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SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

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Spinor Condensates: Experiment

Presented by:

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ICTP
School on Quantum Phase Transitions and Non-Equilibrium Phenomena in Cold Atomic Gases

Trieste, 20.07.2005

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Universität Hamburg

Spinor-BEC

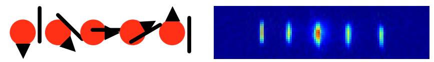
"Conventional" BEC:

Quantum gas with fixed spin orientation

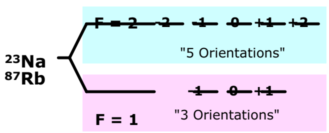
Spin-independent trapping

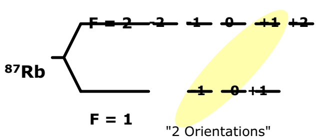
Spinor-BEC:

Bose-Einstein condensate with spin degree of freedom

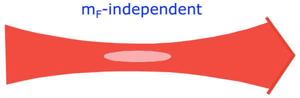


Spinor-Systems



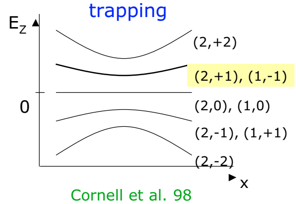


Optical trapping
m_F-independent



Ketterle et al. 98
Sengstock et al. 03
Chapman et al. 03
Hirano et al. 04
Stamper-Kurn et al. 05

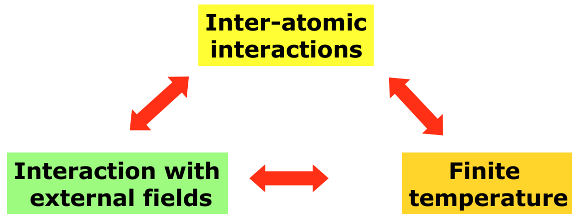
Magnetic trapping

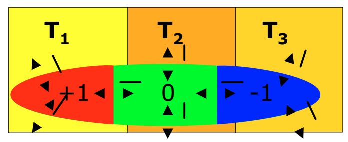


Cornell et al. 98
Hall et al. 04

For theory see talks of T.L. Ho and M. Ueda

Spinor-BEC as a Multi-Component System





Experimental Studies (not complete)

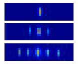
Inter-atomic interactions

^{23}Na F=1 - ground state, spin domains and metastability

- J. Stenger et al., Nature **396**, 345-348 (1998).
- D. M. Stamper-Kurn et al., Phys. Rev. Lett. **83**, 661 (1999)
- H.-J. Miesner et al., Phys. Rev. Lett. **82**, 2228 (1999)

^{87}Rb - ground states and dynamics in F=1 and F=2

- H. Schmaljohann et al., Phys. Rev. Lett. **92**, 040402 (2004)
- M.-S. Chang et al., Phys. Rev. Lett. **92**, 140403 (2004)
- T. Kuwamoto, K. Araki, T. Eno, and T. Hirano, Phys. Rev. A **69**, 063604 (2004)



Interactions with external fields

Coherence in a quasi spin 1/2 system

- D. S. Hall, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **81**, 1543 (1998)
- M. H. Wheeler, K. M. Mertes, J. D. Erwin, and D. S. Hall, Phys. Rev. Lett. **93**, 170402 (2004)

Coreless vortex formation

- A. E. Leanhardt et al., Phys. Rev. Lett. **90**, 140403 (2003)

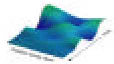
Vortex lattice in a quasi spin 1/2 system

-V. Schweikhard egt al., Phys. Rev. Lett. **93**, 210403 (2004)

Finite temperature

Thermal component spin waves

- J. M. McGuirk et al., Phys. Rev. Lett. **89**, 090402 (2002)
- J. M. McGuirk, D. M. Harber, H. J. Lewandowski, and E. A. Cornell, Phys. Rev. Lett. **91**, 150402 (2003)



Decoherence driven cooling

- H. J. Lewandowski, J. M. McGuirk, D. M. Harber, and E. A. Cornell, Phys. Rev. Lett. **91**, 240404 (2003)

Constant temperature BEC

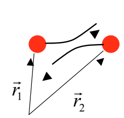
M. Erhard, H. Schmaljohann, J. Kronjäger, K. Bongs, and K. Sengstock, Phys. Rev. A **70**, 031602 (2004)

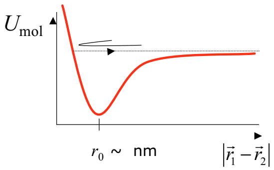
Interactions in spinor Bose-Einstein condensates

Single Component Collisions

Simple collision picture:

Colliding atoms experience the corresponding molecular potential.





Quantum gas parameter regime:

Density below 10^{15} cm^{-3}
→ mean distance > 100 nm >> r_0

Temperature ~ 100 nK
→ low relative velocities

Only s-wave scattering, single parameter: a

$$V_{int}(\vec r_1-\vec r_2)\approx\frac{4\pi\hbar^2}{m}\delta(\vec r_1-\vec r_2)a$$

Gross-Pitaevskii equation

Collisional interactions approximated by effective mean field potential

$$\left(-\frac{\hbar^2}{2m}\Delta+V_{ext}+\frac{4\pi\hbar^2 a}{m}|\psi(\vec r)|^2\right)\psi(\vec r)=E\,\psi(\vec r)$$

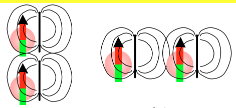
Magnetic Dipole-Dipole Interactions

Dipole-Dipole potential

- long range
- orientation dependence

$$V(\vec r-\vec r')=\frac{\mu_0}{4\pi}(g_F\mu_B)^2\frac{\vec F_1(\vec r)\cdot\vec F_2(\vec r')-3(\vec F_1(\vec r)\cdot\vec e)(\vec F_2(\vec r')\cdot\vec e)}{|\vec r-\vec r'|^3}$$

$$\vec e=\frac{\vec r-\vec r'}{|\vec r-\vec r'|}$$



Spin dependent part

- conversion between angular momentum and spin orientation

$$\vec F_1(\vec r)\cdot\vec F_2(\vec r')-3(\vec F_1(\vec r)\cdot\vec e)(\vec F_2(\vec r')\cdot\vec e)$$
$$=F_{1z}\cdot F_{2z}+\frac{1}{2}(F_{1+}\cdot F_{2-}+F_{1-}\cdot F_{2+})$$
$$-\frac{3}{4}(2e_zF_{1z}+e_-F_{1+}+e_+F_{1-})\cdot(2e_zF_{2z}+e_-F_{2+}+e_+F_{2-})$$
$$e_\pm=e_x\pm ie_y$$

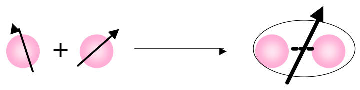
Prefactor:

$g_F\sim1$, $n\sim10^{20}$ m^{-3} → $E_{dd}\sim10^{-33}$ J

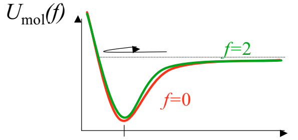
$$\frac{\mu_0}{4\pi|\vec r-\vec r'|^3}(g_F\mu_B)^2\approx\frac{\mu_0 n(g_F\mu_B)^2}{4\pi}\approx 8.6\cdot10^{-54}\,\mathrm{Jm^3}\times n g_F^2$$

Collisional Magnetic Interactions

Classification of collisional interactions by the total spin of the colliding pair.



Molecular potential curves change slightly for different spin configuration in the entrance channel.



Effective mean field now depends on several characteristic scattering lengths: $a_0, a_2,..., a_f$

$$V_{\text{int}}(\vec{r}_1 - \vec{r}_2) \approx \frac{4\pi\hbar^2}{m}\delta(\vec{r}_1 - \vec{r}_2)\sum_f a_f P_f$$

Collisional Interactions for F=1

Rewrite the potential in terms of the spin expectation value for each atom:

$$V_{\text{int}}(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2}{m}\delta(\vec{r}_1 - \vec{r}_2)\sum_f a_f P_f$$
$$= \left(g_0 + g_2 \vec{F}_1 \cdot \vec{F}_2\right)\delta(\vec{r}_1 - \vec{r}_2)$$

Two contributions:

$$g_0 = \frac{4\pi\hbar^2}{m}\frac{2a_2 + a_0}{3} \qquad g_2 = \frac{4\pi\hbar^2}{m}\frac{a_2 - a_0}{3}$$

spin-independent part | spin-dependent part

Magnetic properties:

- determined by spin-dependent part
- total spin projection is conserved
- typically a small difference in the weak quantum gas interactions

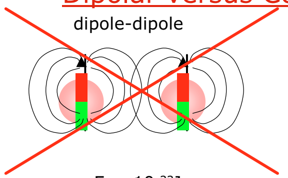
$g_2 > 0$: "anti-ferromagnetic"

$g_2 < 0$: "ferromagnetic"

Example ^{87}Rb: a_0=101.8(2) a_B, a_2=100.4(1)a_B

E.G.M. van Kempen. et al., PRL, **88**, 093201 (2002)

Dipolar versus Collisional Interactions

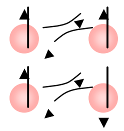


E_{dd}~10^{-33}J

see also: Cr-BEC (μ~6μ_B)

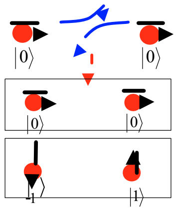
Axel Griesmaier et al., Phys. Rev. Lett. **94**, 160401 (2005)

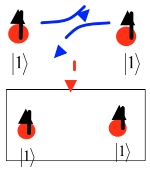
collisional



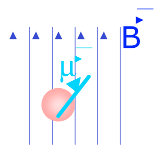
E_{mf}~10^{-32}J

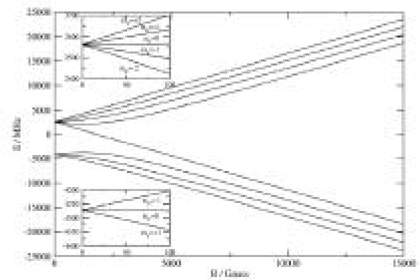
conservation of total spin:





Interactions with an External Magnetic Field



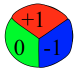


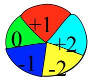
$$V_{\text{Zeeman}} = -p\langle F_z \rangle - q\left(\langle F_z^2 \rangle - 4\right)$$

$p = \mu_B g_F B$ linear Zeeman effect

$q = \pm\frac{(\mu_B B)^2}{4\Delta E_0}$ quadratic Zeeman effect

Interactions in Spinor BEC





single comp. mean field	linear Zeeman	mean field exchange interaction	quadratic Zeeman
n_0 n_{-1} n_{+1} n_{-2} n_{+2} chemical potential ~ 120nK	F=2 F=1 ~35 µK/G but: cancels due to spin conservation	$\lvert 1\rangle$ $\lvert 1\rangle$ F=1: $\propto \vec{F}_1 \cdot \vec{F}_2$ g_2~10nK , g_4~0.2nK	F=2 F=1 ~14nK/G²

Spin-dependent energy functional:

$E_{spin} = (- p \langle F_z\rangle + q \langle F_z^2\rangle + g_2 \langle F\rangle^2 n + g_4 |\langle P_0\rangle|^2 n) n$

lin. Zeeman energy — quadratic Zeeman energy — Spin dependend mean field [1] — additional mean field for F=2 [2]

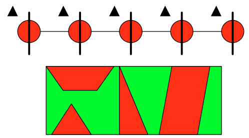
[1] T.-L. Ho, PRL, 81, p.742 (1998)
[2] M. Koashi, M. Ueda, PRL, 84, p.1066 (2000)

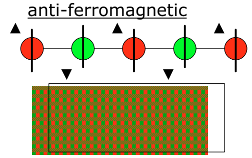
Magnetism

Condensed matter (100-1000 K):

Spins on a lattice and exchange coupling $H_{spin} = -\frac{1}{2} J \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j$

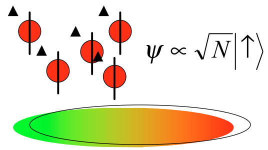
ferromagnetic — anti-ferromagnetic

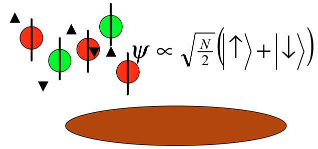




Gas (nK):

"ferromagnetic" — "anti-ferromagnetic"

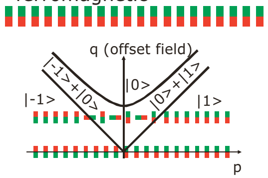




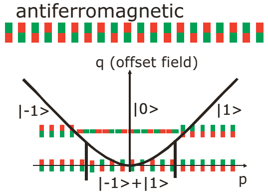
Experimental determination of spinor ground states

F=1 in Na

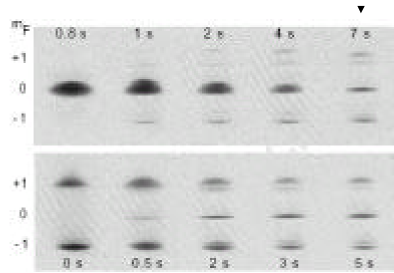
ferromagnetic



antiferromagnetic



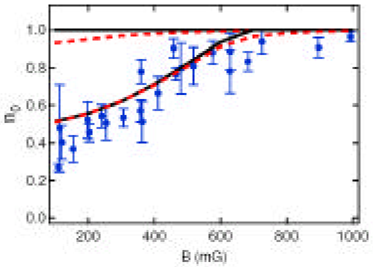
final state

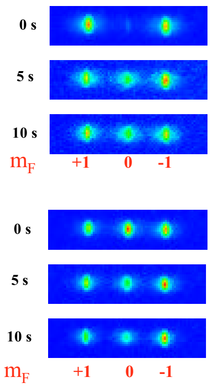


J. Stenger, et al., Nature **396**, 345 (1998).

F=1 in Rb

- Rb F = 1 is ferromagnetic!



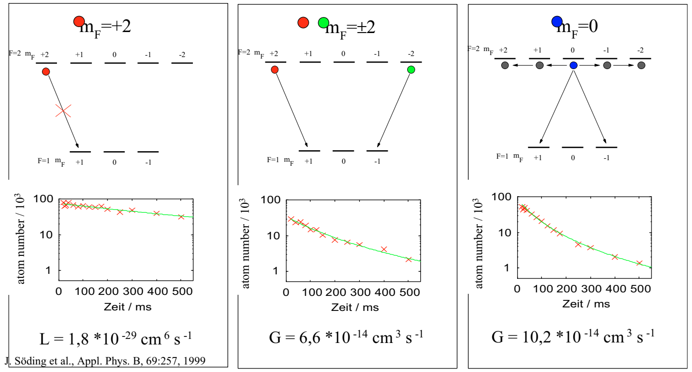


M.-S. Chang et al., PRL **92**, 140403 (2004)

Hamburg (2003)

F=2: Losses

Problem: decay to F=1 possible for non-streched states



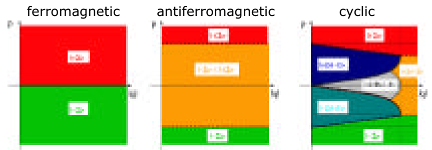
F=2 Spin Dynamics Rates

initially prepared m_F states	initial total spin	initial channels into m_F state G $[10^{-13}\text{cm}^3\text{s}^{-1}]$	finally populated m_F states
$\vert 0\rangle$	0	$\to \vert\pm1\rangle \approx 21.0$	equipartition
$\vert +1\rangle + \vert -1\rangle$	0	$\to \vert 0\rangle \approx 26.9$	equipartition
		$\to \vert\pm2\rangle \approx 4.6$	
$\vert +1\rangle + \vert 0\rangle + \vert -1\rangle$	0	$\to \vert\pm2\rangle \approx 5.0$	equipartition
$\vert +2\rangle + \vert -2\rangle$	0	-	$\vert +2\rangle + \vert -2\rangle$
$\vert +2\rangle + \vert 0\rangle + \vert -2\rangle$	0	$\to \vert\pm1\rangle < 0.1$	$\vert +2\rangle + \vert -2\rangle$
$\vert +2\rangle + \vert -1\rangle$	1/2	-	$\vert +2\rangle$
$\vert +1\rangle + \vert 0\rangle$	1/2	$\to \vert +2\rangle \approx 21.7$	$\vert +2\rangle$
		$\to \vert -1\rangle \approx 19.2$	
$\vert +1\rangle$	1	$\to \vert +2\rangle \approx 22.4$	$\vert +2\rangle$
		$\to \vert 0\rangle \approx 12.2$	
		$(\to \vert -1\rangle \approx 4.7)$	
$\vert +2\rangle$	2	-	$\vert +2\rangle$

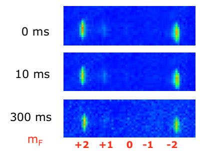
for details see: H. Schmaljohann et al., Phys. Rev. Lett. 92, 040402 (2004).

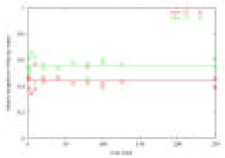
F=2 Ground State

Phase diagram for F=2 spinor Bose-Einstein condensates



Antiferromagnetic ground state is stable for Rb F=2

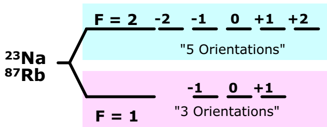




Components superposed **^{87}Rb F = 2 is antiferromagnetic**

Magnetic Ground States in Experiment

Investigated Elements:



	^{23}Na	^{87}Rb
F = 2 ("5 Orientations")	x	anti-ferro-magnetic
F = 1 ("3 Orientations")	anti-ferro-magnetic	ferromagnetic

Methods:

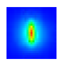
- Relaxation towards the equilibrium state
- Study of spatial overlap and relative population
- Investigation of spin dynamics

Spin Dynamics

Spinor Vector Order Parameter

Single component BEC:
→ scalar order parameter

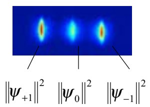
$$\psi(\vec r,t)=\sqrt{n(\vec r,t)}e^{i\phi(\vec r,t)}$$



Spinor-BEC:
→ vector order parameter

$$\Psi(\vec r,t)=\begin{pmatrix}\psi_{m_F=F}(\vec r,t)\\ \vdots \\ \psi_{m_F=-F}(\vec r,t)\end{pmatrix}=\sqrt{n(\vec r,t)}\begin{pmatrix}\zeta_{m_F=F}(\vec r,t)\\ \vdots \\ \zeta_{m_F=-F}(\vec r,t)\end{pmatrix}$$

$\|\psi_{m_F}\|^2$ population in magnetic hyperfine state m_F



The population in each spin component is directly accessible in experiment!
What about the phase?

Spindynamics - Simulation

Coupled GPE (T=0), homogeneous case:

$$\vec\psi(\vec r,t) = \sqrt{n(\vec r,t)}e^{i\phi(t)}\cdot\vec\zeta(\vec r,t)$$

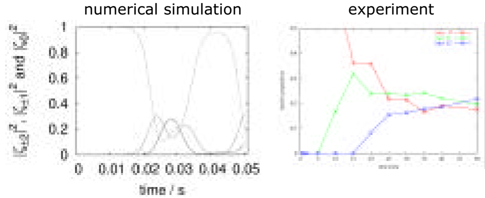
$$i\hbar\frac{\partial}{\partial t}\sqrt{n(\vec r,t)}e^{i\phi(t)} = \left(-\frac{\hbar^2\nabla^2}{2m}+V_{ext.}(\vec r)+g_0 n(\vec r)\right)\sqrt{n(\vec r,t)}e^{i\phi(t)},$$

$$\begin{aligned}i\hbar\frac{\partial}{\partial t}\vec\zeta(\vec r,t) &= \tilde g_1 n(\vec r)\vec S\vec\zeta(\vec r,t)\vec\zeta^*(\vec r,t)\vec S\vec\zeta(\vec r,t)\\ &+\tilde g_2 n(\vec r)\vec S^2\vec\zeta(\vec r,t)\vec\zeta^*(\vec r,t)\vec S^2\vec\zeta(\vec r,t)\\ &-pS_z\vec\zeta(\vec r,t)+q(S_z^2\vec\zeta(\vec r,t)-4).\end{aligned}$$

Results:

- delay
- oscillations
- phase dependency !
- here: no damping

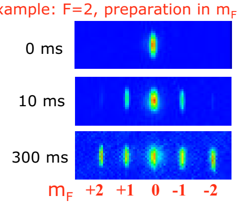
numerical simulation — experiment



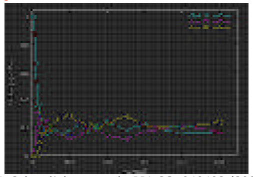
small seed in +/-1 und +/-2 comp.: 10^{-4} H. Schmaljohann et al., PRL **92**, 040402 (2004)

Spin Dynamics ^{87}Rb

- Example: F=2, preparation in $m_F = 0$

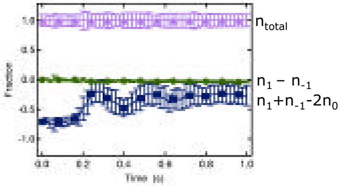


- **Spin-oscillations in F=2**

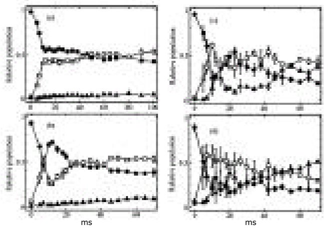


H. Schmaljohann et al., PRL **92**, 040402 (2004)

- **Spin-oscillations in F=1**



M.-S. Chang et al., PRL **92**, 140403 (2004)



T. Kuwamoto et al., PRA **69**, 063604 (2004)

Spin Dynamics: Influence of Magnetic Field

Linear Zeeman-effect has no consequences

$$i\hbar\frac{\partial}{\partial t}\zeta_{+1} = g_2 n\zeta_{-1}^*\zeta_0^2 - p\zeta_{+1} - 3q\zeta_{+1}$$

$$i\hbar\frac{\partial}{\partial t}\zeta_{0} = 2g_2 n\zeta_{0}^*\zeta_{+1}\zeta_{-1} - 4q\zeta_{0}$$

$$i\hbar\frac{\partial}{\partial t}\zeta_{-1} = g_2 n\zeta_{+1}^*\zeta_0^2 + p\zeta_{-1} - 3q\zeta_{-1}$$

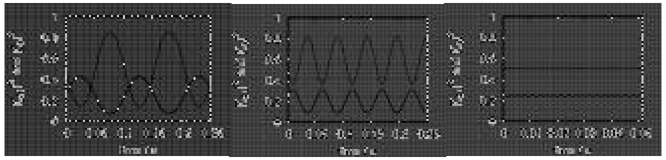
$$\tilde{\zeta}_x \equiv \zeta_x e^{ixp/\hbar} \longrightarrow$$

$$i\hbar\frac{\partial}{\partial t}\tilde{\zeta}_{+1} = g_2 n\tilde{\zeta}_{-1}^*\tilde{\zeta}_0^2 - 3q\tilde{\zeta}_{+1}$$

$$i\hbar\frac{\partial}{\partial t}\tilde{\zeta}_{0} = 2g_2 n\tilde{\zeta}_{0}^*\tilde{\zeta}_{+1}\tilde{\zeta}_{-1} - 4q\tilde{\zeta}_{0}$$

$$i\hbar\frac{\partial}{\partial t}\tilde{\zeta}_{-1} = g_2 n\tilde{\zeta}_{+1}^*\tilde{\zeta}_0^2 - 3q\tilde{\zeta}_{-1}$$

Dephasing by quadratic Zeeman-effect



B = 0 B = 340 mG B = 1700 mG

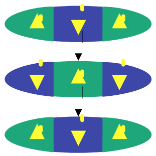
Spin Waves

Quasi-Spin-½:

Spin waves in thermal ensembles, not in the condensate fraction

Origin: interference of collisional amplitudes

Preparation:



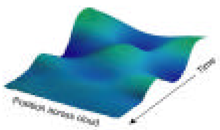


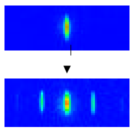
Image: E. A. Cornell, JILA
Phys. Rev. Lett. **89**, 090402 (2002)

Spinor-BEC:

Spin waves in condensate fraction predicted but not yet experimentally confirmed

???

spatial effects in spin dynamics



H. Schmaljohann et al., Appl. Phys. B. 79, 1001 (2004)
T. Kuwamoto et al., Phys. Rev. A **69**, 063604 (2004)

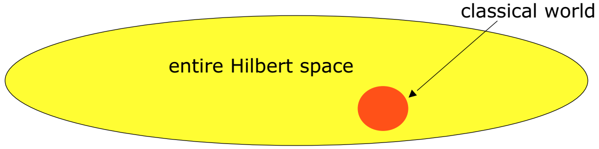
Coherence in Spin Dynamics ?

Preparation
Decoherence
Dephasing
Fragmentation

Goal: General Understanding of Decoherence in Multi-Component Systems

known: - spinor ground states and basic spinor dynamics
- coherent manipulation techniques and external influences

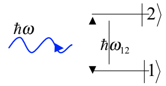
next: - measure and study spinor density matrix in time
- develop understanding of the dependence of decoherence on the dimensionality of the system

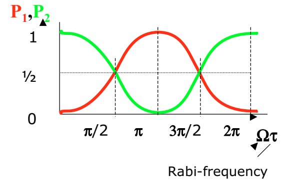


$$\rho_{\mathrm{QM}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow[?]{} \rho_{\mathrm{classical}} = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$$

Electromagnetic Coupling of the Internal States

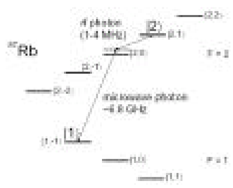
Reminder:
2-level system





Example:
Quasi spin-½

Two-photon microwave-coupling:

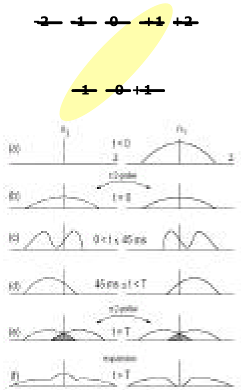


D. S. Hall et al., PRL **81**, 1539-1542 (1998)

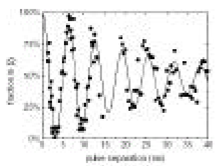
→ 2-level system

Coherence in a Quasi-Spin½-System

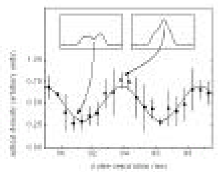
Idea: Ramsey-Interferometer



total population



central population



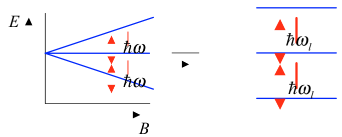
D. S. Hall et al.,
PRL **81**, 1543-1546 (1998)

→ **"phase memory" > 100 ms**

Spinor-BEC Preparation - Two Regimes

"Linear Zeeman regime"

- The "coupling energy" is larger than the quadratic Zeeman splitting
- Simultaneous coupling of all m_F-levels
- Full multi-level system treatment necessary

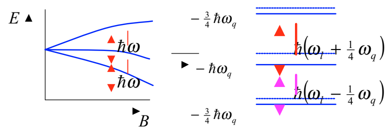


$$\omega_l = \mu_B g_F B \approx 2\pi \cdot 700\,\mathrm{kHz} \cdot \frac{B}{[\mathrm{G}]}$$ (for ^{87}Rb)

"quadratic Zeeman regime"

F=1 in ^{87}Rb:

- The "coupling energy" is smaller than the quadratic Zeeman splitting
- Only coupling of adjacent m_F-levels
- Effective two-level system

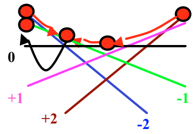


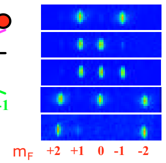
$$\omega_q = \frac{(\mu_B B)^2}{4\hbar\omega_{12}} \approx 2\pi \cdot 72\,\mathrm{Hz} \cdot \frac{B^2}{[\mathrm{G}^2]}$$

Preparation Considerations

High offset field:

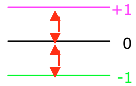
- individual transitions resolved
- <F²> changes
- full state space accessible
- adiabatic passage: phase relations hard to track

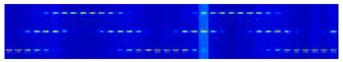




Low offset field:

- simultaneous coupling on all transitions
- <F²> conserved → no spin dynamics ?
- only "classical" states accessible
- phase relations well defined
- "Rabi"-oscillations

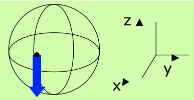




"Classical Spin" Picture

Classical picture:

- Spin-1 sphere in real space
- Arrow represents spin expectation value

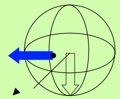


$$\vec{S}_{cl} = \langle \vec{S} \rangle = \zeta^+ \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} \zeta$$

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Example: Rotation

- Rotation by angle ϕ around axis $\vec{n}$: $U = \exp(i\phi\vec{n}\cdot\vec{S})$
- Example: 90° around x-axis ("π/2-pulse"):



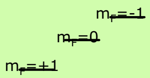
$$U_x\left(\tfrac{\pi}{2}\right) = \exp\left(i\frac{\pi}{2}S_x\right) = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2}i & -1 \\ \sqrt{2}i & 0 & \sqrt{2}i \\ -1 & \sqrt{2}i & 1 \end{pmatrix}$$

$$U_x\left(\tfrac{\pi}{2}\right)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

Linear and Quadratic Zeeman Influence

linear Zeeman effect:

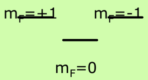
- relative phase between +1 and -1 components
- no change in classical spin length



$$\vec{\zeta}_{\frac{\pi}{2}} = \begin{pmatrix} -\frac{1}{2} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \rightarrow \vec{\zeta}_{\frac{\pi}{2}}(t) = \begin{pmatrix} -\frac{1}{2}e^{i\phi t} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2}e^{-i\phi t} \end{pmatrix} \longrightarrow \vec{S}_{cl}\left(\vec{\zeta}_{\frac{\pi}{2}}(t)\right) = \begin{pmatrix} -\sin(\varphi t) \\ -\cos(\varphi t) \\ 0 \end{pmatrix}$$

quadratic Zeeman effect:

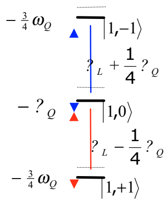
- relative phase between 0 and ±1 components
- periodic de- and rephasing



$$\vec{\zeta}_{\frac{\pi}{2}} = \begin{pmatrix} -\frac{1}{2} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \rightarrow \vec{\zeta}_{\frac{\pi}{2}}(t) = \begin{pmatrix} -\frac{1}{2} \\ \frac{e^{i\varphi t}}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \longrightarrow \vec{S}_{cl}\left(\vec{\zeta}_{\frac{\pi}{2}}(t)\right) = \begin{pmatrix} 0 \\ -\sin(\varphi t) \\ 0 \end{pmatrix}$$

Model System

⁸⁷Rb, F=1



Hamiltonian

$$H = -\omega_L\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} - \omega_Q\begin{pmatrix} 3/4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3/4 \end{pmatrix} - \frac{\Omega}{\sqrt{2}}\cos(\omega_{rf}t)\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

linear Zeeman (~B) | quadratic Zeeman (~B²) | rf-coupling

density matrix time evolution

$$\dot{\rho} = i[H,\rho] - \tfrac{1}{2}\left(L^+L\rho + \rho L^+L - 2L^+\rho L\right)$$

Lindblad operator

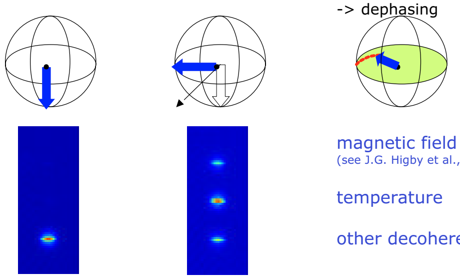
$$L = \frac{1}{\sqrt{t}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

"Ramsey-Experiment" for Spin 1 Systems

initial prep. "π/2"-pulse free evolution "π/2"-pulse

$$\vec{\zeta} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{\zeta} = \begin{pmatrix} -\frac{1}{2} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

detuning
-> rotation in xy-plane
noisy linear Zeeman effect
-> phase randomization
quadratic Zeeman effect
-> dephasing



?

magnetic field gradient influence
(see J.G. Higby et al., cond-mat/0502517)

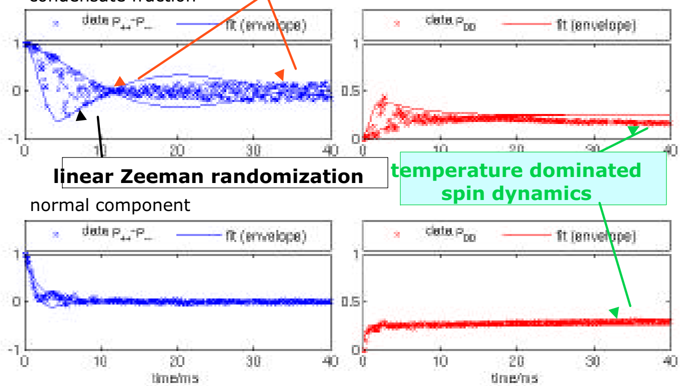
temperature

other decoherence mechanisms

Attention: This is not a Bloch-sphere description !!!

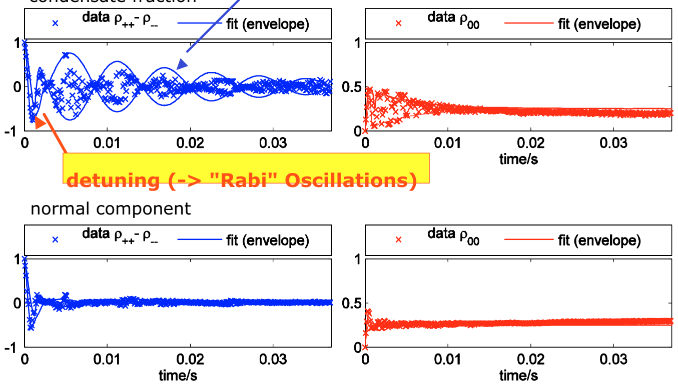
"Ramsey-Experiment" for Spin 1 Systems

magnetic offset field ~ 0.55 G (ν_L~384kHz), ν_R~5kHz, ν_{QZ}~87Hz



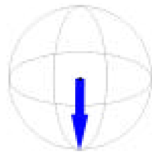
"Ramsey-Experiment" for Spin 1 Systems

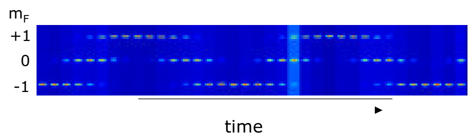
magnetic offset field ~ 1.1 G (ν_L~770kHz), ν_R~5kHz, ν_{QZ}~350Hz



"Rabi"-Oscillation

continosly driven spin





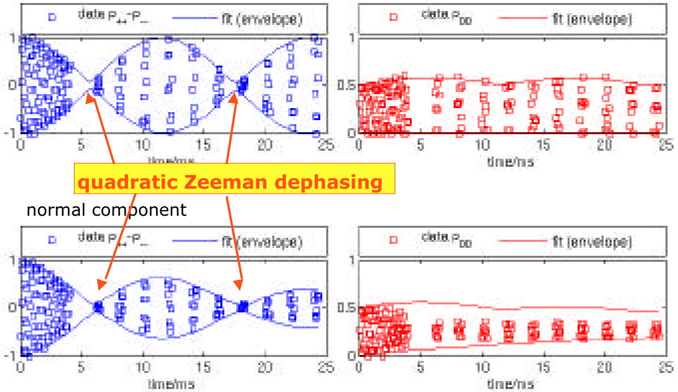
important:
depends on magnetic field gradients only via the "Rabi"-frequency

$$\Omega(\vec{r}) = \sqrt{(\Omega_0(\vec{r}))^2 + (\Delta(B(\vec{r})))^2} \overset{\Omega_0^2 >> \Delta^2}{\approx} \Omega_0(\vec{r})\left[1 + \frac{1}{2}\left(\frac{\Delta(B(\vec{r}))}{\Omega_0(\vec{r})}\right)^2\right]$$

"Rabi"-Oscillation

magnetic offset field ~ 1.1 G (ν_L~770kHz), ν_R~5kHz, ν_{QZ}~350Hz

condensate fraction

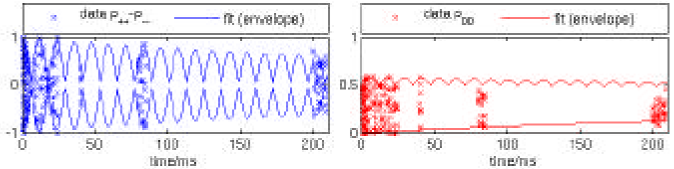


normal component

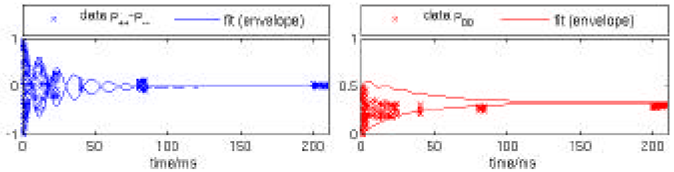
"Rabi"-Oscillation

magnetic offset field ~ 1.1 G (ν_L~770kHz), ν_R~5kHz, ν_{QZ}~350Hz

condensate fraction



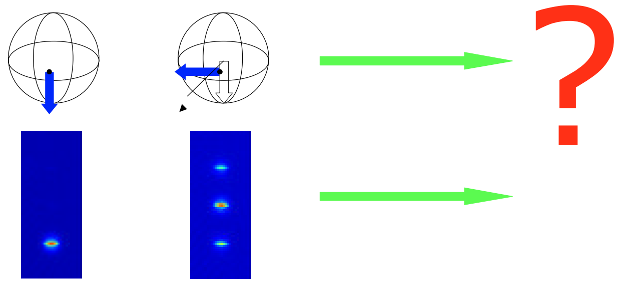
normal component



Incomplete "Ramsey-Experiment"

initial prep. "π/2"-pulse free evolution

$$\vec{\zeta} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{\zeta} = \begin{pmatrix} -\frac{1}{2} \\ \frac{i}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

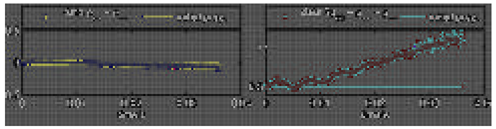


Recent Results from Hamburg

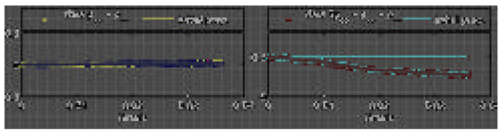
Incomplete Ramsey:

- Rotated streched state "broken" by quadratic Zeeman effect
- Quadratic Zeeman phase dictates the direction of spin dynamics
- ▸ Clear evidence of coherent evolution!
- Thermodynamics dominates for long time scales

Evolution of the condensed fraction

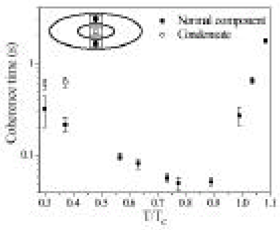


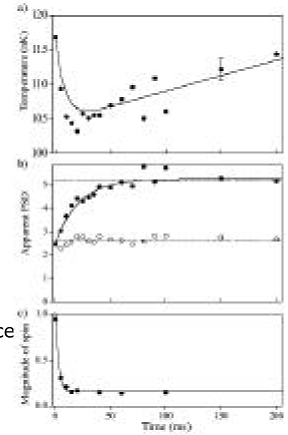
Evolution of the normal component



Coherence in a Quasi-Spin½-System

Now:
condensate+thermal cloud



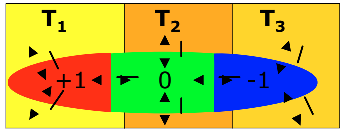


- temperature dependent decoherence
- **decoherence-driven cooling!**

H. J. Lewandowski et al., PRL **91**, 240404 (2003)

Thermodynamics

⇒ How do different quantum gas components at different T do interact with each other and how do they exchange population?



temperature reservoir and particle reservoir

special here, combination of:

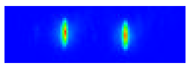
- different time scales for spin dynamics within condensate fraction and thermalization

- total spin conservation allows, e.g.:
  - ⇒ new path to BEC
  - ⇒ condensate melting
  - ⇒ temperature driven magnetization !

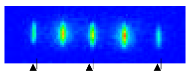
Separation: Condensate - Normal Component

^{87}Rb in F=2:
Fast (coherent) spin dynamics results in a spinor-BEC with a spin projection different from the normal component (thermal atoms).
-> **"Statistically different ensembles"**

0 ms

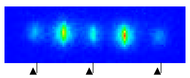


40 ms

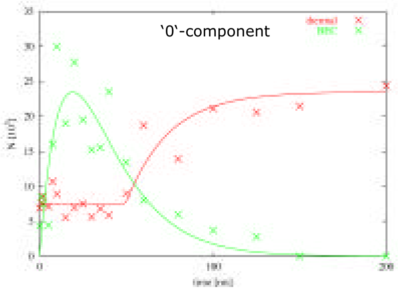


"pure" condensates

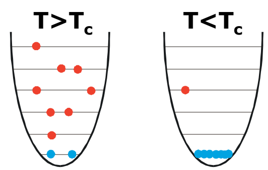
125 ms

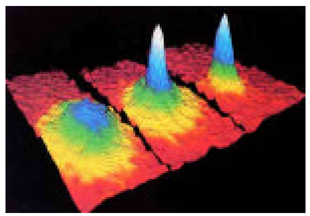


separate ensembles



BEC Phase Transition

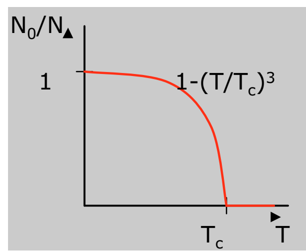




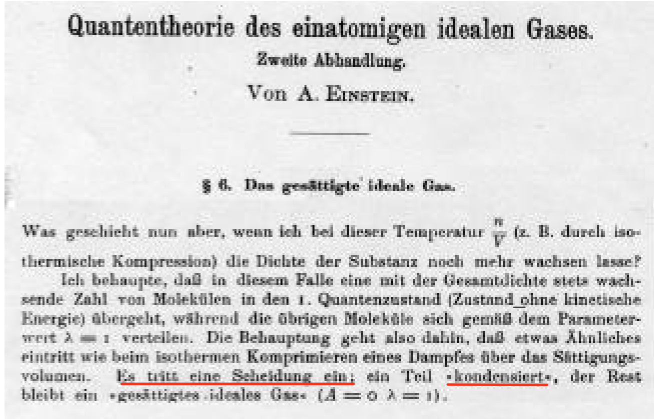
JILA 1995

Condition for BEC:

$$n_0 \Lambda^3_{dB}(T_c) = 2.612...$$



History



Quantentheorie des einatomigen idealen Gases.

Zweite Abhandlung.

Von A. Einstein.

§ 6. Das gesättigte ideale Gas.

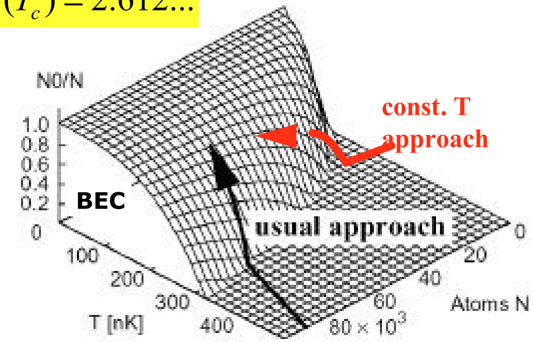
Was geschieht nun aber, wenn ich bei dieser Temperatur $\frac{n}{V}$ (z. B. durch isothermische Kompression) die Dichte der Substanz noch mehr wachsen lasse?

Ich behaupte, daß in diesem Falle eine mit der Gesamtdichte stets wachsende Zahl von Molekülen in den 1. Quantenzustand (Zustand ohne kinetische Energie) übergeht, während die übrigen Moleküle sich gemäß dem Parameterwert $\lambda = 1$ verteilen. Die Behauptung geht also dahin, daß etwas Ähnliches eintritt wie beim isothermen Komprimieren eines Dampfes über das Sättigungsvolumen. Es tritt eine Scheidung ein; ein Teil »kondensiert«, der Rest bleibt ein »gesättigtes ideales Gas« ($A = 0$ $\lambda = 1$).

A. Einstein, Sitzungsber. Preuss. Akad. Wiss., 3, **1925**

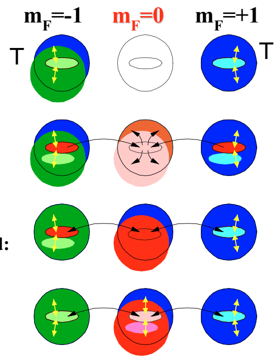
BEC – "new" aspects

$$n_0 \Lambda_{dB}^3 (T_c) = 2.612...$$



Realization in F=1 Spinor BEC

normal components +/-1: temperature reservoir
condensate fractions +/-1:particle reservoir



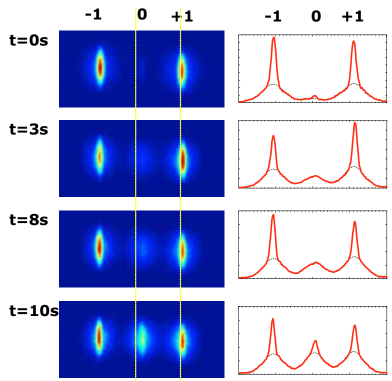
start:
m_F = 0 empty

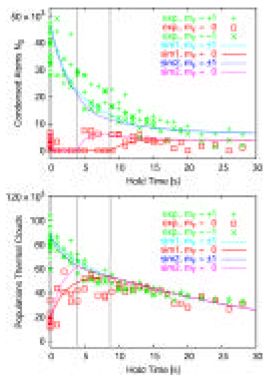
slow spin dynamics
populates mF=0
-> fast thermalization

if critical density
for each component reached:
BEC transition

only population of m_F=0
condensate component

BEC at Constant Temperature





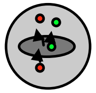
M. Erhard et al., PRA **70**, 031602 (2004)

Thermodynamic Rate Model

description by a rate equation model

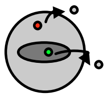
based on 7 variables $\{N_0^-, N_0^0, N_0^+, N_t^-, N_t^0, N_t^+, T\}$

thermalization



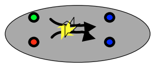
$$\dot N_{0,th}^X = -\tilde\gamma_{th} N_0^X N_t$$
$$\dot N_{t,th}^X = +\tilde\gamma_{th} N_0^X N_t$$
$$\dot T_{th} = -\tilde\gamma_{th} T N_0$$

1-body losses



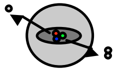
$$\dot N_{0,1b}^X = -\gamma_1 N_0^X$$
$$\dot N_{t,1b}^X = -\gamma_1 N_t^X$$

spin dynamics



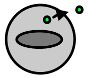
$$\dot N_{0,sp}^{\pm} = \tilde\gamma_{sp1} N_0^0 N_0^0 - \tilde\gamma_{sp2} N_0^+ N_0^-$$
$$\dot N_{0,sp}^0 = -2\tilde\gamma_{sp1} N_0^0 N_0^0 + 2\tilde\gamma_{sp2} N_0^- N_0^+$$

3-body losses



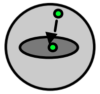
$$\frac{\dot N_{0,3b}^X}{N_0^X} = -L c_3 (N_0)^{4/5}$$

evaporation



$$\dot N_{t,ev}^X = -\gamma_e N_t^X$$
$$\dot T_{ev} = \gamma_e (T - T_e)$$

phase space redistribution

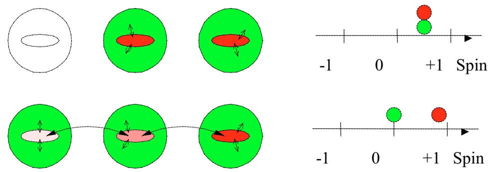


If $(N_t^X > N_c(T))$:

$$N_0^X(t+\Delta t) = N_0^X(t) + N_t^X(t) - N_c$$
$$N_t^X(t+\Delta t) = N_c$$
$$T(t+\Delta t) = T(t)\left(1 + \frac{N_0(t+\Delta) - N_0(t)}{N_t(t+\Delta t)}\right)$$

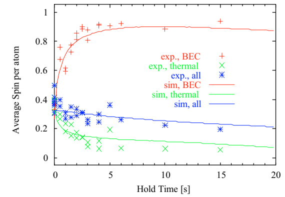
Temperature Dominated Thermodynamics: Magnetization of a BEC

preparation of mixture 0,+1:



- normal components equalize, (via spin dynamics) -> total spin = 0
- condensate spin increases!

temperature driven magnetization of BEC!



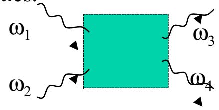
Spinor-BEC: Open Questions

- Interplay of spatial and spinor dynamics?
- How does the transition from a pure state to a mixed state occur?
- Seed effects ?
- Spin waves ?
- Entanglement ?
- ...

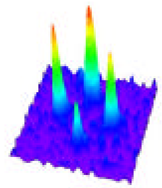
Outlook: Spinor Four Wave Mixing

quantum optics viewpoint ⇒ four-wave-mixing

optics:

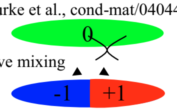


for BEC (Phillips et al.):



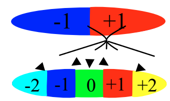
spinor condensates (J. P. Burke et al., cond-mat/0404499):

four wave mixing



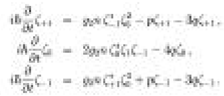
⇒ fully equivalent description
⇒ to populate empty modes:
- seed
- quantum fluctuations

F=2: even more complex



multi mode coupling competing four wave mixing channels

DGLs:

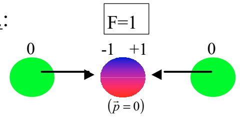


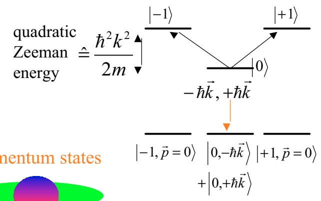
Outlook: Spinor Four Wave Mixing

- **adding kinetic energy plus magnetic fields:**

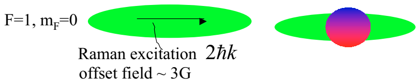
➔ additional processes possible

e.g.:

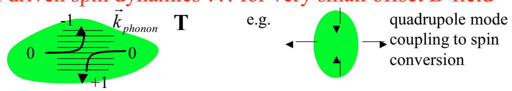




proposed experiment: FWM into zero momentum states

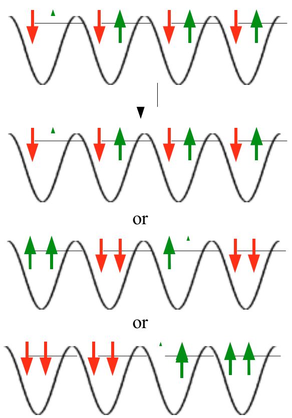


➔ phonon driven spin dynamics ??! for very small offset B-field



➔ coupling of spinor components and finite T excitations ?

Outlook: Spinor-BEC in Optical Lattices



- ^{87}Rb offers:
- ferromagnetic system in F=1
- anti-ferromagnetic system in F=2
- MI-SF phase transition is modified compared to single component BEC

➔ complex new phase diagramms

- Various possibilities for the creation of spin domains
- Dipole-dipole interactions become important

➔ quantum-phase transition to a ferromagnetic state in F=1 theoretically predicted

- Entanglement in OL in various scenarios