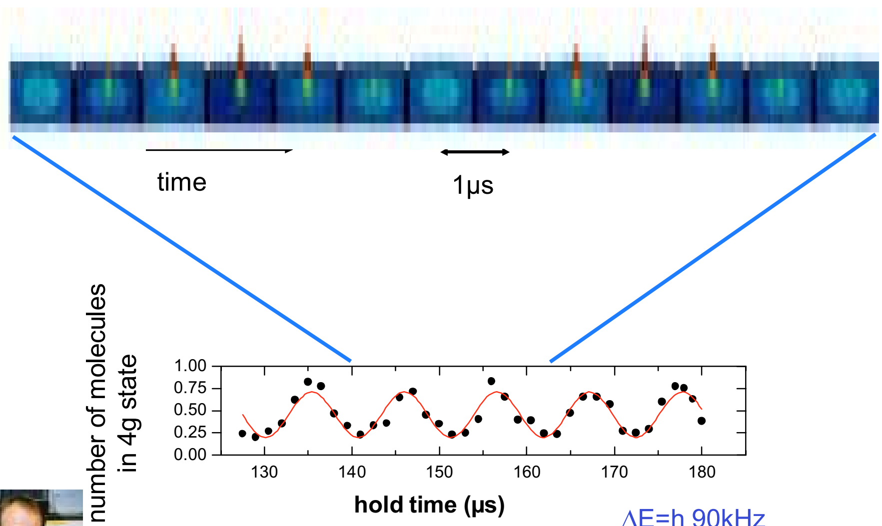


molecular beam splitter: coherence ?

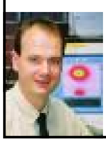
interferometer



After some struggle... It worked!!

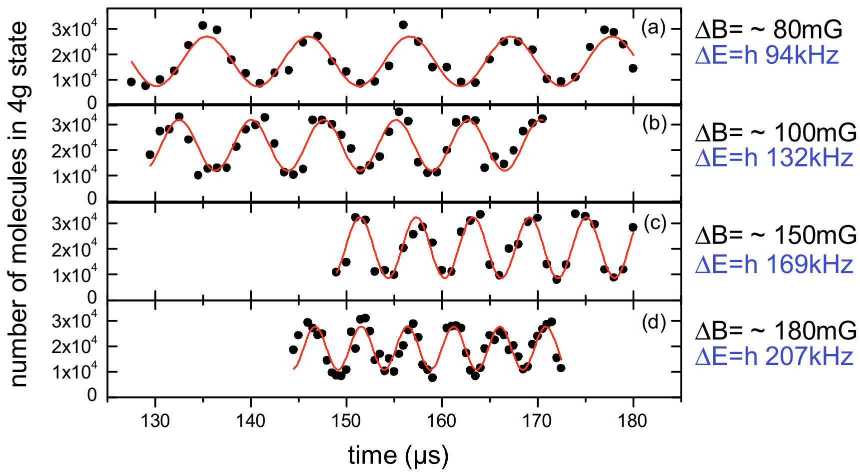


$\Delta E = h\ 90\text{kHz}$
$= E_{4g} - E_{6l}$



"This is called Stückelberg oscillations!"

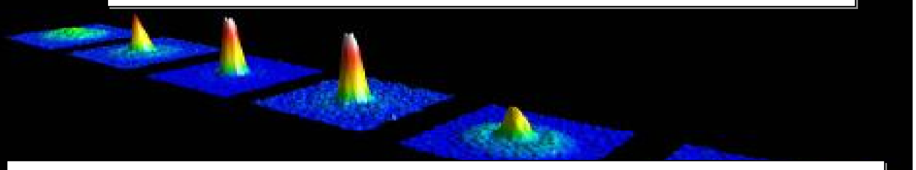
Stückelberg oscillations



molecule interferometer for future precision experiments !

Conclusion

Efficient creation of S-, G- and L-wave molecules
Precision determination of **Ψ(r)**, E_b, μ, **a** of molecules
Tetramer states and molecule interferometry
Molecules in many-body regime.



Cold molecules roadmap

Formation → single molecule → two- and many-body → applications

What is this mysterious state?

Ultracold atoms
quantum gases

Date: Mon, 10 May 2004 14:41:24 -0400

From: Cheng Chin [mailto:Cheng.Chin@uibk.ac.at]
Sent: Wednesday, November 10, 2004 16:09
To: Paul S. Julienne; Eite Tiesinga
Cc: Rudolf Grimm; Charles W. Clark; Christoph Naegerl
Subject: s p d f g h i k l m n o q r t u v w x y z

Hi folks,

I found the reference for the spectroscopy notation in
Phys. Rev. 33, 900.906 (1929), "Report on Notation for Atomic Spectra"
by H. N. Russell, A. G. Shenstone, and Louis A. Turner

It is **s p d f g h i k l m n o q r t u v w x y z**.
There is no j as Charles said.
……
So we have ultracold L-wave molecules. ☺