The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
Energy Agency

SMR 1666 - 25

SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

11 - 22 July 2005

Dipolar gases: Theory

Presented by:

Luis Santos

University of Stuttgart, Germany

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Trieste, 21th July 2005

Dipolar gases: theory

Luis Santos

Universität Stuttgart

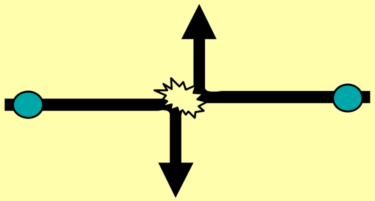
CO.CO.MAT

CONTROL OF QUANTUM CORRELATIONS IN TAILORED MATTER

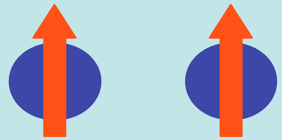
SFB/TR 21 – STUTTGART, ULM, TÜBINGEN

Outline of the talk

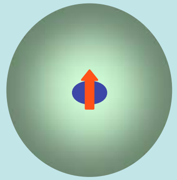
1. The role of the interactions



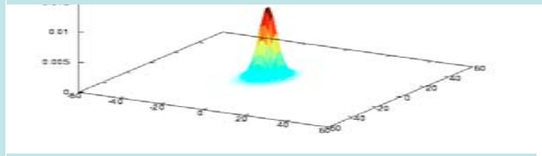
2. Dipolar gases.
The dipole-dipole interaction



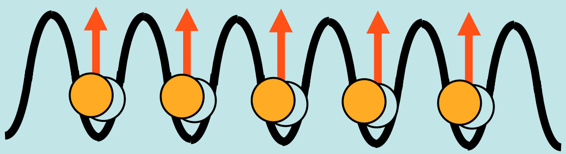
3. Ground-state and stability of a dipolar BEC



4. Nonlocal nonlinearity.
Multidimensional solitons



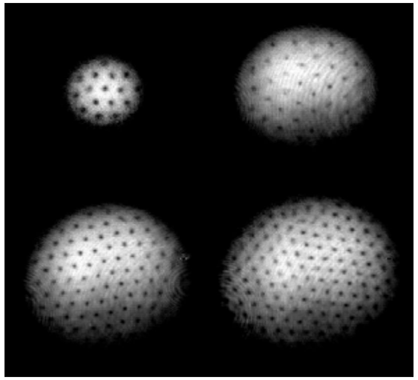
5. Dipolar gases in optical lattices



The role of the interactions

Ultra cold gases are very dilute, with typical densities lower than 10^{14} atoms/cm^3

But the interparticle interactions play a crucial role in the properties of ultra cold gases, as e.g. **superfluidity**

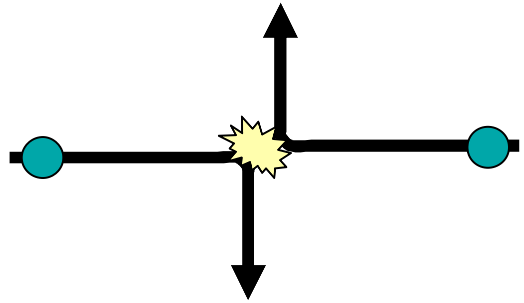


[Raman et al., PRL **87**, 210402 (2001)]

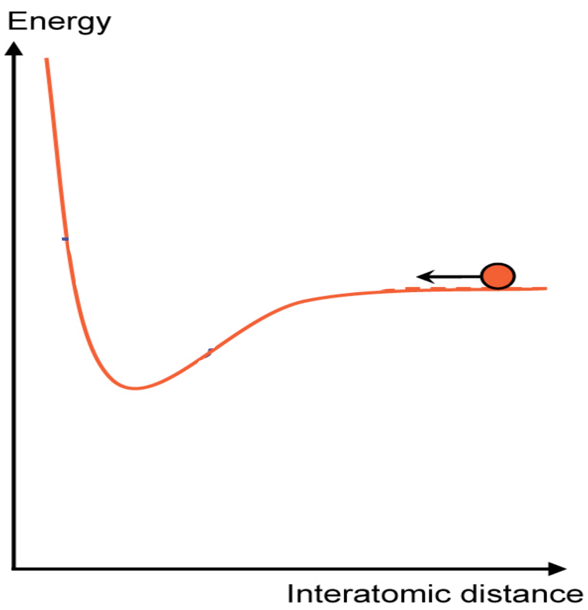
See lectures of S. Stringari and A. Fetter

The role of the interactions

In typical experiments the atoms interact via short-range isotropic interactions



The interaction between ultracold atoms is determined by **the s-wave scattering length “a”**



$$V(\vec{r}-\vec{r}\,') \approx \frac{4\pi\hbar^2 a}{m}\delta(\vec{r}-\vec{r}\,')$$

The role of the interactions

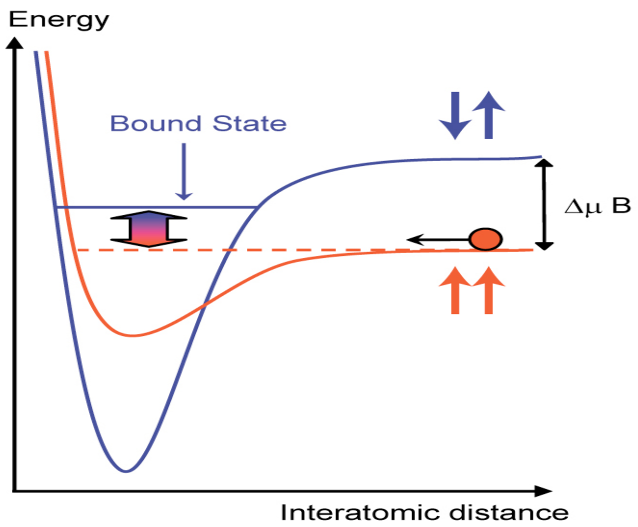
a>0

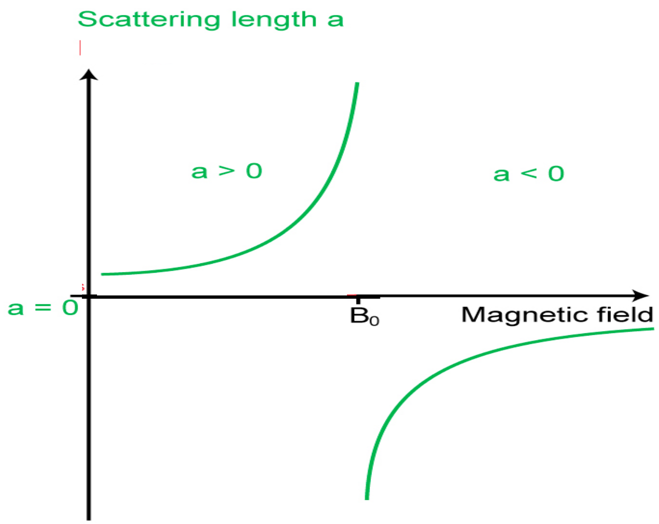
Repulsive interactions

Feshbach Resonances

a<0

Attractive interactions





[Tiesinga et al, PRA **47**, 4114 (1993); Inouye et al., Nature **392**, 151 (1998)]

The role of the interactions

At low Temperatures the BEC physics is given by a
nonlinear Schrödinger equation with local cubic nonlinearity

Gross-Pitaevskii equation

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+U(r,t)+\frac{4\pi\hbar^2 aN}{m}\left|\Psi(r,t)\right|^2\right]\Psi(r,t)$$

External potential (trap) — Atom-Atom interactions

The stability of the BEC depends very much on the sign of a

a >0	a <0
Repulsive interactions	Attractive interactions
STABLE	UNSTABLE IN d>1

The role of the interactions

But...a trapped attractive gas can be stable!

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + U(r,t) + \frac{4\pi\hbar^2 aN}{m} \left| \Psi(r,t) \right|^2 \right] \Psi(r,t)$$

>

The zero-point oscillation compensates the attraction

$$\frac{4\pi\hbar^2 a}{m} n < \hbar\omega$$

Critical number of particles

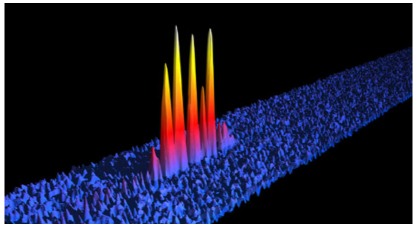
[Bradley et al., PRL **78**, 985 (1997)]

The role of the interactions

Due to the interactions, **<u>BEC physics is intrinsically nonlinear</u>**

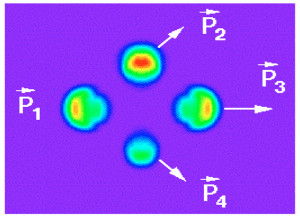
Bright solitons

[Strecker et al., Nature **417**, 150 (2002)]
[Khaykovich et al., Science **296**, 1290 (2002)]



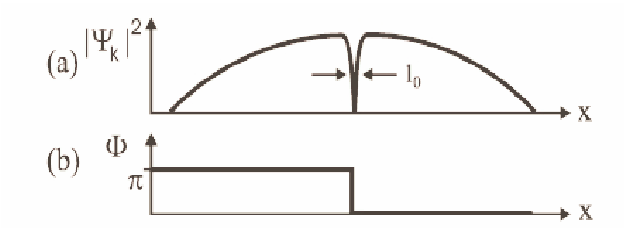
Four-wave mixing

[Deng et al., Nature **398**, 218 (1999)]



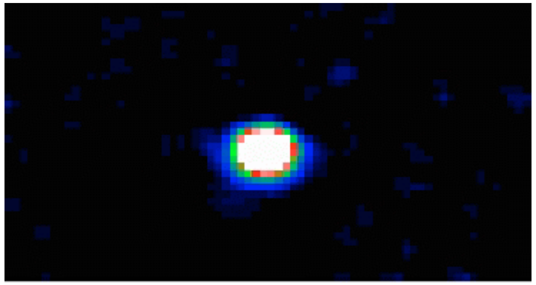
Dark Solitons

[Burger et al, PRL **83**, 5198 (1999)]
[Denschlag et al., Science **287**, 97 (2000)]



Condensate collapse

[E. A. Donley et al. Nature **412**, 295 (2001)]



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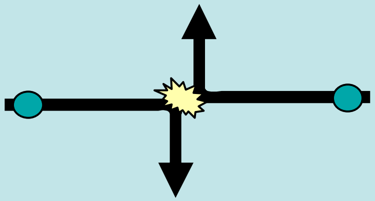

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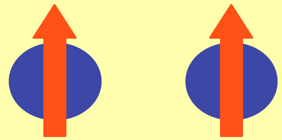

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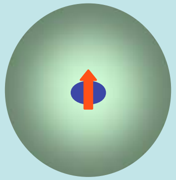

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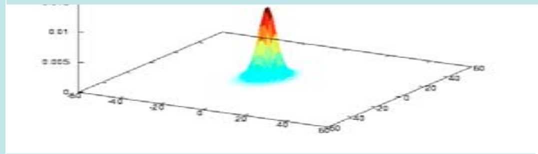

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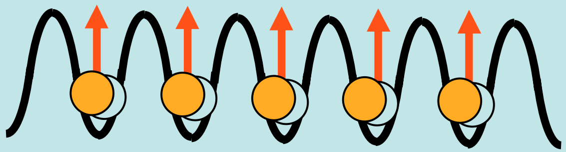
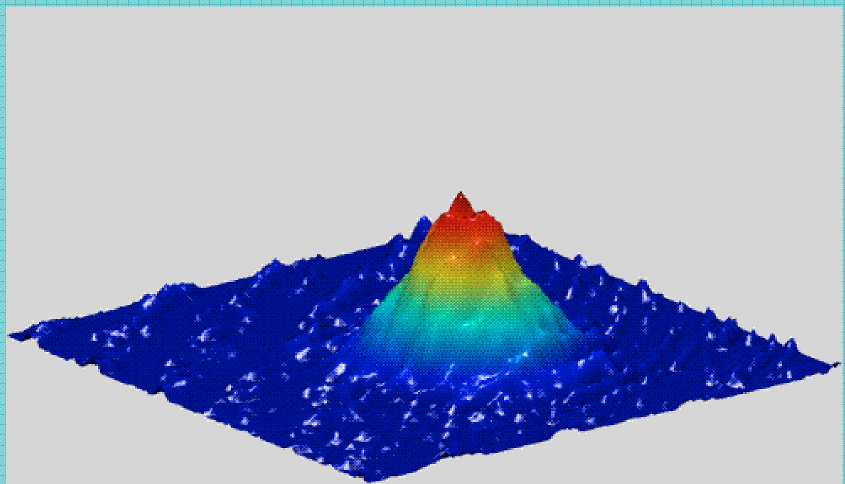


Dipolar gases. The dipole-dipole interaction

Recent experimental developments have opened a novel research area in cold gases: **the analysis of dipolar gases**

BEC in Chromium atoms

[Griesmaier et al., PRL **94**, 160401 (2005)]



Chromium has a large magnetic moment, $\mu=6\mu_B$

See lecture of Jürgen Stuhler

Dipolar gases. The dipole-dipole interaction

Atoms with large magnetic moment

Cooling and trapping of polar molecules
[Bethlem and Meijer, Int. Rev. Phys. Chem. **22**, 73 (2003)]

Feshbach resonances in binary mixtures
[Stan et al. PRL **93**, 143001 (2004); Inouye et al., PRL **93**, 183201 (2004); Petrov et al., cond-mat/0502010]

Photoassociation of polar molecules in optical lattices
[Jaksch et al., PRL **89**, 040402 (2002); Damski et al., PRL **90**, 110401 (2003); Rom et al., PRL **93**, 073200 (2004)]

Laser induced dipole-dipole interaction
[O'Dell et al., PRL **84**, 5687 (2000)]

Rydberg atoms
[Jaksch et al., PRL **85**, 2208 (2000)]

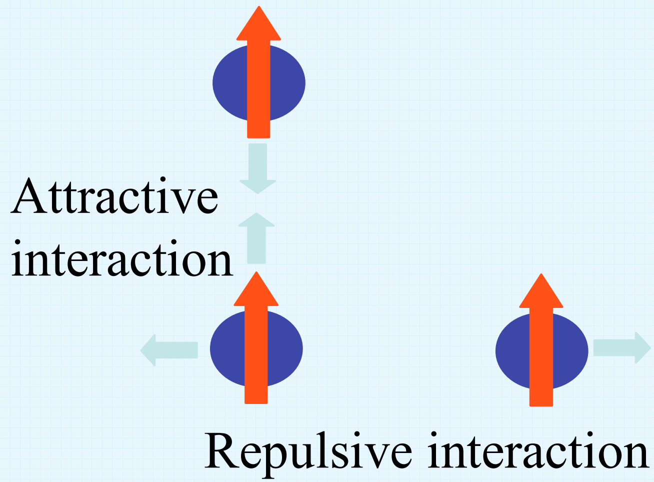
Dipolar gases. The dipole-dipole interaction

Dipole-dipole interaction

$$V(\vec{r}) = \frac{d^2}{r^3}\left(1 - 3\cos^2\theta\right)$$

The interaction is anisotropic
(partially attractive and partially
repulsive!)

Long-range interaction



The condensate properties are critically determined
by the trap geometry

Dipolar gases. The dipole-dipole interaction

At low temperatures the physics of a dipolar BEC is given by a nonlocal nonlinear Schrödinger equation

Nonlocal NLSE

$$i\hbar\frac{\partial}{\partial t}\psi(\vec r,t)=\left\{\begin{array}{l}\dfrac{-\hbar^2}{2m}\nabla^2+V(\vec r,t)+g\left|\psi(\vec r,t)\right|^2+\\ +g_d\displaystyle\int d\vec r\,'\frac{(1-3\cos^2\theta)}{\left|\vec r-\vec r\,'\right|^3}\left|\psi(\vec r\,',t)\right|^2\end{array}\right\}\psi(\vec r,t)$$

$g \propto a$ ⟷ $g_d \propto d^2$

Resemblances to

plasma physics [Litvak et al., Sov. J. Plasma Phys. **1**, 60 (1975)]

physics of nematic liquid crystals [Conti et al., PRL **91**, 073901 (2003)]

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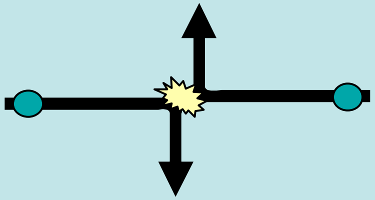

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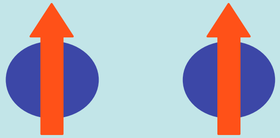

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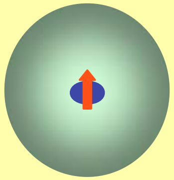

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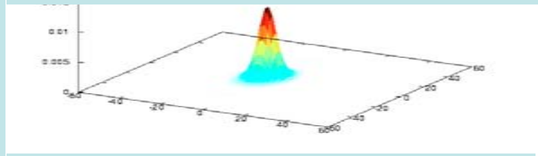
Multidimensional solitons



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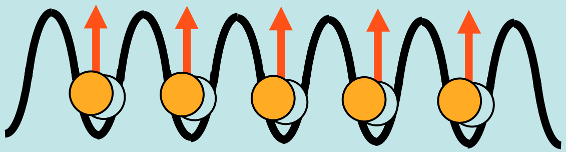


Ground-state and stability of a dipolar BEC

Let's assume that there is no local interaction: g=0

Homogeneous space (no trap)

Like for a<0 the system is unstable

Dispersion law for elementary excitations

$$E(k)=\sqrt{E_k^2+\frac{8\pi}{3}g_d n_0(3\cos\theta_k-1)E_k}$$

Trapped case

Like for a<0 a stable condensate can be found under certain conditions

Contrary to a<0 the sign and value of the mean dipole-dipole interaction is strongly modified by the trapping potential

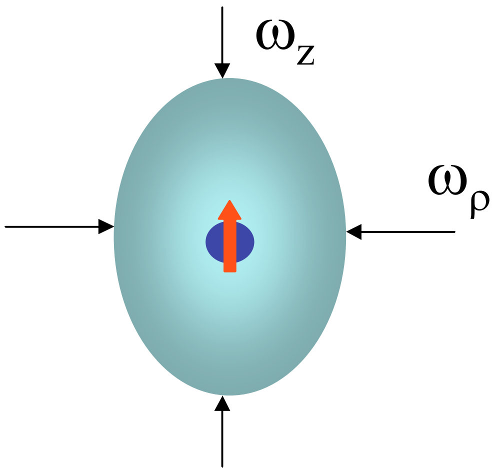
Ground-state and stability of a dipolar BEC

[Yi and You, PRA **61**, 041604 (2000); Góral et al., PRA **61**, 051601 (2000); Santos et al., PRL **85**, 1791 (2000)]

Cylindrical trap: axis in the dipole direction

Trap frequencies ω_ρ and ω_z

Trap aspect ratio $\sqrt{\omega_\rho / \omega_z}$

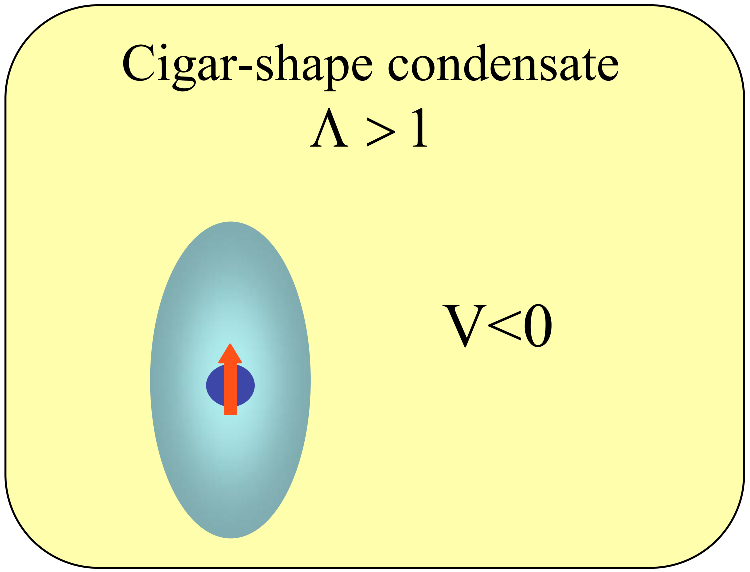


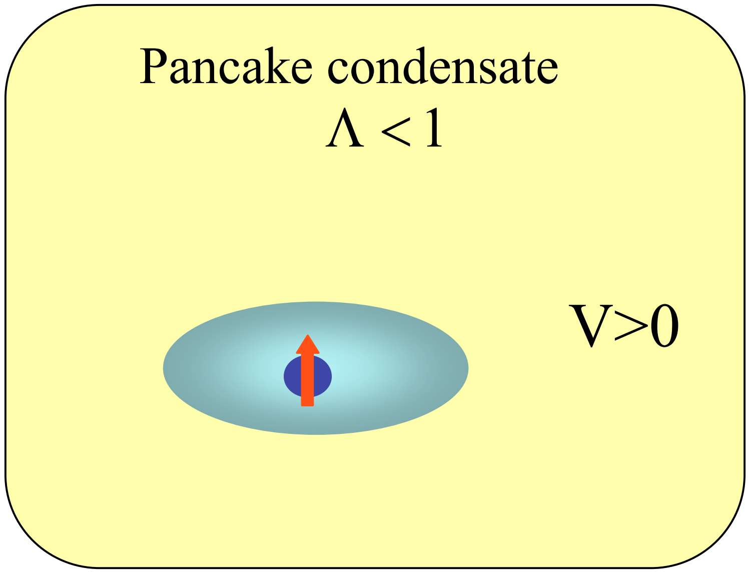
Ground-state and stability of a dipolar BEC

Mean dipole-dipole interaction

$$V = g_d \int d\vec{r}' \frac{1-3\cos^2\theta}{\left|\vec{r}-\vec{r}'\right|^3} \left|\psi\left(\vec{r}'\right)\right|^2 \left|\psi\left(\vec{r}\right)\right|^2$$

$$\psi(\rho, z) \propto \exp\left[-\frac{1}{2L_z^2}\left(\Lambda^2\rho^2 + z^2\right)\right] \qquad V = V_0 \frac{1}{1-\Lambda^2}\left[2-\Lambda^2 - \frac{3\Lambda \tanh^{-1}\left(\sqrt{\Lambda^2-1}/\Lambda\right)}{\sqrt{\Lambda^2-1}}\right]$$





$$\Lambda = L_z / L_\rho$$

Ground-state and stability of a dipolar BEC

Cigar-shape traps ($\omega_z < \omega_\rho$)

V<0 ⇨ Attractive gas

$\hbar\omega_z << |V| << \hbar\omega_\rho$ ⇨ $|V| < \hbar\omega_\rho$ ⇨ $|V| > \hbar\omega_\rho$

Bright-soliton-like BEC — 3D Attractive gas — **Collapse**

Eventually collapse occurs for $\sqrt{\omega_\rho / \omega_z} > 0.41$

Pancake-shape traps with $\sqrt{\omega_\rho / \omega_z} < 0.41$

V>0 and grows with g_d ⇨ Repulsive gas

$\hbar\omega_\rho << V << \hbar\omega_z$ ⇨ $V > \hbar\omega_z$

Quasi2D : Bose Gas with repulsive interaction. Radially Thomas-Fermi

3D Thomas-Fermi

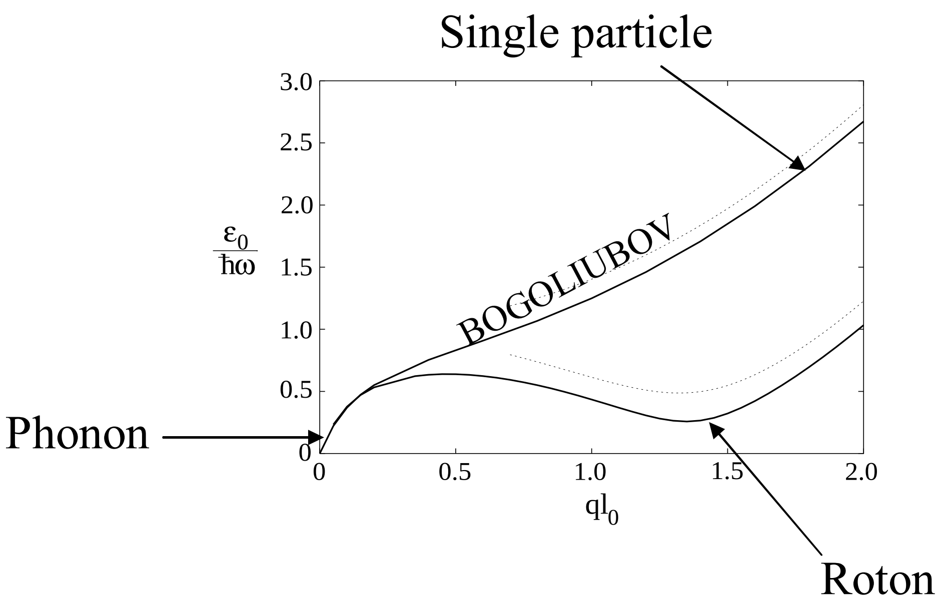
[O'Dell et al., PRL **92**, 250401 (2004)]

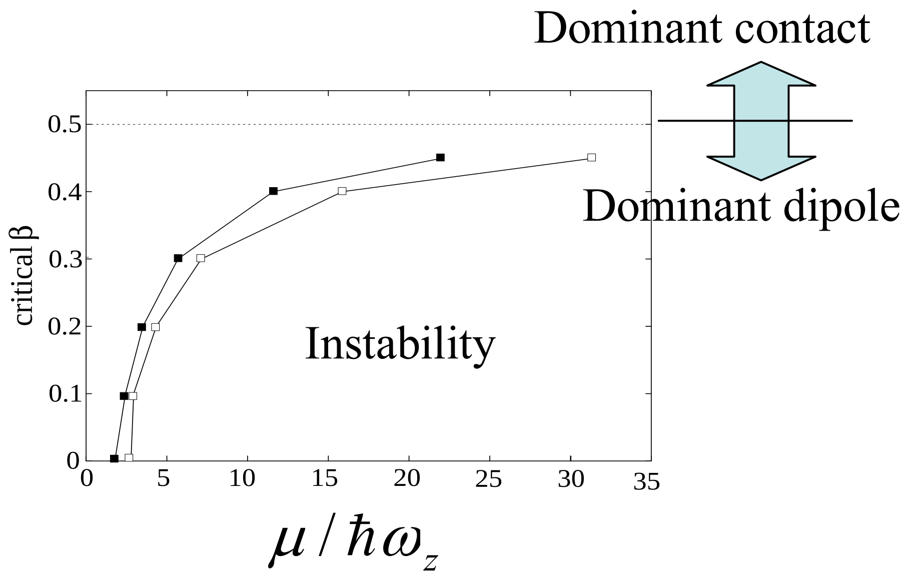
Ground-state and stability of a dipolar BEC

[O'Dell et al., PRL **90**, 110402 (2003); Santos et al., PRL **90**, 250403 (2003)]

What happens if $\sqrt{\omega_\rho / \omega_z} < 0.41$?

The dispersion law acquires a roton-maxon character for sufficiently large dipole-dipole interactions

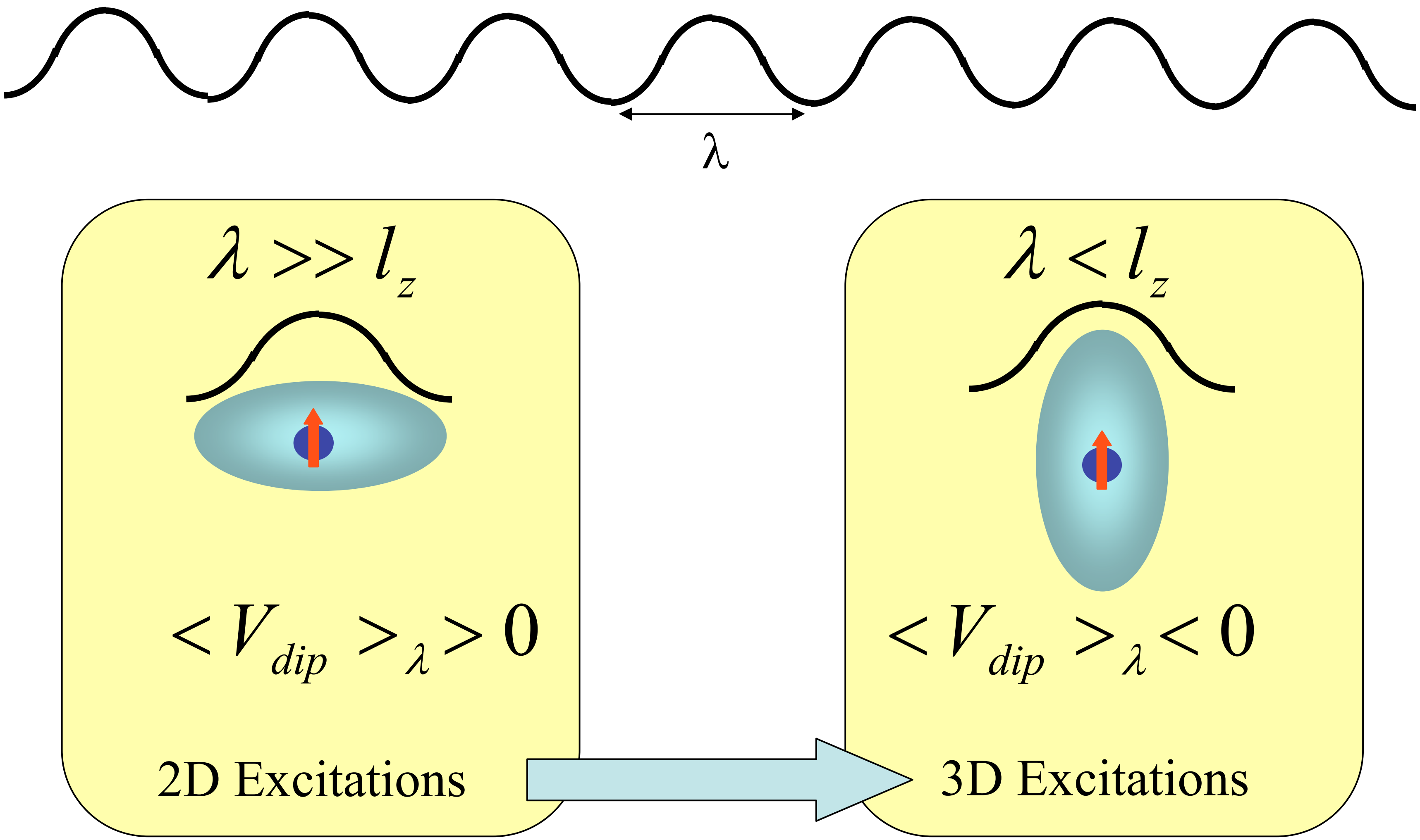




The gas becomes eventually unstable unstable when the roton touches zero

Ground-state and stability of a dipolar BEC

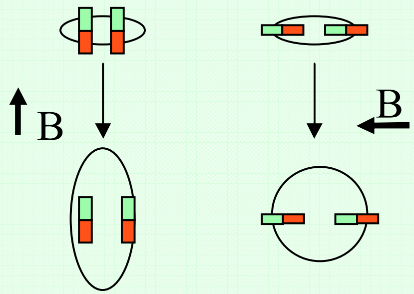
Why do we see a roton in a dipolar gas?



Ground-state and stability of a dipolar BEC

Expansion

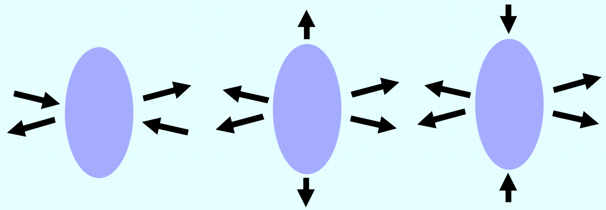
[Góral and Santos, PRA **66**, 023613 (2002);
Yi and You, PRA **67**, 045601 (2003);
Giovanazzi *et al.*, J. Opt. B **5**, 208 (2003)]



See talk of Jürgen Stuhler

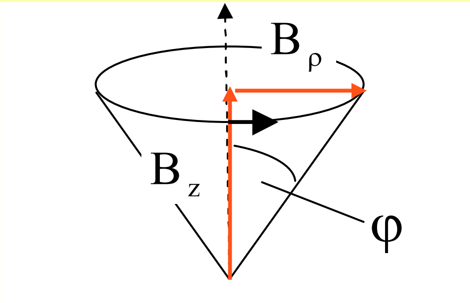
Low-lying excitations

[Yi and You, PRA **66**, 013607 (2002);
Góral and Santos, PRA **66**, 023613 (2002)]

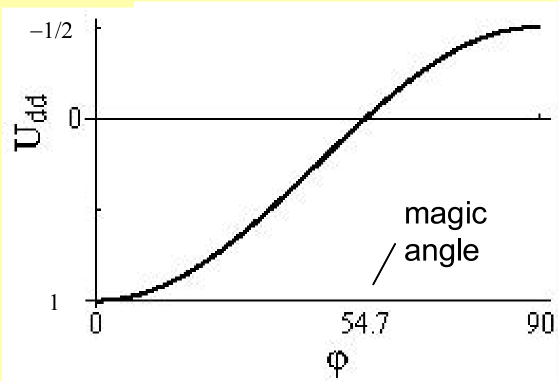


Tunability

[Giovanazzi *et al.*, PRL **89**, 130401 (2002)]



$$V_d(\vec{r}) = \frac{\mu_0 \mu^2}{4\pi}\left(\frac{3\cos^2\varphi - 1}{2}\right)\left(\frac{1-3\cos^2\theta}{r^3}\right)$$



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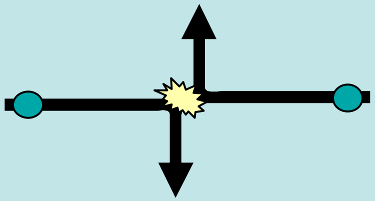

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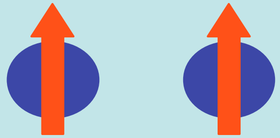

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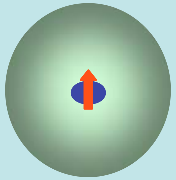

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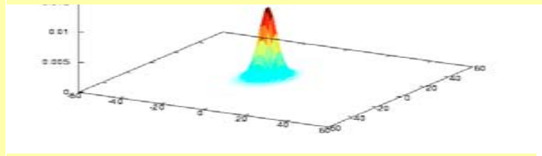

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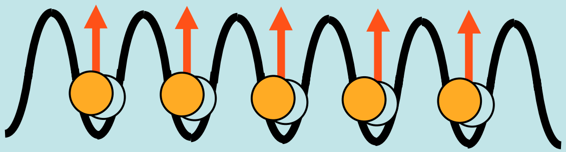


Nonlocal nonlinearity. Multidimensional solitons

„Standard“-BEC physics is described by a NLSE with local cubic nonlinearity

Dark Solitons (a>0) ⟵ 1D NLSE ⟶ **Bright solitons (a<0)**

[Zakharov & Shabat., JETP **34**, 62 (1972)]

Continuous solitons become unstable in 2D and 3D

Periodic potentials: BEC in Optical Lattices

Gap Solitons: Bright solitons with a>0 [Eiermann et al., PRL **92**, 230401 (2004)]

Multidimensional **discrete solitons** in periodic potentials but they **do not move** [Fleischer et al., Nature **422**, 147 (2003)]

2D (3D) mobile solitons are possible combining 1D (2D) lattices with one uniform direction. But **the motion is only in 1D**. [Baizakov et al., PRA **70**, 053613 (2004)]

Nonlocal nonlinearity. Multidimensional solitons

Nonlocal NLSE

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = \left\{ \begin{array}{l} \frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r},t) + g\left|\psi(\vec{r},t)\right|^2 + \\ + g_d \int d\vec{r}\,' \frac{(1-3\cos^2\theta)}{\left|\vec{r}-\vec{r}\,'\right|^3}\left|\psi(\vec{r}\,',t)\right|^2 \end{array} \right\} \psi(\vec{r},t)$$

$g \propto a$ ⟷ $g_d \propto d^2$

Nonlocal nonlinearity is also observed in disparate physical systems

Plasma physics

[Litvak et al., Sov. J. Plasma Phys. **1**, 60 (1975)]

Photorefractive materials

[Shin et al., PRL **78**, 130401 (2002)]

Nematics

[Peccianti et al., Nature **432**, 733 (2004)]

Nonlocal nonlinearity. Multidimensional solitons

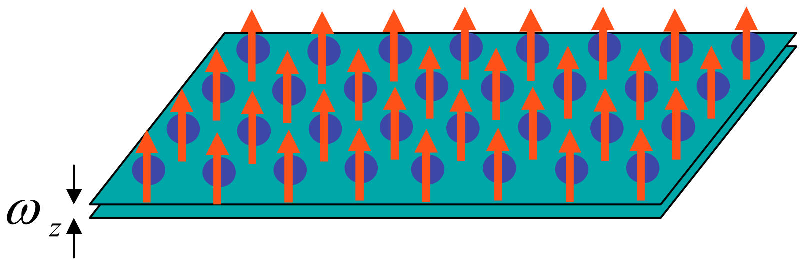
Very active research field in non linear physics

Nonlocal nonlinearity can **stabilize** multidimensional solitons

[Bang et al., PRE **66**, 046619 (2002)]

Multidimensional solitons observed e.g. in Nematics

[Peccianti et al., Nature **432**, 733 (2004)]



Is it possible to observe 2D solitons in dipolar BEC which move truly in 2D?

[P. Pedri and L. Santos, cond-mat/0503019]

Nonlocal nonlinearity. Multidimensional solitons

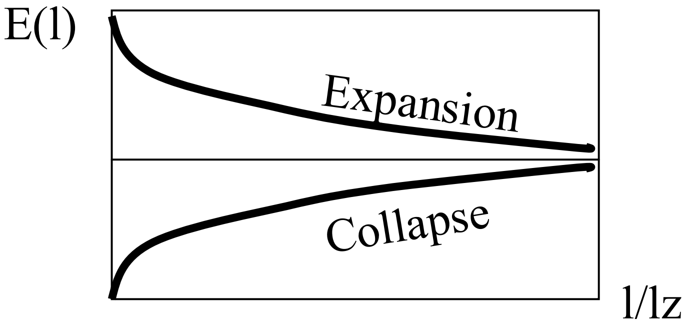
Gaussian Ansatz

$$\psi(\vec{r}) \propto e^{-\rho^2/2l^2} e^{-z^2/2l_z^2}$$

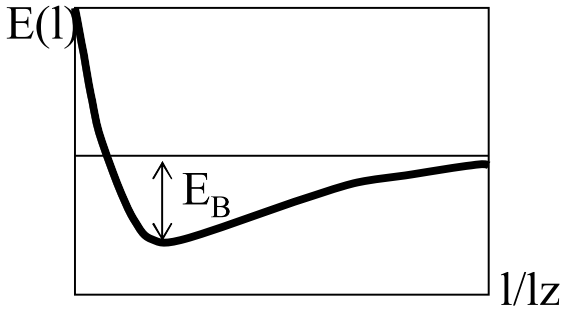
$$E(l) \propto \frac{1}{(l/l_z)^2}\left(1 + \frac{g}{2(2\pi)^{3/2}} + \frac{g_d}{3\sqrt{2\pi}} f\left(\frac{l}{l_z}\right)\right)$$

$$f(x) = \frac{1}{x^2-1}\left[2x^2 + 1 - \frac{3x^2 \arctan\left[\sqrt{x^2-1}\right]}{\sqrt{x^2-1}}\right]$$

No dipole



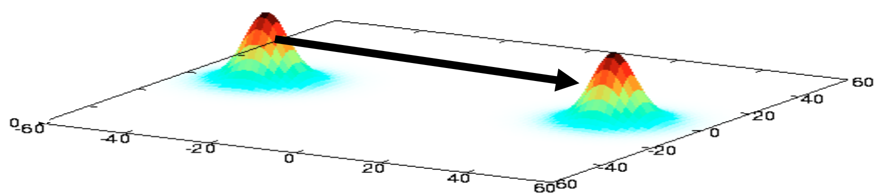
Dipolar gas



Stability condition

$$\frac{g_d}{3\sqrt{2\pi}} < 1 + \frac{g}{2(2\pi)^{3/2}} < \frac{-2g_d}{3\sqrt{2\pi}}$$

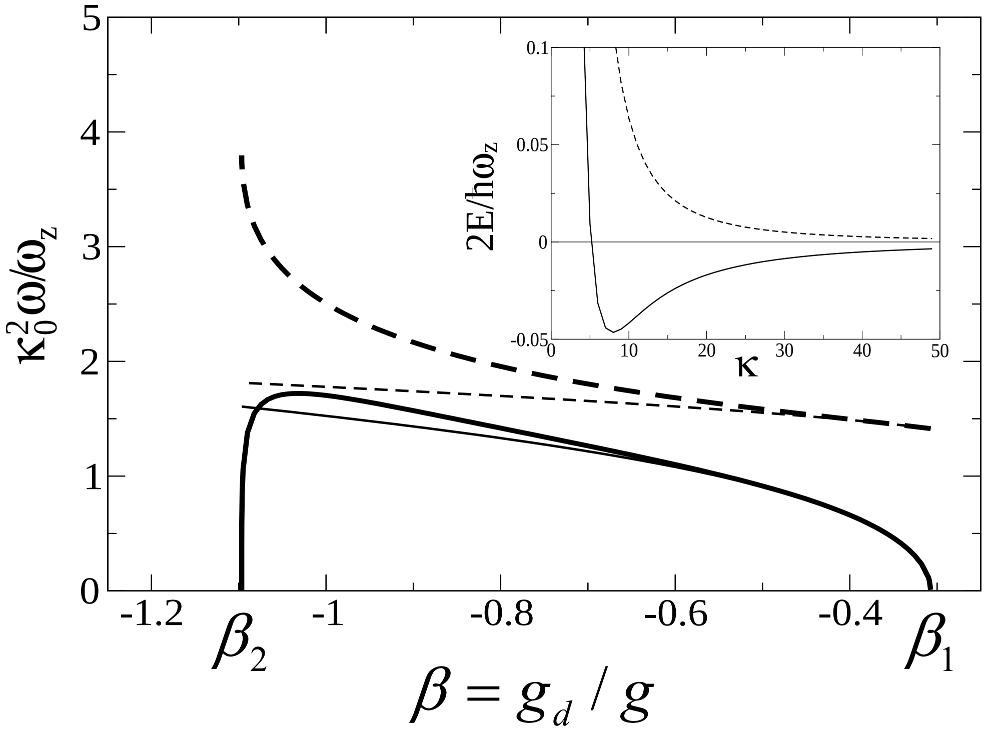
Truly-2D motion without distortion!!



We need $g_d < 0$ ⇨ Tunability

Nonlocal nonlinearity. Multidimensional solitons

3D Analysis of the lowest-lying excitations



Crucial role of the anisotropy

Stability Window

$$|\beta| > |\beta_1|$$

2D instability against expansion for small dipoles

$$|\beta| < |\beta_2|$$

3D instability against collapse for large dipoles

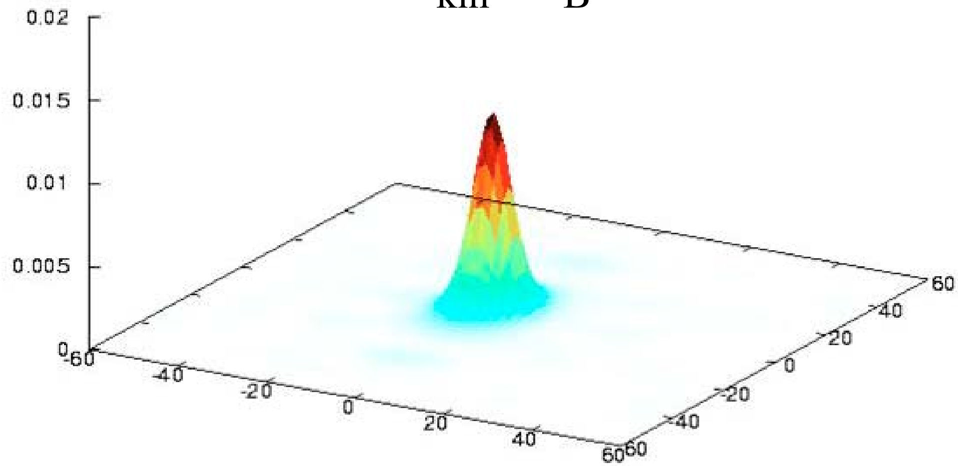
The lowest lying level is always the breathing mode

Nonlocal nonlinearity. Multidimensional solitons

The 2D Solitons scatter inelastically

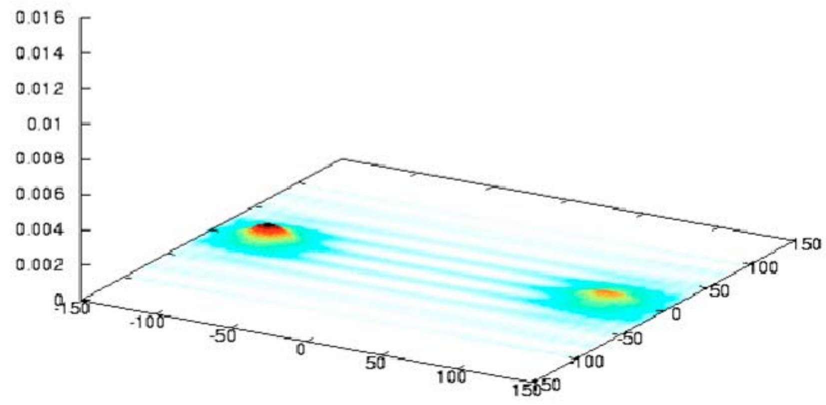
Soliton fusion

$E_{kin} < E_B$



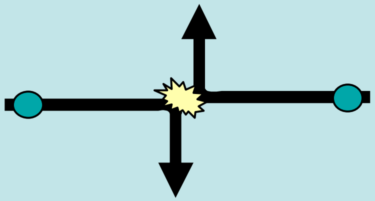
Soliton destruction

$E_{kin} > E_B$

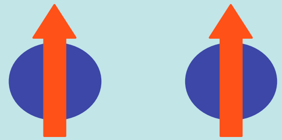


Outline of the talk

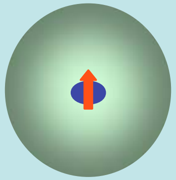
1. The role of the interactions



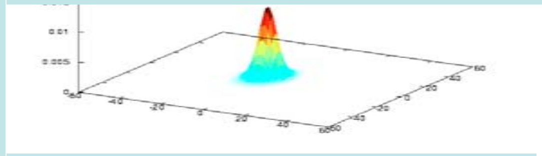
2. Dipolar gases.
The dipole-dipole interaction



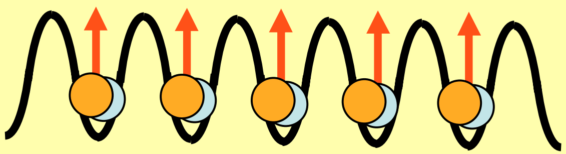
3. Ground-state and stability of a dipolar BEC



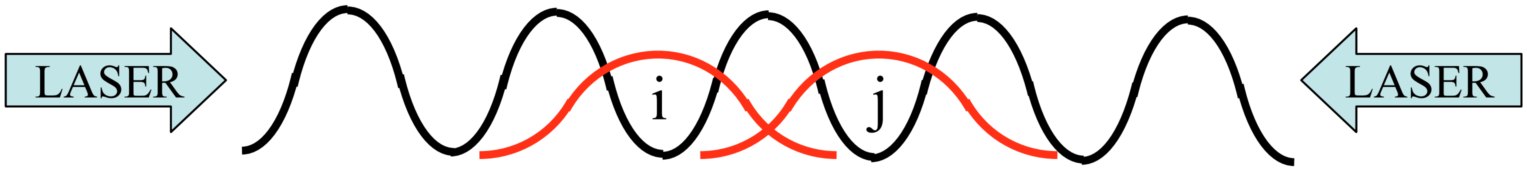
4. Nonlocal nonlinearity.
Multidimensional solitons

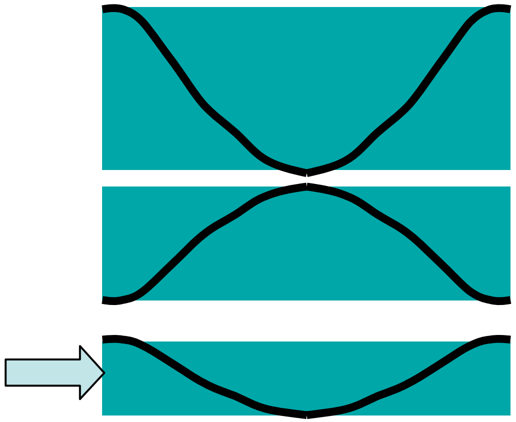


5. Dipolar gases in optical lattices



Dipolar gases in optical lattices





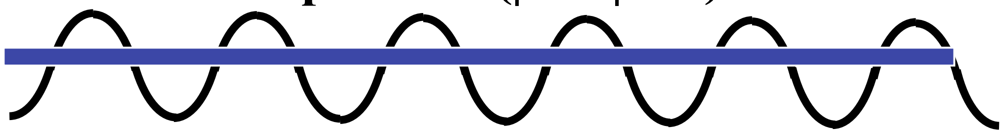
Short-range interacting bosons in lattices

Bose-Hubbard Hamiltonian

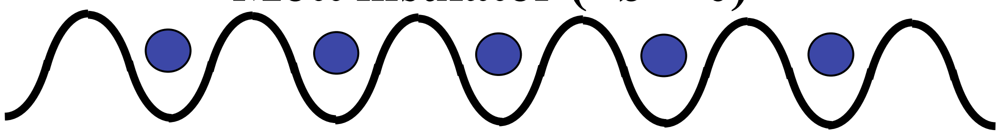
[Fisher *et al.*, PRB **40**, 546 (1989) ; Jaksch *et al.*, PRL **81**, 3108 (1998)]

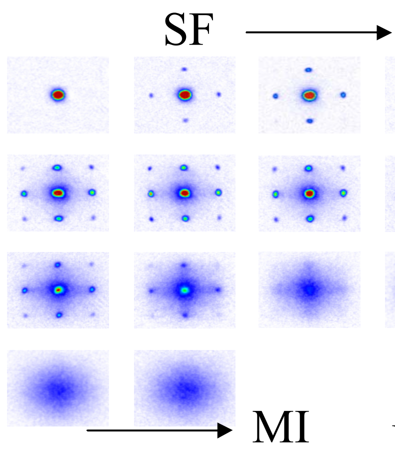
$$H = -J\sum_{<ij>}\left(b^{+}_{i}b_{j} + H.c.\right) + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) - \mu\sum_i n_i$$

Superfluid (|<b>| > 0)



Mott insulator (<b>=0)





[M. Greiner et al., Nature **415**, 39 (2002)]

Dipolar gases in optical lattices

Bose-Hubbard with finite-range interactions

$$H = -J\sum_{<ij>}\left(b^{+}{}_{i} b_j + H.c.\right) - \mu\sum_i n_i + \frac{U_0}{2}\sum_i n_i\left(n_i - 1\right) + \frac{U_1}{2}\sum_{<i,j>} n_i n_j + \frac{U_2}{2}\sum_{<<i,j>>} n_i n_j + \ldots$$

Supersolid

[Andreeev and Lishitz, JETP **29**, 1107 (1969); Chester, PRA **2**, 256 (1970); Leggett, PRL **25**, 1543 (1970)]

Superfluidity + periodic modulation of the density

Experiment in Solid Helium [Kim and Chan, Science **305**, 1941 (2004)]

Still a rather controversial issue [Leggett, Science **305**, 1921 (2004); Prokof'ev and Svistunov, PRL **94**, 1555302 (2005)]

In a square lattice, for hard-core bosons ($U_0 >> U_1$), the supersolid phase is unstable against phase separation [Batrouni and Scalettar, PRL **84**, 1599 (2000)]

The supersolid phase can be stabilized for softcore bosons ($U_0 < 4U_1$) [Sengupta et al., cond-mat/0412338]

Dipolar gases in optical lattices

Bose-Hubbard with dipole-dipole interactions

[K. Góral et al., PRL **88**, 170406 (2002)]

$$J = -\int w^*(\vec{r}-\vec{r}_i)\left[\frac{-\hbar^2}{2m}\nabla^2 + V_{latt}(\vec{r})\right] w(\vec{r}-\vec{r}_j)\, d^3\vec{r}$$ ← Tunneling

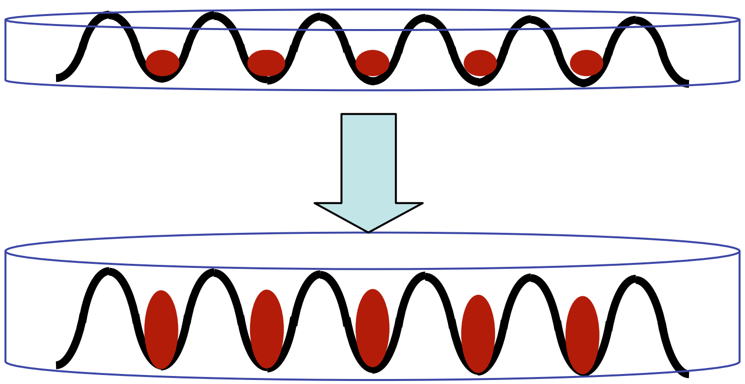
Interaction energy →

$$U_m = \int \left|w^*(\vec{r}-\vec{r}_i)\right|^2 V_{\text{int}}(\vec{r}-\vec{r}\,')\left|w(\vec{r}-\vec{r}_j)\right|^2 d^3\vec{r}\,' d^3\vec{r}$$

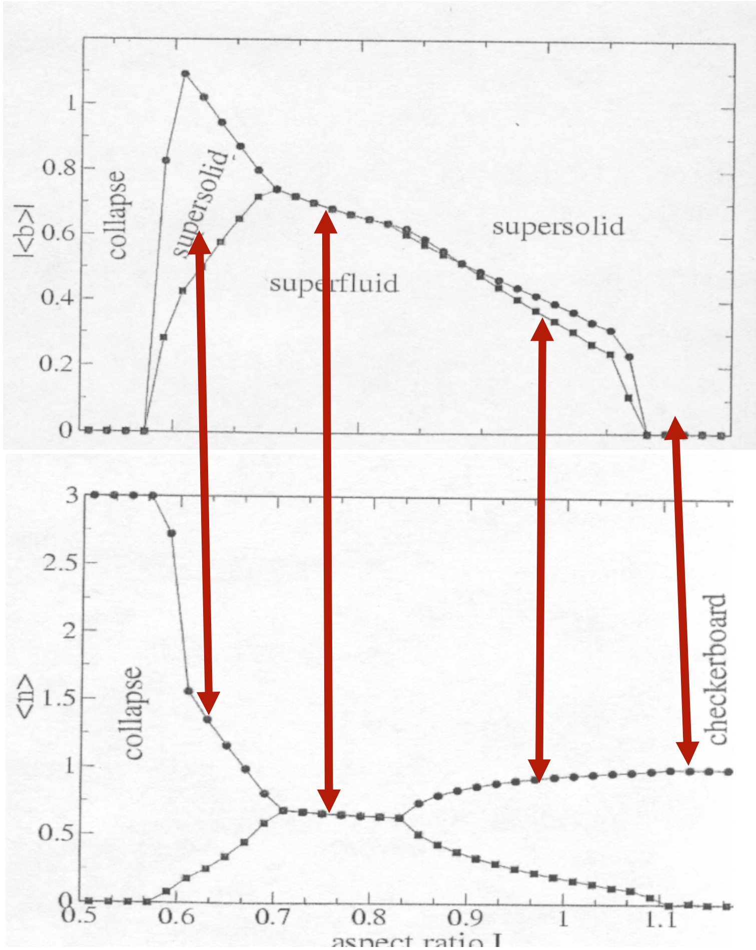
$$V_{\text{int}}(\vec{r}) = \frac{g_d}{r^3}\left(1-3\cos^2\theta\right) + g\delta(\vec{r}) \quad \longrightarrow \quad U_m = U_m^{(d)} + U_m^{(c)}$$

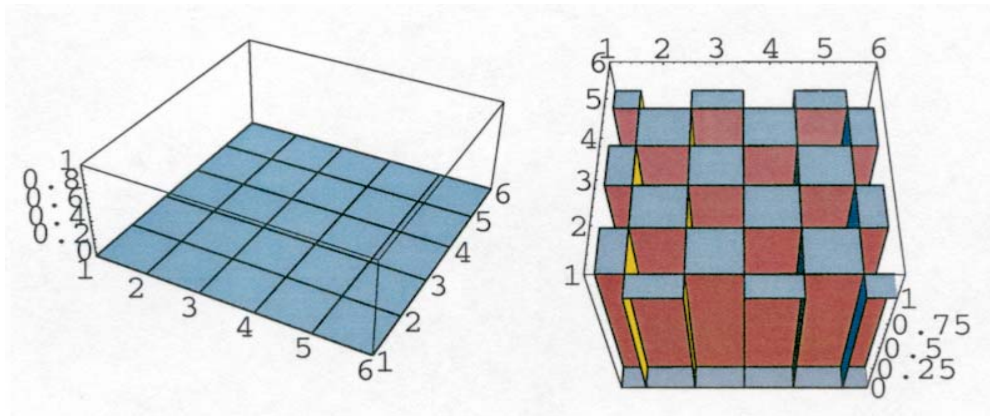
Let a>0 ($U_0^{(c)} > 0$) and a dipole oriented perpendicular to a 2D lattice ($U_{m>0}^{(d)} > 0$)

Changing the tranversal confinement changes the U_m coefficients.
In particular U_1/U_0, this allows an **easy control of the quantum phases**



Dipolar gases in optical lattices





- Similar control by changing the angle between dipole and lattice plane

- Gutzwiller Ansatz, up to 80x80 sites

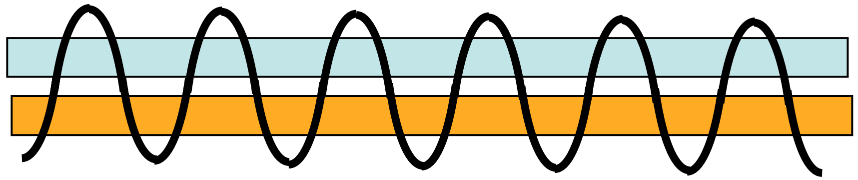
$$|\Psi\rangle = \prod_i \sum_{n_i} f(i, n_i) |n_i\rangle_i$$

Dipolar gases in optical lattices

Creation of a dipolar lattice gas in 3 stages

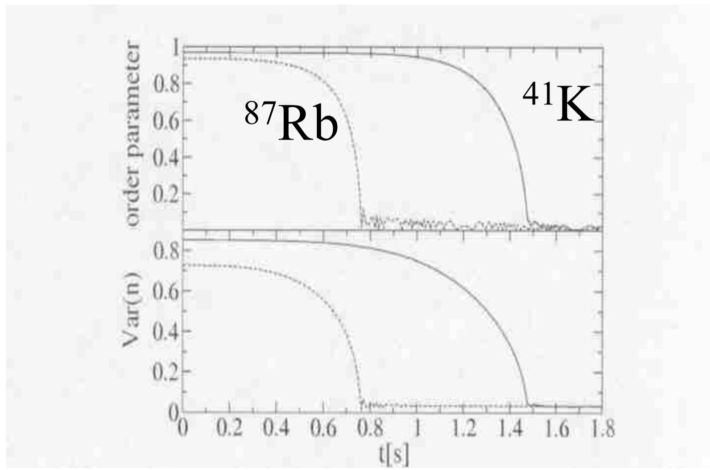
[Jaksch et al., PRL **89**, 040402 (2002); Damski et al., PRL **90**, 110401 (2003)]

1) Adiabatic growing of the lattice



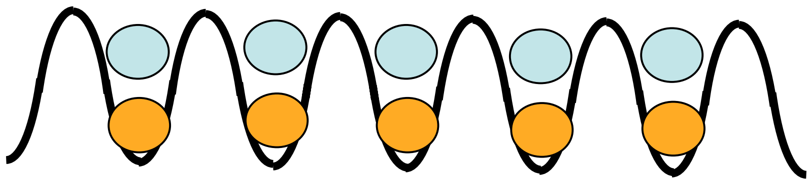
- Two trapped atomic species
  In particular ^{87}Rb-^{41}K
- 2-species BH Hamiltonian:

$$H = \sum_{<ij>} \left[J_a a_i^+ a_j + J_b b_i^+ b_j \right] + U_{ab} \sum_i n_{ai} n_{bi} +$$
$$+ \frac{1}{2} \sum_i \left[U_{0a} n_{ai} (n_{ai} - 1) + U_{0b} n_{bi} (n_{bi} - 1) \right]$$



- We consider miscibility:
  Eventually Feshbash res.
- Dynamical Gutzwiller Ansatz
  [Jaksch *et al*., PRL **89** 040402 (2002)]

t~1-2s



Dipolar gases in optical lattices

2) <u>In-site molecule formation</u>

• Two possible routes:

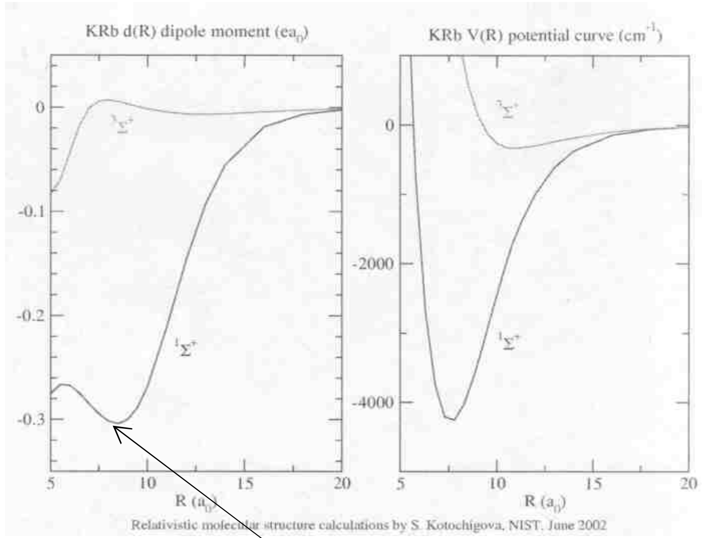
1) Two-color Raman photoassociation

to make ground-state $^1\Sigma^+$ or $^3\Sigma^+$ dimers.

2) Use a pulse of microwave radiation

to adiabatic pass from a free-free state

to a bound one.

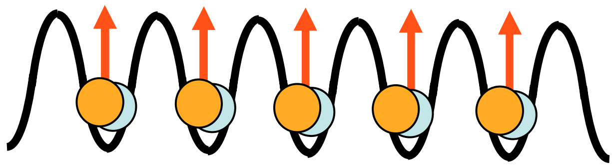


0.76 Debyes

• A succession of Raman pulses transfer to the v=0 state

• For the $^1\Sigma^+$ v=0 level, d=0.76 D has been predicted for RbK

Dipolar gases in optical lattices

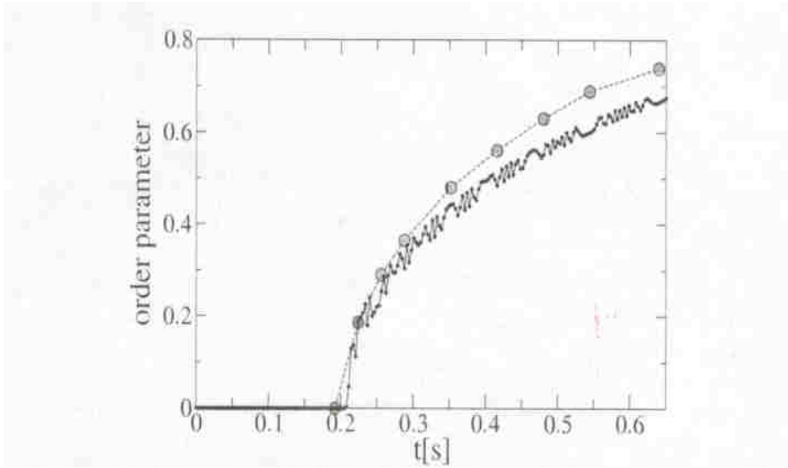


3) Adiabatic melting

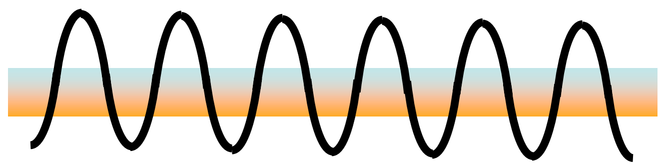
- After the formation: Molecular Mott Applications to QC [Brennen *et al*., PRL **82**, 1060 (1999); DeMille, PRL **88**, 067901]
- Melting dynamics : Gutzwiller Ansatz taking into account the dip-dip interact.
- Two stages in the melting:

  1) Reduce the lattice potential: U_0/J decreases

  2) Reduce the transversal confinement: The anisotropy of the dip-dip helps to further reduce U_0



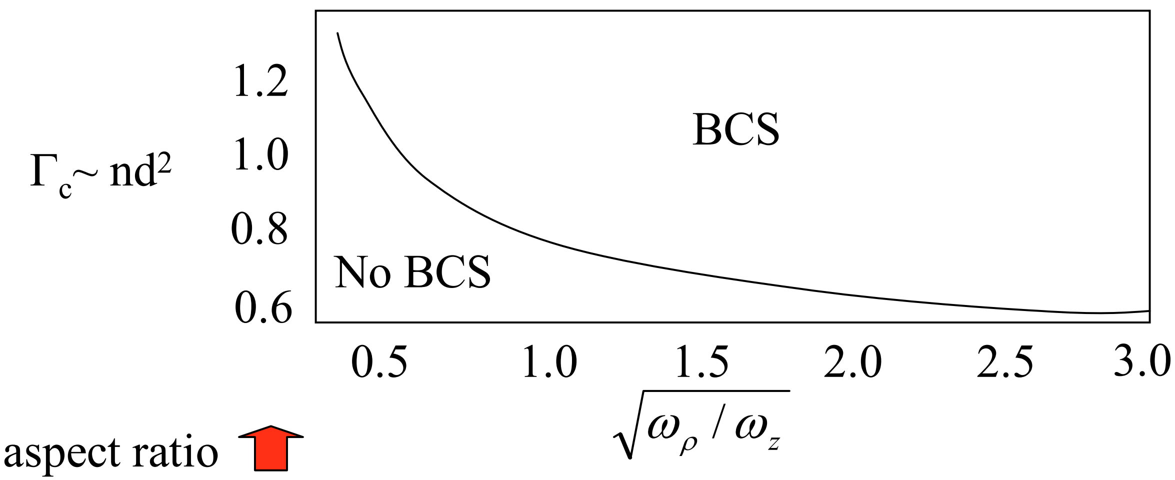
t~1s



Other interesting developments in dipolar gases

Fermionic dipolar gases

[Baranov et al., PRA **66**, 013606 (2002); Baranov et al., PRL **92**, 250403 (2004)]



Critical trap aspect ratio ⬆
(once more!)

Other interesting developments in dipolar gases

Rapidly-rotating dipolar gases

[Baranov et al., PRL **94**, 070404 (2005); Rezayi et al., cond-mat/0507064]

Lecture of Nigel Cooper tomorrow

Dipolar effects in spinor condensates

[Pu et al., PRL **87**, 140405 (2001); Yi and You, PRL **93**, 040403 (2004)]

Quantum information applications

[Brennen et al., PRL **82**, 1060 (1999); Jaksch et al., PRL **85**, 2208 (2000); DeMille et al., PRL **88**, 067901 (2002)]

People

P. Pedri
S. Sinha

M. Baranov
L. Dobrek
B. Damski
M. Lewenstein

G. V. Shlyapnikov

E. Tiemann

S. Giovannazi

K. Góral

S. Kotochigova
P. Julienne

P. Zoller

J. Stuhler
A. Griesmaier
T. Koch
M. Fattori
T. Pfau