The Abdus Salam
International Centre for Theoretical Physics

SMR 1666 - 31

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

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BCS→BEC in p-wave Feshbach Resonance

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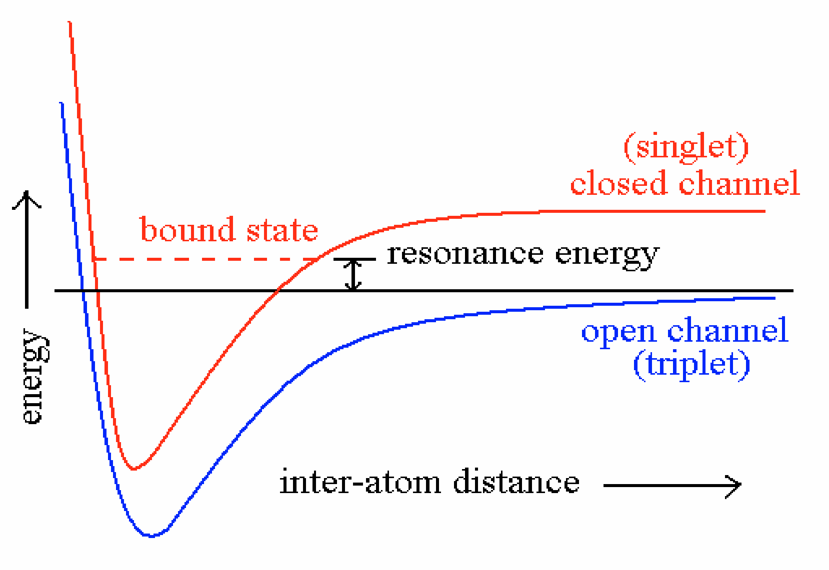
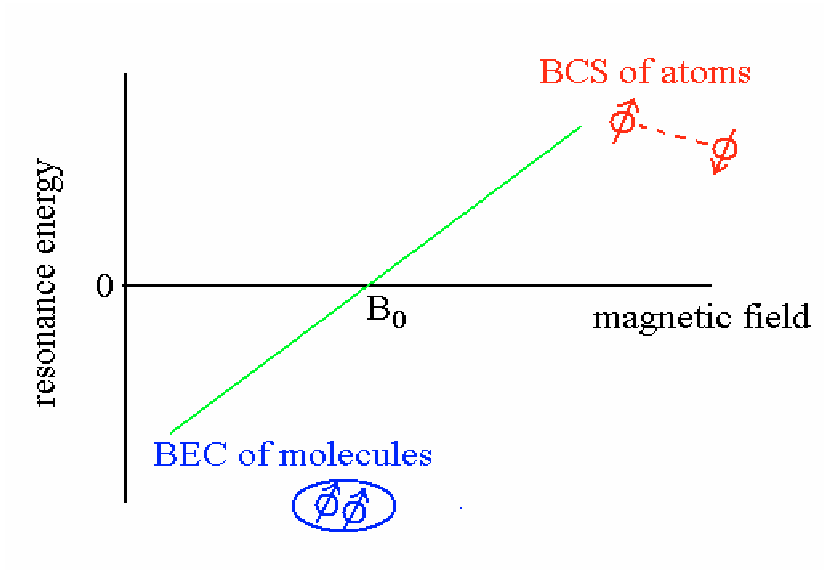
PRL, to appear

with Chi-Ho Cheng





P-wave molecules

p-wave Feshbach resonances:

^{6}Li Paris

^{40}K Colorado $|f, m_f \rangle = |9/2,-7/2\rangle$

suggests: p-wave superfluid !

Other theoretical works:

Ho, Ohashi, Radzihovsky, Volovik ….

Earlier work on BCS-BEC in p-wave (l=1,m=1; ^{3}He-A):

Leggett, Randeria et al, Muzikar and Mermin:

not a cross-over

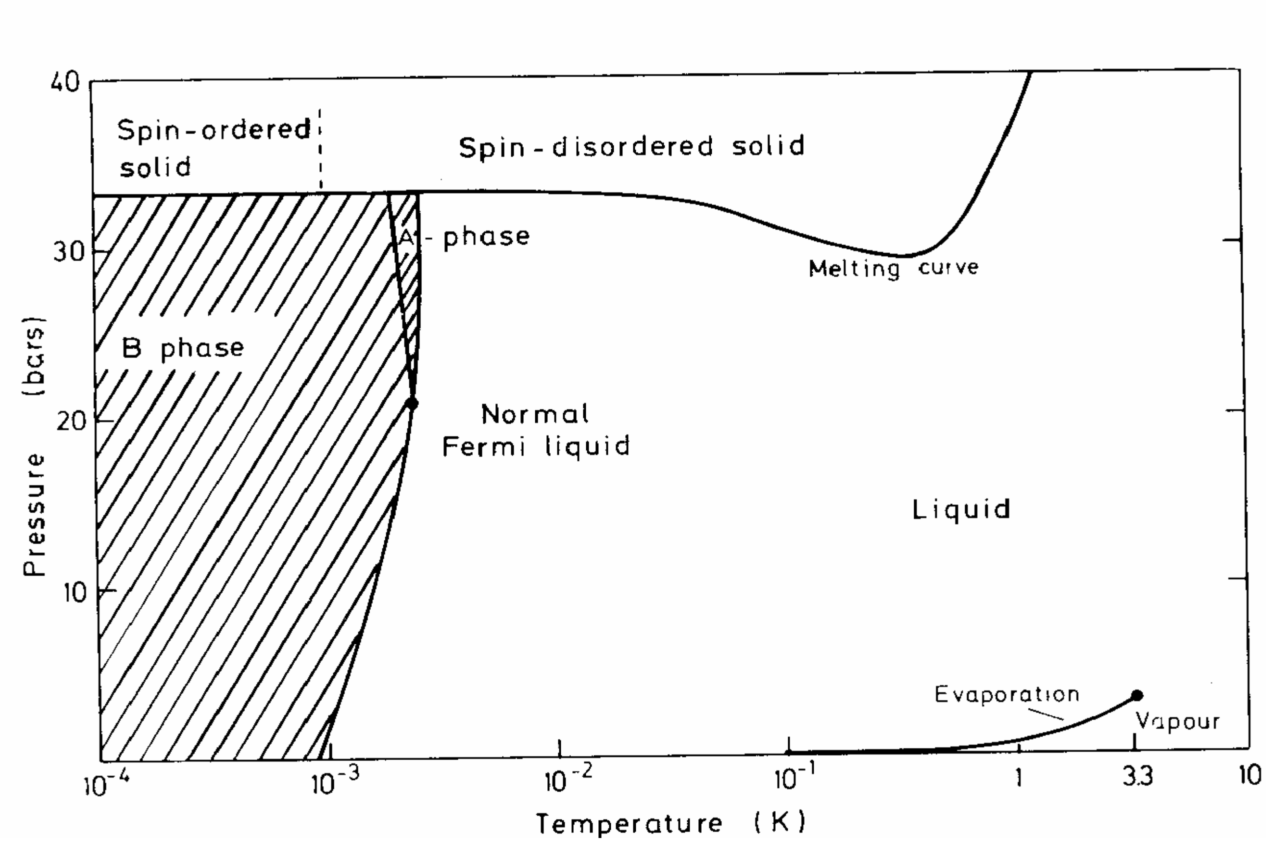
node vs. *nodeless*

→ *singularity when μ crosses zero*

particle current has different forms if direction of l varies

Here: symmetry of ground state (c.f. Ho, Radzihovsky) p-wave superfluid ----

^{3}He !



^{3}He, order parameter:

p-wave

triplet

k_x, k_y, k_z

$|\uparrow\uparrow>, \ |\downarrow\downarrow>, \ |\uparrow\downarrow> + |\downarrow\uparrow>$

or linear comb.

or linear comb.

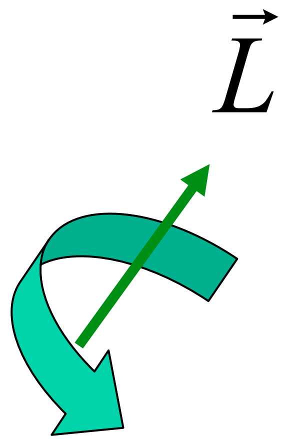
$l = 1, m = -1, 0, 1$

A-phase:

$(k_x + ik_y)[|\uparrow\uparrow> + |\downarrow\downarrow>]$

$|\uparrow\downarrow + \downarrow\uparrow>$

or rotations



Interesting properties of ^{3}He (-A):

(c.f. s-wave)

anisotropic gap, superfluid density ….
gapless excitations
phase of Cooper pair varies with $\hat{k}$:

$$k_x + ik_y = \sin(\theta_k)e^{i\varphi_k}$$

orientation
internal mode

c.f. High Tc (oxide)/ UPt_3 …. superconductors

Differences with ^{3}He:

Resonant field depends on hyperfine sublevel / (hyperfine) spin polarized

→ consider only one species

Remaining question:

orbital part

$$k_x, k_y, k_z$$

$$k_x + ik_y$$

??…

Anderson and Morel:

weak coupling BCS, pairing only between like spins

isotropic system

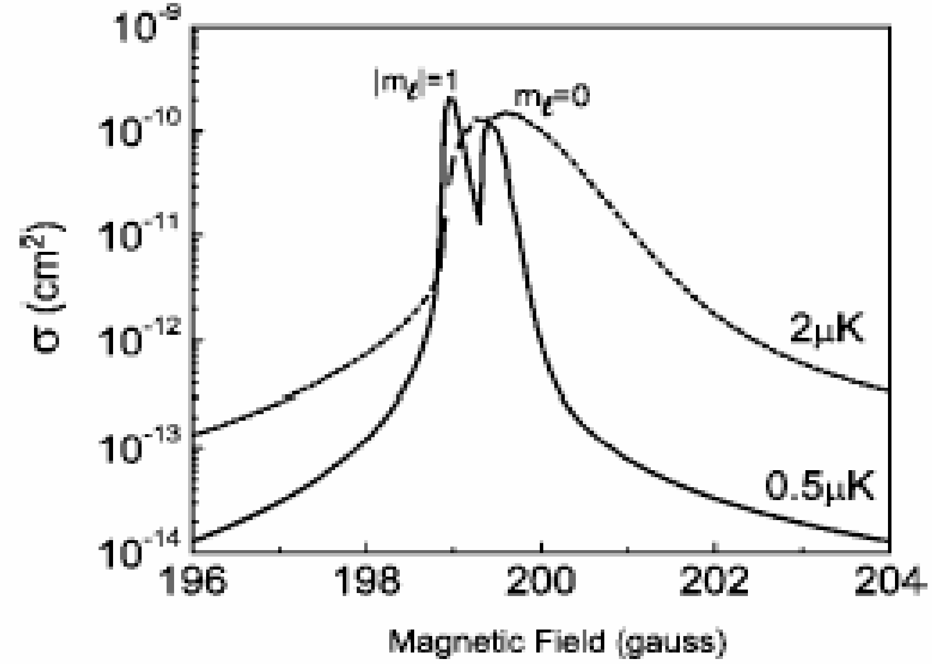
most favorable state is $k_x + ik_y$ or rotations

L=1, m = 1

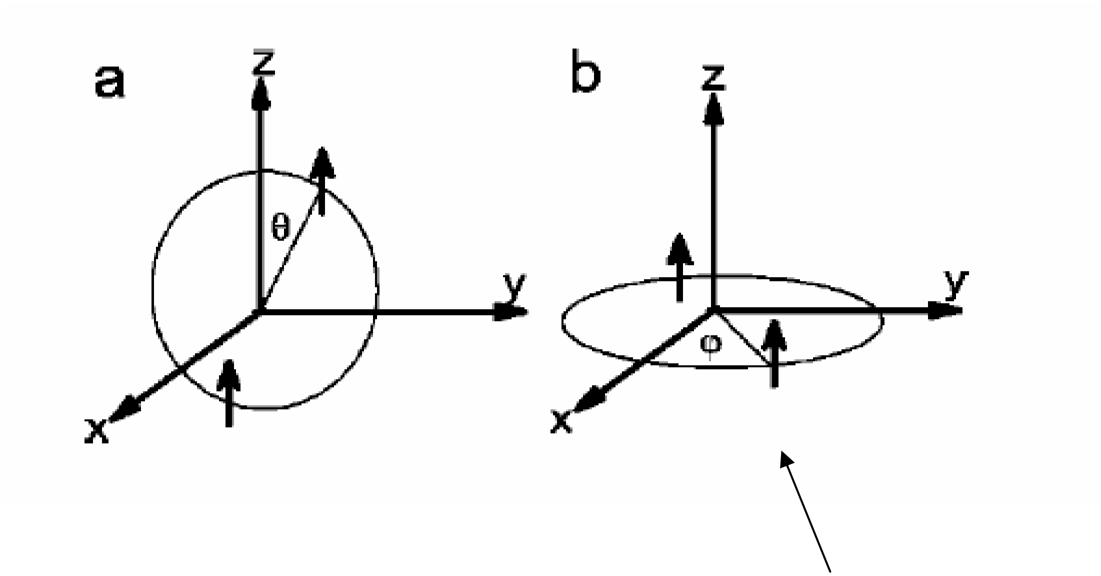
A-phase

Applies also to BEC (Diener and Ho)
(if assume equal detunings for m=-1,0,1)

Ticknor et al (2004):



m_l=0 resonance splits from m_l=±1



Dipole interaction more repulsive in $m_l = \pm 1$ channels p-wave scattering amplitude:

$$f_m(k) = \frac{k^2}{-\frac{1}{v_m} + c_m k^2 - ik^3}$$

↑

m_l dep.

v_m : volume

c_m : L^{-1}

$1/v \propto (B\text{-}B_0)$

S-wave :

$$f(k) = \frac{1}{-\frac{1}{a} + rk^2 - ik}$$

a, r : L

a = a(B)

$1/a \propto (B\text{-}B_0)$

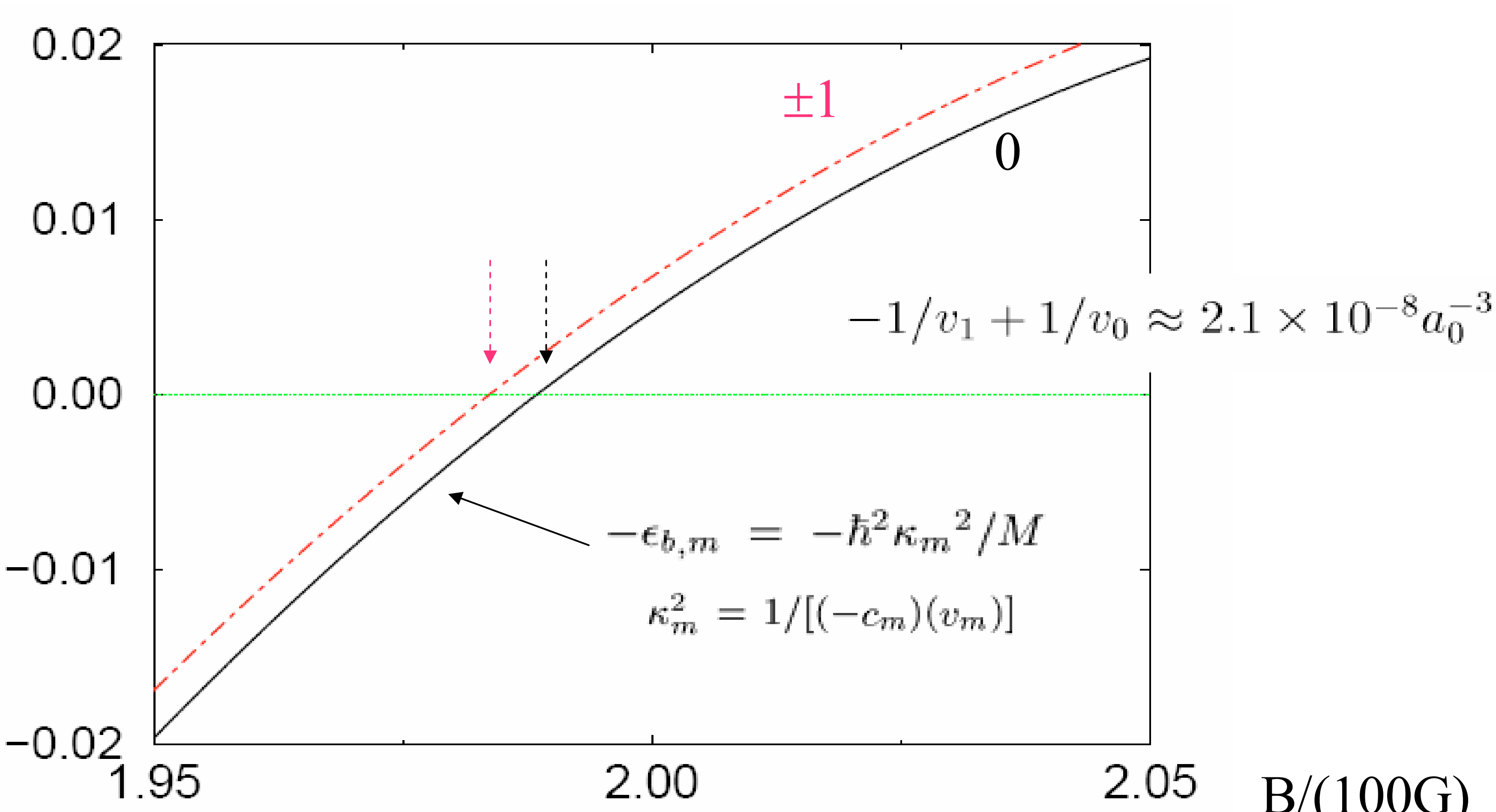
$$f_m(k) = \frac{k^2}{-\frac{1}{v_m} + c_m k^2 - i k^3}$$

$- 0.02 a_o^{-1} < 0$ and roughly constant

^{40}K:

$- 1/v_m$

$(10^{-5}\ a_o^{-3})$



B/(100G)

splitting
~ μK

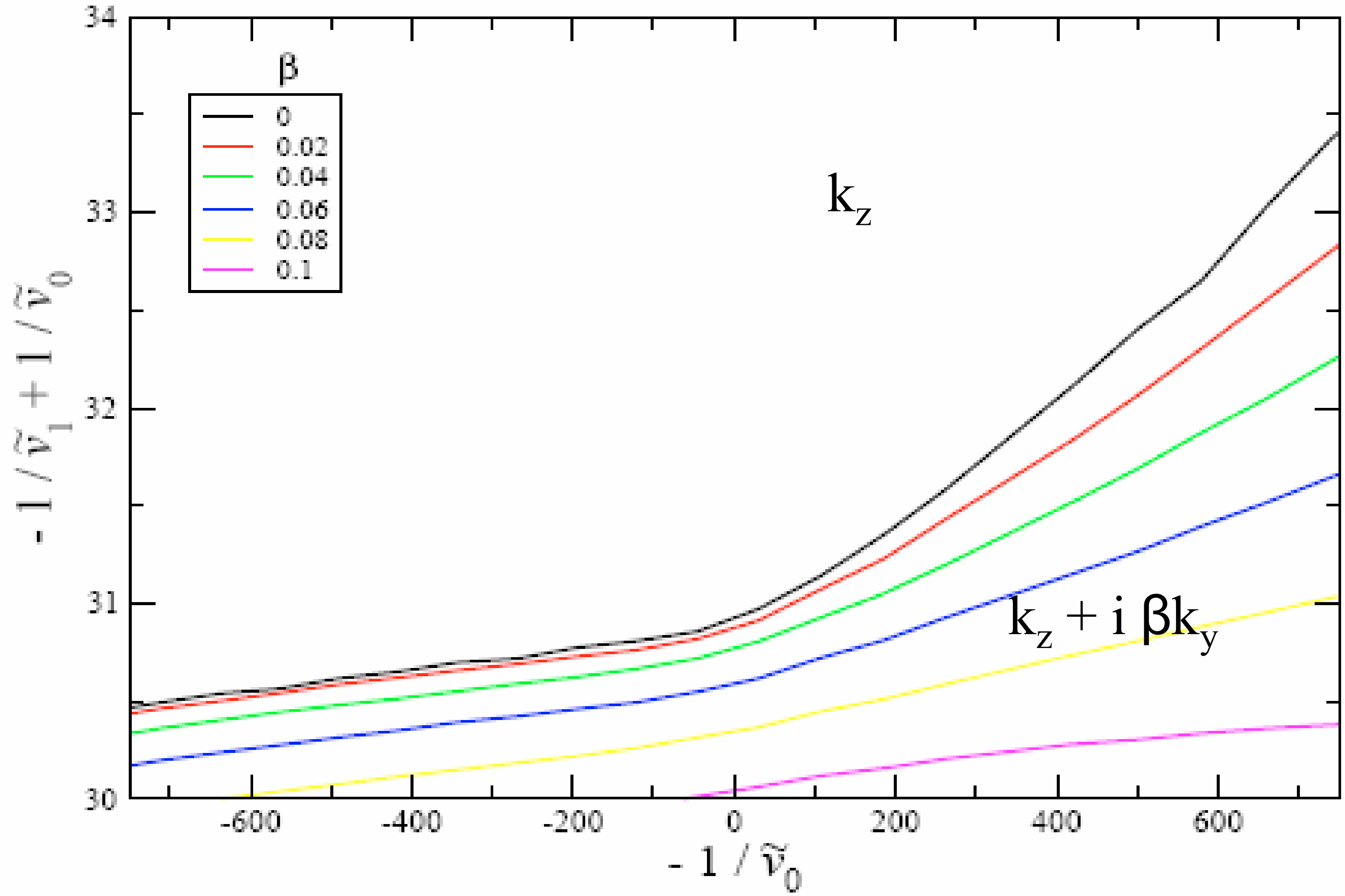
expect:

BEC limit : m_l=0 (k_z) Bosons only

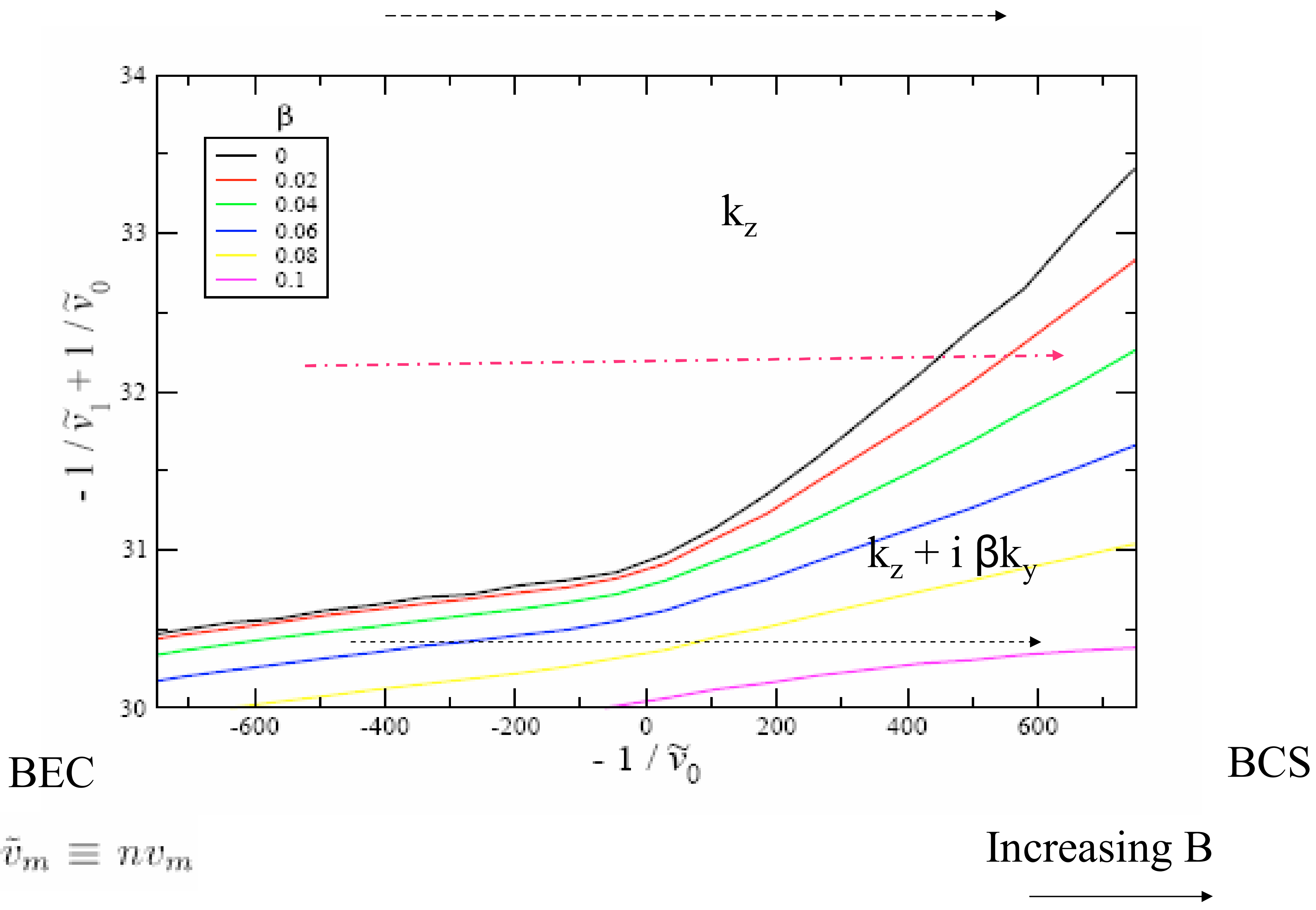
k_z only also in BCS if “large” splitting

~ $k_z + i\,\beta k_y$ if intermediate splitting ($\beta<1$)

note: z defined by magnetic field, not free to rotate



$\tilde{v}_m \equiv n v_m$



$\tilde{v}_m \equiv n v_m$

$$H_f = \sum_{\vec{k}} (\frac{\hbar^2 k^2}{2M} - \mu) a^\dagger_{\vec{k}} a_{\vec{k}}$$

$$H_b = \sum_{\vec{q}} \left(-2\mu + \delta_m + \frac{\hbar^2 q^2}{4M} \right) b^\dagger_{\vec{q},m} b_{\vec{q},m}$$

$$H_a = \frac{1}{L^{3/2}} \sum_{m,\vec{q},\vec{k}} \left\{ b^\dagger_{\vec{q},m} \left(i\sqrt{4\pi} k Y_1^{m*}(\hat{k}) \tilde{\alpha}^*_m \right) a_{-\vec{k}+\vec{q}/2} a_{\vec{k}+\vec{q}/2} + h.c. \right\}$$

two-body scattering:

$$-\frac{1}{v_m} = \frac{4\pi\hbar^2}{M} \left(\frac{\delta_m}{|\tilde{\alpha}_m|^2} - \frac{1}{L^3} \sum_{\vec{k}} \frac{M}{\hbar^2} \right)$$

$$c_m = -\frac{4\pi\hbar^2}{M} \left(\frac{\hbar^2}{M|\tilde{\alpha}_m|^2} + \frac{1}{L^3} \sum_{\vec{k}} \frac{1}{2\epsilon_k} \right)$$

mean field:

$$b_{0,m}(\delta_m - 2\mu) = -\sum_{\vec{k}} \left(\frac{i\sqrt{4\pi}}{L^{\frac{3}{2}}} k Y^*_{1m}(\hat{k}) \tilde{\alpha}^*_m \right) \langle a_{-\vec{k}} a_{\vec{k}} \rangle$$

$$D_m = -i\sqrt{4\pi}\tilde{\alpha}_m b_{0,m}/L^{3/2}$$

$$\Delta_{\vec{k}} = \sum_m D_m k Y_1^m(\hat{k}) \longrightarrow \begin{matrix} D_0 k_z \\ D_1(-k_x + ik_y) \\ D_{-1}(k_x + ik_y) \end{matrix}$$

$$(\delta_m - 2\mu) D_m = \frac{2\pi}{L^3} \sum_k \frac{|\tilde{\alpha}_m|^2 k Y^*_{1m}(\hat{k}) \Delta_{\vec{k}}}{\left[(\epsilon_k - \mu)^2 + |\Delta_{\vec{k}}|^2\right]^{\frac{1}{2}}}$$

$$-\frac{M}{4\pi v_0}D_0 \;+\; \frac{M^2 c_0}{2\pi}\left\{D_0\left[\mu - \frac{3M}{20\pi}(2|D_1|^2 + 3|D_0|^2 + 2|D_{-1}|^2)\right] + \frac{3M}{10\pi}D_0^* D_1 D_{-1}\right\} = \frac{2\pi}{L^3}\sum_{\vec{k},m} D_m Y_1^{0*}(\hat{k})Y_1^{m*}(\hat{k})h(\vec{k})$$

$$-\frac{M}{4\pi v_1}D_1 \;+\; \frac{M^2 c_1}{2\pi}\left\{D_1\left[\mu - \frac{3M}{20\pi}(3|D_1|^2 + 2|D_0|^2 + 6|D_{-1}|^2)\right] + \frac{3M}{20\pi}D_{-1}^* D_0^2\right\} = \frac{2\pi}{L^3}\sum_{\vec{k},m} D_m Y_1^{1*}(\hat{k})Y_1^{m*}(\hat{k})h(\vec{k})$$

(and 1 ↔ -1)

$$n = \frac{1}{L^3}\sum_{\vec{k}}\left(v_{\vec{k}}^2 - \frac{|\Delta_{\vec{k}}|^2}{4\epsilon_k^2}\right) + \frac{M^2}{(4\pi\hbar)^2}\sum_m (-c_m)|D_m|^2$$

$$h(\vec{k}) \equiv \frac{k^2}{\left[(\epsilon_k - \mu)^2 + |\Delta_{\vec{k}}|^2\right]^{1/2}} - \frac{k^2}{\epsilon_k}\left(1 + \frac{\mu}{\epsilon_k} - \frac{|\Delta_{\vec{k}}|^2}{2\epsilon_k^2}\right) .$$

↑

$$\tilde{\mu} \equiv \mu/\epsilon_{\rm F},\ \tilde{D}_m \equiv D_m/v_{\rm F},\ \tilde{c}_m \equiv n^{-1/3}c_m$$

$$\tilde{v}_m \equiv n v_m \qquad k_{\rm F} \equiv 6\pi^2 n$$

$$\epsilon_{\rm F} \equiv \hbar^2 k_{\rm F}^2/2M,\ v_{\rm F} \equiv k_{\rm F}/M,$$

$$\tilde{c}_0 = \tilde{c}_1 = -100$$

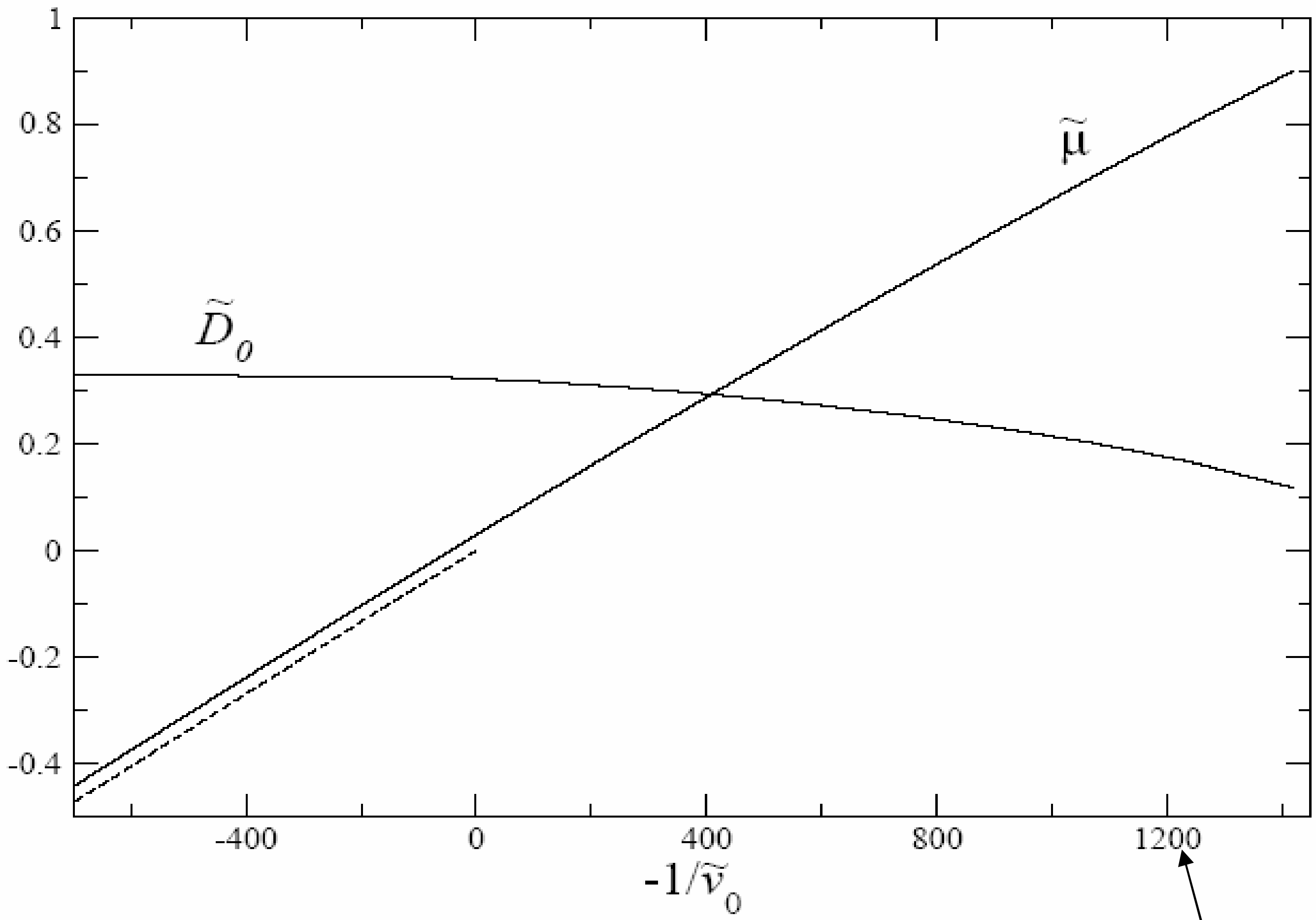
density = $6.7 \times 10^{13}\mathrm{cm}^{-3}$

recall:

$$f_m(k) = \frac{k^2}{-\frac{1}{v_m} + c_m k^2 - ik^3}$$

$\tilde{c}_0 = \tilde{c}_1 = -100$

$(32\pi/3)^{1/3}(-\tilde{c}_0)^{-1/2}$



note scale, $\tilde{c}_m \equiv n^{-1/3} c_m$ arge

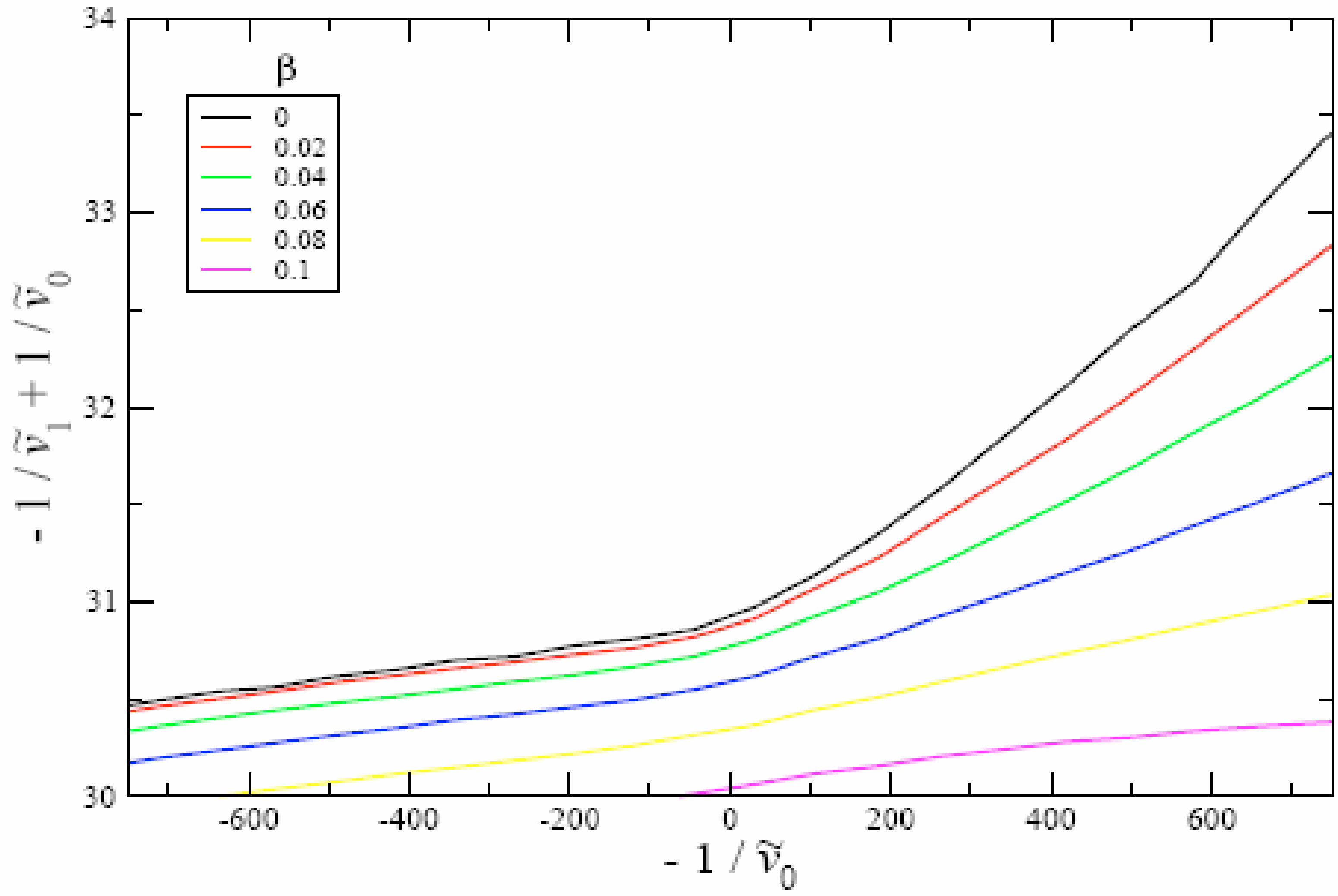
'dilute'

Phase transition to $D_{\pm 1} \neq 0$: linearize in $D_{\pm 1}$

$$-\frac{1}{\tilde{v}_1^*} + \frac{1}{\tilde{v}_0} = \frac{3(6\pi)^{\frac{2}{3}}}{5\pi} \tilde{D}_0^2(-\tilde{c}_0) \quad + 9\pi \int_0^\infty dx \int_{-1}^{1} dy \frac{x^4(1-3y^2)}{[(x^2-\tilde{\mu})^2 + \frac{3}{\pi}\tilde{D}_0^2 x^2 y^2]^{1/2}}$$

BCS: $-1/\tilde{v}_1^* + 1/\tilde{v}_0 \rightarrow 12\pi \approx 37.7.$

BEC: $-1/\tilde{v}_1^* + 1/\tilde{v}_0 \rightarrow 48\pi/5 \approx 30.2$



34
33
32
31
30
β
0
0.02
0.04
0.06
0.08
0.1
-600
-400
-200
0
200
400
600
$-1/\tilde{v}_1 + 1/\tilde{v}_0$
$-1/\tilde{v}_0$

^{40}K: $-1/v_1 + 1/v_0 \approx 2.1 \times 10^{-8} a_0^{-3}$

density = $6.7 \times 10^{13} \mathrm{cm}^{-3}$ $\tilde{c}_0 = \tilde{c}_1 = -100$

$-1/\tilde{v}_1 + 1/\tilde{v}_0 \sim 2000$

most likely, p-wave superfluid, but polar phase; different from ^{3}He

dipole interaction important (also l = 2, 3, …)