Comparative studies of nonlinear ITG and ETG dynamics*

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Outline

- ☐ Ballooning Formalism and the form of nonlinear interactions in a torus
- ☐ Dynamics of Drift Wave - Zonal Flow interactions: ITG - ETG broken symmetry
- ☐ ITG nonlinear dynamics: turbulence spreading and size scaling of transport
- ☐ Nonlinear toroidal mode coupling and ETG nonlinear saturation
- ☐ Conclusions

Ballooning Formalism...

- Ballooning Formalism (BF): Using asymptotic techniques based on scale separation.
- Fourier decomposition of scalar potential fluctuations:

$$\delta\phi = \mathrm{e}^{in\zeta} \sum_m \mathrm{e}^{-im\theta} \delta\phi_m(r,t)$$

- (r, θ, ζ) are field-aligned flux coordinates, with r the radial (flux) variable, θ the poloidal angle and the equilibrium $\mathbf{B}$ field given by the Clebsch representation $\mathbf{B} = \nabla(\zeta - q\theta) \times \nabla\psi_p$ and $q(r) \equiv \mathbf{B} \cdot \nabla\zeta / \mathbf{B} \cdot \nabla\theta$
- Fourier harmonics $\delta\phi_m(r,t)$ have two scale structures:
  - $\approx (nq')^{-1}$ due to $-1 \lesssim k_\parallel qR = (nq - m) \lesssim 1$: $\parallel$ mode-structure
  - $\approx L_A \ll L_p$ due to equilibrium variation: radial envelope

- Multiple scale structure of Fourier harmonics:

$$\delta\phi_m(r,t) = \underbrace{A(r,t)}_{\text{envelope}} \int_{-\infty}^{\infty} e^{-i(nq-m)\eta} \underbrace{\delta\Phi(\eta,r,t)}_{\text{parallel mode structure}} d\kappa$$

$$= \exp i \int nq'\theta_k dr \int_{-\infty}^{\infty} e^{-i(nq-m)\eta} \delta\Phi(\eta,r,t) d\kappa$$

$$\theta_k = -i\frac{1}{nq'}\frac{\partial}{\partial r} \quad (\text{Dewar ; NF81})$$

The form of nonlinear interactions in a torus

- The linear DW mode structures can be described with three degrees of freedom: the toroidal mode number n, the parallel mode structure reflecting the radial width of a single poloidal harmonic m, and radial mode envelope $A(r) = \exp i \int \theta_k d(nq)$.
- Correspondingly, nonlinear interactions can take the following three forms: mode coupling between two n's, distortion of the parallel mode structure, and modulation of the radial envelope. (Z. Lin *et al.* PRL05, PoP05)
- Radial envelope modulation via generation of zonal flows dominates in ITG turbulence. (L. Chen *et al.* PoP00, PRL04, PoP04).
- ETG turbulence is regulated by nonlinear toroidal mode couplings. (Z. Lin *et al.* PRL05, PoP05)

Dynamics of Drift Wave - Zonal Flow interactions: ITG - ETG broken symmetry

- DW-ZF interactions can be systematically derived from the non-linear gyrokinetic equation (Frieman&Chen PoP82)

$$\left(\partial_t + v_\parallel \partial_\parallel + i\omega_d\right)_k \overline{\delta H}_k = \mathrm{i}\,(e/m)\,QF_0 J_0(\gamma)\delta\phi_k - (c/B)\,\mathbf{b}\cdot\left(\mathbf{k}''_\perp \times \mathbf{k}'_\perp\right) J_0(\gamma')\delta\phi_{k'}\overline{\delta H}_{k''} \quad ,$$

$$QF_0 = \omega_k\left(\partial F_0/\partial v^2/2\right) + (\mathbf{k}/\omega_c)\cdot\mathbf{b}\times\nabla F_0 \quad , \qquad \gamma = (k_\perp v_\perp/\omega_c)$$

- Separated adiabatic and non-adiabatic response, $\overline{\delta H}$:

$$\delta F = \frac{e}{m}\delta\phi\frac{\partial}{\partial v^2/2}F_0 + \sum_{\mathbf{k}_\perp}\exp\left(-\mathrm{i}\mathbf{k}_\perp\cdot\mathbf{v}\times\mathbf{b}/\omega_c\right)\overline{\delta H}_k \quad .$$

- Quasi-neutrality conditions:

$$n_0 e^2\left(1/T_i + 1/T_e\right)\delta\phi_k = \left\langle eJ_0(\gamma_i)\overline{\delta H}_i\right\rangle_k - \left\langle eJ_0(\gamma_e)\overline{\delta H}_e\right\rangle_k \quad .$$

- Quasi adiabatic electrons (ions) and fluid ions (electrons).

- Zonal flows dominates in ITG turbulence:
  $m_e/m_i \ll 1$, $\overline{\delta H}_{ez} = -(e/T_e) F_{0e} \delta\phi_z$. $n \neq 0$ quasi-neutrality.

$$\left(n_0 e^2/T_i\right)\left(1 + T_i/T_e\right)\delta\phi_k - \left\langle e J_0(\gamma_i)\overline{\delta H}_i^L\right\rangle_k + \left\langle e J_0(\gamma_e)\overline{\delta H}_e^L\right\rangle_k =$$
$$-(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')\,\delta\phi_{k'} J_0(\gamma_e'')\overline{\delta H}_{ek''}\right\rangle_k - \left\langle e J_0(\gamma_e)\overline{\delta H}_e^{NL}\right\rangle_k$$
$$-(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')\left[J_0(\gamma_i)J_0(\gamma_i') - J_0(\gamma_i'')\right]\delta\phi_{k'}\overline{\delta H}_{ik''}\right\rangle_k \;,$$

- ETG turbulence is regulated by nonlinear toroidal mode couplings:
  $k_\perp \rho_i \propto (m_i/m_e)^{1/2} \gg 1$, $\overline{\delta H}_{iz} = 0$. $n \neq 0$ quasi-neutrality.

$$\left(n_0 e^2/T_e\right)\left(1 + T_e/T_i\right)\delta\phi_k - \left\langle e J_0(\gamma_i)\overline{\delta H}_i^L\right\rangle_k + \left\langle e J_0(\gamma_e)\overline{\delta H}_e^L\right\rangle_k =$$
$$+(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')\,\delta\phi_{k'} J_0(\gamma_i'')\overline{\delta H}_{ik''}\right\rangle_k + \left\langle e J_0(\gamma_i)\overline{\delta H}_i^{NL}\right\rangle_k$$
$$+(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')\left[J_0(\gamma_e)J_0(\gamma_e') - J_0(\gamma_e'')\right]\delta\phi_{k'}\overline{\delta H}_{ek''}\right\rangle_k \;.$$

- Zonal Flows are NLy generated via Reynolds-Stress. However, ITG-ZF polarizability is $\chi_{iz} \simeq 1.6 q^2 \epsilon^{-1/2} k_z^2 \rho_i^2$ (Rosenbluth&Hinton PoP98), while ETG-ZF polarizability is $\chi_{ez} \simeq (T_e/T_i)$ (Z. Lin *et al.* PRL05, PoP05).

- ITG - ETG broken symmetry:
  - ZF coupling to ETG is weaker than for ITG because of Hasegawa-Mima rather than $\mathbf{E} \times \mathbf{B}$ dominant nonlinearity
  - ZF generation rate by ETG is slower than by ITG because of stronger polarizability

Modulational instability growth rate

ITG

$$\gamma_M^2 = \left(\frac{c}{B} k_\theta k_z \right)^2 \frac{(T_i/T_e)}{\omega_0 \partial_{\omega_0} D_{Ri}} \frac{k_z^2 \rho_i^2 \alpha_i}{\chi_{iz}} |A_0|^2$$

$$\alpha_i = \delta P_{\perp i}/(e n_0 \delta\phi) + 1$$

ETG

$$\gamma_M^2 = \left(\frac{c}{B} k_\theta k_z \right)^2 \frac{(T_i/T_e)}{\omega_0 \partial_{\omega_0} D_{Re}} k_z^2 \rho_e^2 \alpha_e^2 \times \left(\left\langle\left\langle k_\perp^2 \right\rangle\right\rangle - k_z^2 \right) \rho_e^2 |A_0|^2$$

$$\alpha_e = \delta P_{\perp e}/(e n_0 \delta\phi) - 1$$

ITG nonlinear dynamics

- Multiple time scales enter the problem: $\delta\Phi(\eta;r)$ forms on a $R/|v_{\mathrm{gr},\parallel}| \approx \omega^{-1}$ time scale; while the envelope slowly propagates radially on $\tau_A \approx L_A/|v_{\mathrm{gr},r}|$.
- Sufficiently close to marginal stability, such that $|\gamma_L/\omega| \ll 1$ (*e.g.* ITG), parallel mode structure forms without significant nonlinear distortions: characteristic nonlinear time scale is $\tau_{\mathrm{NL}} \approx \gamma_L^{-1}$.
- Only linear wave dispersive properties need to be taken into account for determining $\delta\Phi(\eta;r)$ and $D(r,\omega,\theta_k)$, with $\theta_k \equiv (-i/nq')\partial_r$ (Dewar NF81).
- NL interactions reflect on the radial envelope only, for which one can systematically derive nonlinear equations, assuming a hierarchy among NL wave-wave interactions, where the $\tau_{\mathrm{NL}} \approx \gamma_L^{-1}$ is set by ITG-ZF interactions. (L. Chen *et al.* PoP00, PRL04, PoP04)

- ZF as ITG envelope modulation: standard NL equations (L. Chen *et al.* PoP00).

$$\mathcal{L}_P P = 2S\partial_x Z \qquad \mathcal{L}_P = \partial_\tau - \bar{\gamma}_P - 2\delta^{1/2}\partial_x + i\Gamma(\lambda+\xi) + i\partial_x^2$$

$$\mathcal{L}_S S = -P\partial_x Z \qquad \mathcal{L}_S = \partial_\tau - \bar{\gamma}_S - 2\delta^{1/2}\partial_x + i\Gamma(\lambda+\xi) + i\partial_x^2$$

$$\mathcal{L}_Z Z = 2\mathrm{Re}\left[P^*\partial_x S - S\partial_x P^*\right] \qquad \mathcal{L}_Z = (\partial_\tau + \bar{\gamma}_z) \quad .$$

- $(P,S,Z)e^{-i\omega t}$ are suitably normalized ITG, sideband (in complex conjugate pairs) and ZF amplitudes.

$$\left\{\overbrace{\omega^{-1}\partial_t - \frac{\gamma}{\omega}}^{\text{drive/damping}} - \underbrace{\frac{\xi}{nq'\theta_k}\partial_r}_{\text{group vel.}} + \overbrace{i(\lambda+\xi)}^{\text{potential well}} + \underbrace{\mathrm{i}\frac{\lambda}{(nq'\theta_k)^2}\partial_r^2}_{\text{(de)focusing}}\right\} A(r,t) \quad = \quad \text{NL TERMS}$$

Size scaling of transport I

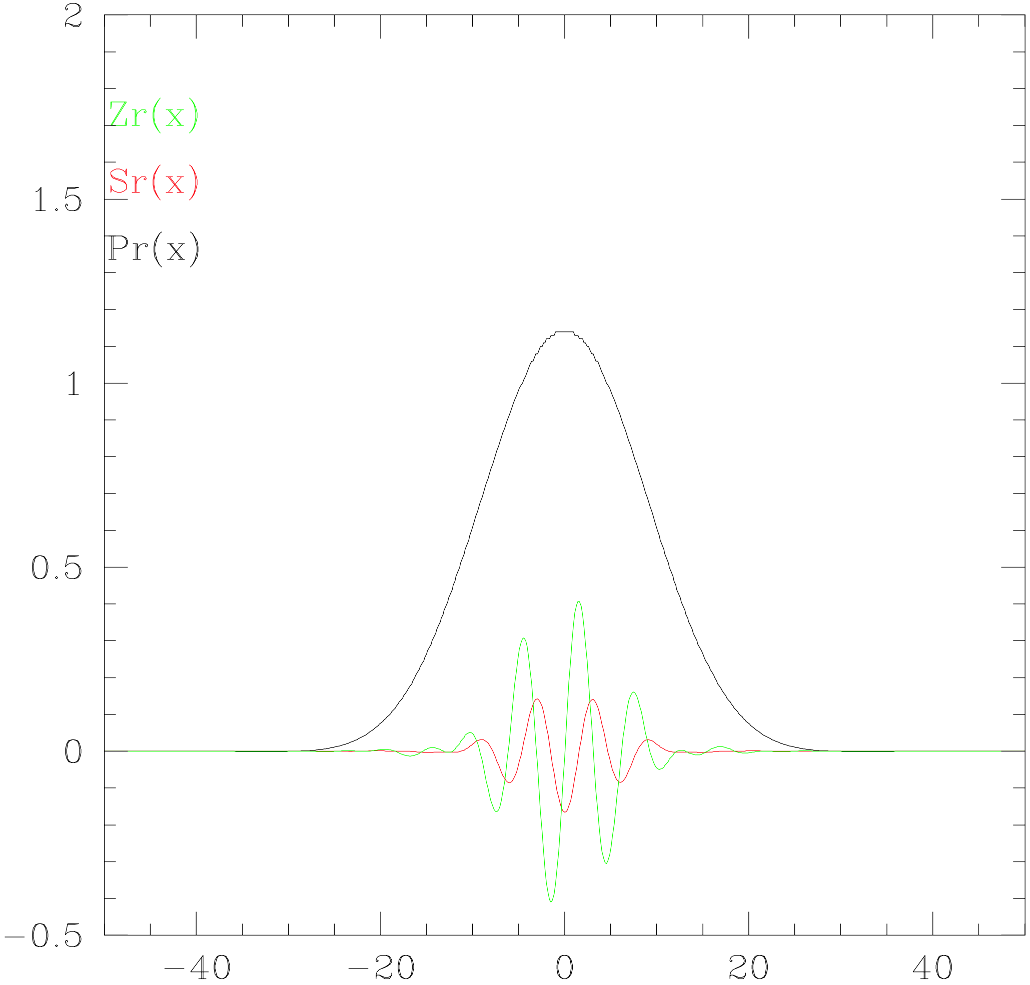
- Transport is a local process which may depend on global equilibrium profiles via dependencies of the turbulence intensity, $I = \langle |P|^2 + 2|S|^2 \rangle$, on the system size. $\langle ... \rangle$ = average on 1/5 of the linear unstable domain for P.

- For large systems, $L_p/\rho_i \gg 1$, gyrokinetic simulations indicate gyro-Bohm transport. In the present model we expect $\chi = \chi_{GB}(I/I_\infty)$. Size scaling of transport is monitored by size scaling of $I = \langle |P|^2 + 2|S|^2 \rangle$.

- Turbulence intensity dependencies on global plasma equilibrium are related with turbulence spreading (Kim *et al.* NF03, Hahm *et al.* PPCF04, Lin *et al.* PoP04, Chen *et al.* PRL04 PoP04, Gurcan *et al.* PoP05).

- Here, we assume a simple paradigm case, with quadratic dispersiveness, $D_{Ri} = \omega/\omega_0 - 1 + \theta_k^2 + V(x)$, $V(x) = 1 - \exp(-x^2/\bar{L}_p^2)$, $\bar{\gamma}_P = A\exp(-x^2/\bar{L}_p^2) - 1$, $\bar{L}_p = |ndq/dr|L_p|\omega/\gamma_{P\infty}|$.

Modulational Instability



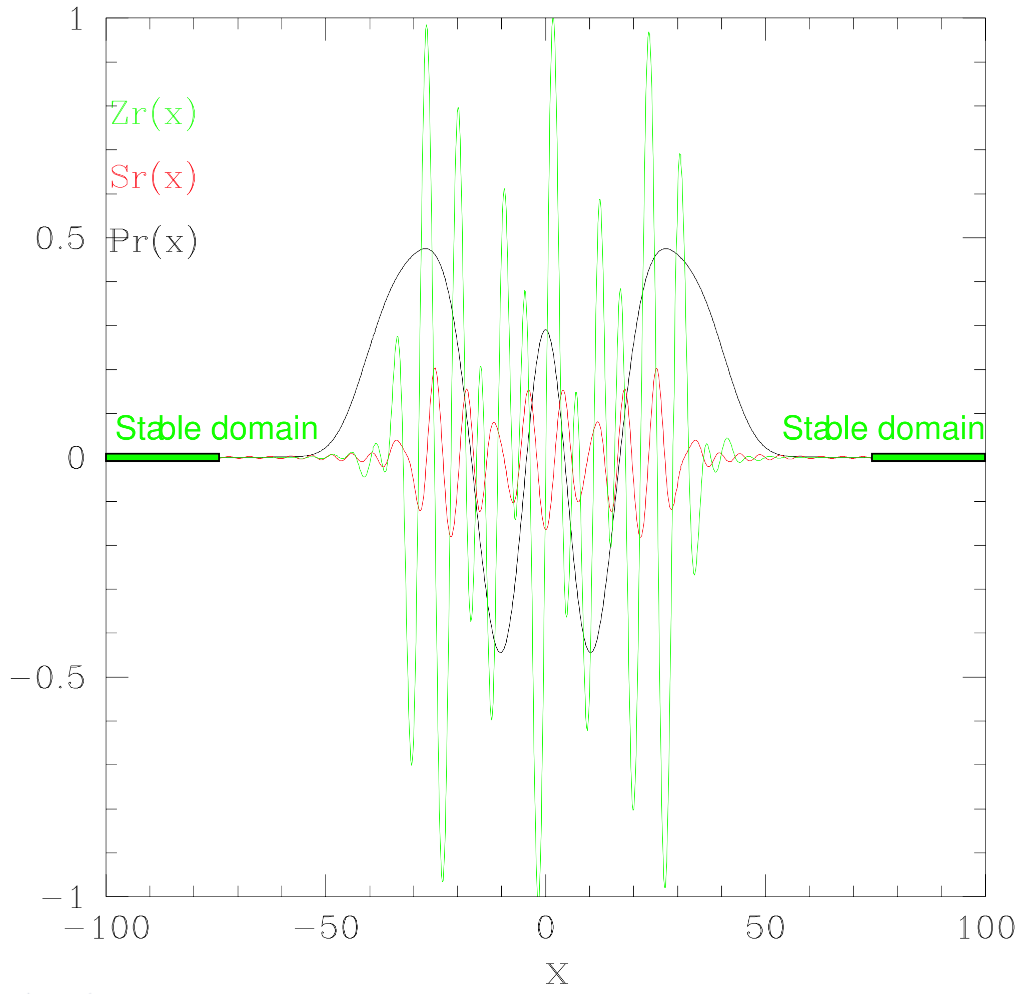
$$\tau = 20 \quad ; \quad A = 1.15$$

$$\gamma_z = 0.1 \quad ; \quad \bar{\gamma}_S = -\bar{\gamma}_d = -1$$

$$\bar{\gamma}_P = A \exp(-x^2/\bar{L}_p^2) - 1$$

$$\bar{L}_p = |ndq/dr| L_p \left(\frac{\omega}{|\gamma_P(x=\infty)|} \right)^{1/2}$$

Turbulence Spreading I

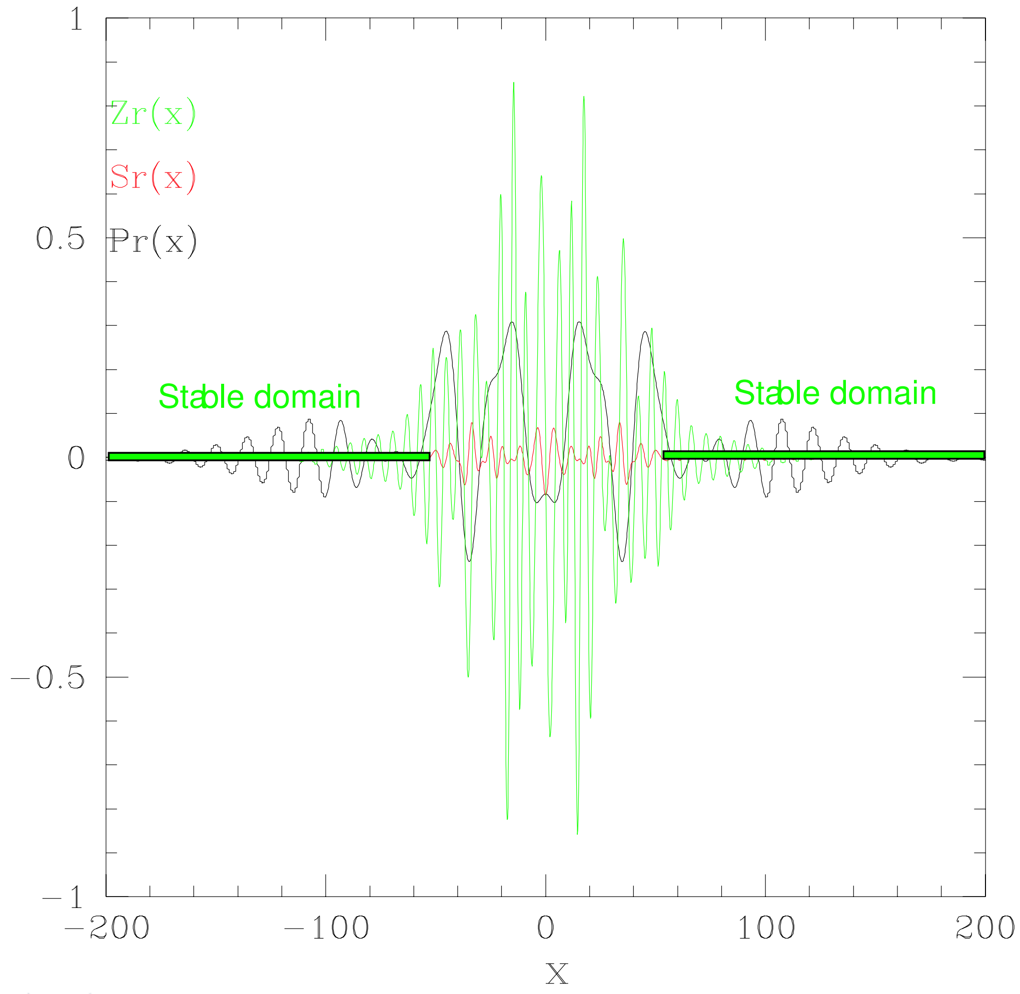


$$\tau = 50 \ \ ; \ \ A = 1.15$$

$$\gamma_z = 0.1 \ \ ; \ \ \bar{\gamma}_S = -\bar{\gamma}_d = -1$$

$$\bar{\gamma}_P = A \exp(-x^2/\bar{L}_p^2) - 1$$

Turbulence Spreading II



$\tau = 125 \; ; \; A = 1.15$

$\gamma_z = 0.1 \; ; \; \bar{\gamma}_S = -\bar{\gamma}_d = -1$

$\bar{\gamma}_P = A \exp(-x^2/\bar{L}_p^2) - 1$

Size scaling of transport II: Gyro-Bohm

- Transport is a local process which may depend on global equilibrium profiles via dependencies of the turbulence intensity, $I = \langle |P|^2 + 2|S|^2 \rangle$, on the system size. $\langle ... \rangle$ = average on 1/5 of the linear unstable domain for P.
- For sufficiently strong growth rate, the mode grows at the local growth rate and NL saturates before any linear radial mode structure can form.
- The same happens for a sufficiently large system, when NL interactions become important before the ITG traveling radial wave-packets sample varying equilibrium, because of either linear or NL induced wave spreading.
- In such conditions, the system behaves as an infinite and uniform medium and turbulent transport is gyro-Bohm: fixed point solutions of NL-Eqs. give (White *et al.* PoP05)

$$I = \left\langle |P|^2 + 2|S|^2 \right\rangle = I_f = \bar{\gamma}_z \frac{(\bar{\gamma}_d + 2\bar{\gamma}_{P0})}{|\bar{\gamma}_d - \bar{\gamma}_{P0}|} \left(2 + \frac{2\Gamma^{1/2}}{5\bar{\gamma}_{P0}\bar{L}_p} \right) \quad .$$

Size scaling of transport III: Bohm-like

- For either sufficiently small system or weak growth rate, ITG traveling radial wave-packets sample regions of varying equilibrium and turbulent transport is Bohm-like.
- For $\bar{L}_p \bar{\gamma}_{P0} \Gamma^{1/2} \approx 1$, the turbulence intensity scales with the system size as (White *et al.* PoP05)

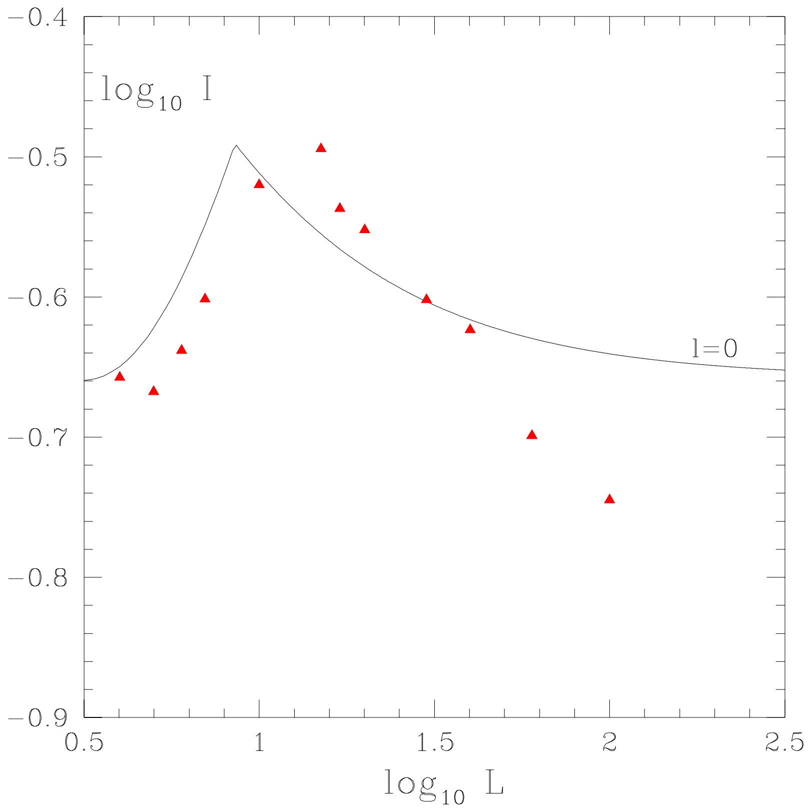
$$I \simeq I_0 = \frac{\bar{\gamma}_z \bar{\gamma}_d \bar{L}_p}{\sqrt{2\Gamma}} \left(1 + \frac{2}{\bar{\gamma}_d \Gamma^{1/2} \bar{L}_p} \right)^{-1} \left(1 + \frac{4\Gamma}{\bar{\gamma}_d^2 \bar{L}_p^2} \right) \quad .$$

- The control parameter from Bohm-like to gyro-Bohm transition is $\bar{L}_p \bar{\gamma}_{P0} \Gamma^{1/2}$, which is also the number of linearly unstable radial eigenmodes of the pump ITG. (L. Chen *et al.* PRL04, PoP04).

Spatiotemporal Chaos in ITG-ZF NL dynamics

- In the transition from small to large system size, fixed point solutions become unstable: chaotic behavior is reached via an infinite set of period doubling bifurcations. (L. Chen *et al.* PoP00)

- Fixed point solutions (unstable), still retain the main features to account for Bohm-like to gyro-Bohm transition in turbulence intensity behavior and agree well with global gyrokinetic simulations.(White *et al.* PoP05)

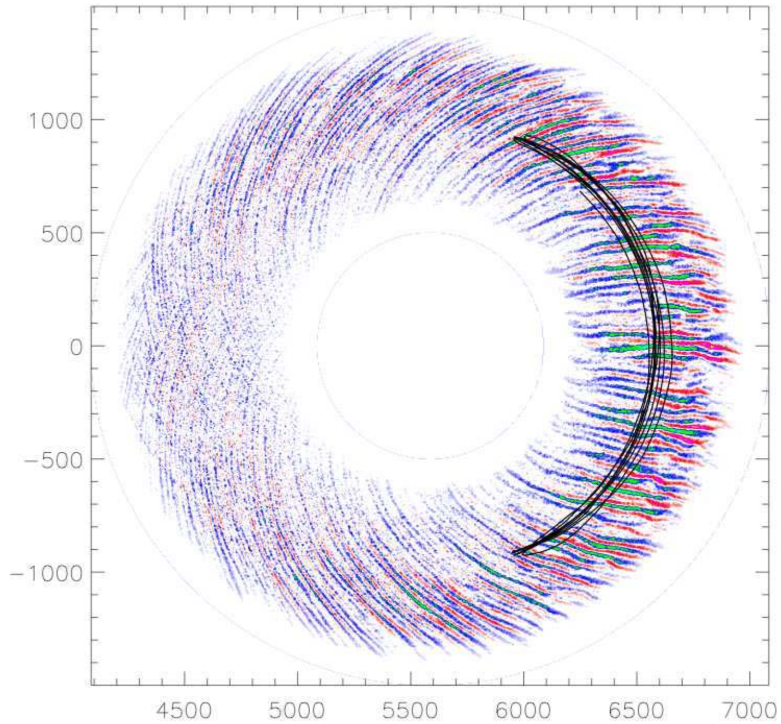


ETG saturation via nonlinear toroidal coupling

- Nonlinear ETG-ZF dynamics is of negligible importance for ETG saturation and for setting the level of turbulent transport.
- ETG saturation can be set only by distortions of the parallel mode structures or by nonlinear coupling between different n's. This situation was recently studied by dedicated numerical simulations (Z. Lin*et al.* PoP05, PRL05).
- Analyses of the nonlinear evolution of a single-n ETG demonstrated that saturation occurs via the generation of an $(m = \pm 1, n = 0) = (m, n)^* \times (m \pm 1, n)$ mode, which broadens the radial width of poloidal harmonics.
- The $|k_\parallel|$ increase corresponds to more ballooning $\Phi(\theta; r)$ due to θ-space potential well modification by the $(\pm 1, 0)$ mode; enhanced Landau damping.
- Elongated ETG eddies at saturation (streamers) are not appreciably altered: weak ZF effects on ETG and no excitation of a slab-like secondary KH instability (F.Jenko *et al.* PoP00, W. Dorland *et al.* PRL00).

- Analyses with multiple-n ETG show a much lower saturation level than in the single-n case: nonlinear coupling between two different n's is the dominant process in the ETG saturation.

- Particle transport is diffusive. No evidence of slab-like secondary KH instability.(Z. Lin *et al.* PoP05, PRL05)



- This coupling is a truly toroidal process, since the Hasegawa-Mima term is $\propto \mathbf{b} \cdot \mathbf{k}'_{\perp} \times \mathbf{k}''_{\perp}$. Coupling of two elongated streamers with $\mathbf{k}'_r, \mathbf{k}''_r \simeq 0$ is possible only because of localized radial structure of the single poloidal harmonics on a $\approx 1/|nq'|$ scale.

- Efficient nonlinear coupling between two different n's, n_0 and n_1, imposes that poloidal harmonics be localized near the same radial position: low order rational surface r_s, where $m_0/n_0 \simeq m_1/n_1 \simeq q(r_s) \equiv m_l/n_l$.

- Take $n_l = n_0 - n_1 \approx n_0^{1/2}$. The low-$n$ beat waves are quasi modes since they do not satisfy three-wave resonant conditions due to $1 > (\gamma_{L0}/\omega_0) \sim (\gamma_{L1}/\omega_1) \sim k_{\perp}\rho_e \sim n_0^{-1/4} > |\omega_0 - \omega_1|/\omega_0 \sim n_0^{-1/2}$.

- Toroidal geometry and nonlinear nature $\Rightarrow$ quasi modes are characterized by highly localized radial structures as well as long parallel wavelength, $k_{\parallel} \sim 1/(n_0^{1/2} q R_0)$ (not ballooning).

- From the quasineutrality for n_0, n_1, n_l modes, one systematically calculates the parallel mode structure of the quasi modes, and then derives the evolution equations for the normalized amplitudes $a_0(t)$, $a_1(t)$ and $a_l(t)$, where $a = eA/T_e$. (Z. Lin *et al.* PoP05, PRL05)

$$(\partial_t - \gamma_{L0})a_0 = -\gamma_{NL1}a_1a_l \ , \qquad (\partial_t - \gamma_{L1})a_1 = \gamma_{NL0}a_0a_l^* \ , \qquad \partial_t a_l = \gamma_{NLl}a_0a_1^* \ ,$$

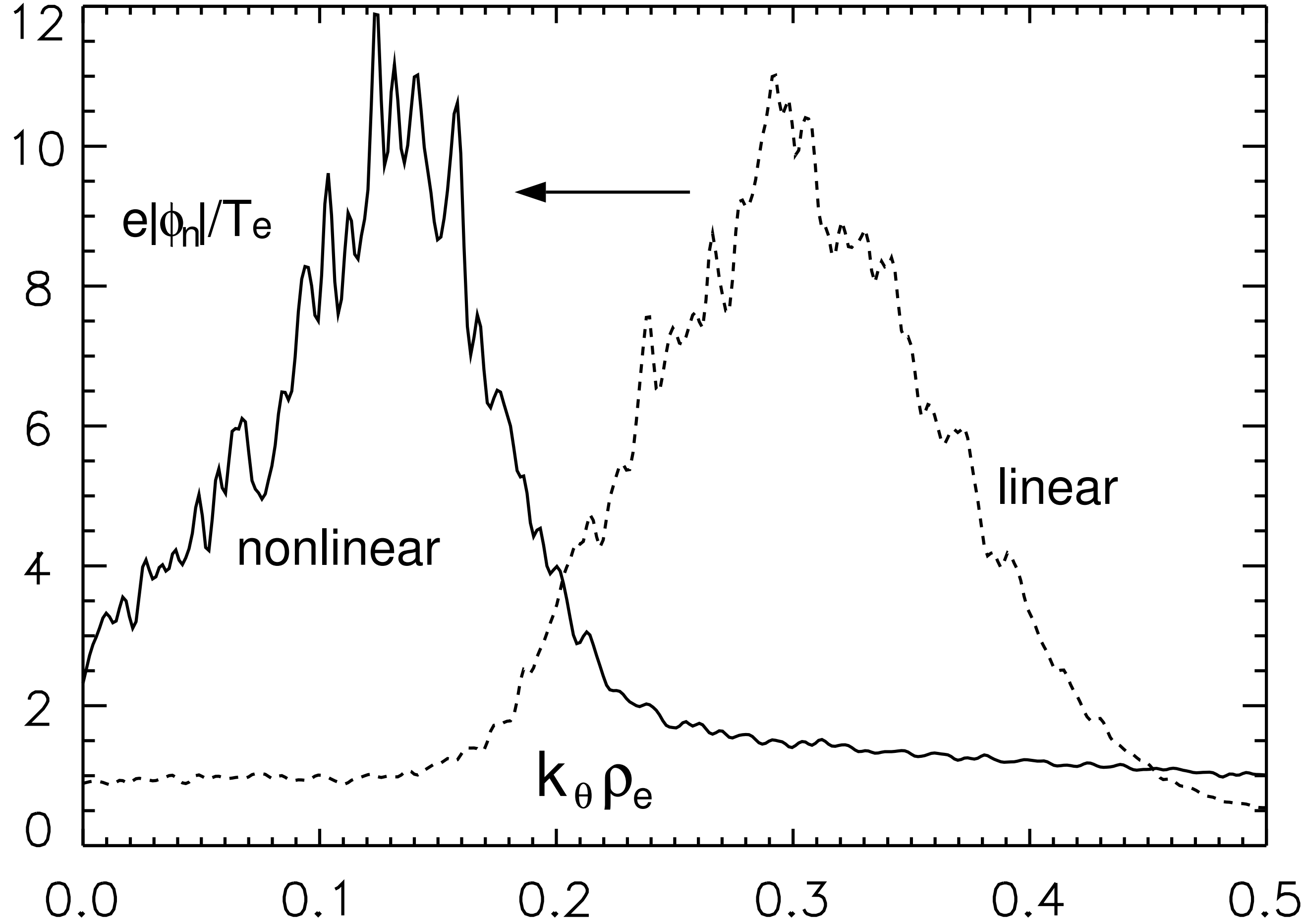
$$\gamma_{NL0,1} = (\left\langle\left\langle k_{\perp 0,1}^2\right\rangle\right\rangle / W_l^2) k_{\theta 0,1}^2 s\alpha_e|\omega_{ce}|(T_i/T_e)\rho_e^4$$

$$\gamma_{NLl} = (2n_l/n_0)k_\theta^4 s\alpha_e|\omega_{ce}|(T_i/T_e)\rho_e^4 \qquad \theta_\ell \equiv \theta + 2\pi\ell$$

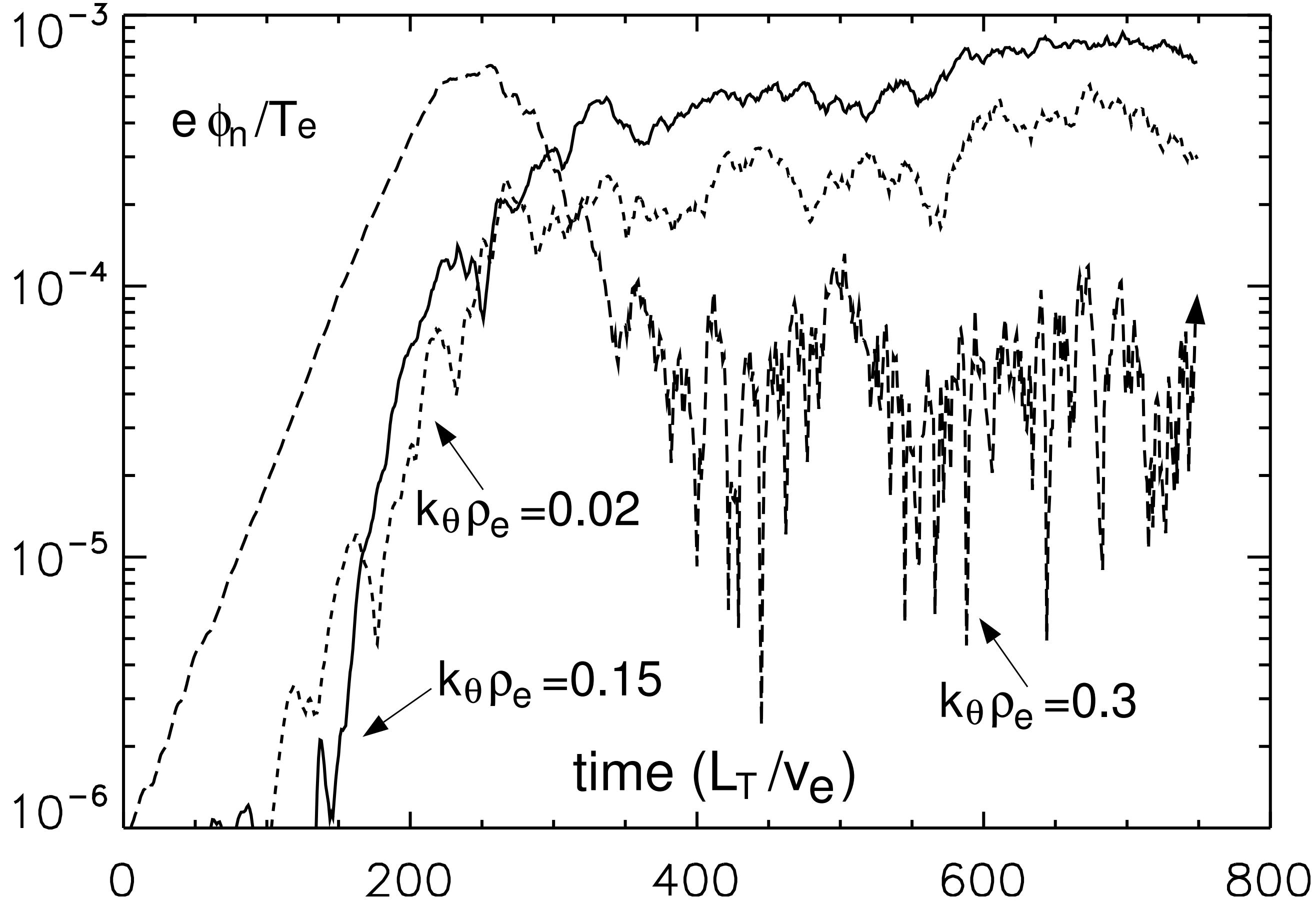
$$\frac{\langle\langle k_\perp^2\rangle\rangle}{k_\theta^2 W_l^2} = 4\pi^2\sum_\ell \ell^2 e^{2\pi i\ell nq}\int_{-\infty}^{\infty}\left[4\pi^2\ell^2 - \left(1 + s^2\left\langle\left\langle\theta^2\right\rangle\right\rangle\right)\right]\left[1 + s^2\theta_\ell^2\right]\Phi^*(\theta)\Phi(\theta_\ell)d\theta \ .$$

- From these equations, the spectral transfer is toward longer poloidal wavelengths.

□ Spectral transfer toward longer poloidal wavelengths... (Z. Lin *et al.* PoP05, PRL05)



□ ...is non-local in *k*-space. (Z. Lin *et al.* PoP05, PRL05)



- Since $n_0 \sim n_1 \gg n_l$, we can analyze the multiple n case in the continuum limit. Introducing $I_n = |a_n|^2/2$ and $v_n = -\gamma_{NL} n_l |a_l|$

$$(\partial_t - 2\gamma_{Ln}) I_n + v_n \partial_n I_n = 0 \ , \qquad (\partial_t + \gamma_l)|a_l| = 4\alpha_e |\omega_{ce}| (T_i/T_e) \rho_e^4 q' \int k_{\theta n}^3 I_n dn \ ,$$

- Here, γ_l is the damping rate of the forced n_l-mode via $k_\parallel v_\parallel$ Landau damping.
- Since $v_n < 0$, ETG energy is gradually transferred to longer poloidal wavelengths via scattering off the low-n quasi modes, till saturation takes place due to enhanced damping and/or decreased drive.
- Turbulent transport level is smaller than values of experimental relevance, as demonstrated in global gyrokinetic simulations (Z. Lin *et al.* PoP05, PRL05).

- Low-n quasi modes have a crucial role as mediators of the nonlocal spectral energy transfer: necessary to properly treat the dynamics of these low mode numbers, which are characterized by highly localized radial structures and are very extended along the field lines, $k_{\parallel} \sim 1/(n_0^{1/2} q R_0)$.

- Underestimating the quasi mode amplitude or occupation number implies underestimating v_n, resulting in a larger ETG saturation level and turbulent transport. This point could help resolving the discrepancy between flux tube and global gyrokinetic particle simulation. (F.Jenko *et al.* PoP00; W. Dorland *et al.* PRL00; B. Labit *et al.* PoP03; J. Li *et al.* PoP04; Z. Lin *et al.* PoP05, PRL05)

Conclusions

- ITG-ETG symmetry is nonlinearly broken due to the different response of respectively electrons and ions to Zonal Flows (ZF).
- ITG is dominated by ITG-ZF nonlinear interactions and turbulence spreading, resulting in size-scaling of the associated turbulent transport.
- Nonlinear toroidal mode coupling dominates ETG saturation and ETG-ZF interactions enter on the longest nonlinear time scale only. The ETG turbulent transport level is smaller than values of experimental relevance.