The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR 1666 - 22

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Collective modes and pairing gaps of a strongly Interacting Fermi Gas

Presented by:

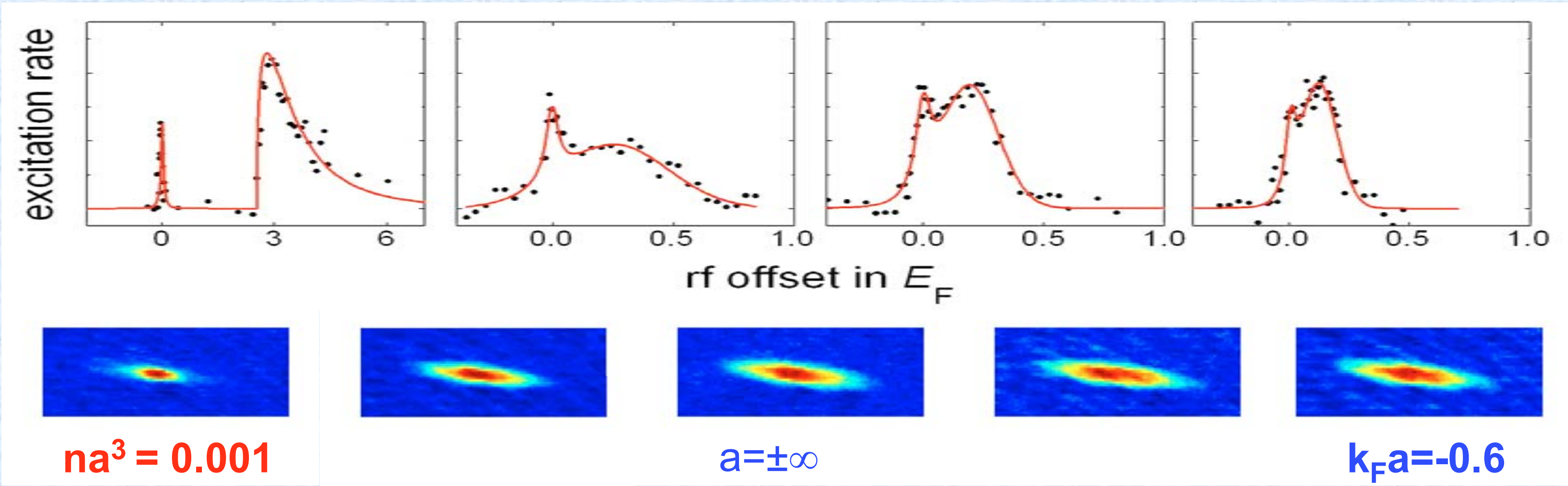
Cheng Chin

University of Chicago

School on Quantum Phase Transitions and Non-Equilibrium Phenomena in Cold Atomic Gases

Trieste, 15 July 2005

Collective modes and pairing gaps of a strongly Interacting Fermi Gas



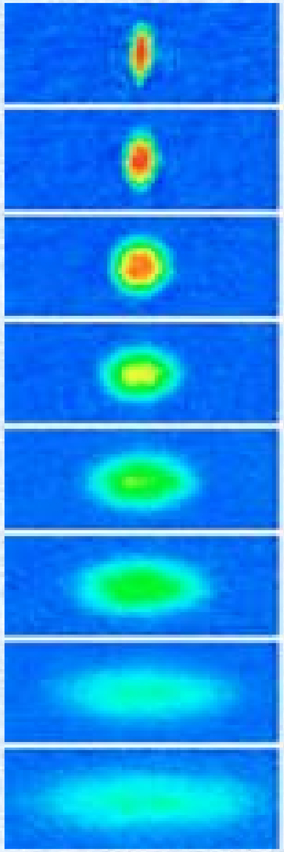
Cheng Chin

Experimentalphysik, Innsbruck University, Austria
James Franck Institute, Physics Dept., Univ. of Chicago

A surprising merge of two interesting fields

Interacting Fermi gas

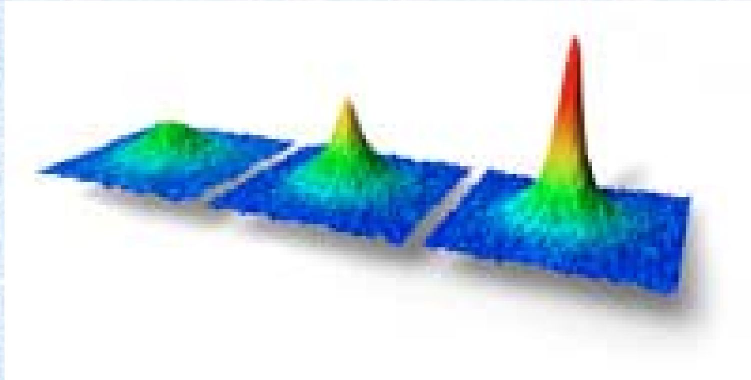
(Duke, JILA, Innsbruck...)

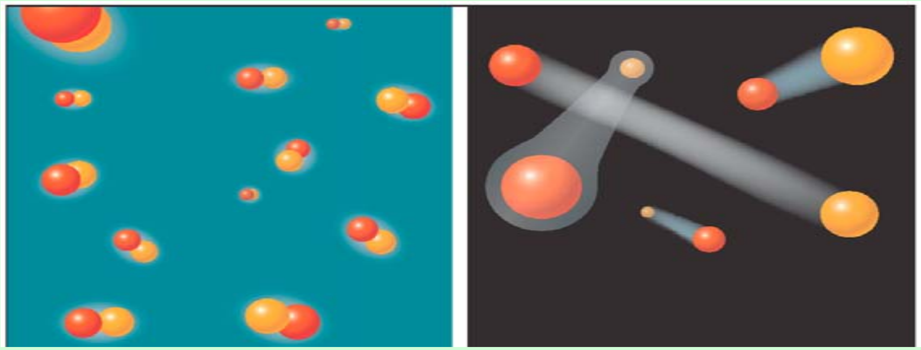


Hydrodynamic expansion

Ultracold molecules

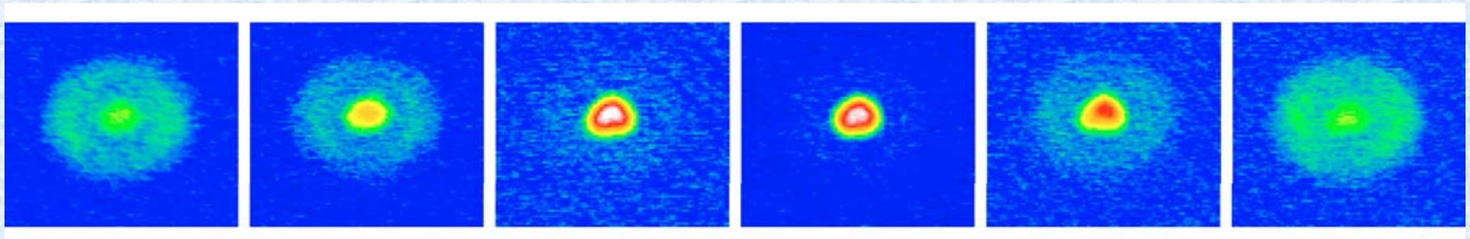
(Innsbruck, JILA, MIT...)



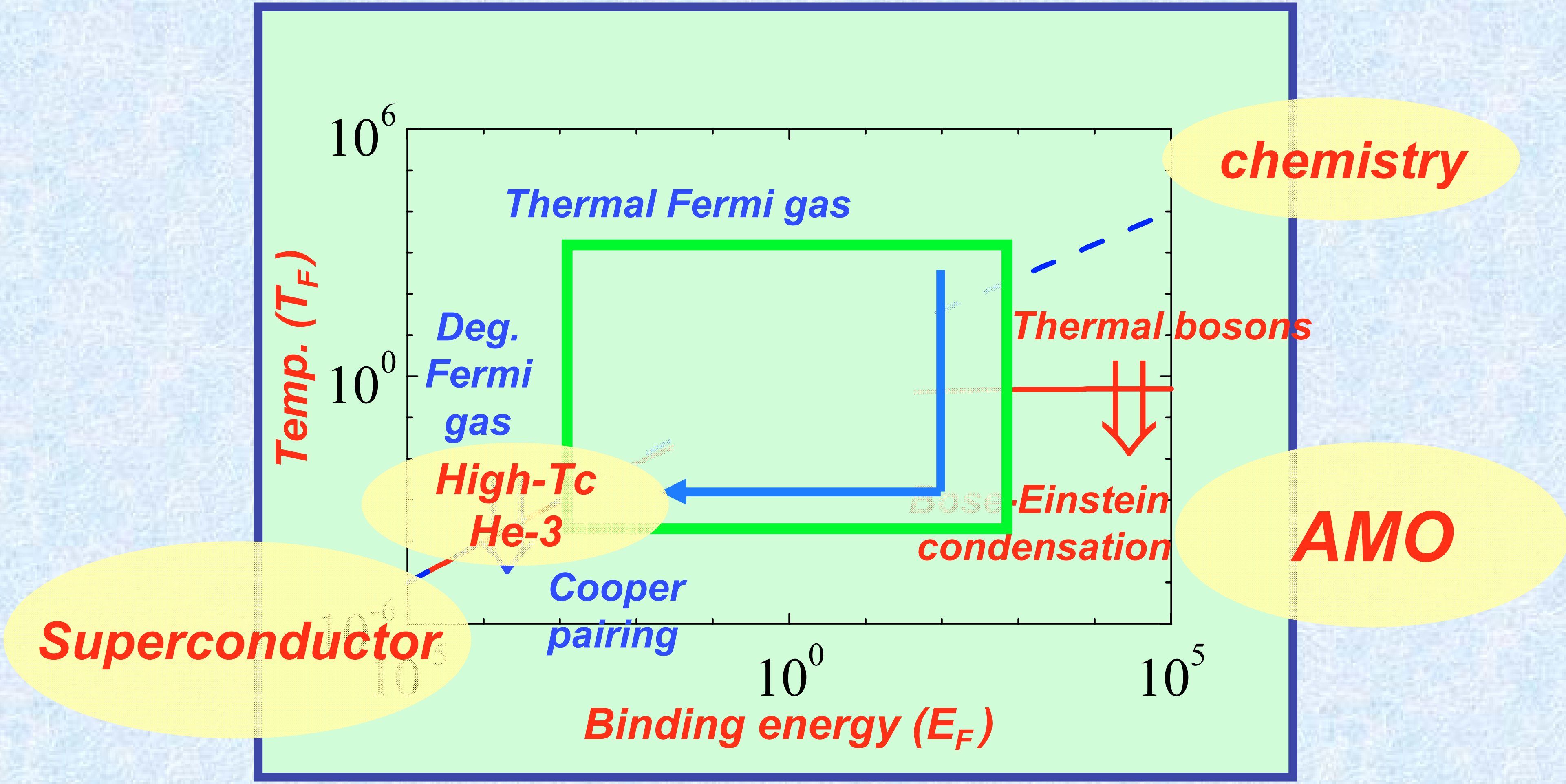


BEC-BCS crossover ***(Science 04)***

Molecule Stueckelberg oscillation (Innsbruck)



Phase diagram of a two-component Fermi gas



Eagles, Leggett

Fermi gas experiments in Innsbruck

- Bose-Einstein condensation of Li_2

  atoms → molecules → mol. BEC

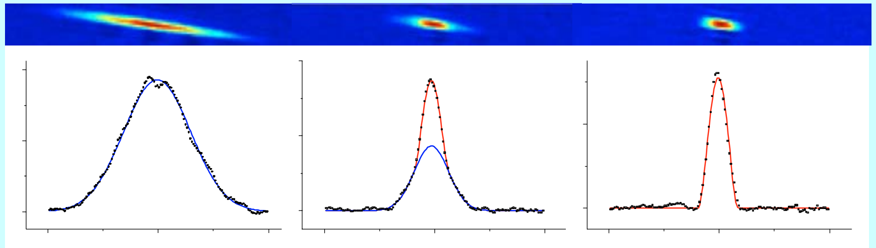
- Adiabatic BEC-BCS crossover
  mol. BEC → BCS

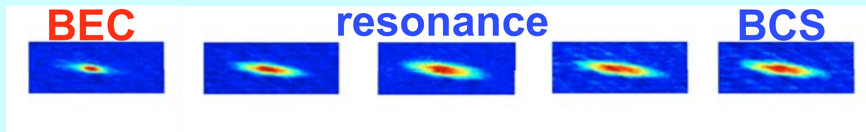
- Surprises in collective modes

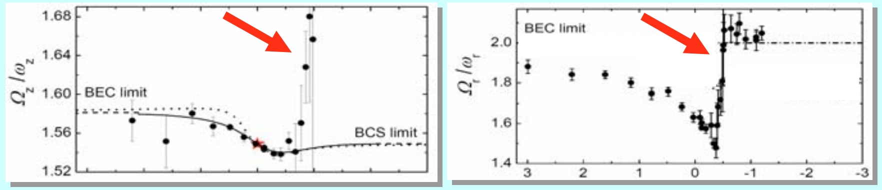
- Observation of pairing gap

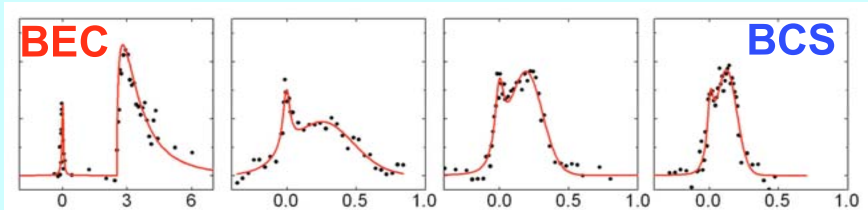
- A bosonic mean-field model

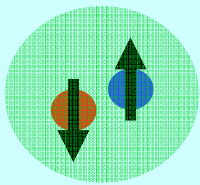
- Recent experiment



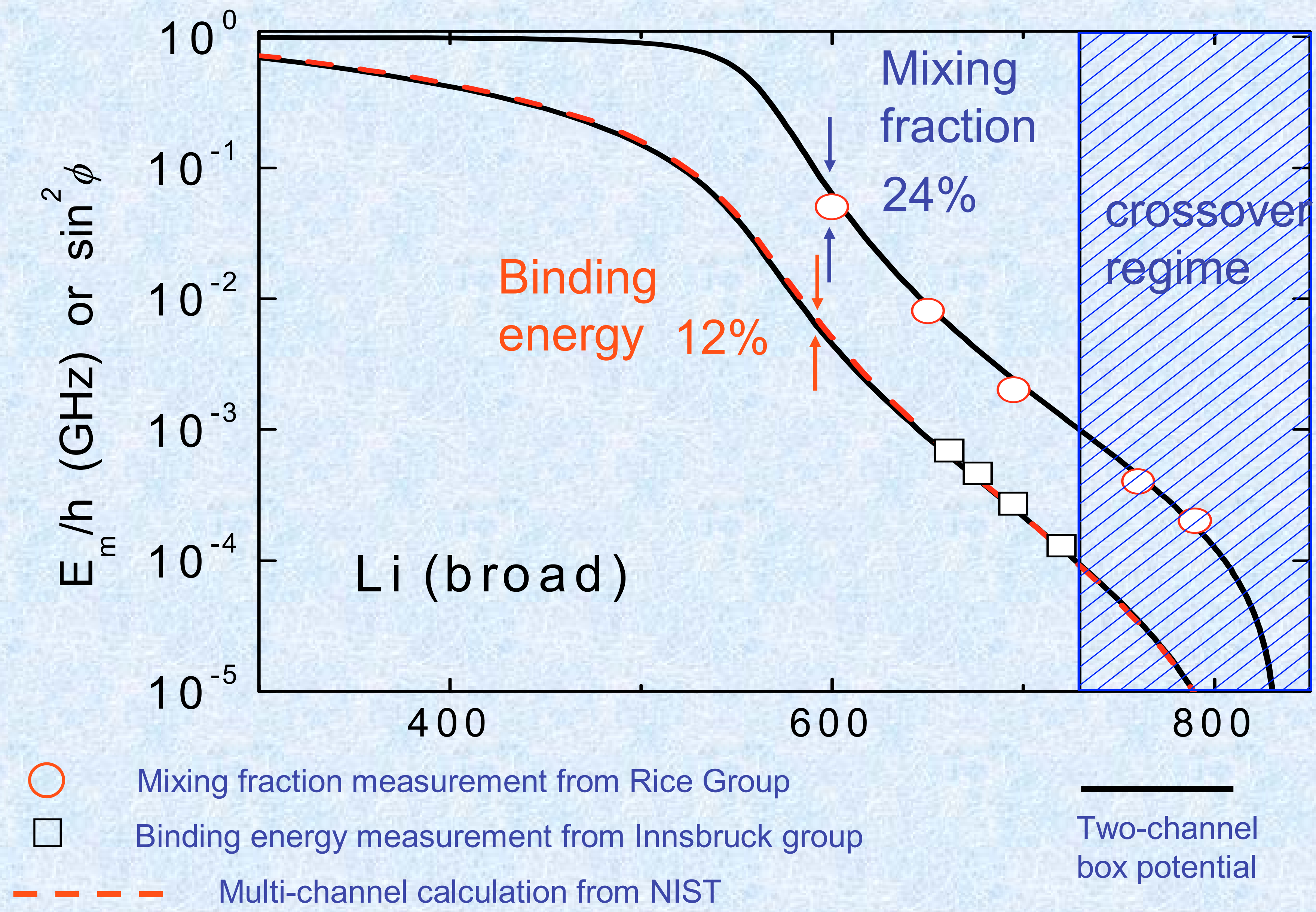






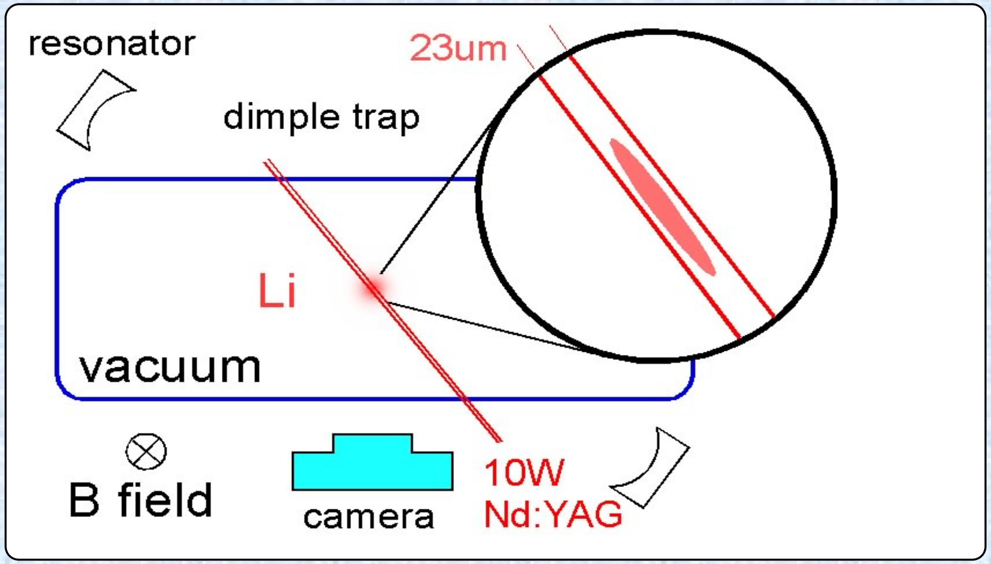


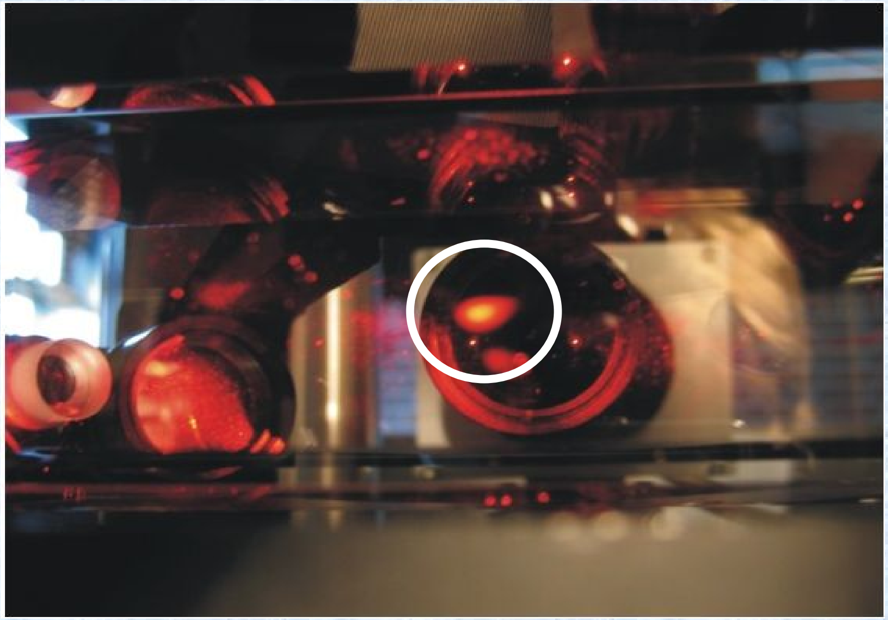
Li broad resonance: theory and experiment



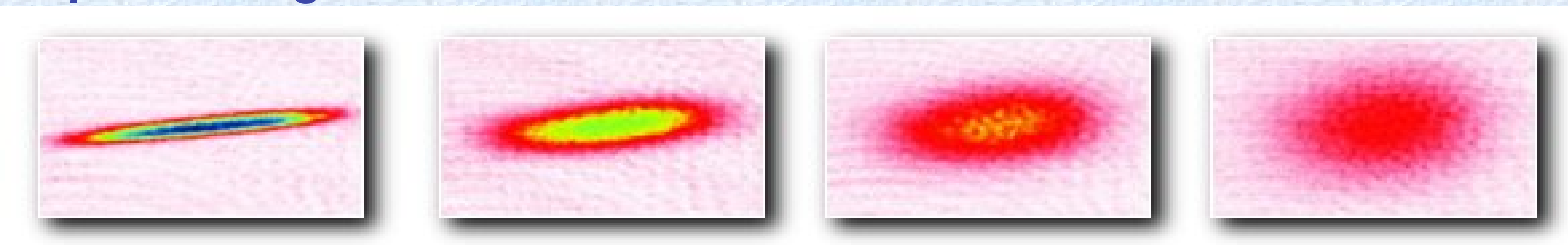
Li Experimental Setup

- Resonator set up at Brewster's angle. Opt. Lett 26, 1837 (2001)
- Efficient loading into the optical trap: ~ 10^7 thermal atoms



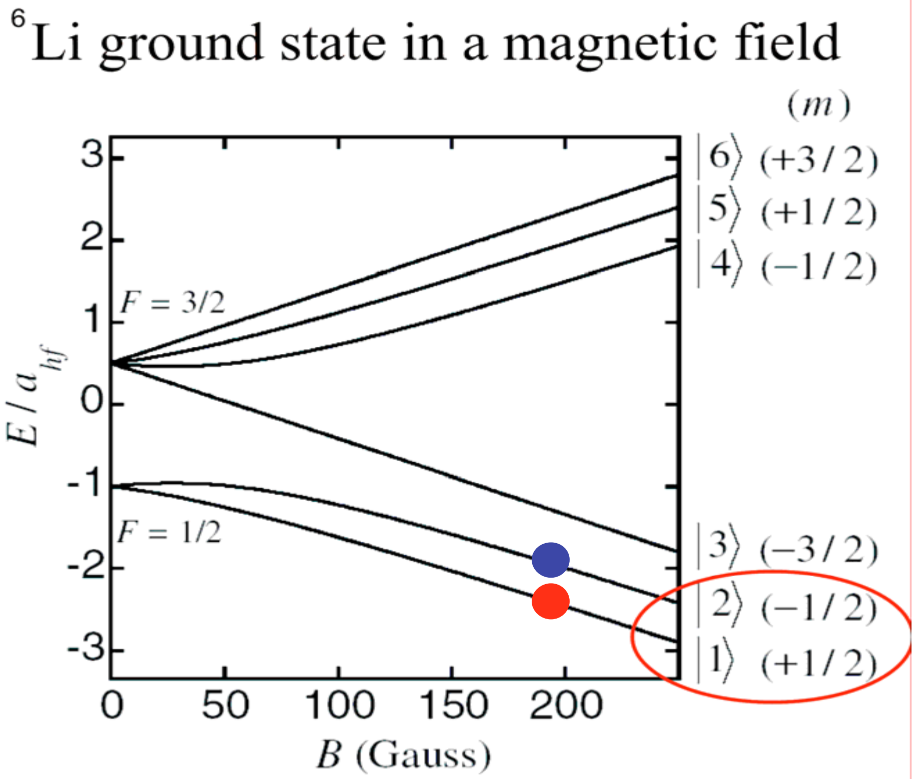


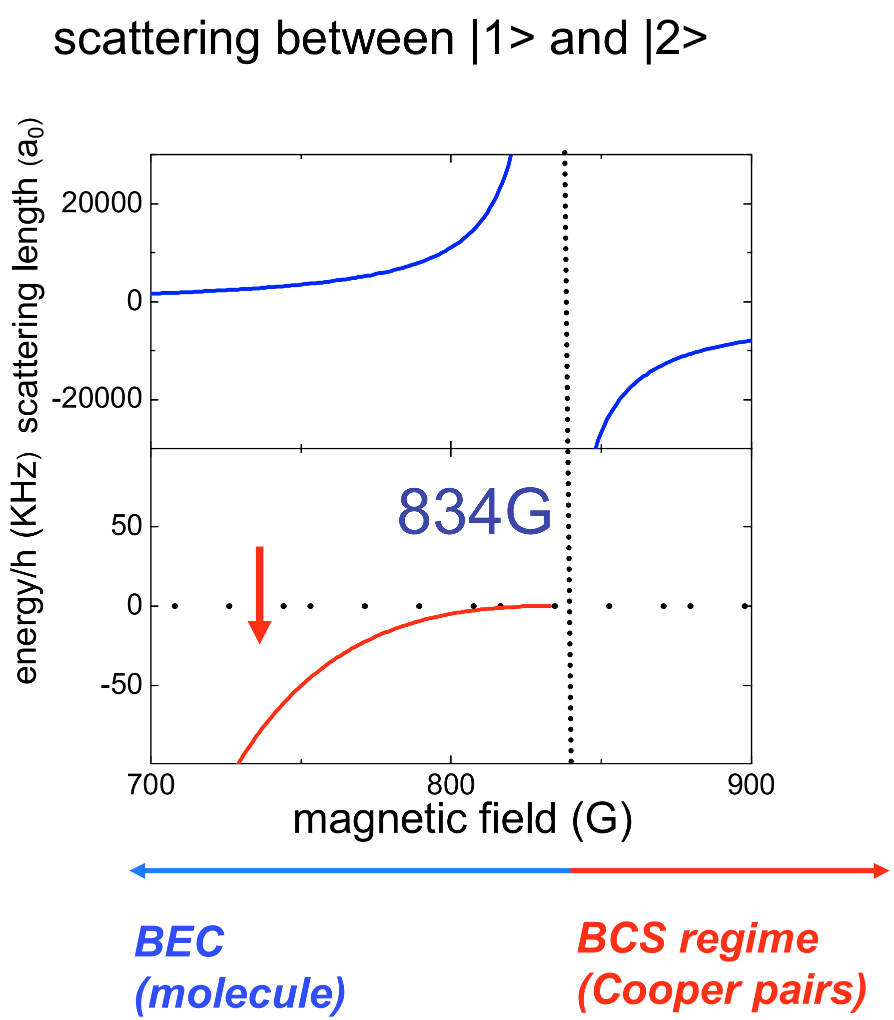
Absorption images



Time Evolution

From atoms to molecules to molecular condensate





At high fields,

|2> = |2s, m_s=-1/2, m_i=0>

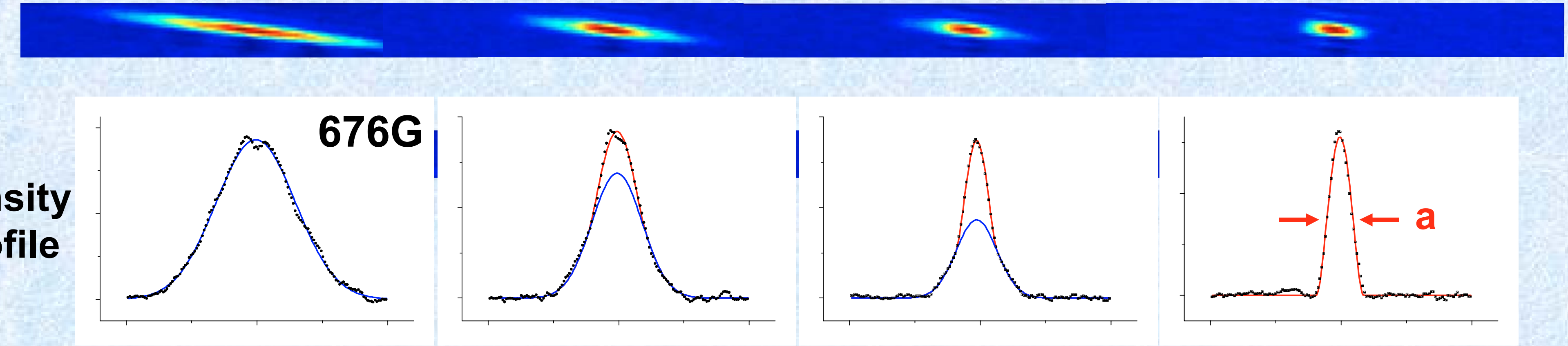
|1> = |2s, m_s=-1/2, m_i=-1>

Bose-Einstein condensation of Li_2

Evaporative cooling in an optical trap

Time **2.6 s** **3 s** **3.6 s** **4.0 s**

Density profile



Pure Thermal gas

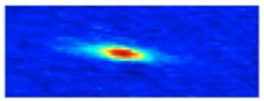
1mm

Pure molecular condensate

Jochim et al., Science (2003), Bartenstein et al., PRL (2004)

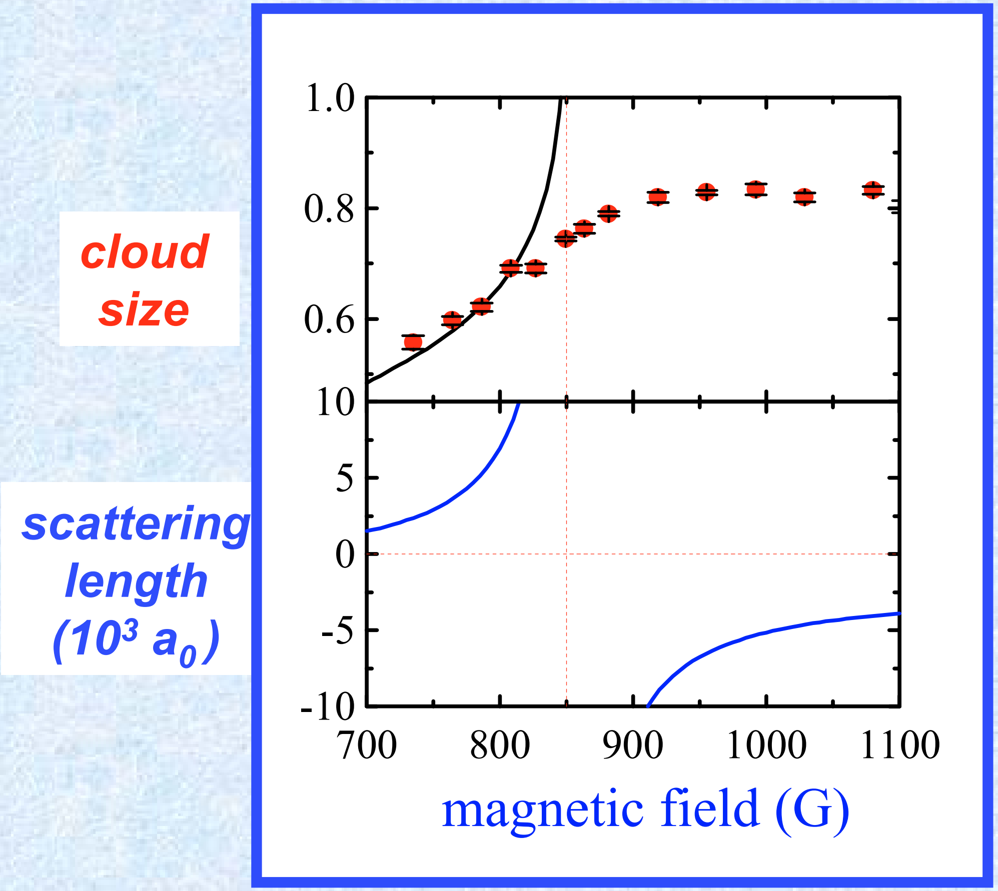
Explore the BEC-BCS crossover

BEC regime



$na^3 = 0.001$

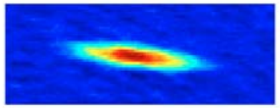
Molecular BEC



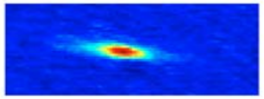
Converting a BEC into a Fermi gas!

What characterize the BEC-BCS crossover?

BEC regime



$na^3 = 0.28$

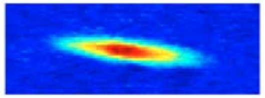


$na^3 = 0.001$

Molecular BEC

$\mu \sim na$

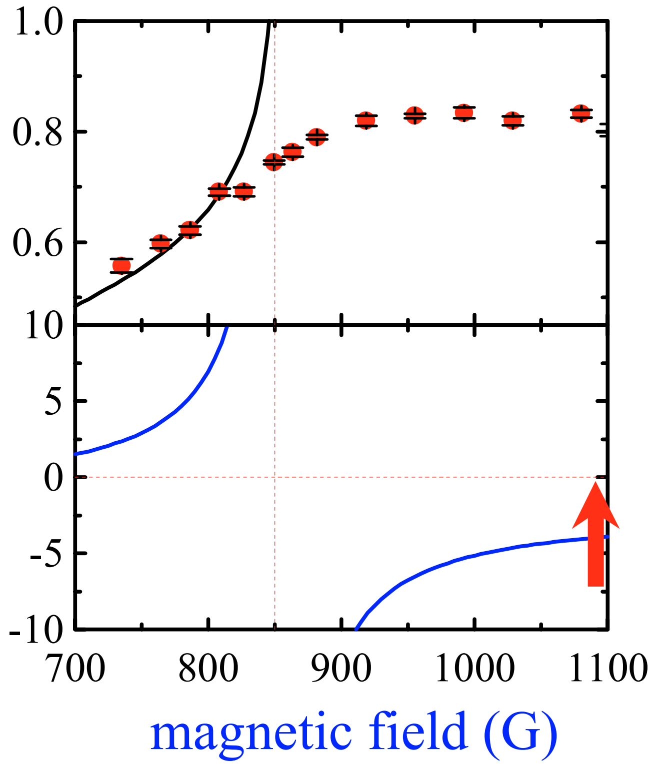
$T' = 0.2\ T_F$



$|a| \rightarrow$ ◎

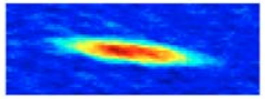
Nor. cloud size

scattering length *($10^3\ a_0$)*

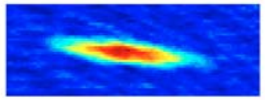


Bartenstein et al., PRL (2004)

BCS regime

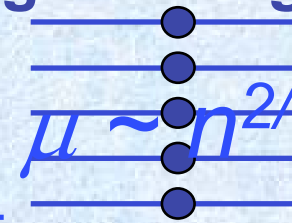


$k_F|a| = 6$



$k_F|a| = 0.6$

Deg. Fermi gas

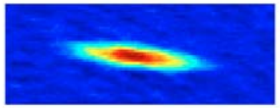


$\mu \sim n^{2/3}$

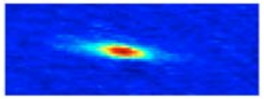
$T = 0.03 T_F$

What characterize the BEC-BCS crossover?

BEC regime



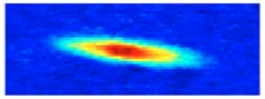
$na^3 = 0.28$



$na^3 = 0.001$

Molecular BEC

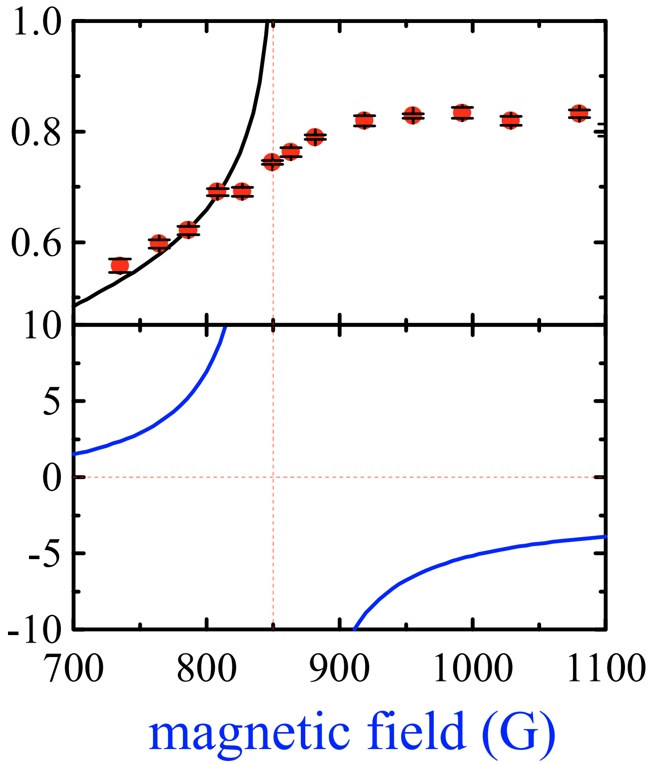
$\mu \sim na$



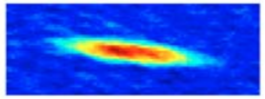
$|a| \to \infty$

Nor. cloud size

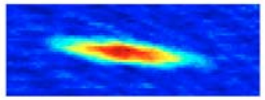
scattering length ($10^3\ a_0$)



BCS regime



$k_F|a| = 6$



$k_F|a| = 0.6$

Deg. Fermi gas

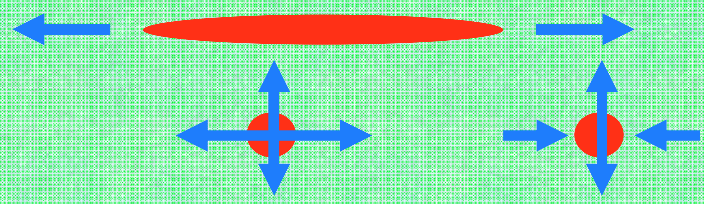
$\mu \sim n^{2/3}$

Collective modes in a Fermi gas

- Equation of hydrodynamics
  - Equation of state

$$\frac{\partial v}{\partial t} = -\nabla\left(\frac{v^2}{2} + \frac{V_{ext}(r)}{m} + \frac{\mu(n)}{m}\right)$$

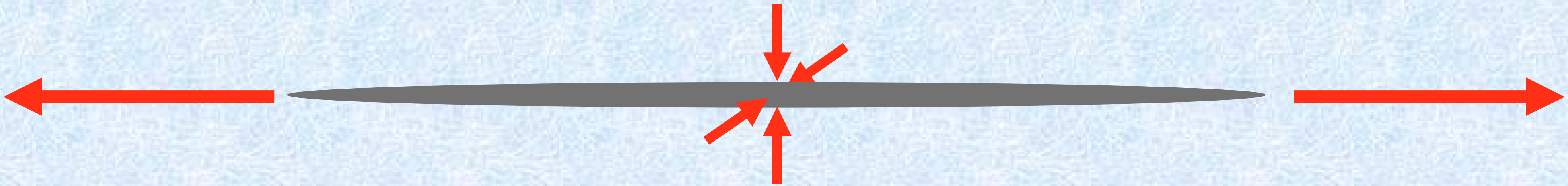
- Collective mode measurement?
  - **Axial quadrupole mode**
  - **Radial compression mode**
  - **Radial surface mode (ongoing)**



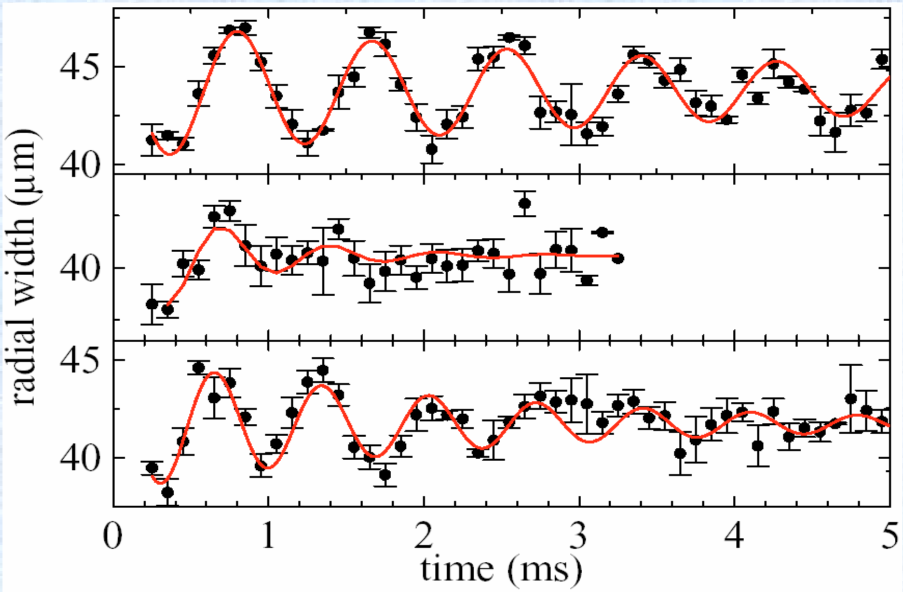
- Results
  - **Surprises in the observation (are they resolved?)**

Cooperation: Stringari

Collective Mode Excitations



Excitation of the lowest quadrupole mode



Determination of collective mode frequency and damping

Experiment:

Inpulse response

Reference:

Bartenstein, PRL 92, 203201 (2004)

axial quadrupole mode

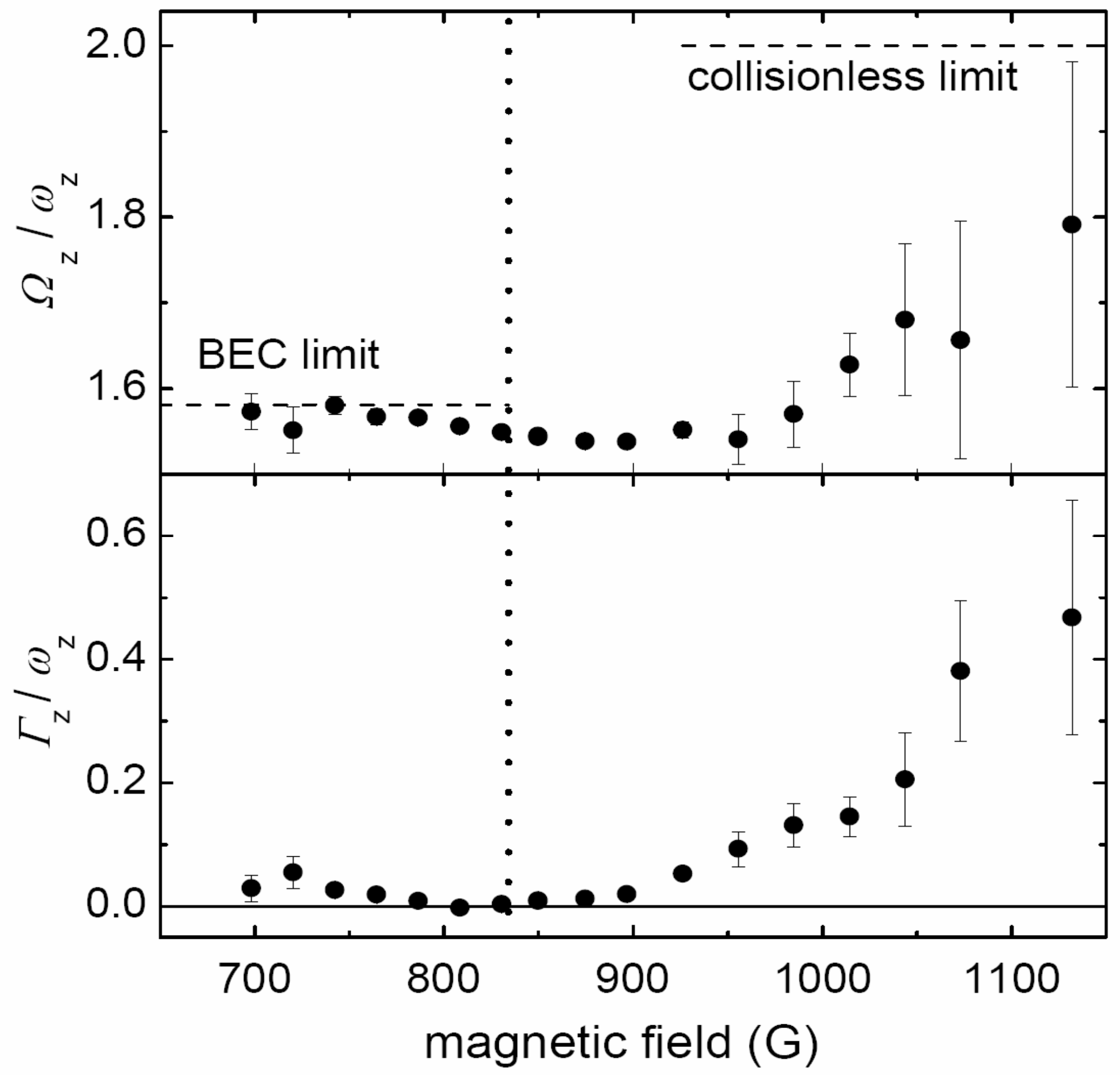
M. Bartenstein et al., PRL **92**, 203201 (2004), PhD thesis M. Bartenstein

frequency

$\mathrm{BEC}: \sqrt{5/2}$

$\mathrm{Unitarity}: \sqrt{12/5}$

damping rate

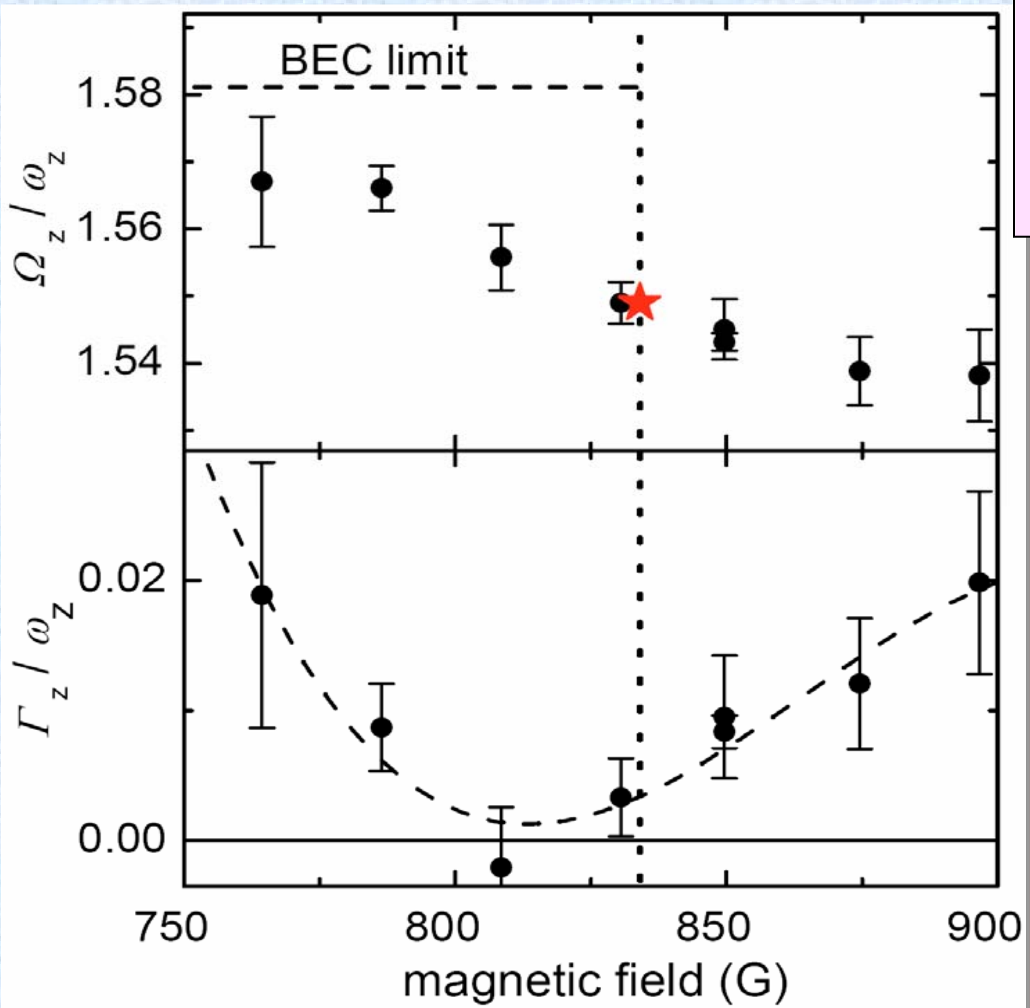


axial mode: resonance region

M. Bartenstein et al., PRL **92**, 203201 (2004), updated data analysis: PhD thesis M. Bartenstein

frequency (normalized to sloshing mode)

damping rate



unitarity point:

$$\Omega_z / \omega_z = \sqrt{12/5}$$

confirms equation of state

$$\mu \propto n^{2/3}$$

extremely weak damping in resonance region !!

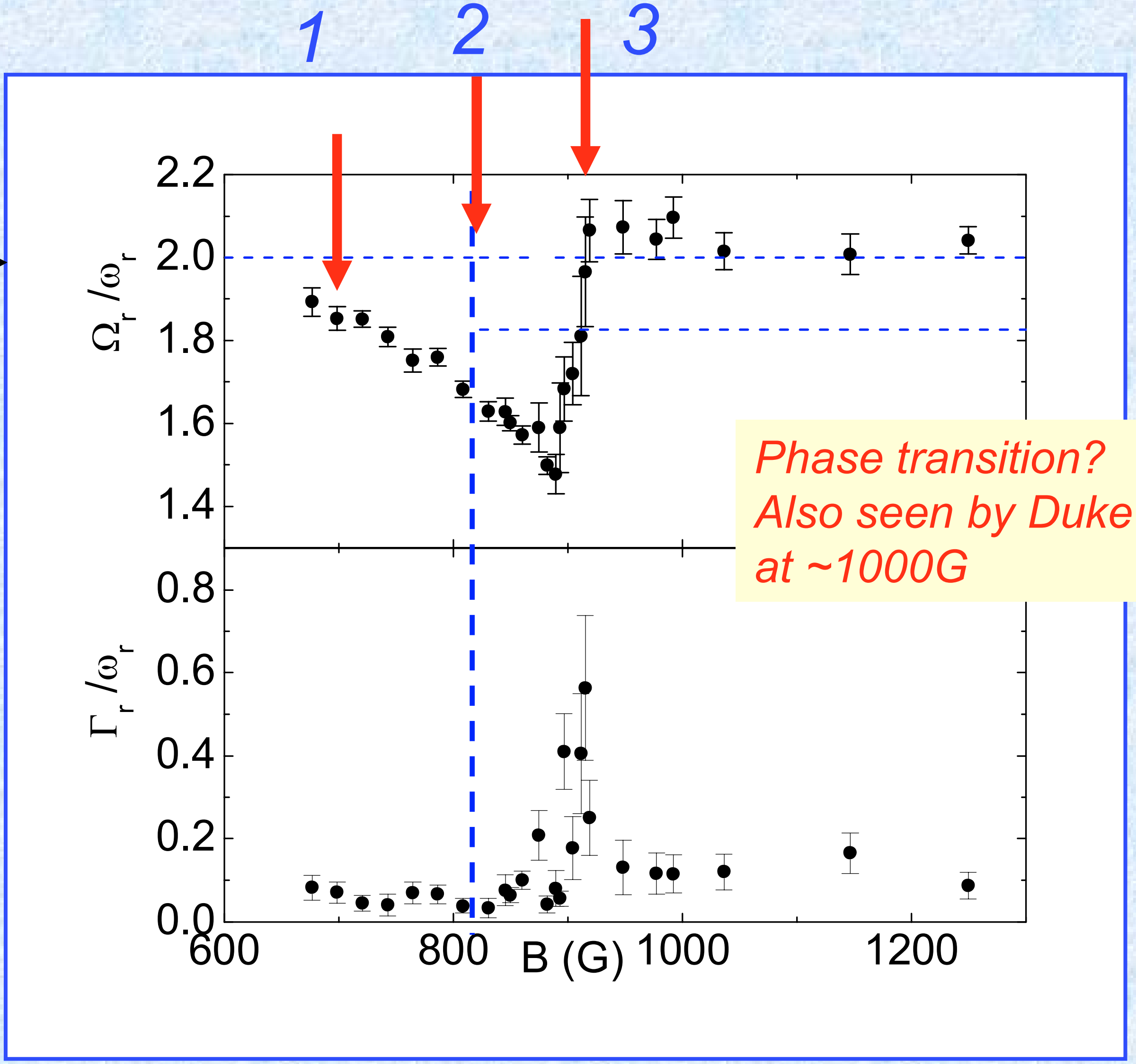
superfluidity?

Radial compression modes

BEC limit

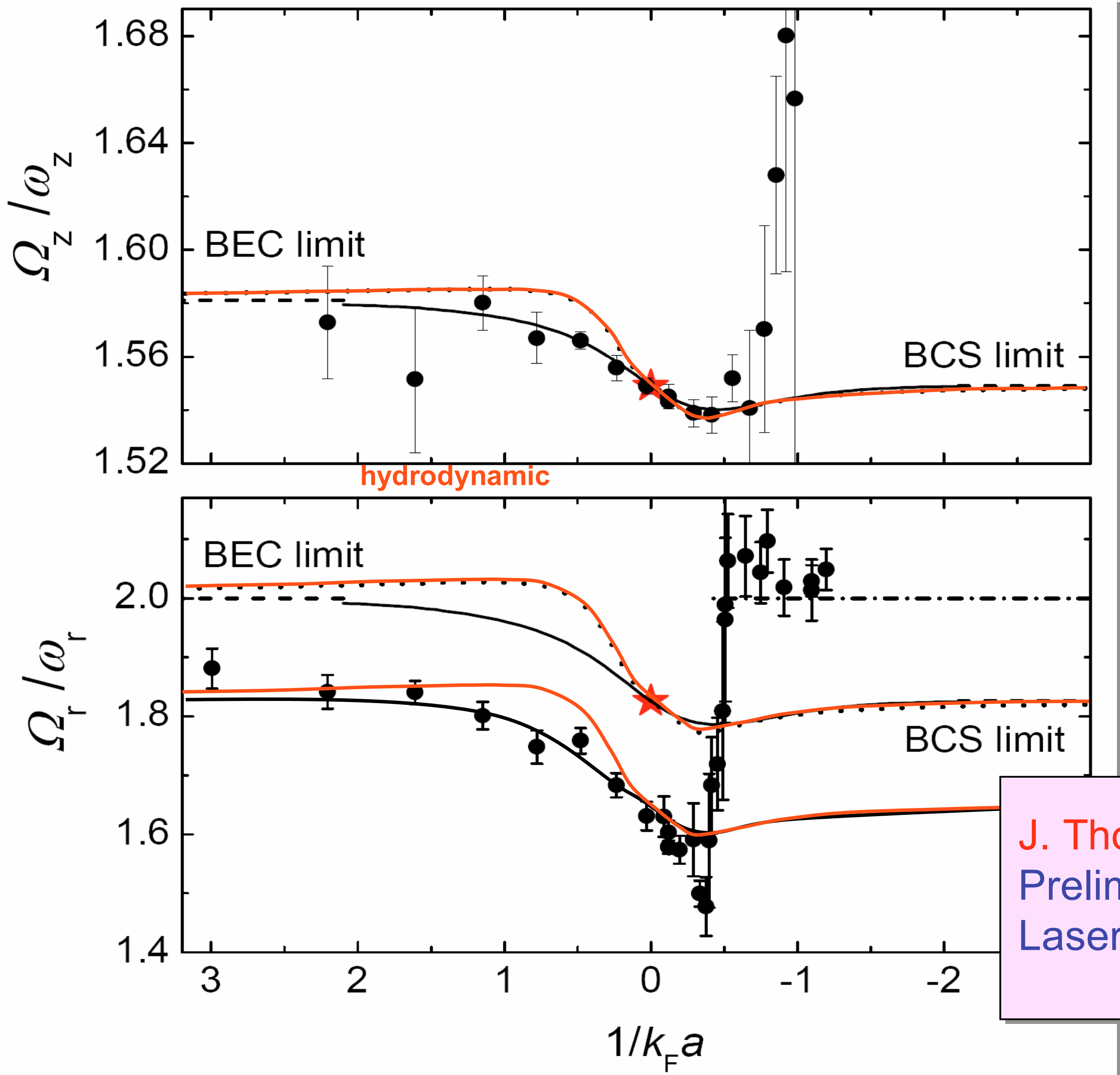
BEC : 2

Unitarity : $\sqrt{10/3}$



Phase transition?
Also seen by Duke group at ~1000G

Comparison with theory



Hu et al.,
PRL 93, 190403 (03)
(Leggett mean-field model)

Manini et al.,
cond-mat/0407039
(Giorgini's QMC calc.)

J. Thomas: 1.83 on resonance
Preliminary test
Laser beam ellipticity of 15~20%.

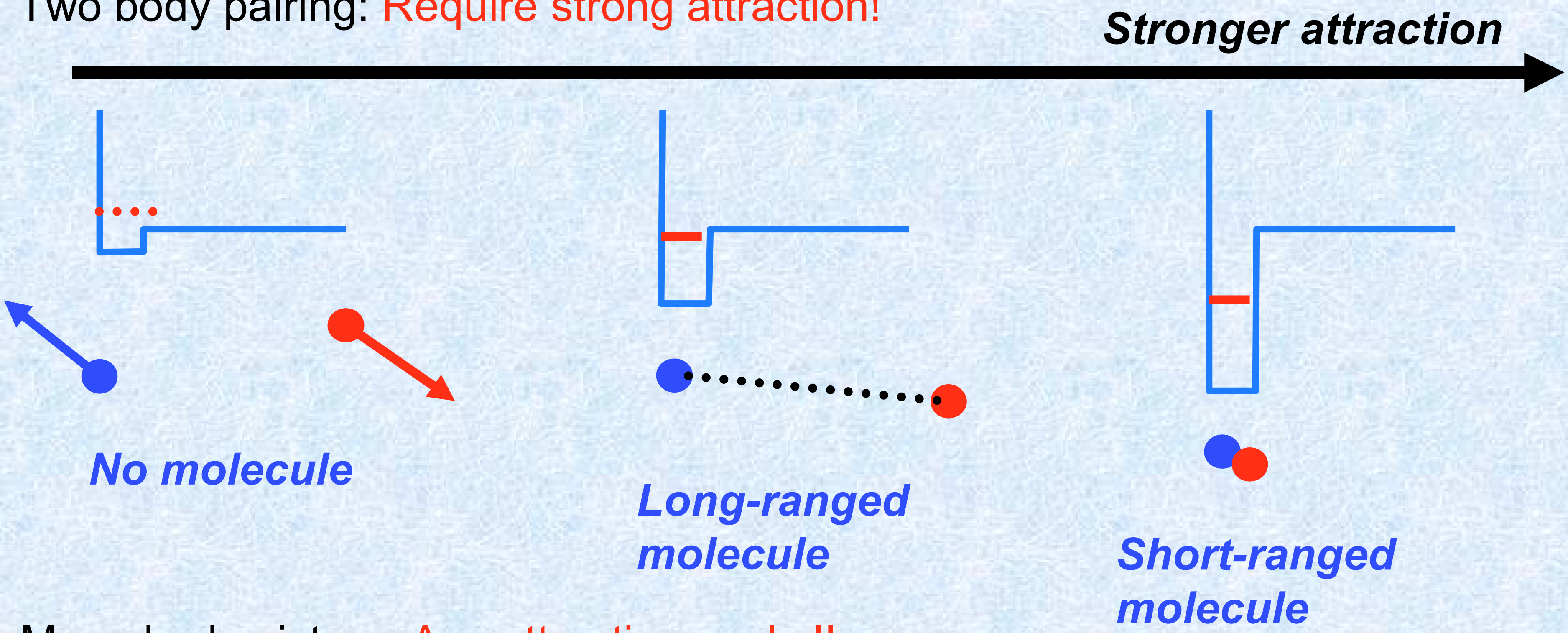
Pairing of fermionic ^{6}Li atoms

- Molecular pairing and Cooper pairing
  - Two-body pairing and many-body pairing

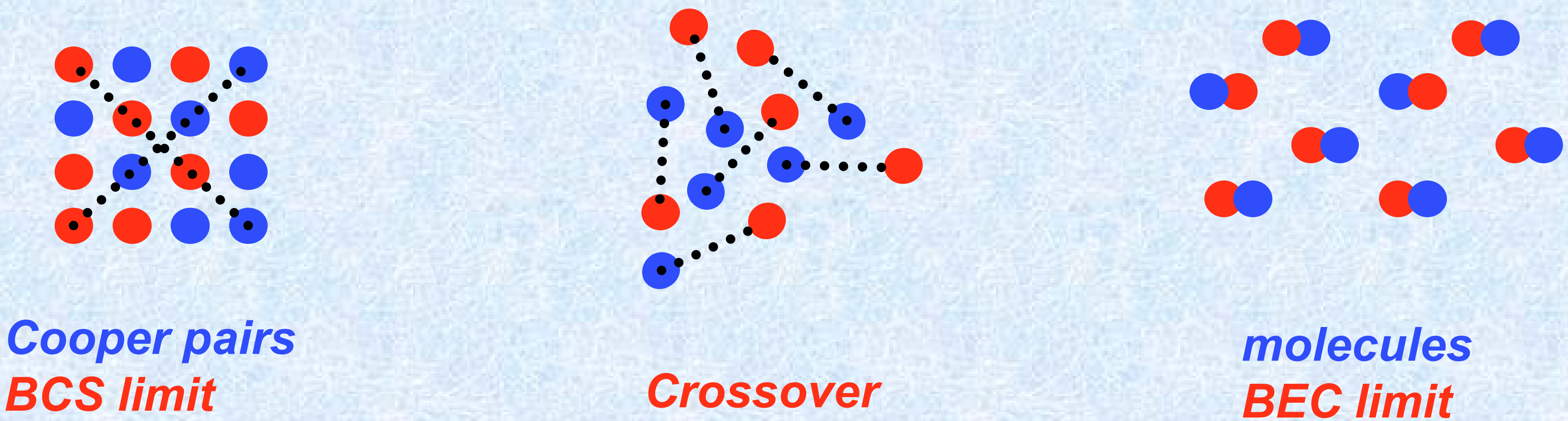
- How do we measure the pairing energy??
  - Molecule picture and BCS picture

- Comparison with theory
  - *Paivi Torma (Finland), Kathy Levin (USA), Allan Griffin (Canada), Giancarlo Strinati (Italy).*

Two body pairing: Require strong attraction!

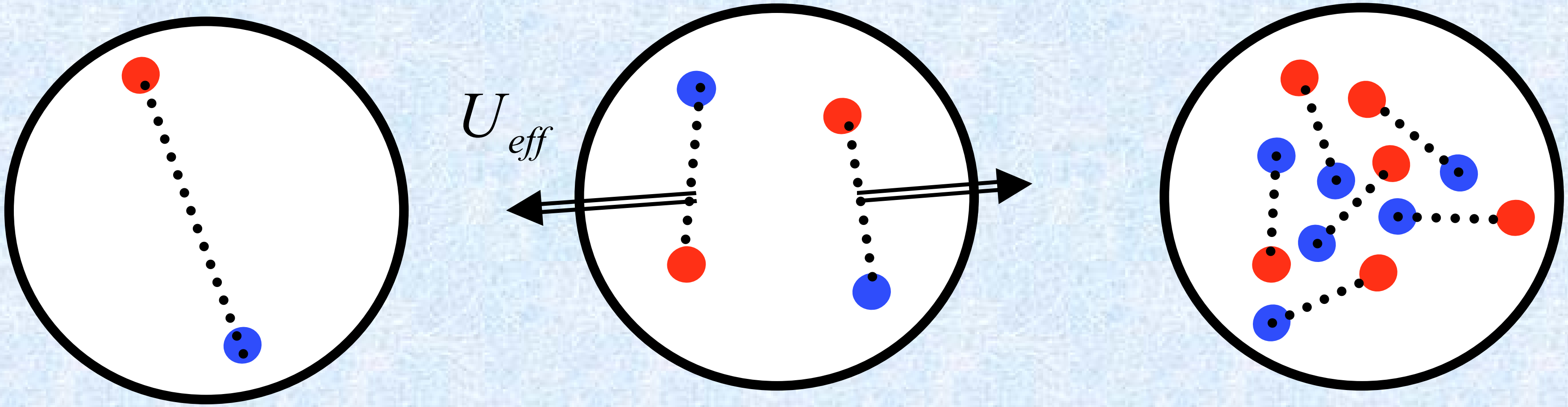


Many-body picture: Any attraction works!!



Enhanced pairing: Four-body picture

Petrov et al.



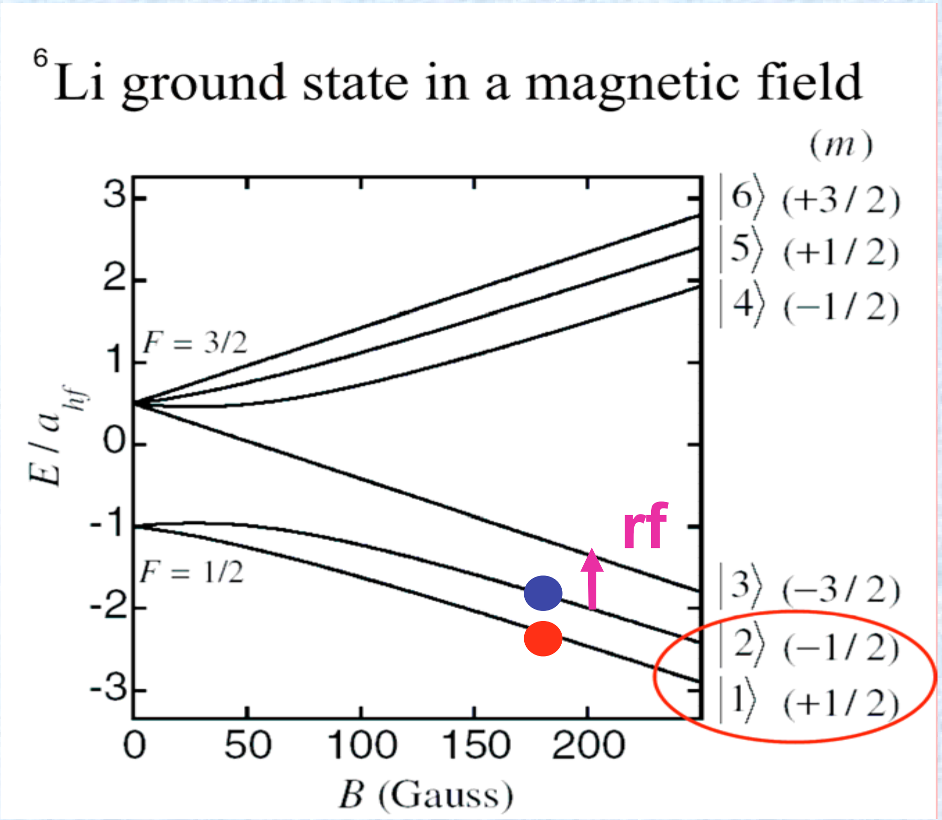
Molecular wave function $\frac{1}{r}e^{-r/a}$

Molecular interaction $U_{eff} \sim +\frac{1}{R}e^{-2R/a}$ (for R > a)

High density enhances pairing *Strinati ´00*

BCS limit: Even weak attraction can pair Fermion.

RF excitation: Molecule picture



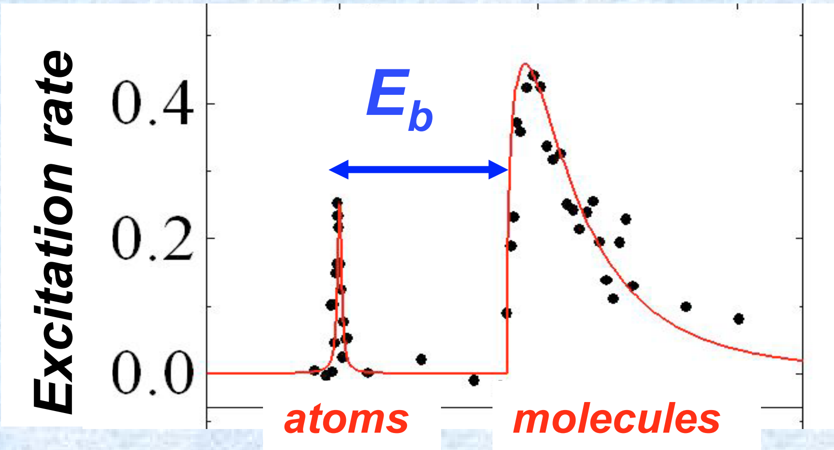
State 3 is initially unpopulated

Proposal: Törma and Zoller, PRL `00

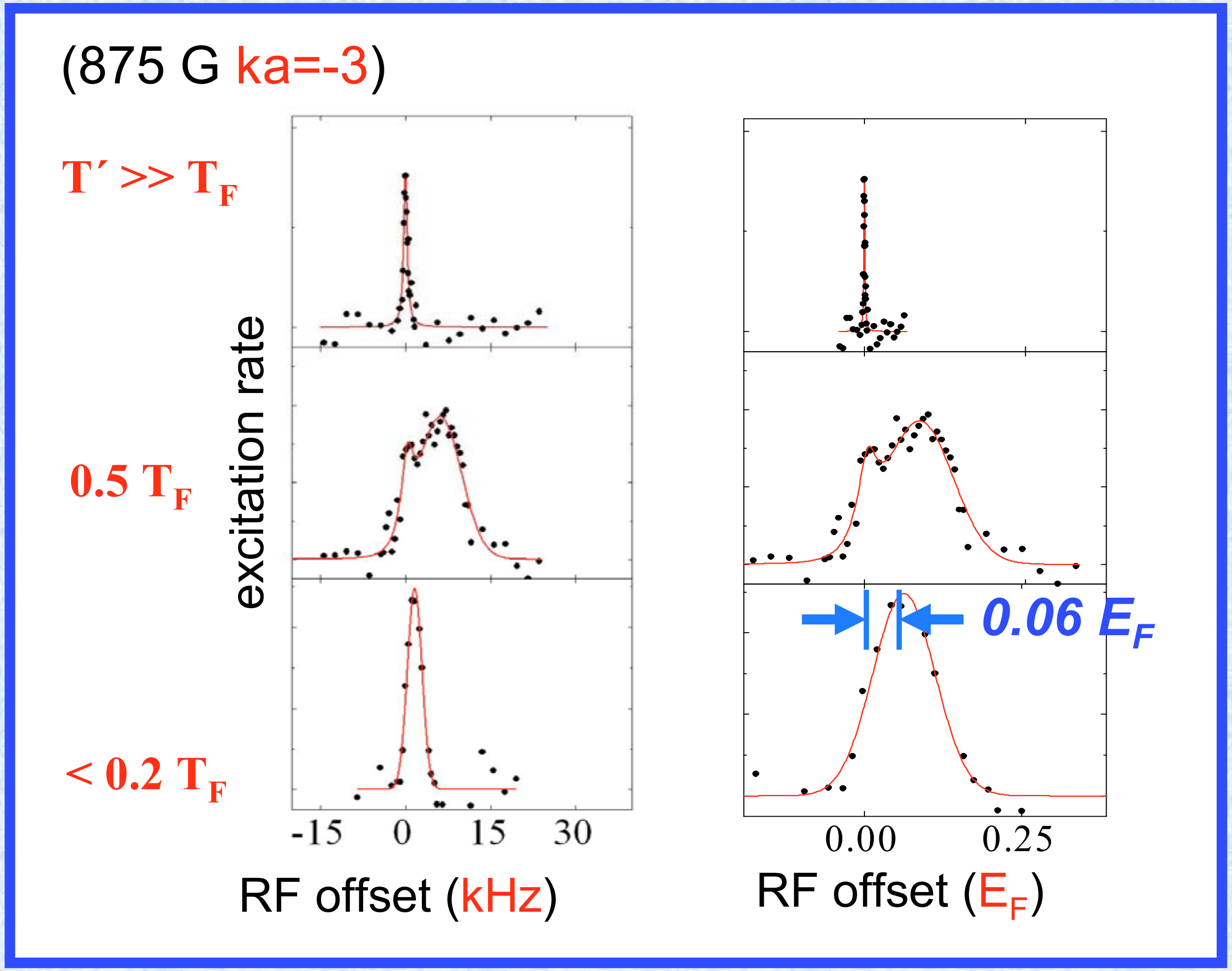
In the molecule picture

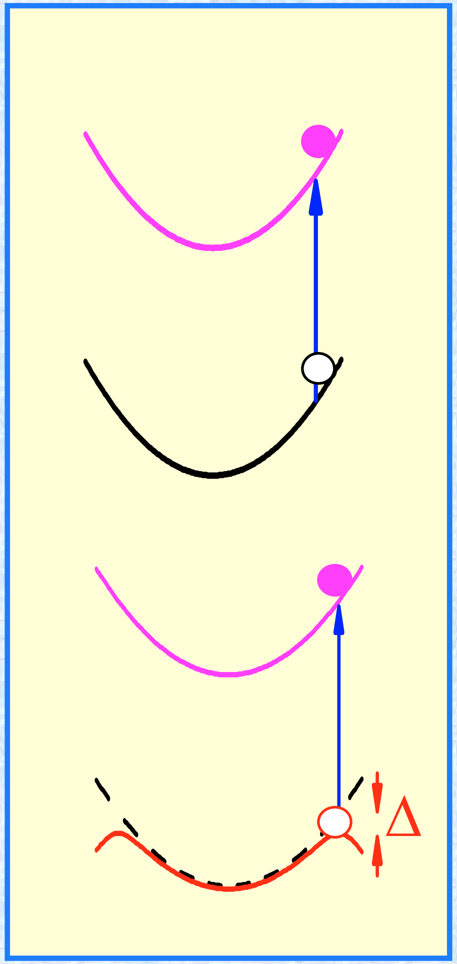
$f_{atom} = E_{23}$

$f_{molecule} = E_{23} + E_b + 2E_K$



RF excitation: many-body picture

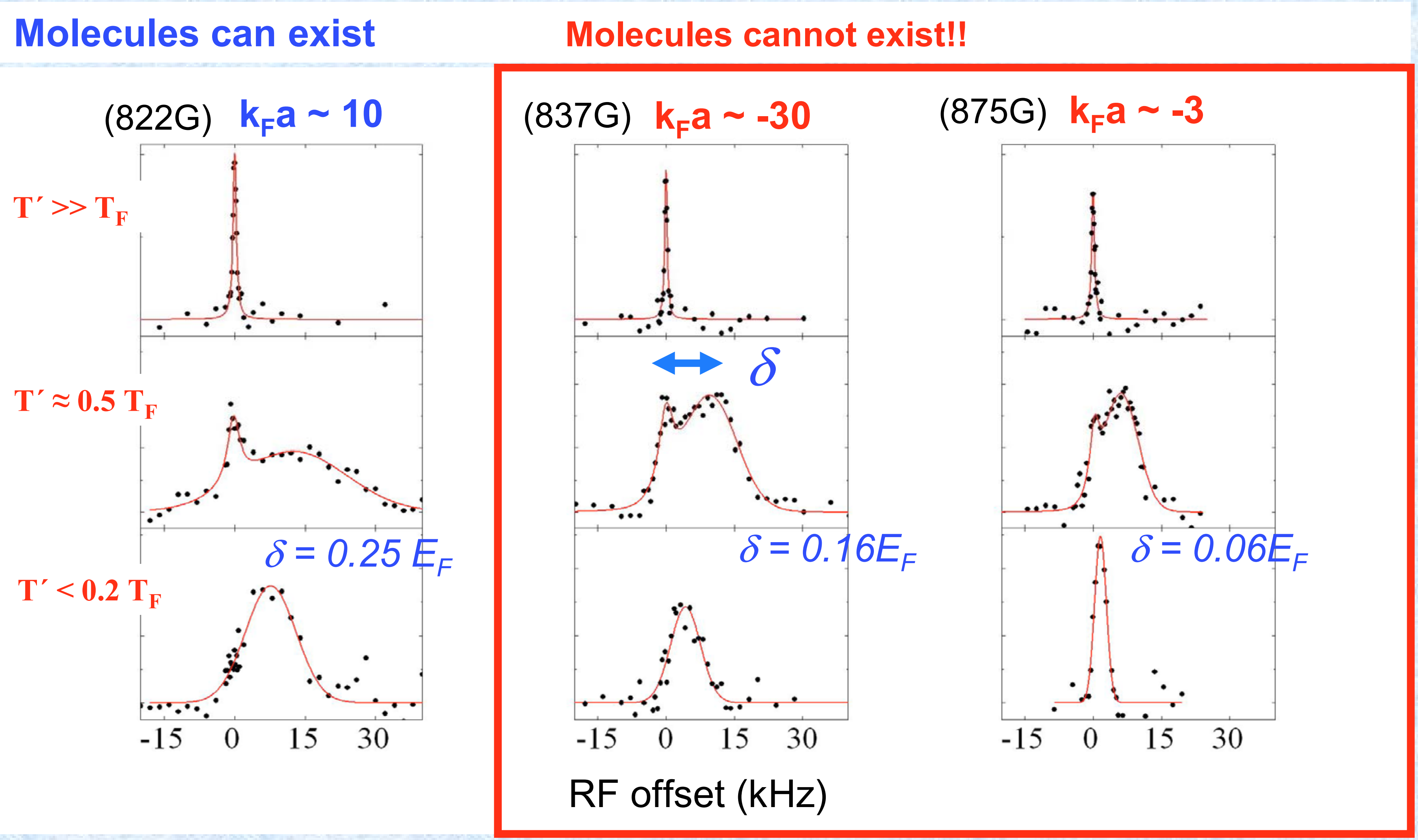




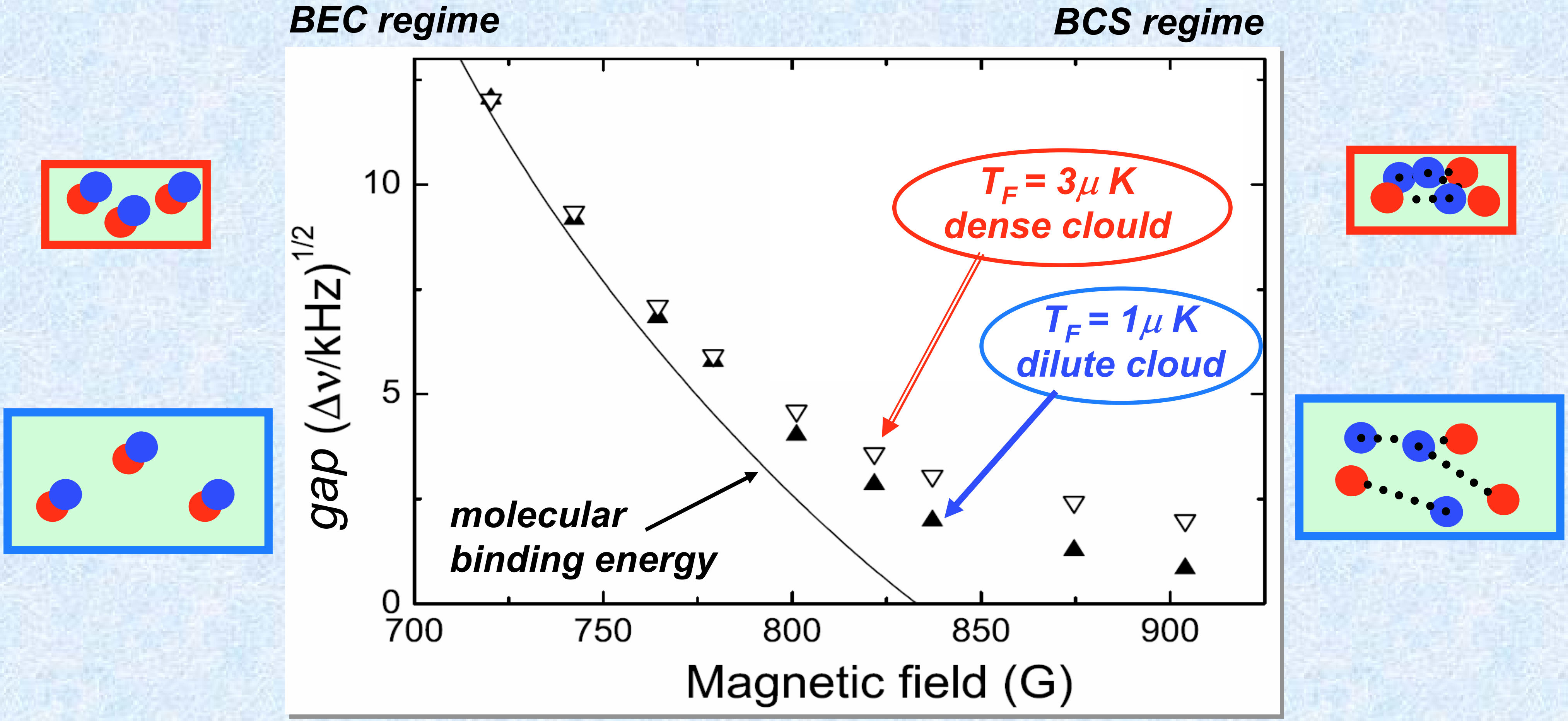
Creation of a hole in state 2 and then a particle in state 3 with the same momentum k.

Energy cost: $\sqrt{(\mu - E_k)^2 + \Delta^2} - (\mu - E_k)$

Pairing gap in the crossover

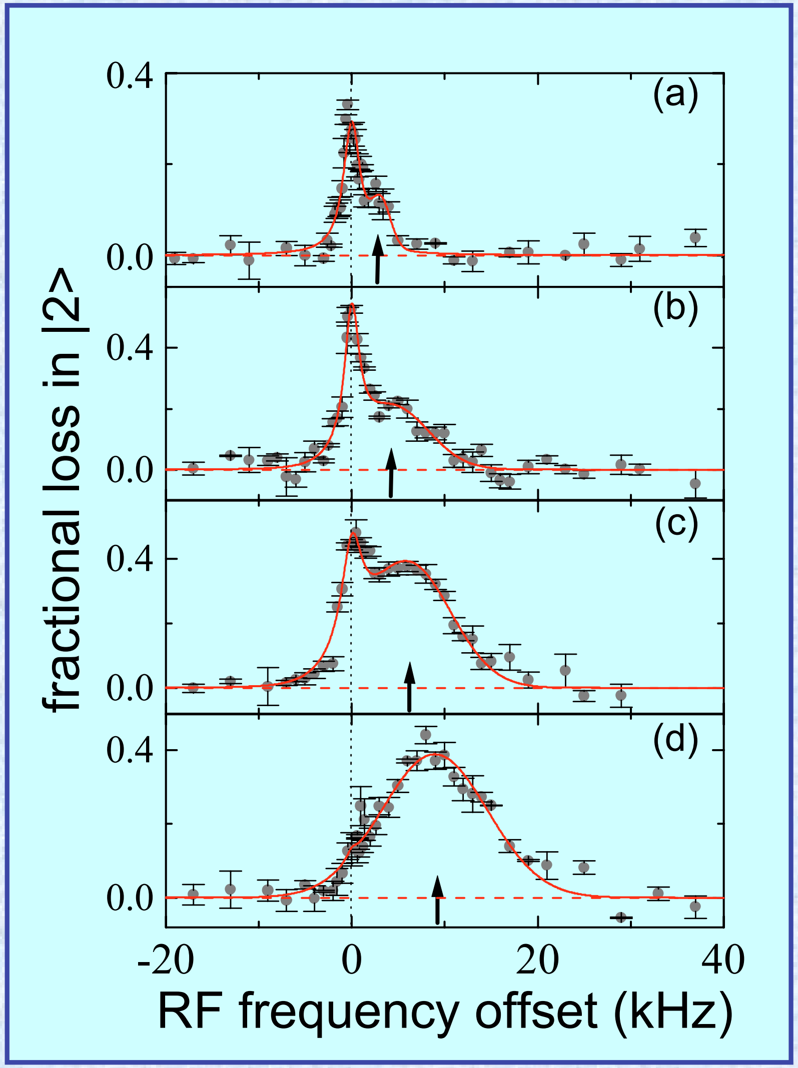


Pairing gaps vs. Fermi energy (density)

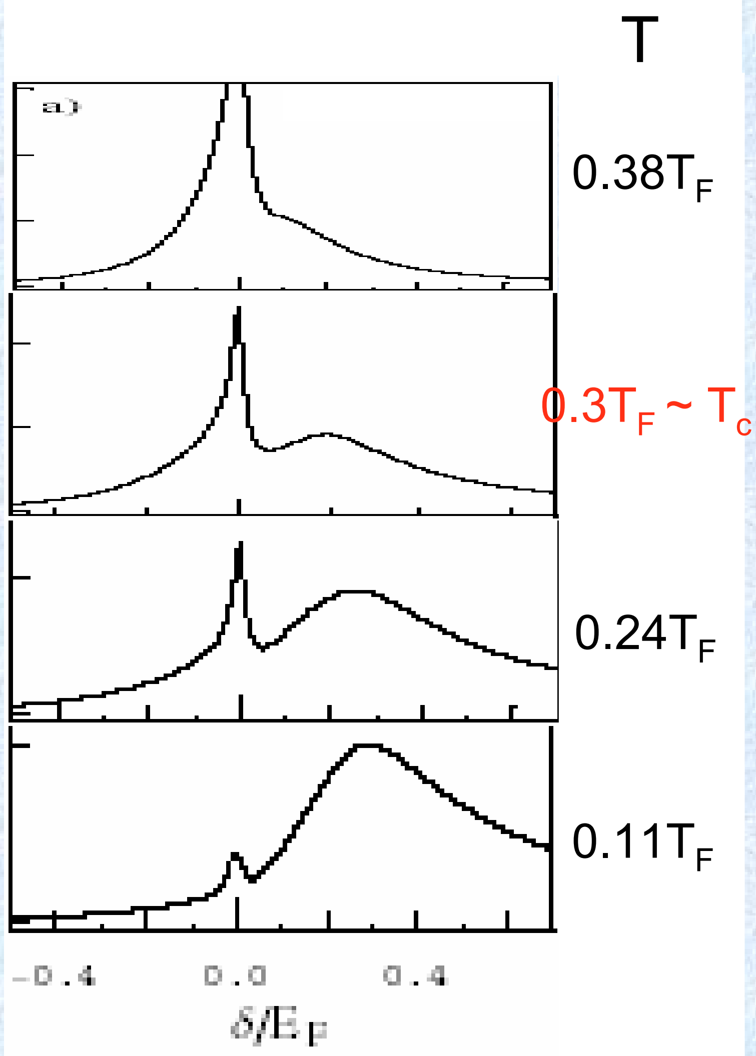


Very close to resonance (837G)

T'/T_F
0.8
0.7
0.45
< 0.2



Theory *(Päivi Törma)*



The absence of narrow peak suggests all atoms are paired.

Compare with theory

Possible reasons:
- *Trap geometry and LDA?*
- *If $a_{13}=a_{12}$, the broad peak will disappear*

A. Leggett

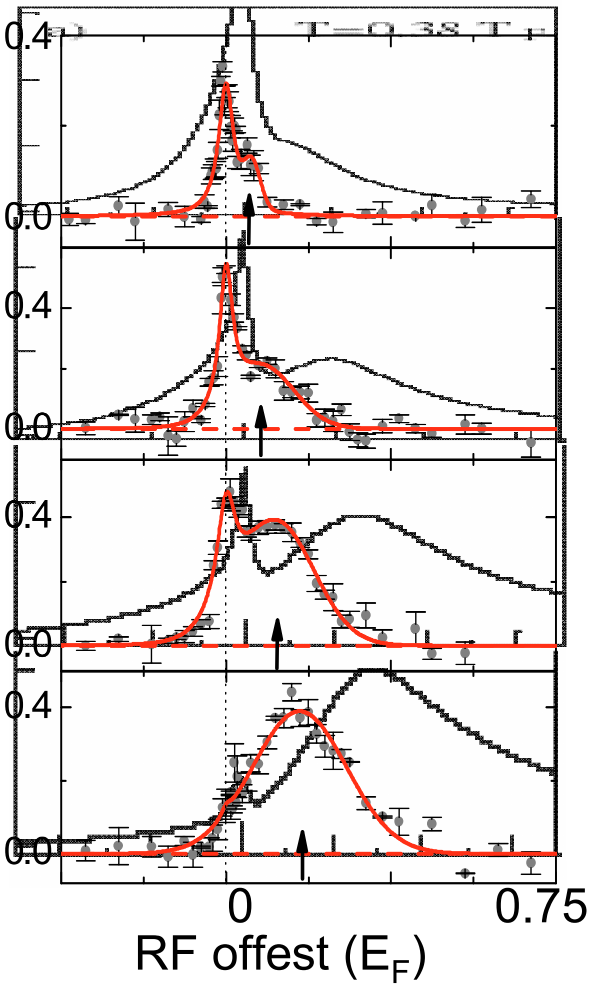
T_{BEC}/T_F

0.8

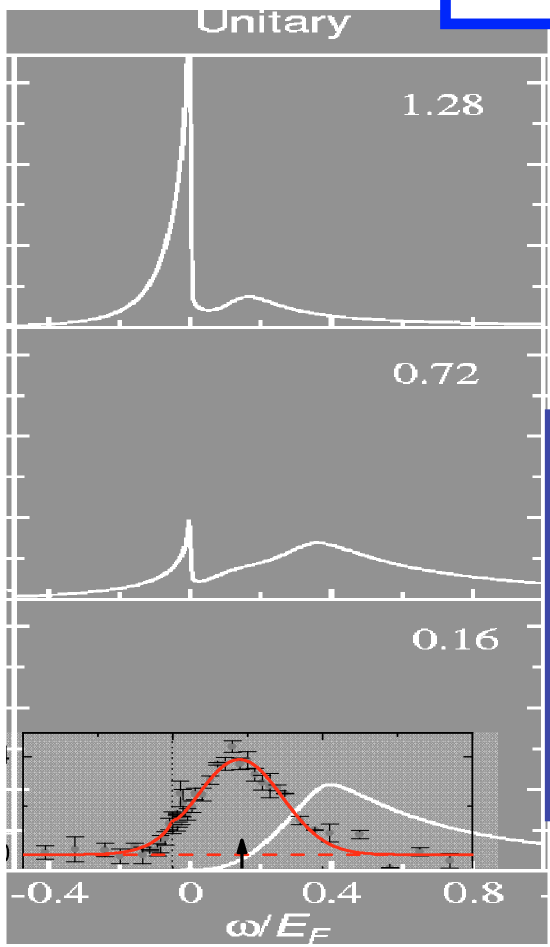
0.7

0.45

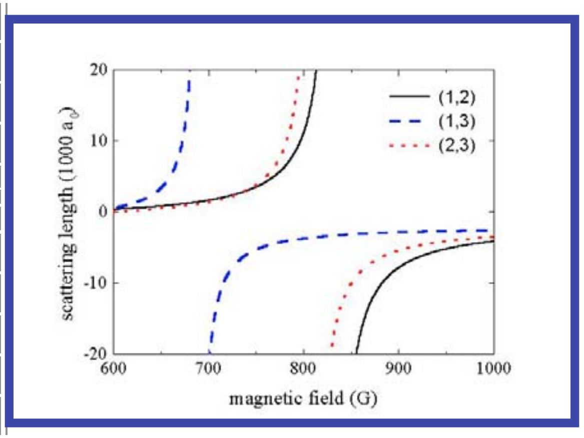
< 0.2



***Theory** (Päivi Törma)*

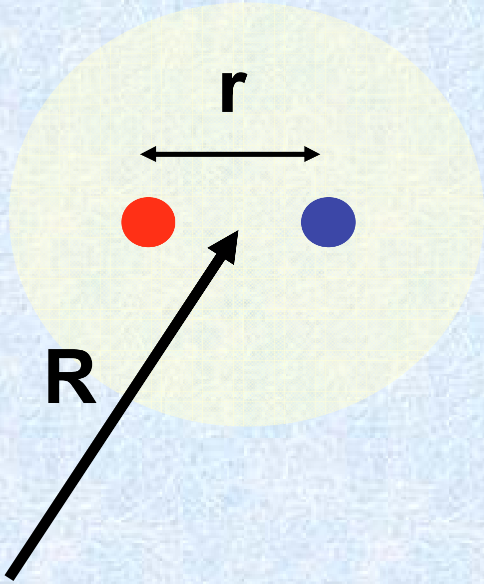


***Theory** (Kathy Levin)*



A mean-field model

Construction:
CM motion : bosonic GP equation
Relative motion: pair correlation.



Generalized mean-field model

$$(-\frac{\hbar^2}{4m}\nabla_R{}^2 - \frac{\hbar^2}{m}\partial_r{}^2 + V_{\text{int}} + g\,|\Psi(R)|^2)\Psi(R)\psi(r) = 2\mu\Psi(R)\psi(r)$$

In the dilute gas limit: LDA $|\Psi(R)|^2$=n and V_{int}~δ(r), we get

$$(-\frac{\hbar^2}{4m}\partial_r{}^2 + gn)\psi(r) = 2\mu\psi(r)$$
$$-a\psi'(0) = \psi(0)$$

Determination of the mean-field term g

Molecular mean-field is determined by atomic interaction. *(Petrov et al. 04)*
We assume ***g=g(r).***

BEC limit: $< gn >= \frac{4\pi\, n\, a_m \hbar^2}{m}$

BCS limit: $< gn >= 2\mu = 2E_F$

$\Rightarrow$

$$g = 4\pi \frac{a_m}{a} \frac{\hbar}{m} r$$

$$g = 6\pi^2 \alpha^{-3/2} \frac{\hbar}{m} r$$

*Both are linear in **r**.*

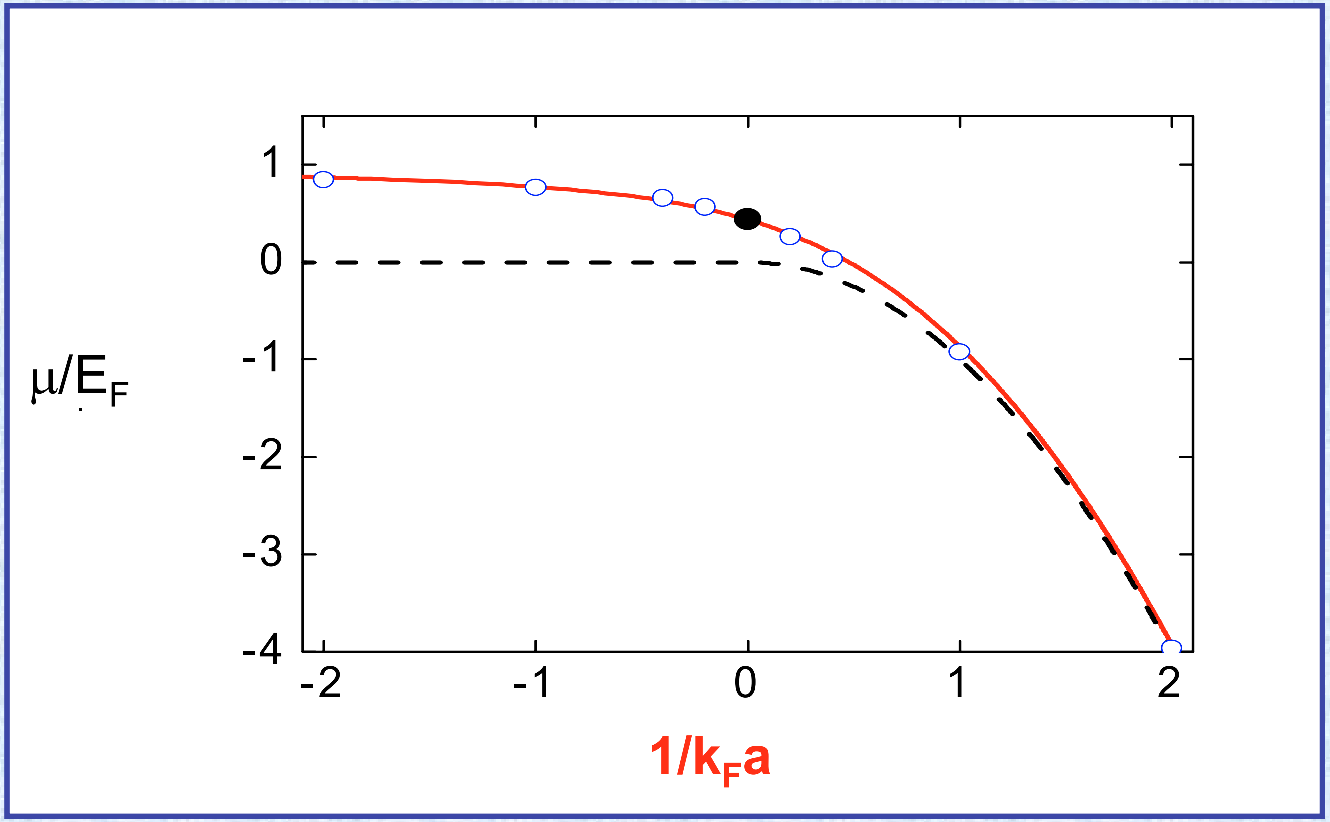
α=2.338.. Is the 1st zero of Airy´s function Ai(x)

In the crossover regime, we assume the BCS form:

$$\text{pair wavefunction}: \psi(r) \sim \mathrm{Ai}\left(\frac{x}{\sqrt{\alpha}} - \frac{\alpha\mu}{E_F}\right)$$

$$\text{chemical potential}: \frac{k_F a}{\sqrt{\alpha}} \mathrm{Ai}'\left(\frac{\alpha\mu}{E_F}\right) = -\mathrm{Ai}\left(\frac{\alpha\mu}{E_F}\right)$$

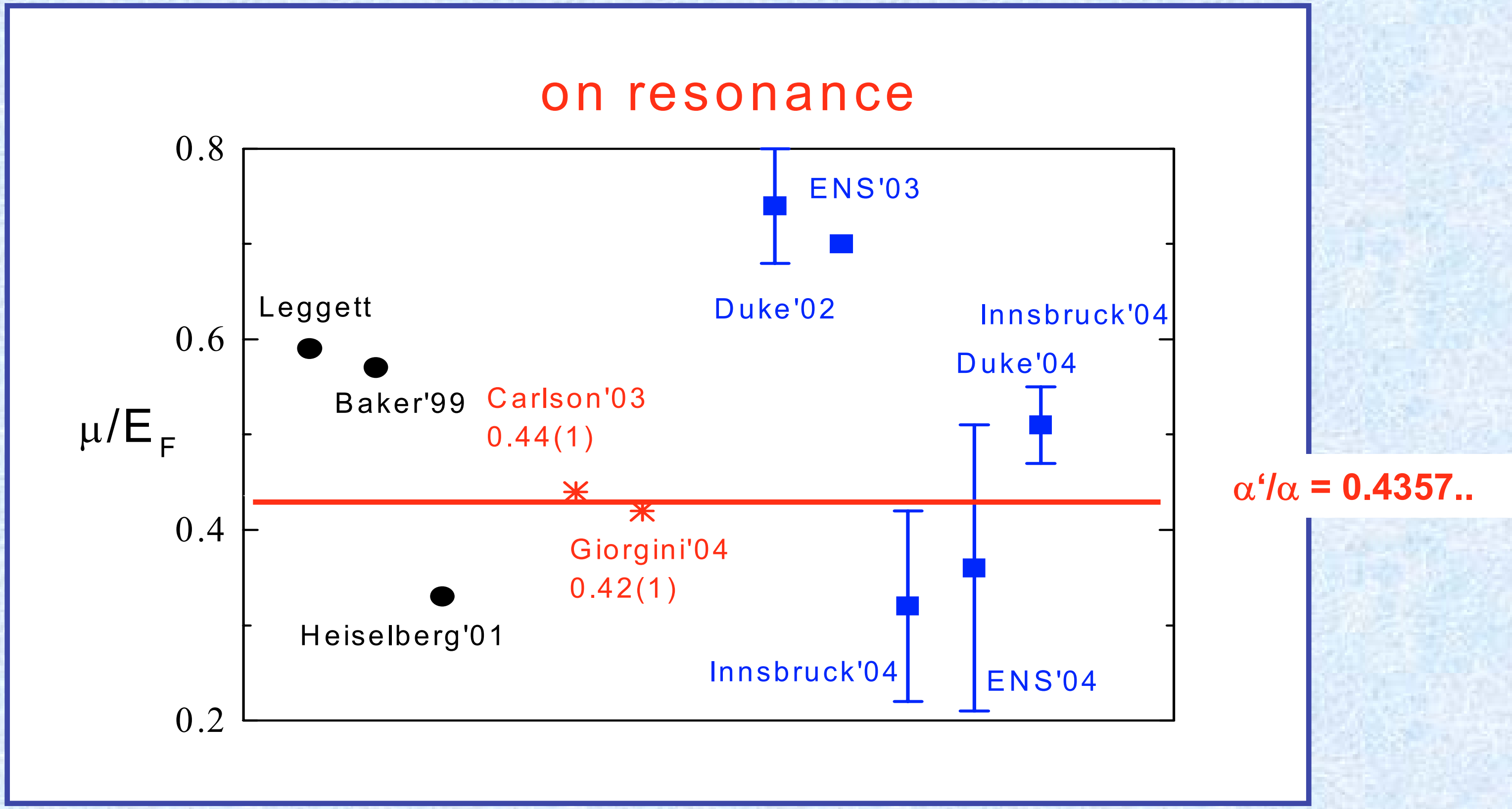
Chemical potential



QMC calculation from Giorgini et al., 2004

QMC calculation from Carlson et al., 2003

Calculation in the crossover regime



• **BCS mean-field theory**

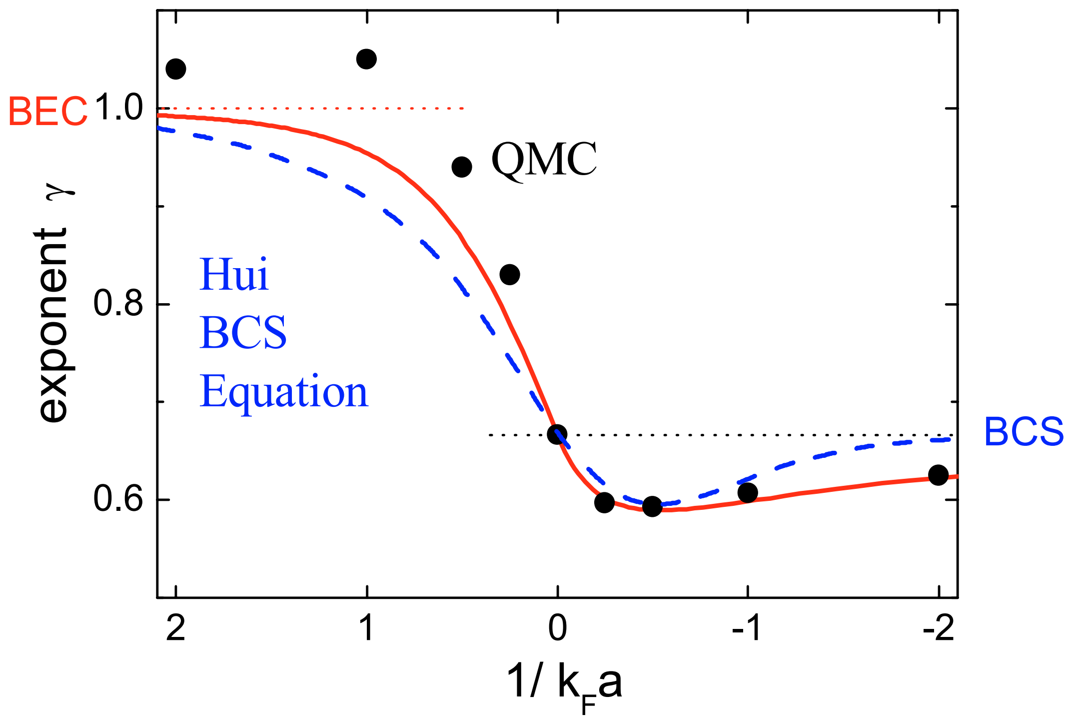
* **Quantum Monte Carlo calculations**

• **Experiments**

Results: Polytropic exponent of the equation of states

$\langle U \rangle \sim \langle nr \rangle \sim n^{\gamma}$

$\mu \sim n^1$



$\mu \sim n^{2/3}$

$$\gamma = \frac{2}{3}\left(1+\frac{E_b}{\mu_m}\right)^{-1}\left(1+\frac{2\alpha^{-3/2}k_F a}{(\mu_m/E_F)^2 k_F^2 a^2 + 2(\mu_m/E_F)}\right)$$

Puzzles

Collective modes: (both axial and radial)

BCS mean-field equation fits better than QMC?

Does trap geometry induce the sharp transition?

Pairing gap:

The gap is smaller than prediction?

Can rf profile determine the pair wave function?

Gap vanishes and collective mode changes?

Ongoing experiment

Radial surface mode excitation and the creation of vortex…

^{6}Li team in the University of Innsbruck

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