The Abdus Salam
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SCHOOL ON QUANTUM PHASE TRANSITIONS
AND
NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Theory of entanglement

Presented by:

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Quantum Information Theory, Quantum Phase Transitions and Cold Atoms

Wanderin' quantum optics theory (Warsaw, Saclay, Hannover, Barcelona)

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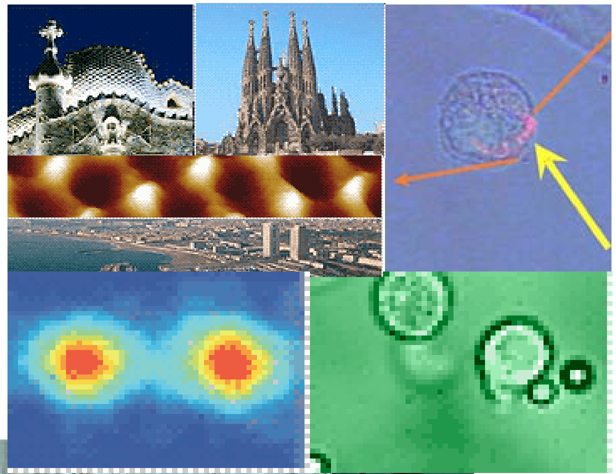
Theoretical Quantum Optics

at the University of Hannover
at ICFO



Cold atoms and cold gases:

- Weakly interacting Bose and Fermi gases (solitons, vortices, phase fluctuations, atom optics, quantum engineering)
- Dipolar Bose and Fermi gases
- Collective cooling, CW atom laser, quantum master equation
- Strongly correlated systems in AMO physics

Quantum Information:

- Quantification and classification of entanglement
- Quantum cryptography and communications
- Implementations in quantum optics

Matter in strong laser fields:

- High harmonics generation, above threshold ionization, multielectron ionization
- Attophysics
- Analogies: Super-intense laser-atom physics and nonlinear atom optics

Three Tales on Quantum Information Theory, Quantum Phase Transitions and Cold Atoms

- Lecture I – Introduction to QIT – theory of entanglement, entanglement criteria and measures, multiparty entaglement, entaglement detection, distillability (literature: bruss.pdf, lewen.pdf, lecture1.ppt)
- Lecture IIa – Entanglement and quantum phase transitions - entaglement in simple integrable models at the criticality, localizable entanglement, entanglement versus correlations (lecture2.pps).
- Lecture IIb – Generation of entanglement in many body systems, generation via quantum phase transitions, generation in complex and disordered systems.
- Lecture IIIa – Entaglement based codes, matrix product states, PEPS (Projected Entangled-Pair States) (lecture3.ppt, armand.pdf).
- Lecture IIIb – Examples – Spin ½ XY chain in a random X-oriented field

Lecture I – based on extracts of the:

Theory of entanglement

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0. Introduction
   - a) The central role of entanglement in quantum information
   - b) Reminder: fundamentals of quantum theory
     - i) Single quantum states
     - ii) Composite quantum systems

I. Separability versus entanglement
   - a) Bipartite case: pure states
     - i) Schmidt decomposition
     - ii) Applications (superdense coding, quantum teleportation)
   - b) Bipartite case: mixed states
     - i) Entanglement criteria
     - ii) Entanglement measures
   - c) Multipartite case: pure states
     - i) Inequivalent entanglement classes
   - d) Multipartite case: mixed states

II. Distillability

a) The distillability problem

b) Filtering method

c) Bound entanglement

   i) UPB states

   ii) NPT bound entangled states?

d) Summary of distillability properties

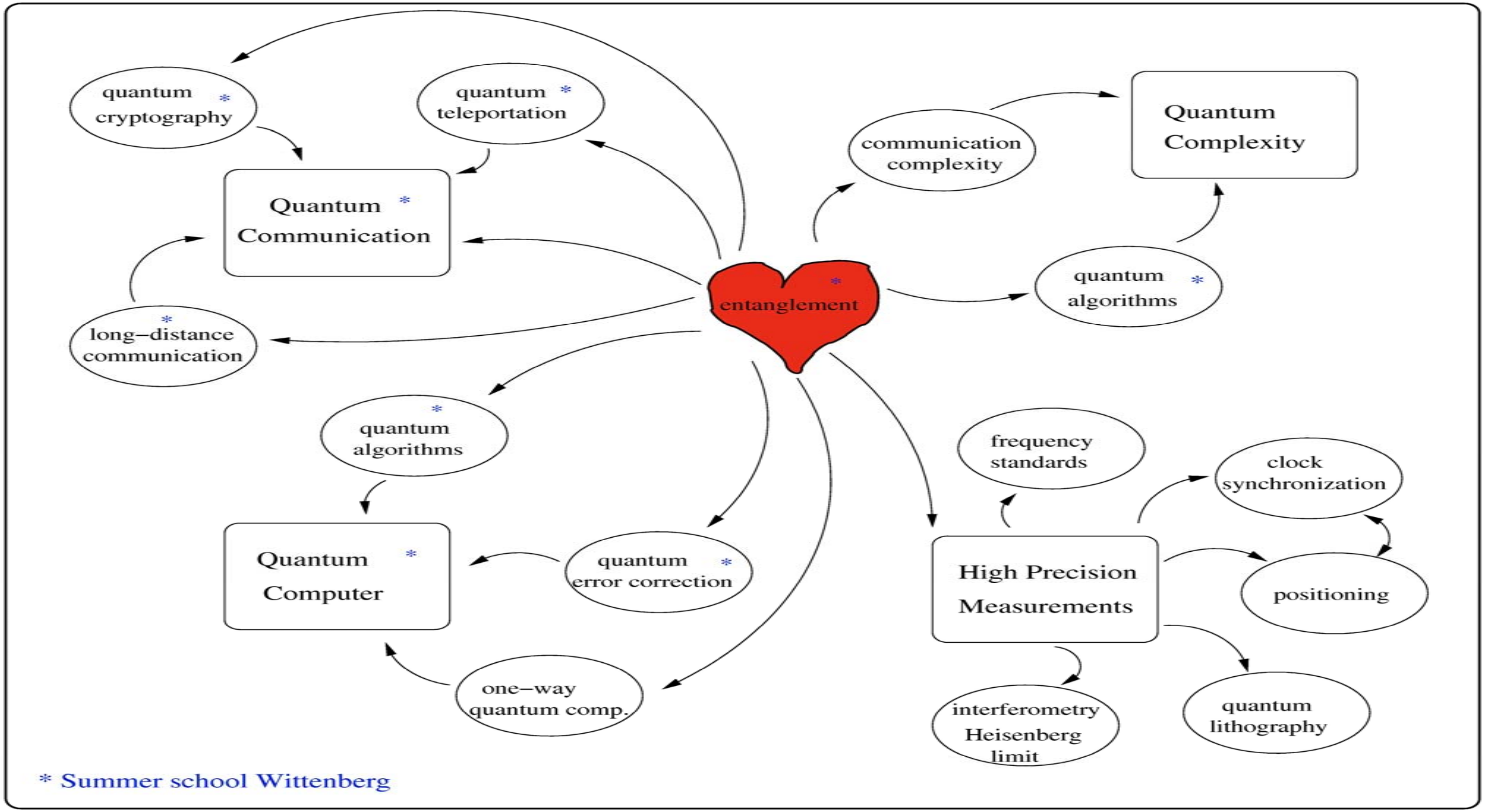
III. Detection of entanglement

a) Bell inequalities

b) Entanglement witnesses

c) Local decompositions of witnesses

Entanglement is at the heart of quantum information



- Entanglement plays an important role in quantum phase transitions and, in general, in many body quantum physics!!!

0.a) Entanglement and quantum information

["Verschränkung": *E. Schrödinger, Naturwissenschaften* **23**, *807 (1935)*]

Quantum teleportation: Instantaneous transmission of properties of a quantum state over a long distance.

Entanglement swapping: Entangle particles that have never interacted with each other.

Superdense coding:
Encode 2 bits in 1 two-level system (qubit).

Quantum cryptography: Entanglement enables detection of an eavesdropper (Ekert protocol).

Shor algorithm: Polynomial complexity of factorisation, algorithm uses entangled quantum state.

Quantum cloning: Imperfect quantum cloning involves entanglement.

Precision measurements: Entanglement allows higher precision in frequency standards, gyroscopes etc.

$\leadsto$ Entanglement is viewed as a resource for quantum information protocols.

$\leadsto$ we need to understand entanglement!

I. Separability versus entanglement

Bipartite systems $H = H_A \otimes H_B$

Pure states:

- $|\psi\rangle = |a\rangle \otimes |b\rangle \rightsquigarrow$ separable
- $|\psi\rangle \neq |a\rangle \otimes |b\rangle \rightsquigarrow$ entangled
- example (separable): $|\psi\rangle = |00\rangle$
- example (entangled): Bell states

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Mixed states:

- $\varrho = \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i| \rightsquigarrow$ separable
- $\varrho \neq \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i| \rightsquigarrow$ entangled

  $0 \leq p_i \leq 1, \ \sum_i p_i = 1, \ \langle a_i | a_j \rangle \neq \delta_{ij}, \ \langle b_i | b_j \rangle \neq \delta_{ij}$
- example (separable):

  $\varrho = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$
- example (entangled): Werner state

  $\varrho_W = (1-p)\frac{1}{4}\mathbb{1} + p|\Phi^+\rangle\langle\Phi^+|$ with $1/3 < p \leq 1$

Decomposition of Werner state

$$\begin{aligned} \varrho_W &= (1-p)\tfrac{1}{4}\mathbb{1} + p|\,\Phi^+\rangle\langle\Phi^+\,| \\ &= \frac{2}{3}p\sum_{i=1}^{3}|\,f_i f_i\rangle\langle f_i f_i\,| \\ &\quad +\frac{1}{4}(1-p)\{|\,00\rangle\langle 00\,| + |\,11\rangle\langle 11\,|\} \\ &\quad +\frac{1}{4}(1-3p)\{|\,01\rangle\langle 01\,| + |\,10\rangle\langle 10\,|\} \end{aligned}$$

with

$$\begin{aligned} |\,f_1\rangle &= \frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle) \\ |\,f_2\rangle &= \frac{1}{\sqrt{2}}(|\,0\rangle + e^{i\frac{2\pi}{3}}|\,1\rangle) \\ |\,f_3\rangle &= \frac{1}{\sqrt{2}}(|\,0\rangle + e^{-i\frac{2\pi}{3}}|\,1\rangle) \end{aligned}$$

Simple conjecture:
separability/entanglement from spectral properties???

Counter example:

M. Nielsen and J. Kempe; Phys. Rev. A **86**, 5184 (2001)

separable:

$$\sigma = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{eigenvalues: } \lambda_\sigma = \{\frac{2}{3}, \frac{1}{3}, 0, 0\}$$

$$\text{and } \sigma_A = \sigma_B = \frac{1}{3}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

entangled:

$$\rho = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{eigenvalues: } \lambda_\rho = \{\frac{2}{3}, \frac{1}{3}, 0, 0\}$$

$$\text{and } \rho_A = \rho_B = \frac{1}{3}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

ρ and σ have same eigenvalues, also ρ_A and σ_A.

$\rightsquigarrow$ conjecture is wrong!

Although, there is some “truth” in it…

Operational separability criteria

$\varrho \in \mathcal{H}_A \otimes \mathcal{H}_B$, dim $\mathcal{H}_A = d_A$, dim $\mathcal{H}_B = d_B \geq d_A$

Pure states:

Schmidt decomposition, Schmidt rank $r \leq d_A$:

$|\,\psi^r\rangle = \sum_i^r a_i |\,e_i\rangle |\,f_i\rangle$, $\quad a_i > 0$, $\sum_i^r a_i^2 = 1$,

$\langle e_i |\, e_j\rangle = \delta_{ij} = \langle f_i |\, f_j\rangle$

$|\,\psi\rangle$ separable $\Leftrightarrow r = 1$ [nec. + suff. condition]

Mixed states:

1) Peres-Horodecki criterion (pos. partial transpose = PPT):

$(\varrho^{T_A})_{m\mu,n\nu} = \varrho_{n\mu,m\nu}$

$\varrho_{sep}^{T_A} = \sum_i p_i\, \varrho_{A,i}^T \otimes \varrho_{B,i}$

ϱ separable $\Rightarrow$ $\varrho^{T_A} \geq 0$

A. Peres; Phys. Rev. Lett. **77**, 1413 (1996)

$\Leftarrow$ for dim 2×2 and 2×3

M. + P. + R. Horodecki; Phys. Lett. A **223**, 1 (1996)

[$2 \times 2, 2 \times 3$: nec. + suff. condition;
general: nec. condition, $\exists$ PPT ent. states
= bound ent. states]

Positive and completely positive maps

- **Let M be a linear, self-adjoint map acting from the space of operators (density matrices) acting in H_B to the space of operators acting in H_C**

$$M(\rho_B) = \rho_C$$

- **The map M is positive, iff it maps positively definite operators into positively definite operators. The map is completely positive, iff for any extension**

$$I_A \otimes M(\rho_{AB}) = \rho_{AC} \ ,$$

the extended map is positive! Example: Trasposition is positive, but not CP; CP maps have the form $M(\rho)=\sum_i C_i \, \rho C_i^+$

Non-operational separability criteria

1) Positive maps:

ϱ is separable $\Leftrightarrow$ for *any* positive map Λ (not CP) **acting on Bob**

$$(\mathbb{1} \otimes \Lambda)\varrho \geq 0$$

[nec. + suff. condition]

M. Horodecki, P. Horodecki and R. Horodecki; Phys. Lett. A **223**, 1 (1996)

2) Entanglement witnesses:

ϱ is entangled $\Leftrightarrow$ $\exists\, \mathcal{W}$ (Hermitian) with

$$\mathrm{Tr}(\mathcal{W}\varrho) < 0 \,,$$
$$\mathrm{Tr}(\mathcal{W}\varrho_{sep}) \geq 0 \,.$$

[nec. + suff. condition]

B. Terhal; Phys. Lett. A **271**, 319 (2000)

Correspondence between these criteria:

A. Jamiolkowski; Rep. Math. Phys. **3**, 275 (1972)

$$\mathcal{W} = (\mathbb{1} \otimes \Lambda)P_+$$

with $P_+ = \frac{1}{d}(\sum_{i=1}^{d} |\, ii\rangle)(\sum_{j=1}^{d} \langle jj\,|)$

I.b) ii) Entanglement measures

Requirements for entanglement measures E:

1) ϱ separable $\Rightarrow E(\varrho) = 0$

2) Normalization:

$$E(P_+^d) = \log d$$

3) No increase under LOCC:

$$E(\Lambda_{LOCC}(\varrho)) \leq E(\varrho)$$

4) Continuity:

$$E(\varrho) - E(\sigma) \longrightarrow 0 \quad \text{for} \quad ||\varrho - \sigma|| \longrightarrow 0$$

5) Additivity:

$$E(\varrho^{\otimes n}) = n\, E(\varrho)$$

6) Subadditivity:

$$E(\varrho \otimes \sigma) \leq E(\varrho) + E(\sigma)$$

7) Convexity:

$$E(\lambda\varrho + (1-\lambda)\sigma) \leq \lambda E(\varrho) + (1-\lambda)E(\sigma)$$

Important entanglement measure

- Logarithmic Negativity (LN)

LN of bipartite state ρ_{AB} :

$$E(\rho_{AB}) = \log_2 \| \rho_{AB}^{T_A} \|$$

$\|.\|$: trace norm (sum of abs. values of eigenvalues)

$\rho_{AB}^{T_A}$:partial transpose of ρ_{AB} w.r.t. A.

In 2x2, the state is entangled (and distillable) if and only if $E(\rho_{AB}) \neq 0$.

Other important entanglement measures

Whole system is in a pure state!

von Neumann Entropy = -Tr $\rho_1 \log_2 \rho_1 \quad \leftrightarrow \quad 4 \det \rho_1$

Coffman, Kundu, Wootters PRA 61, 052306 (2000)

von Neumann Entropy of a block = -Tr $\rho_{block} \log_2 \rho_{block}$

- **CONCURRENCE**

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4,$ are square roots of the eigenvalues of

$$R = \rho_2 \cdot (\sigma_y \otimes \sigma_y) \cdot \rho_2^* \cdot (\sigma_y \otimes \sigma_y)$$

Some important entanglement measures

Entanglement cost:

$$E_C(\varrho) = \inf_{\{\Lambda_{LOCC}\}} \lim_{n_\varrho \to \infty} \frac{n^{in}_{|\Phi+\rangle}}{n^{out}_\varrho}$$

Entanglement of formation:

$$E_F(\varrho) = \inf_{\{dec\}} \sum_i p_i S(|\psi_i\rangle\langle\psi_i|)_{red}$$

Relative entropy of entanglement:

$$E_R(\varrho) = \inf_{\sigma \in S} \mathrm{tr}[\varrho(\log \varrho - \log \sigma)]$$

Distillable entanglement:

$$E_D(\varrho) = \sup_{\{\Lambda_{LOCC}\}} \lim_{n_\varrho \to \infty} \frac{n^{out}_{|\Phi+\rangle}}{n^{in}_\varrho}$$

Relations:

$$E_D(\varrho) \le E(\varrho) \le E_C(\varrho) \quad , \quad E_F(\varrho) \stackrel{?}{=} E_C(\varrho)$$

Properties:

	E_C	E_F	E_R	E_D
continuity	?	$\surd$	$\surd$	?
additivity	$\surd$	?	no[a]	$\surd$
convexity	$\surd$	$\surd$	$\surd$	no (?)[b]

[a] K. Vollbrecht and R. Werner; quant-ph/0010095

[b] P. Shor, J. Smolin and B. Terhal; Phys. Rev. Lett. **86**, 2681 (2001)

Structure of pure 3-qubit states

Tripartite system: $H = H_A \otimes H_B \otimes H_C$

Separable state:

$$|\psi_S\rangle = |\phi_A\rangle \otimes |\phi_B\rangle \otimes |\phi_C\rangle$$

Bi-separable state:

Entanglement of only two of the three systems, e.g. A–BC:

$$|\psi_B\rangle = |\phi_A\rangle \otimes \sum_{i=1}^{2} a_i |e_i\rangle |f_i\rangle$$

3-qubit correlated state:

$\exists$ two classes of inequivalent states:

$$\begin{aligned} |\psi_{GHZ}\rangle &= \tfrac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\ |\psi_W\rangle &= \tfrac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle) \end{aligned}$$

Any 3-qubit correlated pure state $|\psi\rangle$ can be transformed into either $|\psi_{GHZ}\rangle$ or $|\psi_W\rangle$ by *local* reversible operations $\mathcal{A} \otimes \mathcal{B} \otimes \mathcal{C}$ with probability > 0.

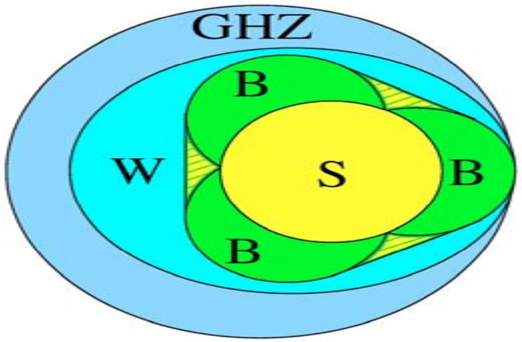
W. Dür, G. Vidal and J. I. Cirac, Phys. Rev. A ***62****, 062314 (2000)*

Classification of mixed 3-qubit systems

A. Acín, D. Bruß, M. Lewenstein and A. Sanpera, Phys. Rev. Lett. **87**, *040401 (2001)*

Introduce convex compact sets:

(ϱ is convex combination of pure states)



Constructing 3-qubit witnesses:

GHZ witness: $\mathcal{W}_{GHZ} = \frac{3}{4}\mathbb{1} - P_{GHZ}$

W witness: $\mathcal{W}_{W} = \frac{2}{3}\mathbb{1} - P_{W}$

$\rightsquigarrow$ Set of W $\setminus$ B–states is not of measure 0

(contrary to pure case)

Properties of "bound entangled" states:

evidence for $\varrho_{BE} \notin$ GHZ $\setminus$ W

Entanglement measures in N qubit systems

- **Entaglement in reduced density matrices**

Let ρ $(|\Psi\rangle\langle\Psi|)$ be a many body state of the system

we define the two-, three-, … qubit reduced density matrices:

$$\rho_{ij} = \mathrm{tr}_{k \neq ij}(\rho)$$

$$\rho_{ijk} = \mathrm{tr}_{l \neq ijk}(\rho)$$

We apply then entanglement measures (or witnesses, positive maps etc.) to the two-, three-, … qubit reduced density matrices:

$$E(\rho_{ij}),\ E(\rho_{ijk})\ \text{etc.}$$

Entanglement measures in N qubit systems

- **Entanglement of assistance**

Let ρ $(|\Psi\rangle\langle\Psi|)$ be a many body state of the system and $\mathbf{M}$ - a measurement acting on qubits $\mathbf{k \neq ij}$.

We define

$$\rho_{ij}(M) = M_{k\neq ij}(\rho)$$

$$E_{as}(ij) = \sup_M E(\rho_{ij}(M))$$

- **Localizable entanglement**

The definition is the same as above, except that now $\mathbf{M}$ is a local measurement acting locally on qubits $\mathbf{k \neq ij}$.

III.a) Detection of entanglement via Bell inequalities

CHSH inequality:

Clauser, Horne, Shimony, and Holt, Phys. Rev. Lett. **23**, *880 (1969)*

assume: a, b, c, d variables with values ± 1

$$\rightsquigarrow (a+c)b + (-a+c)d = \pm 2$$

in each run:

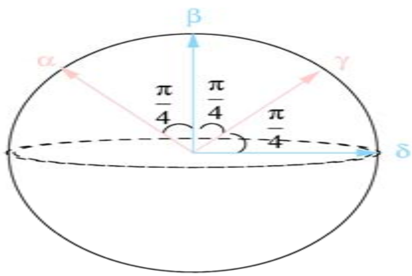
$$(a_i + c_i)b_i + (-a_i + c_i)d_i = \pm 2$$

average:

$$\rightsquigarrow |\langle (a+c)b + (-a+c)d \rangle| \leq 2$$

$S = |\langle ab \rangle + \langle bc \rangle + \langle cd \rangle - \langle da \rangle| \leq 2$ CHSH inequality

Quantum mechanics: $\langle ab \rangle = \mathrm{Tr}[(\vec{\alpha} \cdot \vec{\sigma}_A) \otimes (\vec{\beta} \cdot \vec{\sigma}_B)\varrho]$



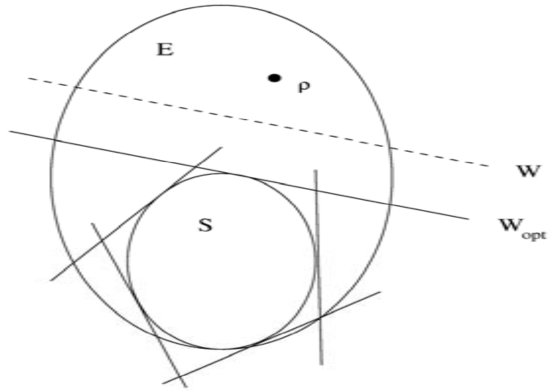
Bloch sphere

Singlet: $S = 2\sqrt{2} \geq 2$

III.b) Entanglement witnesses

Consequence of Hahn-Banach theorem:

Let S be a convex, compact set; $\varrho \notin S$: there exists a hyper-plane that separates ϱ from S.



"Witnesses" $\mathcal{W}$:

- construction of witnesses
- optimisation of witnesses
- infinitely many witnesses needed to characterise S

Generic form for witness:

$$\begin{aligned} W &= P + Q^{T_A} - \epsilon \mathbf{1} \, , \\ \epsilon &= \inf_{|\, e, f\rangle} \langle e, f \,| P + Q^{T_A} |\, e, f \rangle \, , \end{aligned}$$

with $R(P) = K(\rho)$ and $R(Q) = K(\rho^{T_A})$

M. Lewenstein, B. Kraus, I.J. Cirac and P. Horodecki; Phys. Rev. A **62**, 052310 (2000)

Example for entanglement witness

given ϱ with non-positive partial transpose:

$$\mathcal{W} = (|\,\phi\rangle\langle\phi\,|)^{T_A}$$

with

$$\varrho^{T_A}|\,\phi\rangle = \lambda_{min}|\,\phi\rangle$$

is an entanglement witness that detects ϱ:

$$\begin{aligned} Tr(\mathcal{W}\varrho) &= Tr((|\,\phi\rangle\langle\phi\,|)^{T_A}\varrho) \\ &= Tr(|\,\phi\rangle\langle\phi\,|\varrho^{T_A}) \\ &= \langle\phi\,|\varrho^{T_A}|\,\phi\rangle \\ &= \lambda_{min} < 0 \end{aligned}$$

and

$$\begin{aligned} Tr(\mathcal{W}\varrho_{sep}) &= Tr((|\,\phi\rangle\langle\phi\,|)^{T_A}\varrho_{sep}) \\ &= Tr(|\,\phi\rangle\langle\phi\,|\varrho_{sep}^{T_A}) \\ &= \langle\phi\,|\varrho_{sep}^{T_A}|\,\phi\rangle \geq 0 \end{aligned}$$

Another example for an entanglement witness

given an entangled pure state $|\psi\rangle$:

$$\mathcal{W}_{|\psi\rangle} = x\mathbb{1} - |\psi\rangle\langle\psi|$$

with

$$x = \max_{|\varphi_{sep}\rangle} |\langle\psi|\varphi_{sep}\rangle|^2$$

is an entanglement witness that detects $|\psi\rangle\langle\psi|$:

$$\begin{aligned} Tr(\mathcal{W}|\psi\rangle\langle\psi|) &= x - |\langle\psi|\psi\rangle|^2 \\ &= x - 1 < 0 \end{aligned}$$

and

$$\begin{aligned} Tr(\mathcal{W}\varrho_{sep}) &= x - \langle\psi|\varrho_{sep}|\psi\rangle \\ &\geq 0 \end{aligned}$$

Remark:

- $\mathcal{W}$ also detects $|\psi\rangle\langle\psi|$ plus some noise
- How to find x?

  $|\psi\rangle = \sum_{ij} C_{ij}|ij\rangle$

  $|\varphi_{sep}\rangle = |A\rangle|B\rangle$ with $|A\rangle = \sum_i A_i|i\rangle, \; |B\rangle = \sum_j B_j|i\rangle$

$$x = \max_{|A\rangle,|B\rangle} |\sum_{ij} A_i^* B_j^* C_{ij}|^2 = \lambda_{max}^2(C)$$

Recent reviews on quantum entanglement

- *"Mixed-state entanglement and quantum communication"*,
  M. Horodecki, P. Horodecki and R. Horodecki;
  in: "*Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments* (Springer Tracts in Modern Physics, 173),
  Eds. G. Alber, T. Beth, M.+P.+R. Horodecki, M. Rötteler, H. Weinfurter, R. Werner and A. Zeilinger,
  Springer-Verlag (April 2001).
- *"The uniqueness theorem for entanglement measures"*,
  M. Donald, M. Horodecki and O. Rudolph;
  J. Math. Phys. 43, 4252 (2002).
- *"Detecting quantum entanglement"*,
  B. Terhal; J. Theor. Comp. Sc. 287(1), 313 (2002).
- *"Separability and distillability in composite quantum systems – a primer"*,
  M. Lewenstein, D. Bruß, J. I. Cirac, B. Kraus, M. Kuś,
  J. Samsonowicz, A. Sanpera and R. Tarrach;
  J. Mod. Opt. 47, 2841 (2000).
- *"Characterizing entanglement"*,
  D. Bruß; J. Math. Phys. 43, 4237 (2002).

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