Theory of the Anomalous Transport Reduction

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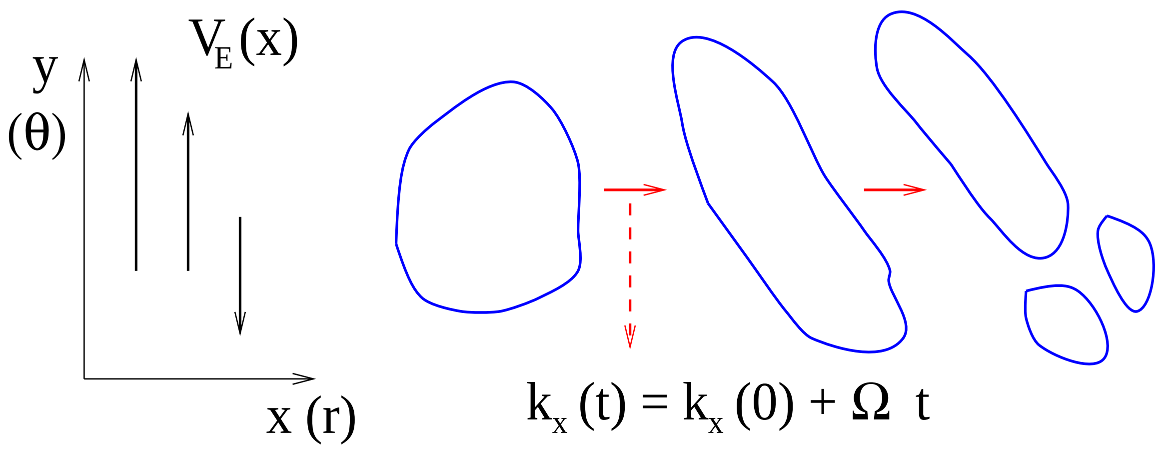
Turbulence regulation by $\mathbf{V}_E$ shear flow

Shear decorrelation

↗ ↖

Mean $\mathbf{E} \times \mathbf{B}$ flow $\langle V_E \rangle$ | **Zonal flow $\tilde{V}_E$**



- **Mean flow (coherent shearing):**

$$\langle V_E \rangle = \langle V_\theta \rangle - \frac{B_\theta}{B} \langle V_\phi \rangle - \frac{1}{eB_z n} \frac{\partial p_i}{\partial r} + \tau$$

- **Zonal flows (random shearing):**

$$\partial_t \phi_{ZF} = \langle \tilde{v}_x \tilde{v}_y \rangle - \nu \phi_{ZF}$$

ISSUES

• **Shearing by both mean flow (Ω) and zonal flows (Ω_{rms}) suppresses turbulent transport Γ [Zonal flows trigger L-H transition]**

• **Formation of transport barrier $\Rightarrow$ Quantitative prediction for the reduction of Γ by both [cf. L-H transition when $\omega_{\mathbf{E}\times\mathbf{B}} \sim |\partial_r V_E| \gtrsim \gamma_{max}$?]**

• **Strong reduction of the flux Γ with Ω and/or Ω_{rms}? [e.g., $\Gamma \propto \Omega^{-\alpha}$ with $2 \lesssim \alpha \lesssim 3.6$ (Boedo et al '02)]**

• **Universal scalings v.s. model dependence of Γ?**

OUTLINE

I. Comparison between mean and random shearing

II. Reduction in turbulent (anomalous) transport

1. Passive scalar fields

2. Particle transport in interchange turbulence

III. Intermittent transport

IV. Conclusions

I. Coherent vs Random Shearing

$$k_x(t) = k_x(0) + k_y \int^t \Omega(t')dt'$$

1. Coherent shearing with constant Ω ($k_x^2 \propto t^2$)

$$\Rightarrow \qquad D \int^t dt' k_x^2(t') \propto D k_y^2 \Omega^2 t^3$$

$$\Rightarrow \qquad \tau_\Delta = (\tau_\eta / \Omega^2)^{1/3} \qquad [\tau_\eta = 1/D k_y^2]$$

2. Random shearing with correlation time τ_{ZF}

$$\Rightarrow \qquad D \int^t dt' k_x^2(t') \propto D k_y^2 \tau_{ZF} \Omega_{rms}^2 t^2$$

$$\Rightarrow \qquad \tau_D = (\tau_\eta / \tau_{ZF} \Omega_{rms}^2)^{1/2} = (\tau_\eta / \Omega_{eff})^{1/2}$$

- **If** $\tau_{ZF} < \Omega_{rms}^{-1}$**,** $\Omega_{eff} = \tau_{ZF} \Omega_{rms}^2 < \Omega_{rms}$
- $\tau_\Delta \leq \tau_D$ **for** $\Omega = \Omega_{rms}$
- **For** $\tau_{ZF} \gg \tau_D$**,** $\Omega(t) \sim \mathrm{const} \to \tau_D = \tau_\Delta$

II. Turbulent transport

1. Passive scalar field χ

$$(\partial_t + U\partial_y)\chi' = -v_x \partial_x \chi_0 + D\nabla^2 n$$

- $\mathbf{v}$: random (prescribed) turbulent flow
- $U(x) = -x\Omega(t)$
- $\chi = \chi_0 + \chi'$
- **Flux** $\Gamma = \langle \tilde{\chi} v_x \rangle = \sum_{\mathbf{k}} |\tilde{\chi}(\mathbf{k})||v_x(-\mathbf{k})| \cos \delta_{\mathbf{k}}$

$\Rightarrow$ **For a given** v_x**, how do** Γ**,** $|\tilde{\chi}(\mathbf{k})|$**, and cross phase** $\cos \delta_{\mathbf{k}}$ **scale with** Ω **in the strong shear limit?**

- **For mean flow or zonal flow with $\tau_\Delta \ll \tau_{ZF}$ ($\Omega_{rms} \sim \Omega$)**

	$\tau_c < \tau_\Omega \ll \tau_\Delta$	$\tau_\Omega < \tau_c < \tau_\Delta$
$\langle \chi' v_x \rangle$	Ω^0	Ω^{-1}
$\langle \chi'^2 \rangle$	$\tau_\Delta \propto \Omega^{-1}$	$\tau_\Delta \Omega^{-1} \propto \Omega^{-5/3} D^{-1/3}$

- **For zonal flow with $\tau_{ZF} < \tau_D$ and Gaussian PDFs:**

	$\tau_c < \tau_{ZF} < \tau_\Omega \ll \tau_D$	$\tau_\Omega < \tau_c < \tau_{ZF} \ll \tau_D$
$\langle\langle \chi' v_x \rangle\rangle$	Ω_{rms}^0	Ω_{rms}^{-1}
$\langle\langle \chi'^2 \rangle\rangle$	$\tau_D \propto \Omega_{rms}^{-1}$	$\tau_D \Omega_{rms}^{-1} \propto \Omega_{rms}^{-2} D^{-1/2}$

Note: $\langle\langle \chi'^2 \rangle\rangle / \langle \chi'^2 \rangle = \tau_D / \tau_\Delta > 1$

Conclusions from passive scalar fields

• **Model dependence of the flux and turbulence amplitude**

• **Cross phase $\cos\delta$ is very weakly reduced**

• **Exact scaling with Ω depends on the propery of random turbulent flow**

• **Effect of random shearing of zonal flows on transport and fluctuation levels correlation time τ_{ZF}**

• $\langle\langle\chi'^2\rangle\rangle_{ZF} \propto \tau_D \Omega_{rms}^{-1} \propto \Omega_{rms}^{-2} > \langle\chi'^2\rangle\rangle \propto \tau_\Delta \Omega^{-1} \propto \Omega^{-5/3}$ **is due to LONGER decorrelation time ($\tau_D > \tau_\Delta$) induced by finite τ_{ZF}**

• **Limitation of scalar field model: random turbulent flow is arbitrary given (i.e., No shearing effect on turbulent flow)**

$\Rightarrow$ **Scalings of Γ and Q in a self-consistent model (Effect of Ω on $|v_x|$)?**

2. Particle transport in interchange turbulence

$$(\partial_t + U\partial_x)n = -v_x\partial_x N_0 + D\nabla^2 n + S$$

$$(\partial_t + U\partial_x)\omega = -g\frac{\partial_y n}{N_0} + \nu\nabla^2\omega$$

where $\mathbf{u} = \mathbf{v} + U(x)\hat{y}$ **and** $N = N_0 + n$

- $U = -x\Omega(t)$
- $\omega\hat{z} = \nabla \times \mathbf{v}$ **evolves dynamically, subject to shearing**
- $D = \nu$
- **Total noise** $f = S - v_x\partial_x N_0 + ...$ **(corr. time** τ_f**)**

$\tau_D \ll \tau_{ZF}$			$\tau_D \gg \tau_{ZF}$
	$\tau_f < \tau_\Omega$	$\tau_f > \tau_\Omega$	$\tau_f < \tau_\Omega$
$\langle n v_x \rangle$	$\Omega^{-2} \ln(\tau_\Delta \Omega)$	$\Omega^{-3} \ln \Omega$	$\tau_D \Omega_{eff}^{-1} \propto \Omega_{rms}^{-3}$
$\langle n^2 \rangle$	$\tau_\Delta \propto \Omega^{-2/3}$	$\Omega^{-5/3}$	$\tau_D \propto \Omega_{rms}^{-1}$
$\langle v_x^2 \rangle$	Ω^{-3}	Ω^{-4}	$\tau_D \Omega_{eff}^{-2} \propto \Omega_{rms}^{-5}$
$\langle v_x v_y \rangle$	$-\Omega^{-3} \ln \Omega$	$-\Omega^{-4} \ln \Omega$	

- **A strong reduction in the transport of particles results from a severe reduction in the amplitude of velocity.**

- **The reduction in cross-phase is very weak ($\propto \Omega^{-1/6} \ln \Omega$) [agrees with Falchetto and Ottaviani, PRL. 92, 025002 (2004)]**

- **A mean flow is generated through Reynolds stress with negative**

viscosity while this Reynolds stress driving itself is reduced by shear as its amplitude becomes large.

III. Intermittent Transport

1. Coherent structure $U_s(y)\hat{x} = |U_s| \cos{(p_y y + \omega_s t)}\hat{x}$

$$[\partial_t + U_s(y)\partial_y]n = D\nabla^2 n$$

$\Rightarrow n = n_0(x) + n_s(y)$

$\Rightarrow \langle n_s U_s \rangle$ **gives** $D_{eff} = DU_s^2 p_y^2 / [\omega_s^2 + (Dp_y)^2]$

2. Coherent structure (n_s, U_s) + turbulence + mean shear flow $U_0(x)\hat{y} = -x\Omega\hat{y}$

$\bullet D \rightarrow D_T$

• **Shearing by Ω:**

$$D_T \propto \Omega^{-1}, \langle n_s U_s \rangle \propto D_T \Omega^{-2}$$

$$\Rightarrow \boxed{D_{eff} \propto \Omega^{-3}}$$

IV. Conclusions

• **Model dependent reduction in the flux and turbulence amplitude $\rightarrow$ Stronger reduction in interchange turbulence due to the suppression of velocity amplitude**

• **In all cases, cross phase $\cos\delta$ is very weakly reduced**

• **Effect of random shearing of zonal flows on transport and fluctuation levels correlation time τ_{ZF}**

• **Random shearing can lead to significant reduction in interchange turbulence (larger transport as compared to coherent shearing)**

• **Effect of random shearing of zonal flows on transport and fluctuation levels of scalar fields crucially depends on the zonal flow pattern and correlation time τ_{ZF}.**

• $\langle\langle\chi'^2\rangle\rangle \propto \Omega_{rms}^{-2} D^{-1/2}$ ($\tau_\Omega < \tau_c < \tau_{ZF} \ll \tau_D$) is due to a LONGER effective decorrelation time of fluid elements induced by finite τ_{ZF}.

• Important to determine both the frequency spectrum (in particular, τ_{ZF}) and the probability distribution function (PDF) of zonal flows in both simulations and physical experiments.

• A useful estimate on τ_{ZF} from $\tau_{ZF} = \int_0^\infty dt \langle V_E(\tau) V_E(\tau+t)\rangle / \langle V_E(\tau)^2\rangle$, or from the width of the $m=0$ frequency spectrum.