The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR 1666 - 19

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Experiments on a Bose gas of rubidium atoms in 2 and 3 dimensions

Presented by:

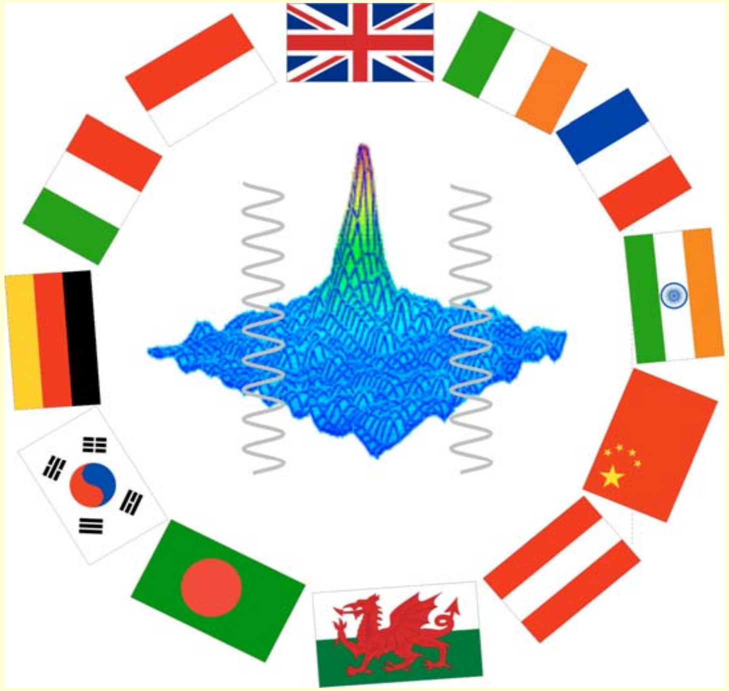
Christopher Foot

University of Oxford, UK

Experiments on a Bose gas of rubidium atoms in 2 and 3 dimensions

Christopher Foot, University of Oxford, UK

ICTP Summer School, July 2005



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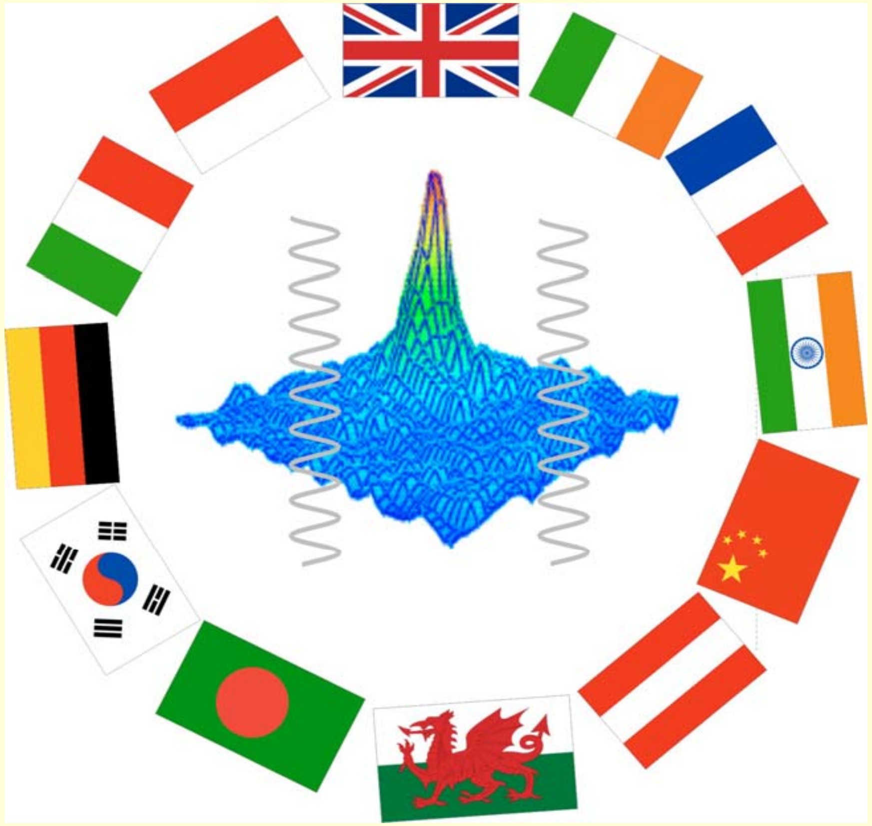


Reference books on BEC

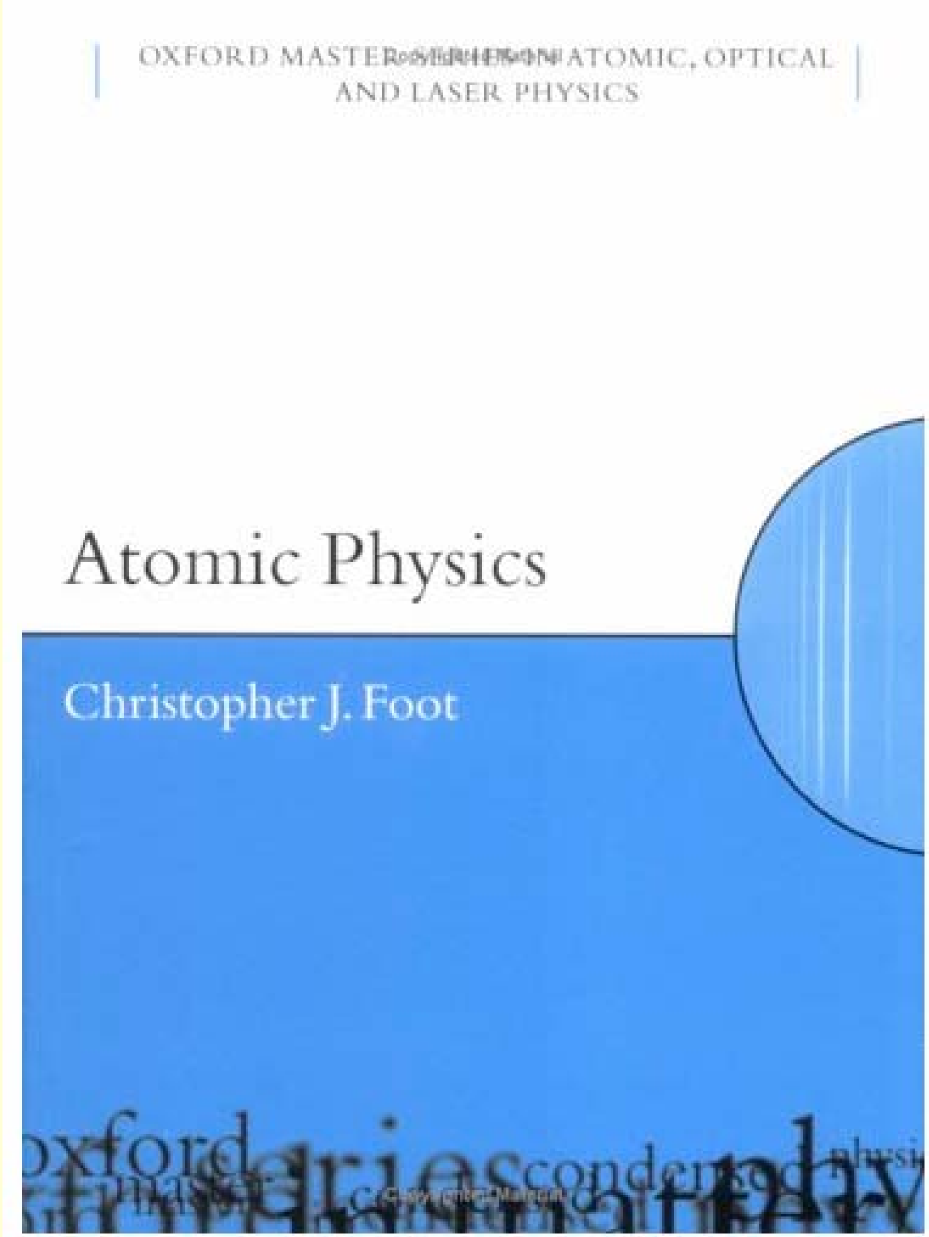
- * Pethick & Smith, CUP
- Pitaevskii & Stringari, OUP

- * Pethick & Smith, Chapter 9 has five problems on vortices very relevant to this lecture.

For first year graduates/
fourth year undergrads.
Chapters on:
Magnetic trapping
& evaporative cooling,
BEC,
Laser cooling,

Published by OUP, 2005

Contains problems on 3D condensates etc. with some solutions on the web site.



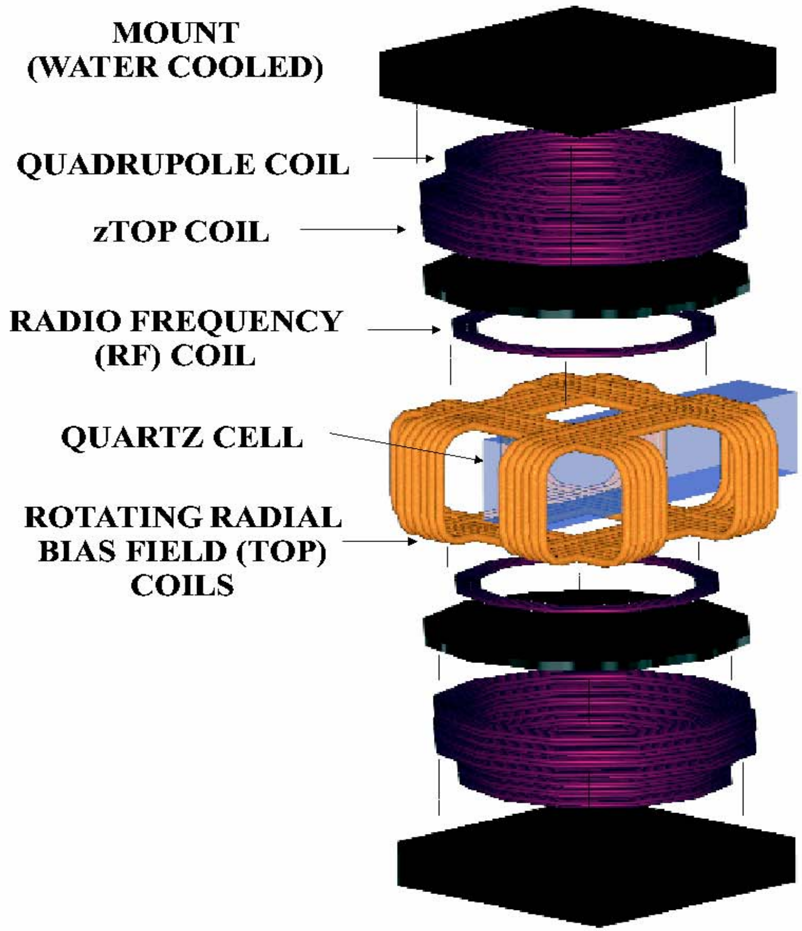
View of the magnetic trap and quartz vacuum cell

BEC

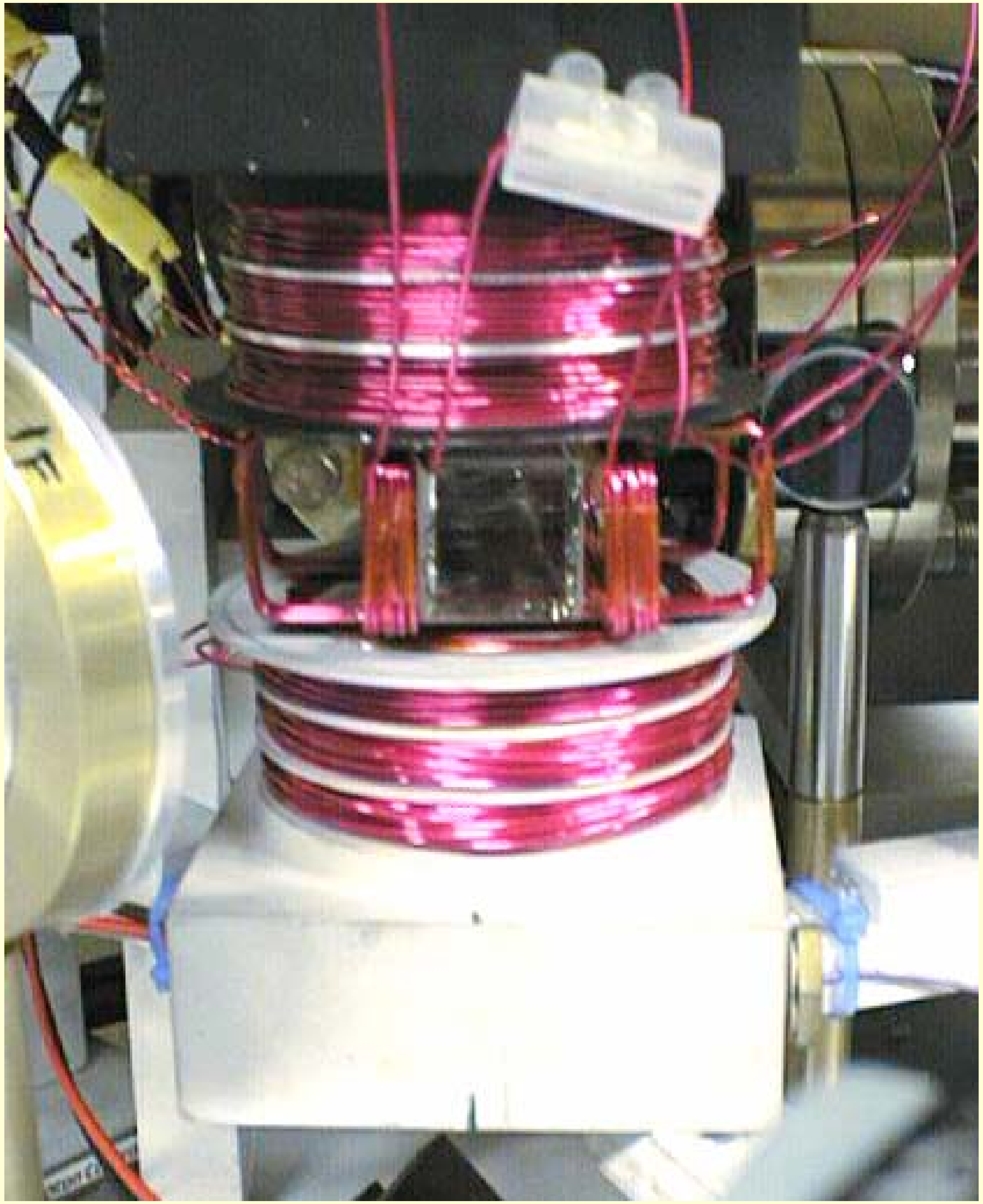
10^5 rubidium atoms.

Temperature ~ 50 nK

Density ~ 10^{14} cm^{-3}

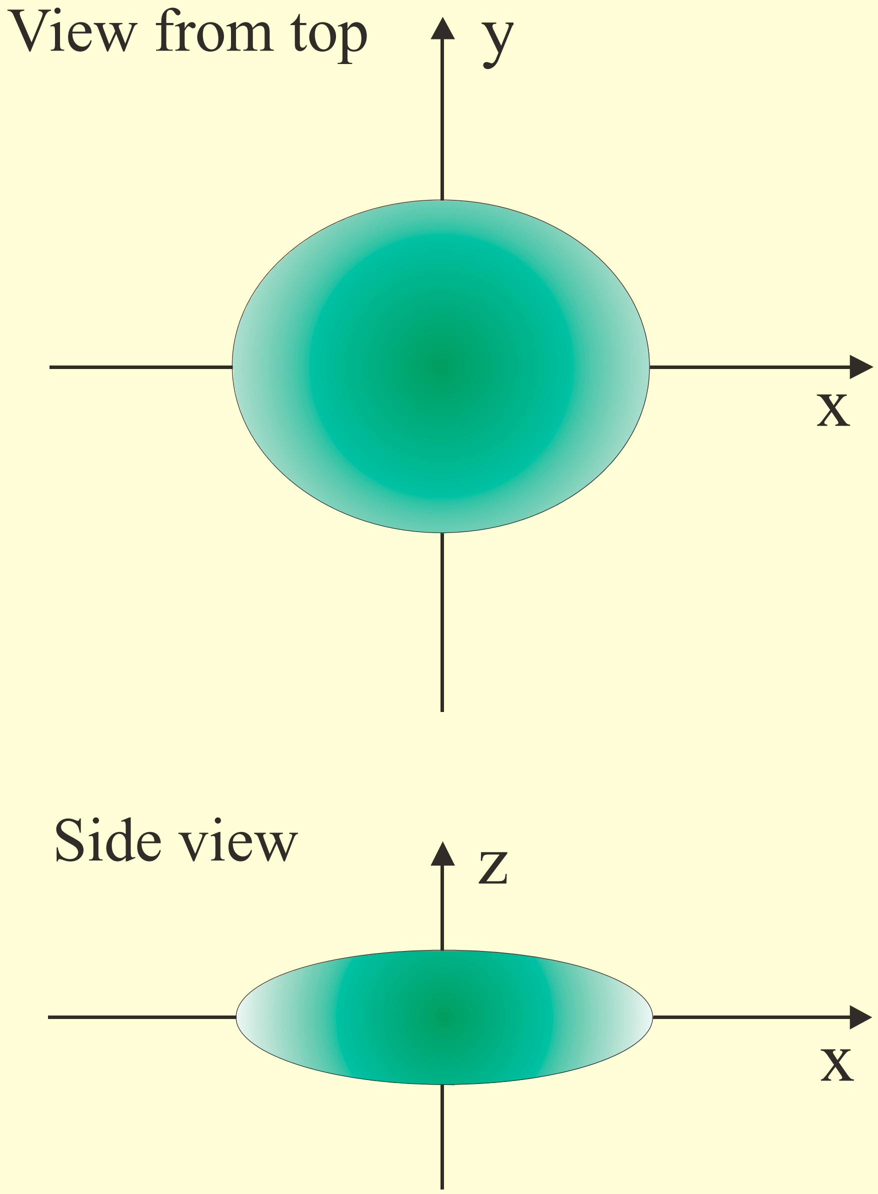


Magnetic trap and quartz vacuum cell



Shape of BEC in a TOP trap

More like a pancake than a cigar (as in Ioffe traps, e.g. Jean Dalibard's expts.)



Outline: talk about experiments on the superfluidity of a Bose gas

- Scissors mode – irrotational flow
- Nucleation of vortices – hysteresis
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Quantised circulation in a quantum fluid

- The velocity field ∝ phase gradient

$$\underline{v} = \frac{\hbar}{M} \underline{\nabla} \psi (\underline{r})$$

- Hence velocity field is irrotational

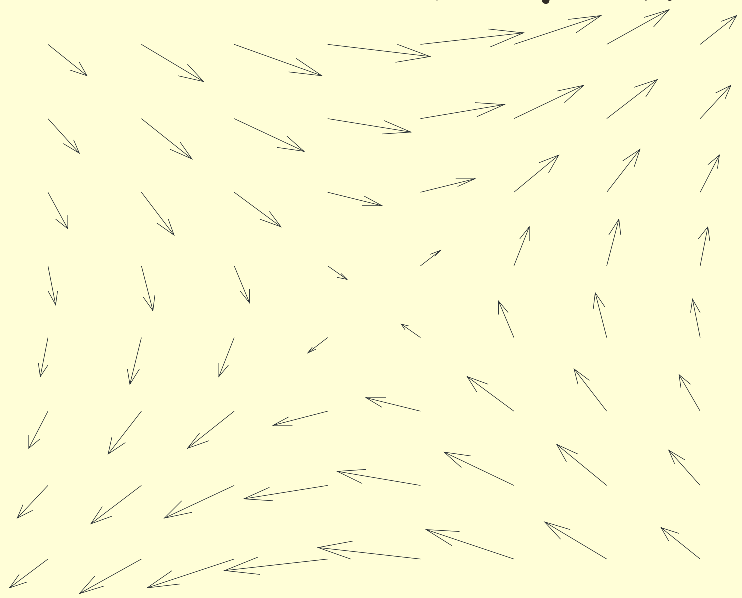
$$\underline{\nabla} \times \underline{v} = 0$$

- Circulation around a closed contour is quantised

$$\oint_c \underline{v} . \underline{dl} = \frac{\hbar}{m} \oint_c \underline{\nabla \psi} . \underline{dl} = \frac{\hbar}{m} 2 \pi \times \text{integer}$$

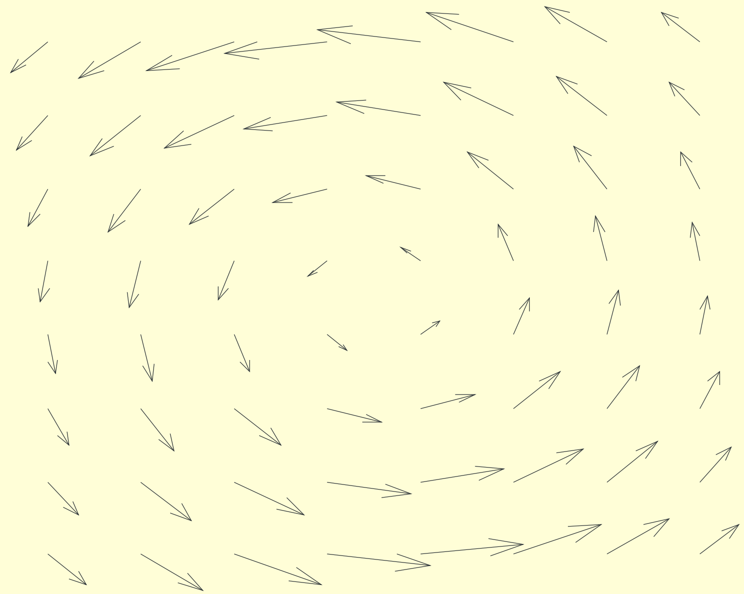
- Zero circulation = irrotational flow
- Non-zero circulation = vortices

Irrotational flow



$$\underline{\nabla} \times \underline{v} = 0$$

Rotational flow



$$\underline{\nabla} \times \underline{v} \neq 0$$

Excitation of scissors mode

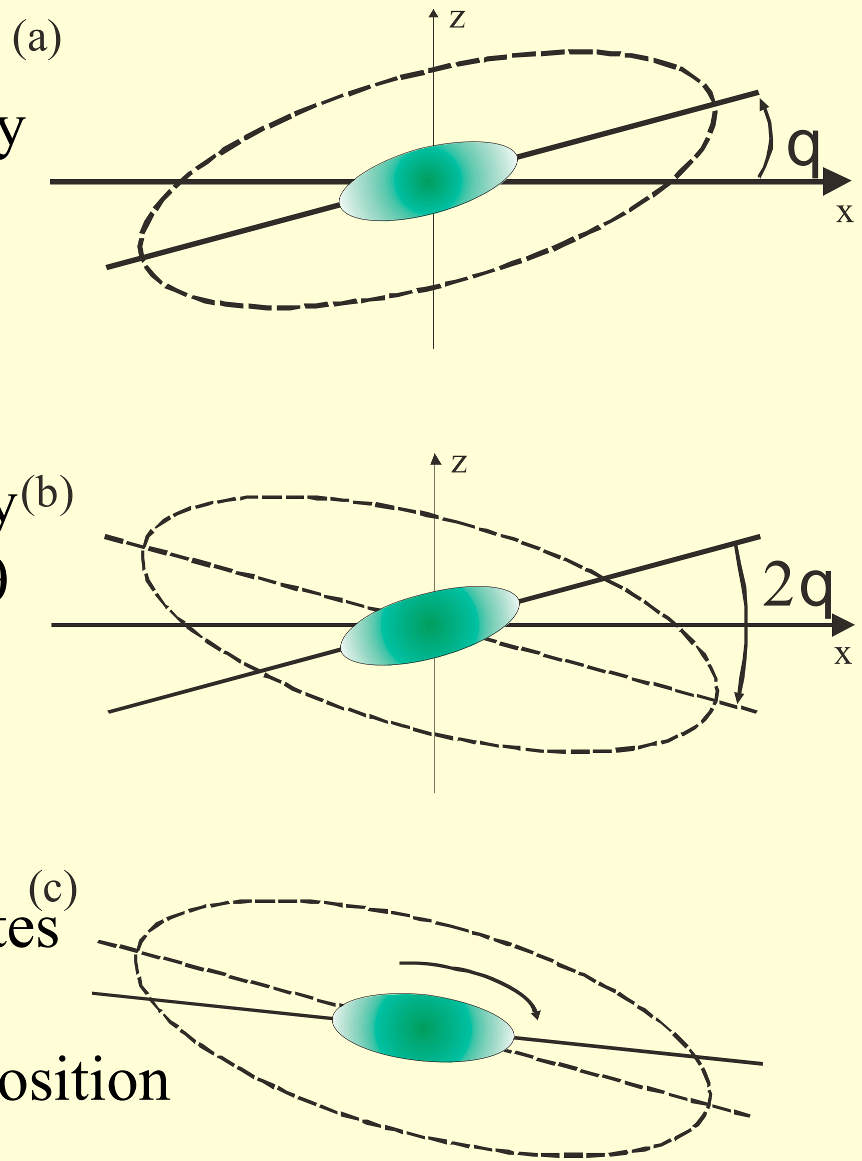
Trap tilted adiabatically to angle θ

θ

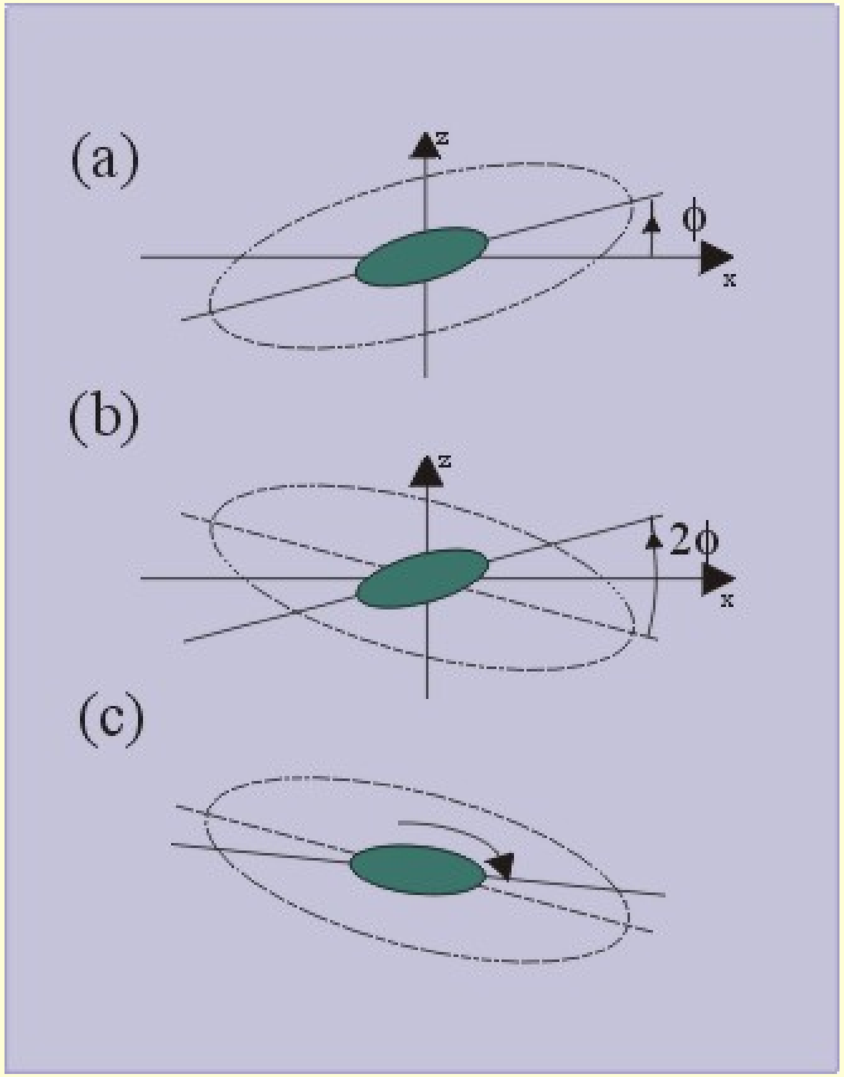
Trap suddenly rotated by -2θ

2θ

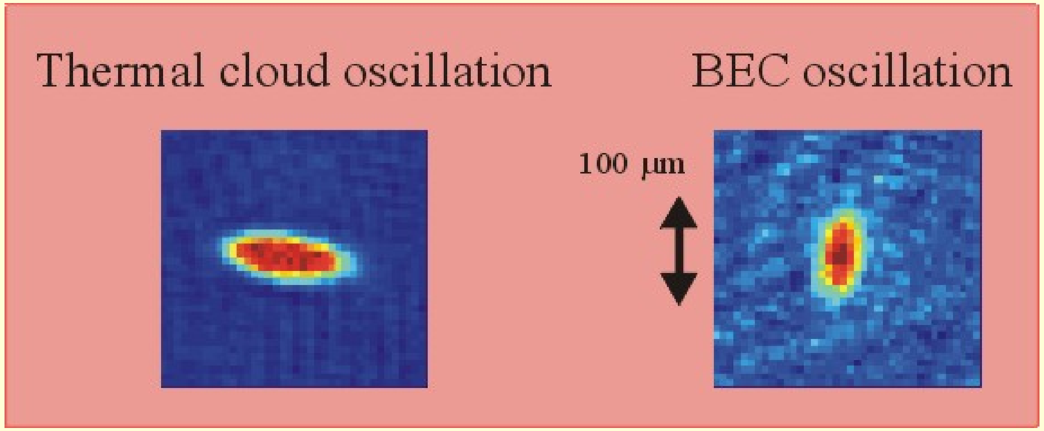
Cloud oscillates about new equilibrium position



Exciting the scissors mode oscillation



Angle extracted from a 2D Gaussian fit

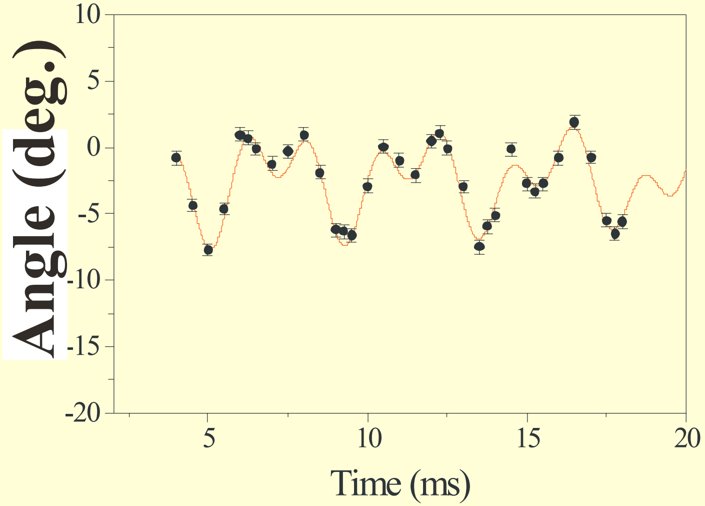


In trap

15 ms TOF

Scissors mode results

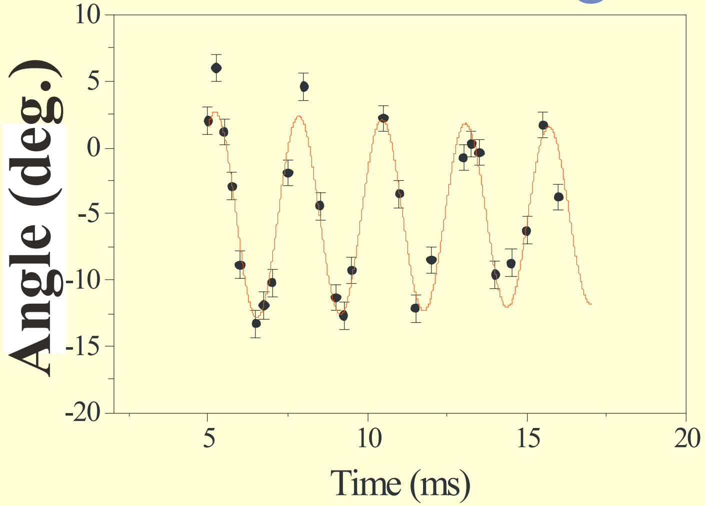
Thermal cloud - two frequencies



f_1=481(1) Hz
f_2=226.4(0.6) Hz

Γ=13(3) Hz

Condensate - single frequency



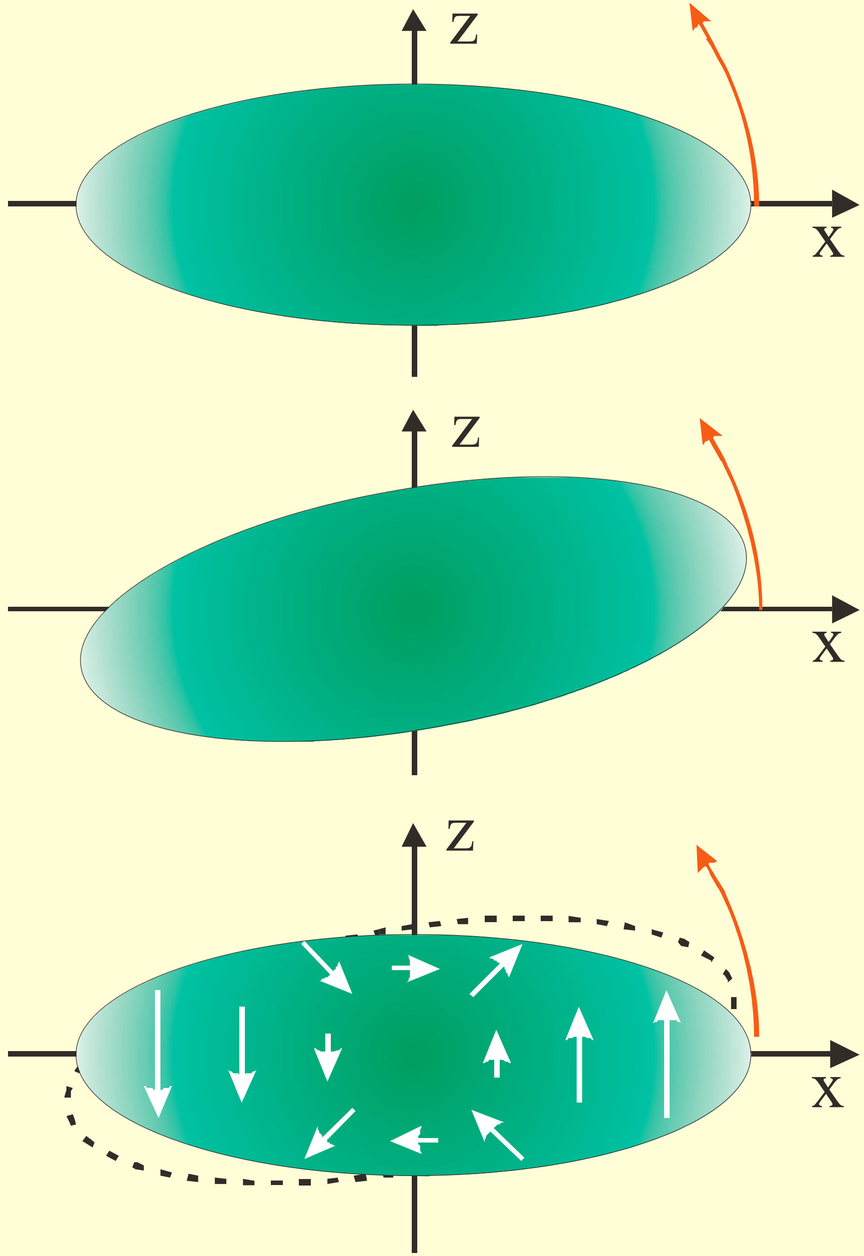
f_{scis}=382(1) Hz

Γ=15(5) Hz

f_{radial}=127.6(0.9) Hz

Described in Pitaevskii & Stringari's book

Irrotational Flow



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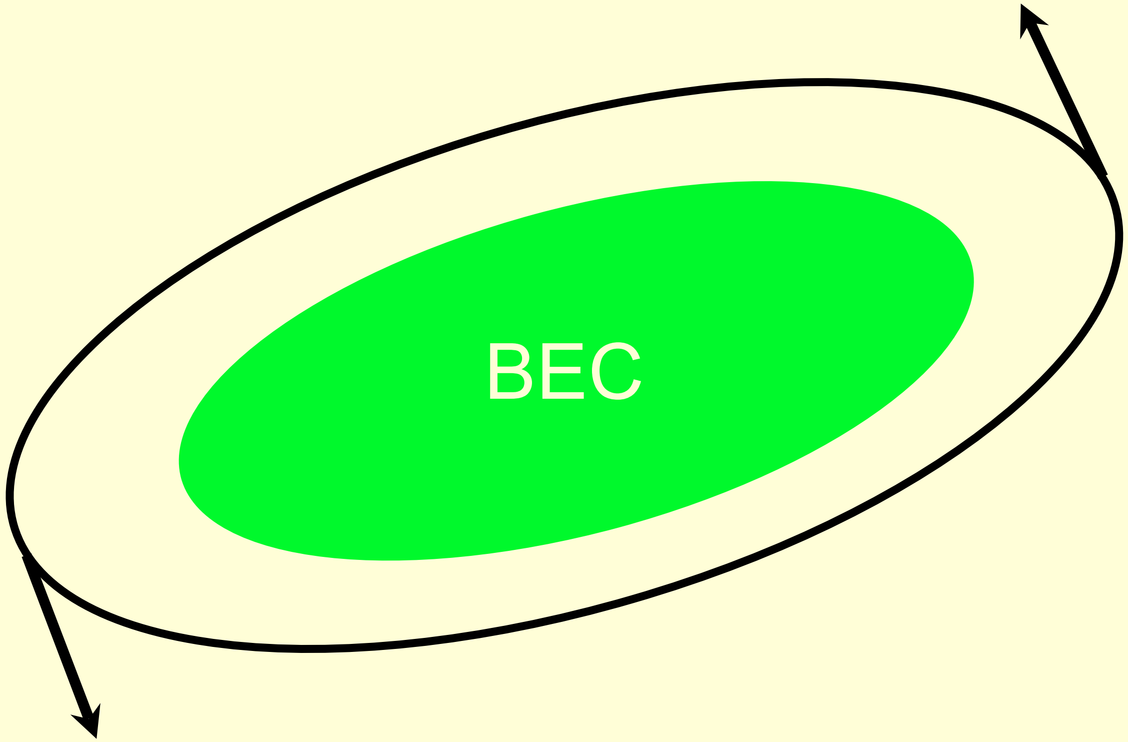
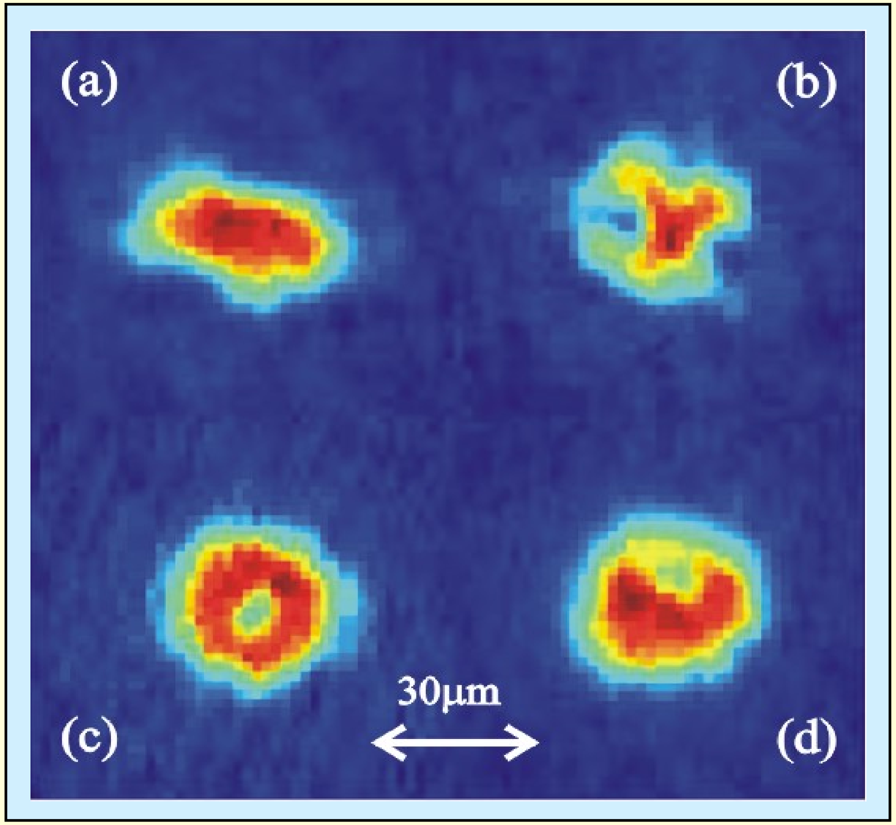
Nucleation of a vortex

- c.f. Jean Dalibard’s talk (first week of Summer School) ?

Rotation of the confining magnetic potential
to impart angular momentum



Producing a single, centred vortex



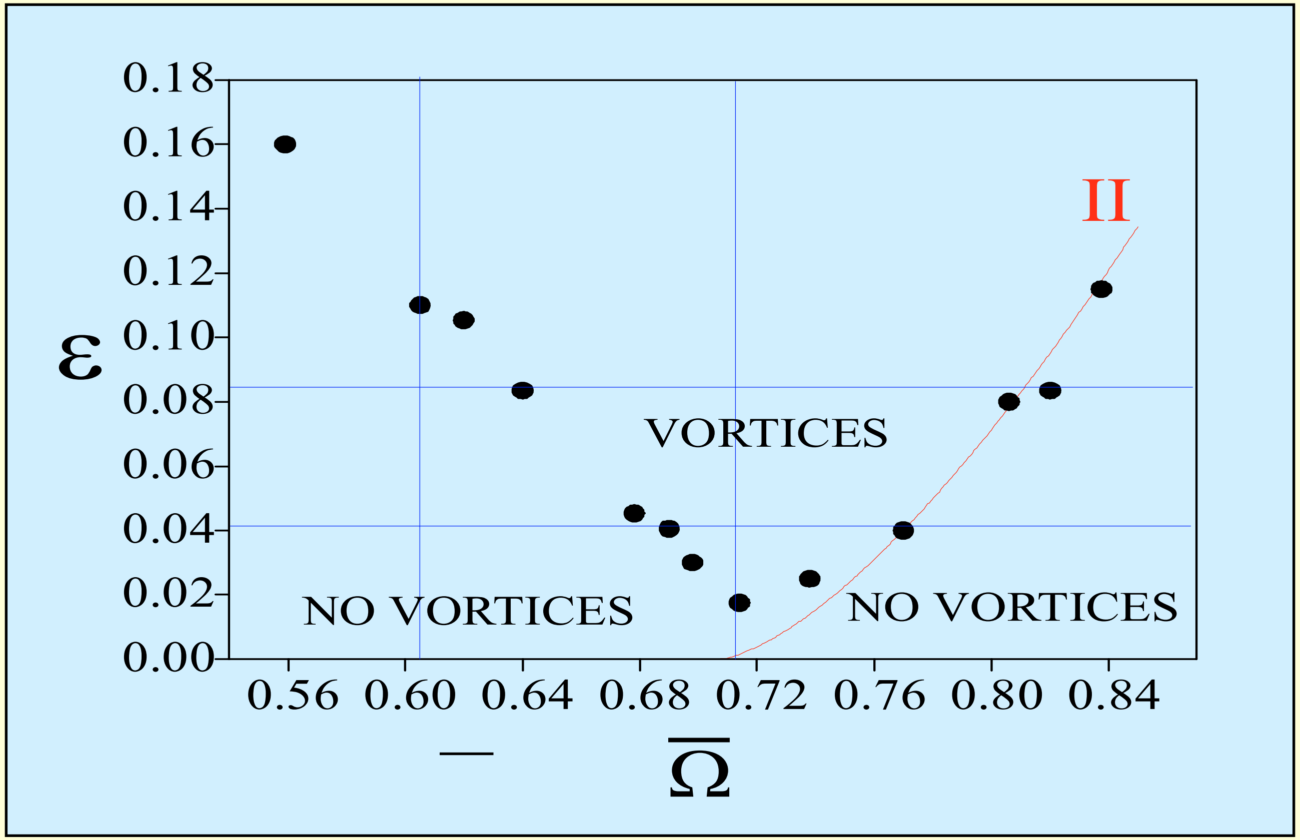
- Condensate at T= 0.5 Tc
- Spin up (0.2s) :

$$\frac{\omega_x}{\omega_y} \rightarrow 1.04, \ \overline{W} = 0.73$$

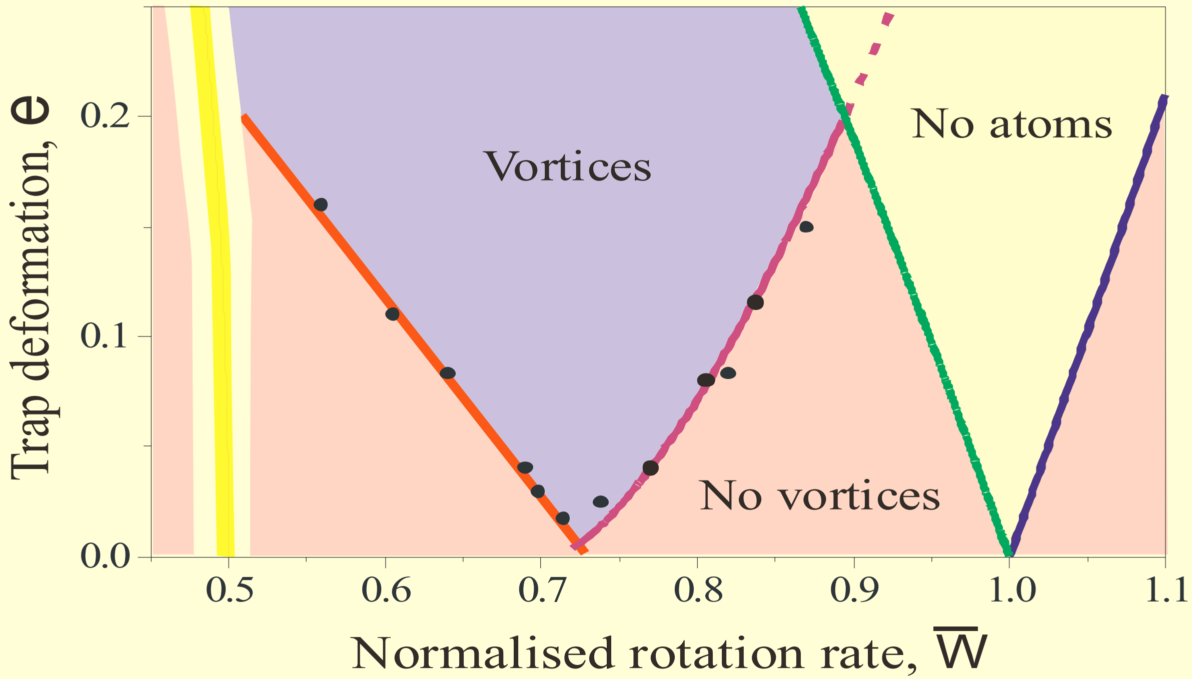
- Spin hold (1s)
- Spin down (0.4s) :

$$\frac{\omega_x}{\omega_y} \rightarrow 1, \ \overline{W} = 0$$

Thresholds for vortex nucleation



- Critical frequency $\Omega_c = 1/\sqrt{2}$
- Line II : stability boundary for the quadrupole II branch.
- Vortices nucleated below Ω_c.



Vortex nucleation exhibits hysteresis

Key

- Dipole driving resonance, $\varepsilon = (1-4\bar{\Omega}^2)^{1/2}$
- Lower vortex nucleation boundary (experimental), $\varepsilon = -0.91\bar{\Omega} + 0.66$
- Stability boundary of Quadrupole II branch, $\varepsilon = \frac{2}{\bar{\Omega}}\left(\frac{2\bar{\Omega}^2 - 1}{3}\right)^{3/2}$
- Stability boundary of Quadrupole I branch, $\varepsilon = 1 - \bar{\Omega}^2$
- Stability boundary of Quadrupole III branch, $\varepsilon = \bar{\Omega}^2 - 1$

No atoms.
Left: Parametric driving of dipole oscillation in rotating frame
Right: Centre of mass unstable

Vortices nucleated

Condensate stable but no vortices

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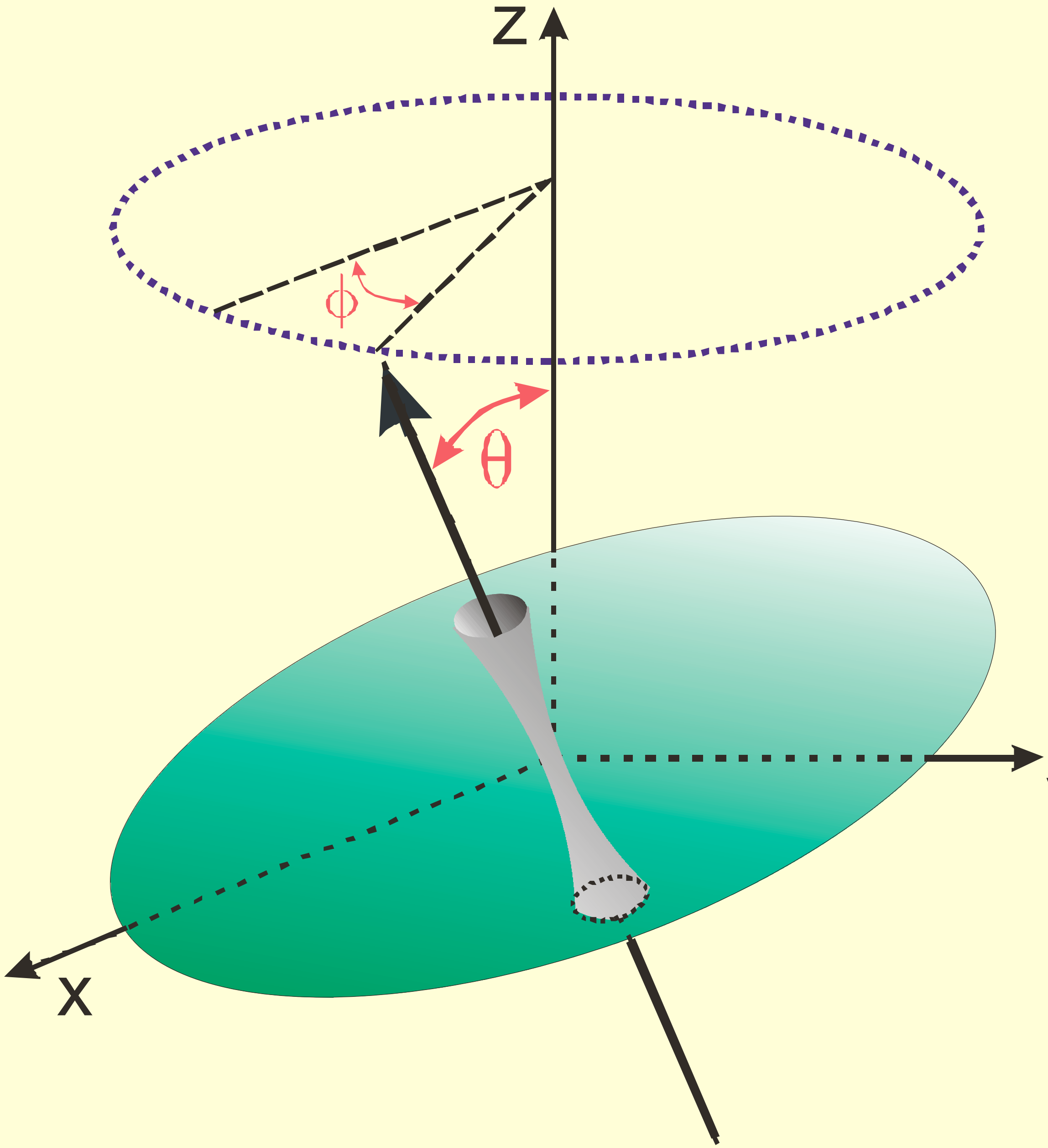
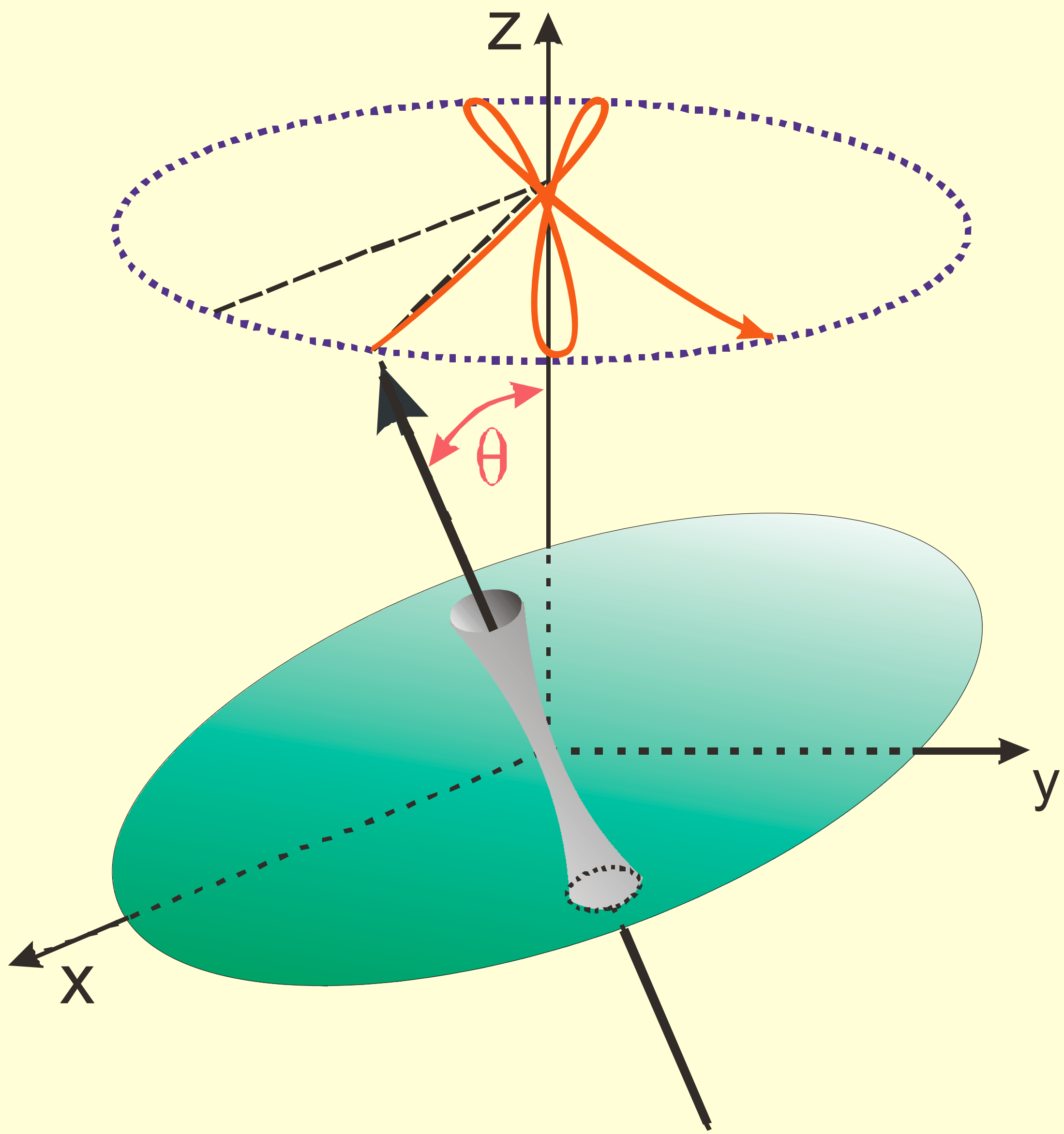
Initial condition for observing the superfluid gyroscope

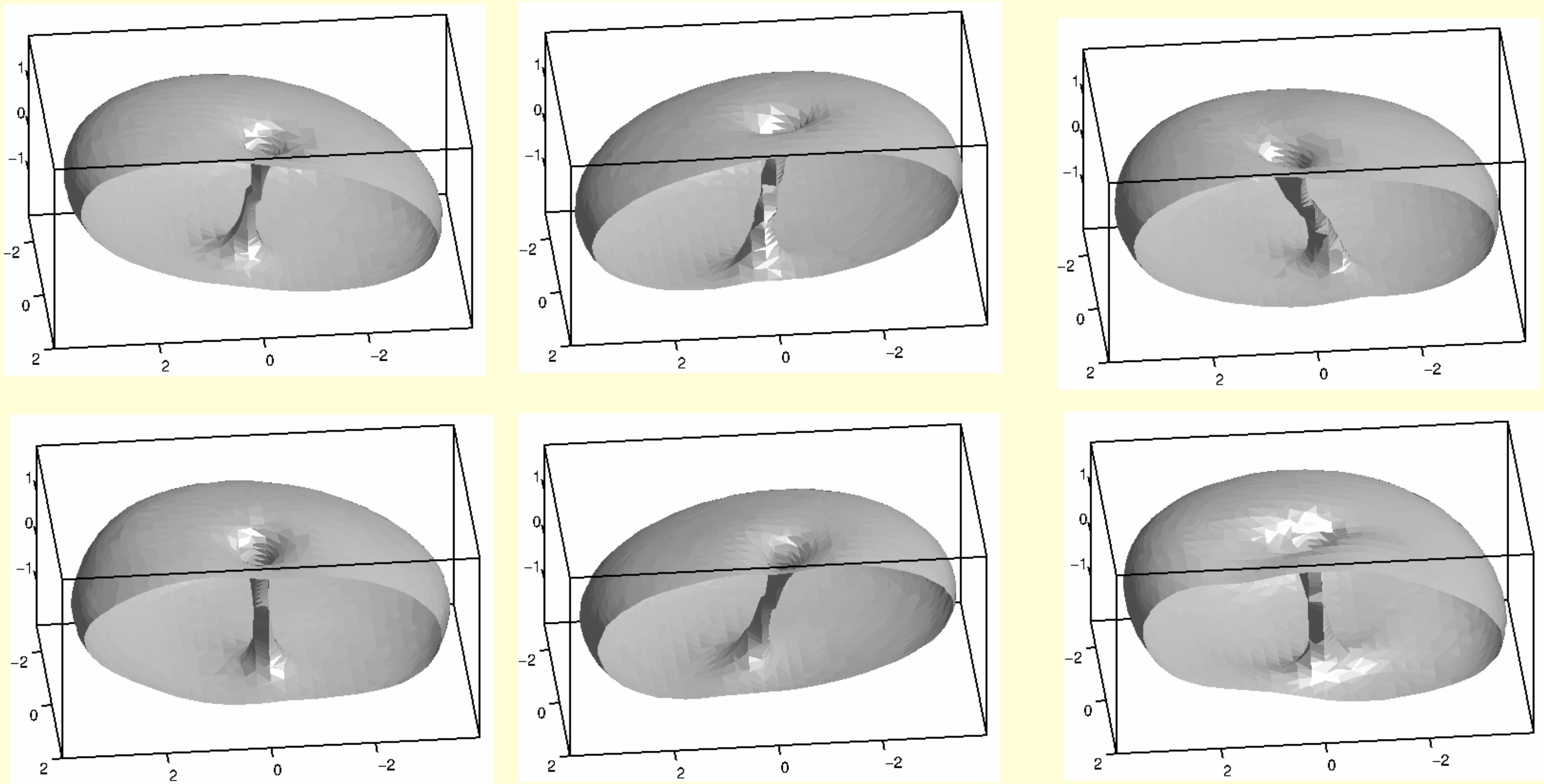


Theory:
Sandro Stringari

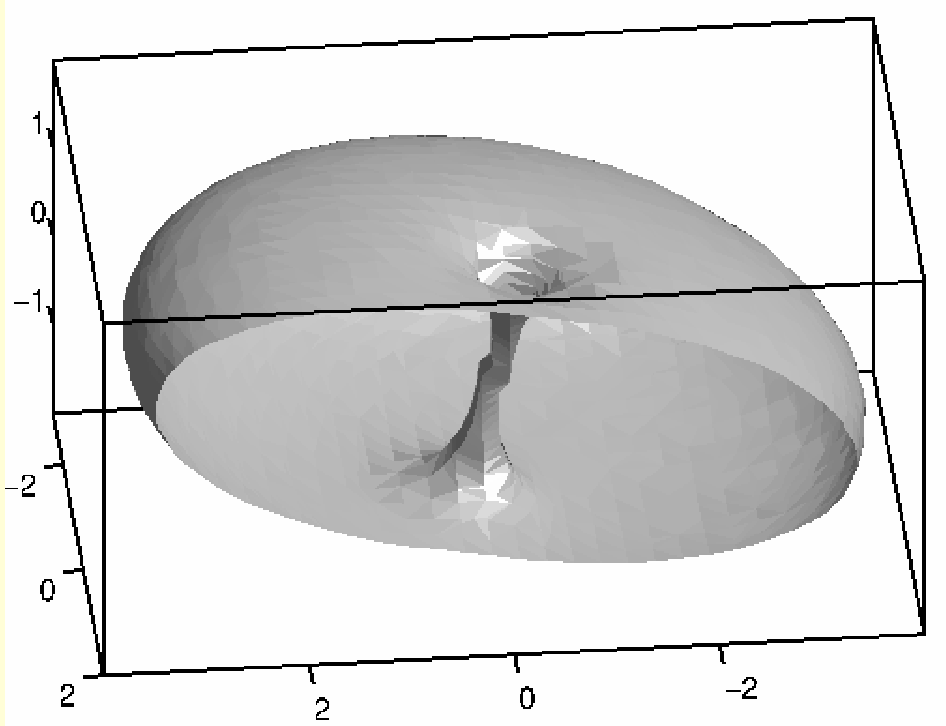
Superfluid gyroscope



Superfluid gyroscope – scissors mode + vortex



Nilsen, McPeake & McCann, Queens University Belfast



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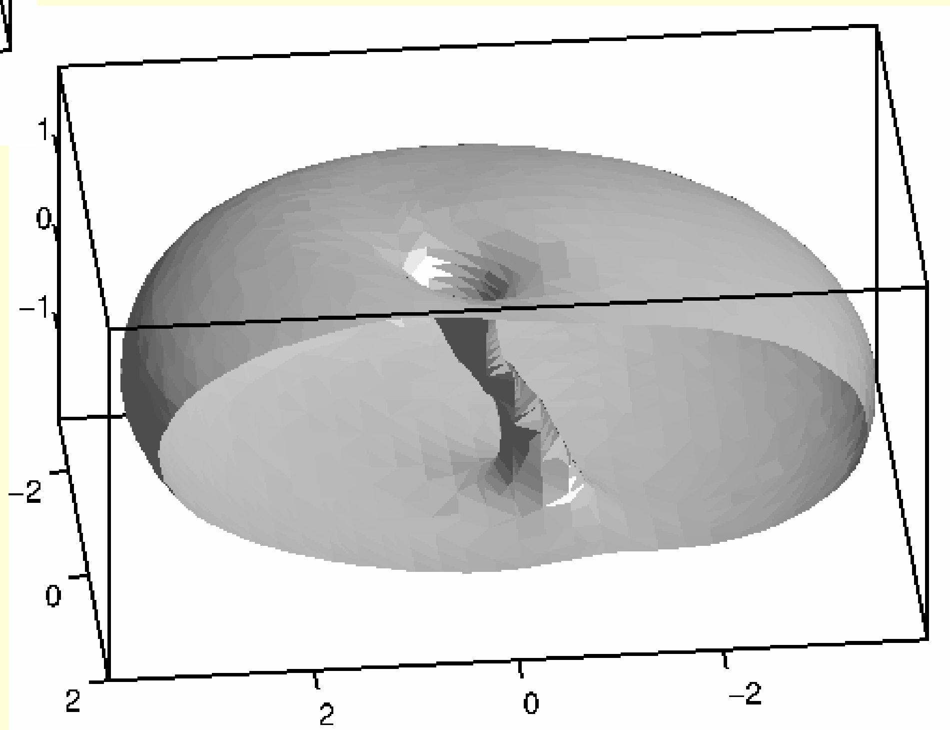
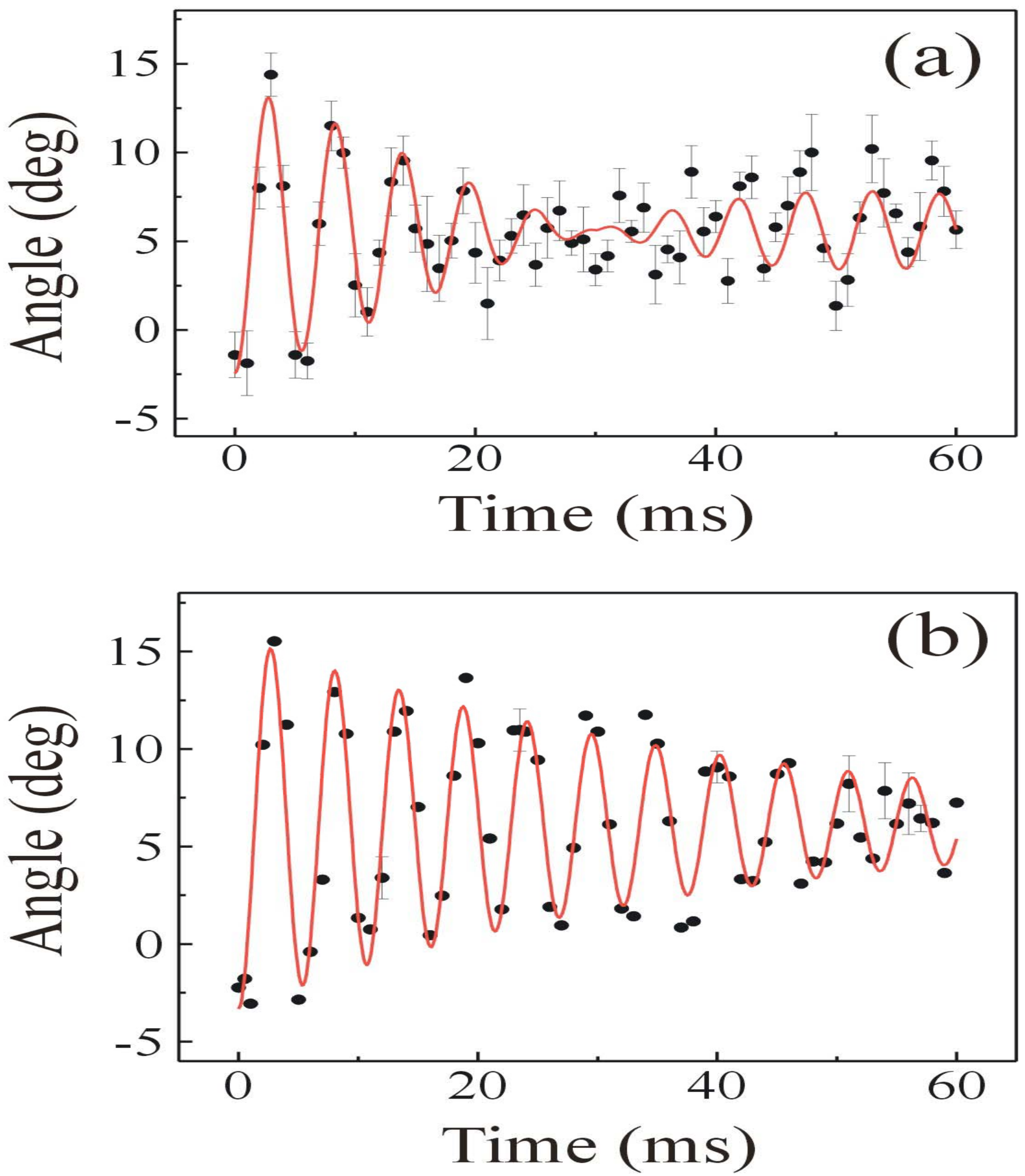




Precession of a condensate containing a single vortex

(*xz* gyroscope)

Final result of gyro expt

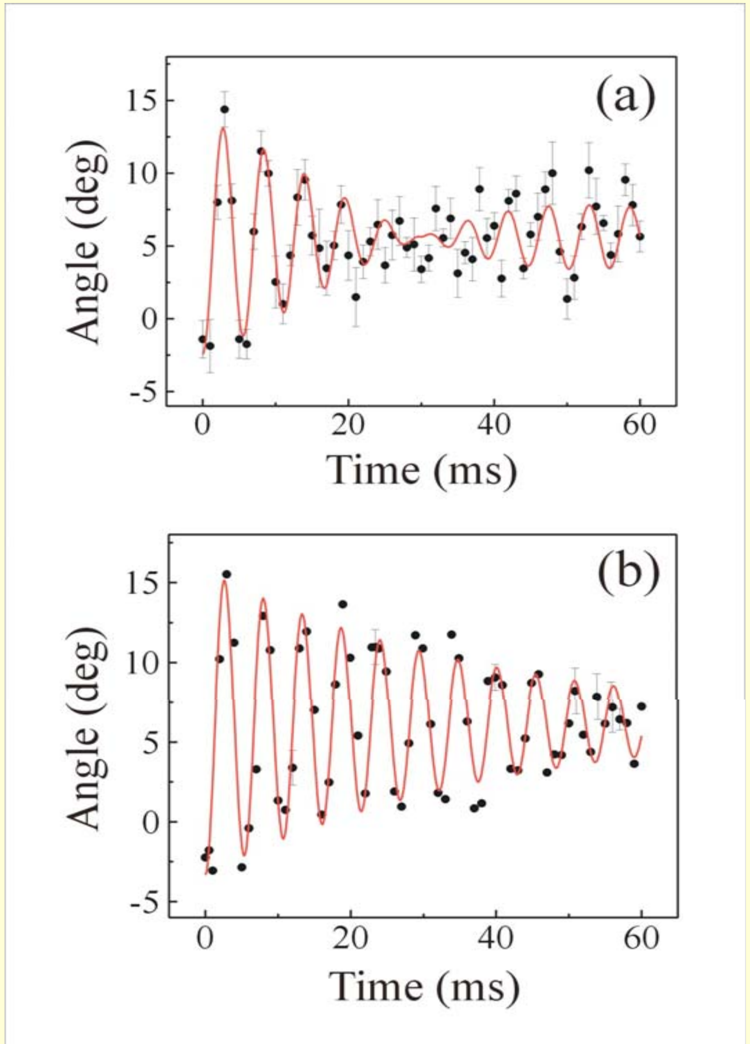
$$\left\langle l_z \right\rangle = 1.07 \pm 0.18\,\hbar$$

Theory: Sandro Stringari

$$\left\langle l_z \right\rangle = \frac{2\Omega}{7\omega_c}\hbar\,\frac{\left(1+\lambda^2\right)^{3/2}}{\lambda^{5/3}}\left(15N\frac{a}{a_{ho}}\right)^{2/5}$$

Described in Pitaevskii & Stringari's book

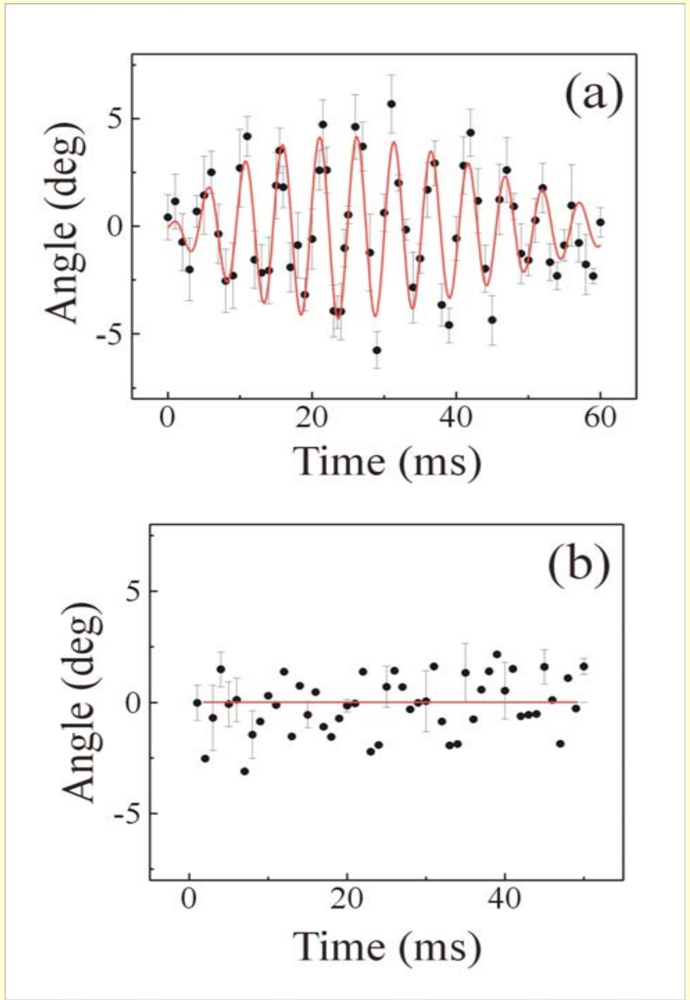
XZ Gyroscope Results



$$\theta_{XZ} = \theta_{eq} + \theta_0 \left|\cos \Omega t\right| \left(\cos \omega_{sc} t\right) e^{-\gamma t}$$

	Expt.	Control
θ_{eq} (deg)	5.6	6.2
$\omega_{sc}/2\pi$ (Hz)	179	186
$\Omega/2\pi$ (Hz)	8.3 ± 0.7	0
γ (Hz)	23.2 ± 6.7	25.1 ± 4.5

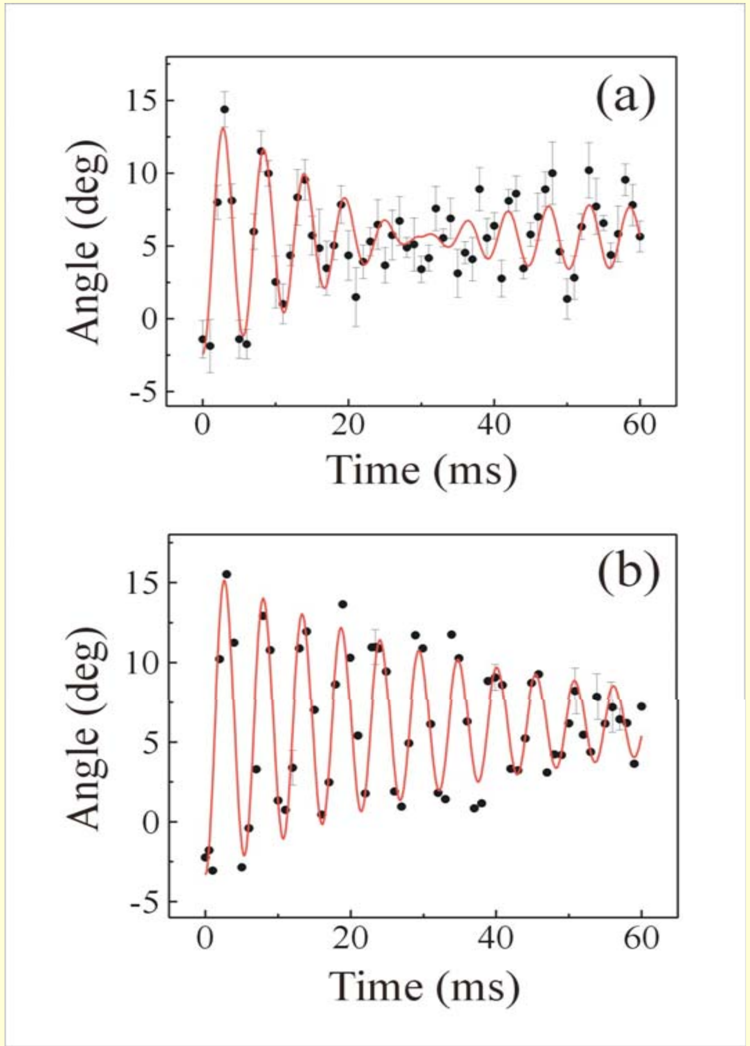
YZ Gyroscope Results



$$\theta_{XZ} = \theta_{eq} + \theta_0 \left|\sin \Omega t\right| \left(\cos \omega_{sc} t\right) e^{-\gamma t}$$

	Expt.	Control
θ_{eq} (deg)	-0.1	0.2
$\omega_{sc}/2\pi$ (Hz)	194	-
$\Omega/2\pi$ (Hz)	7.2 ± 0.6	-
γ (Hz)	24.2 (fixed)	-

XZ Gyroscope Results



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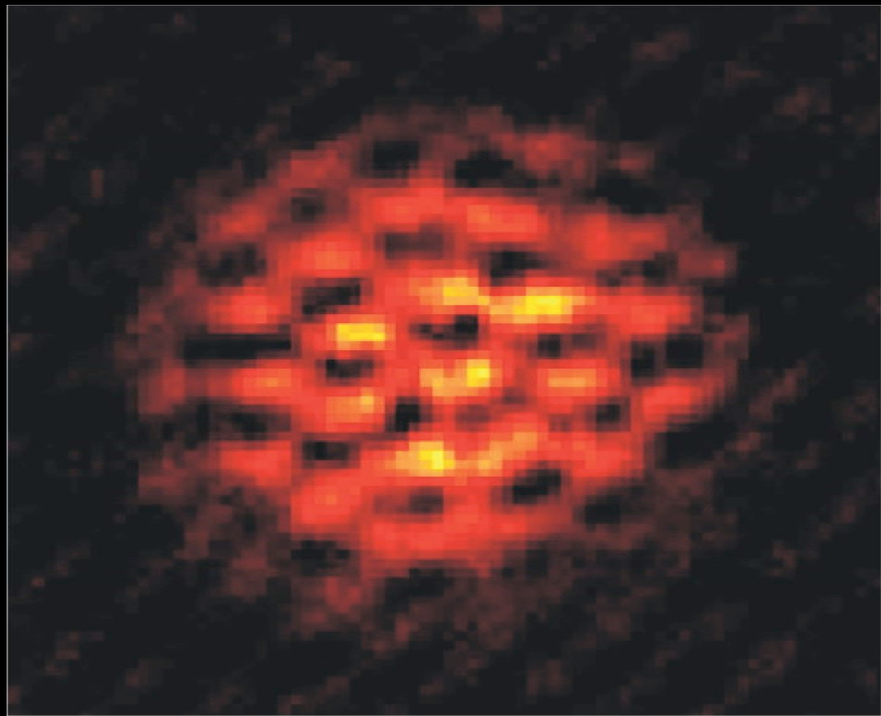
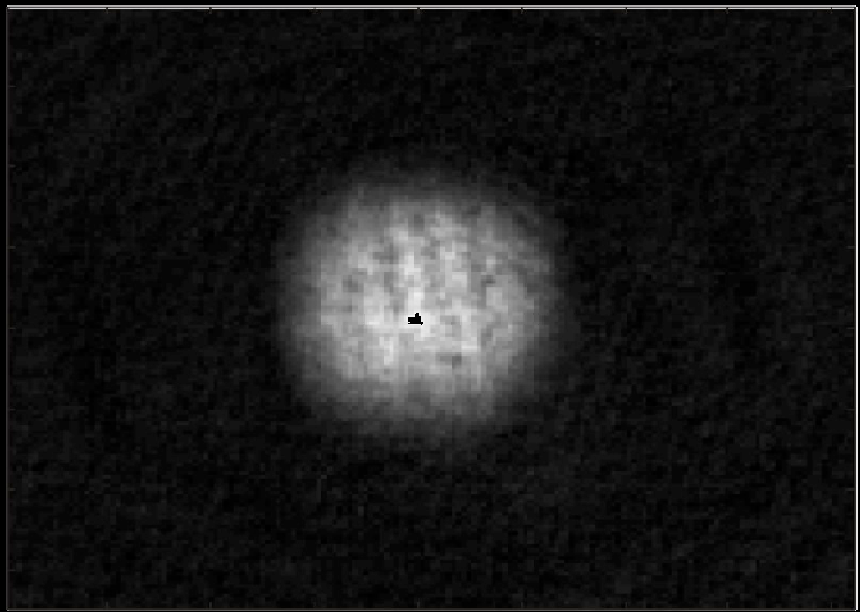
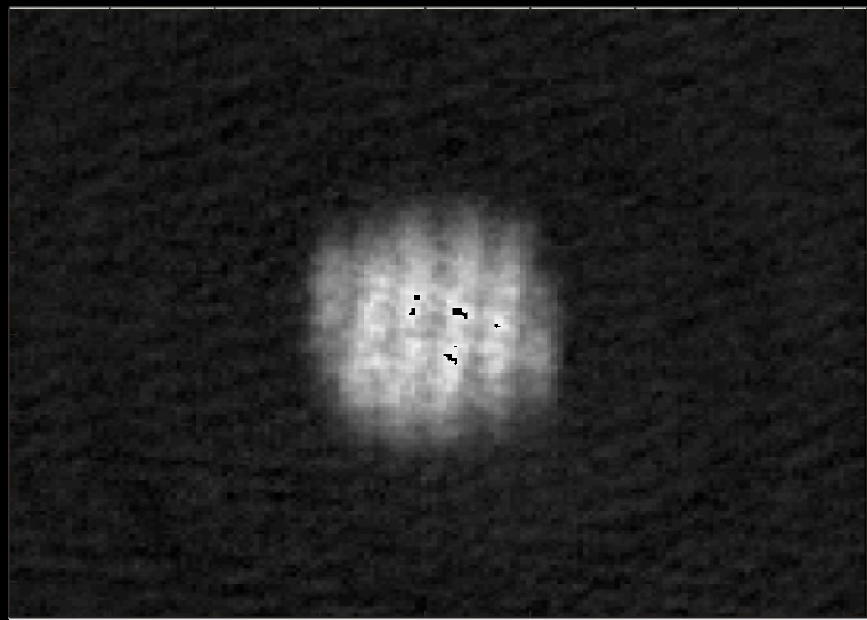
Nucleation of a Vortex Array

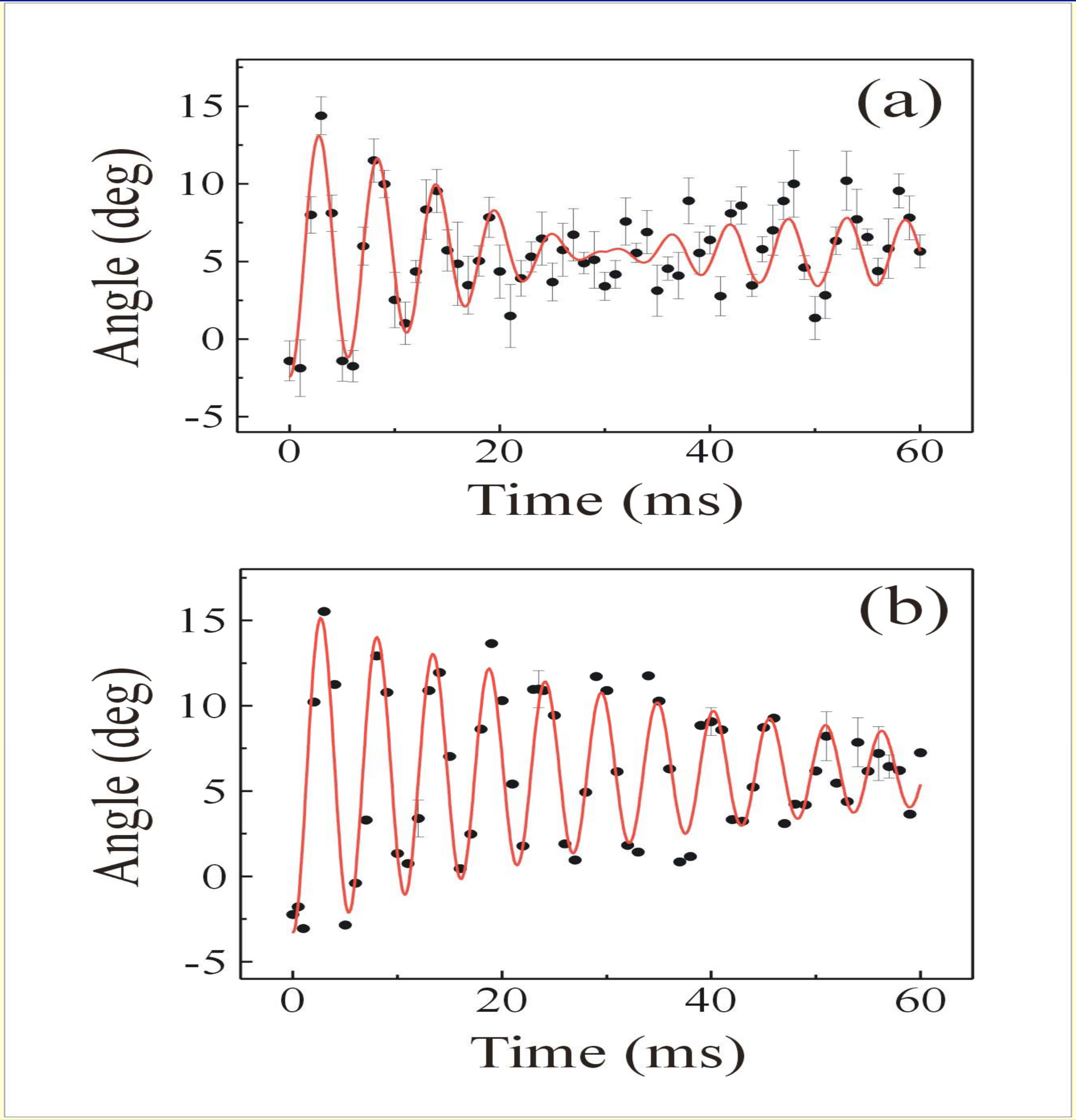


c.f. other expts at ENS, MIT, JILA

Observing the Tilting Mode
(side view of the vortex array)



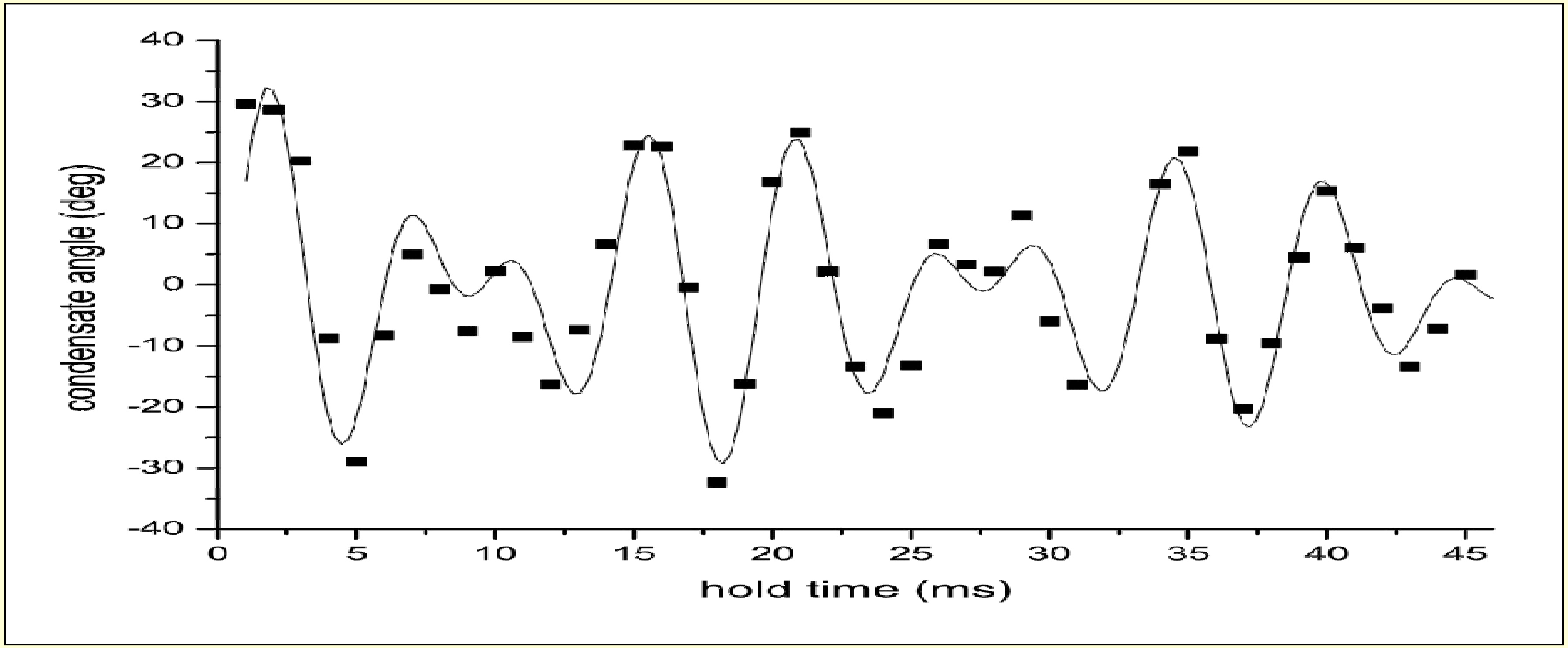




Precession of a condensate containing a single vortex

(*xz* gyroscope)

Precession of the condensate angle as the scissors mode evolves in the presence of a vortex lattice.

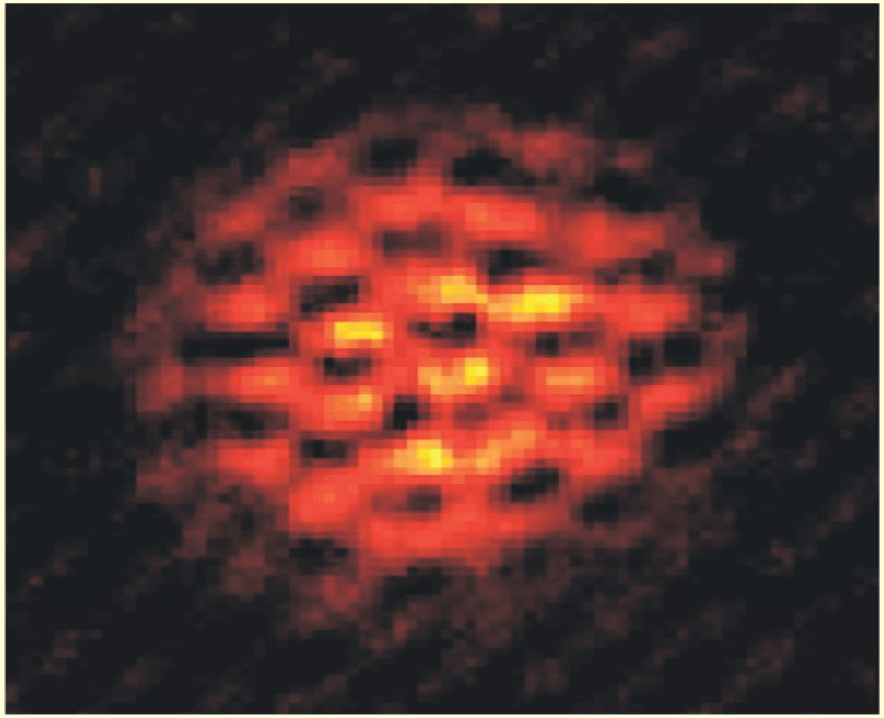


$$\Omega_{precession} = \frac{\omega_+ - \omega_-}{2} = \frac{\langle l_z \rangle}{2m\langle x^2 + z^2 \rangle} = 27.75 \text{ Hz}$$

- $\langle l_z \rangle = 8.4 \pm 0.4\ \hbar$

Agrees with expected value from spatial distribution of vortices

Nucleation of a Vortex Array



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Quantised circulation in a quantum fluid

$$\oint_c \underline{v}.\underline{dl} = \frac{\hbar}{m}\oint_c \underline{\nabla\psi}\ .\underline{dl} = \frac{\hbar}{m} 2\pi\ell$$

$$l = \text{integer}$$

- The wavefunction must vary as $e^{i\ell\varphi}$
- Hence velocity is $v_\varphi = \ell \frac{\hbar}{M\rho}$
- Velocity decreases as ρ, the distance from the centre of the vortex, increases.

Wavefunction for condensate with vortex

$$\psi(\underline{r}) = f(\rho, z)e^{i\ell\varphi}$$

- Substitute into the Gross-Pitaevskii equation to obtain and equation with terms depending on ρ and *z*. (Details in BEC books by Pethick & Smith, and Pitaevskii & Stringari.)

- Consider a uniform potential, $V(\rho, z)=0$. Resulting equation has no dependence on *z;* a purely radial equation where the vortex centre is a line parallel to the *z*-axis.

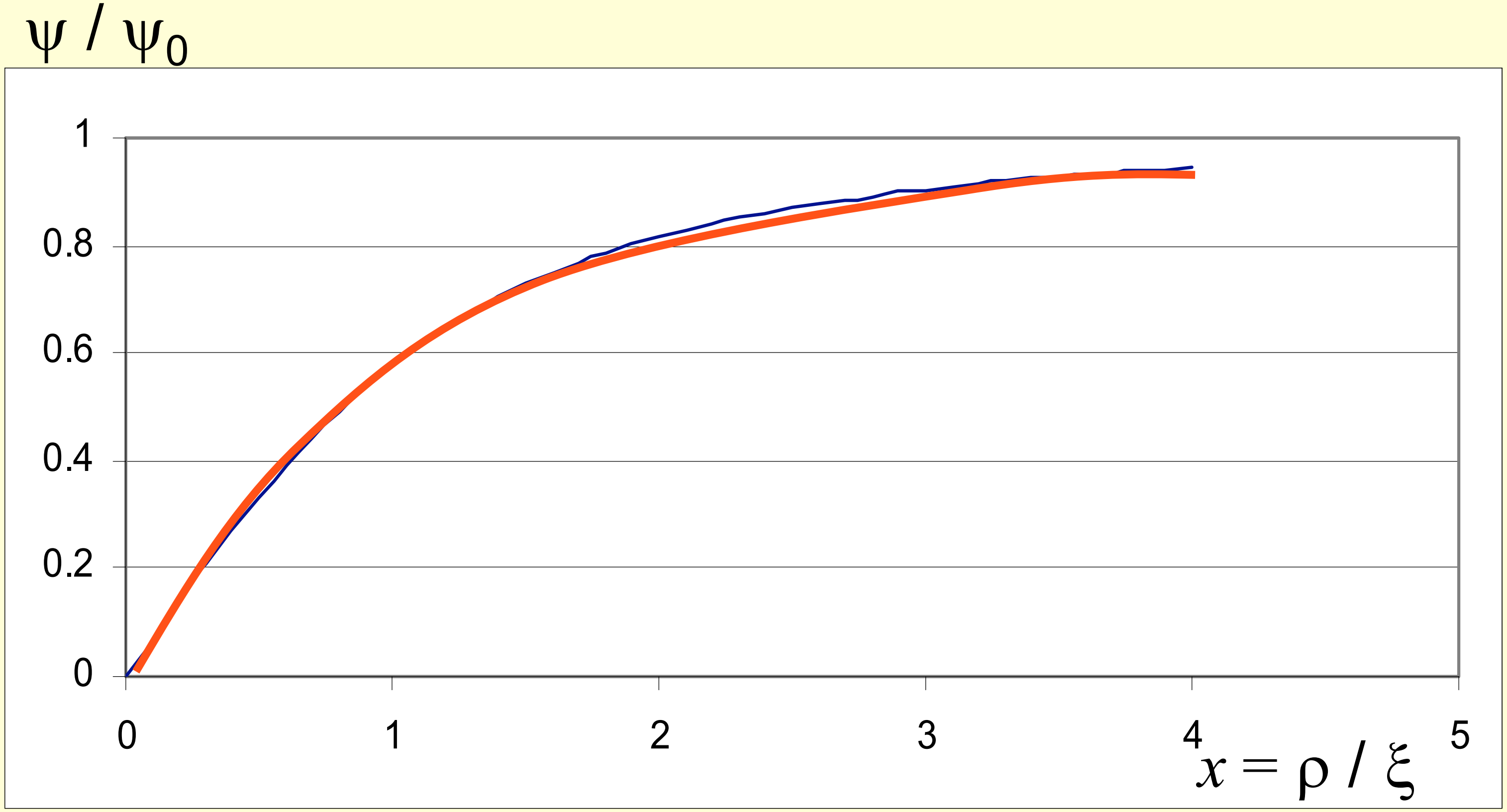
- Radial equation can be solved (to a good approximation) by the variational method using a trial wavefunction of the form

$$\chi(x) = \frac{x}{(\alpha + x^2)^{1/2}}$$

• Variational method with this trial wavefunction gives minimum energy for $\alpha = 2$.

Vortex wavefunction

$$\frac{\psi}{\psi_0} = \frac{x}{\left(2 + x^2\right)^{1/2}}$$



Definition of healing length ξ

Minimum, distance over which the wavefunction can change appreciably

$$\frac{\hbar^2}{2M\xi^2} = \mu$$

Chemical potential

$$\xi^2 = \frac{1}{8\pi n a}$$

n = number density
a = scattering length

(defined in talk by Shlyapnikov, I assume)

Wavefunction for vortex

$$\chi(x) = \frac{x}{\left(\alpha + x^2\right)^{1/2}}$$

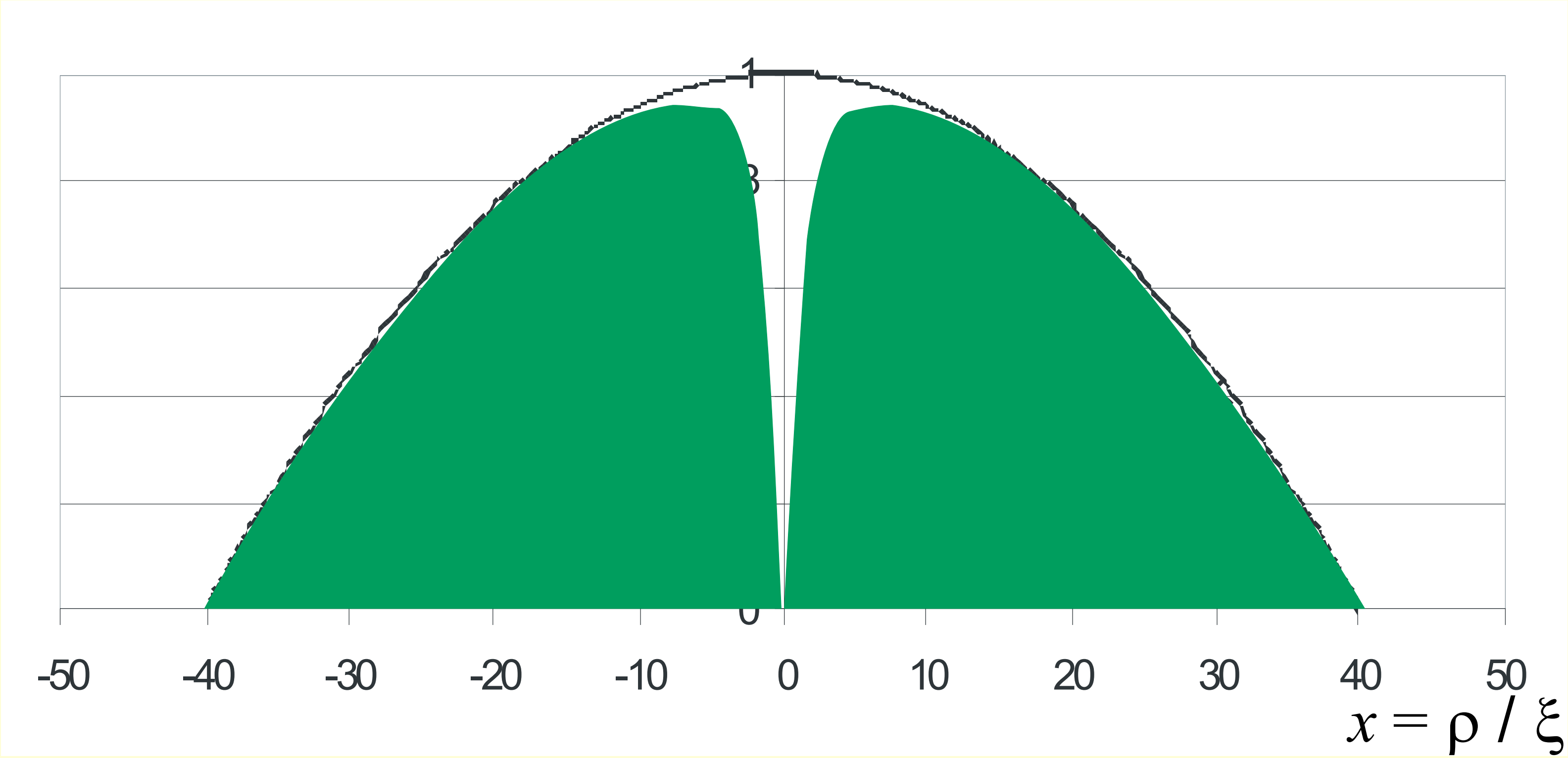
- Variational method with this trial wavefunction gives minimum energy for $\alpha = 2$.

$$U_{\text{vortex}} = \pi n L \frac{\hbar^2}{M} \ln\left(1.5\frac{R}{\xi}\right)$$

- Energy of vortex of length L in the z direction (proportional to <u>length of core).</u>

(see talks later in week by Fetter and Stringari)

Density profile of a BEC with a vortex



$$\frac{\psi}{\psi_0} = \frac{\frac{\rho}{\xi}}{\sqrt{2 + \frac{\rho^2}{\xi^2}}} \left(1 - \frac{\rho^2}{R^2} \right)$$

Calculation of vortex energy by integrating the kinetic energy of the flow for vortex with unit circulation

$$U_{\text{vortex}} = n \iiint \frac{1}{2} M v^2 \rho \, d\rho \, d\varphi \, dz$$

$$= \pi n \iint M v^2 \rho \, d\rho \, dz$$

$$= \pi n \iint \frac{\hbar^2}{M \rho^2} \rho \, d\rho \, dz$$

$$= n a_z \frac{\hbar^2}{M} \pi^{3/2} \int_{\xi}^{R} \frac{1}{\rho} d\rho$$

$$= n a_z \frac{\hbar^2}{M} \pi^{3/2} \ln\left(\frac{R}{\xi}\right)$$

Integrate the k.e. of flow

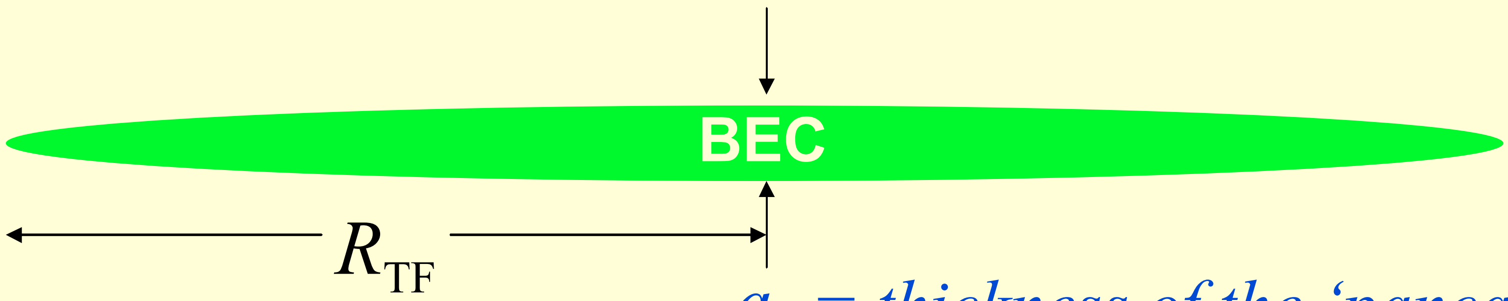
Integration along z

Proportional to <u>length of core</u>

Calculation of the stability of vortex pairs in a 2D Bose gas

Number density distribution

$$N_0|\psi|^2 = n_0\left(1 - \frac{r^2}{R_{\mathrm{TF}}^2}\right)\exp\left(-\frac{z^2}{a_z^2}\right)$$



Thomas-Fermi radius

a_z = *thickness of the 'pancake'*

Gaussian distribution =
usual harmonic oscillator wavefn.

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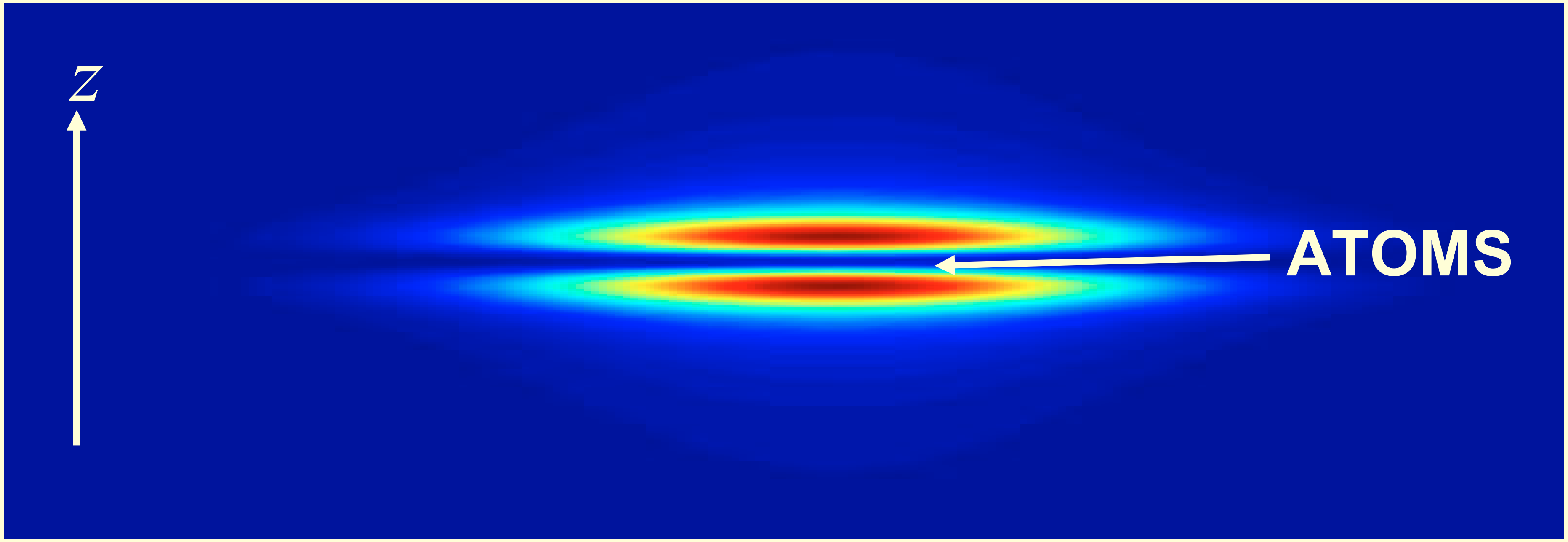
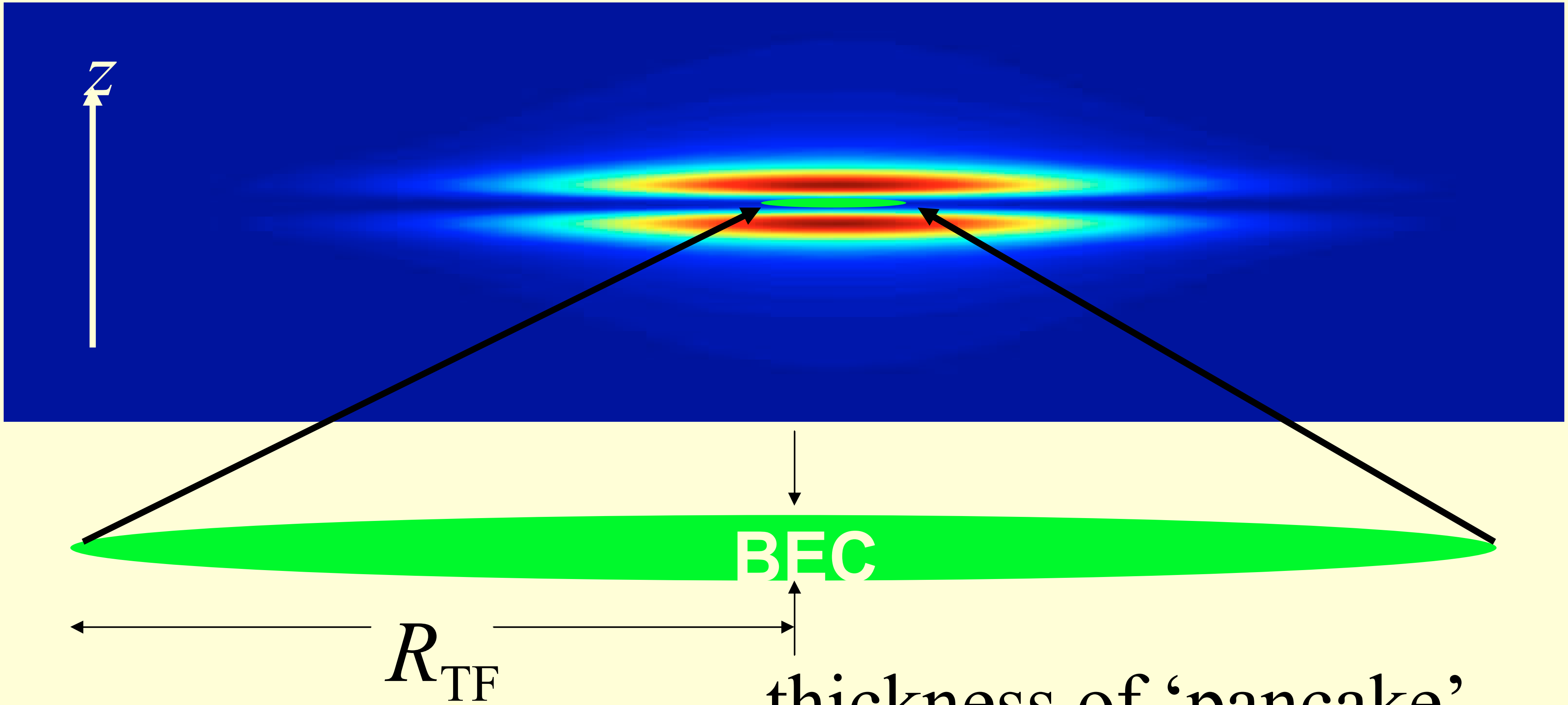
- Squeeze atoms between two sheets of light
  (that create a repulsive potential)
- Weak radial confinement by the magnetic trap
- Creates a thin sheet of atoms = 2D Bose gas



Intensity contours of the light that creates a repulsive potential combined optical and magnetic potentials create a thin sheet of atoms



Radius= 40 μm

for N = 100 000 atoms

thickness of 'pancake'

$a_z = 0.2\ \mu m$

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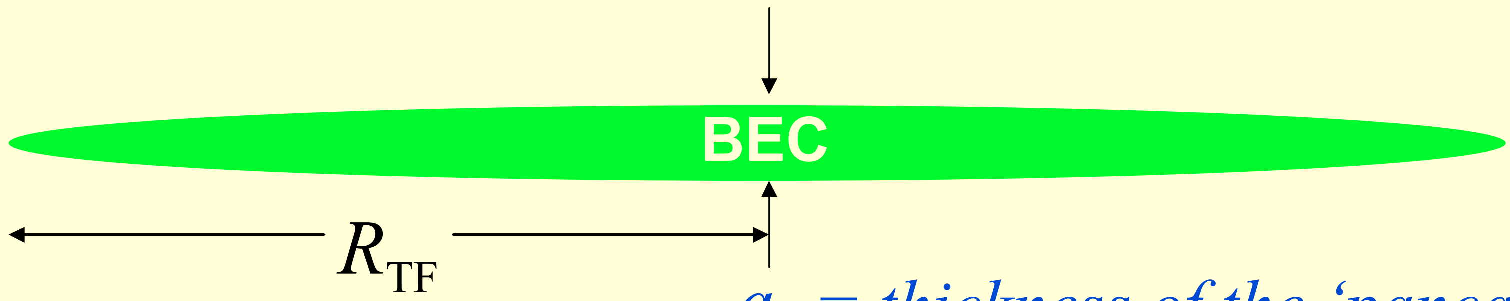

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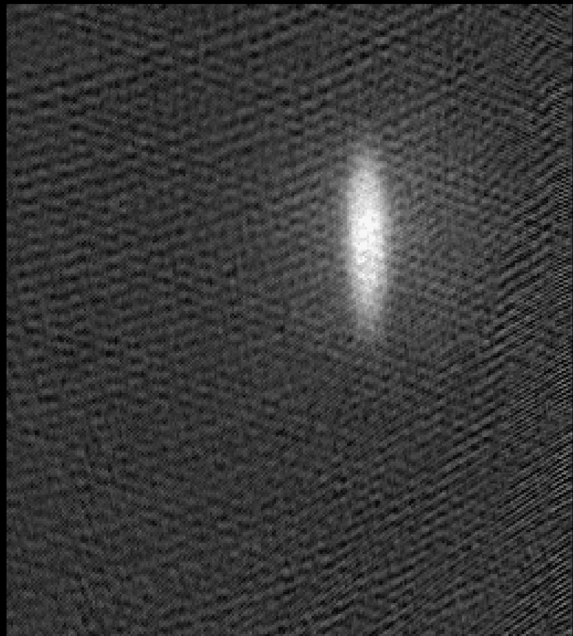
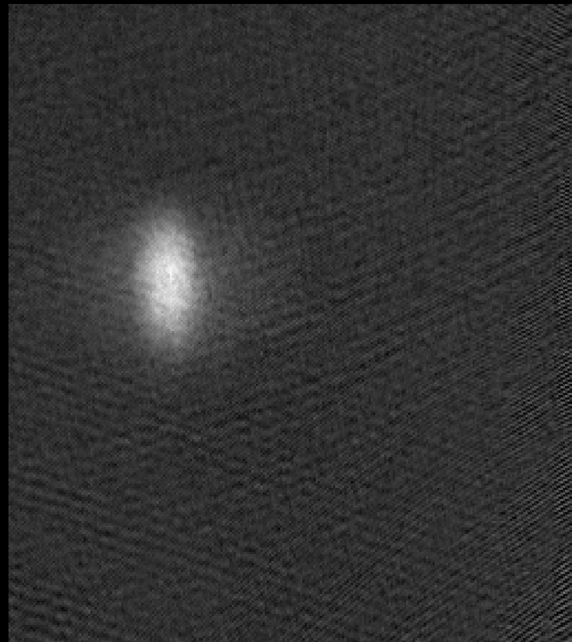
Images of the Bose gas after a time-of-flight expansion

BEC



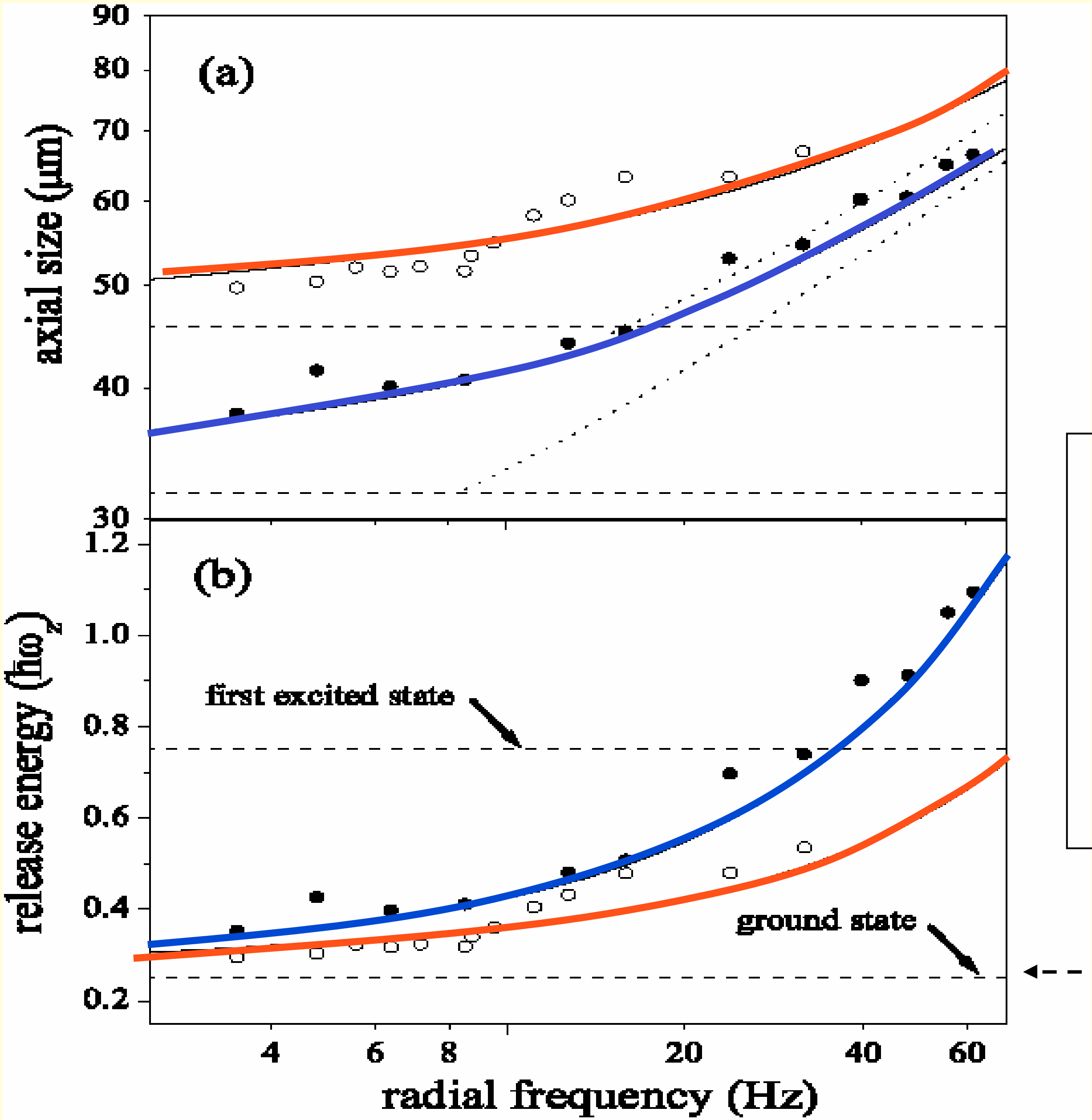


Released from dipole potential but NOT in the Quasi-2D trapping regime

Heisenberg-limited expansion in the Quasi-2D trapping regime

Initially = thin pancake of atoms

Finally = long cigar of atoms



$f_z = 1$ kHz

$f_z = 2$ kHz

$\frac{\hbar\omega}{k_B} \equiv \frac{h\,f}{k_B} = 100$ nK

for $f_z = 2$ kHz

$\frac{1}{4}\,\hbar\omega_z$

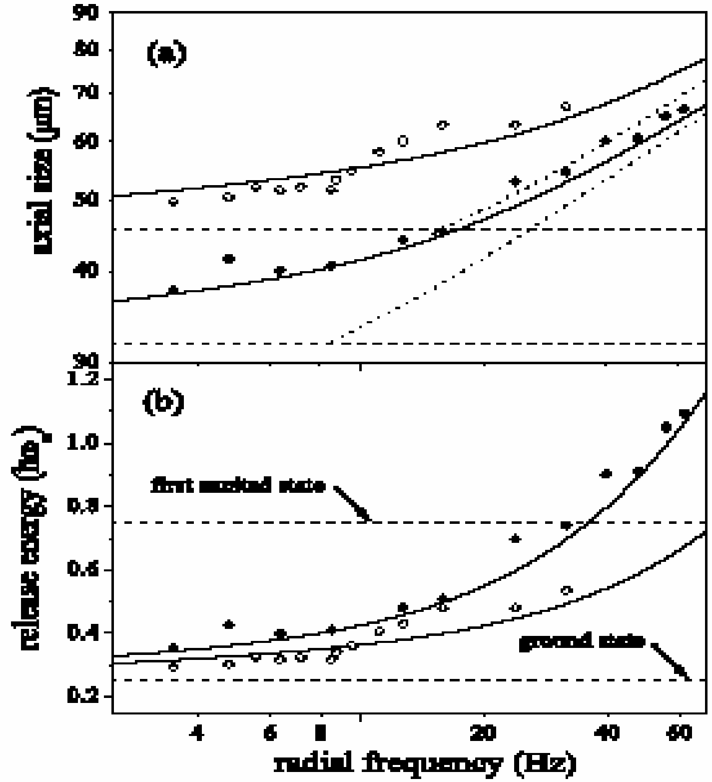


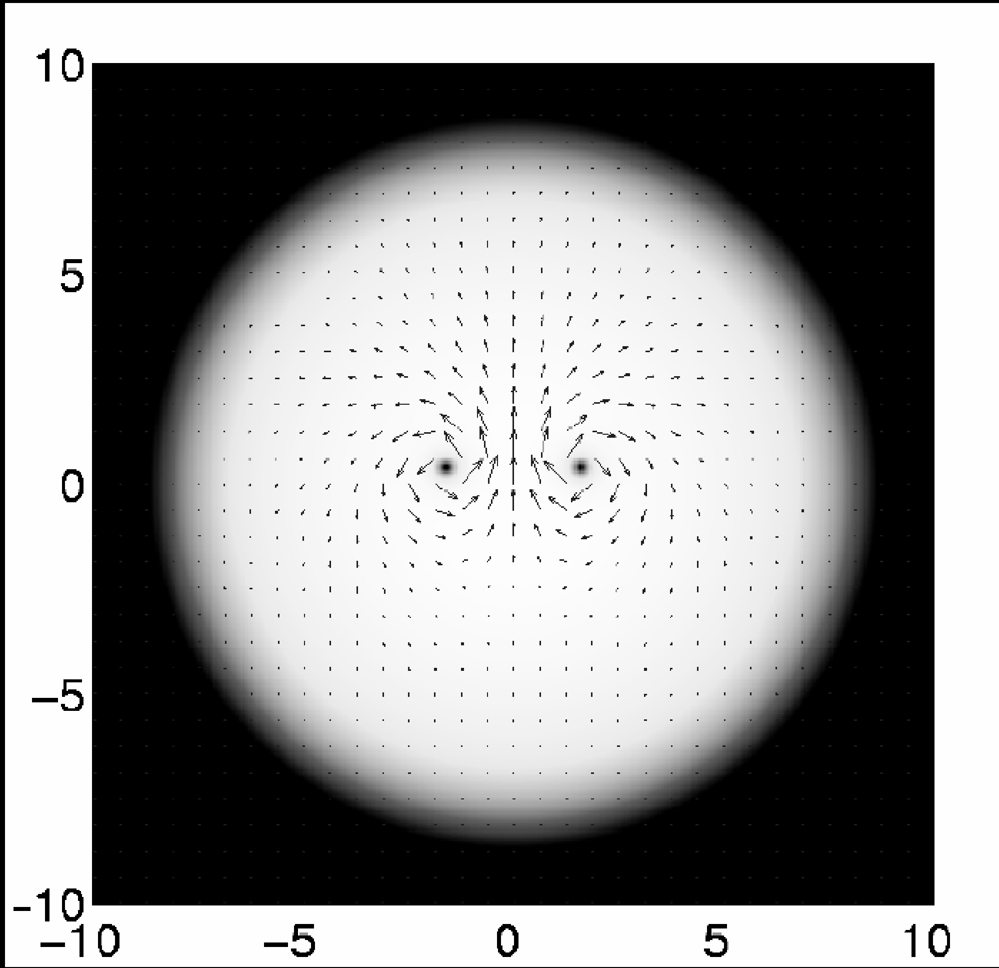
FIG. 10: (a) The axial expansion of the condensate after 15 ms of free expansion for various trap geometries and atom numbers. Solid lines indicate theoretical variational predictions, dashed lines indicate the ideal gas limit and dotted lines the hydrodynamic limit. The data are taken for traps with $\omega_z/2\pi = 1990$ Hz (open circles) and 960 Hz (filled circles). The atom numbers are 8×10^4 and 1.1×10^5, respectively. (b) The release energy of the condensate derived from the expansion measurements in (a). The energy tends towards the vertical zero-point kinetic energy as ω_x is reduced.

Preprint by
Nathan Smith *et al*

Outline: talk about experiments on the superfluidity of a Bose gas

- Scissors mode – irrotational flow
- Nucleation of vortices – hysteresis
- Superfluid gyroscope – vortex causes precession
- Tilting mode of a vortex array – Kelvin wave
- Simple theory of vortex in a quantum fluid
- Experimental creation of a 2D Bose gas
- Predictions for a two-dimensional Bose gas

Numerical simulation of a vortex pair in a BEC of Rb atoms



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Cond-mat/0412512

Transition from Bose-Einstein Condensate to Berezinskii-Kosterlitz-Thouless Phase

healing length = 1 μm, TF radius = 40 μm, for N = 10 000 atoms

From the simple theory in previous part of this talk

$$U_{\text{vortex}} = n a_z \frac{\hbar^2}{M} \pi^{3/2} \ln\left(\frac{R}{\xi}\right)$$

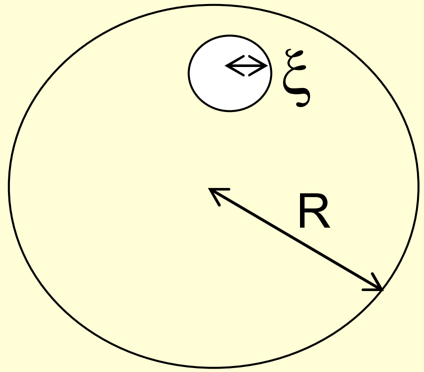
$$= \rho_s \frac{\hbar^2}{M^2} \pi \ln\left(\frac{R}{\xi}\right)$$

where the two-dimensional superfluid density is

$$\rho_s = M n a_z \sqrt{\pi}$$

Entropy of a vortex

$$S_{\text{vortex}} = k_B \ln\left(\frac{R^2}{\xi^2}\right) = 2k_B \ln\left(\frac{R}{\xi}\right)$$



The free energy of a single vortex is

$$F = U - TS$$

$$= \rho_s \frac{\hbar^2}{M^2}\pi \ln\left(\frac{R}{\xi}\right) - 2k_B T \ln\left(\frac{R}{\xi}\right)$$

$$= \left\{\frac{\rho_s \hbar^2 \pi}{M^2} - 2k_B T\right\} \ln\left(\frac{R}{\xi}\right)$$

The free energy equals zero when

$$F = U - TS = \left\{ \frac{\pi \rho_s \hbar^2}{M^2} - 2 k_B T \right\} \ln\left(\frac{R}{\xi} \right) = 0$$

Thus vortices are entropically favourable when the two-dimensional density is

$$k_B T_{BKT} = \rho_s \frac{h^2}{8 \pi M^2}$$

This is the well known criterion for the **Berezinskii-Kosterlitz-Thouless transition** that has been verified in liquid helium films

See Pitaevskii & Stringari's book

Criterion for the

Berezinskii-Kosterlitz-Thouless transition

$$k_B T_{BKT} = \rho_s \frac{h^2}{8\pi M^2}$$

$$\rho_s = Mn\sqrt{\pi}\, a_z$$

$$n = \frac{2M}{\pi^{3/2} a_z \hbar^2} k_B T$$

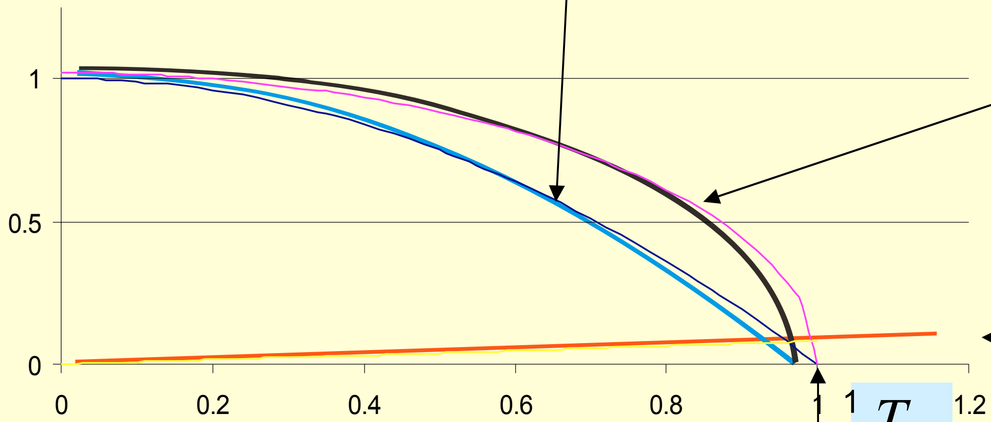
Straight line of plot of number density vs. Temperature

Number of atoms in a 2D BEC

$$\frac{N_0}{N} = 1 - \frac{T^2}{T_0^2}$$

Number density of 2D BEC

$$\frac{n}{n_{\max}} \propto \left(\frac{N_0}{N}\right)^{1/2}$$



$$\frac{T}{T_0}$$

$$n = \frac{2 M k_B}{\pi^{3/2} a_z \hbar^2} T$$

BKT criterion

$$T_{\mathrm{BKT}} \approx T_0$$

Plotted for $N=10^5$ atoms but similar conclusion for the experimentally accessible range $N=10^3$ - 10^6

Energy of single vortex

$$U_{\text{vortex}} = n a_z \frac{\hbar^2}{M} \pi^{3/2} \ln\left(\frac{R}{\xi}\right)$$

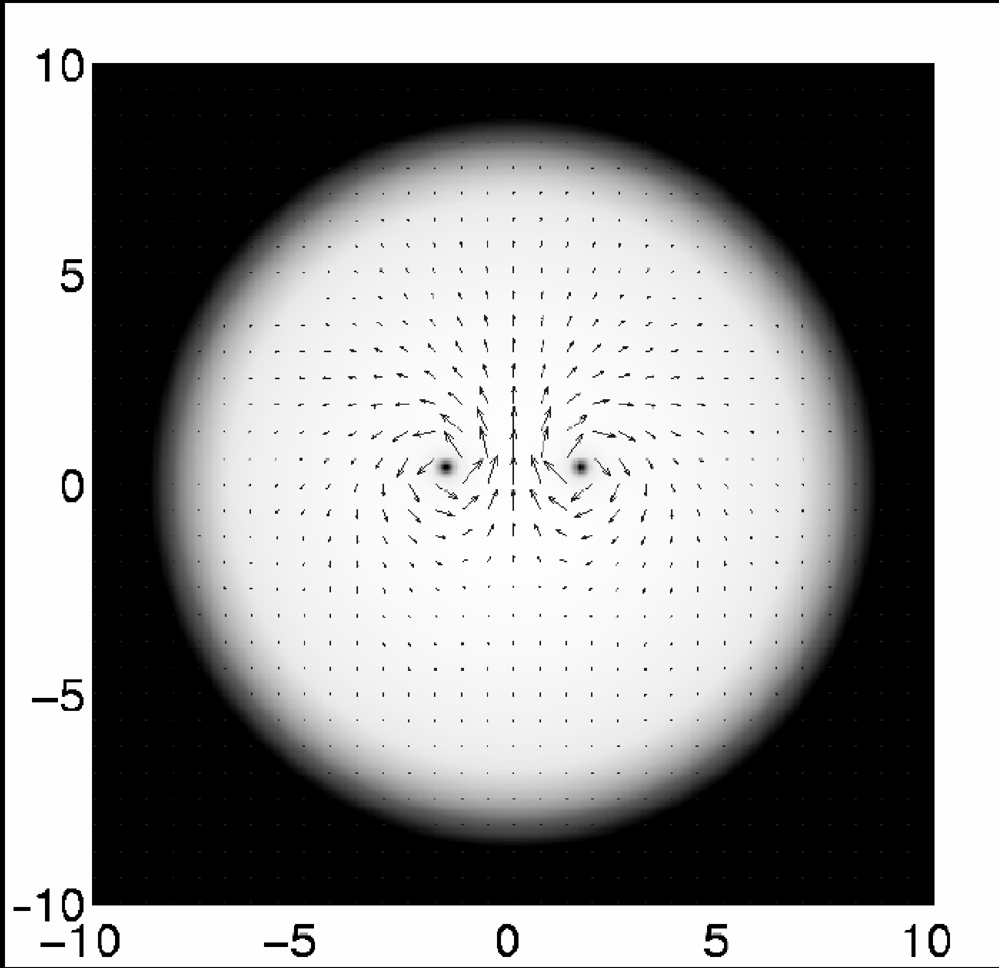
Energy of vortex-antivortex pair

$$U_{\text{pair}} = 2 n a_z \frac{\hbar^2}{M} \pi^{3/2} \ln\left(\frac{R_{12}}{\xi}\right)$$

Minimum vortex-antivortex separation $R_{12} \approx 2\xi$.
In our experiment $R \approx 40\xi$, hence

$$\frac{U_{\text{vortex}}}{U_{\text{pair}}} \approx \frac{\ln(40)}{2\ln(2)} = 2.7$$

Numerical simulation of a vortex pair in a BEC of Rb atoms



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Entropy of a vortex

$$S_{\text{vortex}} = k_B \ln\left(\frac{R^2}{\xi^2}\right) = 2k_B \ln\left(\frac{R}{\xi}\right)$$

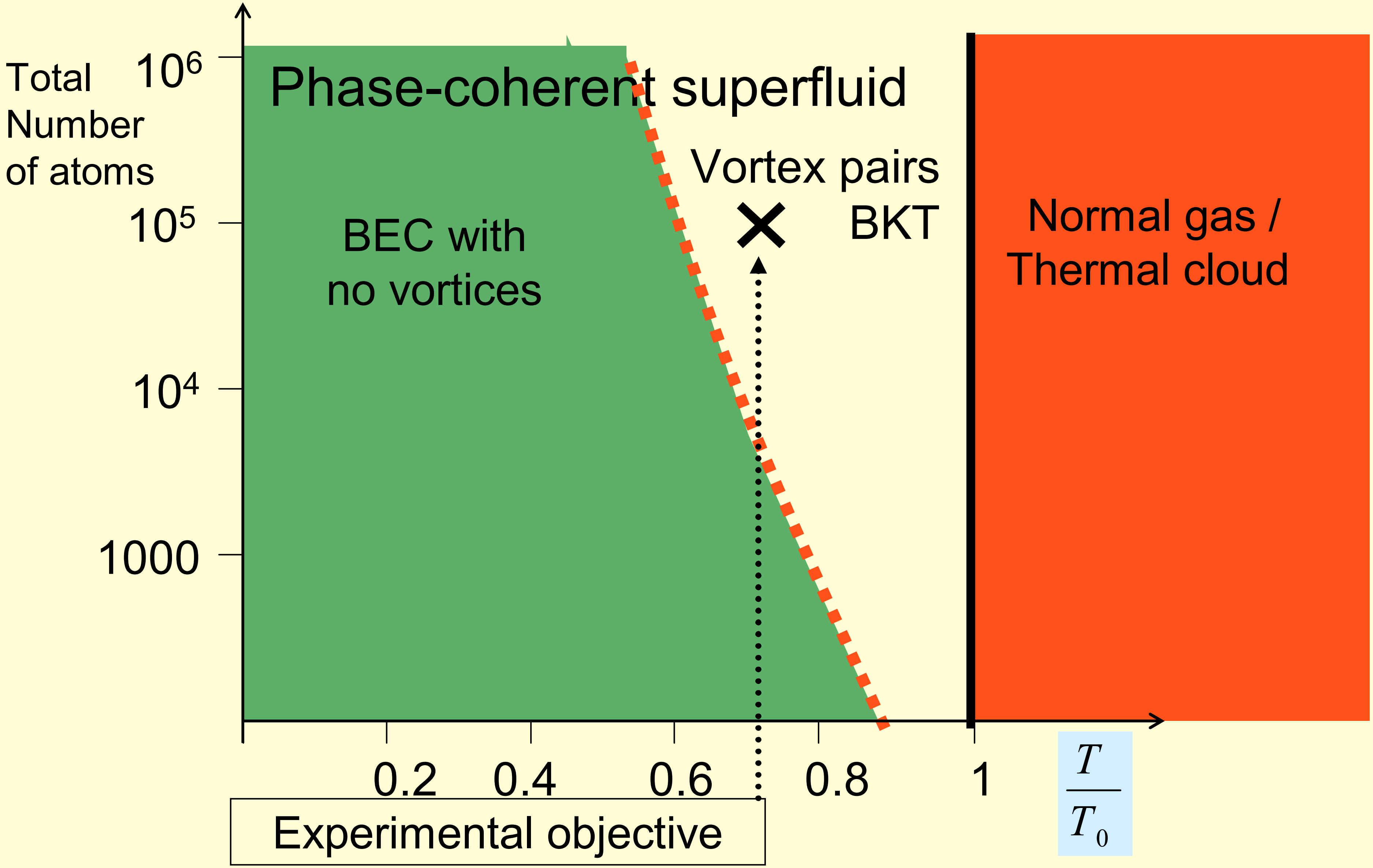
Entropy of pair

$$S_{\text{pair}} = k_B \ln\left(2\pi\frac{R^2}{\xi^2}\right)$$

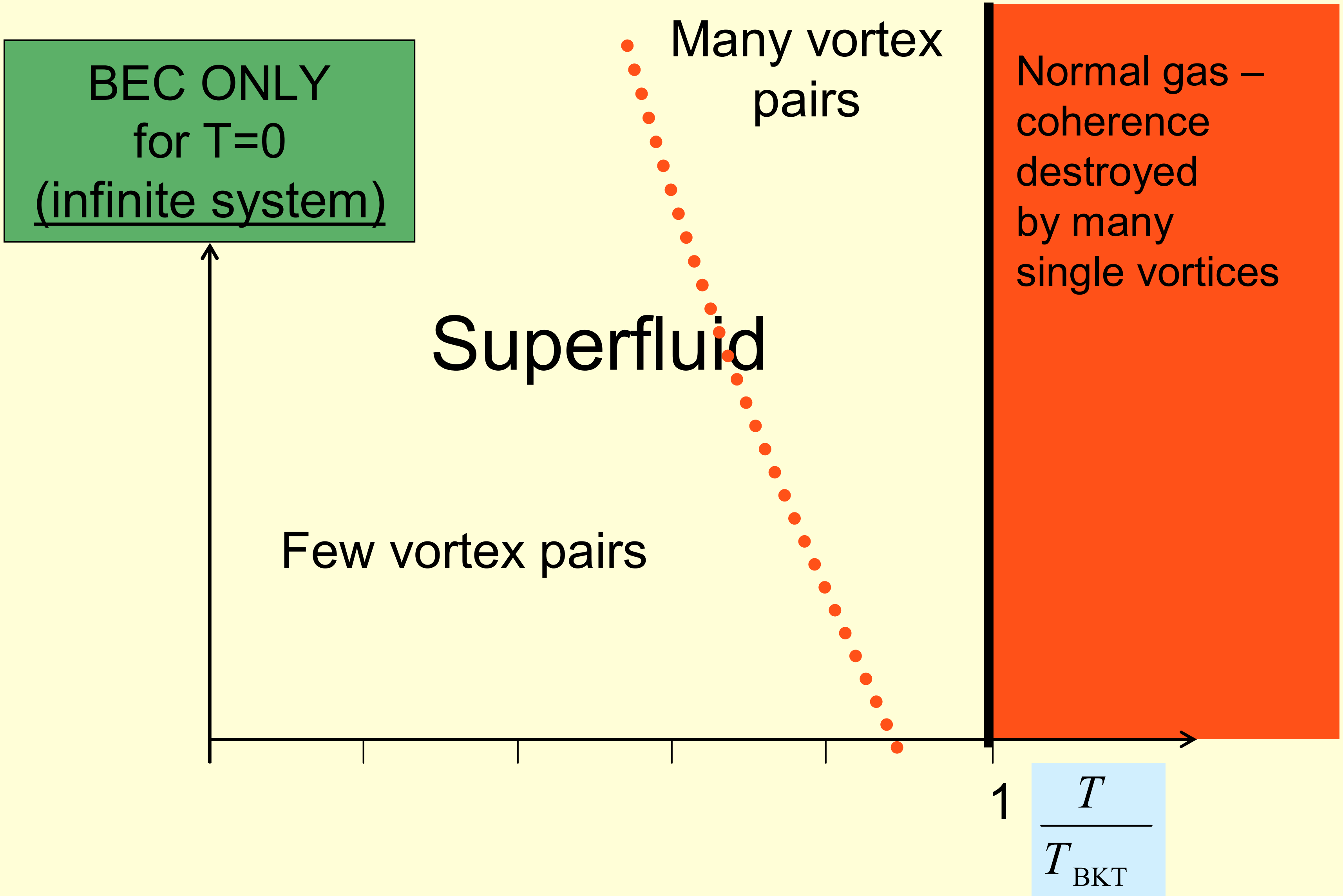
$$= 2k_B \ln\left(\frac{R}{\xi}\right) + 2k_B \ln\left(\sqrt{2\pi}\right)$$

Typically condensed matter the system size R >> ξ, hence these entropies are similar and it the energy U that makes the difference. Not so simple for finite size system like our trapped 2D Bose gas.

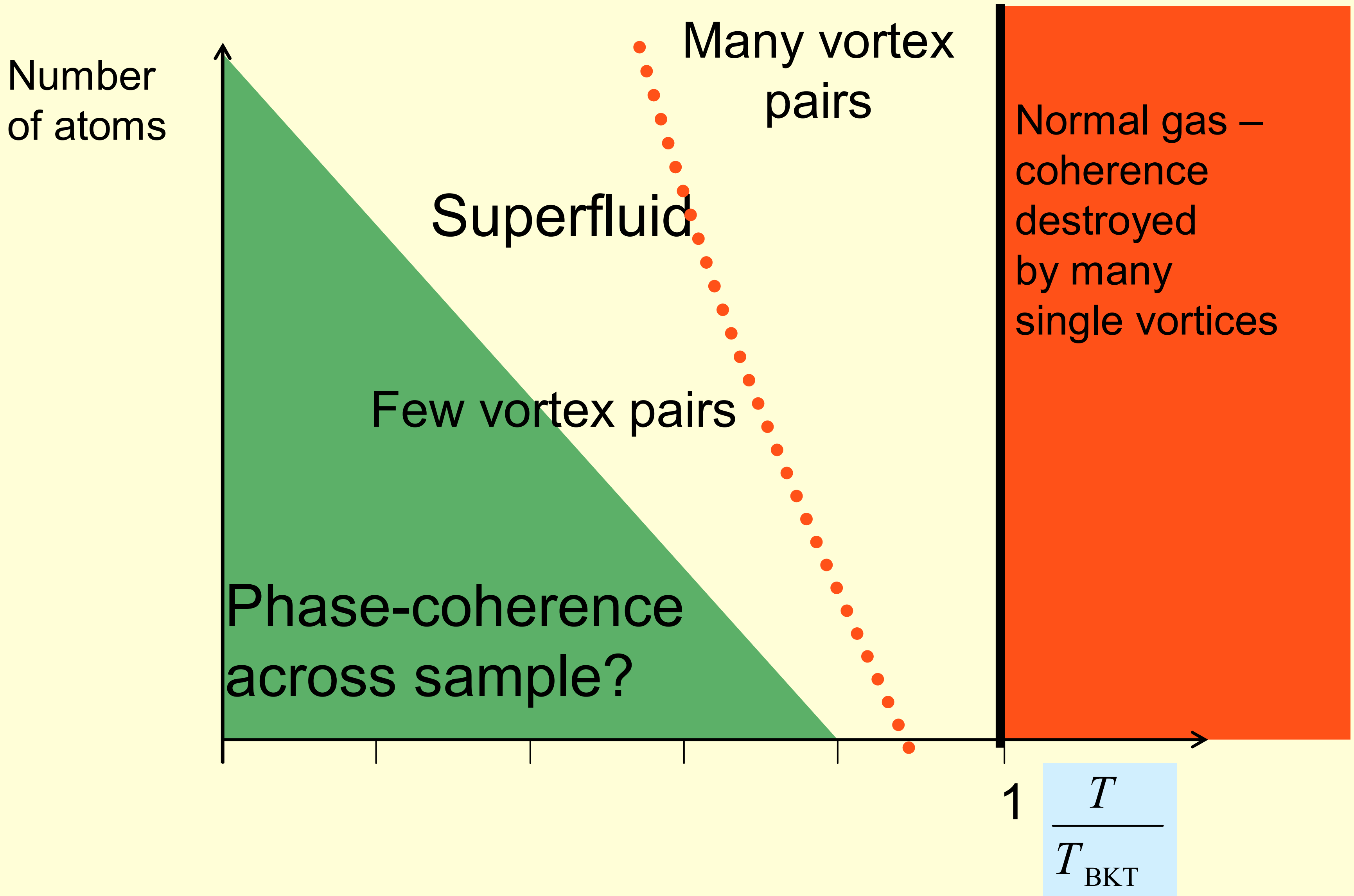
Theoretical prediction for 2D gas in harmonic trap



Theory for the 2D Bose gas in uniform potential?



Speculation for a finite system with uniform potential



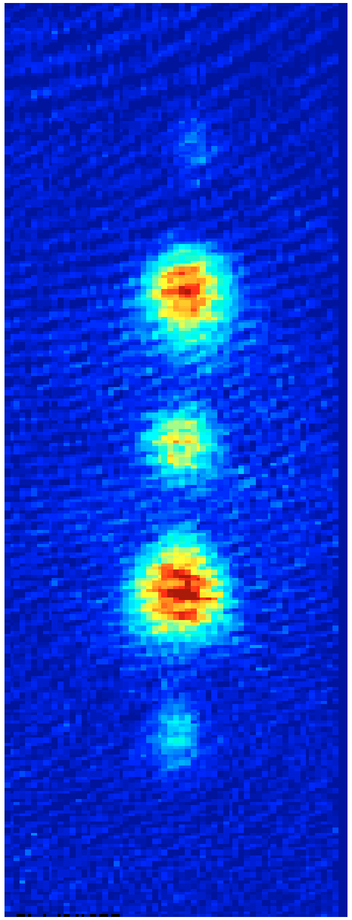
Experiments with a 2D Bose gas

- Observe a Berezinskii-Kosterlitz-Thouless phase ?

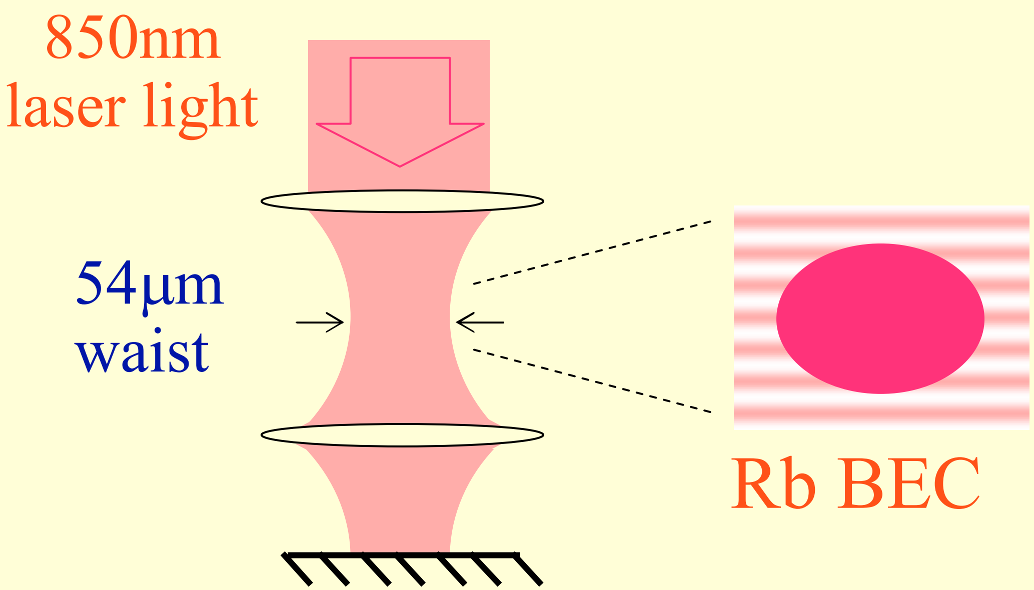
(a) observe formation of vortex-antivortex pairs

(b) measure decrease in coherence of system (quantum fluctuations more important in two dimensions than in three) – see next slide.

Diffraction of atoms from an optical standing wave (we have only done this in 1D so far – see below)



One-dimensional optical lattice



Spacing of peaks corresponds to two photon momentum, $2\hbar k$

Deduce coherence of atoms from contrast of the diffraction pattern

Experiments with a 2D Bose gas

- Observe a Berezinskii-Kosterlitz-Thouless phase ?

(a) Observe formation of vortex-antivortex pairs

(b) Measure decrease in coherence of system (quantum fluctuations more important in two dimensions than in three).

(c) Detect change in compressibility by measuring oscillation frequency of quadrupole modes.

- Bose gas in a 2D lattice

Talk about experiments on the superfluidity of a Bose gas

- Scissors mode – irrotational flow
- Nucleation of vortices – hysteresis
- Superfluid gyroscope – vortex precession
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Condition for quantum degeneracy in 2D gas

$$n a_z \lambda_{dB}^2 \approx 1$$

$$\lambda_{dB} = \frac{h}{Mv}$$

$$k_B T = M v^2$$

$$n a_z \lambda_{dB}^2 = n a_z \frac{h^2}{M^2 v^2} = n a_z \frac{h^2}{M k_B T} \approx 1$$

$$\rho_s \frac{h^2}{M^2} \approx k_B T$$

c.f. BKT condition

$$k_B T_{BKT} = \rho_s \frac{h^2}{8 \pi M^2}$$

Free energy of a vortex pair: $F = U_{\text{pair}} - T\,S_{\text{pair}}$

$$U_{\text{pair}} = \frac{2\pi\hbar^2\rho_{\text{s}}}{m^2}\ln\frac{R_{12}}{\xi}$$

where
m = mass of the Bose-Einstein condensed atom,
R_{12} = separation of the two vortices, and
ξ = healing length.
The two-dimensional superfluid density ρ_{s} is related to number density n_0 by

$$\rho_{\text{s}} = mn_0\sqrt{\pi}a_z\,.$$

We assume that $R_{12} \simeq 2\xi$, hence

$$U_{\text{pair}} = 2\pi^{3/2}\hbar^2 \times \frac{n_0 a_z}{m}\ln 2$$

From $F = U_{\rm pair} - T_2 S_{\rm pair} = 0$ we find the transition temperature is given by

$$T_2 = \frac{U_{\rm pair}}{S_{\rm pair}} \simeq \frac{\hbar\omega_\perp}{S_{\rm pair}} \left(\frac{N_0 a_z}{a_{\rm scatt}}\right)^{1/2}$$

th

where
$a_{\rm scatt}$ = scattering length that characterises the strenght of the interactions,
N_0 = number of condensed atoms.

Comparison with the BEC transition temperature in 2D

$$k_{\rm B} T_0 \simeq \hbar\omega_\perp N^{1/2},$$

shows that

$$\frac{T_2}{T_0} \simeq \left(\frac{N_0}{N} \frac{a_z}{a_{\rm scatt}}\right)^{1/2} \times \frac{k_{\rm B} \ln 2}{S_{\rm pair}}. \quad (1)$$

A vortex pair has entropy

$$S_{\text{pair}} = k_{\text{B}} \ln\left(\frac{2\pi R_{\text{TF}}^2}{\xi^2}\right).$$

Thus

$$\frac{T_2}{T_0} \simeq \left(\frac{N_0}{N}\frac{a_z}{a_{\text{scatt}}}\right)^{1/2} \times \frac{\ln 2}{\ln\left(\frac{2\pi R_{\text{TF}}^2}{\xi^2}\right)}.$$

N_0, ξ and R_{TF} depend on N and T in a known way, hence we calculate T_2/T_0 as a function of N.

Summary

- We have Quasi-2D confinement of a Bose gas in a combined optical and magnetic potential
- Observe a Berezinskii-Kosterlitz-Thouless phase?

(a) directly observe formation of vortex-antivortex pairs

(b) measure decrease in coherence of system.

- Experiments on 2D Bose gases in periodic potentials.
- Direct Quantum Simulation of quantum magnetism.