



The Abdus Salam  
International Centre for Theoretical Physics



SMR 1666 - 30

**SCHOOL ON QUANTUM PHASE TRANSITIONS  
AND  
NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES**

**11 - 22 July 2005**

***Fermi condensates***

Presented by:

**Markus Greiner**

University of Colorado at Boulder, USA

ICTP SCHOOL ON QUANTUM PHASE TRANSITIONS AND  
NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES 2005

# Fermi condensates

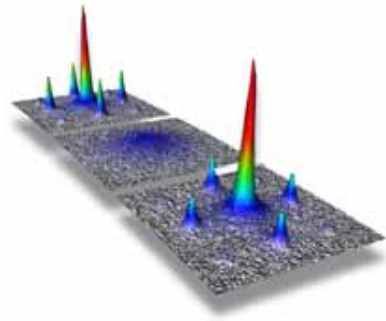
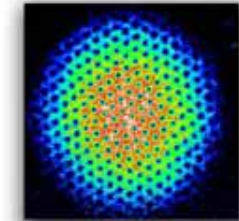
Markus Greiner

JILA, Group of D. Jin;  
Coworkers: C. Regal and J. Stewart  
NIST and the University of Colorado,  
Boulder



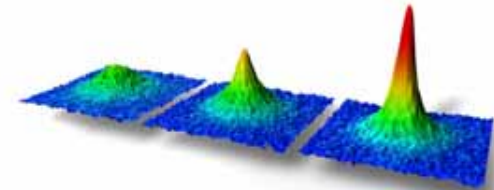
# Highly controlled many-body quantum systems

**Weakly interacting Bose gases:**  
Coherence, superfluid flow, vortices ...



**Strongly correlated Bose systems:**  
Superfluid to Mott insulator transition

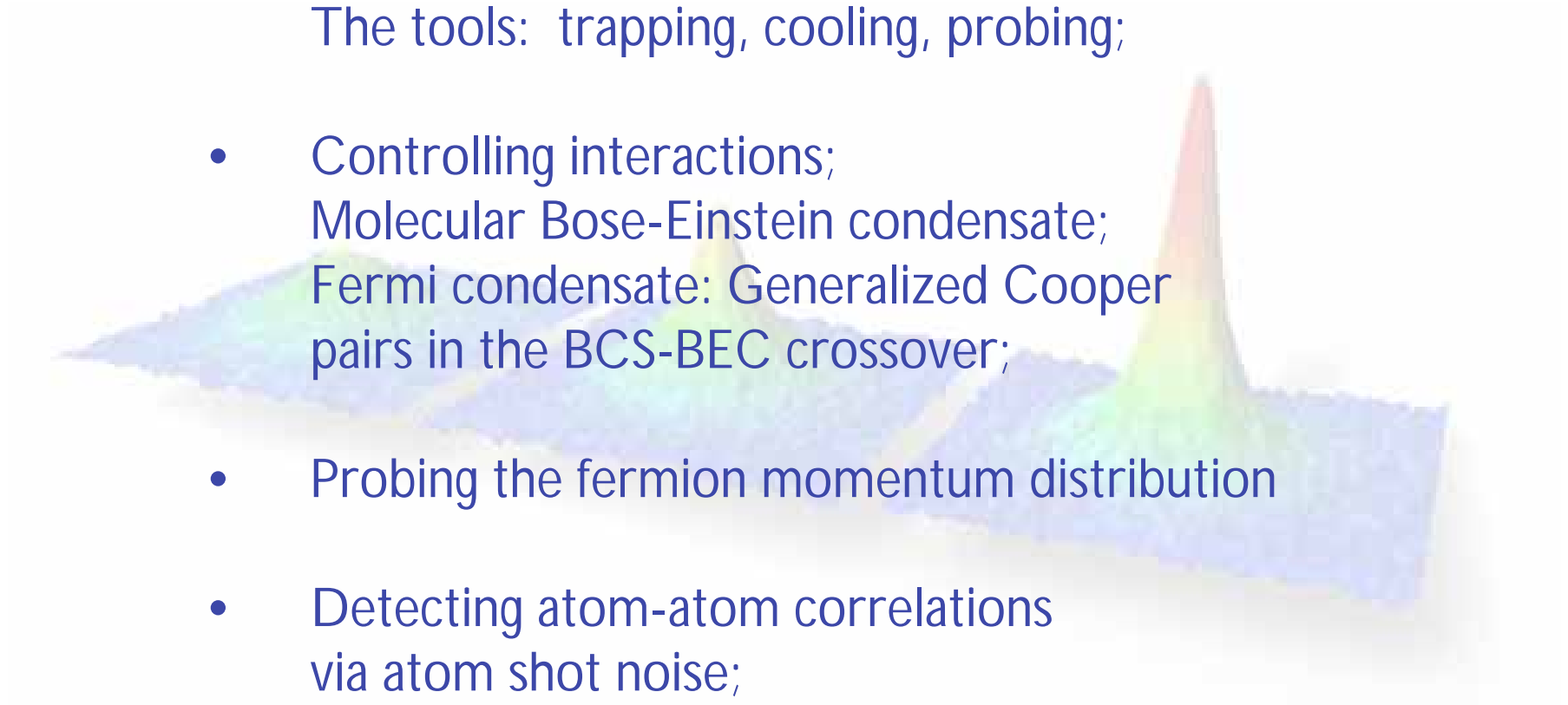
**Fermionic superfluidity:**  
BCS-BEC crossover physics



- **Condensed matter physics** studied with an atomic physics system

# Outline:

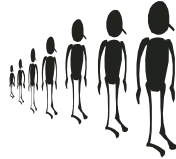
- Fermionic superfluidity;  
The tools: trapping, cooling, probing;
- Controlling interactions;  
Molecular Bose-Einstein condensate;  
Fermi condensate: Generalized Cooper pairs in the BCS-BEC crossover;
- Probing the fermion momentum distribution
- Detecting atom-atom correlations  
via atom shot noise;



# Bosons



# Fermions

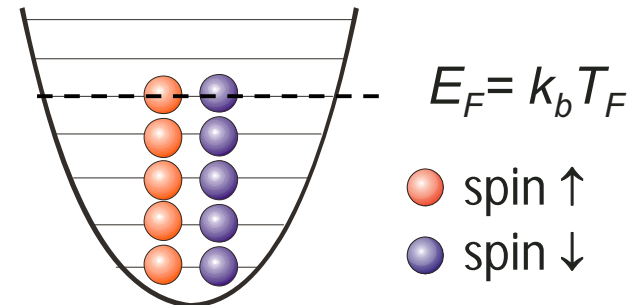
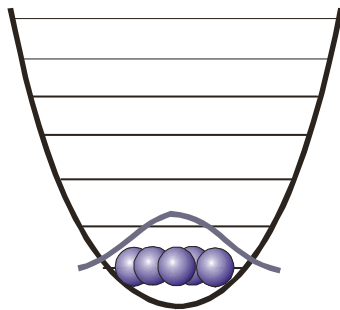


integer spin

$\Psi_{1,2} = \Psi_{2,1}$   
→ Bosonic  
enhancement

half-integer spin

$\Psi_{1,2} = -\Psi_{2,1}$   
→ Pauli exclusion  
principle



1995: Bose-Einstein condensation  
e.g.  $^{87}\text{Rb}$ ,  $^{23}\text{Na}$ ,  $\text{H}$ ,  $^{39}\text{K}$  ...  
photons, liquid  $^4\text{He}$

1999: Fermi sea of atoms  
 $^{40}\text{K}$ ,  $^6\text{Li}$   
electrons, protons, neutrons

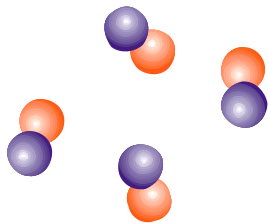
# Pairing and Superfluidity

→ Spin is additive: Fermions can pair up and form effective bosons:

$$\Psi(1, \dots, N) = \hat{A} [ \phi(1,2) \phi(3,4) \dots \phi(N-1,N) ]$$

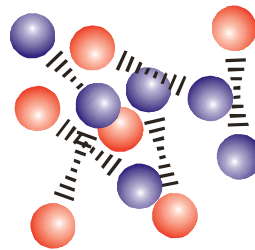
● spin ↑  
● spin ↓

Molecules of fermionic atoms



BEC of weakly bound molecules

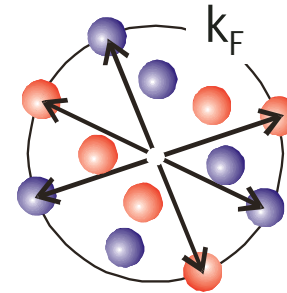
Generalized Cooper pairs of fermionic atoms



BCS - BEC crossover



Cooper pairs



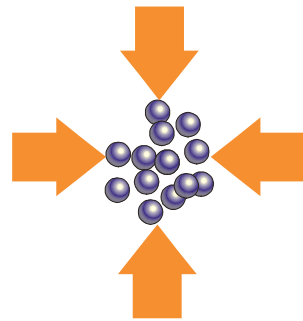
BCS superconductivity  
Cooper pairs: correlated momentum-space pairing

BCS-BEC crossover for example:

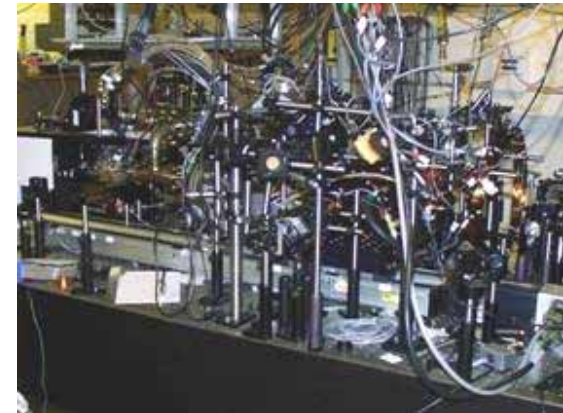
Eagles, Leggett, Nozieres and Schmitt-Rink, Randeria, Strinati, Zwerger, Holland, Timmermans, Griffin, Levin ...

# Cooling a gas of fermionic $^{40}\text{K}$ atoms

## 1. Laser cooling and trapping of $^{40}\text{K}$

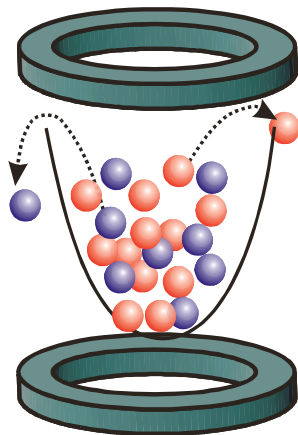


300 K to 1 mK  
 $\sim 10^9$  atoms



## 2. Magnetic trapping and evaporative cooling

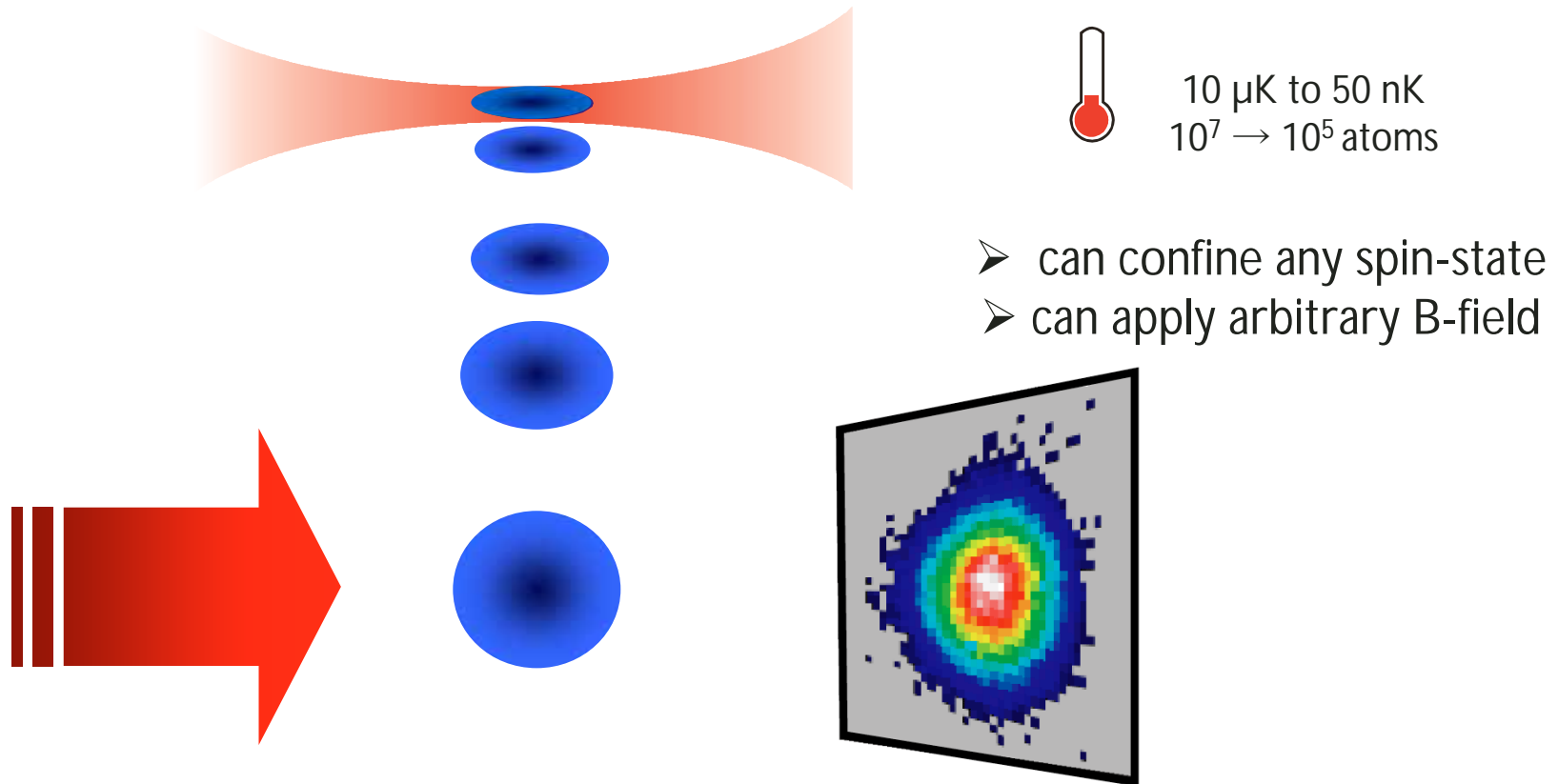
● spin  $\uparrow$   
● spin  $\downarrow$



1 mK to 10  $\mu\text{K}$   
 $\sim 10^9 \rightarrow 10^7$  atoms

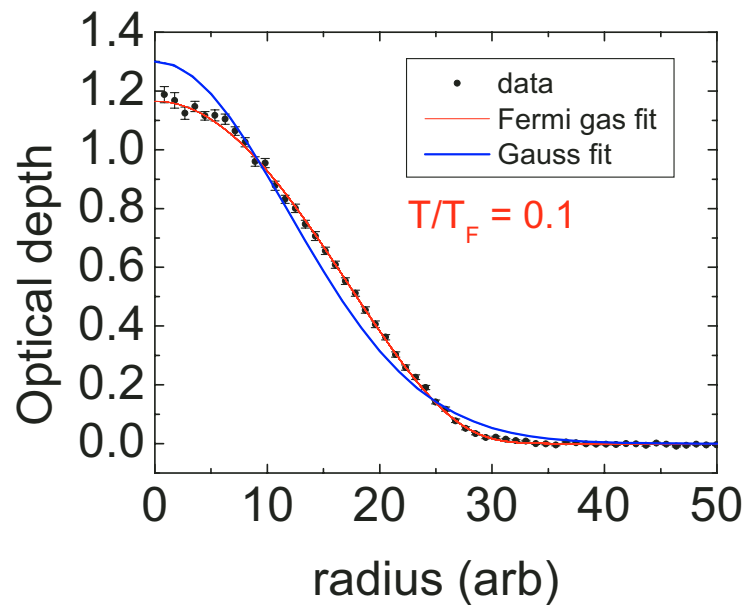
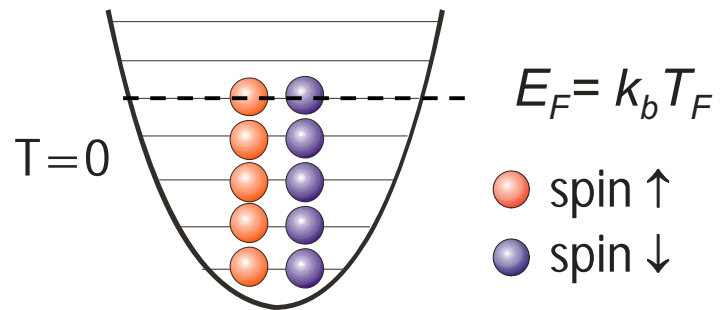
# Cooling a gas of fermionic $^{40}\text{K}$ atoms

## 3. Optical trapping and evaporative cooling

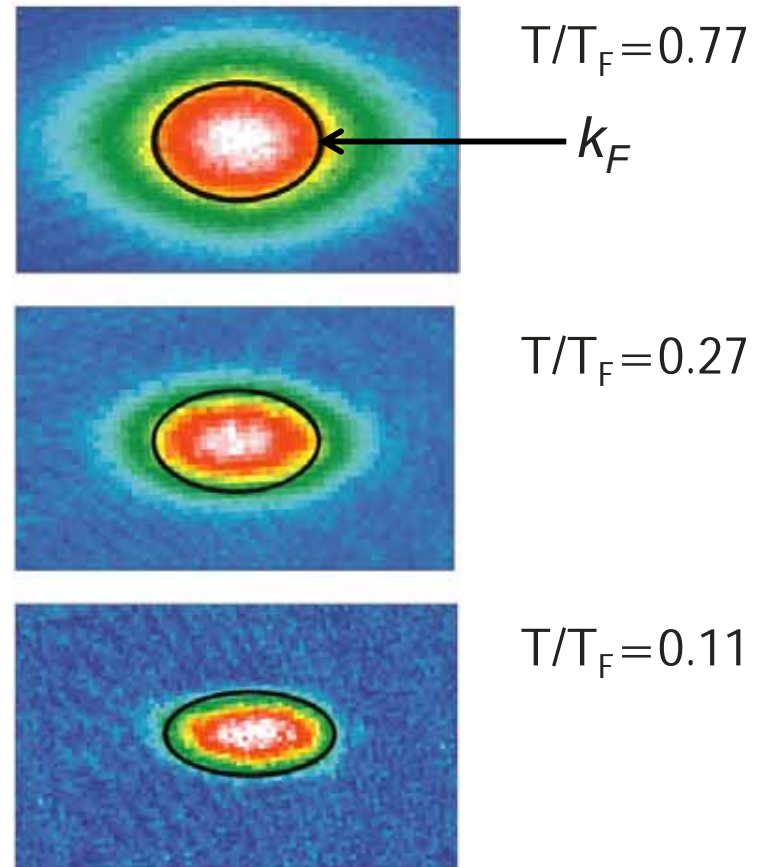




# Quantum degeneracy



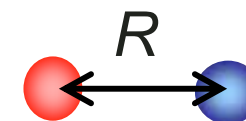
## momentum distributions



# Controlling interaction

Magnetic Feshbach resonance

Turning the knob: a **new molecular bound state** appears as  $B$  is varied



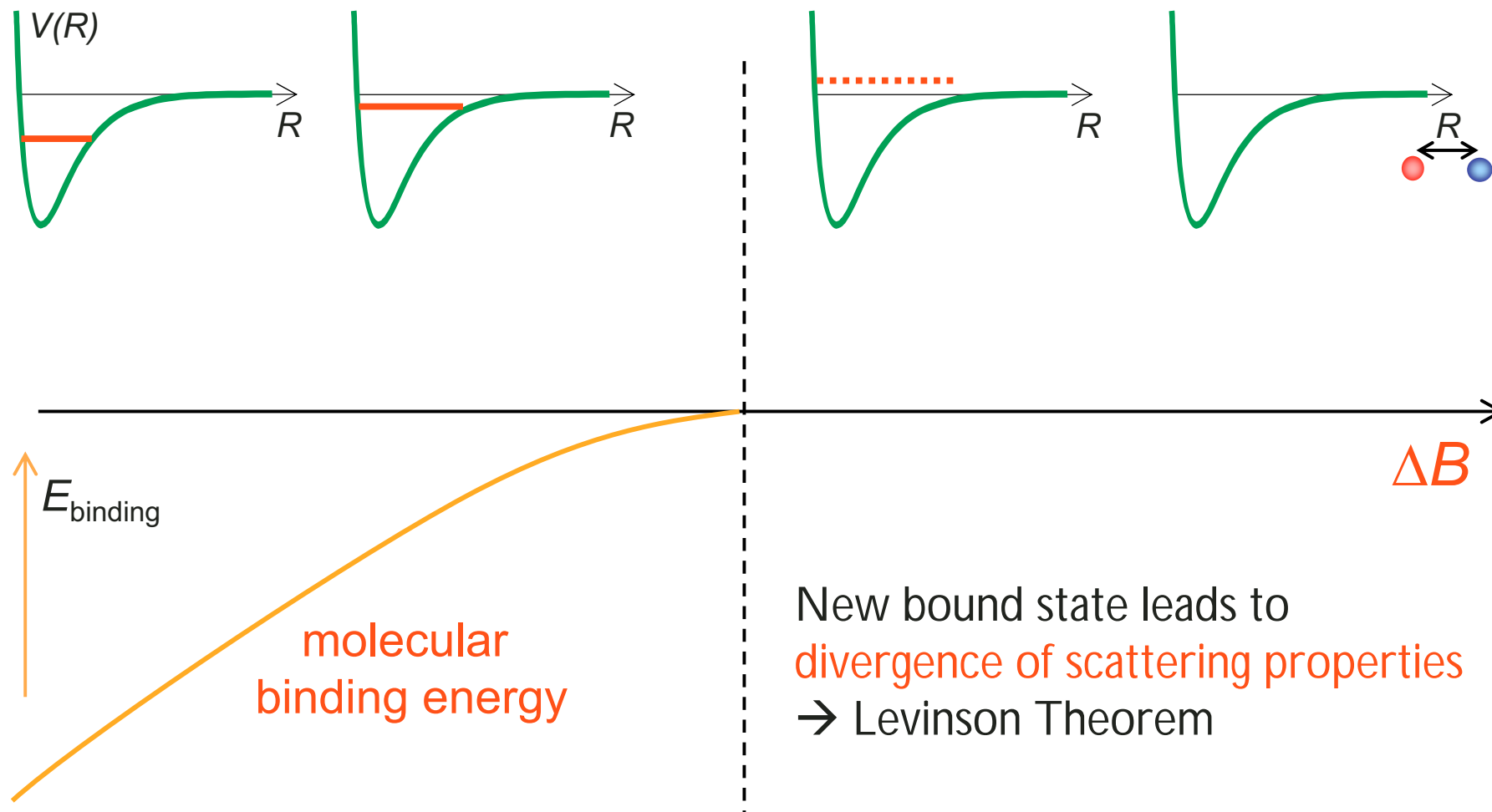
internuclear separation



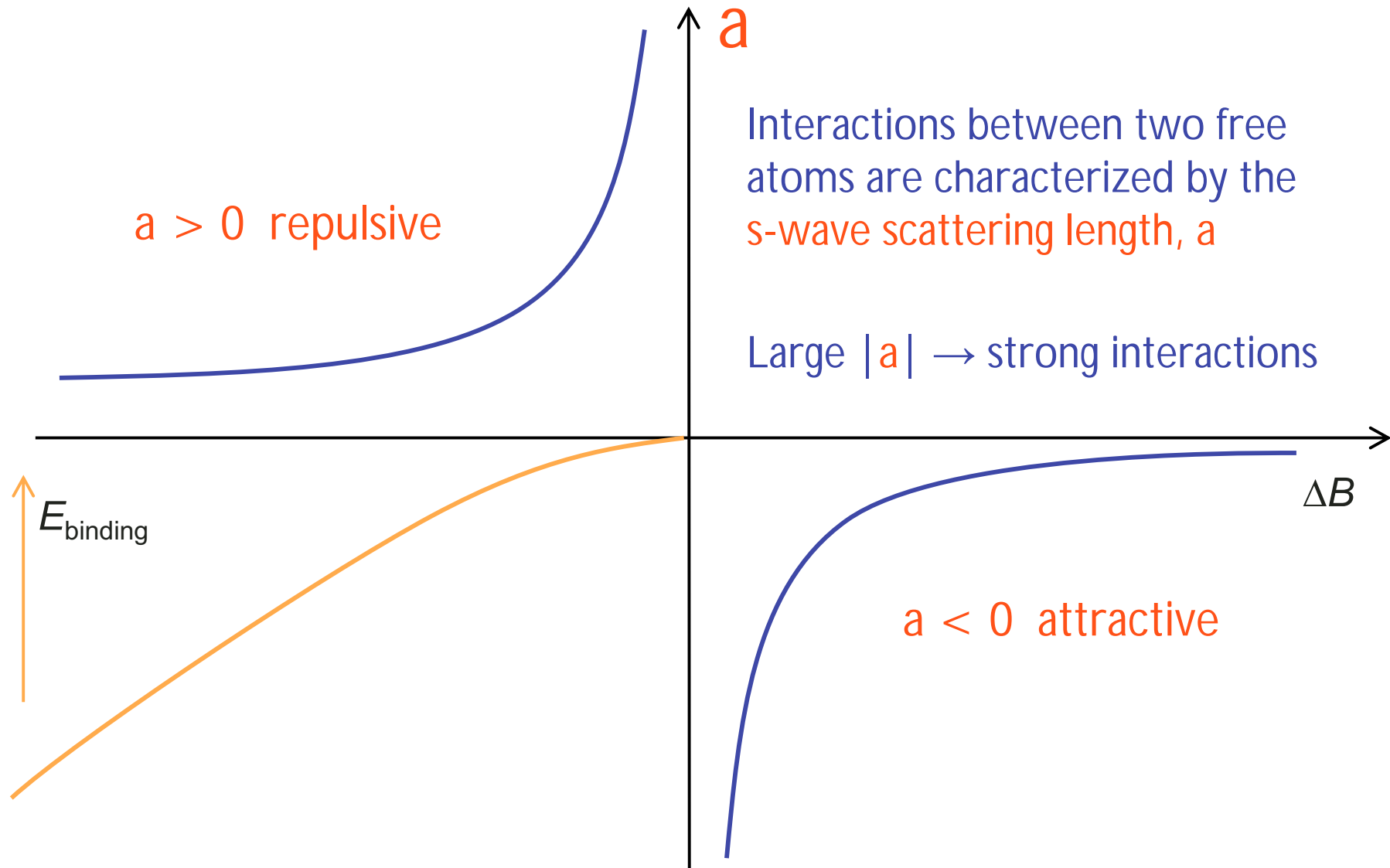
$B$ -field

# Controlling interaction

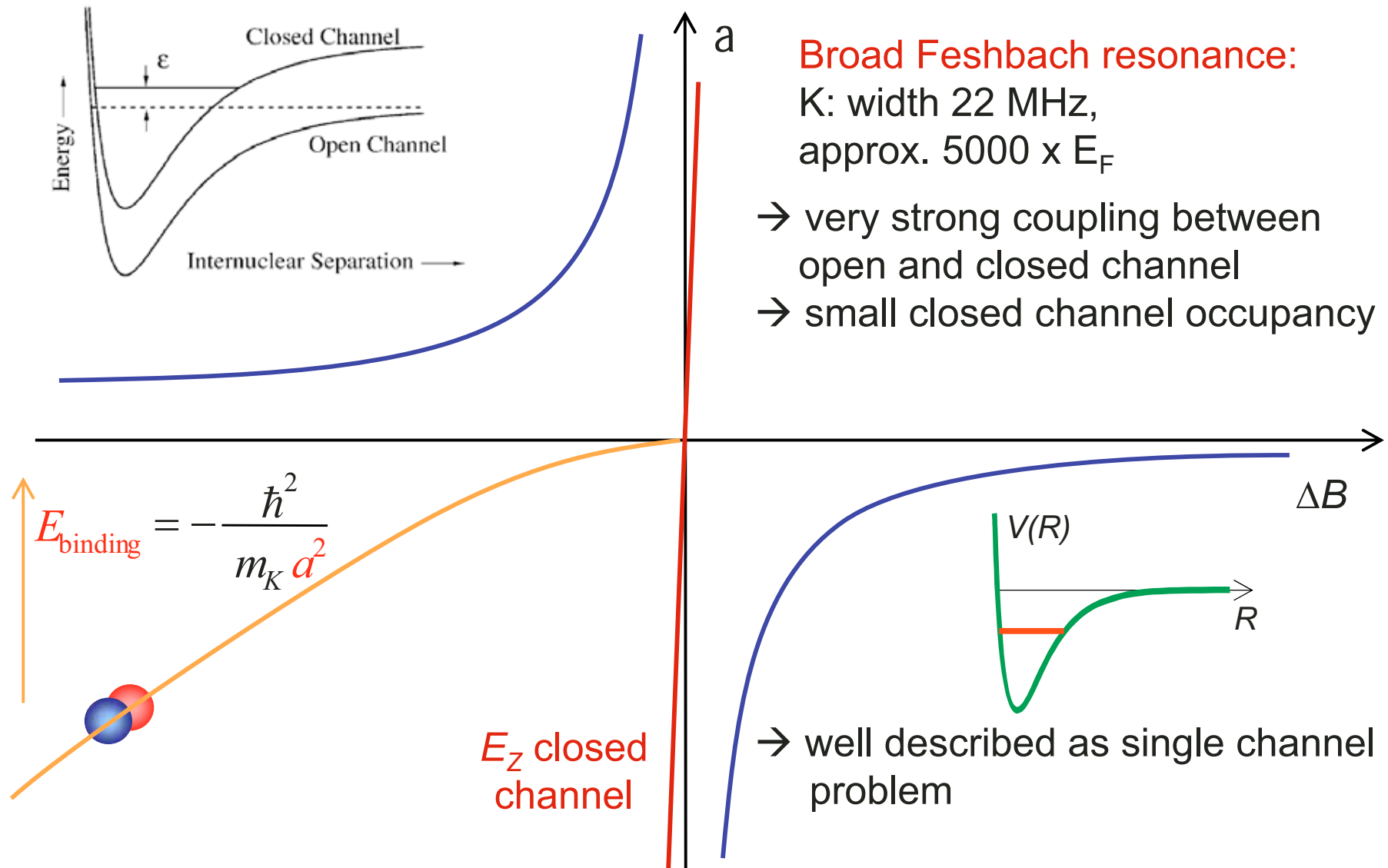
Magnetic Feshbach resonance: a **new molecular bound state** appears as  $B$  is varied



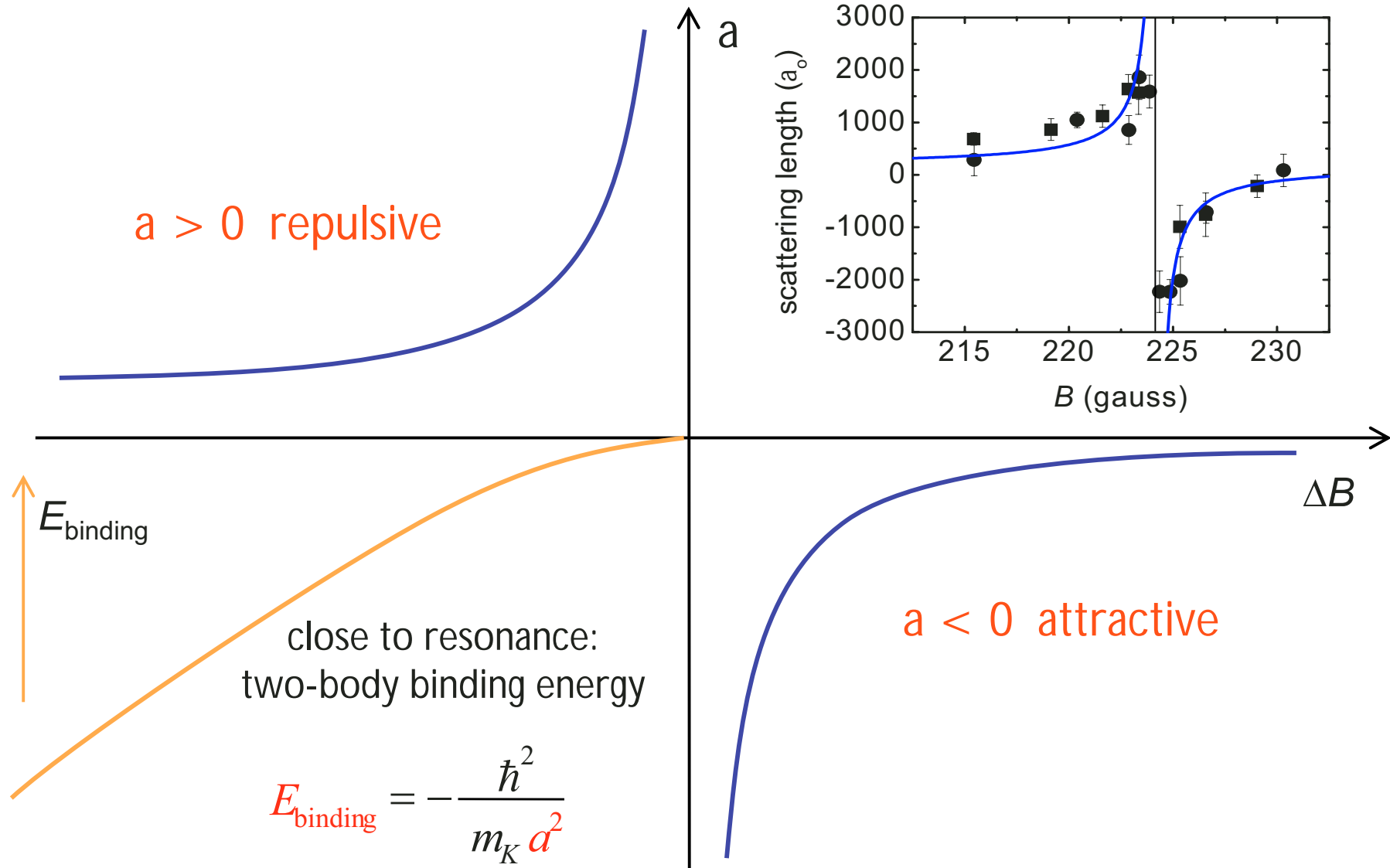
# Divergence of scattering length



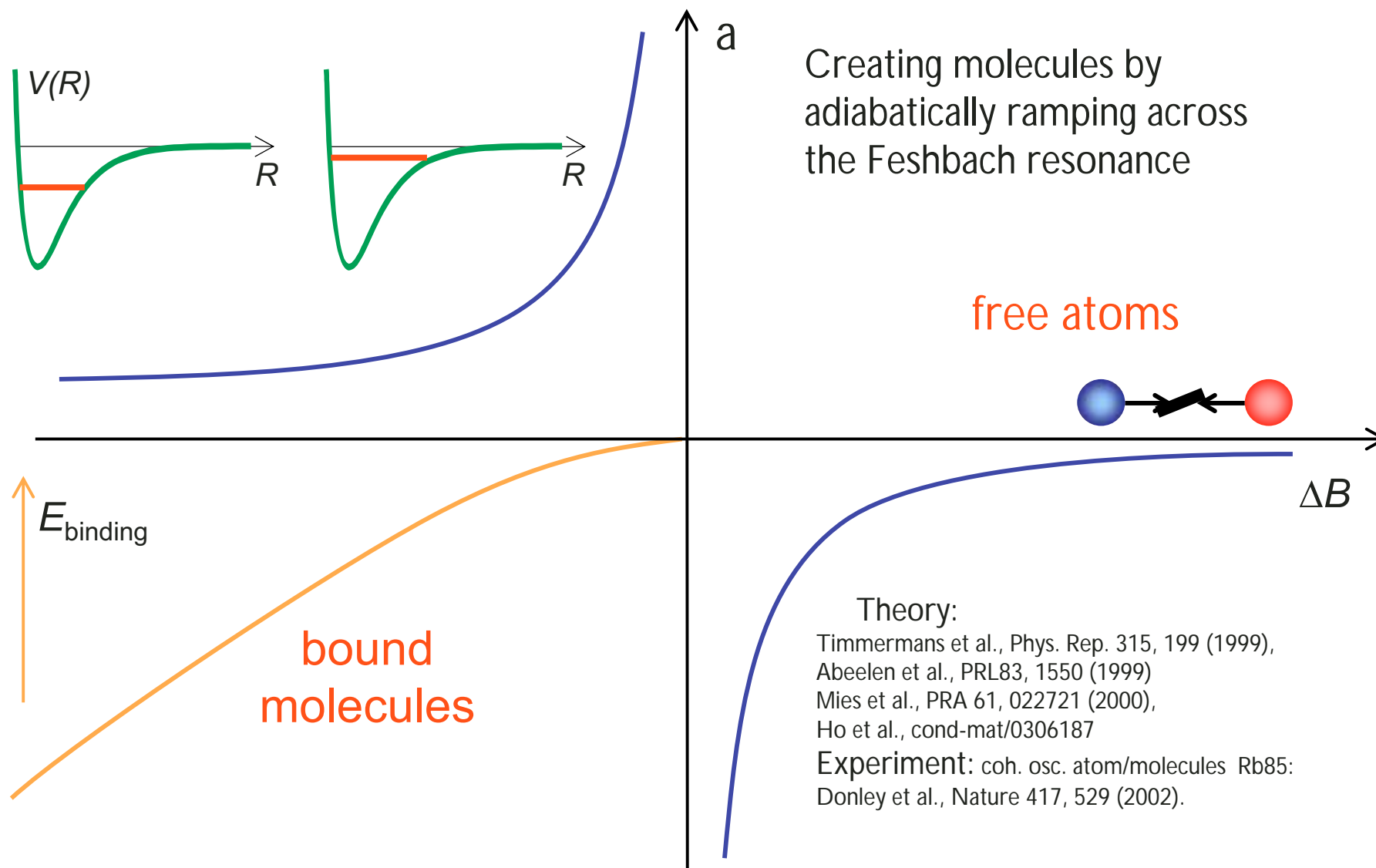
# Broad feshbach resonance



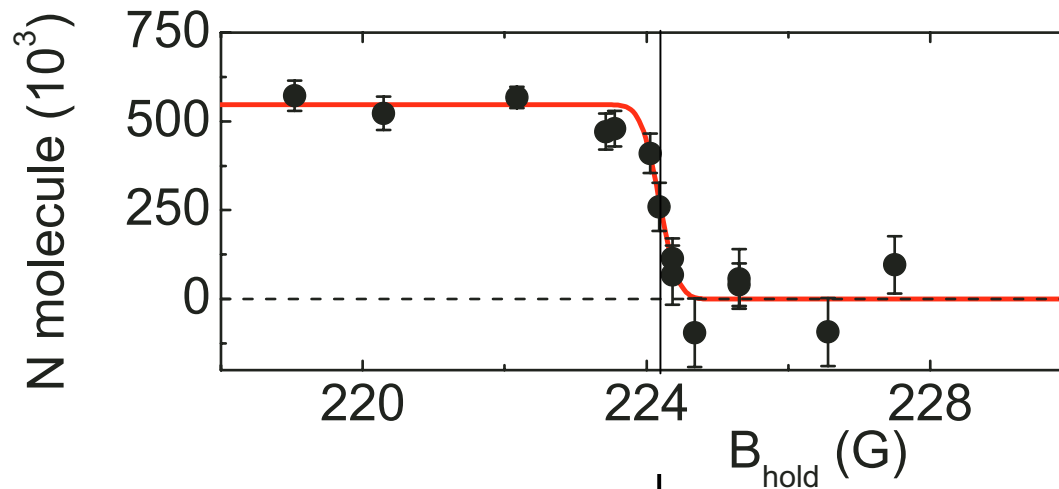
# Measurement of scattering length



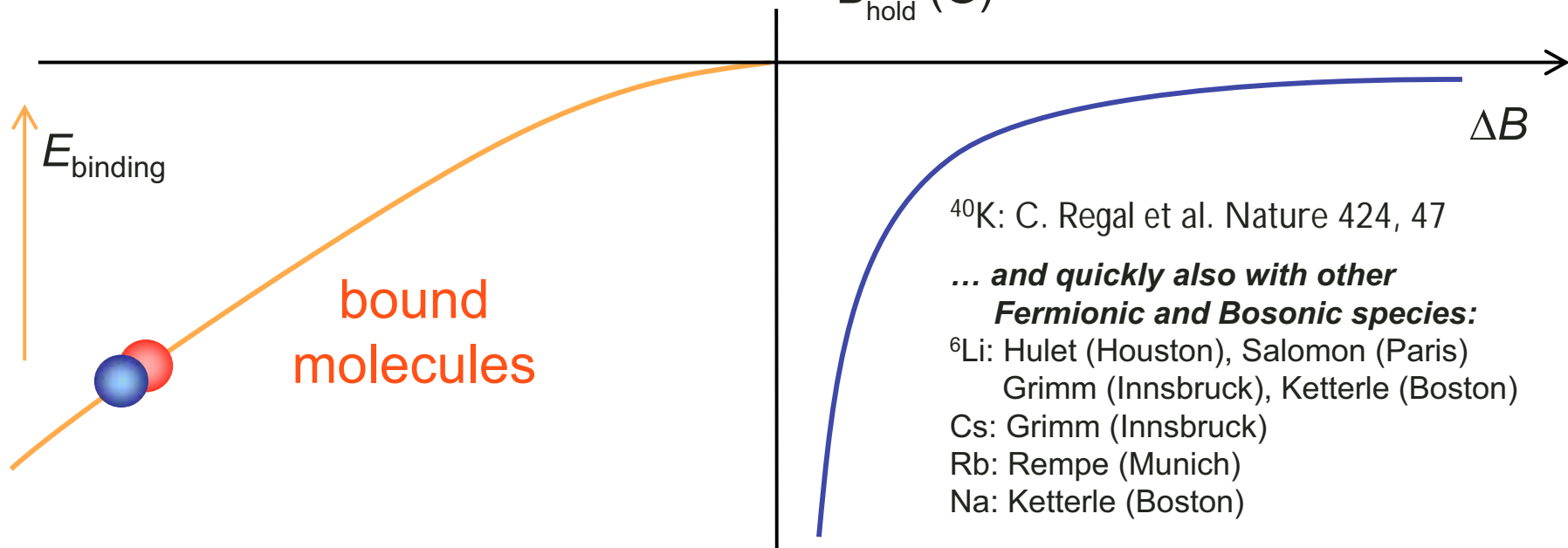
# Creating molecules



# Creating molecules

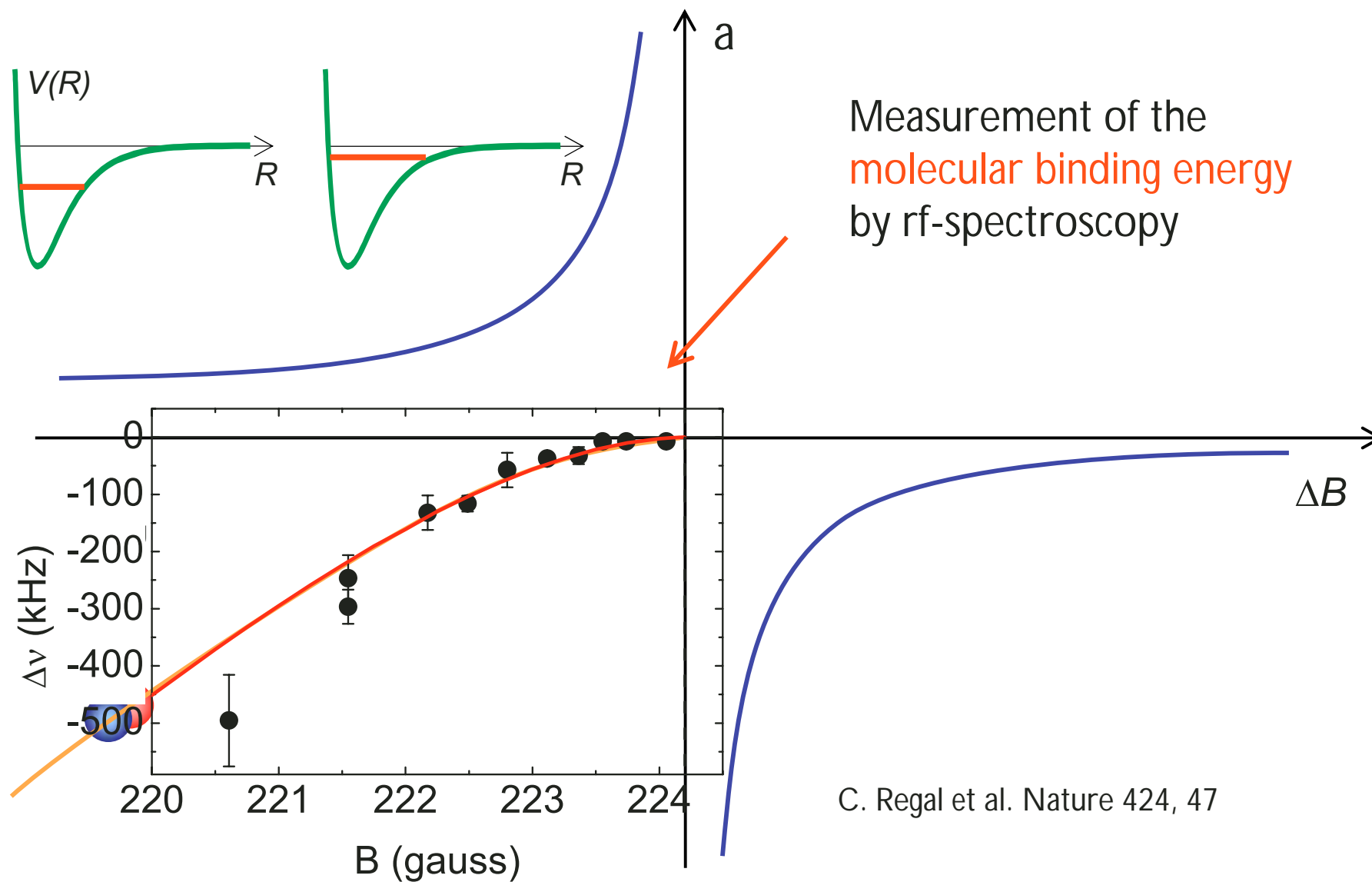


measured  
molecule  
number





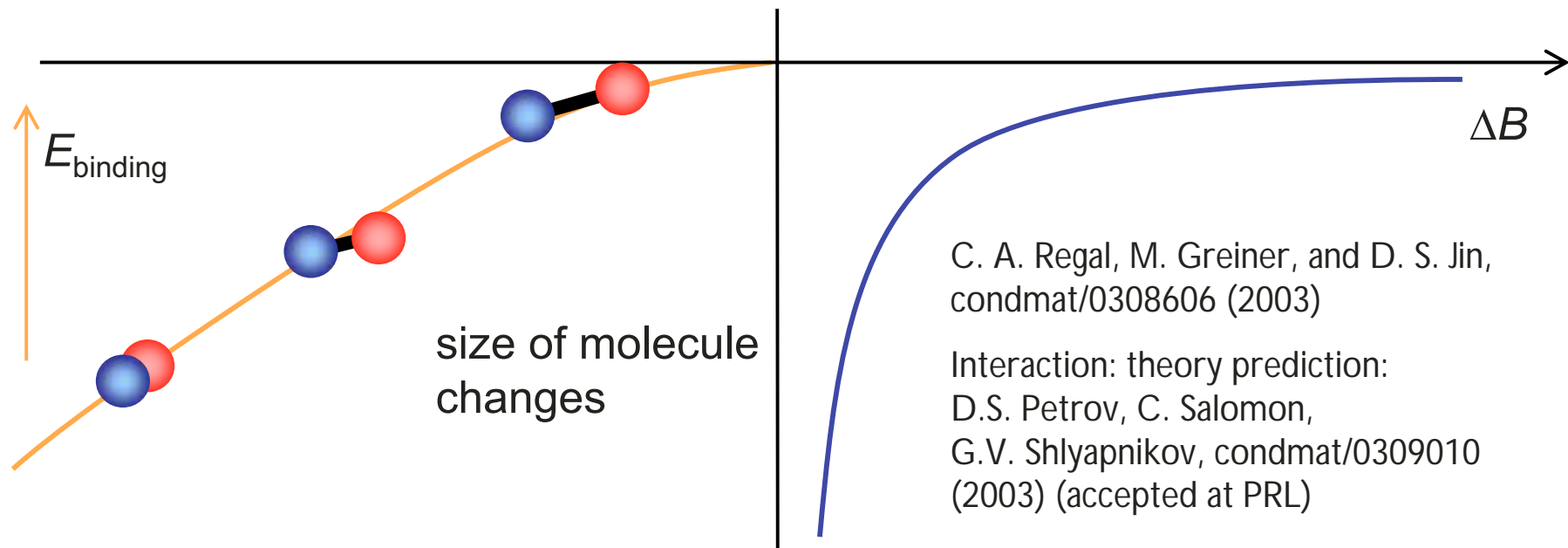
# Molecule binding energy



C. Regal et al. Nature 424, 47

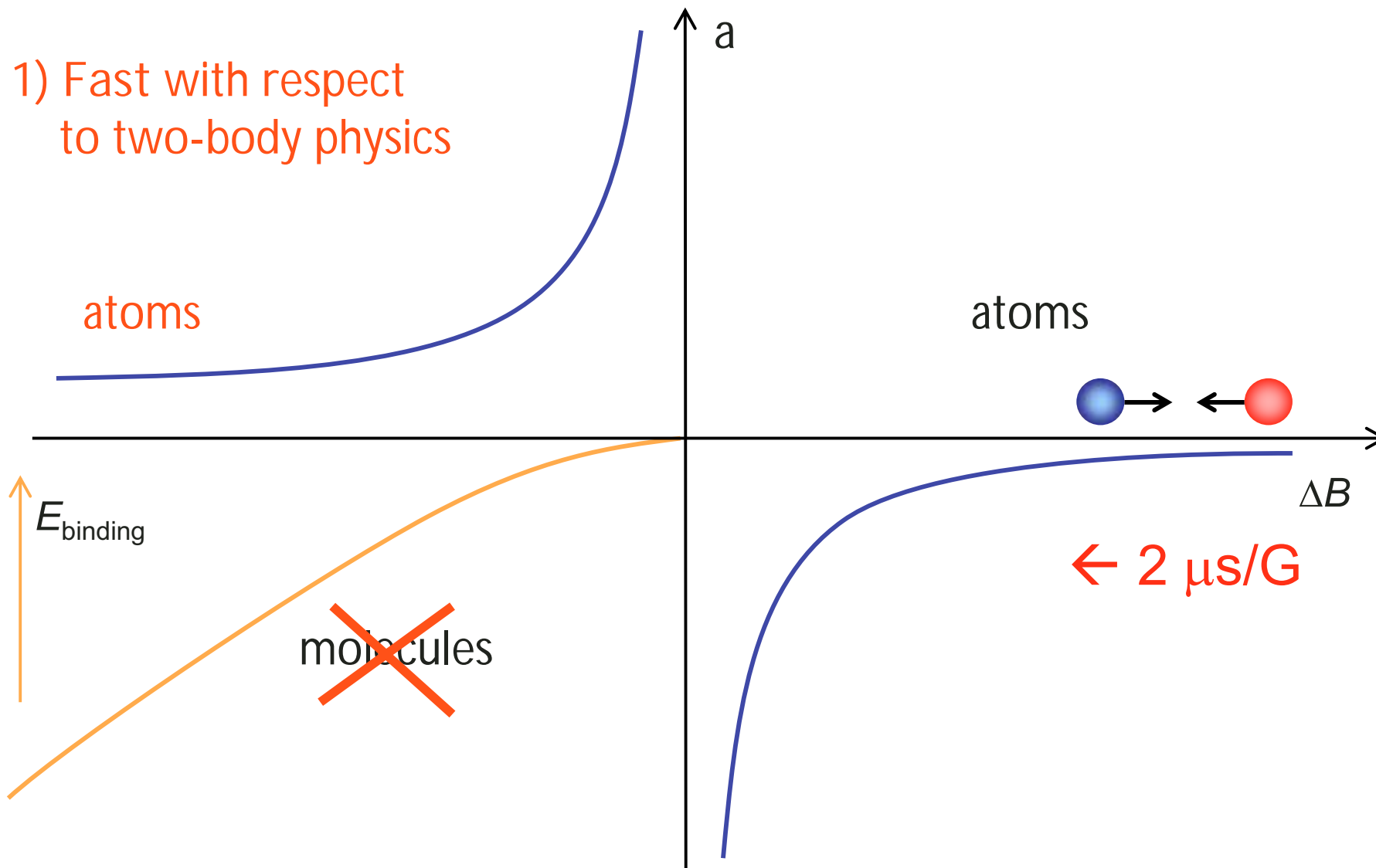
# Molecule properties

- extremely weakly bound
- large, molecule size  $\approx a$
- **but:** ridiculously stable close to Feshbach resonance



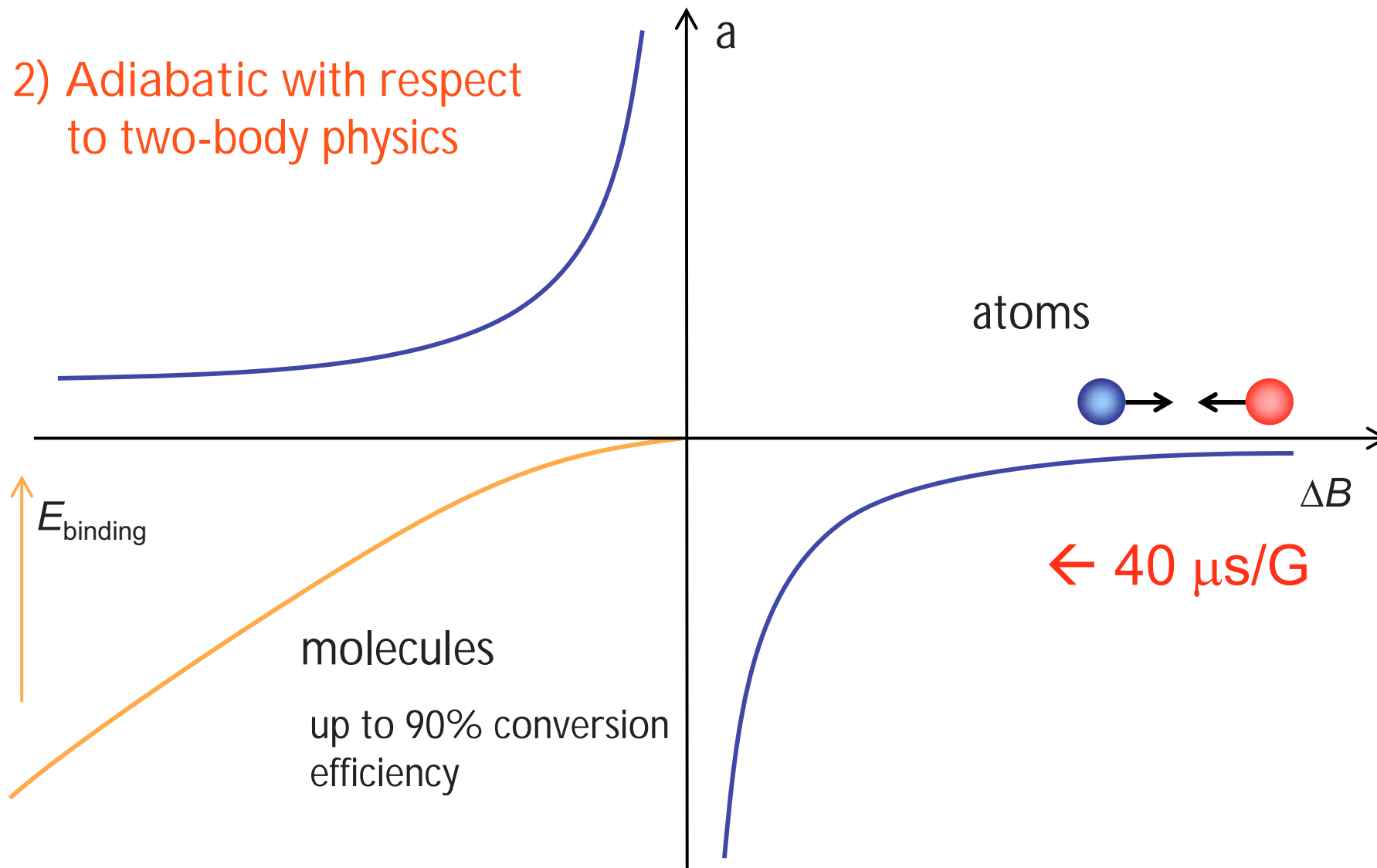
# Timescale of B-ramp

1) Fast with respect to two-body physics



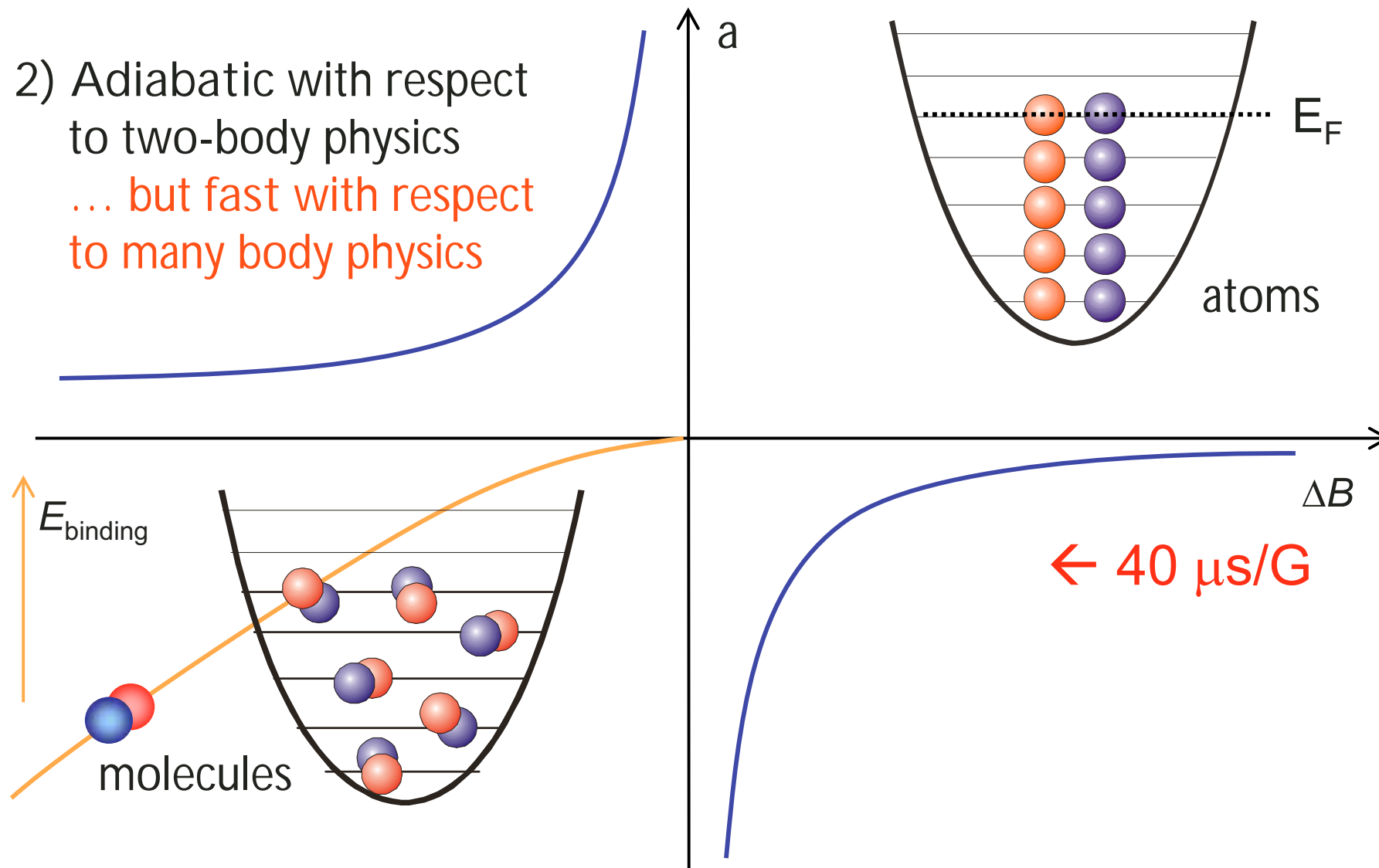
# Timescale of B-ramp

2) Adiabatic with respect to two-body physics



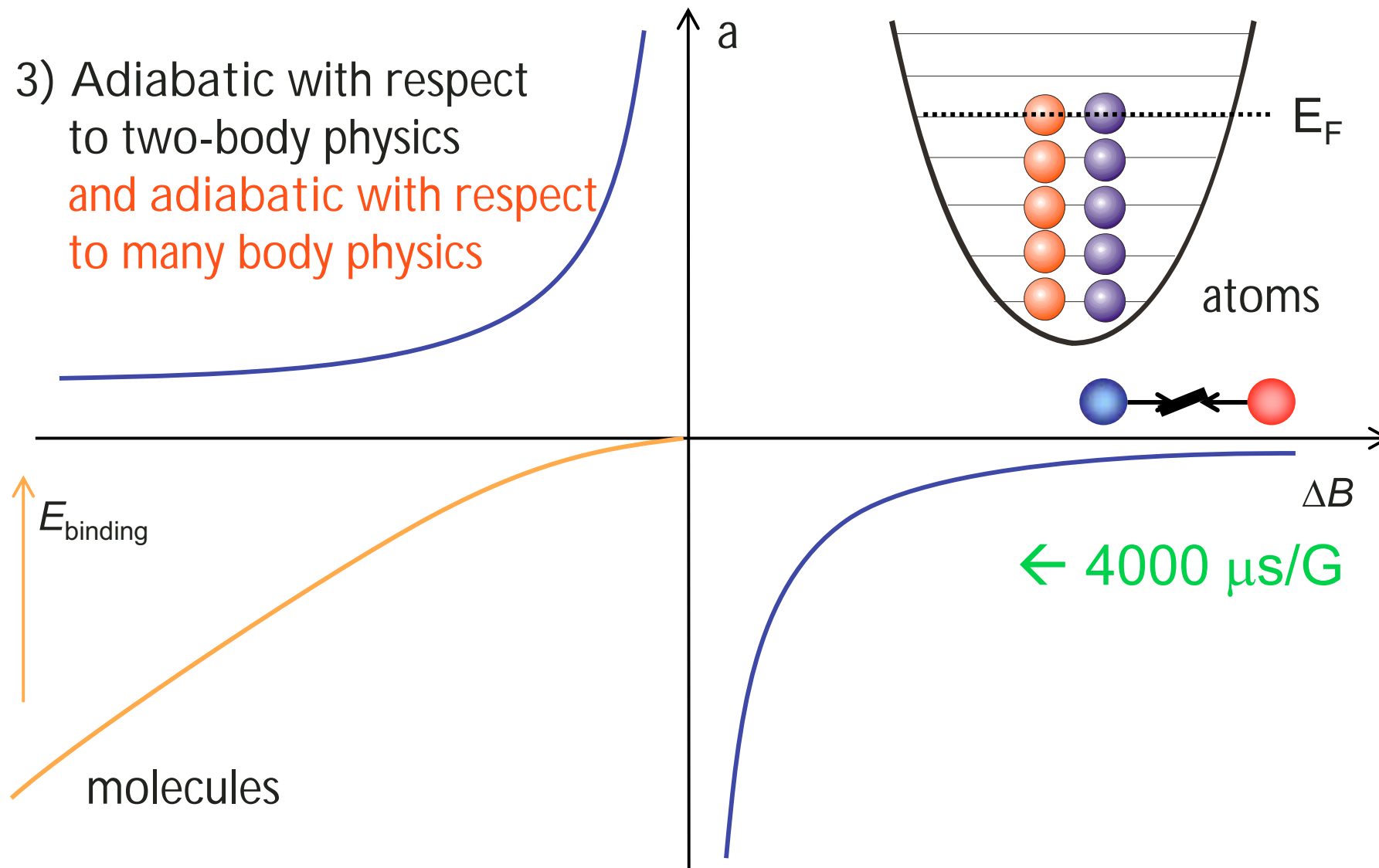
# Timescale of B-ramp

2) Adiabatic with respect to two-body physics  
... but fast with respect to many body physics



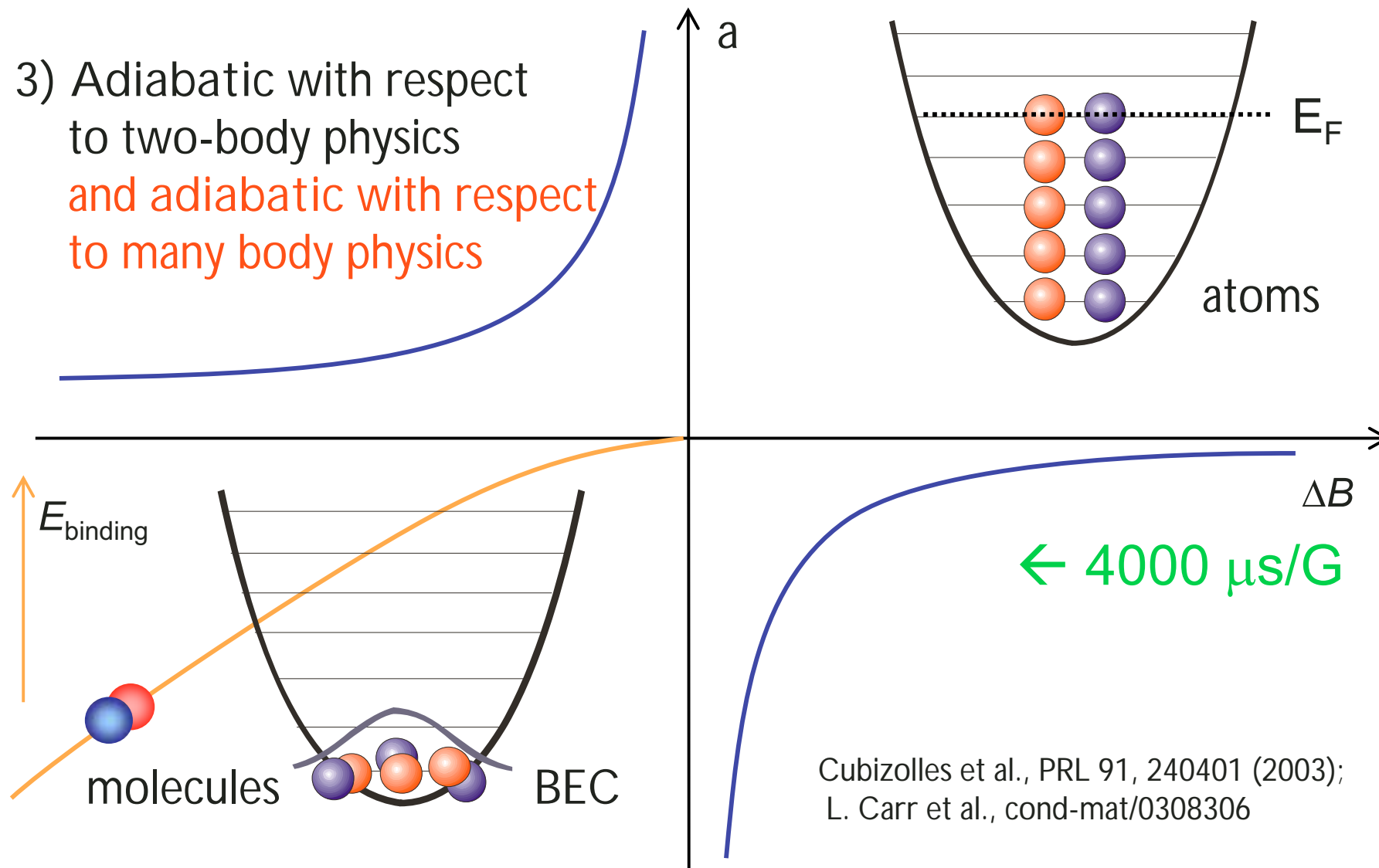
# Timescale of B-ramp

3) Adiabatic with respect to two-body physics  
and adiabatic with respect to many body physics



# Timescale of B-ramp

3) Adiabatic with respect to two-body physics  
and adiabatic with respect to many body physics



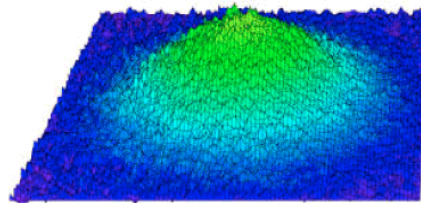
Cubizolles et al., PRL 91, 240401 (2003);  
L. Carr et al., cond-mat/0308306

# Molecular Bose-Einstein condensate

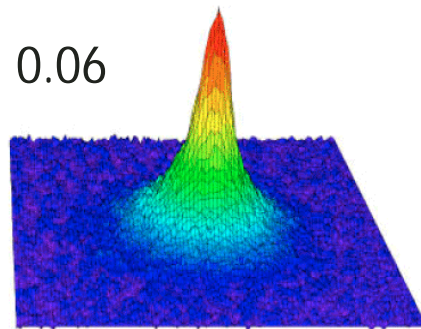
A molecular BEC  
emerges from a  
Fermi sea!

↑ a

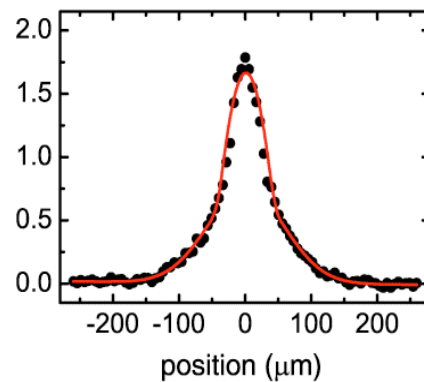
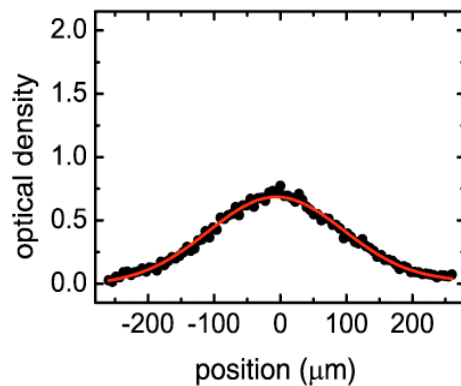
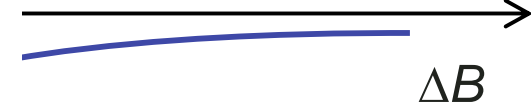
$T/T_F = 0.19$



0.06



Time of flight  
absorption image



profile

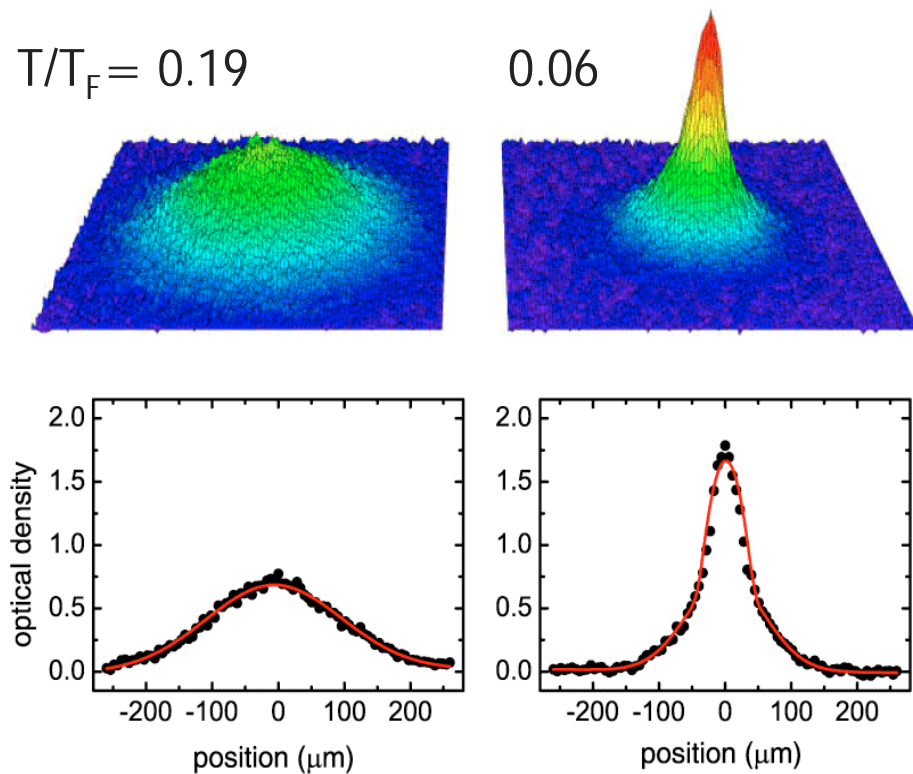
M. Greiner, C. A. Regal, and D. S. Jin,  
Nature 426, 537 (2003)

${}^6\text{Li}$ :  
Jochim et al., Science 302: 2101(2003),  
M. Zwierlein et al.,  
Phys. Rev. Lett. 91, 250401 (2003).  
T. Bourdel et al., cond-mat/0403091



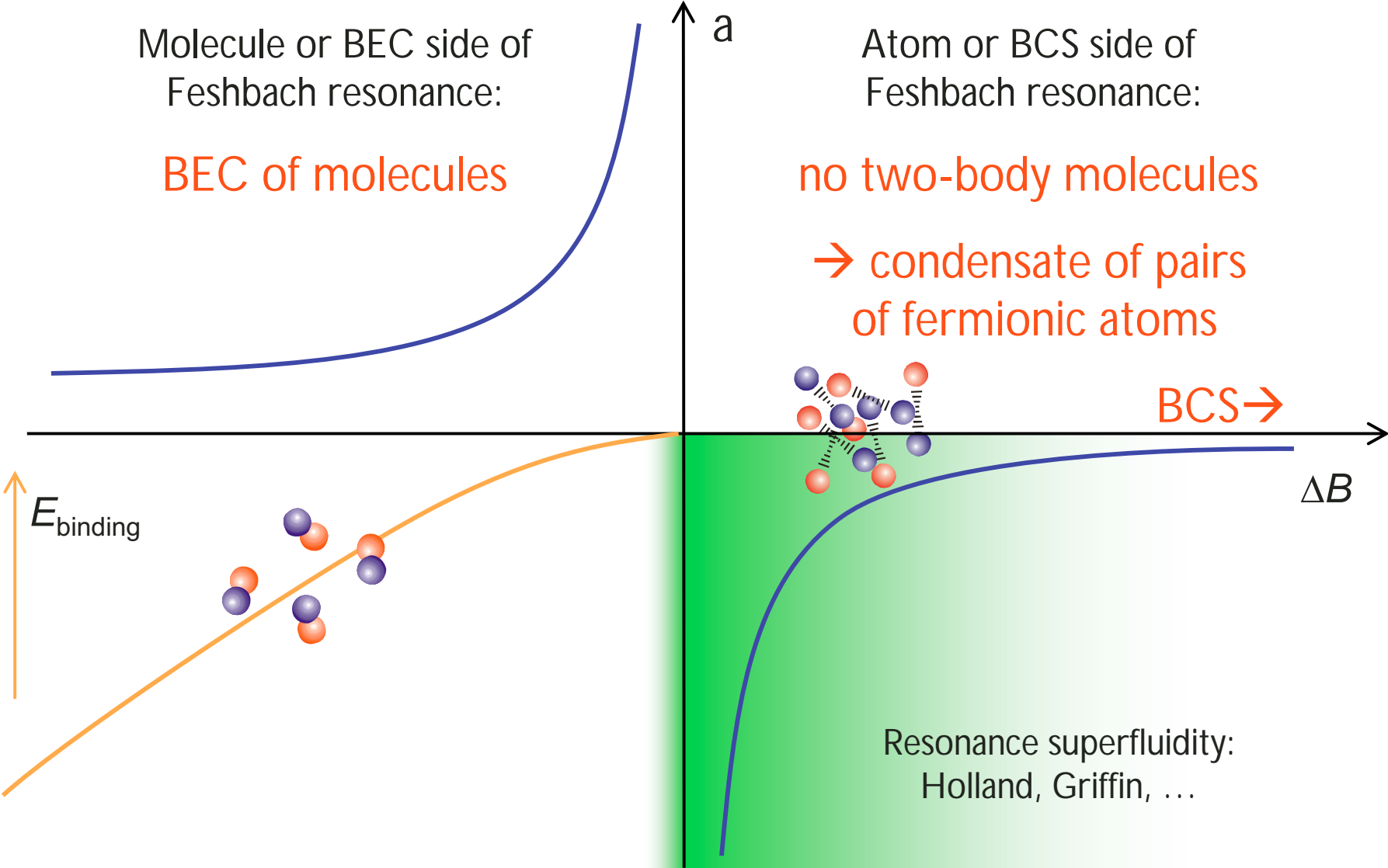
# Molecular Bose-Einstein condensate

A molecular BEC  
emerges from a  
Fermi sea!



→ timescale for many-body  
adiabaticity is 100x slower  
than for two-body adiabaticity

# Condensation of pairs of fermionic atoms



# Detecting a Fermi condensate

2) rapidly ramping across the Feshbach resonance to project atoms pair wise onto molecules

→ fast compared to many body physics

$E_{\text{binding}}$

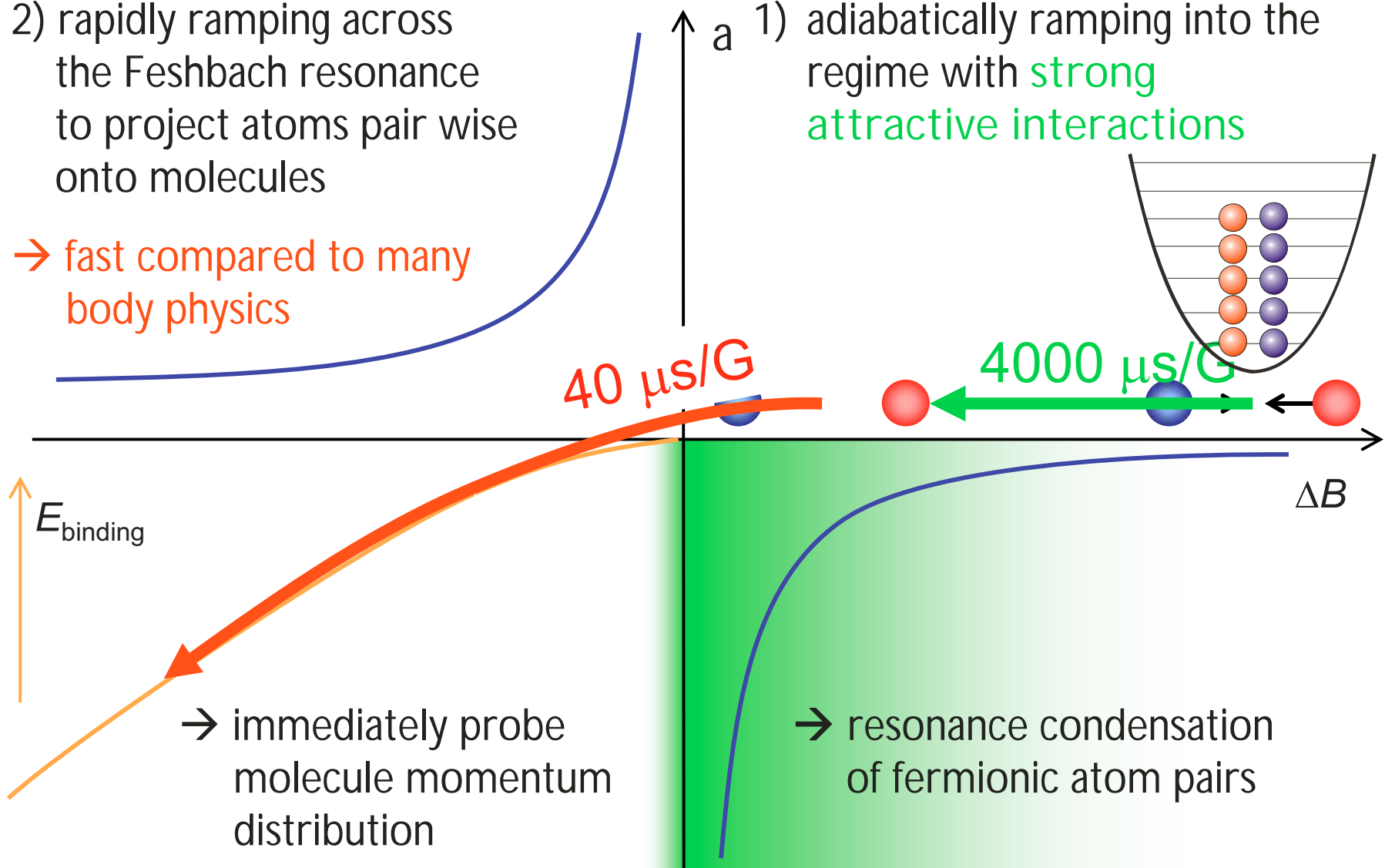
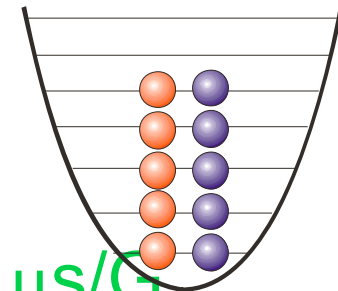
→ immediately probe molecule momentum distribution

$40 \mu\text{s/G}$

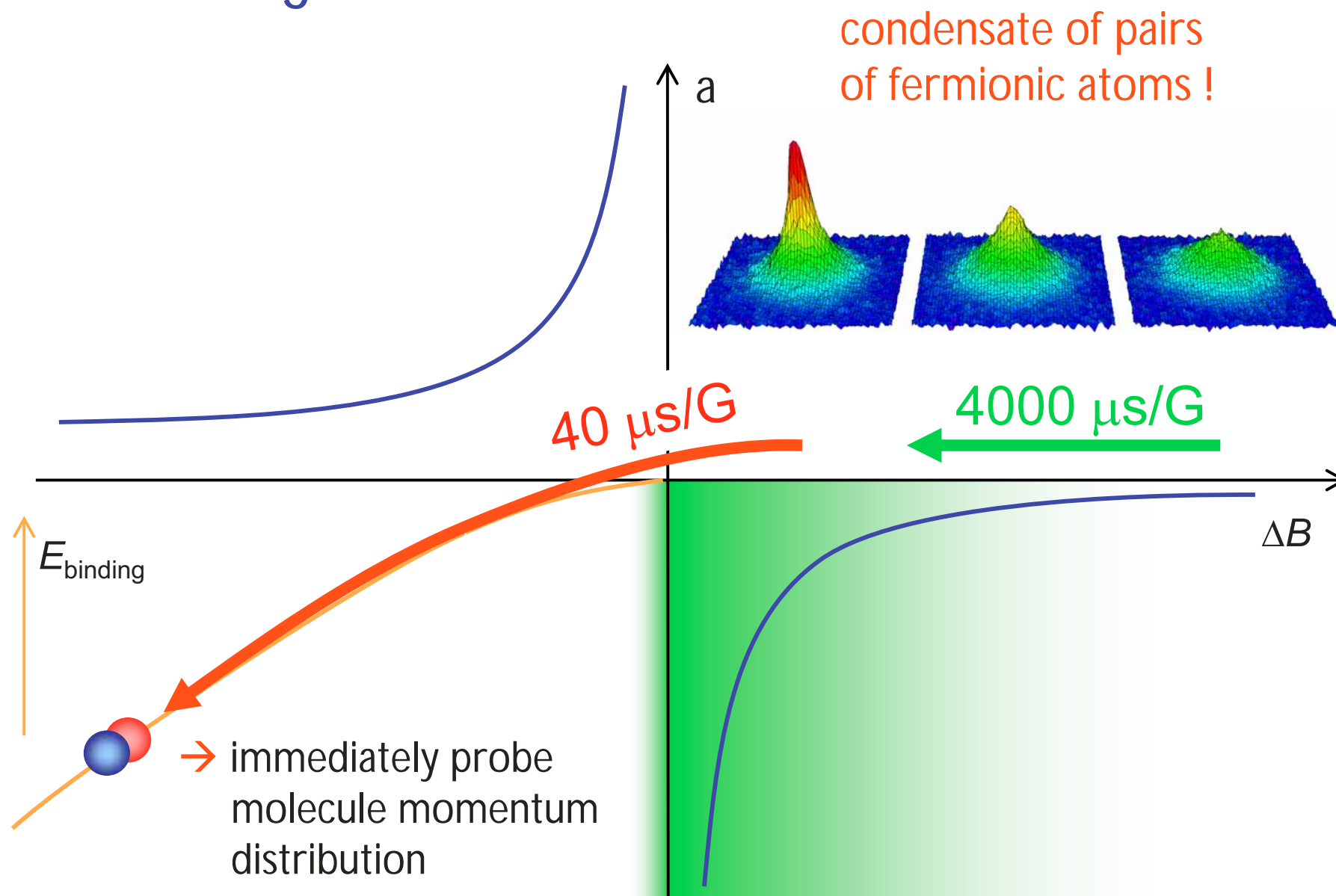
1) adiabatically ramping into the regime with strong attractive interactions

$4000 \mu\text{s/G}$

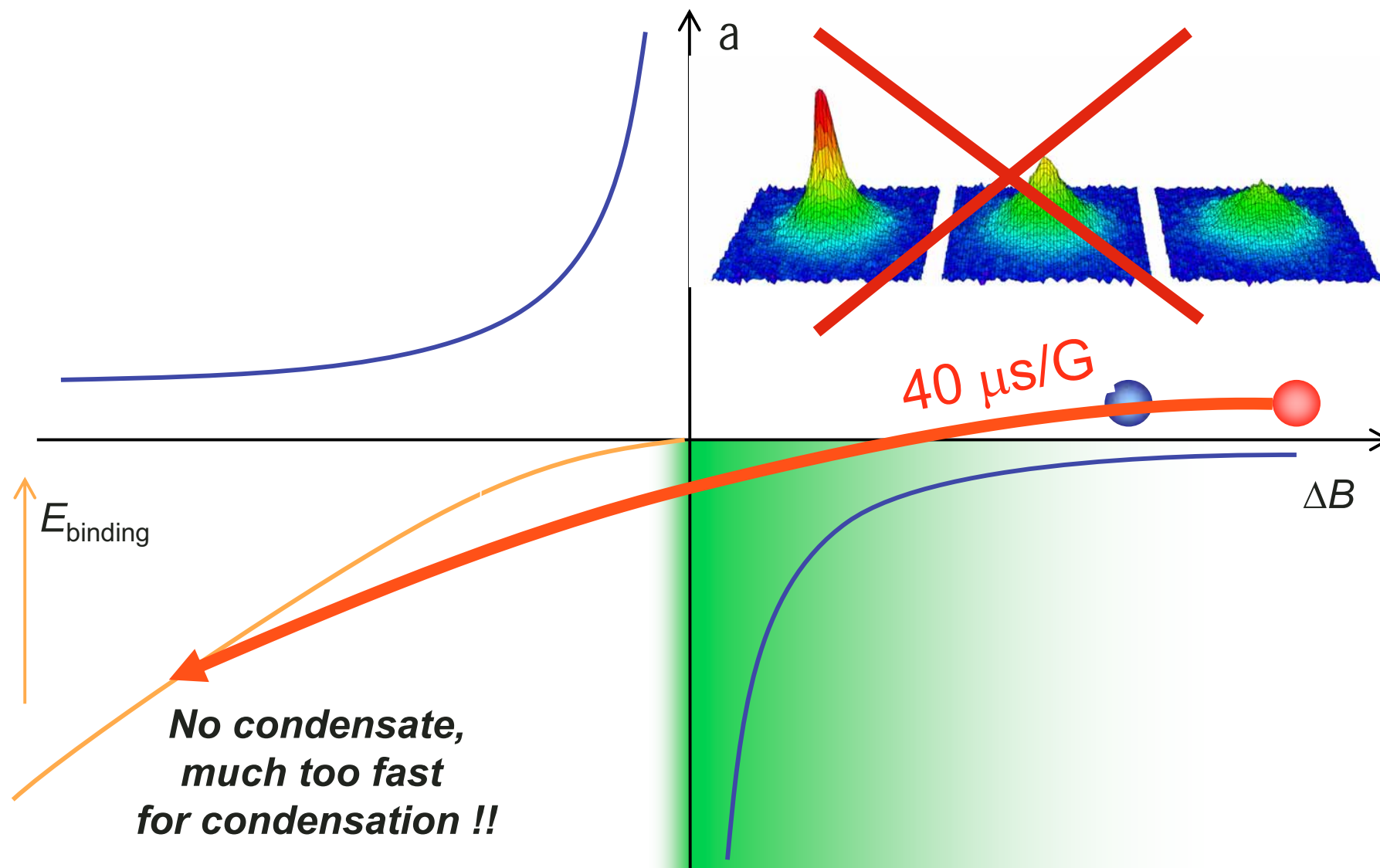
→ resonance condensation of fermionic atom pairs



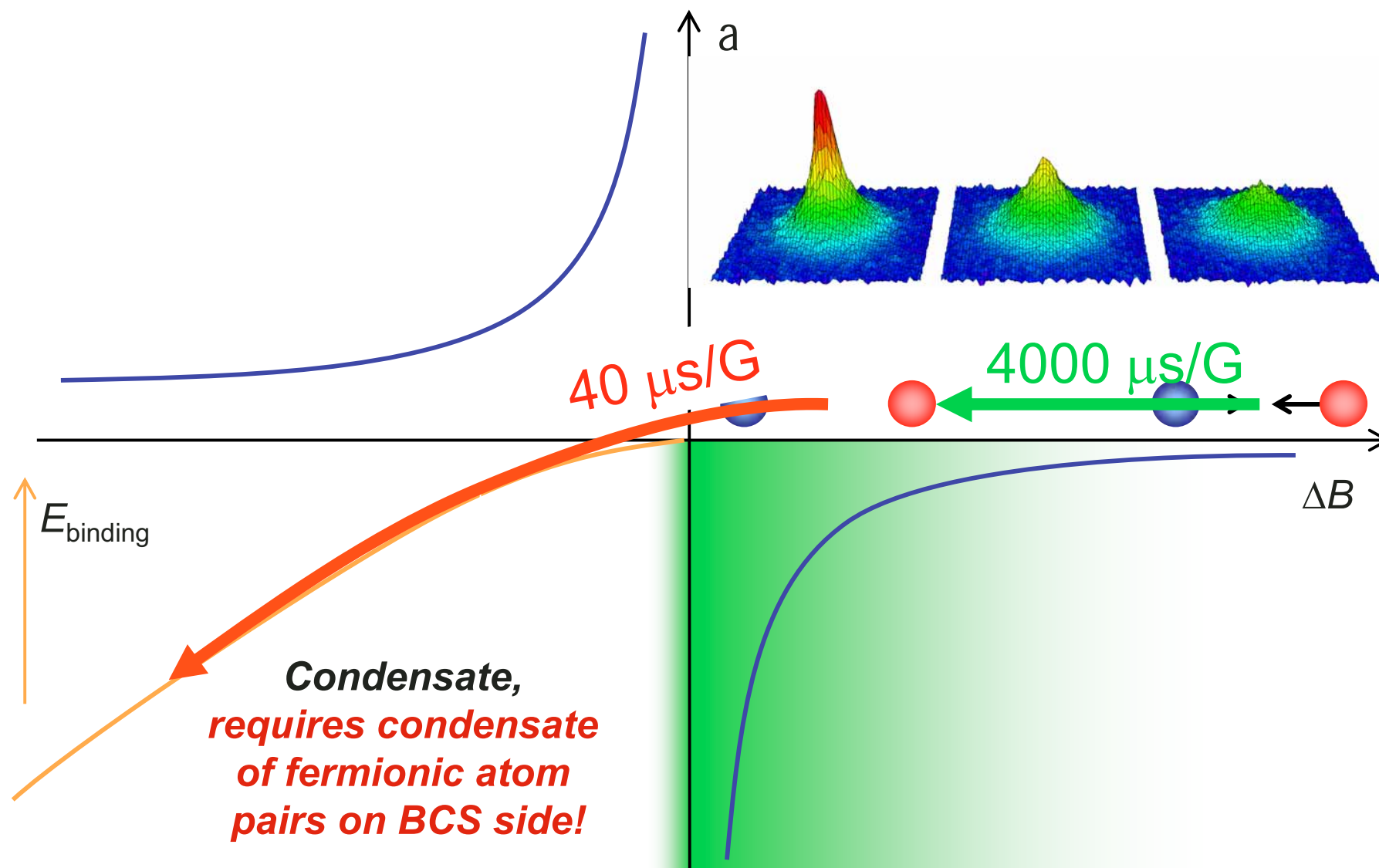
# Detecting a Fermi condensate



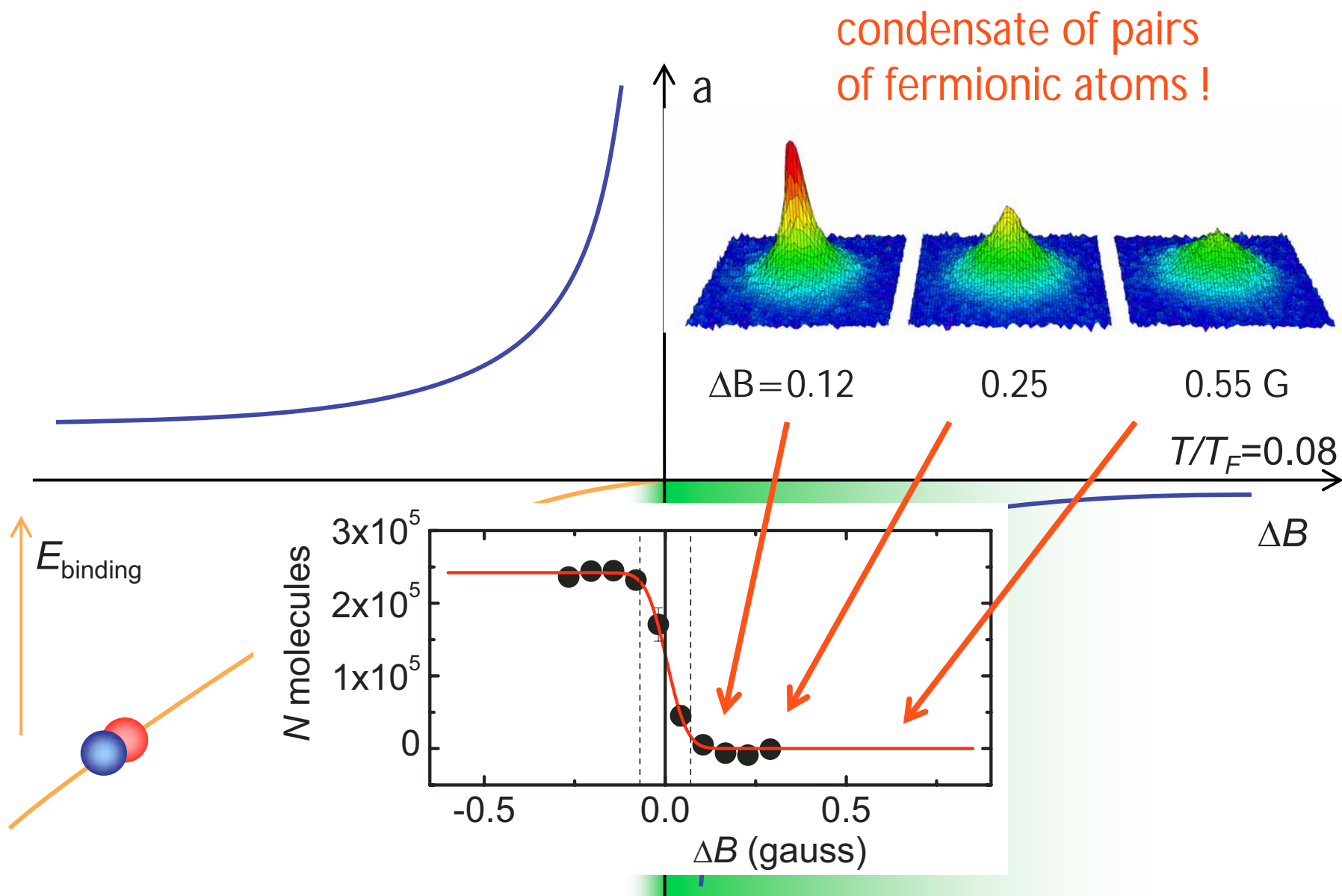
# Detecting a Fermi condensate



# Detecting a Fermi condensate

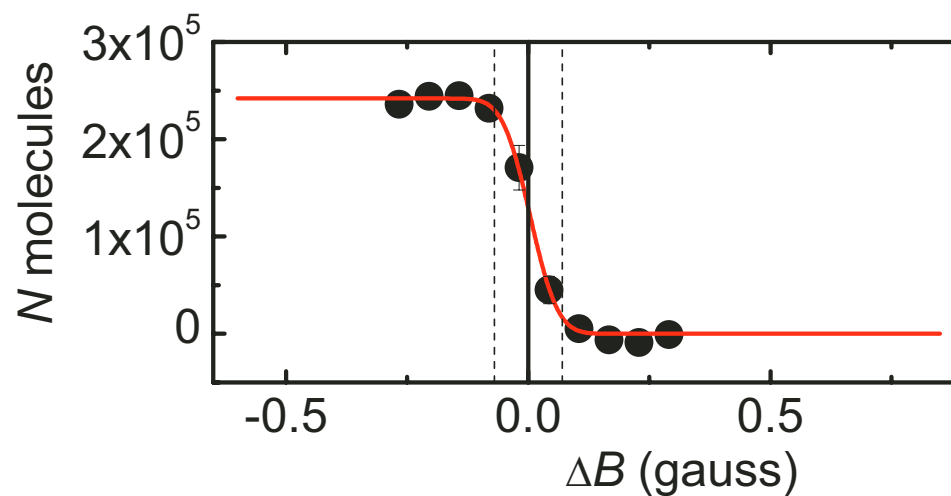
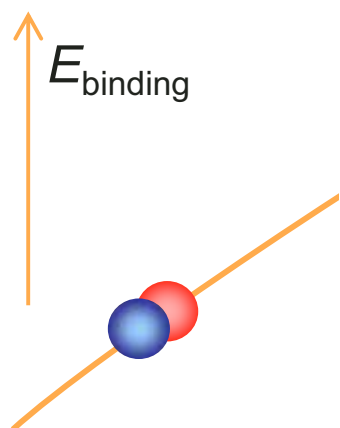
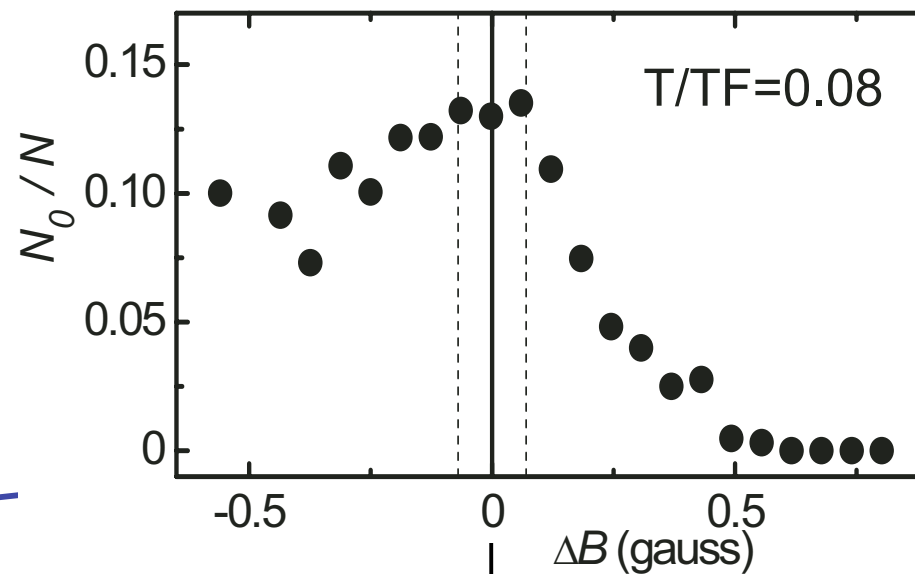


# Fermionic condensate



# Fermionic condensate

condensate fraction

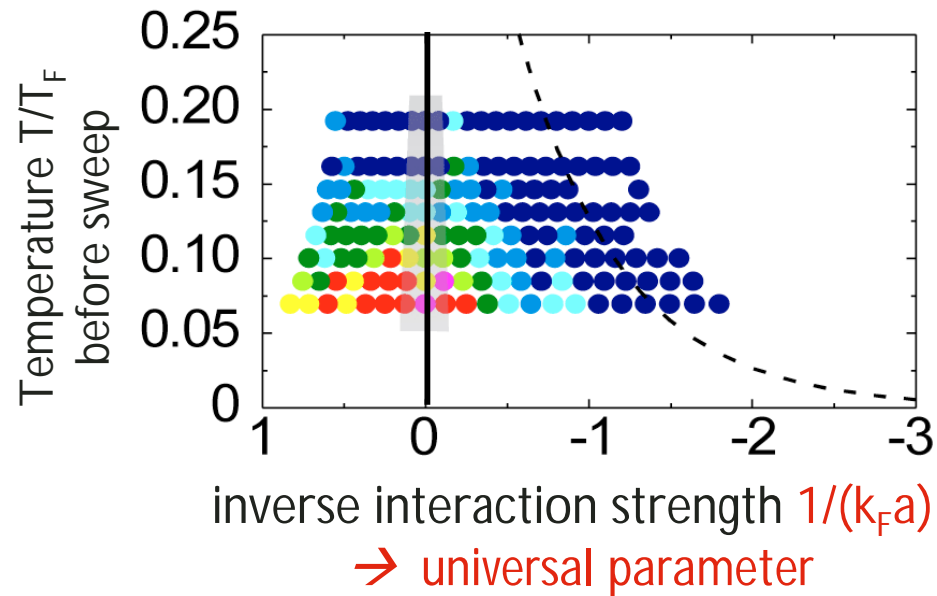


$\Delta B$



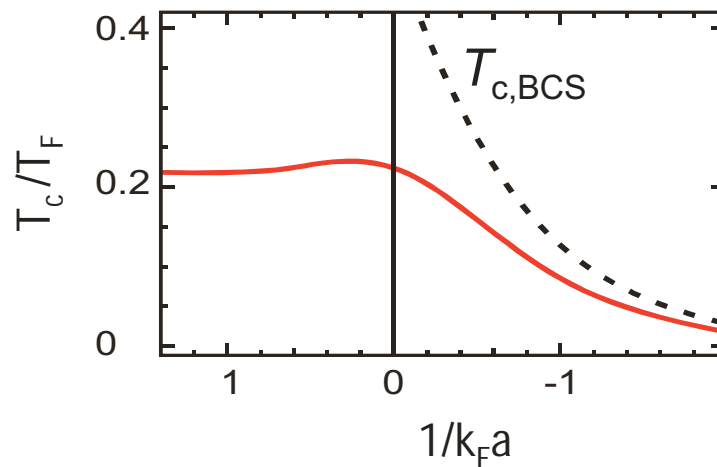
# BCS-BEC crossover

**BEC**  
of  
molecules



**BCS**  
of  
Cooper pairs

Figure:  
M. Randeria



$$T_{c,BCS} \approx e^{-\frac{\pi}{2k_F|a|}}$$

BEC-BCS crossover  
theory for example:  
Eagles, Leggett,  
Nozieres et al., Randeria,  
Holland et al., Timmermans  
et al., Ohashi et al.,  
Stajic et al. ...

# BCS-BEC crossover

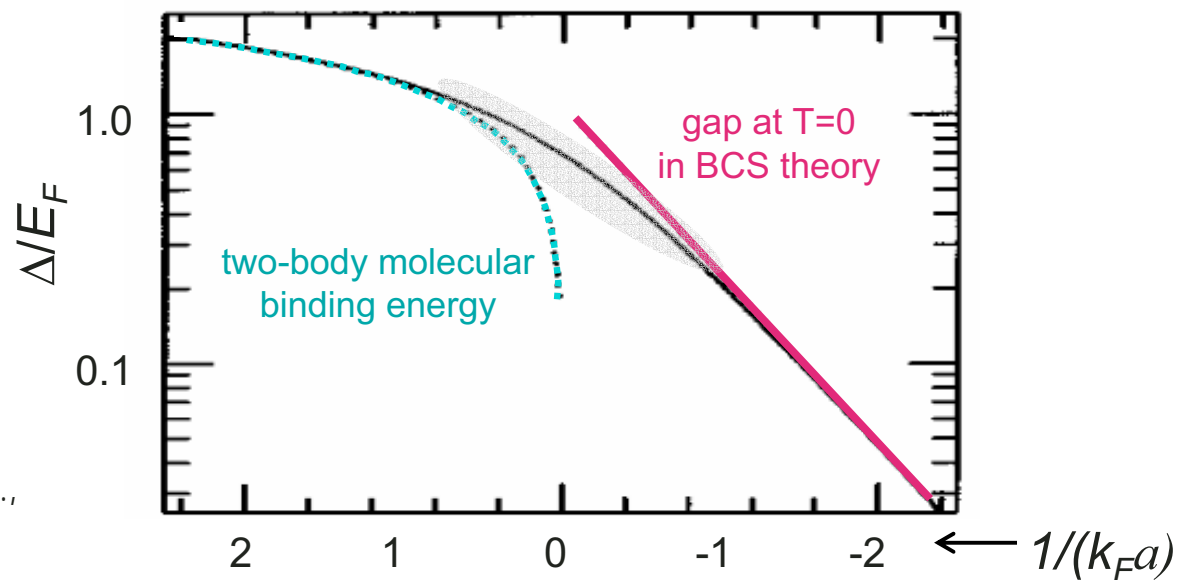
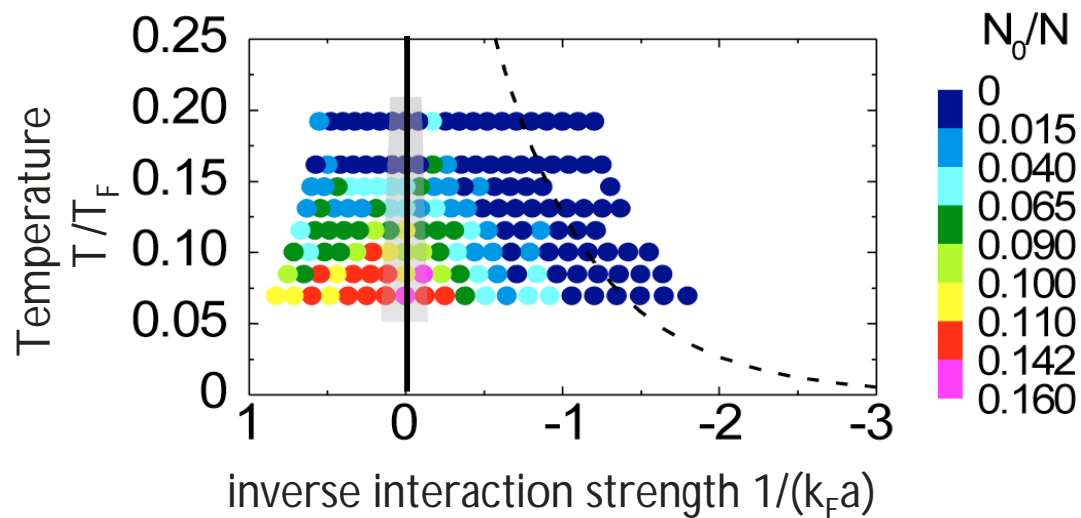
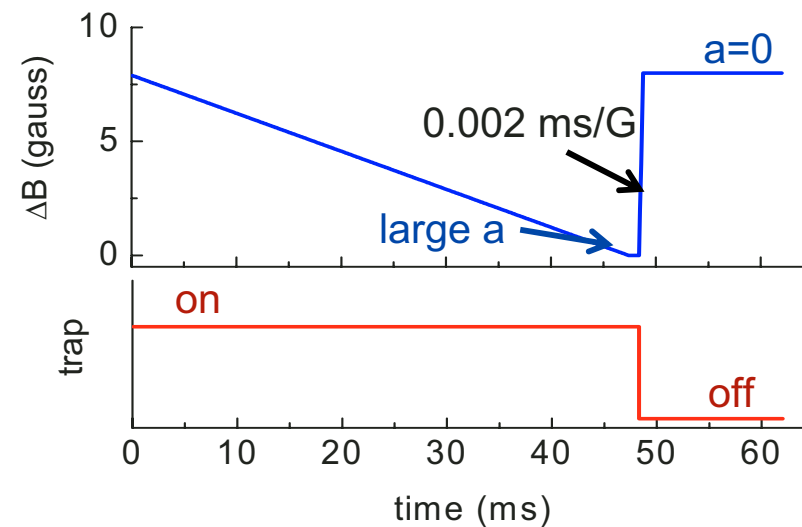
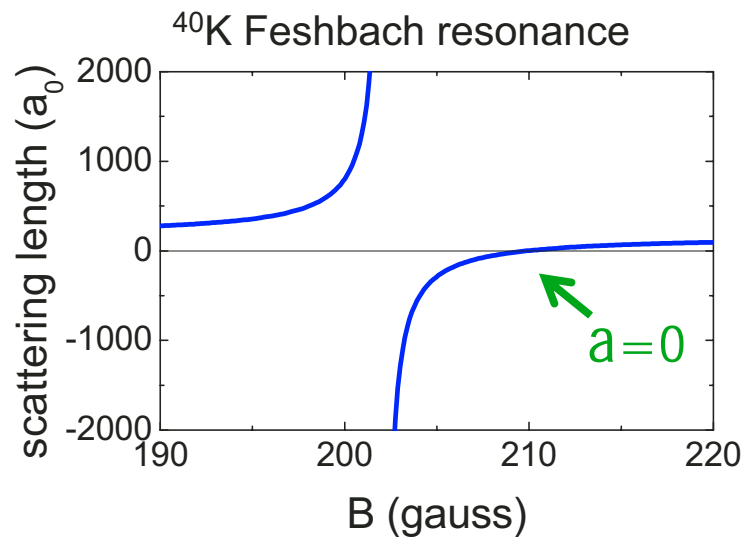


Figure: J. R. Engelbrecht et al.,  
PRB 55, 15153 (1997)



# Probing atom momentum distribution

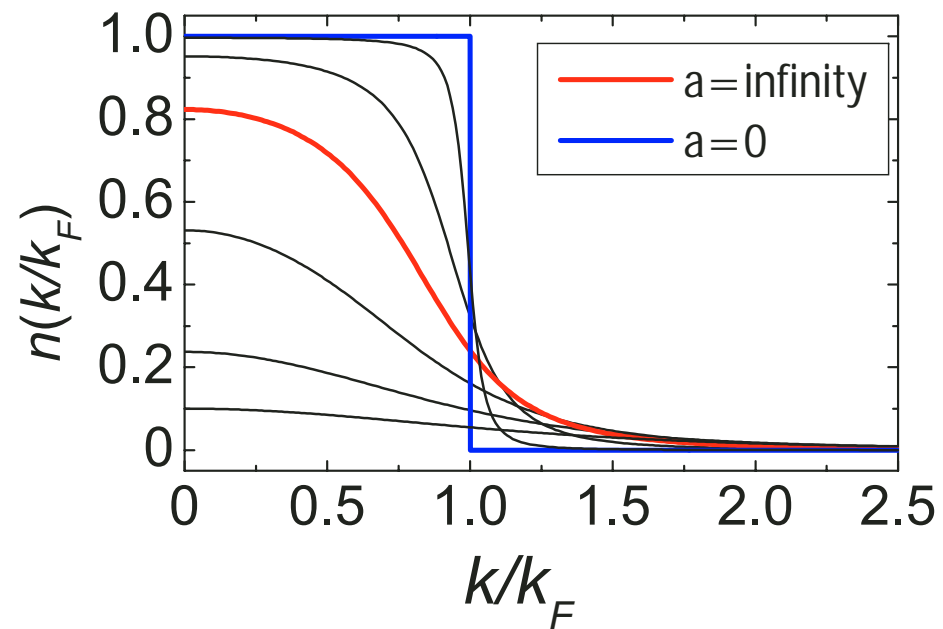
rapidly switching off interactions before TOF expansion:  
→ pairs dissociate, momentum distribution of fermions is measured



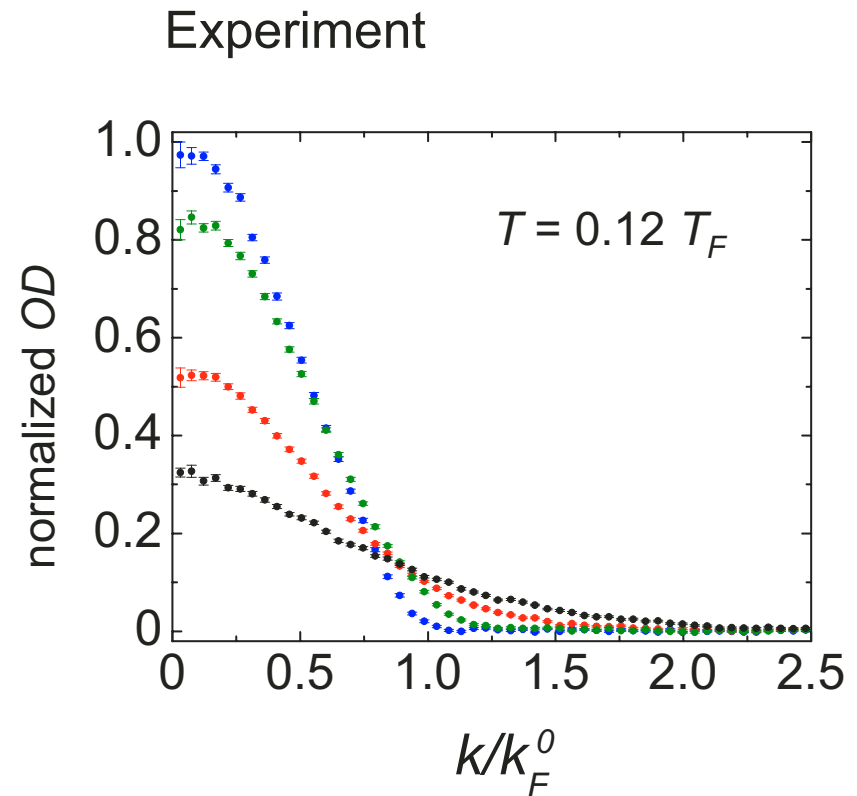
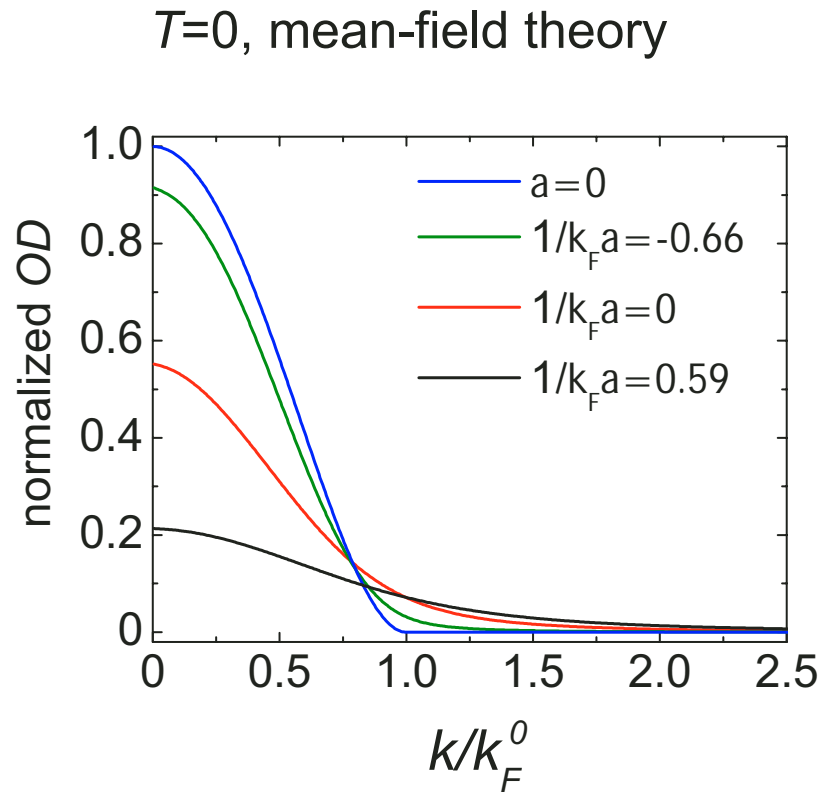
# BCS-BEC crossover theory

Homogeneous gas,  $T=0$ :

Momentum distribution broadens because of pairing

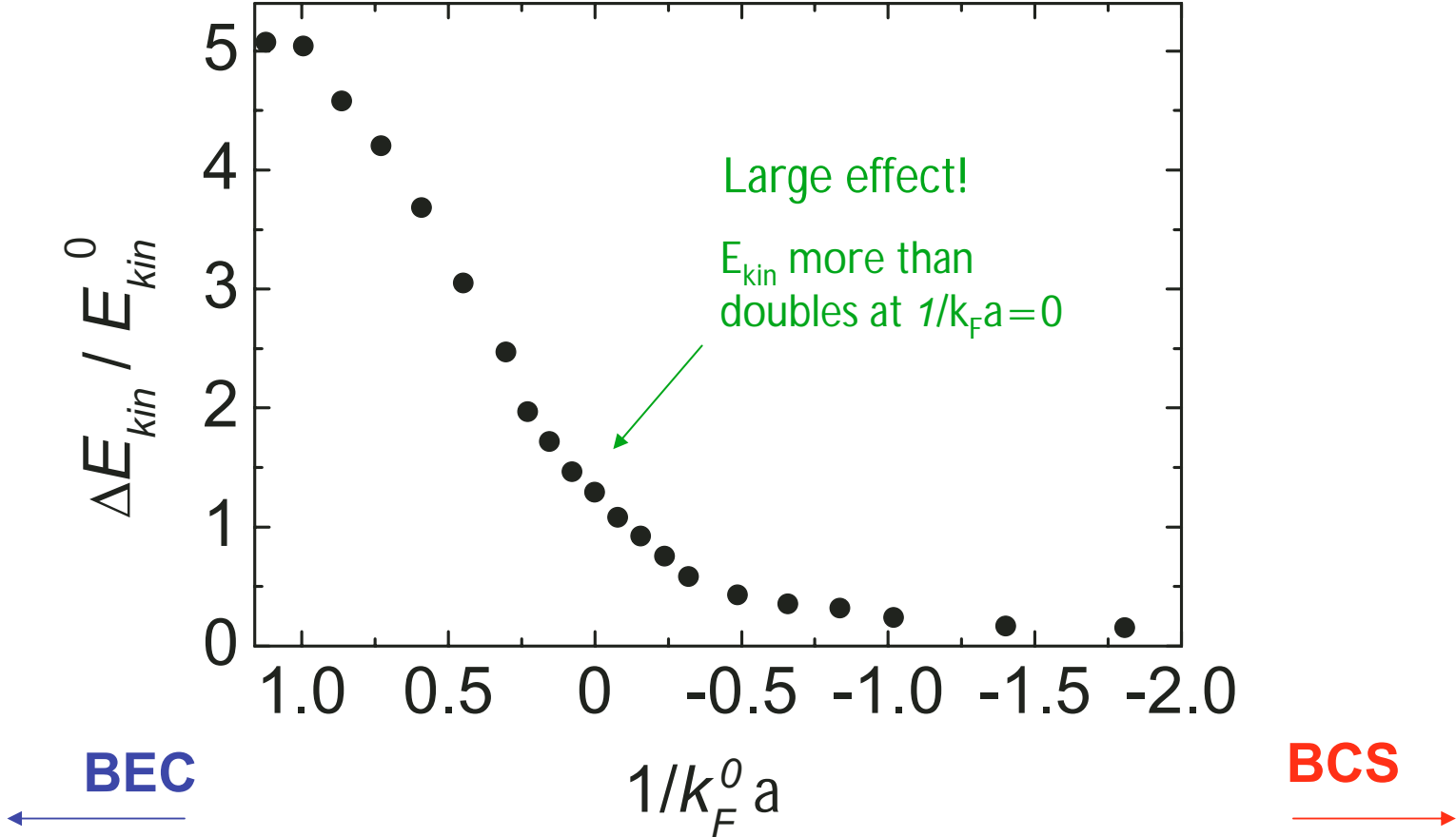


# Momentum distributions of trapped gas

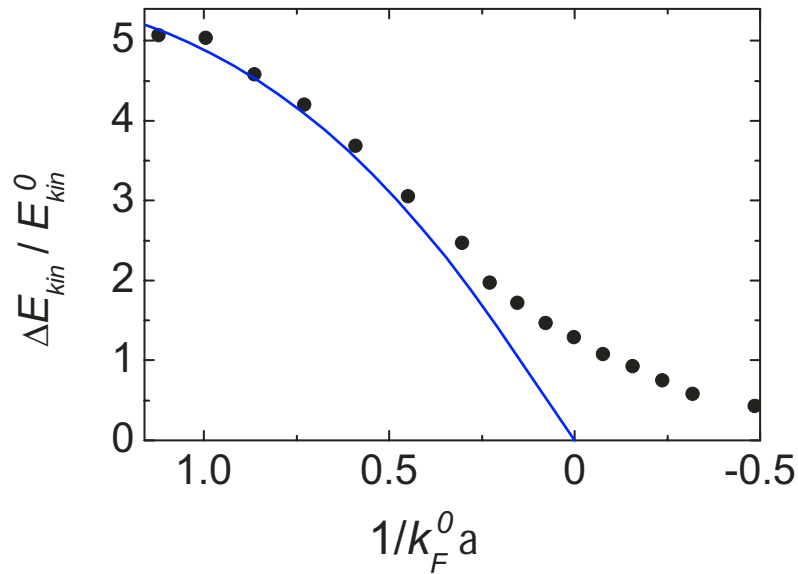


# Kinetic energy

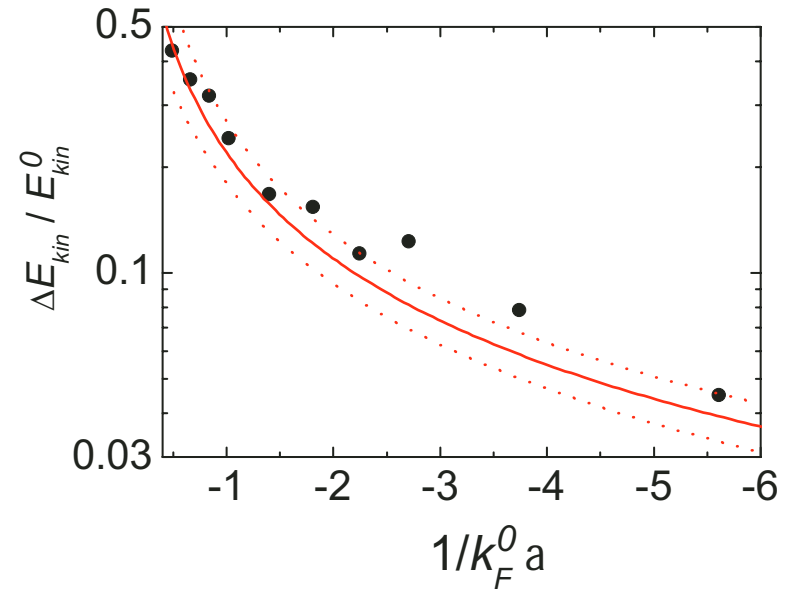
change in  $E_{kin}$  normalized to  $E_{kin}$  at  $a=0$



# Two limits: exact theories



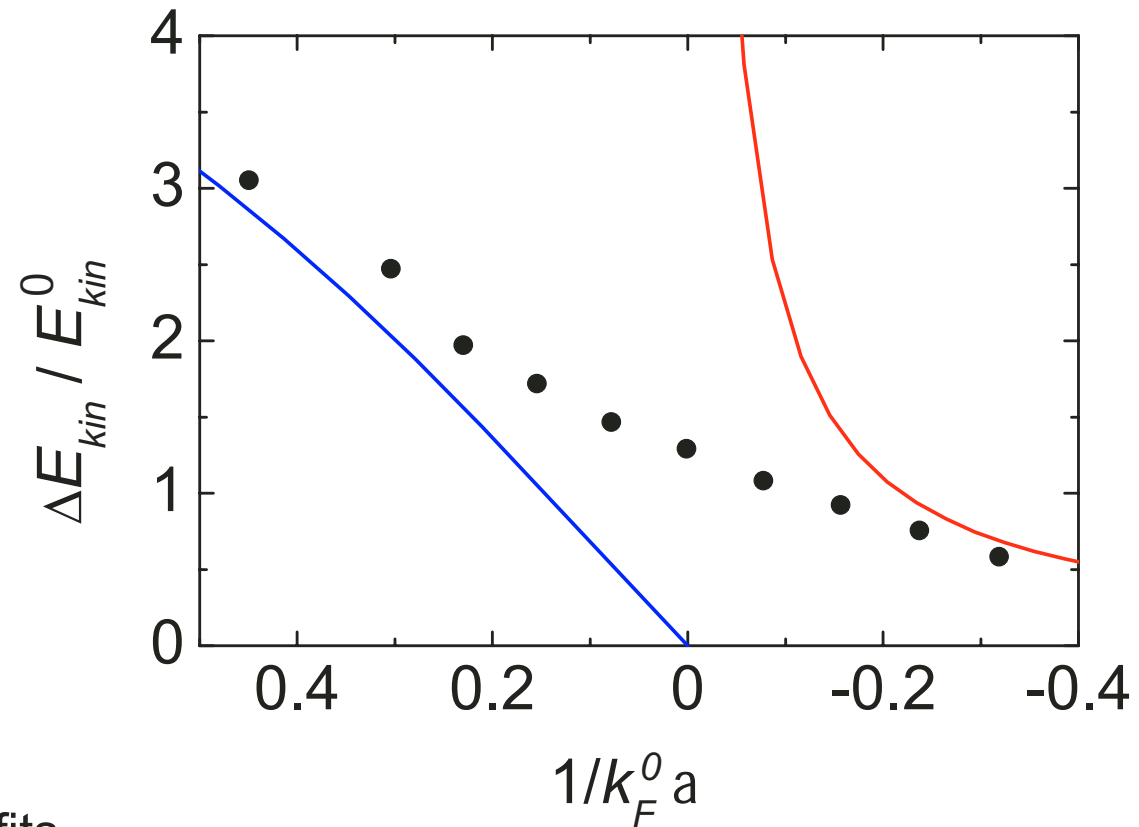
Time-evolve wavefunction  
of isolated molecule



Calculate change due to  
weak attractive interactions  
in the normal state

Theory: Murray Holland and Stefano Giorgini

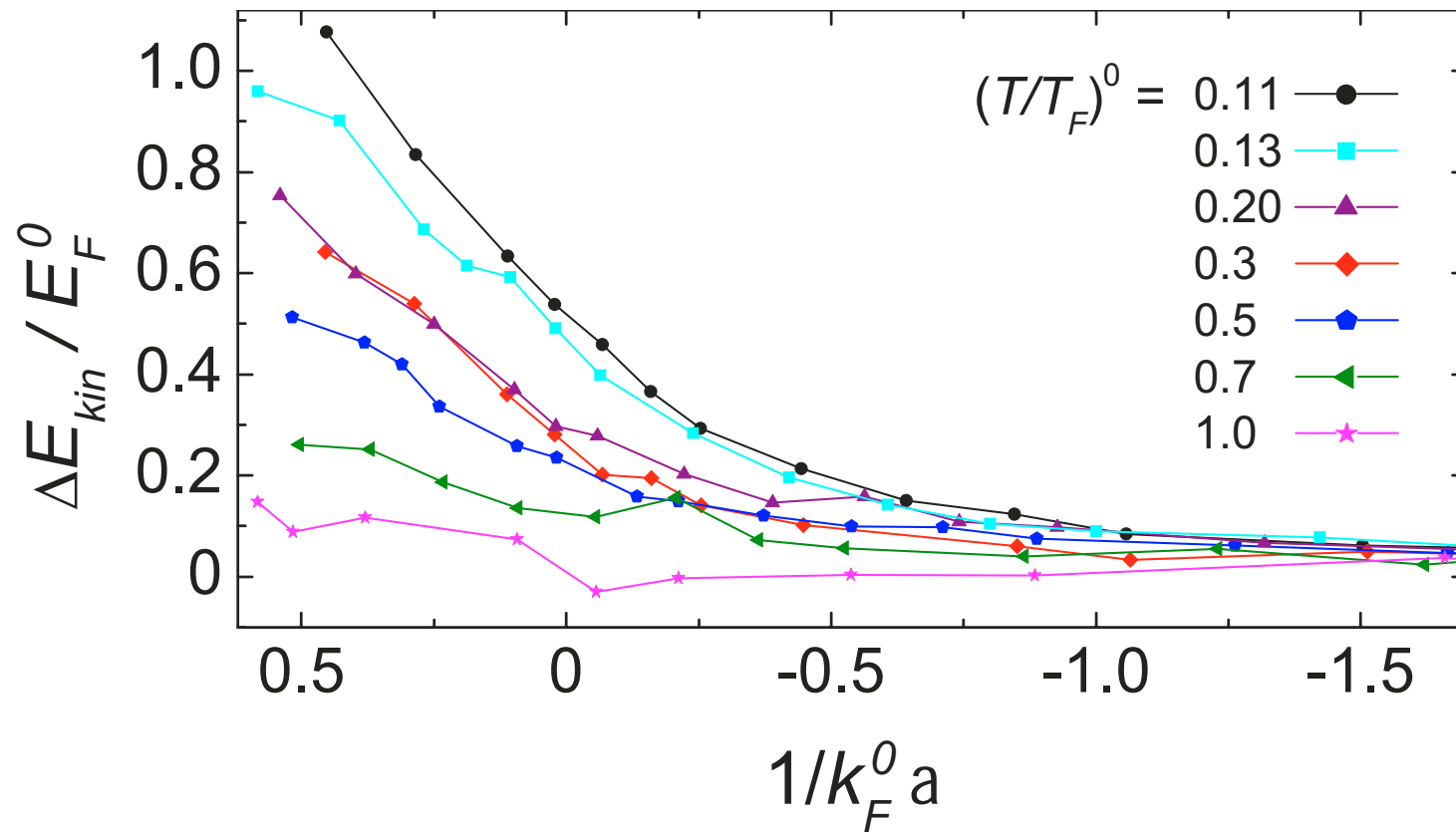
# In between



- Neither theory fits
- probe of pairing in crossover
- future goal: compare to full crossover theory



# Varying $T/T_F$



•  $T_F$  is roughly constant but  $n$  changes

# Thanks:

Deborah Jin  
Cindy Regal

Jayson Stewart

*... I am starting a research  
group this summer,*

***PhD students and  
postdocs welcome ...***



Markus Greiner  
Deborah Jin    Cindy Regal

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# *Correlations in atom shot noise*

Markus Greiner

JILA, Group of D. Jin;  
Coworkers: C. Regal and J. Stewart  
NIST and the University of Colorado,  
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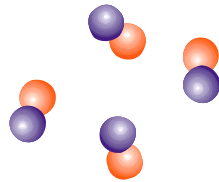


# Detecting atom-atom correlations

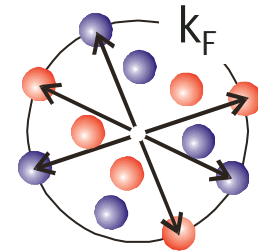
New experimental systems show interesting atom-atom correlations:

- atom pair correlations:

spatial:  
molecules

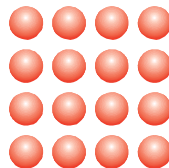


in momentum space:  
Cooper pairs

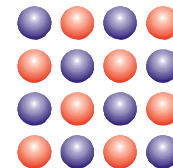


- in lattices:

Mott insulator:



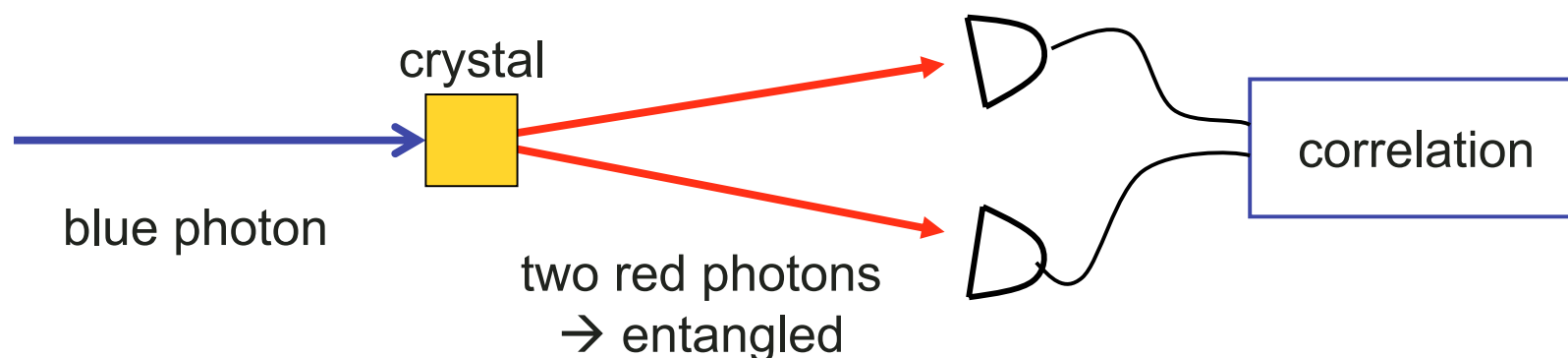
Anti-ferromagnetic  
phases:



- presently only the overall density distribution is measured in time-of-flight absorption imaging  
→ no information about atom-atom correlations

# Photon pair detection in quantum optics

*Parametric down conversion:*



practically all experiments detecting non-classical states of light are based on the detection of **photon-photon correlations**

... papers too good to be published ... ;-)

Proposal on the detection of correlations in atom shot noise  
by Ehud Altman *et al.*, PRA 70, 013603

### Probing many-body states of ultra-cold atoms via noise correlations

Ehud Altman, Eugene Demler, and Mikhail D. Lukin  
Physics Department, Harvard University, Cambridge, MA 02138  
(Dated: June 9, 2003)

We propose to utilize density-density correlations in the image of an expanding gas cloud to probe complex many body states of trapped ultra-cold atoms. In particular we show how this technique can be used to detect superfluidity of fermionic gases and reveal broken spin symmetries in Mott states of atoms in optical lattices. The feasibility of the method is investigated by analysis of the relevant signal to noise ratio including experimental imperfections.

Much of the excitement in the field of Bose-Einstein condensation owes to the clear demonstration it provides of the wave character of matter. The condensed state of bosons involves macroscopic occupation of a delocalized single particle state. Consequently, it is characterized by sharp density peaks in the freely expanding gas cloud after it is released from the trap[1]. Patterns that appear when two or more superfluid clouds interfere[2], are a direct probe of the single particle coherence, amplified by macroscopic occupation.

Recent experiments open intriguing directions for studying many body phenomena beyond single particle coherence. For example, observation of the superfluid to Mott insulator transition [3], as well as experiments in-

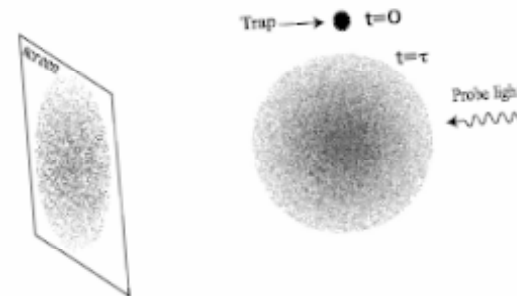
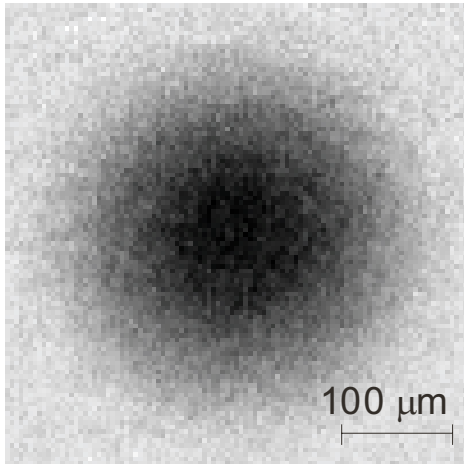


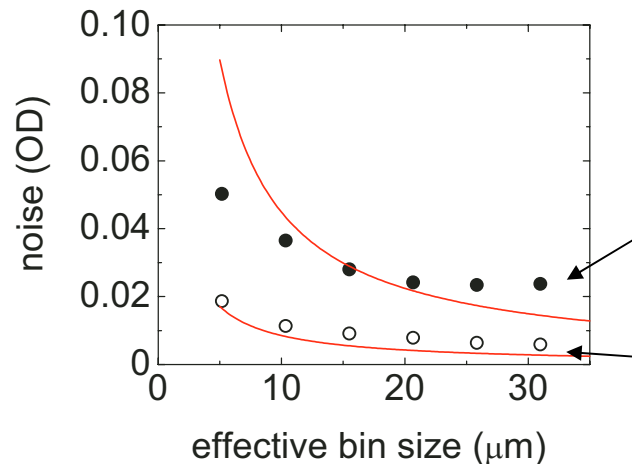
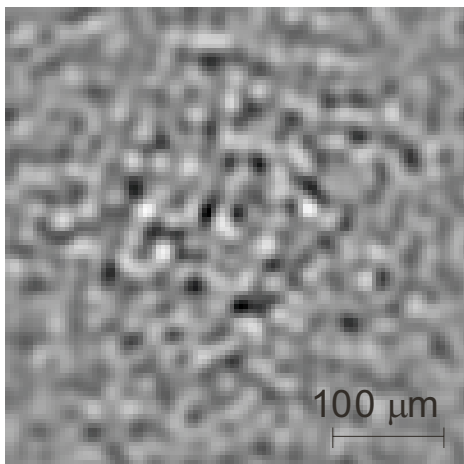
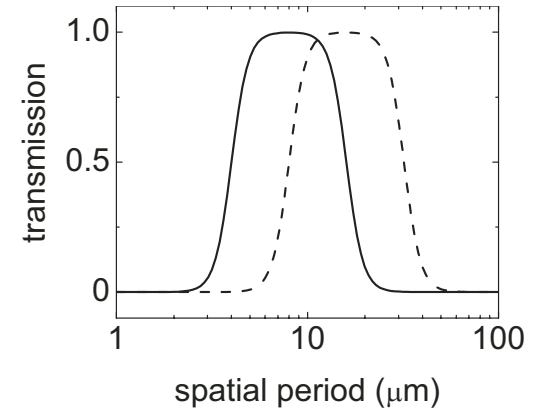
FIG. 1: Time of flight imaging. The atoms are released from the confining potential at time  $t = 0$  and the density of the expanding cloud is imaged at a later time. Spatial noise correlations in this image can be used to probe the quantum state at  $t = 0$ .

# Atom shot-noise limited imaging

→ proposed by E. Altman, E. Demler, and M.D.Lukin, PRA 70, 013603 (2004)



- TOF absorption image
- take fit residual
- spatial filter to “bin” picture on variable length scale



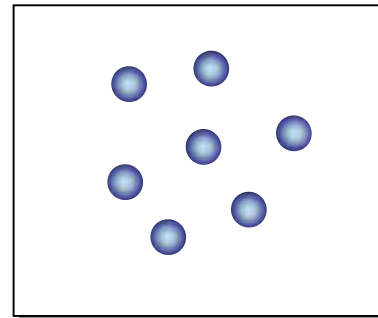
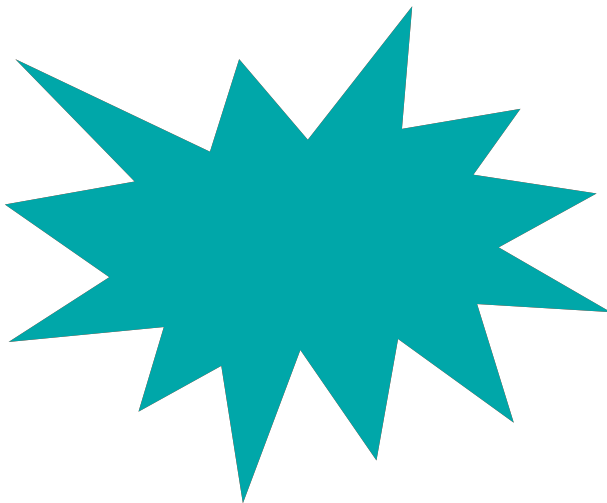
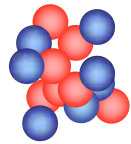
**Atom shot-noise limited image:**

measured noise at OD=1, Poisson noise

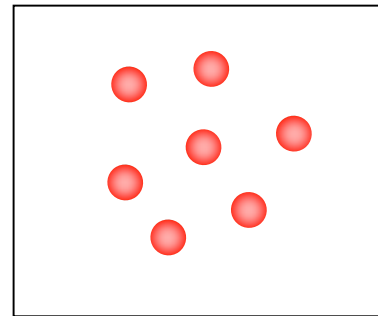
background noise, expected photon SN

# Spatial shot-noise correlations

- TOF absorption image in two spin states after molecule dissociation



$$m_f = -9/2$$

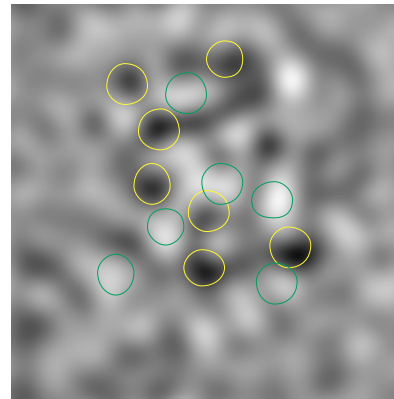
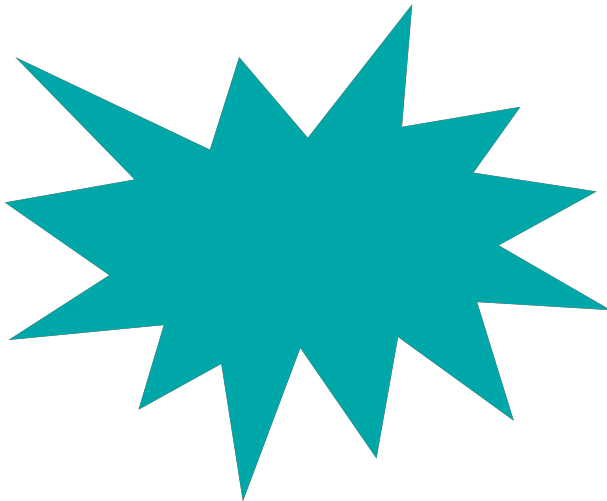
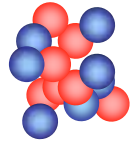


$$m_f = -7/2$$

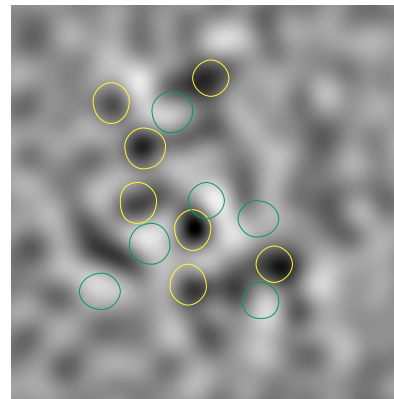


# Spatial shot-noise correlations

- TOF absorption image in two spin states after molecule dissociation



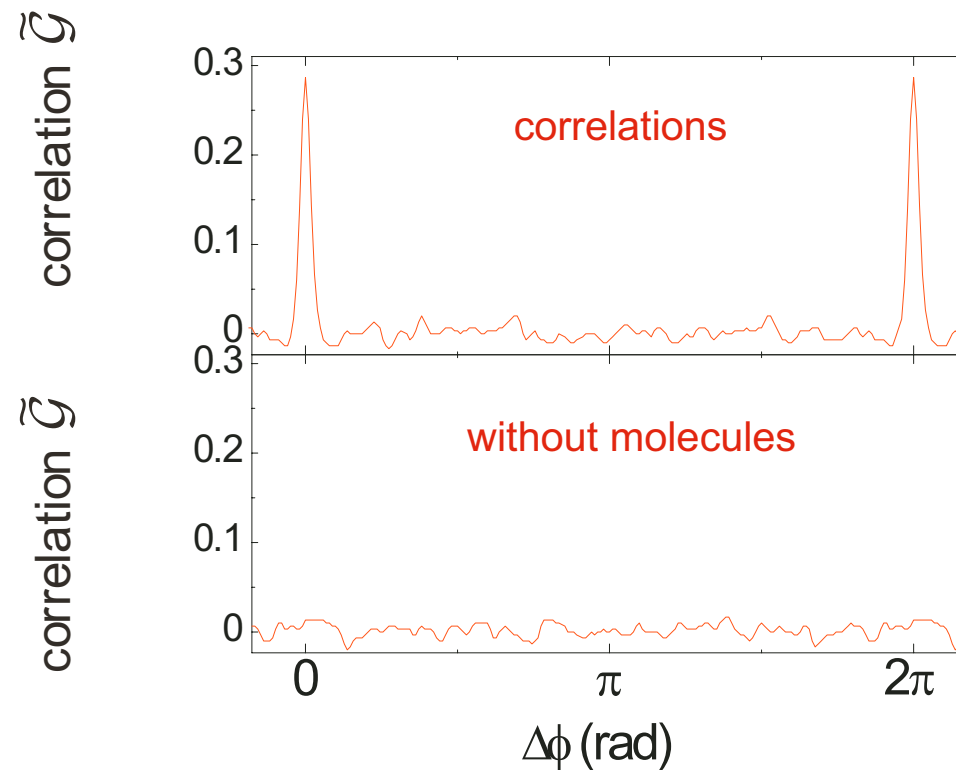
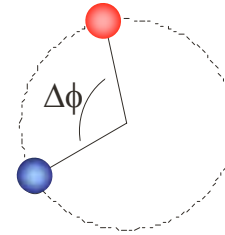
$$m_f = -9/2$$



$$m_f = -7/2$$

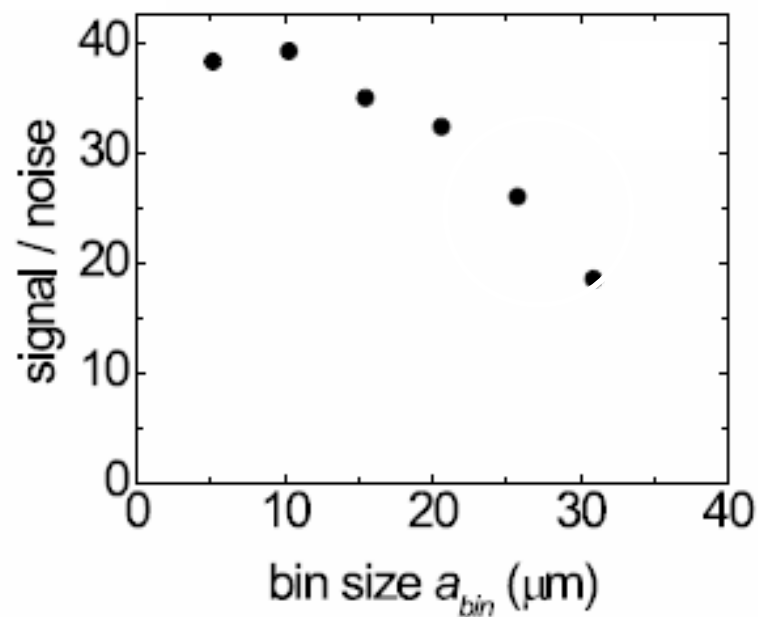
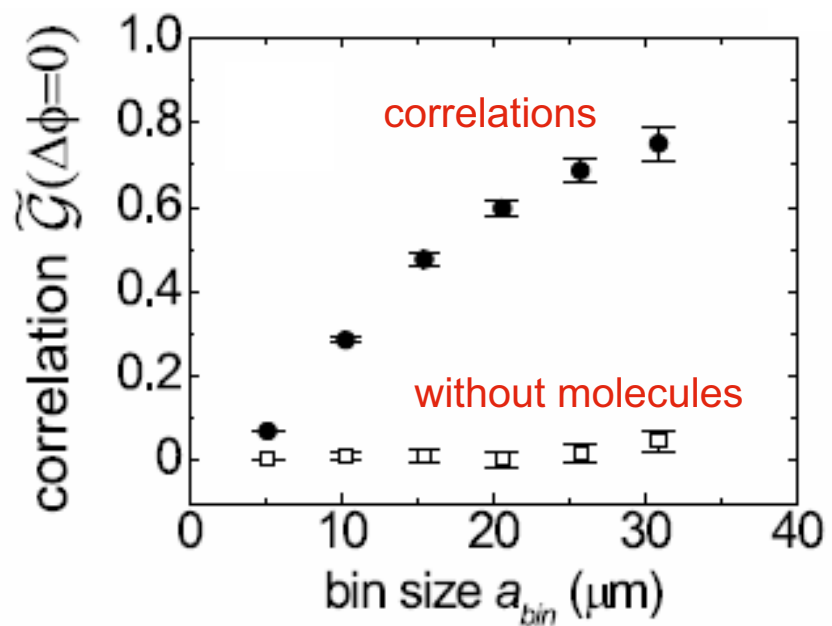
# Finding shot-noise correlations

$$\tilde{\mathcal{G}}_{(-7/2,-9/2)}(\Delta\phi) = \left\langle \frac{\langle \delta N_{-7/2}(r, \phi) \cdot \delta N_{-9/2}(r, \phi + \Delta\phi) \rangle_{\phi}}{\sqrt{\bar{N}_{-7/2}(r) \bar{N}_{-9/2}(r)}} \right\rangle_r$$



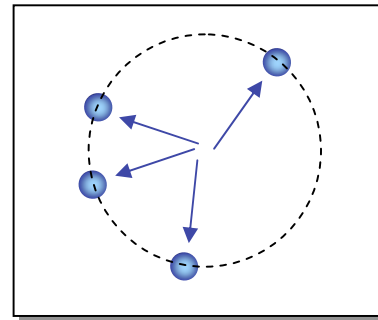
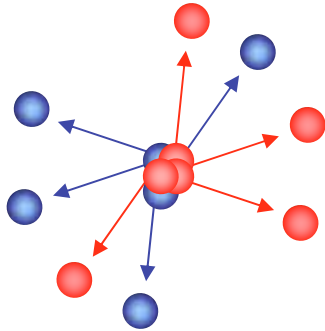
# Finding shot-noise correlations

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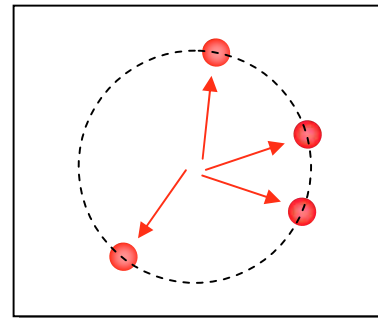
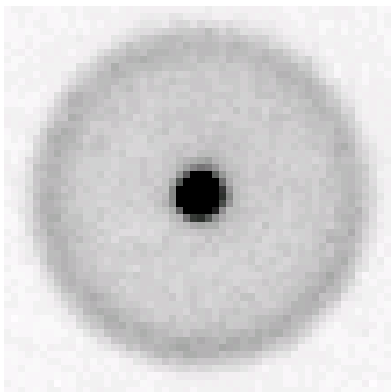


# Shot-noise correlations in momentum space

- Correlations of atoms with equal momentum in opposite directions:

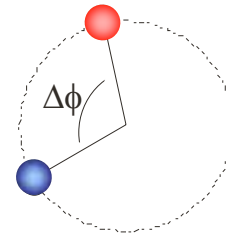
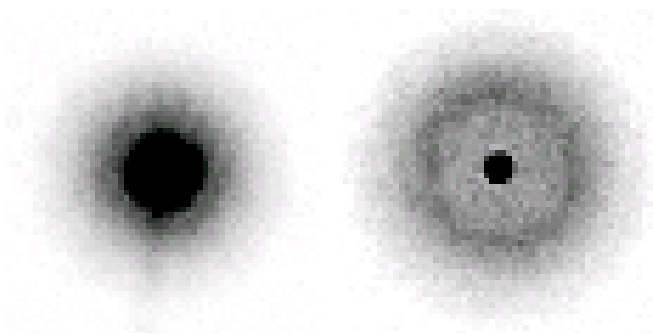


$$m_f = -9/2$$

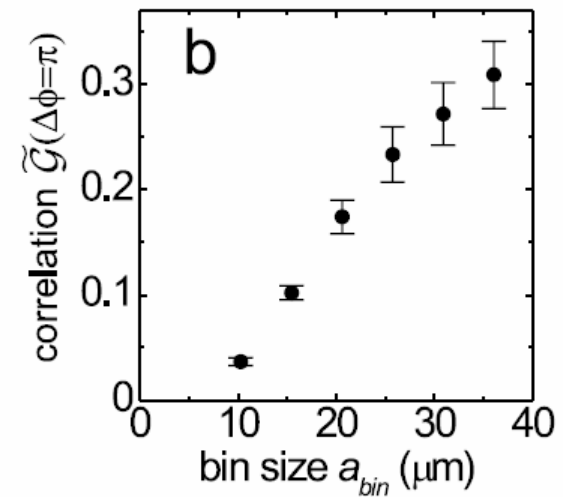
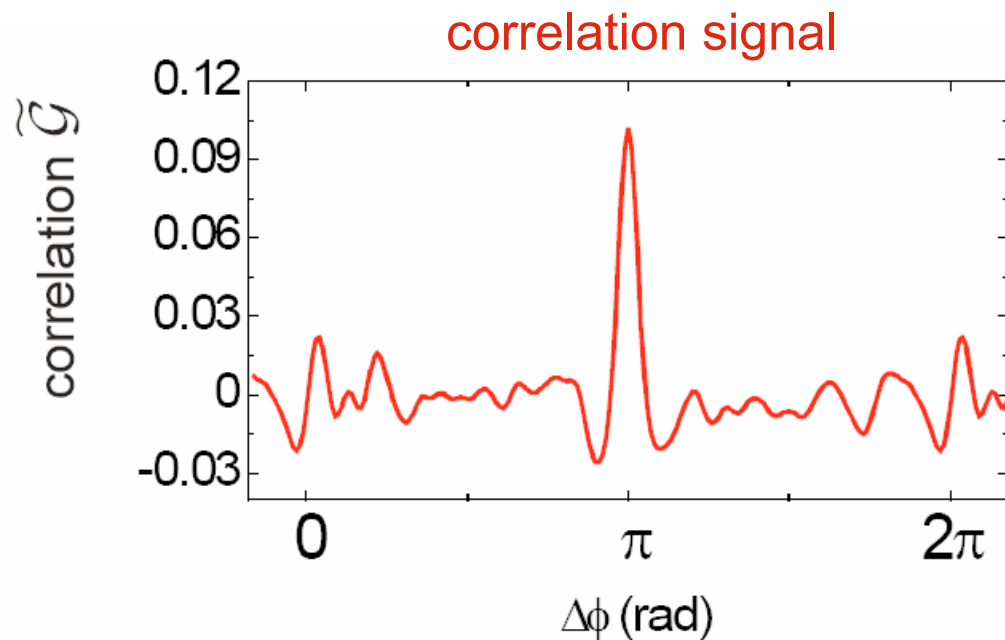


$$m_f = -5/2$$

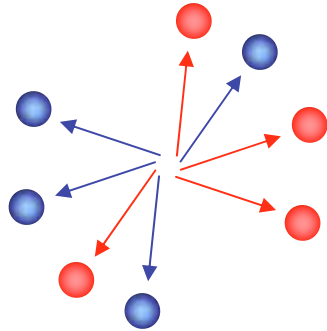
# Noise correlations of nonlocal singlet pairs



M. Greiner *et al.*,  
PRL (2005)



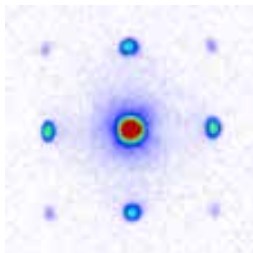
# Future applications:



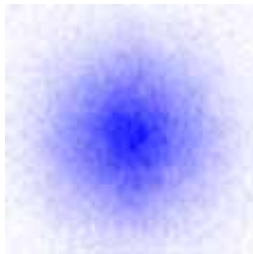
- Detect condensed pairs, should work in the BCS limit;  
Things to optimize:
  - switch off interaction
  - optimize ratio between relative and center of mass motion
  - optimize condensate fraction
- Pairs are entangled (singlet molecules)
  - EPR pairs
  - study Bell inequalities and entanglement

# Noise correlations in optical lattices

- *Work by Simon Fölling, Fabrice Gerbier, Artur Widera, Olaf Mandel, Tatjana Gericke and **Immanuel Bloch in Mainz***  
*Proposed by Ehud Altman et al.*

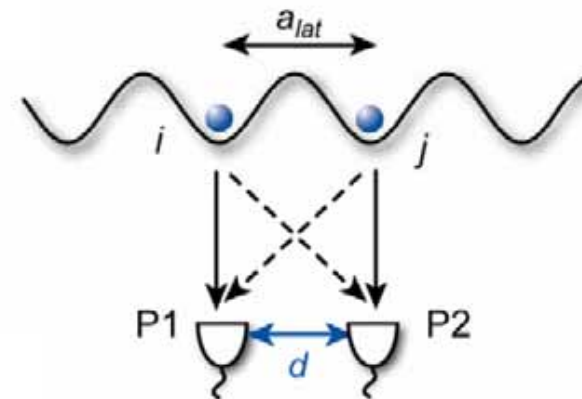
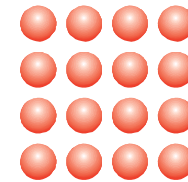


Superfluid phase:  
long range phase  
coherence

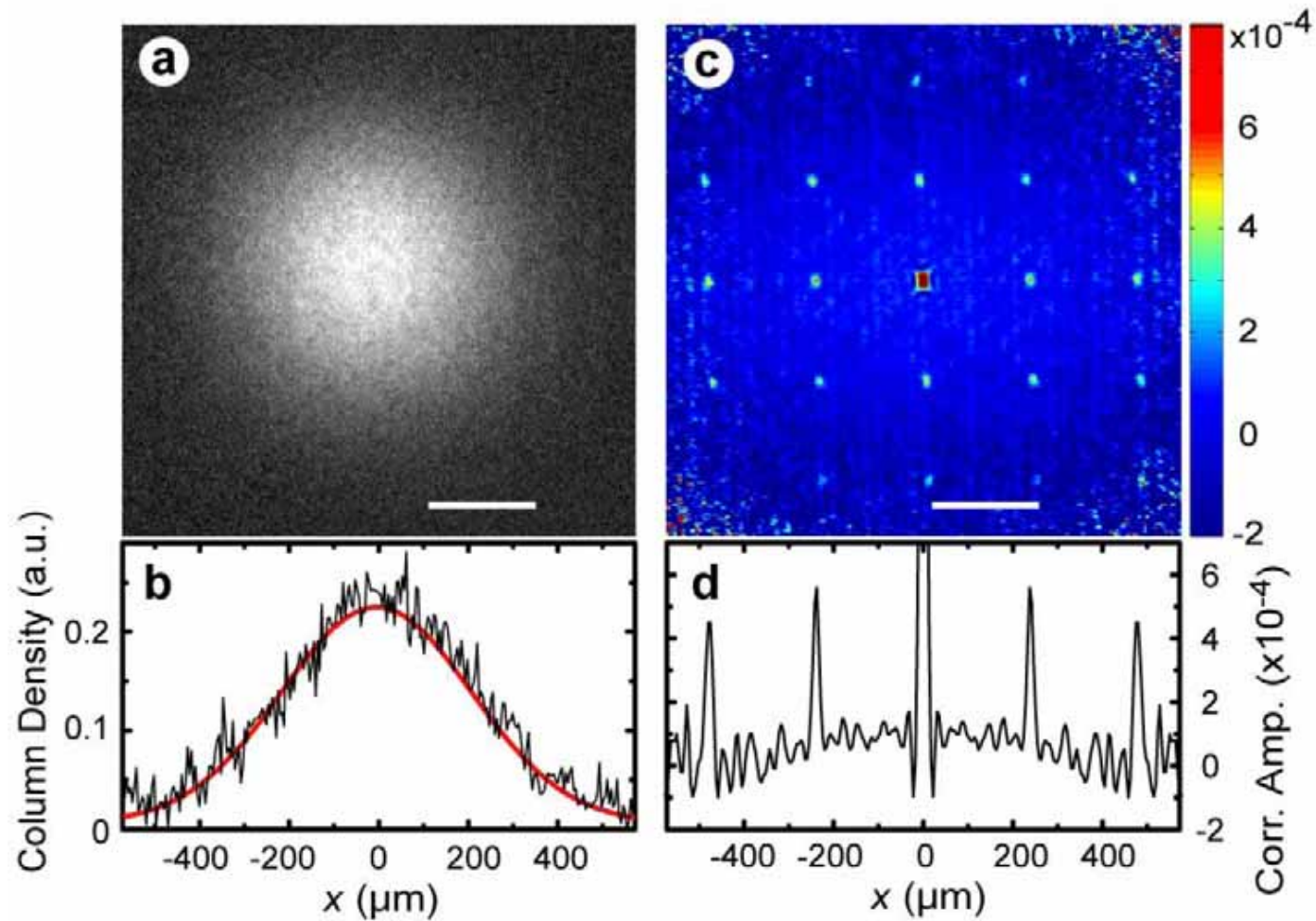


Mott insulator:  
no first order phase  
coherence  
→ no interference  
pattern

Hanbury Brown Twiss  
(HBT) experiment:  
Measure correlations  
of fluctuations



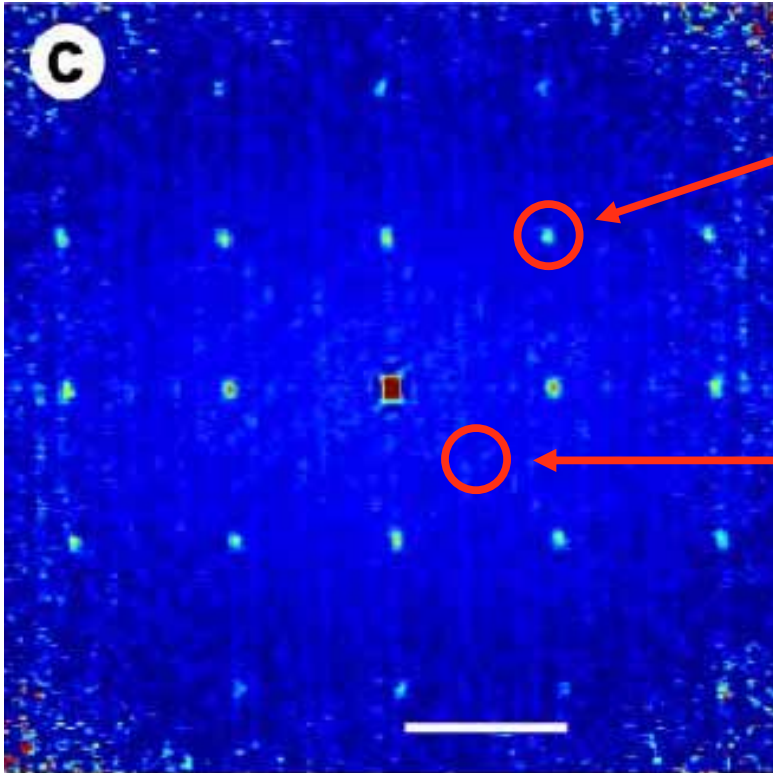
# Noise correlations in optical lattices (Mainz)



Foelling et al., Nature (2005)

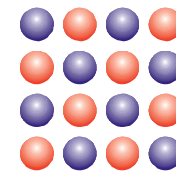


# Noise correlations in optical lattices (Mainz)



Fermions:  
anti correlations  
→ peaks should be negative

Anti-ferromagnetic state:  
additional  
correlations  
peaks



Spin waves etc. ...

# Thanks:

Deborah Jin  
Cindy Regal

Jayson Stewart

*... I am starting a research  
group this summer,*

***PhD students and  
postdocs welcome ...***



Markus Greiner  
Deborah Jin    Cindy Regal