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SCHOOL ON QUANTUM PHASE TRANSITIONS AND NON-EQUILIBRIUM PHENOMENA IN COLD ATOMIC GASES

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Vortex Liquids in Rotating Atomic Bose Gases

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Research Council

Outline

- Motivation + Formulation of the Problem
- Mean-field Theory
- Quantum Fluctuations & Quantum Melting
- Quantum Liquids of Vortices
- Effects of Dipolar Interactions
- Summary

Rotating Atomic Bose Gases

Weak interactions:

s-wave scattering length, $a_s \sim 5$nm
mean particle separation, $\bar{a} \sim 100$nm

Healing length: $\xi \equiv \frac{1}{\sqrt{8\pi\bar{n}a_s}} \sim 0.5\mu$m $\Rightarrow$vortex cores are large.

Vortex density: $n_V = \frac{2m\Omega}{h}$ $\Rightarrow$vortex separation $a_V \equiv \sqrt{\frac{\hbar}{m\Omega}}$.

The regime of high vortex density $a_V \lesssim \xi$ can be achieved.

[Schweikhard *et al.*, PRL **92**, 040404 (2004)]

Are there novel *uncondensed* many-vortex states?

[Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

Formulation of the Problem

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{1}{2} m\omega_\perp^2 (x_i^2 + y_i^2) + \frac{1}{2} m\omega_\parallel^2 z_i^2 \right] + \eta \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j) \qquad \left[\eta = \frac{4\pi\hbar^2 a_s}{m} \right]$$

Rotating Frame: $H_\Omega = H - \vec{\Omega} \cdot \vec{L}$

<u>One-Body Terms</u>

$$\begin{aligned} H_\Omega^{(1)} &= \frac{|\vec{p}|^2}{2m} + \frac{1}{2} m\omega_\perp^2 (x^2 + y^2) + \frac{1}{2} m\omega_\parallel^2 z^2 - \vec{\Omega} \cdot \vec{r} \times \vec{p} \\ &= \frac{|\vec{p} - m\vec{\Omega} \times \vec{r}|^2}{2m} + \frac{1}{2} m(\omega_\perp^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2} m\omega_\parallel^2 z^2 \end{aligned}$$

$$\boxed{q^* \vec{B}^* = 2m\vec{\Omega}}$$

+ Interactions

$\eta\bar{n} \ll \hbar\omega_{\|}, \hbar\omega_{\perp}$ $\Rightarrow$ Single particle states restricted to 2D & the lowest Landau level [Wilkin, Gunn & Smith (1998)]

$$\langle x, y|m\rangle \propto z^m e^{-|z|^2/2} \qquad \left[z \equiv (x+iy)/a_{\perp}; a_{\perp} \equiv \sqrt{\frac{\hbar}{m\omega_{\perp}}}\right]$$

Total angular momentum (in units of $\hbar$)

$$L = \sum_i m_i$$

What are the groundstates as a function of total angular momentum?

[For $\hbar\omega_{\|} \ll \eta\bar{n} \ll \hbar\omega_{\perp}$, *i.e.* the 3D LLL regime, see T.L. Ho, PRL **87**, 060403 (2001) for mean-field theory]

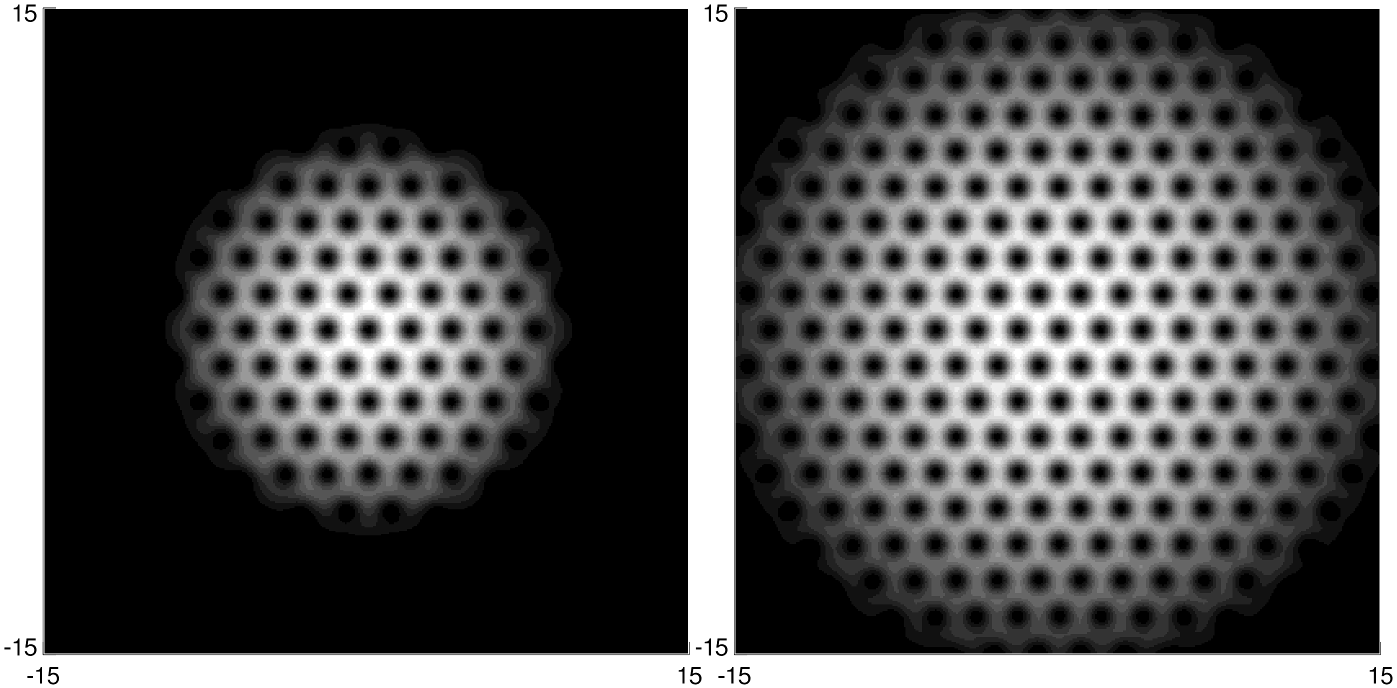
Gross-Pitaevskii Mean-Field Theory

[Butts & Rohksar, Nature **397**, 327 (1999)]

$$\Psi(\{\vec{r_i}\}) = \prod_{i=1}^{N} \psi(\vec{r_i}) \qquad \psi(\vec{r_i}) \propto \prod_{\alpha=1}^{N_V} (z - Z_\alpha) \quad \left[\times e^{-|z|^2/2}\right]$$

Minimise the expectation value of the interaction energy, for fixed N and L.

L/N=30 *L/N=90*



[NRC, Komineas & Read, PRA **70**, 033604 (2004)]

Vortex Lattice vs. Vortex Liquid

[NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

The Filling Factor

FQHE: $\nu = n_e \frac{h}{eB}$

Here: $\nu = n_{2\mathrm{d}} \frac{h}{q^* B^*} = n_{2\mathrm{d}} \frac{h}{2m\Omega}$

$$\nu = \frac{n_{2\mathrm{d}}}{n_V} = \frac{N}{N_V}$$

Quantum fluctuations

[Fetter, Phys. Rev. **162**, 142 (1967); Haldane & Wu, PRL **55**, 2887 (1985)]

Magnus force dynamics for a 2D vortex:

$$\rho_s \kappa_0 \dot{Y} + F_X^{\rm ext} = 0 \qquad \left[\rho_s \kappa_0 = (n_{2\rm d} m)\frac{h}{m} = n_{2\rm d} h\right]$$

$$-\rho_s \kappa_0 \dot{X} + F_Y^{\rm ext} = 0$$

Lagrangian:

$$L = -n_{2\rm d} h\, \dot{X} Y - V(X,Y) \qquad \left[\vec{F}^{\rm ext} = -\vec{\nabla} V\right]$$

Quantise:

$$\Pi_X \equiv \frac{\partial L}{\partial \dot{X}} = -n_{2\rm d} h\, Y$$

$$[\hat{X}, \hat{\Pi}_X] = i\hbar \;\Rightarrow\; [\hat{X}, \hat{Y}] = -\frac{i}{2\pi n_{2\rm d}}$$

$$\Delta X \Delta Y \geq \frac{1}{4\pi n_{2\rm d}}$$

$$\Delta X^2 + \Delta Y^2 \geq \frac{1}{2\pi n_{2\rm d}}$$

Lindemann criterion for (quantum) melting:

$$\sqrt{\Delta X^2 + \Delta Y^2} \sim \frac{1}{\sqrt{n_{\rm 2d}}} \quad \geq \quad c_L \times a_V \sim c_L \frac{1}{\sqrt{n_V}}$$

$$\Rightarrow \text{Filling factor} \quad \nu \equiv \frac{n_{\rm 2d}}{n_V} \quad \leq \quad \nu_c \quad \simeq 14$$

Exact diagonalisations, $\nu_c \simeq 6$ [NRC, Wilkin & Gunn PRL **87**, 120405 (2001)]

Quantum Liquids of Vortices for $\nu < \nu_c$

Laughlin State [Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

$$\Psi_L(\{z_i\}) = \prod_{i<j}^{N} (z_i - z_j)^2 \qquad \left[\times e^{-\sum_i |z_i|^2/2}\right]$$

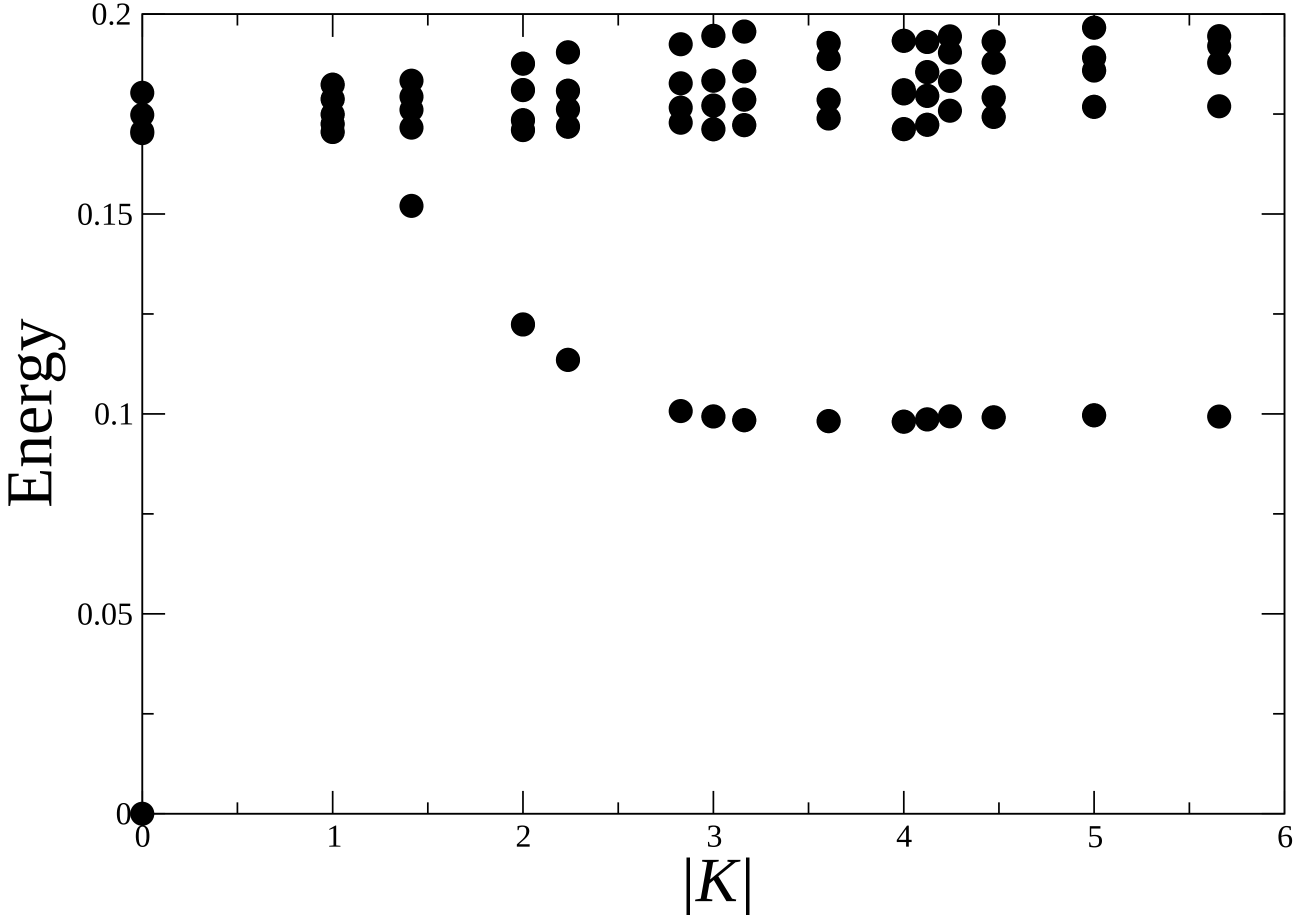
Exact groundstate with total angular momentum $L = N(N-1)$.

Describes a filling factor $\nu = \frac{N}{N_V} = 1/2$.

An incompressible fluid – gapped collective excitations in the bulk.
Particle-like excitations have: fractional particle number;
fractional statistics ("anyons").

[See articles by Laughlin & Haldane in "The Quantum Hall Effect", eds. Prange & Girvin]

$\nu = 1/2$



$(N = 8, N_V = 16)$

Composite Fermion States

[NRC & Wilkin, PRB **60**, R16279 (1999)]

Composite Fermion $=$ Boson $+$ Vortex

[Girvin, Read, Zhang, Jain...]

$$\Psi(\{\vec{r}_k\}) = \mathcal{P}_{\mathrm{LLL}} \prod_{i<j} (z_i - z_j) \psi_{\mathrm{CF}}(\{\vec{r}_k\})$$

Treat the composite fermions as *non-interacting*

- “Vortex liquids”;
- Cannot be described by a condensed wavefunction;
- Describe FQHLs at $\nu = 1/2, 2/3, 3/4, \ldots 3/2, 2$.

[See also Regnault & Jolicoeur, PRL **91**, 030402 (2003), PRB **69**, 235309 (2004); Chang *et al.*, cond-mat/0412253]

Read-Rezayi "Parafermion" states

[Read & Rezayi, PRB **59**, 8084 (1999)]

Incompressible states, with quasiparticle excitations which obey "non-abelian" exchange statistics.

$$\Psi^{(k)}(\{z_i\}) = \mathcal{S}\left[\prod_{i<j\in A}^{N/k}(z_i - z_j)^2 \prod_{l<m\in B}^{N/k}(z_l - z_m)^2 \ldots\right]$$

$$\nu^{(k)} = \frac{k}{2}$$

- The dominant sequence of incompressible states found in exact diagonalisations
- Large overlaps with the two-body groundstates (up to $\nu = 3$).

[NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

Experimental Status and Implications

$\mu/(2\hbar\omega_\perp) \lesssim 1$ $\Rightarrow$LLL
$\nu \gtrsim 500$ $\Rightarrow$vortex lattice

[Schweikhard *et al.*[JILA], PRL **92**, 040404 (2004)]

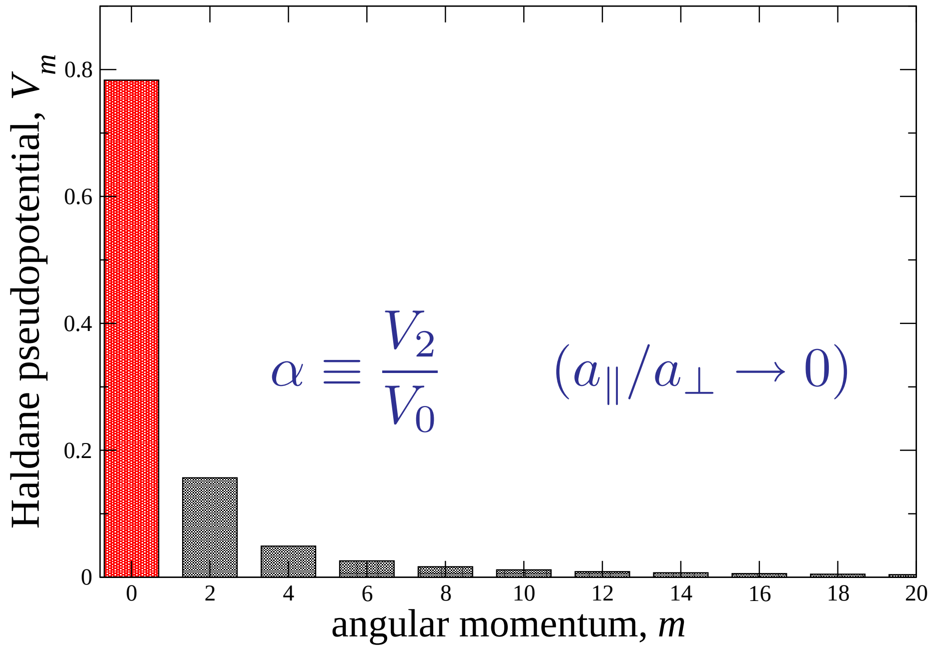
It will require further special efforts to access the regime of quantum-melted vortex liquids at $\nu < \nu_c \simeq 6$.

What would one look for?
– density distribution; [NRC, van Lankvelt, Reijnders & Schoutens, cond-mat/0409547]
– collective excitations; [M. Cazalilla, PRA 2003]
– fractional statistics; [B. Paredes *et al.*, PRL 2001]
– vanishing condensate fraction; [J. Sinova *et al.*, PRL 2003]
– density correlation functions. [N. Read & NRC, PRA 2003]

Dipolar Interactions

BEC of ^{52}Chromium [A. Griesmaier *et al.*, Phys. Rev. Lett. **94**, 160401 (2005)]

$$V(\boldsymbol{r}) = \frac{4\pi\hbar^2 a_s}{m}\,\delta^3(\boldsymbol{r}) + C_d \frac{x^2+y^2-2z^2}{(x^2+y^2+z^2)^{5/2}}$$

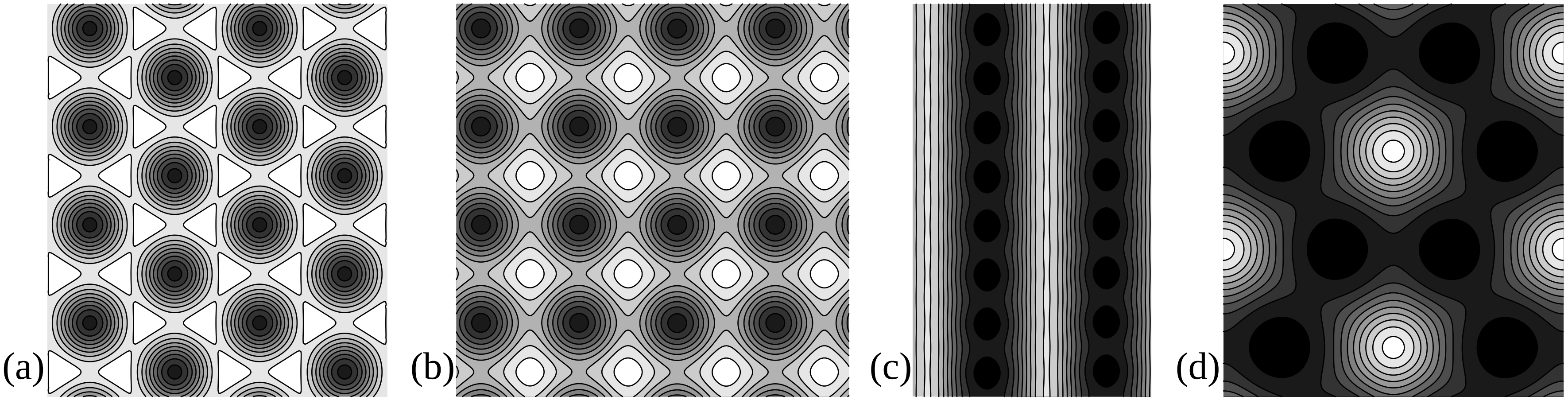


[For rotating dipolar fermions, see M. Baranov *et al.*, Phys. Rev. Lett. **94**, 070404 (2005)]

Vortex Lattices in Dipolar Bose Gases

[NRC, E.H. Rezayi & S.H. Simon, cond-mat/0505759]

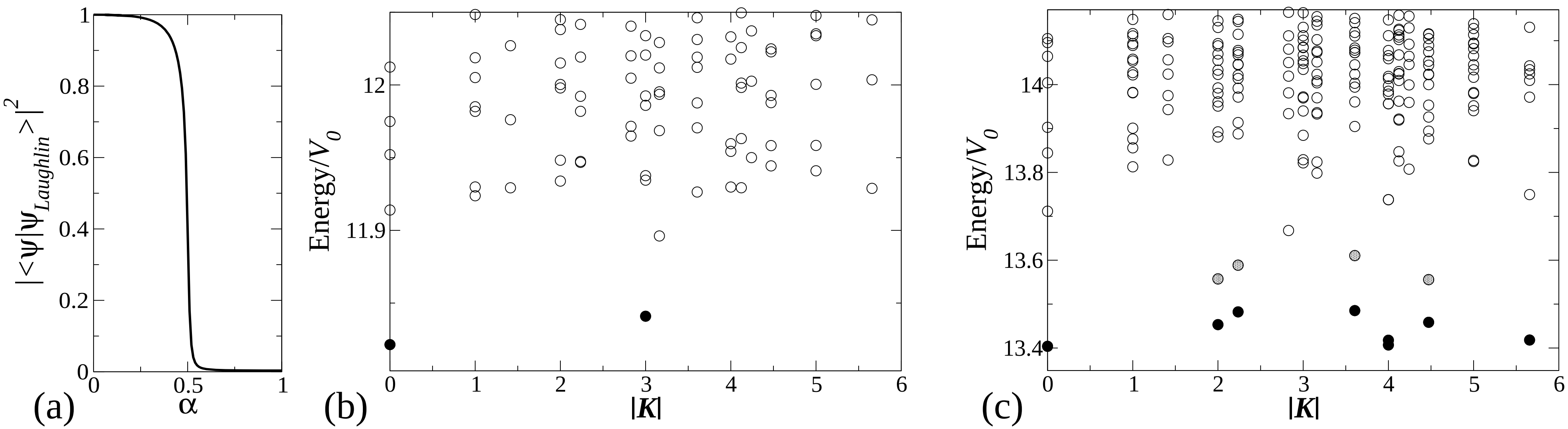
GP Mean-field theory, $\nu \to \infty$



$\alpha = 0 \to 0.20$ $\quad$ $0.20 \to 0.24$ $\quad$ $0.24 \to 0.60$ $\quad$ $0.60 \to$

Stripe crystal $\quad$ "Bubble" crystals

Exact Diagonalisations on a torus, $\nu = 1/2$.



Stripe, $\alpha = 0.528$

Bubble crystal, $\alpha = 0.758$

The stripe and bubble phases are stable to quantum fluctuations to very low filling factors.

Summary

- In the limit of weak interactions a rotating atomic Bose gas in a harmonic well occupies states in the lowest Landau level.

- The crucial parameter controlling the nature of the groundstate is the filling factor, $\nu \equiv N/N_V$.

- For large $\nu > \nu_c \simeq 6$ the groundstate is a *vortex lattice*.

- For $\nu < \nu_c$ quantum fluctuations of the vortices lead to *vortex liquid* groundstates.

- The vortex liquids involve the binding of vortices to the particles. They are closely related to fractional quantum Hall liquids.

- Dipole interactions can lead to a transition of the mean-field vortex lattice to square, "stripe crystal" and "bubble crystal" phases. The stripe and bubble states are stable to quantum fluctuations even down to $\nu = 1/2$.