The Abdus Salam International Centre for Theoretical Physics

SMR 1666 - 11

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Fermions and bosons in low dimensions

Presented by:

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Fermions and bosons in low dimensions

T. Giamarchi [lectures I & II]

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UNIVERSITÉ DE GENÈVE

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Fonds national suisse
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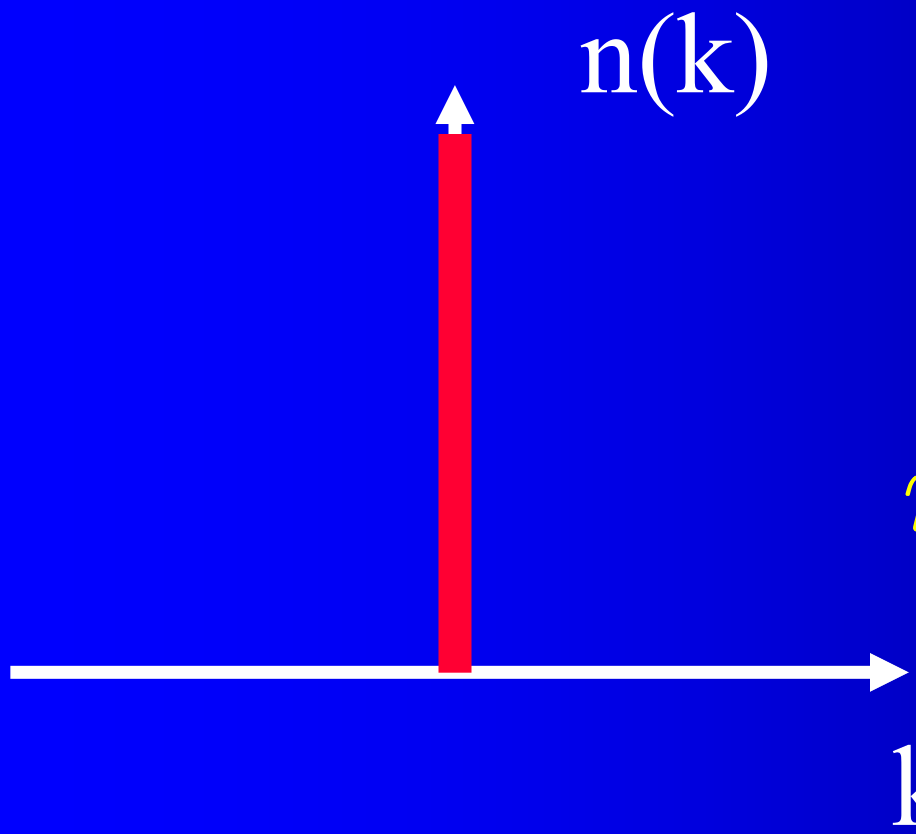
Free bosons : crash course

- Free particles: condensation in k=0 state

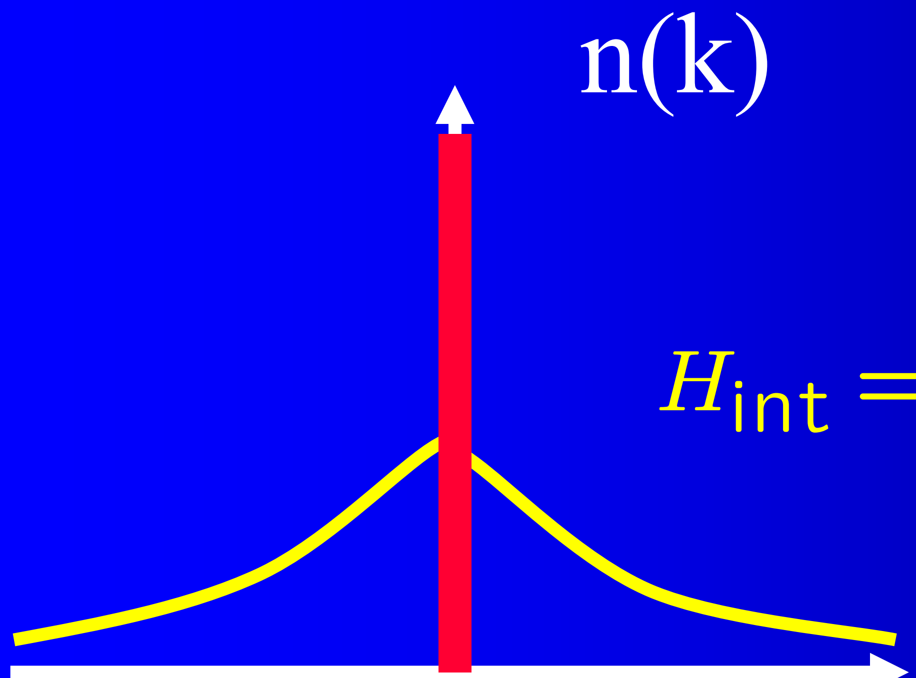


$$H_0 = \sum_k \xi_k b_k^\dagger b_k$$

$$n(k) = \langle 0 | b_k^\dagger b_k | 0 \rangle$$

• Excitations: single particles with momentum k

• Do interactions change this ?

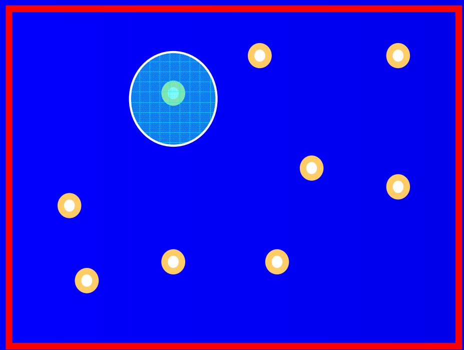


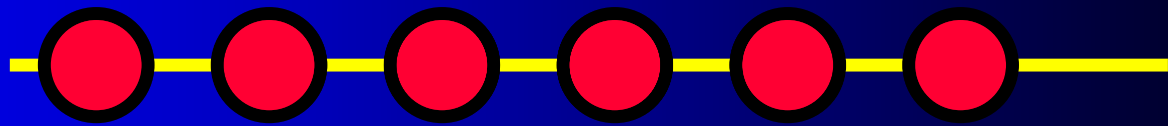
$$H_{\text{int}} = \frac{1}{\Omega} \sum_{k,k',q} V(q) b^\dagger_{k+q} b^\dagger_{k'-q} b_{k'} b_k$$

Not much !

One dimension is different

- No individual excitation can exist (only collective ones)





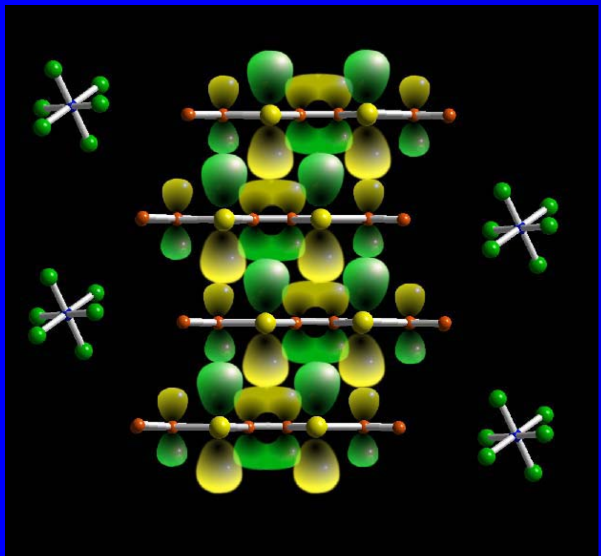
- Strong quantum fluctuations

$$\psi \to \psi e^{i\theta}$$

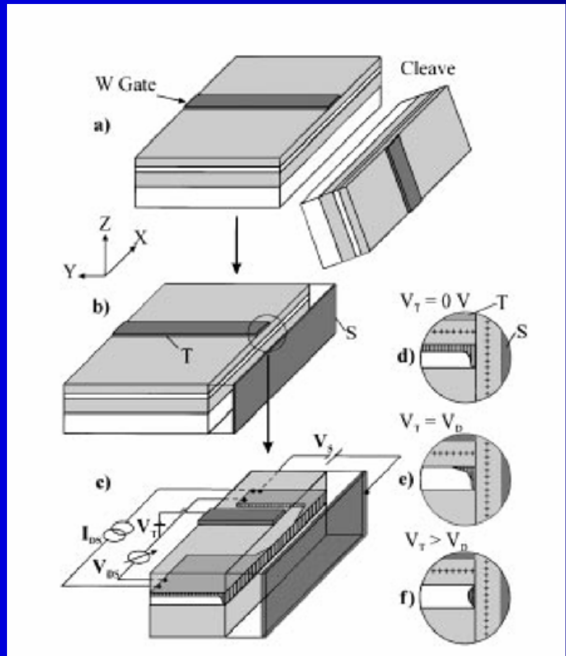
Continuous symmetry

Does one dimension exists ?

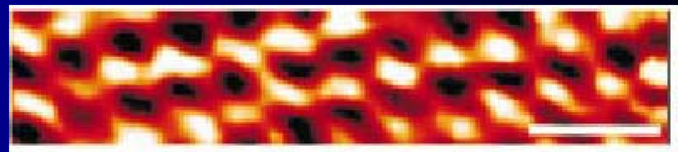
Hard to realize in condensed matter



Organic conductors



Quantum wires



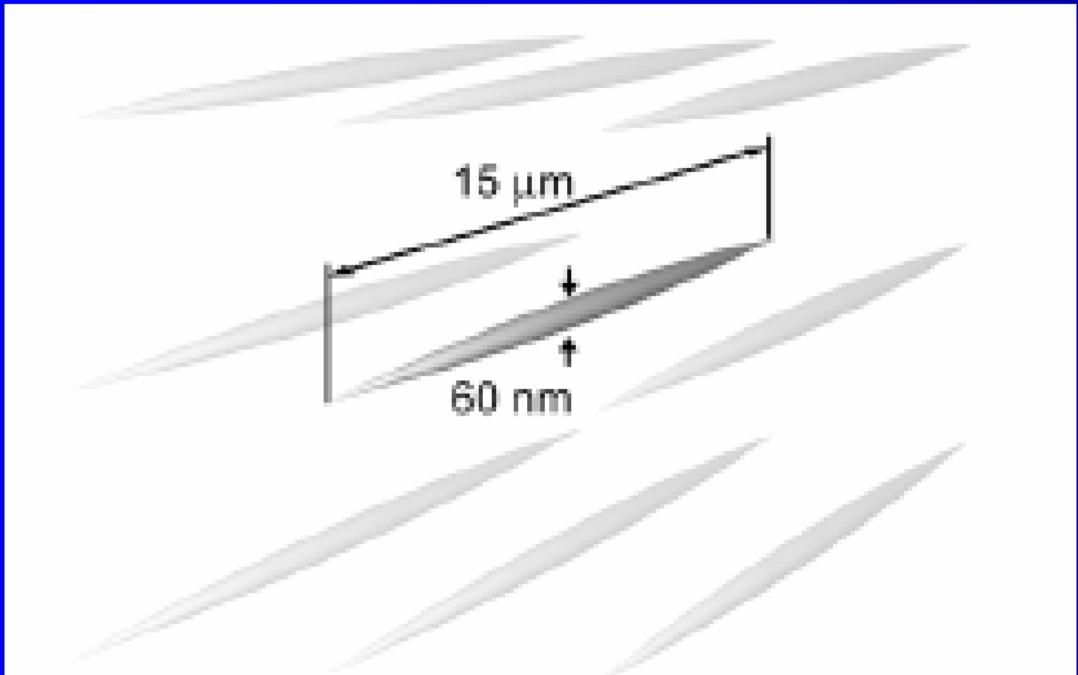
Nanotubes

- Josephson junctions
- Ladders
- Edge states in FQHE

Cold atoms: ideal systems

Optical lattices, Chips

$N_0 \sim 10$ to 10^3 atoms



T. Stoferle *et al.* PRL **92** 130403 (2004)

M. Greiner et al. PRL (2001)

W. Haensel et al. Nature (2002)

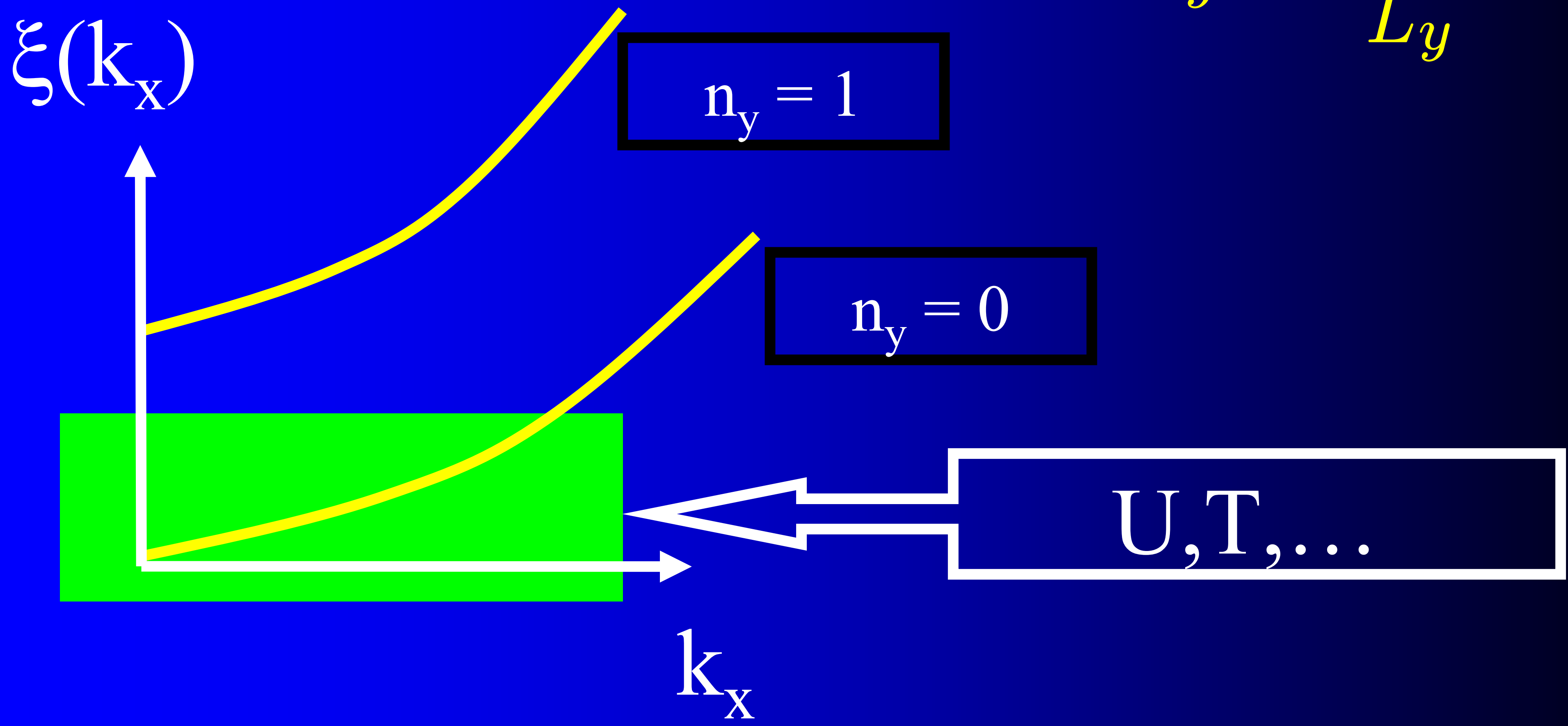
……

What does 1D means ?

$$\xi(k) = \frac{k_x^2}{2m} + \frac{k_y^2}{2m}$$

$$k_x \sim \frac{2\pi n_x}{L_x}$$

$$k_y \sim \frac{2\pi n_y}{L_y}$$

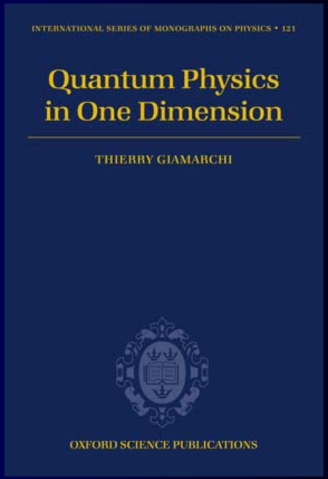


Questions

- How to deal with interactions/quantum fluctuations
- What is the new physics in 1D ?
  - Change of nature of the « particles »
  - New phases (Mott insulators)
- How to go from 1D to higher dimensions

General references on 1D

• Will follow closely:
TG, Quantum physics in one dimension, Oxford (2004)



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- Gogolin, A. O., Nersesyan, A. A., and Tsvelik, A. M. (1999). Bosonization and Strongly Correlated Systems. Cambridge University Press, Cambridge.
- Schonhammer, K. (2002). J. Phys. C, 14, 12783.
- Senechal, D. (2003). CRM Series in Mathematical Physics, Springer. cond-mat/9908262.

How to study

- Exact methods (Bethe Ansatz)

Exact

spectrum; limited to very special models

- Numerics

``Exact''

special models, size limitations,
quantities specific to models

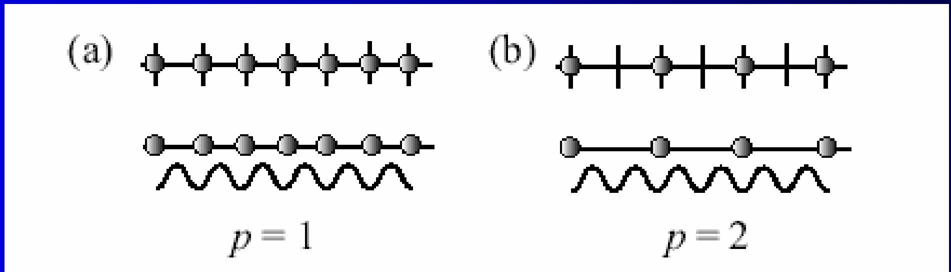
- Low energy methods methods

Models

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu\int dx\, \rho(x)$$

- Lattice:

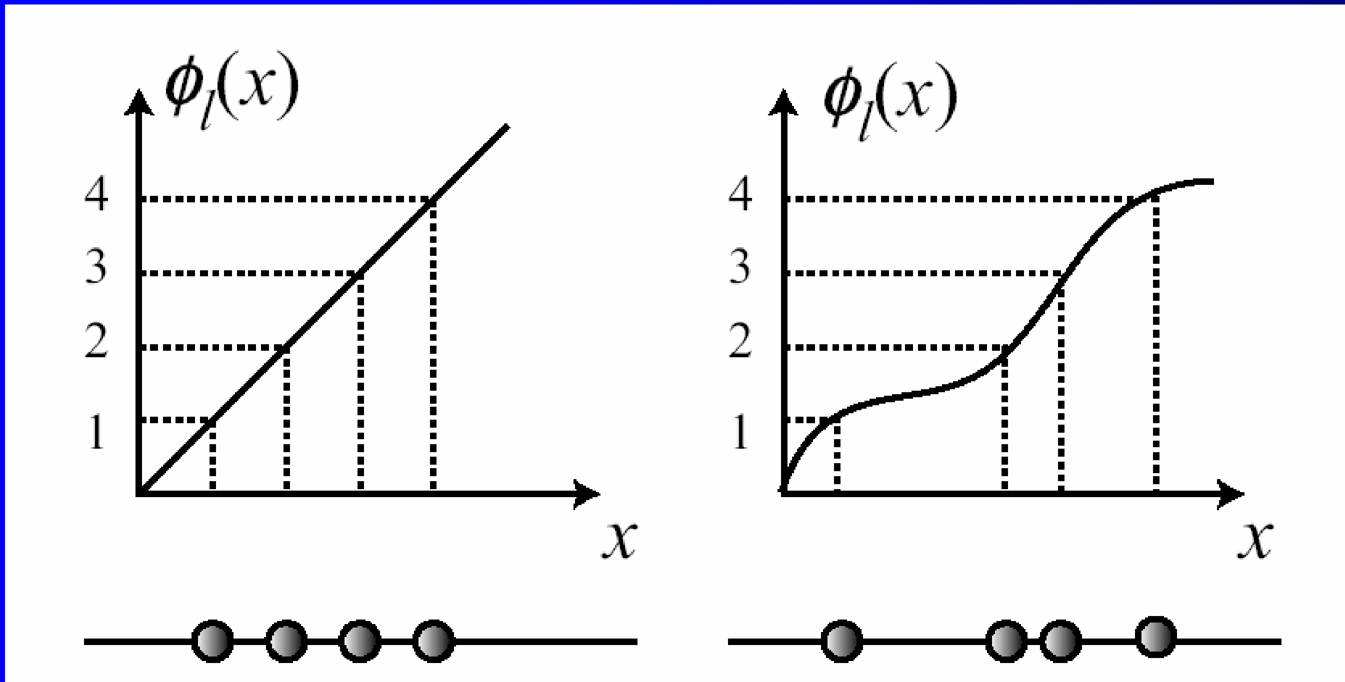


$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

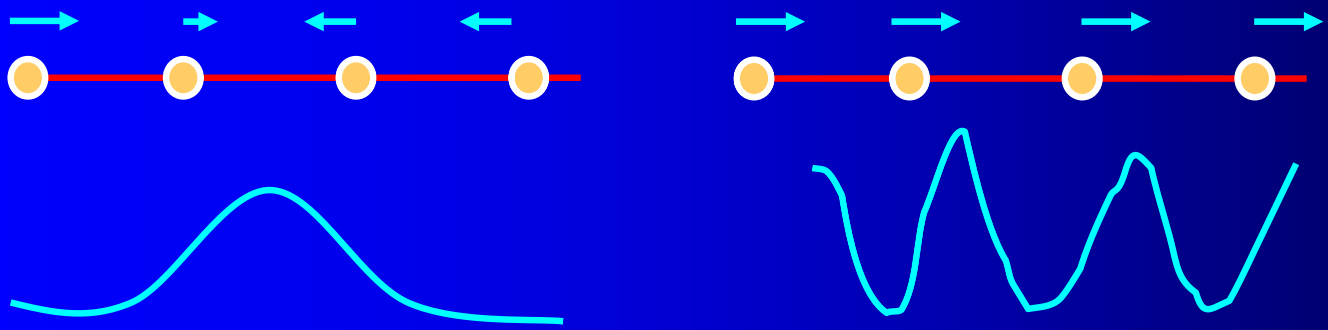
1D: unique way of labelling



$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly



CDW

$q \sim 0$ $\qquad$ $q \sim 2\pi\rho_0$

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$\psi_B^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i2p(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

All short distance properties: form of the operators. ϕ, θ: smooth fields

Hamiltonian

$$\int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} \rightarrow \frac{\rho_0}{2M}\int dx(\nabla\theta(x))^2$$

$$\frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') \rightarrow \frac{U}{2\pi^2}\int dx(\nabla\phi(x))^2$$

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

Luttinger liquid concept

- How much is perturbative

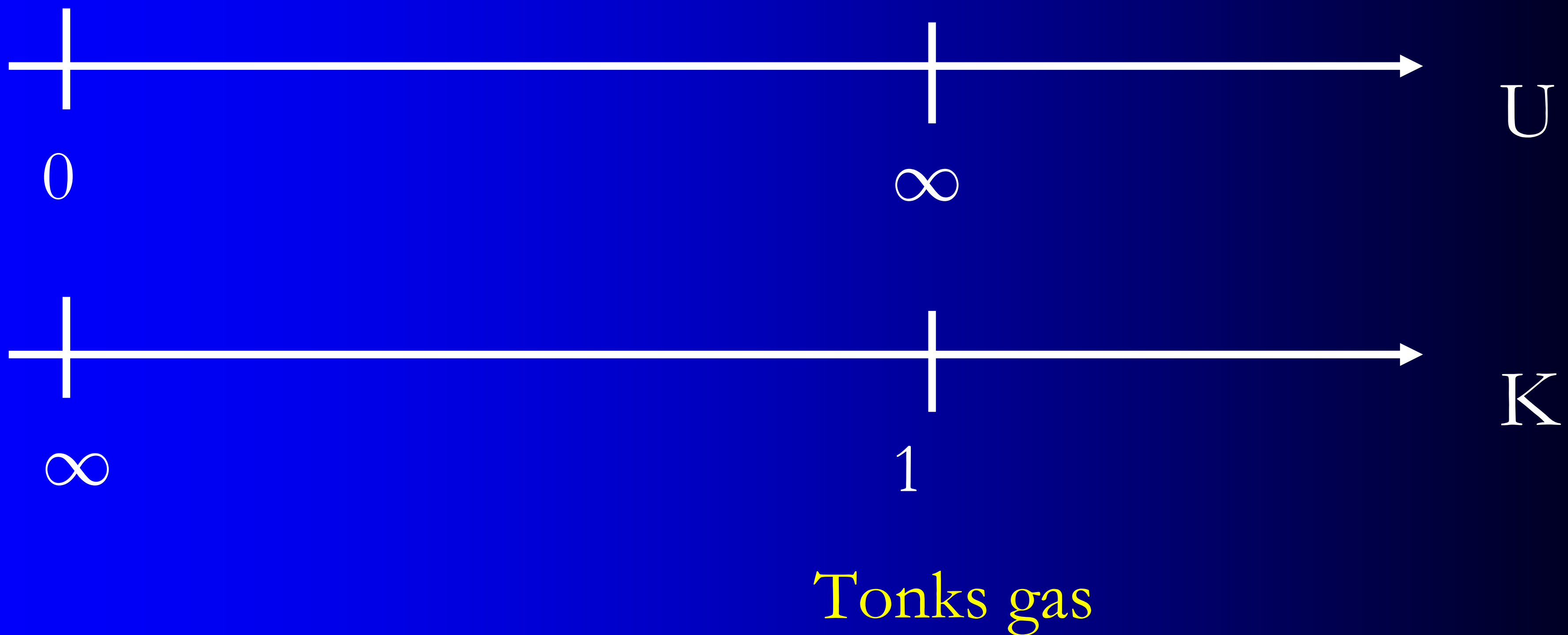
- Nothing provided the correct u,K are used (Haldane)

- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

Luttinger parameters

u: velocity of collective excitations

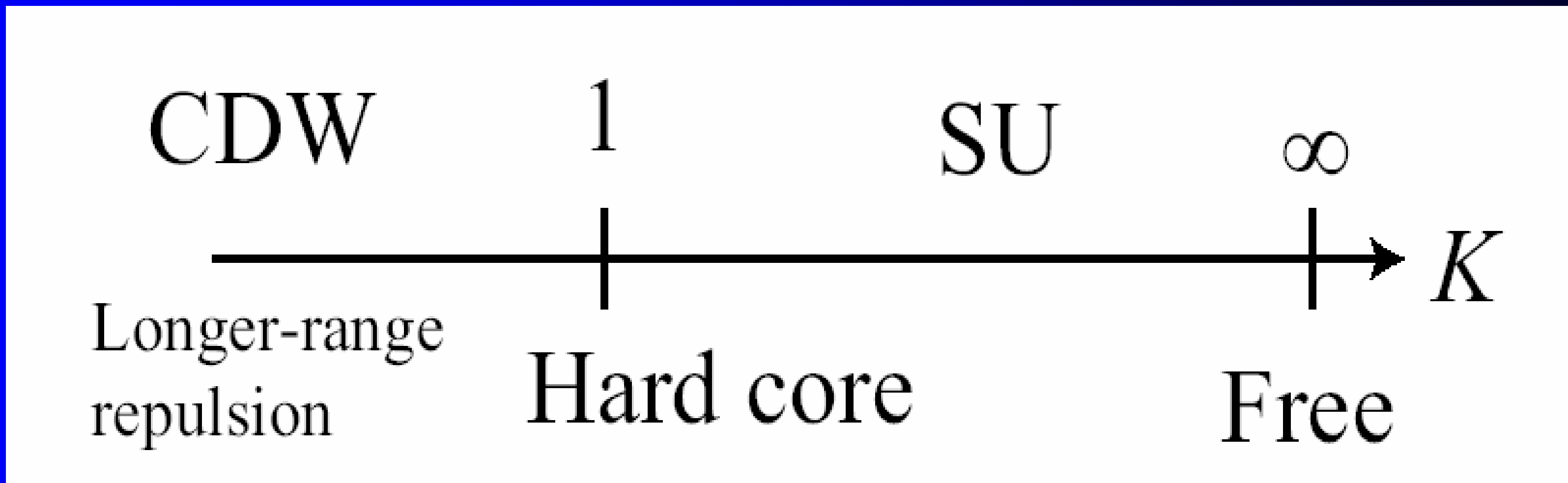
K: dimensionless parameter

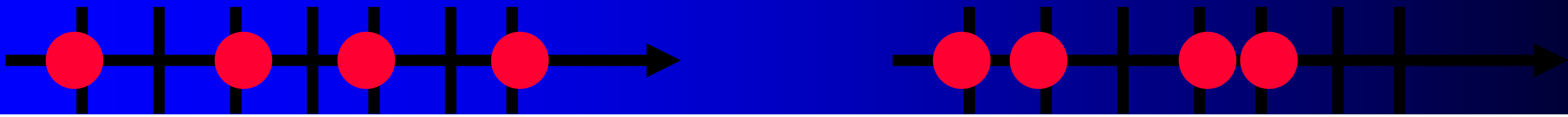


Correlations

$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$



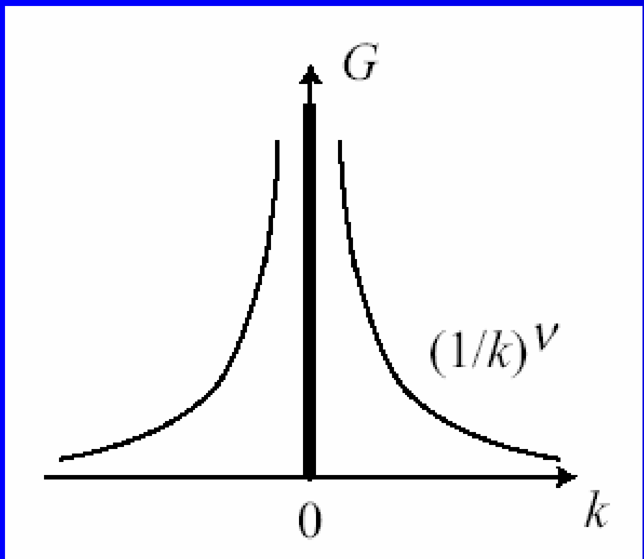


Condensate ?

$$G(x,\tau) = -\langle T_\tau \psi(x,\tau)\psi^\dagger(0,0)\rangle$$

$$n(k) = -\int dx e^{ikx} G(x,\tau = 0^-)$$

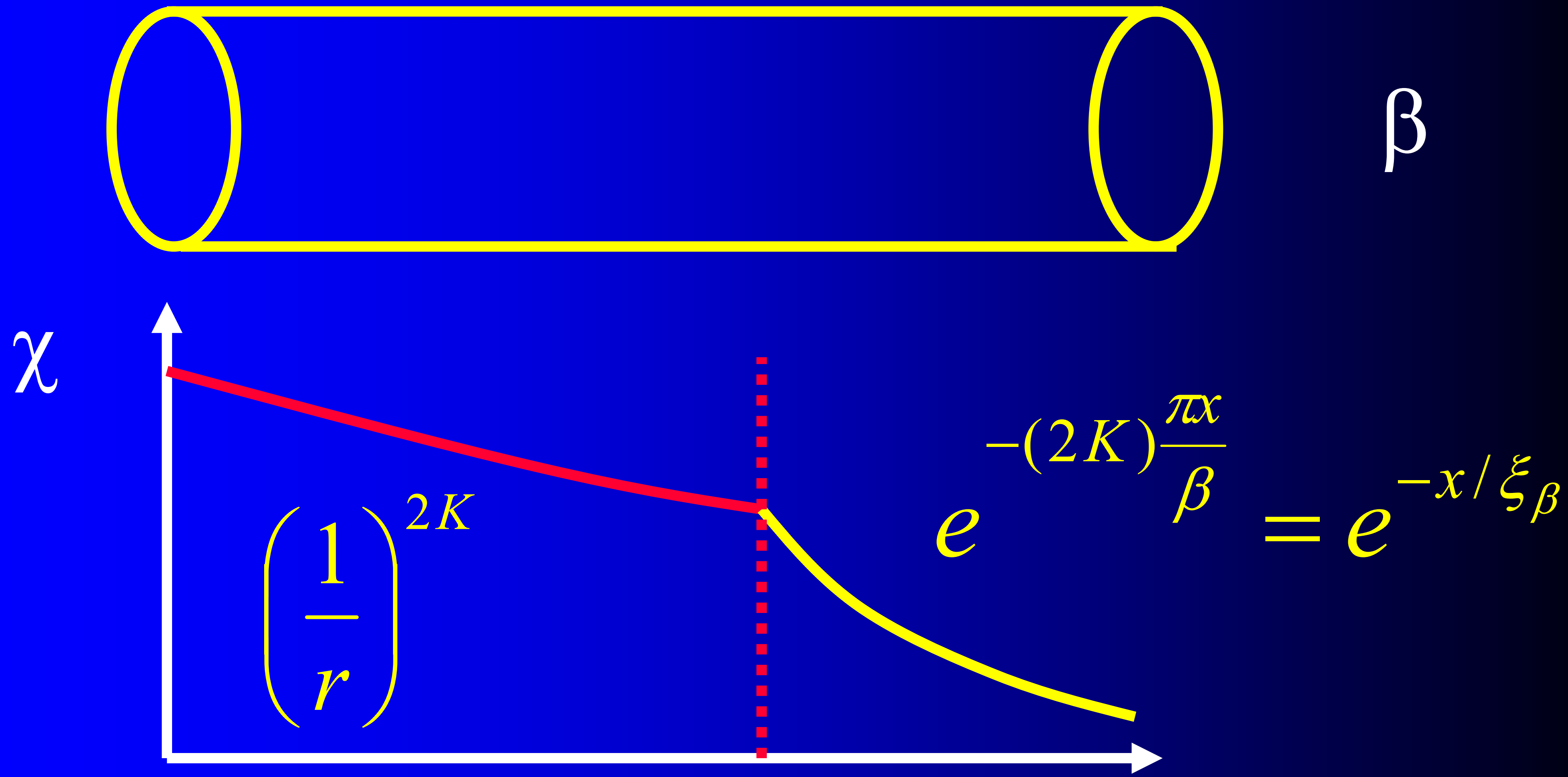
$$\nu = \frac{1}{2K} - 1$$



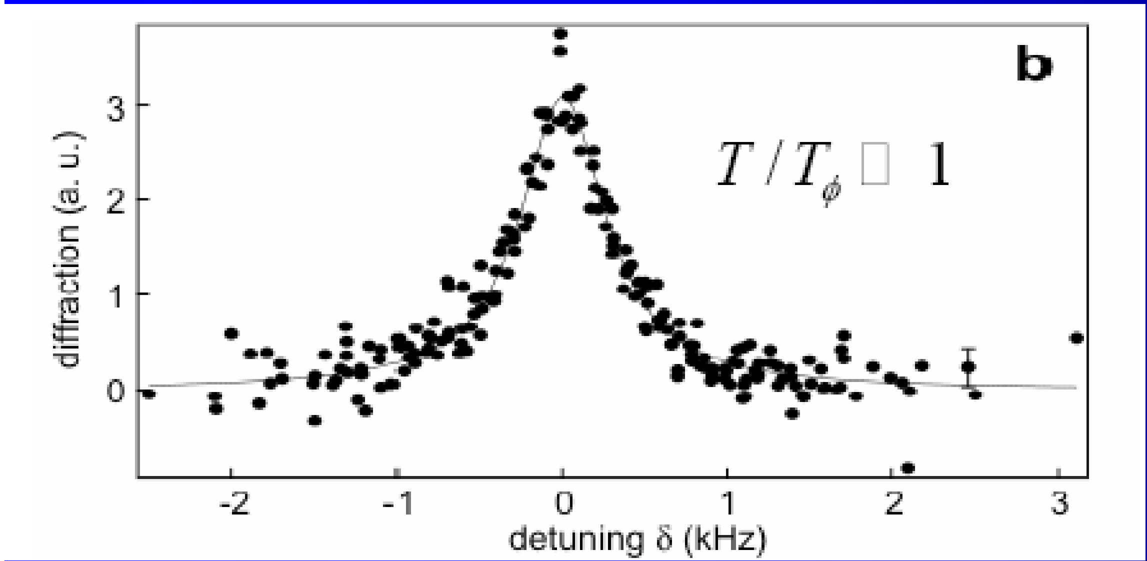
No true condensate
$(K \neq 0)$

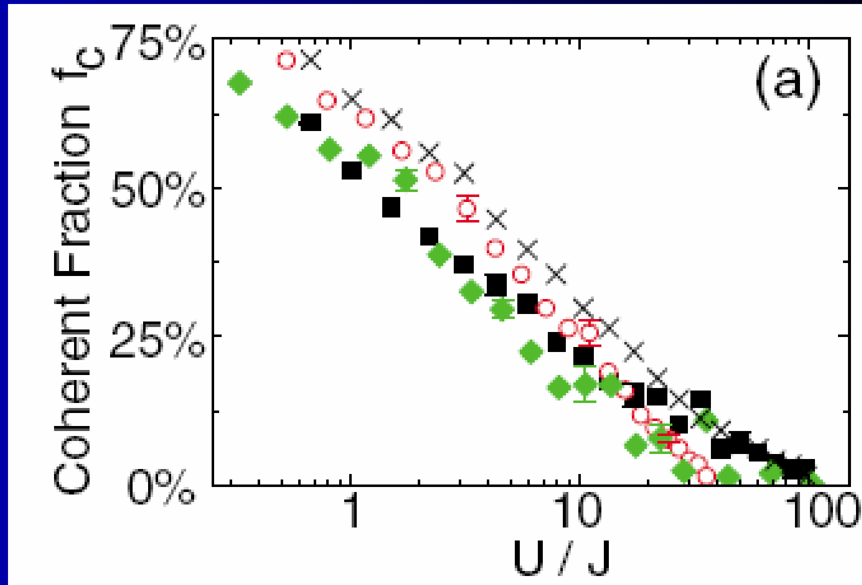
Finite temperature

Conformal theory



Quantum depletion of condensate



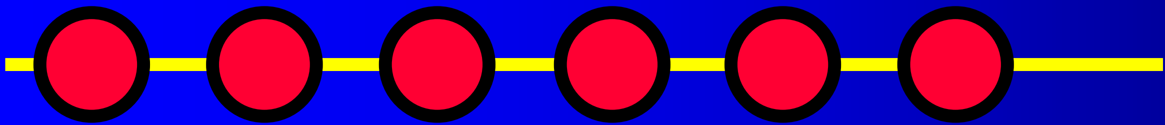


P. Bouyer *et al.* (2004)

T. Stoferle *et al.* (2004)

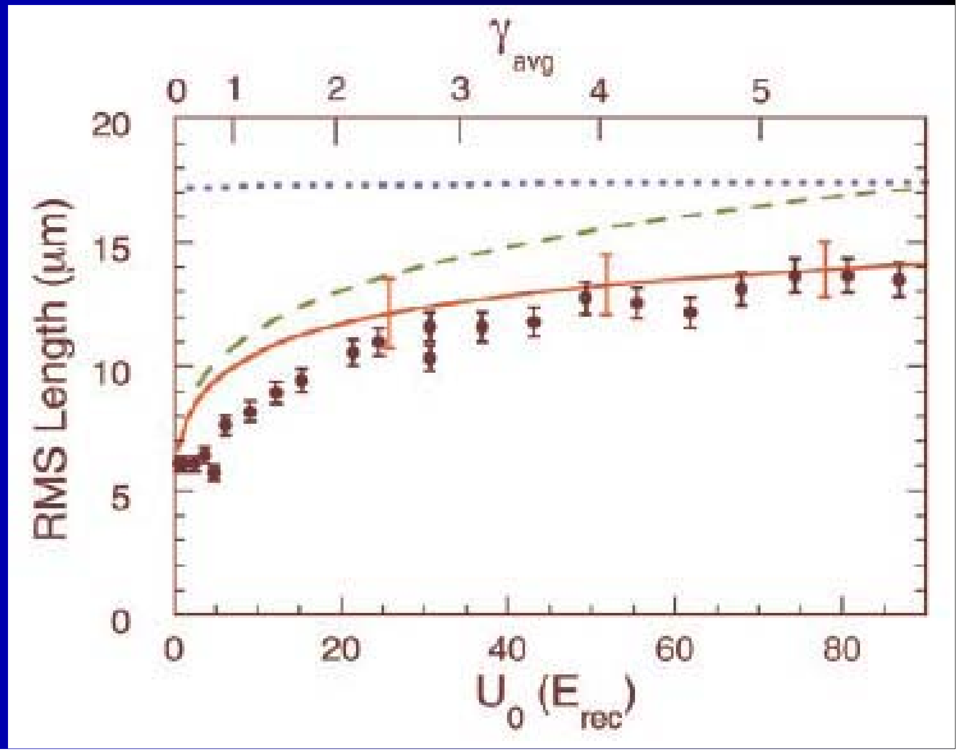
Good qualitative agreement

Tonks limit



$U = \infty$: spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

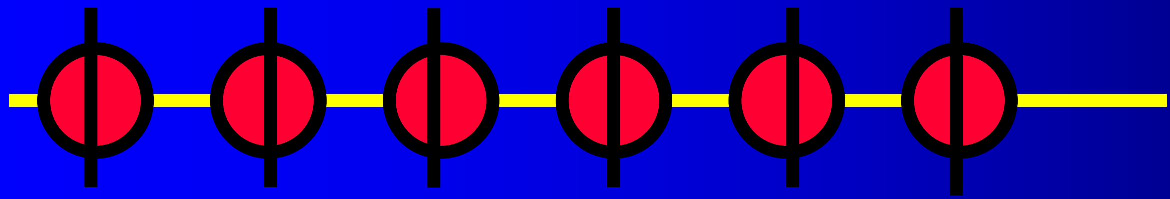


B. Paredes et al Nature (2004)

T. Kinoshita et al. Science (2004)

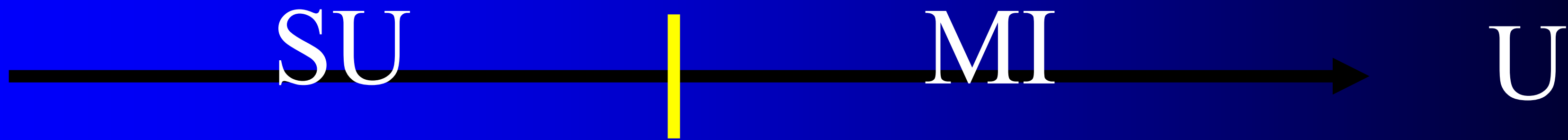
M. Kohl et al. cond-mat (2004)

Lattice: Mott transition

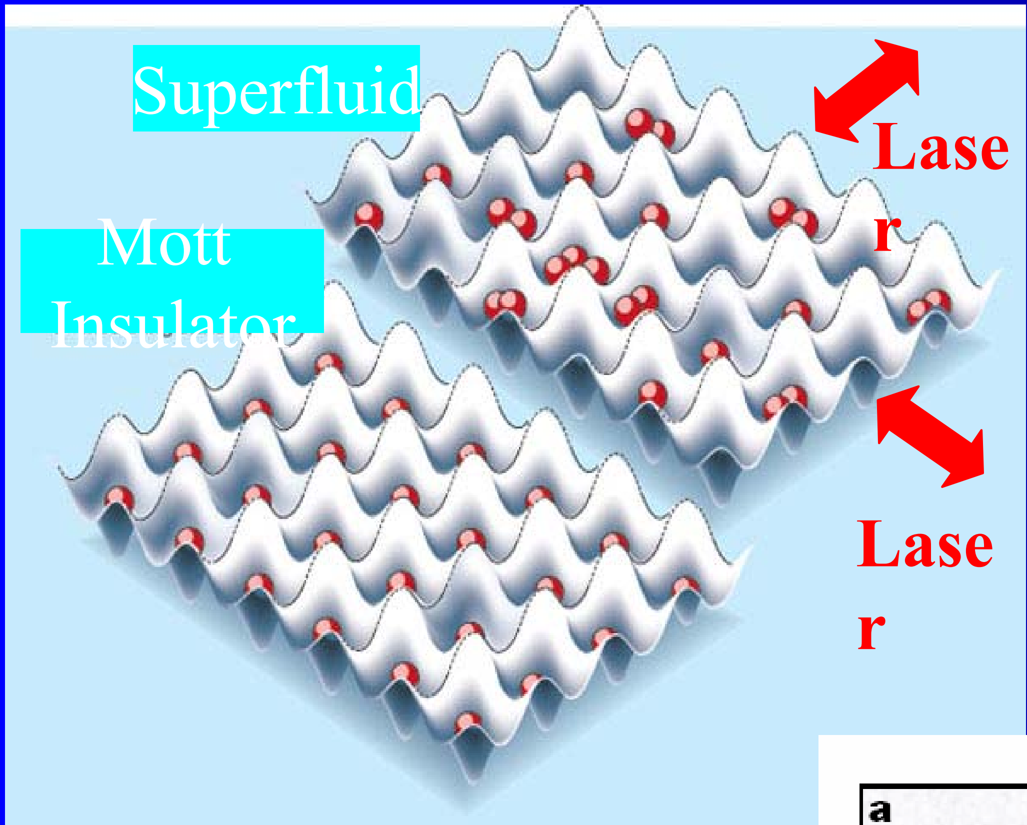


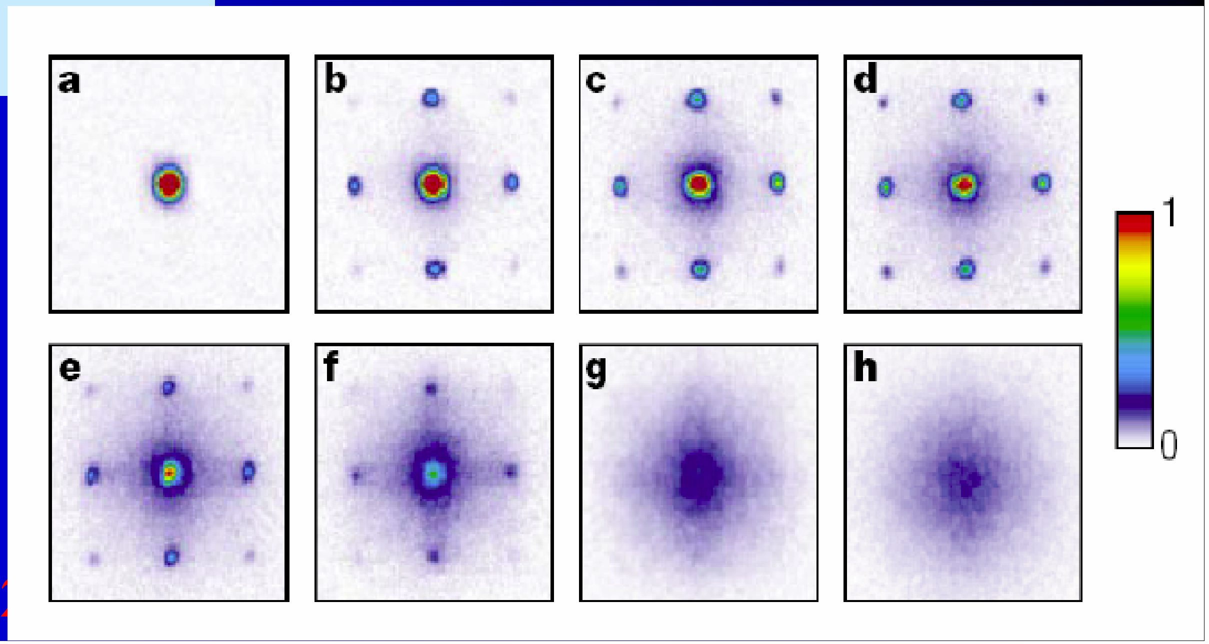
Costs U

Quantum phase transition



Mott transition and cold atoms



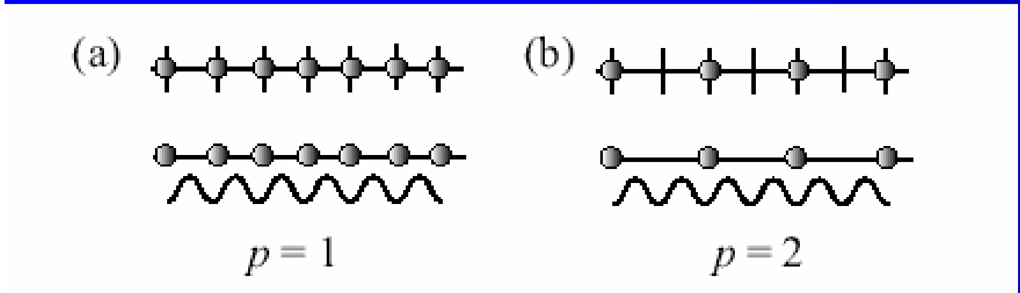


Superfluid to Mott insulator transition in a 3D optical lattice

[D. Jaksch et al. PRL 81 (1998)]

[M Greiner *et al.* Nature, 415 (200

How to treat?



$$H_L = \int dx V(x) \rho(x)$$

$$= \int dx V(x) e^{i2p(\pi \rho_0 x - \phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H_L = \int dx \cos(2\phi(x) + \delta x)$$

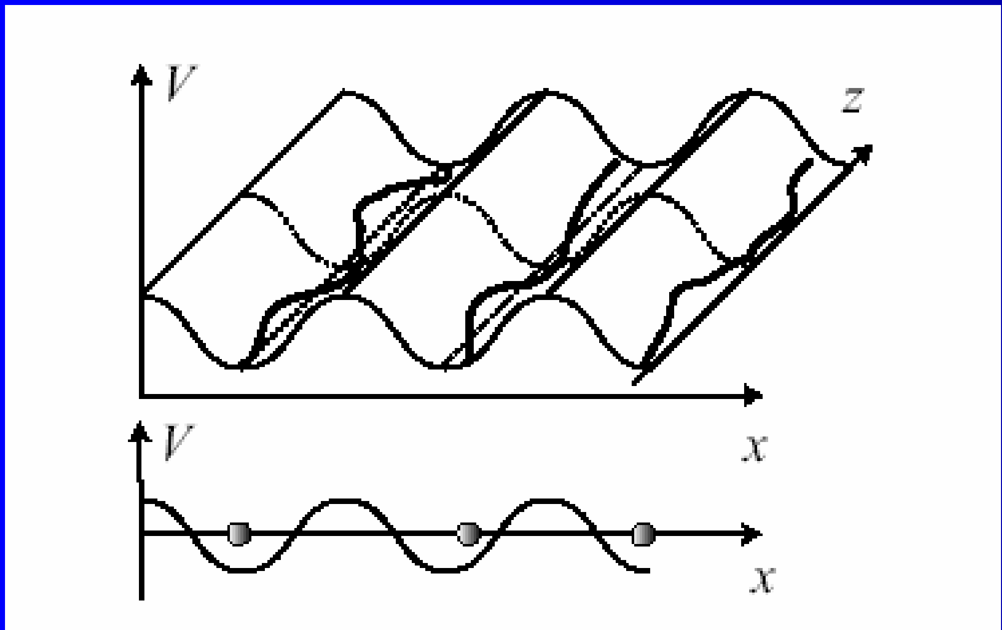
- Commensutate: $Q = 2\pi\rho_0$

$$H_L = \int dx \cos(2\phi(x))$$

Competition

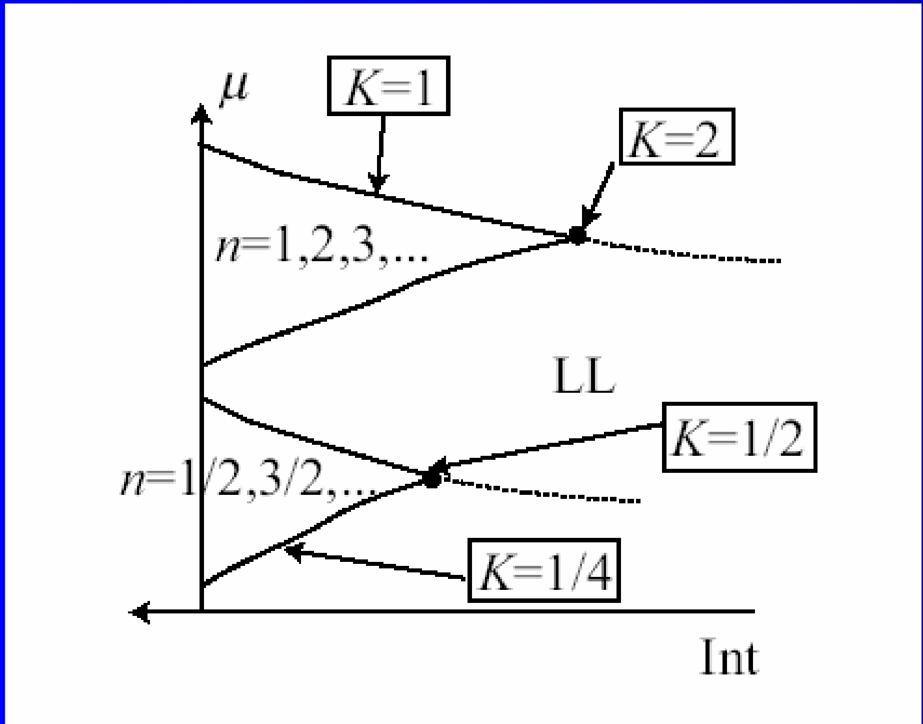
$$S = \int \frac{dx d\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \phi(x,\tau))^2 + u(\partial_x \phi(x,\tau))^2]$$

$$S_{\mathrm{lat}} = \int dx d\tau V \cos(2\phi(x,\tau))$$



Kosterlitz-Thouless transition

K=2



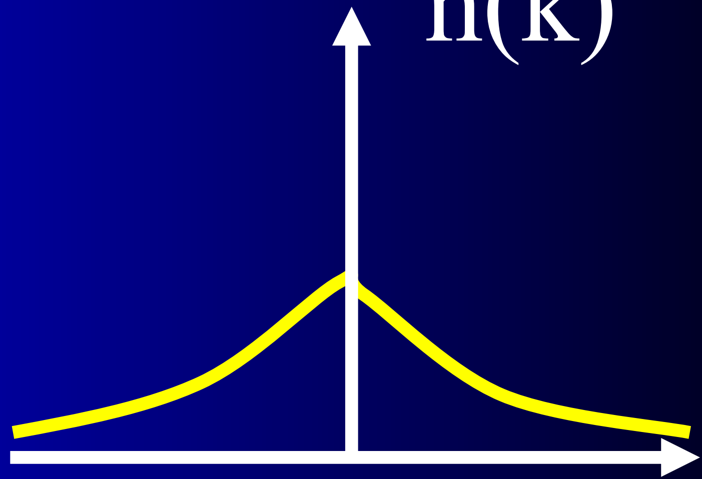
Mott insulator:

ϕ is locked

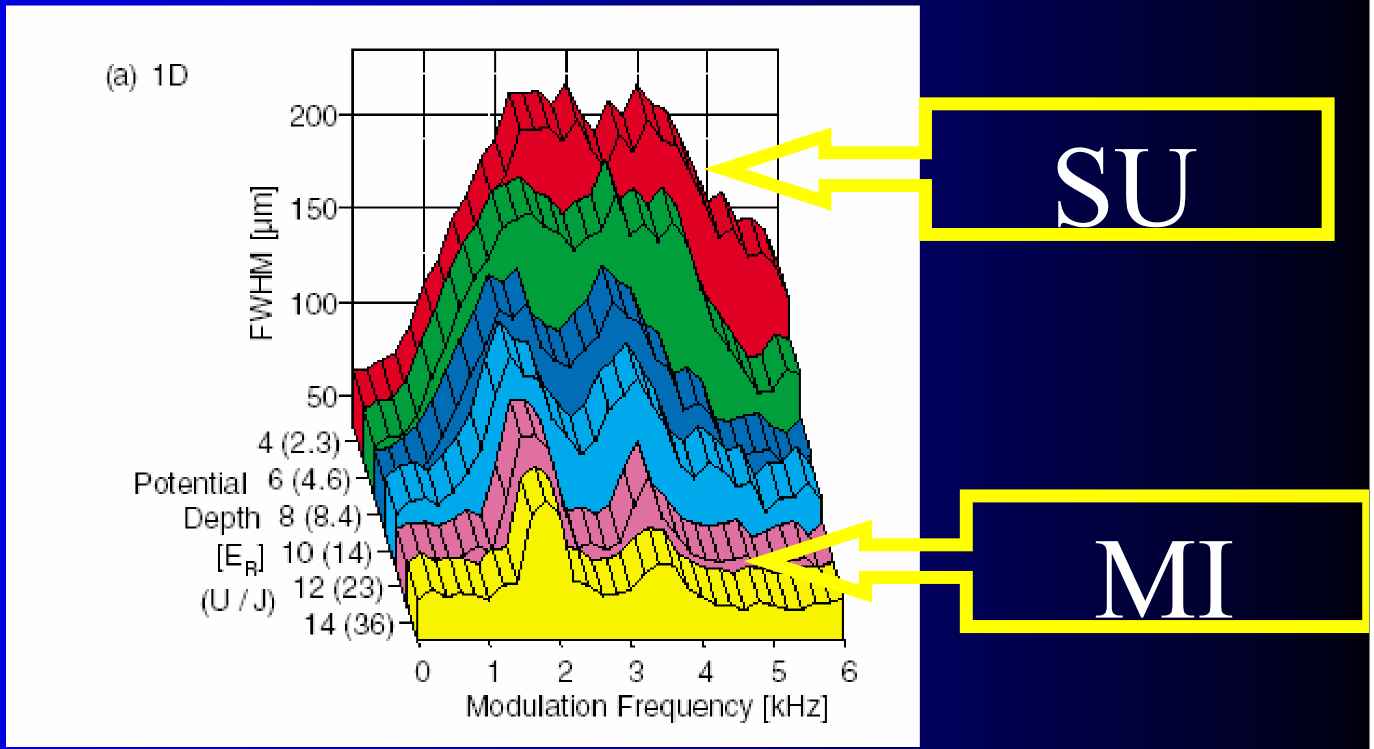
Density is fixed

Gap in the excitation spectrum

$$G(x) \propto \exp[-|x|/\xi]$$



Energy absorbed



T. Stoferle *et al.* PRL **92** 130403 (2004)

$$V(x) = [V_0 + A\sin(\omega t)]\cos(Qx)$$

Bosons 1D

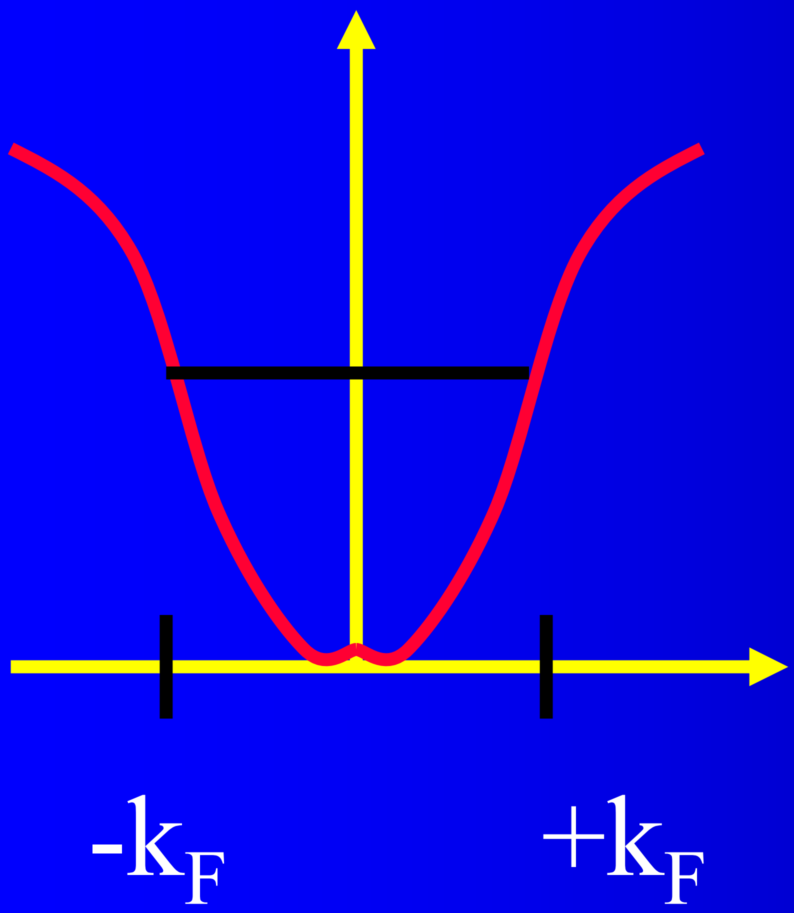
- Luttinger liquid physics
- New phases : Mott insulator
- Good qualitative agreement with exp.
- Correlation functions !

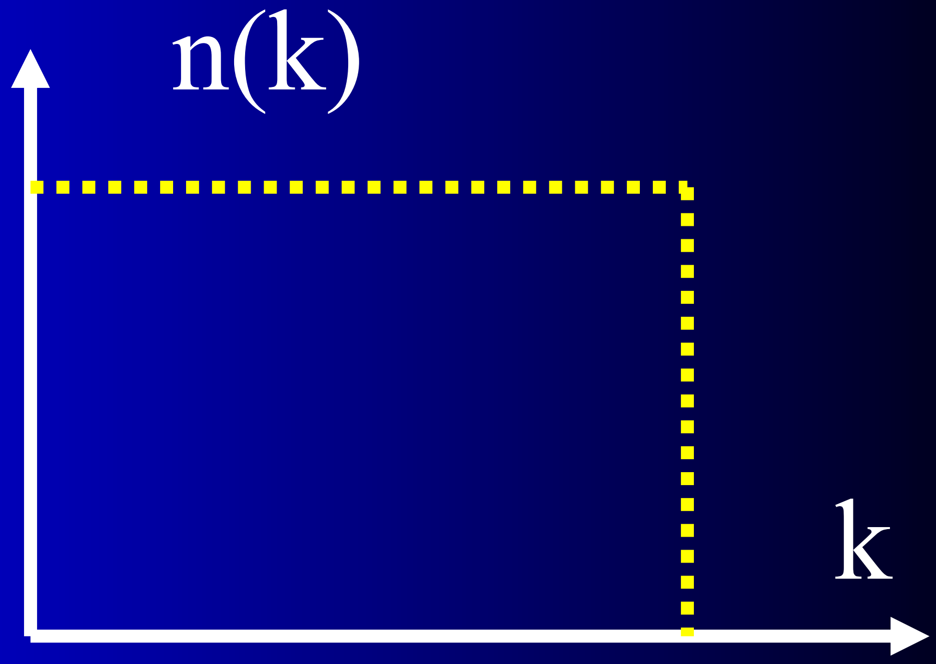
Many open points

- Confining potential
- Dynamics
- Disorder
- Etc……

Fermions in low d

Free electrons : basics

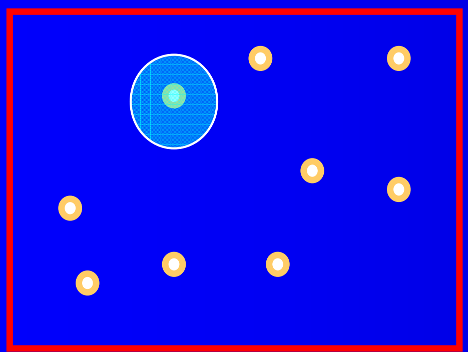


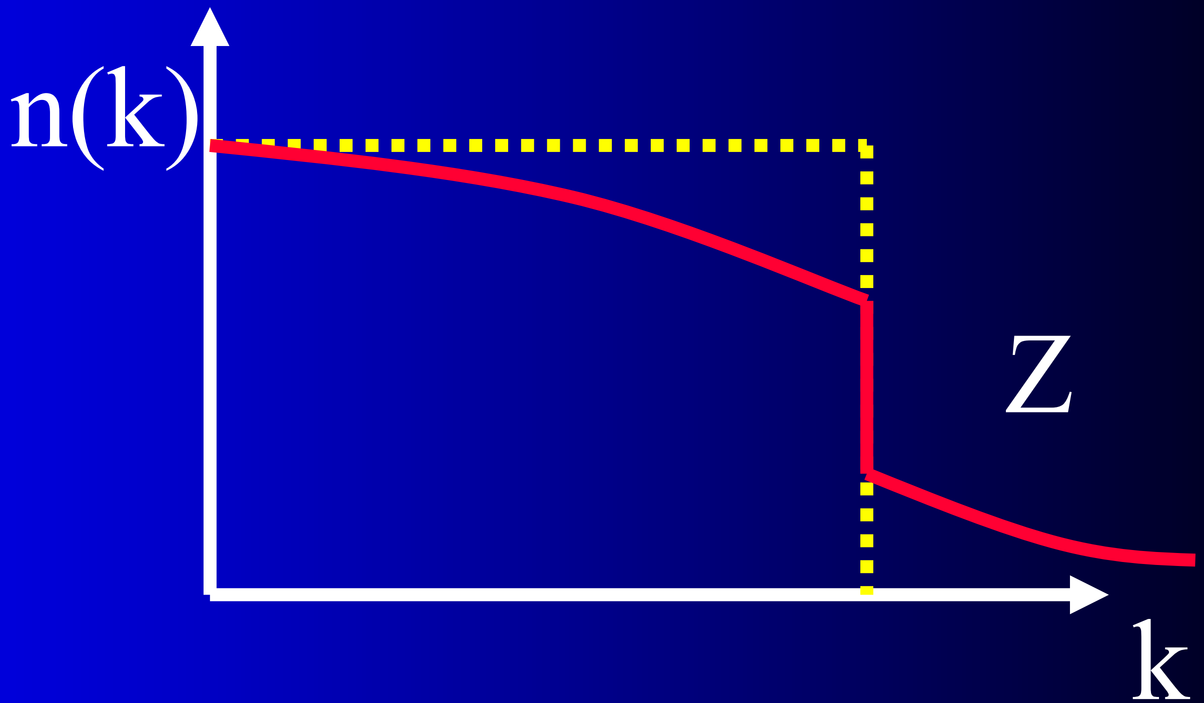


Individual excitations: fermions (particles or holes)

Fermi liquid : crash course

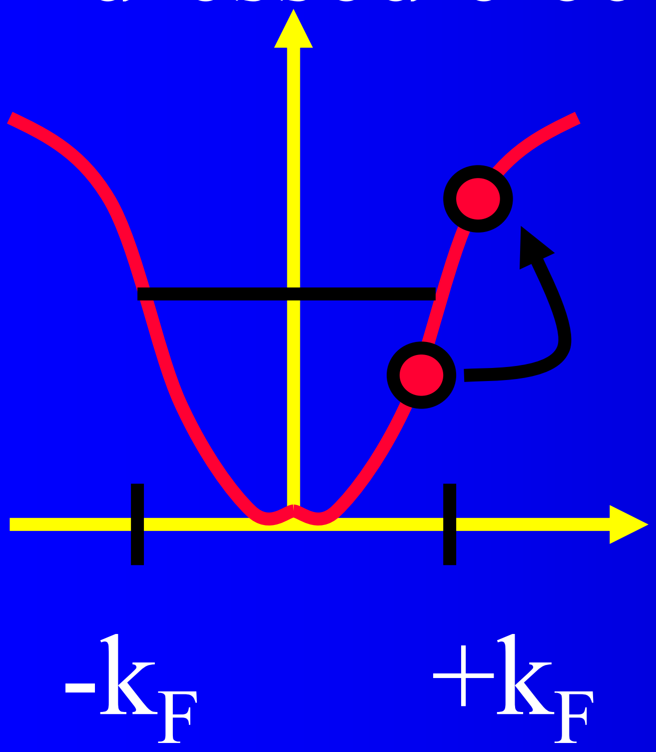
- Individual fermionic excitations exist (as for free electrons)

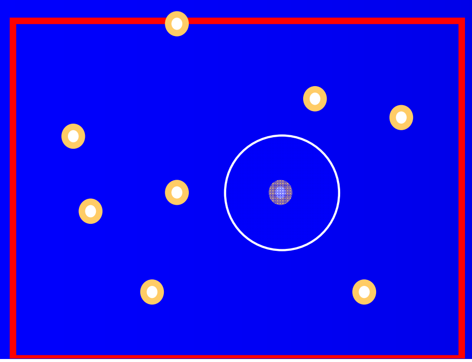


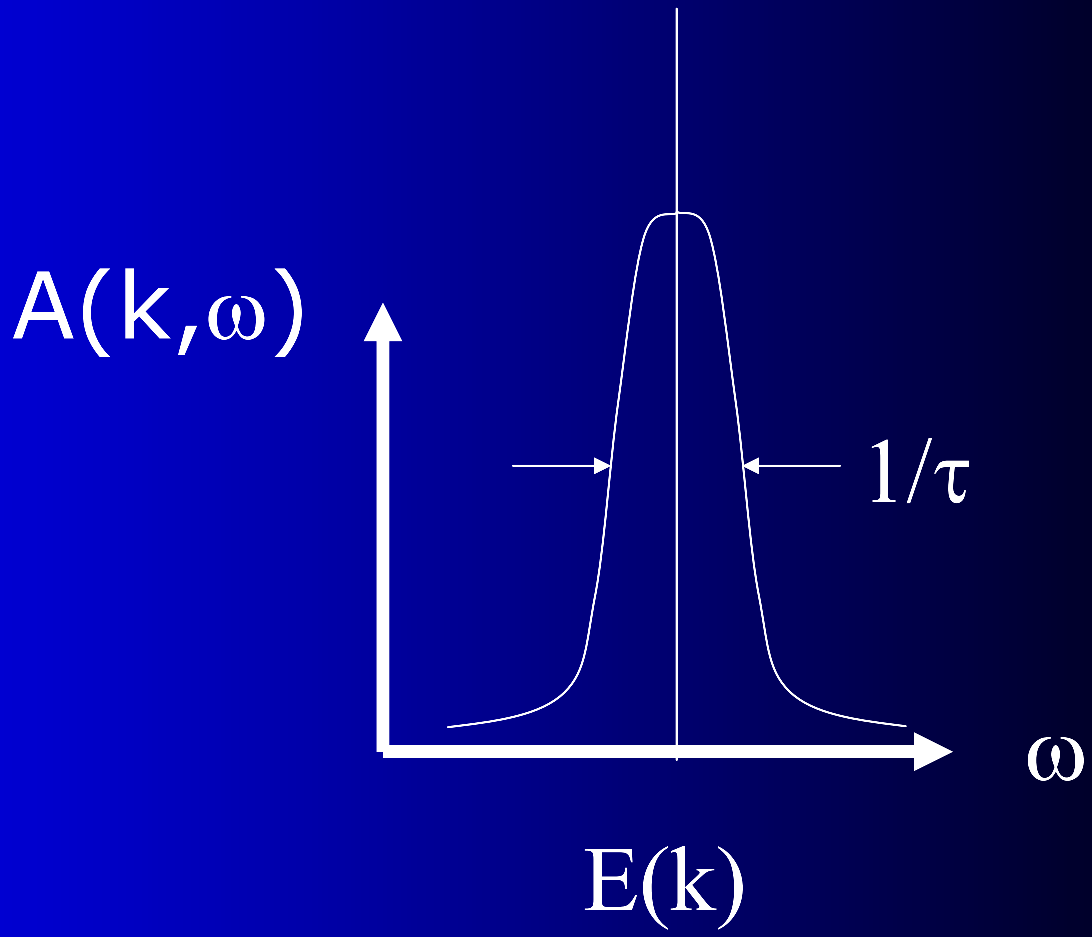


Excitations

- dressed electron (quasiparticles)





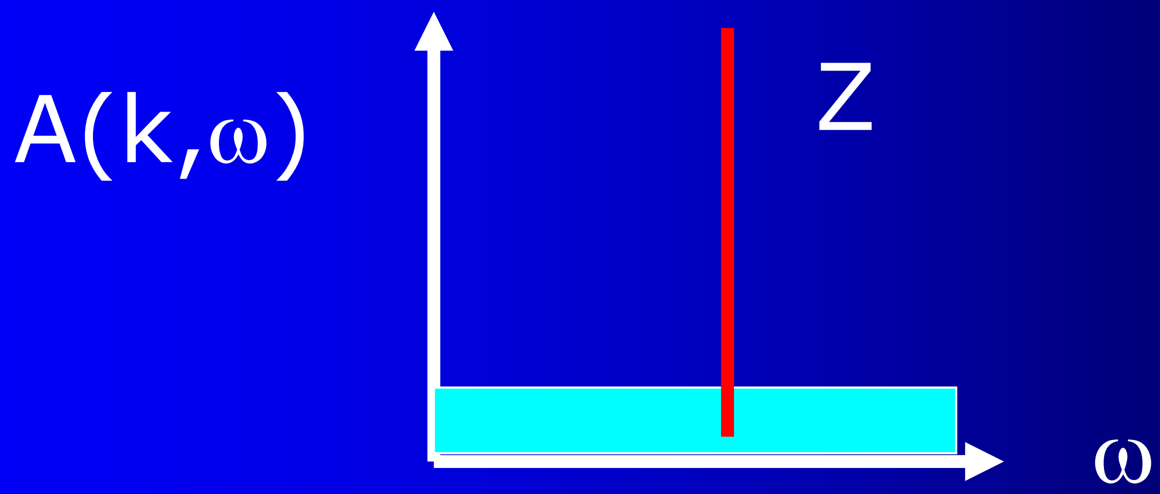


Properties

- Lifetime larger than average energy

$$\psi \propto e^{iE(k)t - t/\tau} \qquad 1/\tau \propto \omega^2$$

- QP are sharp (nearly free) excitations close to the Fermi surface



Fermi liquid theory

- Shown perturbatively in U
- Much more general and robust

Element	m*/m	χ/χ_0
Nb	2	1
3He	6	20
Heavy fermion	100	100

How to break this boring stuff

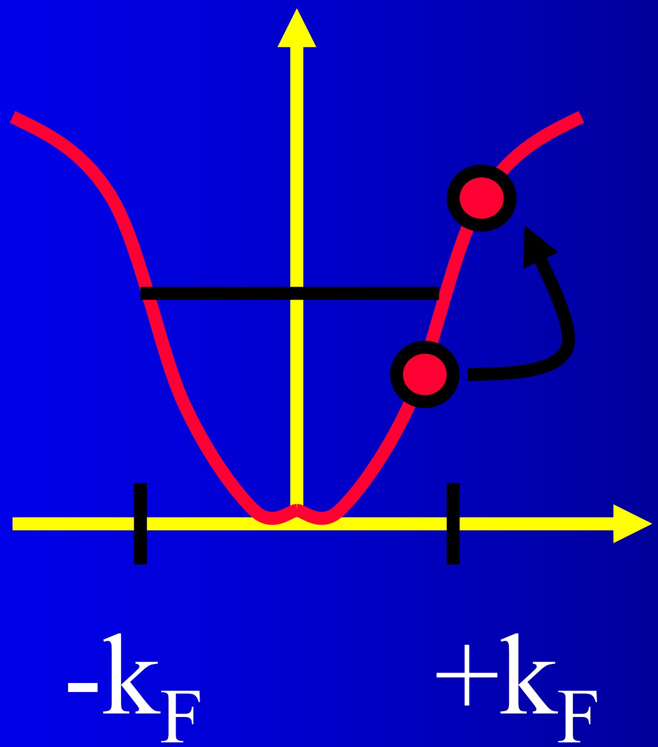
- Strong or unusual interactions (localized electrons, BCS, …)

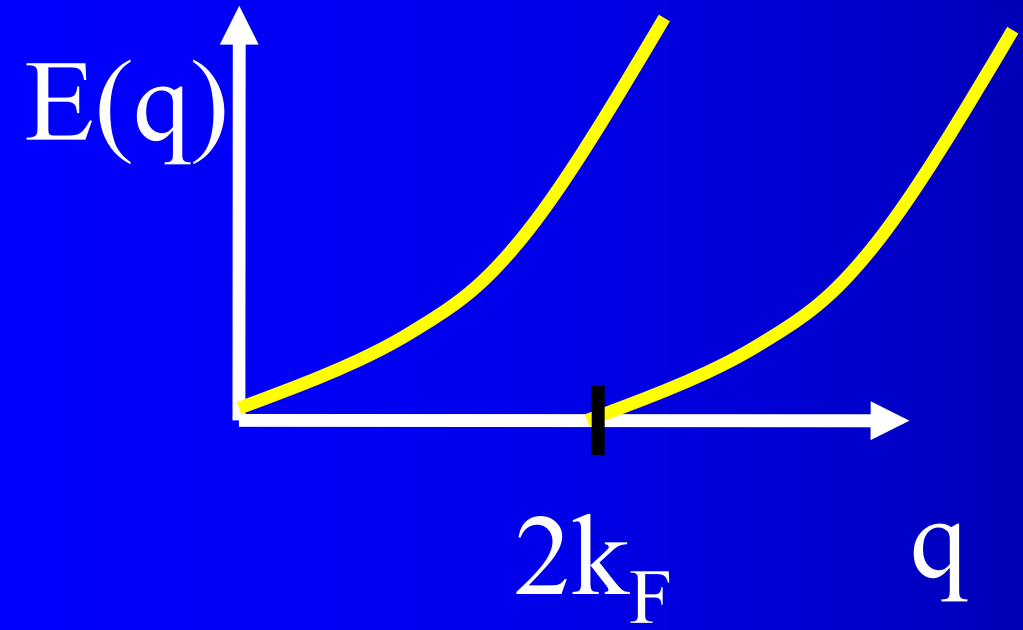
- Special fermi surfaces (nesting, singularities at E_F)

- Special dimensions (d=1, d=2 (?))

Particle hole excitations

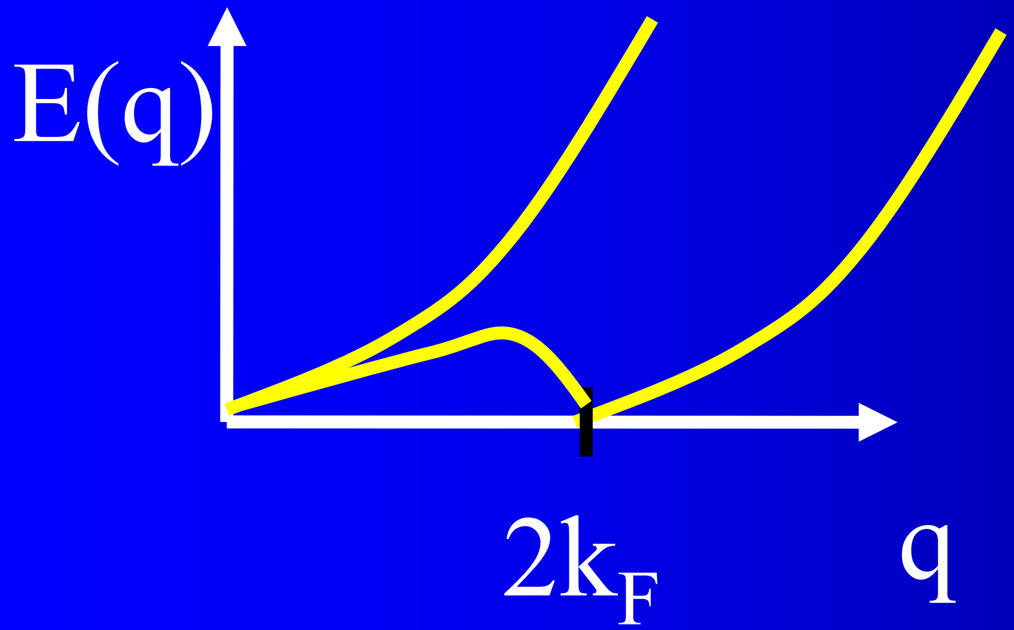
$$E(k,q) = \varepsilon(k+q) - \varepsilon(k)$$





D>1:

continuum



D=1:

Well defined excitations

$$E(q) = v_F q$$

Same method than for bosons

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$2k_F = 2\pi\,\rho_0$

$$H = \frac{\hbar}{2\pi}\int dx\left[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2\right]$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2}\sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))}e^{-i\theta(x)}$$

No interactions K= 1

Interactions

$$\rho(x)\rho(x') \approx (\nabla\Phi(x))^2$$

$$H = \int \frac{dx}{2\pi}[uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\Phi(x))^2]$$

u velocity of sound

K< 1 : repulsive

K>1 : attractive

$$O_{SU}(x) = \psi^*_R(x)\psi^*_L(x) \approx e^{i2\Theta(x)}$$

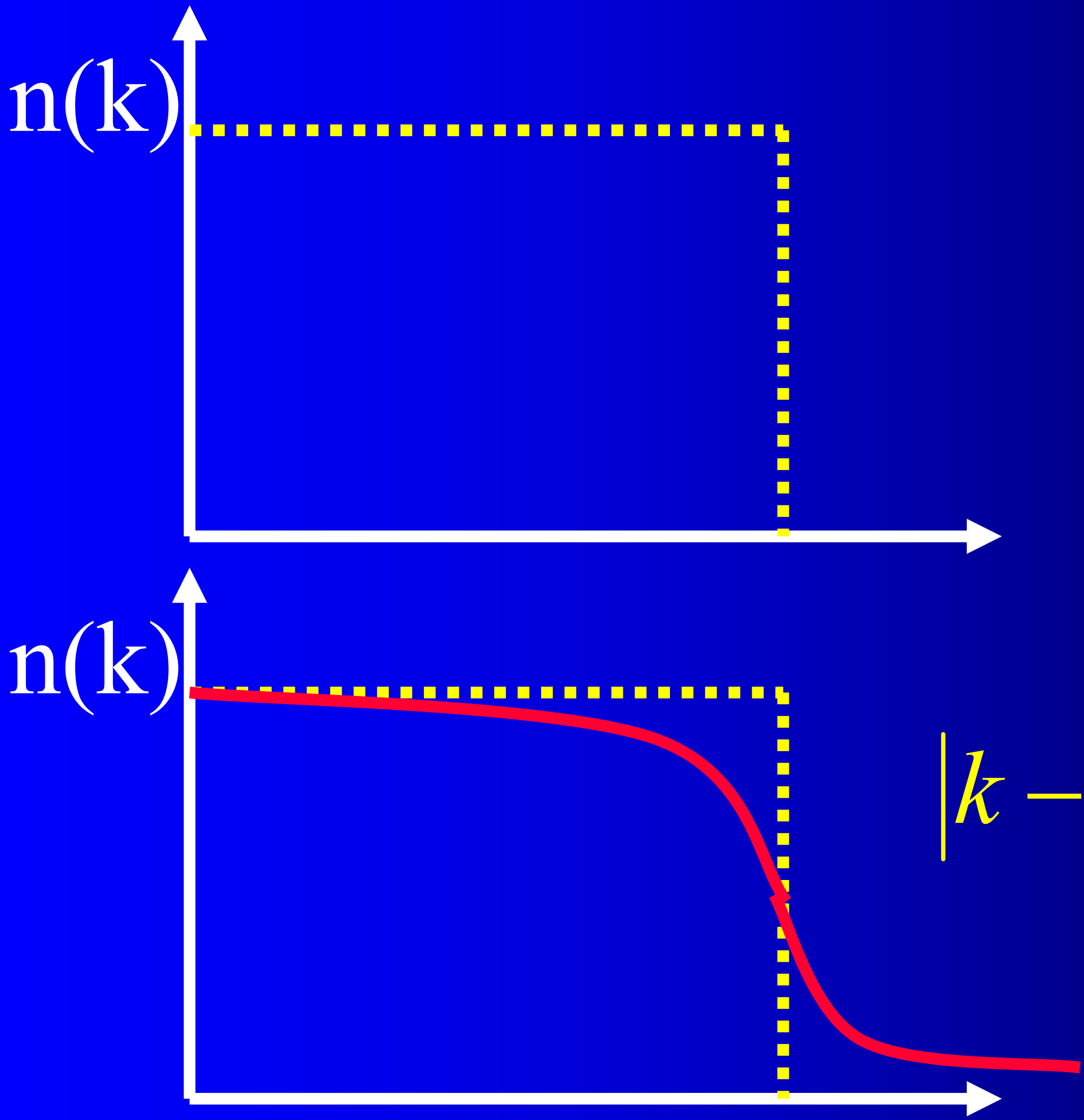
$$\langle O_{SU}(x) O_{SU}(0) \rangle = \left(\frac{1}{x}\right)^{1/2K}$$

Single particle excitations

$$\langle \psi_R(x)\psi^*_R(0) \rangle = \left(\frac{1}{x}\right)^{\frac{1}{2}[K+K^{-1}]} \quad e^{iArg(\tau/x)}$$

$$K = 1 \qquad \langle \psi_R(x)\psi^*_R(0) \rangle = \frac{1}{x - v_F\tau}$$

No Landau Quasiparticles

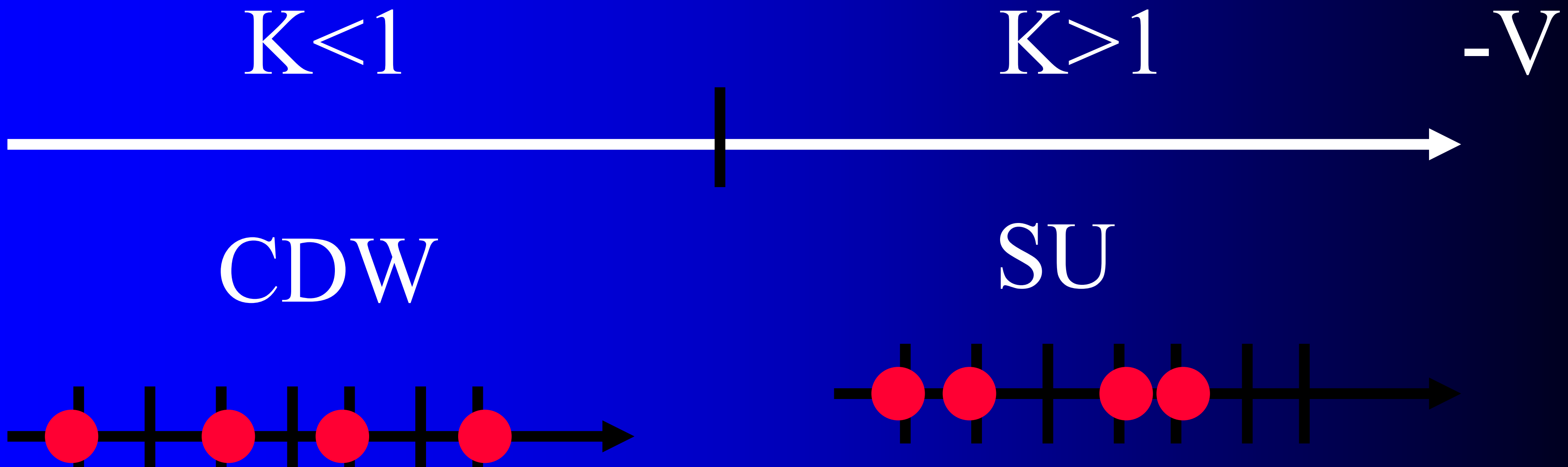


$$\left|k - k_F\right|^{\frac{1}{2}[K+K^{-1}]-1}$$

Phase diagram

$$\chi(q,\omega) = \int dx\,d\tau \; e^{i(qx+\omega\tau)} \chi(x,\tau) \qquad \chi \approx \omega^{\eta-2}$$

Most divergent fluctuations



System with spin

Same treatment

$$\rho_\uparrow \to \nabla\Phi_\uparrow \qquad \rho_\downarrow \to \nabla\Phi_\downarrow$$

More convenient

$$\rho = \frac{1}{\sqrt{2}}(\rho_\uparrow + \rho_\downarrow) \qquad \sigma = \frac{1}{\sqrt{2}}(\rho_\uparrow - \rho_\downarrow)$$

$$H_{kin} = H_\uparrow + H_\downarrow = H_\rho + H_\sigma$$

$$H_{\text{int}} = U \sum_i \rho_\uparrow \rho_\downarrow = U(\rho + \sigma)(\rho - \sigma)$$

$$= U(\rho\rho - \sigma\sigma)$$

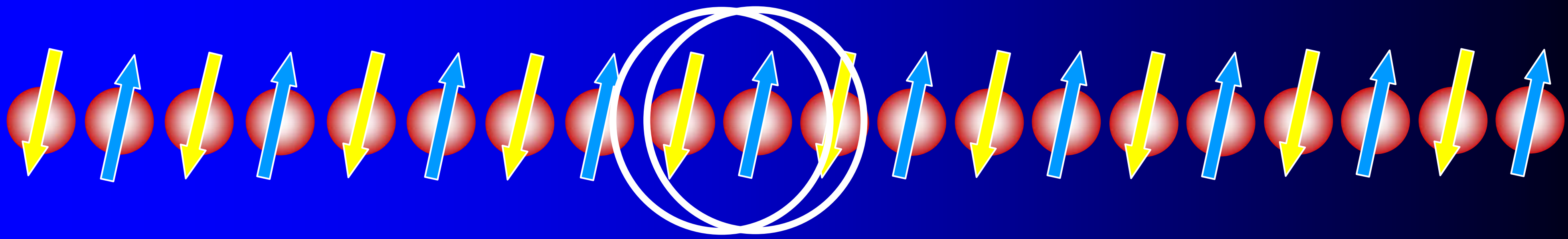
$$H = H_\rho + H_\sigma$$

(u_ρ, K_ρ) Charge excitations

(u_σ, K_σ) Spin excitations

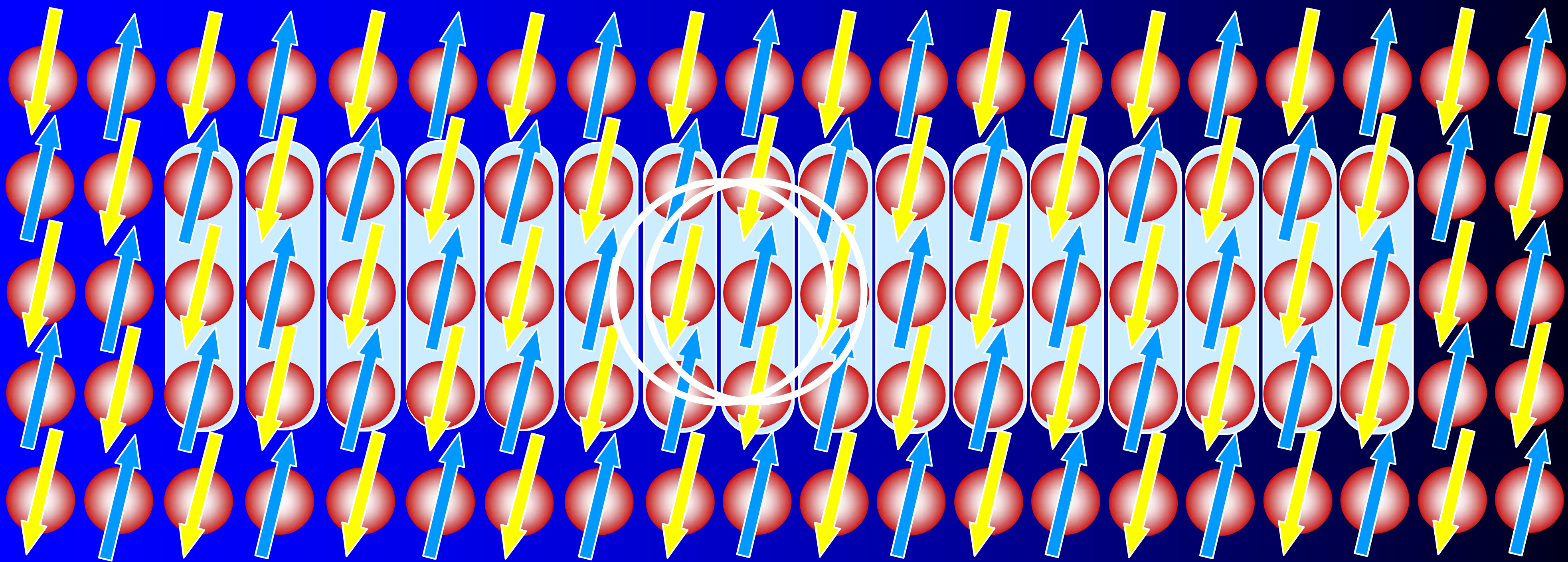
Charge-spin separation

Spin-Charge Separation



Spin-Charge Separation – higher D ?

Spin Charge



Loose order: Energy increases with spatial separation

→ Force that binds spin and charge

Correlation functions

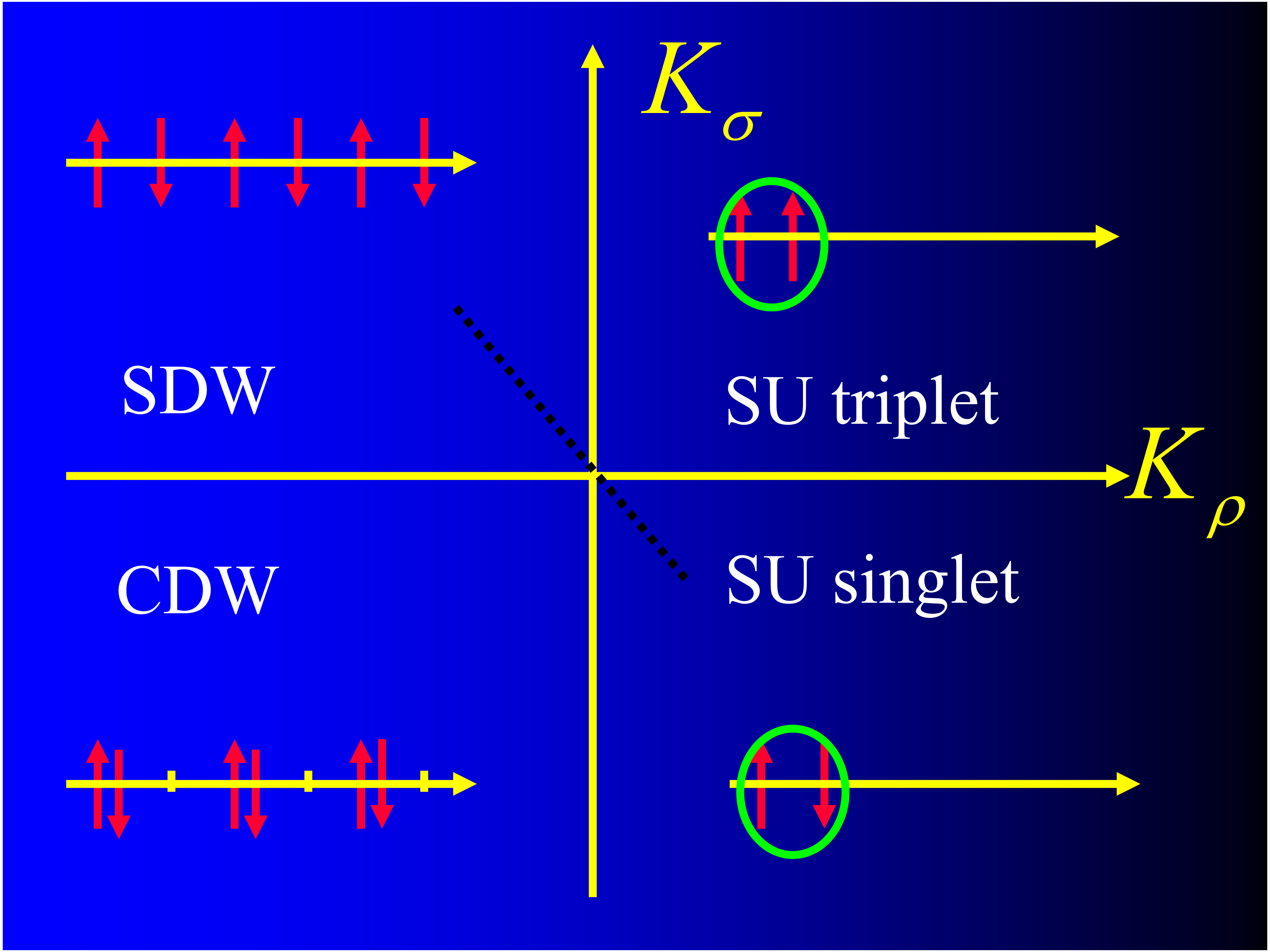
$$\langle S(x)S(0)\rangle = \frac{1}{x^2} + \cos(2k_F x)\left(\frac{1}{x}\right)^{K_\sigma + K_\rho}$$

- Perturbation (small U)

$$u_\rho K_\rho = u_\sigma K_\sigma = v_F$$

$$u_\rho / K_\rho = v_F + U / \pi$$

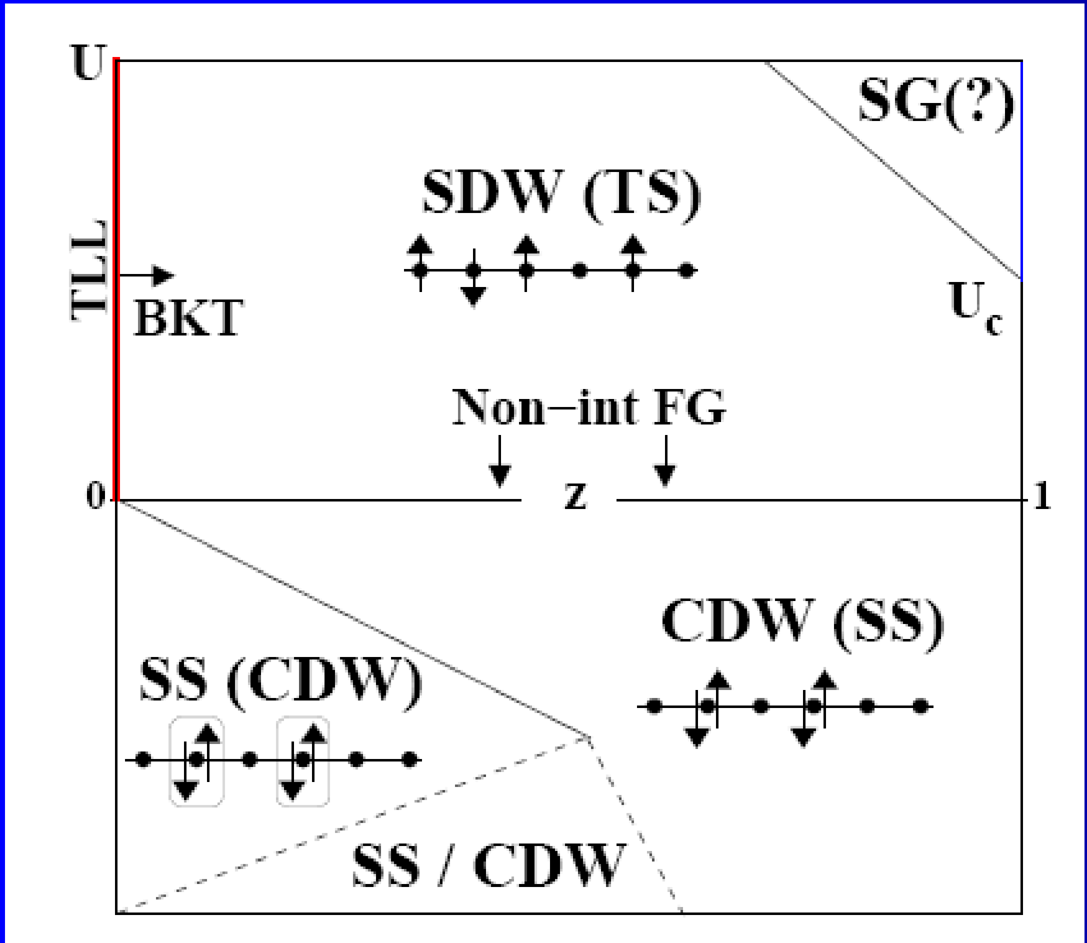
$$u_\sigma / K_\sigma = v_F - U / \pi$$



K_σ
SDW
SU triplet
K_ρ
CDW
SU singlet

Different hoppings

M.A. Cazalilla, A.F. Ho, TG (2005)



Spin gap

$$S(q \sim 0, \omega) = \langle S^+ S^- \rangle = \sqrt{\omega^2 - \Delta^2}$$

$$z = |v_\uparrow - v_\downarrow| / (v_\uparrow - v_\downarrow)$$

Fermions

- Luttinger liquid physics + Mott
- New physics ($v_\uparrow \neq v_\Downarrow$)
- Possibility of pairing (singlet/triplet)
- n(k) is useless !! Needs correlations