The case of the trapped singularities

or: A unified dynamical model for plasma confinement transitions

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Very simple research on a very complex system

- A case study in bifurcation and stability analysis
  - — unifies two different views of the physics of **plasma confinement transitions**
  - — provides new intelligence on the big issues of **shear flow suppression of turbulence** and **oscillatory régimes**
  - — suggests new design, control, and optimization strategies for experiments.

Ball, R. 2005 Preprint,
http://wwwrsphysse.anu.edu.au/∼rxb105/rb.html

What are confinement transitions?

Occur in fusion plasma containment systems such as tokamaks and stellarators

✘

highly turbulent

anomalous transport to edge and walls

degradation of coherent structures

hopelessly disruptive

poor energy and particle confinement

L−regime

✔

quiescent

reduced transport

development of stable shear and zonal flows

well −behaved

good confinement

H−regime

Why are they so?

- Confi nement (L–H) transitions have been the subject of intensive experimental, *in numero,* and theoretical and modelling investigations since the 1980s.
- Two major strands in the literature:
  1. A quasi two-dimensional flow phenomenon
     - —occur spontaneously when energy flux from small-scale turbulence to large-scale coherent structures exceeds the nonlinear dissipation rate;
  2. Radial electric fi eld bifurcation
     - —ion orbit losses near the plasma edge or induced biasing cause an electric fi eld, which drives large-scale shear flows nonlinearly.

The most promising approach

to predictive modelling of confinement transitions uses **low-order** or **reduced** dynamical descriptions —

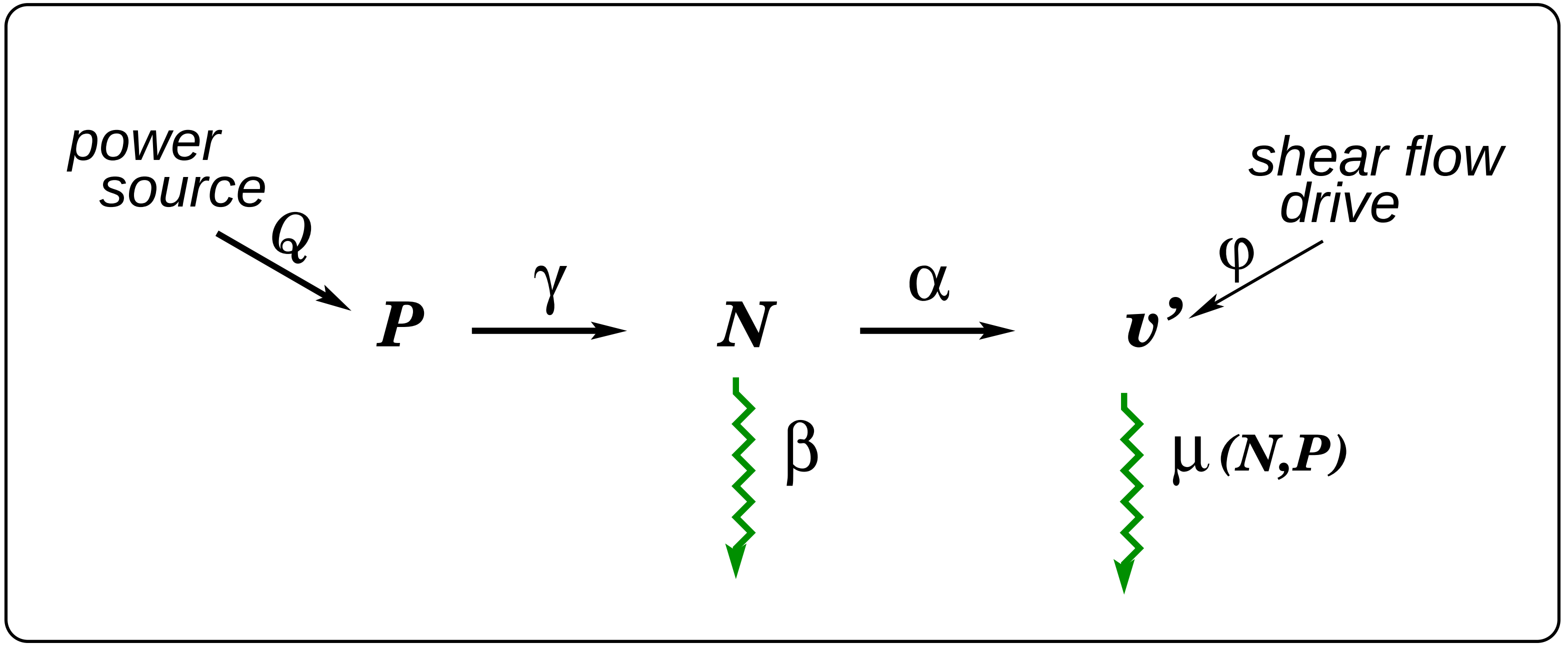
- systems of coupled ODEs in a few ($\sim$2–5) dynamical variables or mode coefficients and parameters.
- This type of modelling averages over spatial or mode spectrum structure, single-particle dynamics, etc
  - —but we can track qualitative features of the collective dynamics, such as bifurcations and stability changes, broadly over the parameter space.
- Motivated by the need for improved control of the (mostly bad) behaviour of fusion plasmas in magnetic containers. **New applications in industry**

Charney, J. and DeVore, J. 1979 *J. Atm. Sci.* 36, 1205. Itoh, S.-I. and Itoh, K. 1988 *Phys. Rev. Lett.* **60**, 2276. Shaing, K. and Crume, E. J. 1989 *Plasma Phys. Control. Fusion* **41**, 1357. Hinton, F. L. 1991 *Phys. Fluids B* **3**, 696. Dnestrovskij, A. Y. et al. 1992 *Plasma Phys. Control. Nucl. Fus. Res.* **2**, 371. *Phys. Rev. Lett.* **72**, 2565. Carreras, B. et al. 1994 *Plasma Phys. Control. Fusion* **36**, A93. Pogutse, O. et al. 1994 *Plasma Phys. Control. Fusion* **36**, 1963. Vojtsekhovich et al. 1995 *Nuclear fusion* **35**, 631–640. Sugama, H. and Horton, W. 1995 *Plasma Phys. Control. Fusion* **37**, 345. Lebedev, V. B. et al. 1995 *Phys. Plasmas* **2**, 3345. Haas, F. A. and Thyagaraja, A. 1995 *Plasma Phys. Control. Fusion* **37**, 415. Drake, J. F. et al. 1996 *Phys. Rev. Lett.* **77**, 494. Beyer, P. and Spatschek, K. H. 1996 *Phys. Plasmas* **3**, 995. Hu, G. and Horton, W. 1997 *Phys. Plasmas* **4**, 3262. Takayama, A. et al. 1998 *Plasma Phys. Control. Fusion* **40**, 775. Peeters, A. G. 1998 *Physics of Plasmas* **5**, 2399. Kardaun, O. J. W. F. et al. 1998 *ECA* **22C**, 1975–1978. Staebler, G. M. 1999 *Nuclear Fusion* **39**, 815–820. Thyagaraja, A. et al. 1999 *Physics of Plasmas* **6**, 2380. Ödblom, A. et al. 1999 *Physics of Plasmas* **6**, 3521. Ball, R. and Dewar, R. L. 2000 *Phys. Rev. Lett.* **84**, 3077. Ball, R., Dewar, R. L. and Sugama, H. 2002 *Phys. Rev. E* **66**, 066408-1. Del-Castillo-Negrete, D. and Carreras, B. A. 2002 *Physics of Plasmas* **9** 118. Franck, C. M., Grulke, O. and Klinger, T. 2003 *Physics of Plasmas* **10**, 323. Kim, E-J., Diamond, P. H. and Hahm, T. S. 2004 *Phys. Plasmas* **11**, 4554. Ball, R. 2005 Preprint, http://wwwrsphysse.anu.edu.au/∼rxb105/rb.html; Submitted to *Phys. Rev. Lett.*

Method

1. Find and interrogate trapped degenerate singularities that occur in the simplest dynamical model for confinement transitions;
2. Unfold the singularities **smoothly** in physically meaningful ways;
3. Interrogate any new singularities that appear;
4. Repeat steps 2 and 3 until the model is **free of pathological or persistent degenerate singularities, is self-consistent, reflects observations in experiments, and is therefore predictive.**

Three energy subsystems



$\boldsymbol{P}$	potential energy of the pressure gradient
$\boldsymbol{N}$	kinetic energy of the turbulence
$\boldsymbol{F} \equiv \pm v'^2$	shear fbw kinetic energy
v'	averaged background shear or zonal fbw velocity

Energy flux diagram → dynamical system

$$\varepsilon \frac{dP}{dt} = Q - \gamma N P$$

$$\frac{dN}{dt} = \gamma N P - \alpha v'^2 N - \beta N^2$$

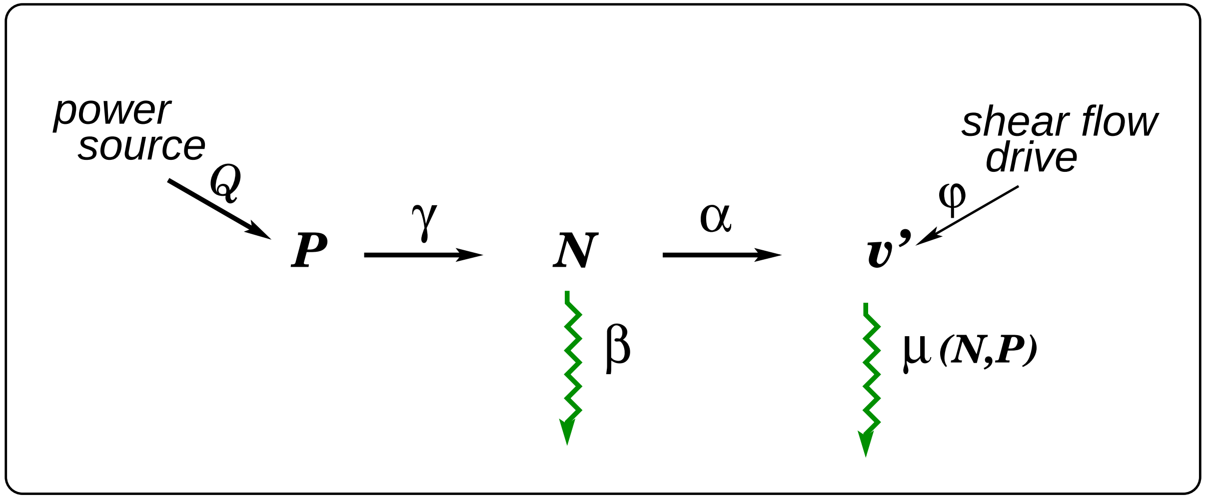
$$2 \frac{dv'}{dt} = \alpha v' N - \mu v' + \varphi$$

$F \equiv \pm v'^2$

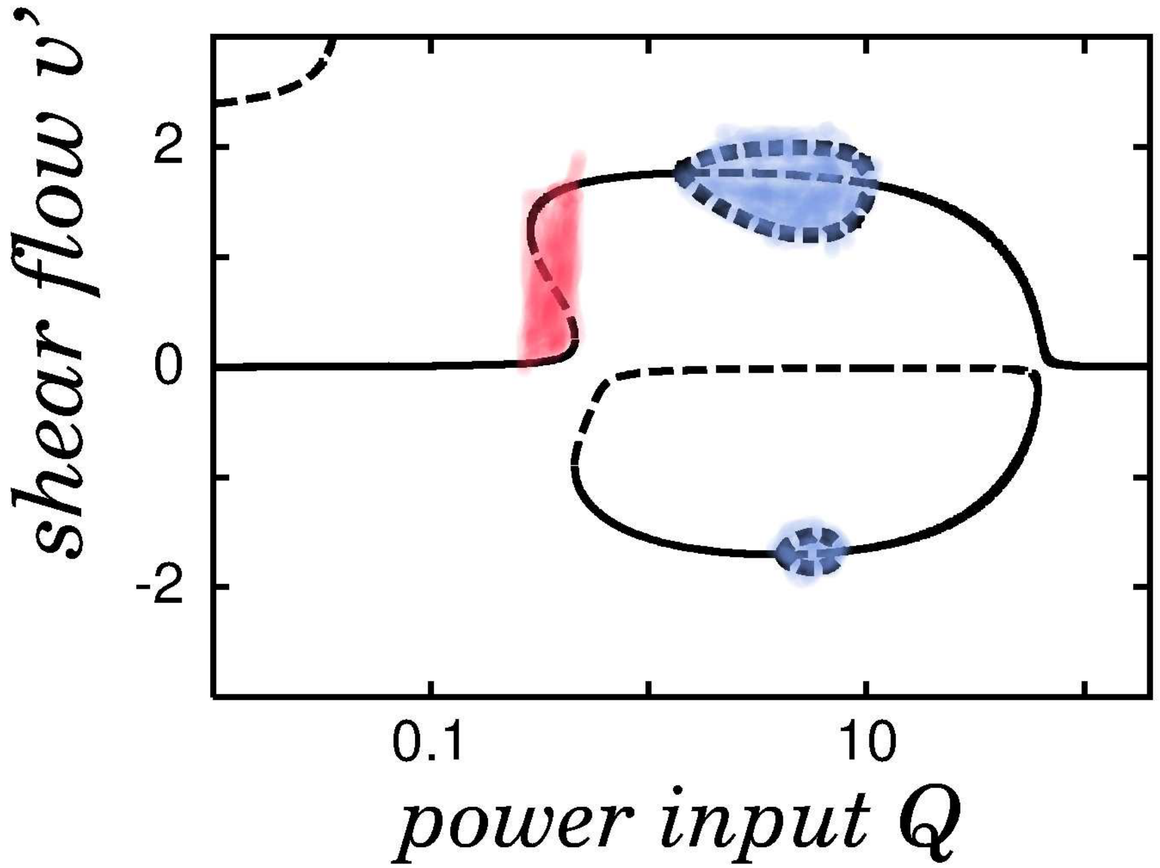
$\mu = \mu(P,N)$

$= \mu_{ne} P^{-3/2} + \mu_{an} P N$

Ball, Dewar & Sugama, Physical Review E 66, 066408, 2002

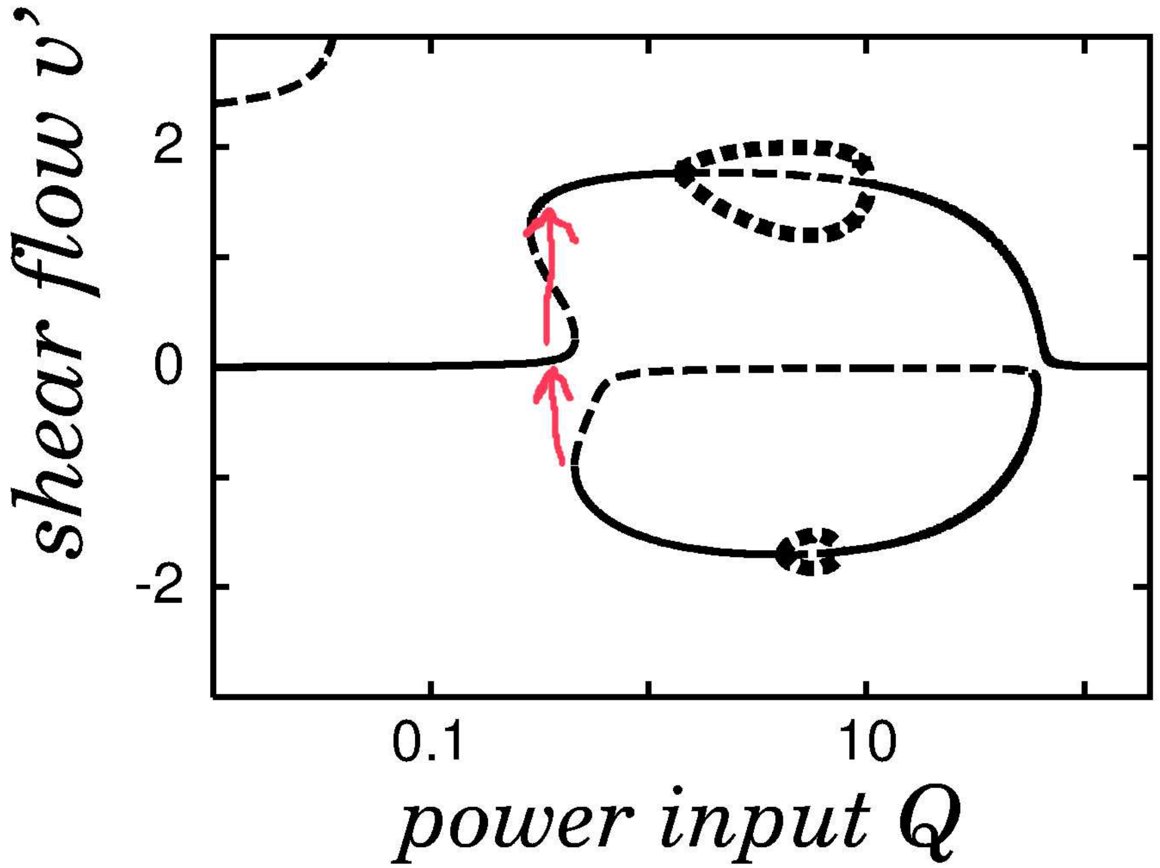


A walk along untrodden ways



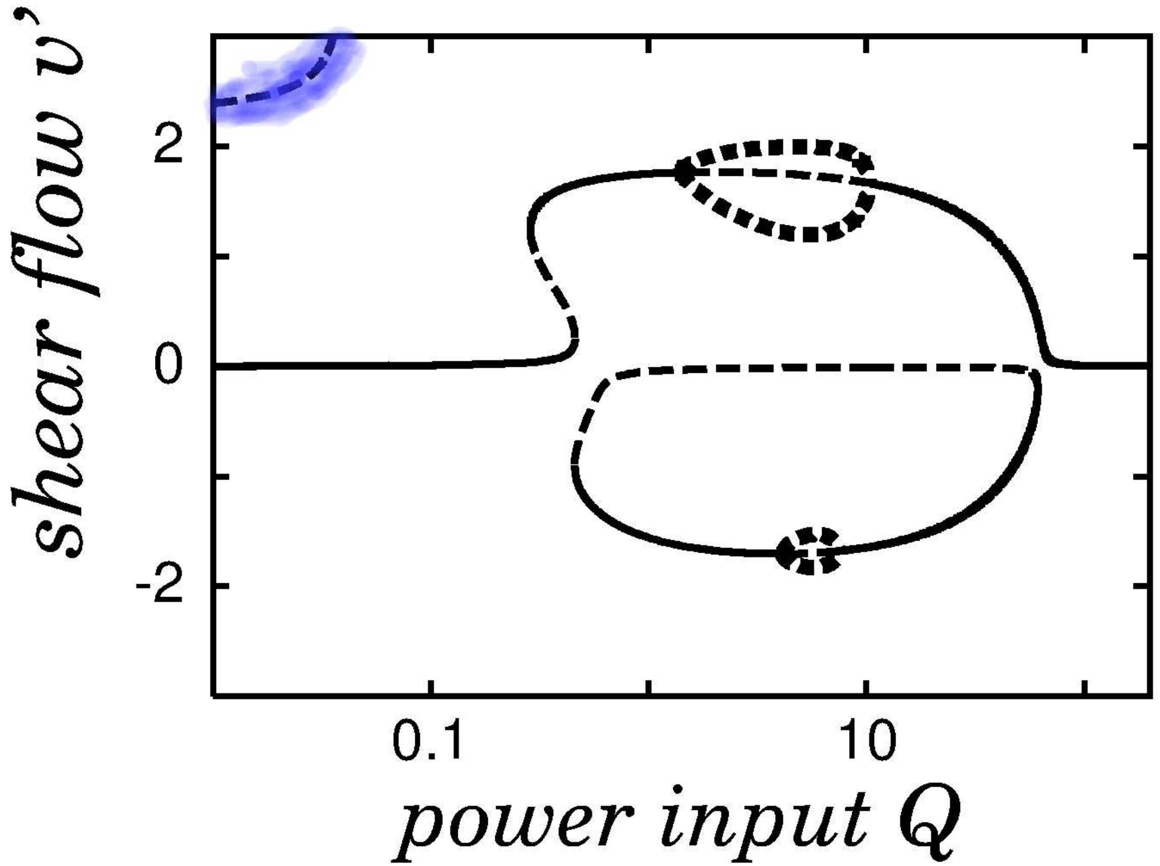
☛ Hysteresis and limit cycles can occur.

A walk along untrodden ways



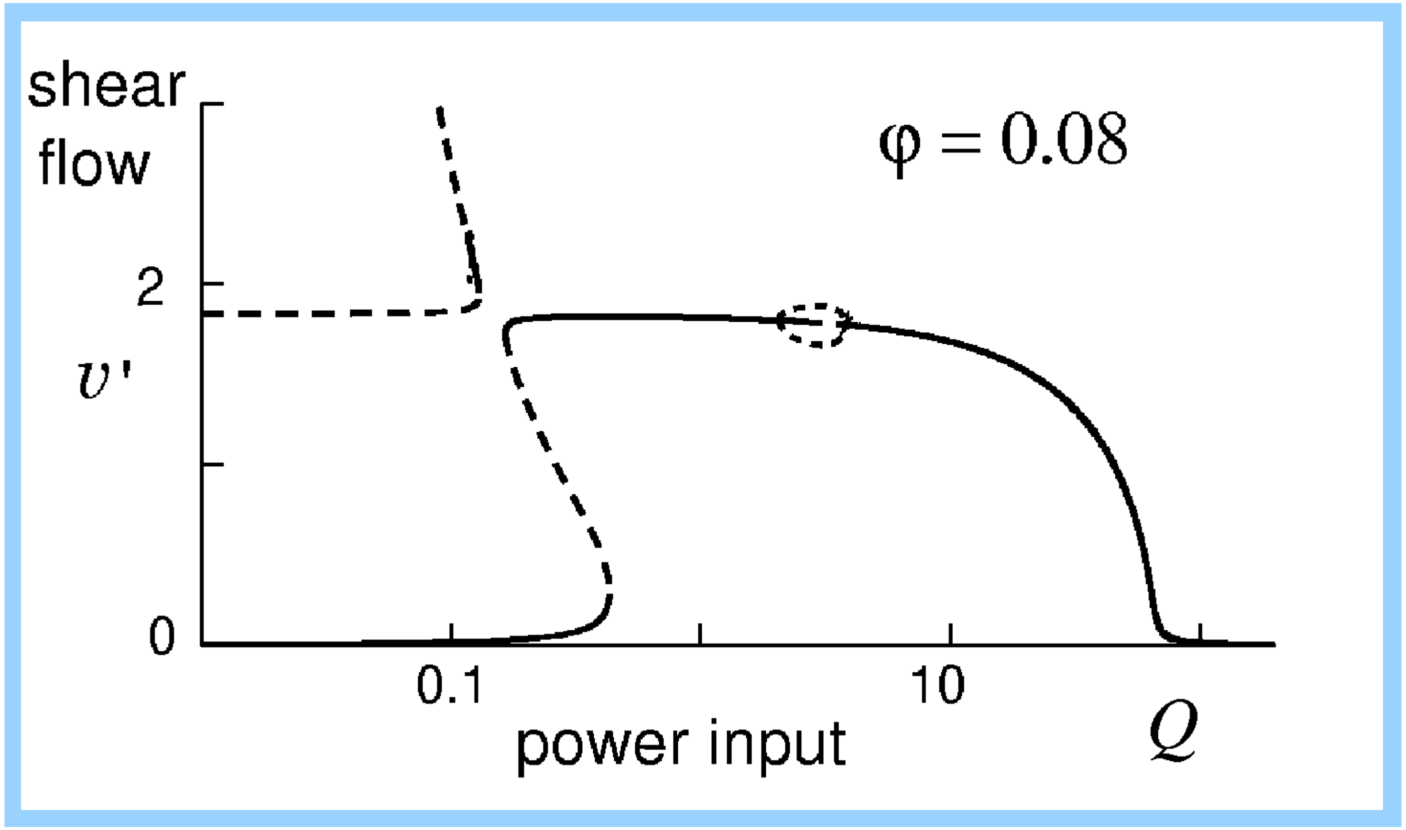
☛ The fbw can spontaneously reverse direction.

A walk along untrodden ways



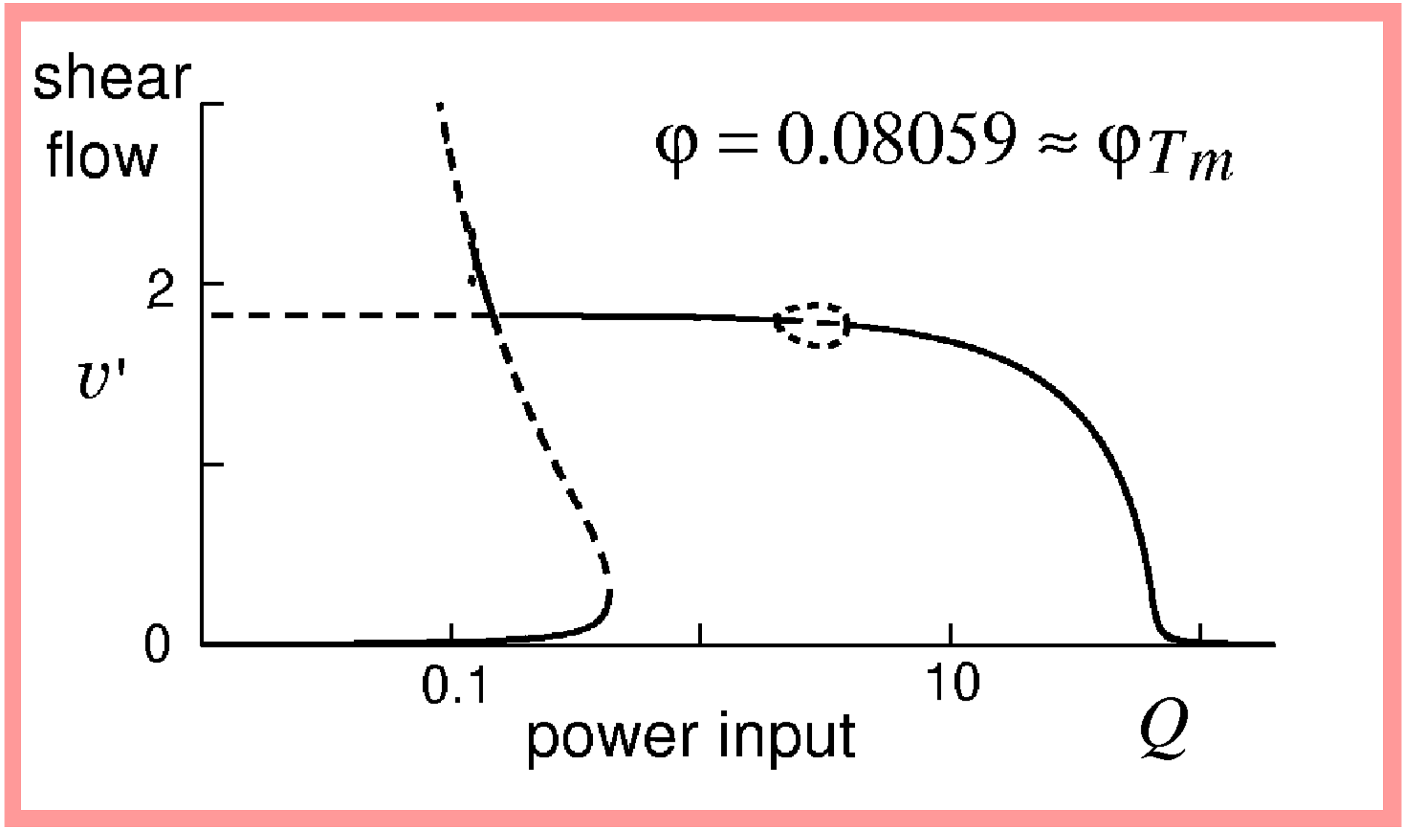
☛ Symmetry-breaking has global as well as local effects. **For $\varphi \neq 0$ a branch of solutions is released from a trap at infinity.**

The plot thickens



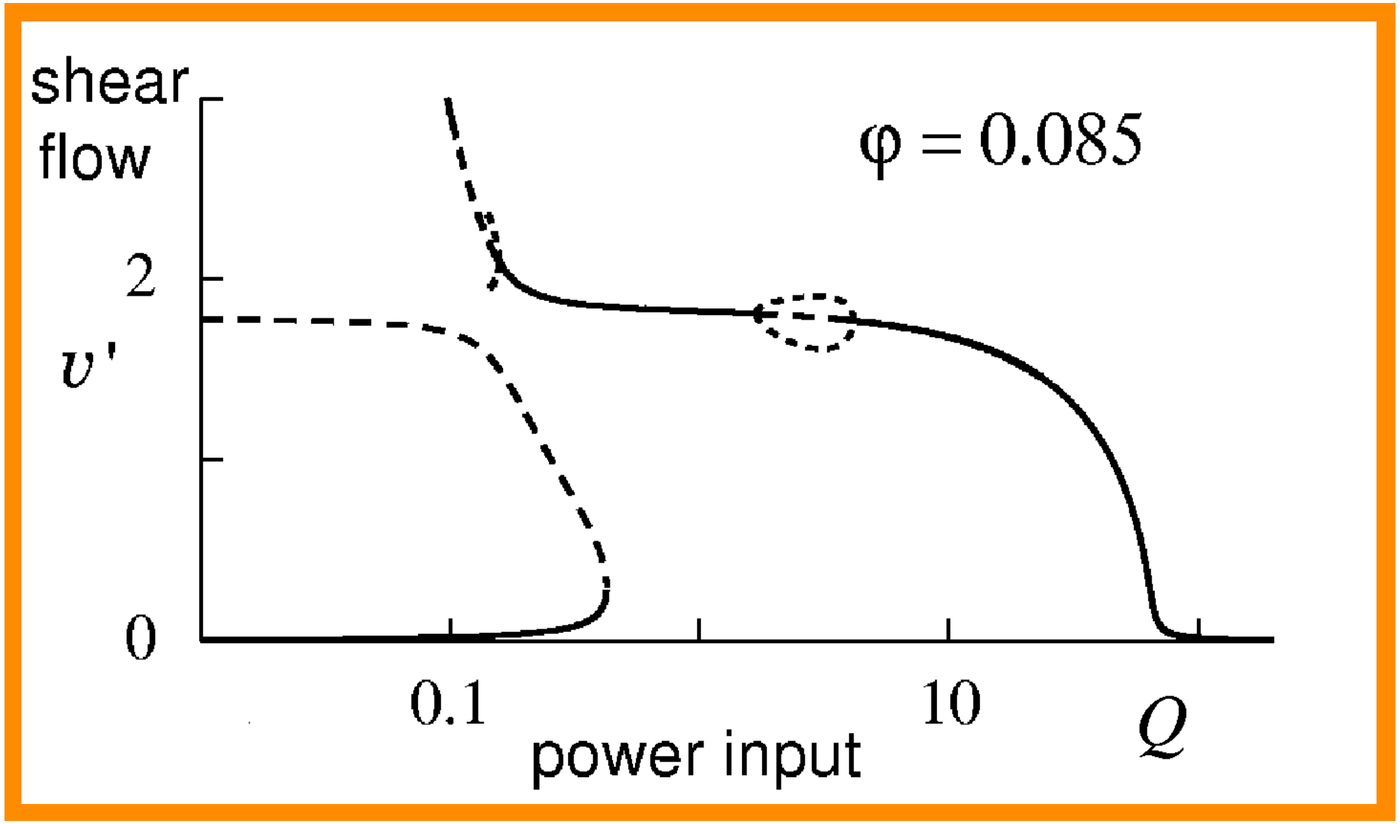
As you increment the driving rate φ the “new” branch develops a branch of limit cycles . . .

The plot thickens



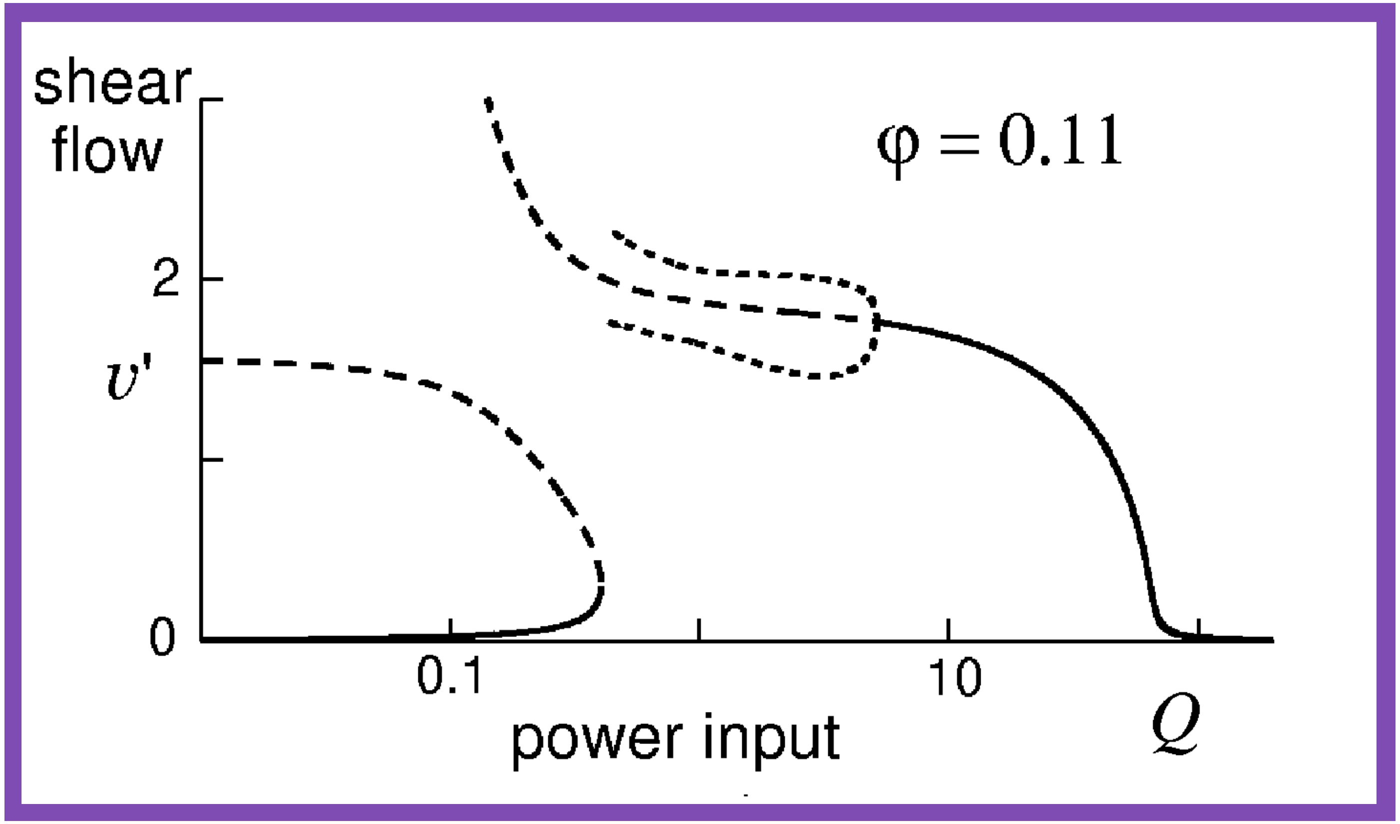
At a critical value of φ the new and old branches exchange at a **non-symmetric** transcritical bifurcation.

The plot thickens



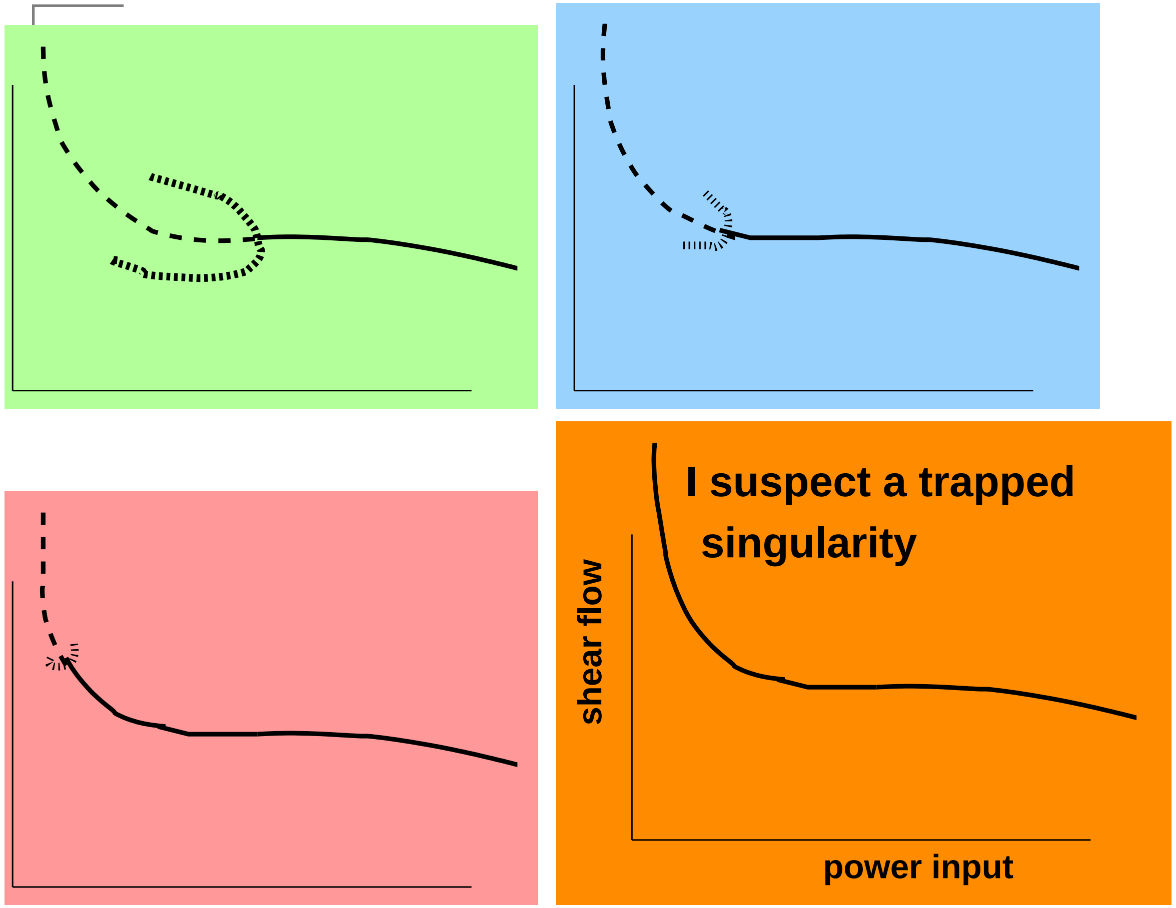
There has been a complete metamorphosis of the dynamics!

The plot thickens



Two Hopf bifurcations annihilate each other at a DZE . . .

The plot thickens



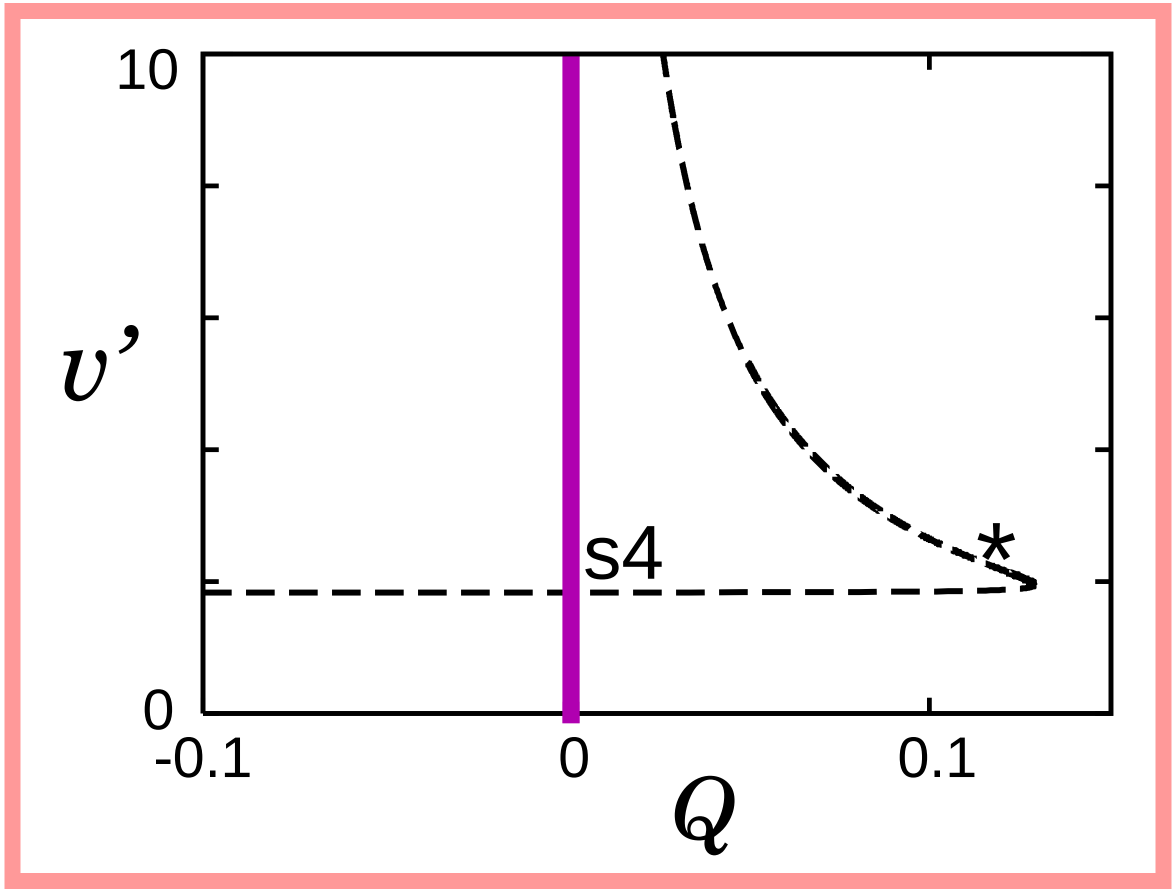
. . . and the remaining Hopf bifurcation is captured by a DZE at $(Q, v') = (0, \infty)$

—which implies **infinite growth of shear flow as the power input *falls!***

Some important physics is still missing from the model.

Find the trap and release the singularity

On a suspiciously degenerate branch of equilibria at $Q = 0$ a trapped degenerate turning point, s4, is found ...



Physics: shear flows feed turbulence as well as suppress it

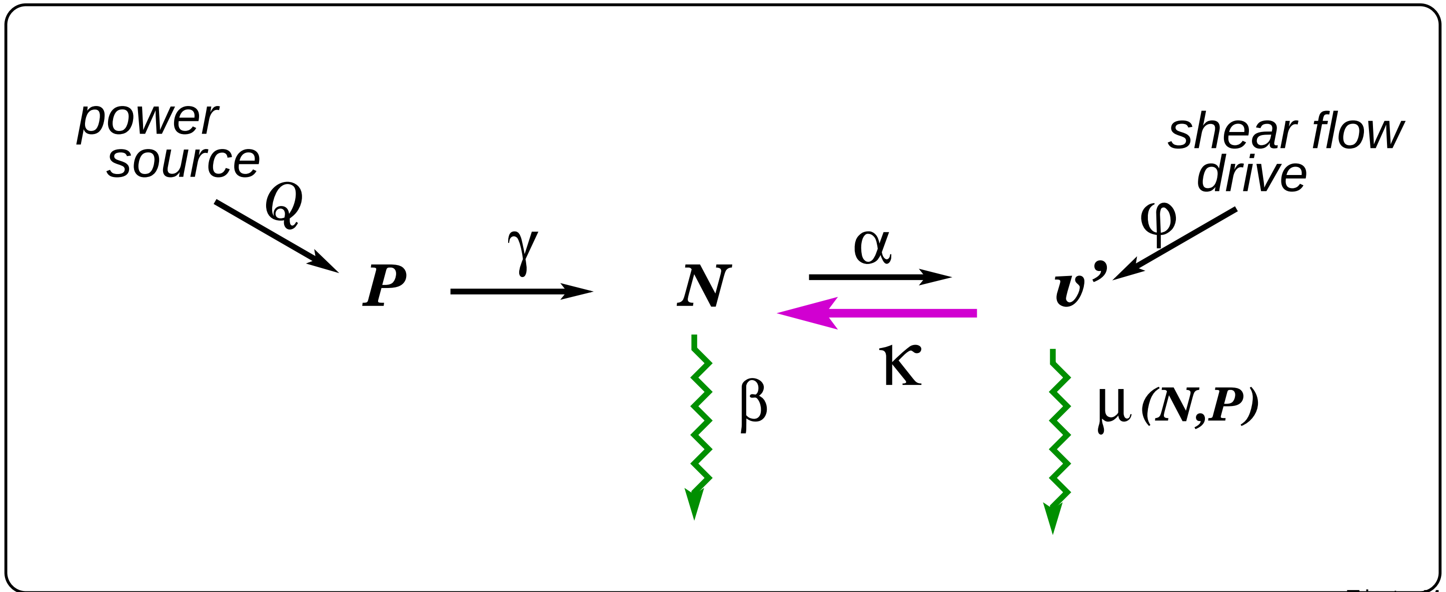
- In a strongly two-dimensional velocity fi eld there is strong tendency to upscale energy transfer, or **inverse energy cascade**, but the net rate of energy transfer to high wavenumbers, or **Kolmogorov cascade**, is not negligible —
  - **kinetic energy in large-scale structures inevitably feeds the growth of turbulence at smaller scales, as well as vice versa.**
- What amounts to an ultraviolet catastrophe in the physics **maps to a trapped degenerate singularity** in the mathematical structure of the model when when energy flux to high wavenumbers is neglected.

The trapped singularity s4 is unfolded smoothly

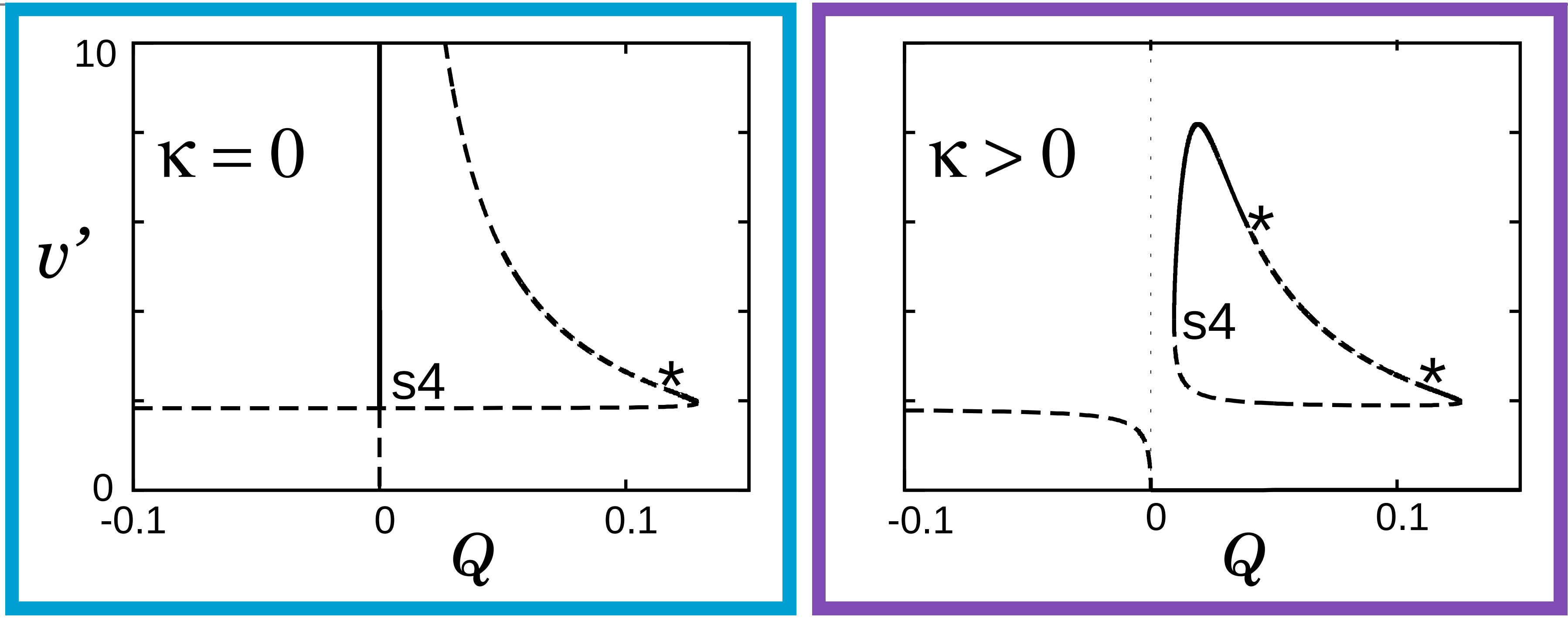
by including a simple, conservative, back-transfer rate between the shear fbw and turbulence subsystems:

$$\frac{dN}{dt} = \gamma NP - \alpha v'^2 N - \beta N^2 + \kappa v'^2$$

$$2\frac{dv'}{dt} = \alpha v' N - \mu(P,N)v' + \varphi - \kappa v'.$$

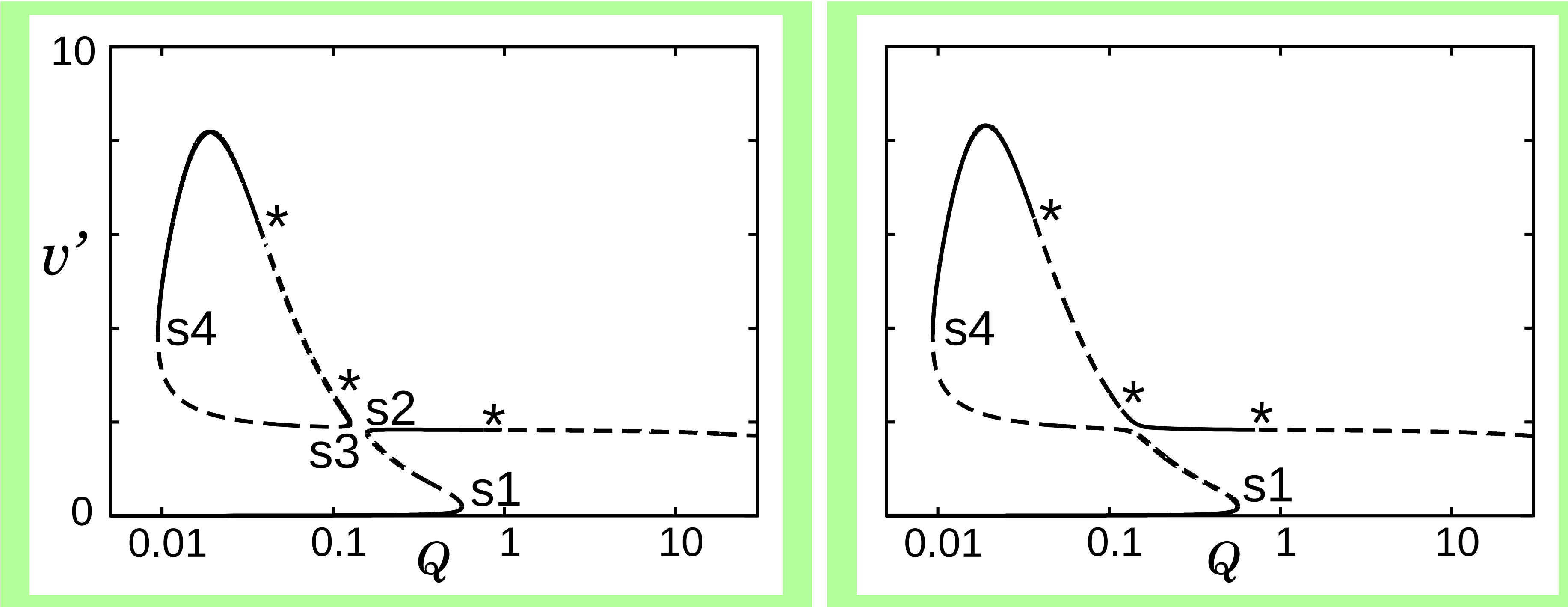


A salutary lesson: unphysical solutions should not be ignored!



- The unfolding creates a **maximum** in the shear fbw
- A **fourth** h.b. is released from a trap at infi nity
- An **isola** of steady-state solutions is formed —but the bifurcation diagram is a slice of a three-dimensional surface of equilibria.

Two slices of the bifurcation surface

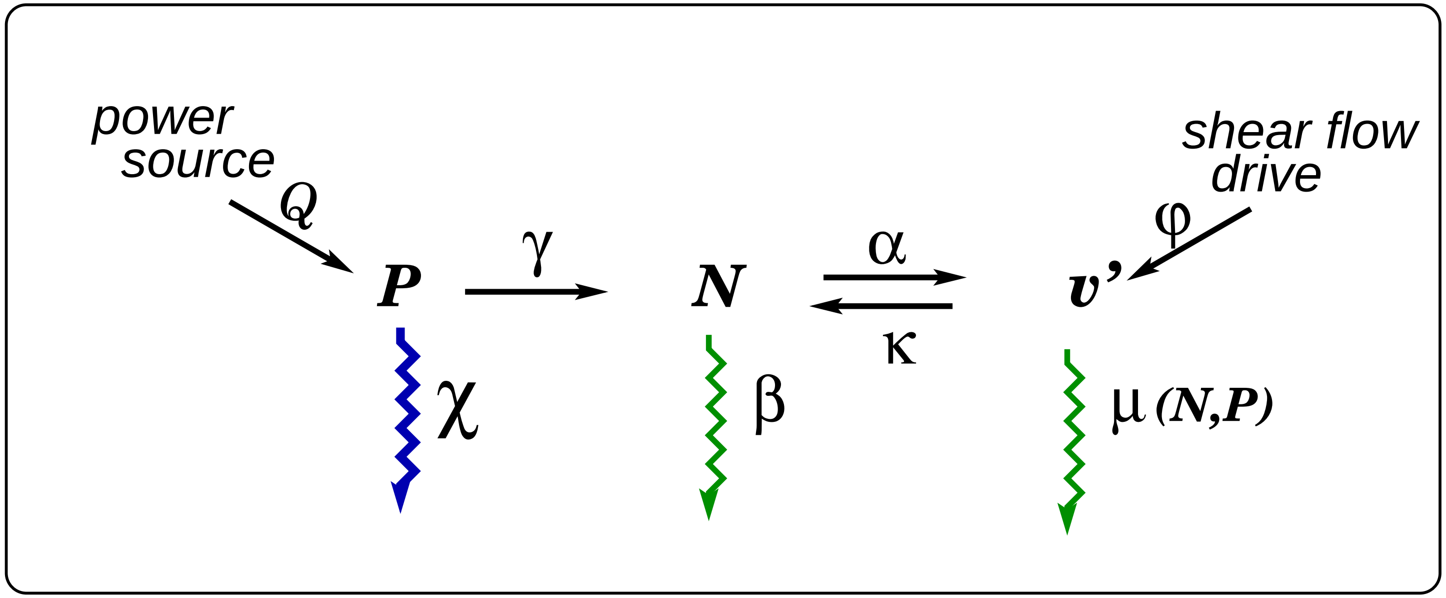


Predictions:

- ✔ Low, intermediate, and high shear flow states.
- ✔ **Two** possible back-transitions.
- ✔ **The shear flow can actually grow as the power input is withdrawn**.

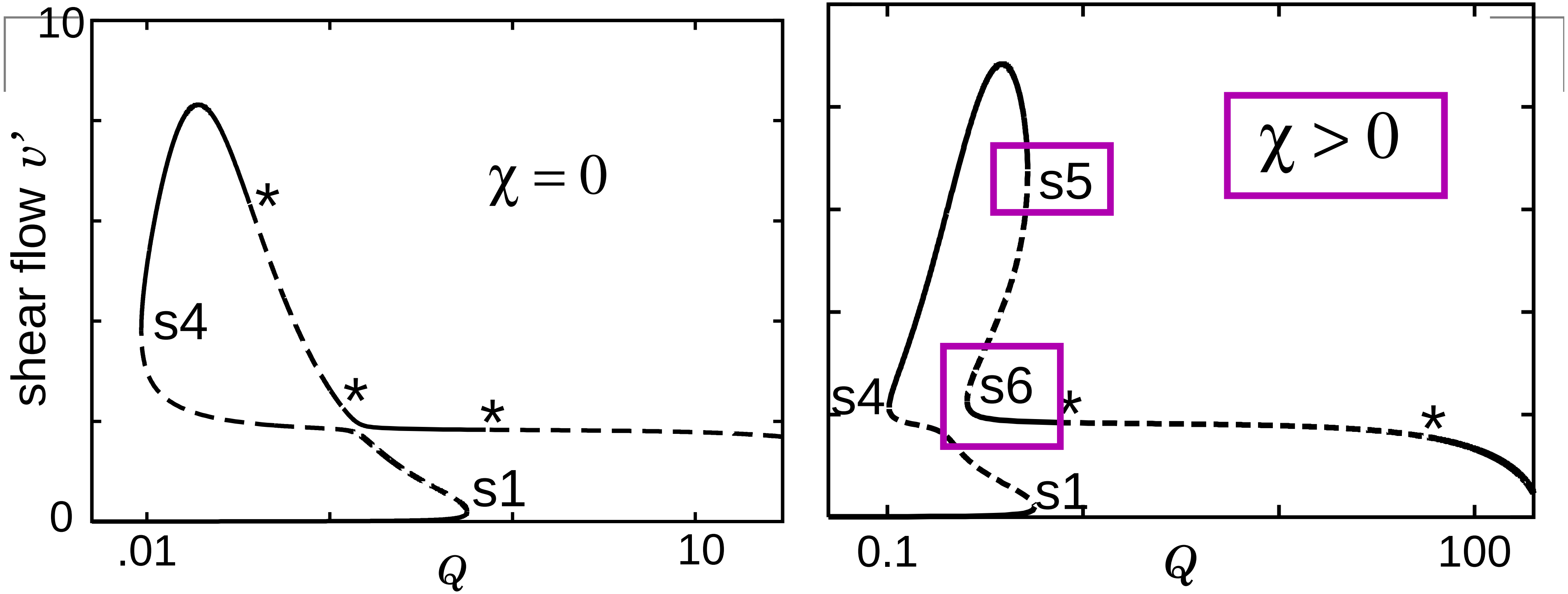
Potential energy dissipation

$$\varepsilon\frac{dP}{dt} = Q - \gamma NP \boxed{-\chi P}$$



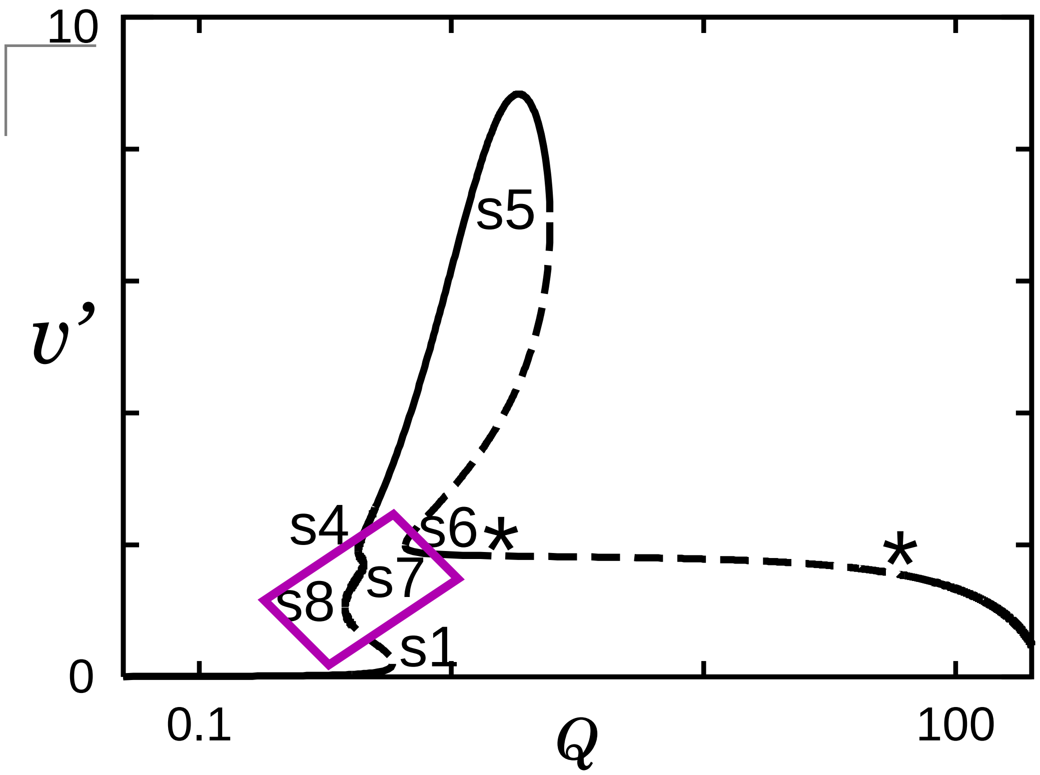
χP represents all non-turbulent or residual losses such as cross-fi eld thermal diffusivity and radiative losses.

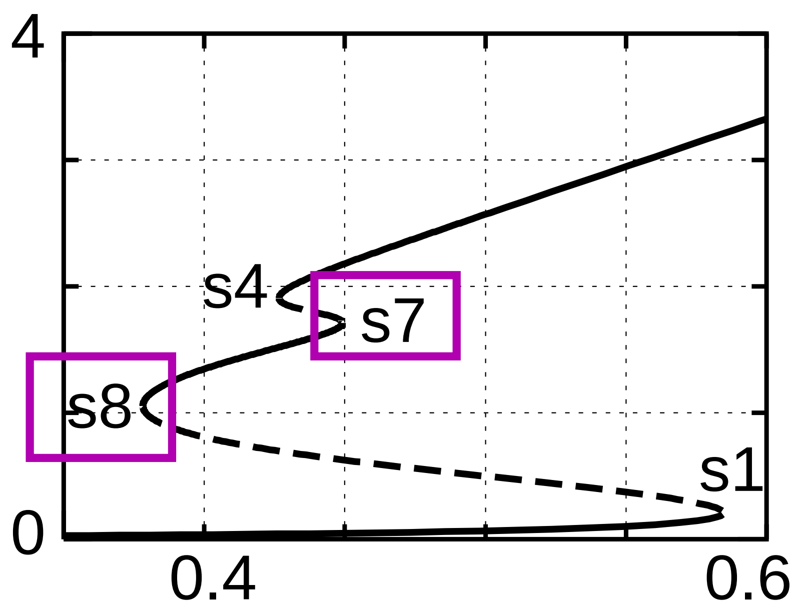
Dramatic changes to bifurcation structure



- Two new turning points s5 and s6 are born from a local cusp singularity.
- Transition to high shear flow state at s6 is now discontinuous!
- System has fivefold multiplicity between s5 and s6.

Dramatic changes to bifurcation structure

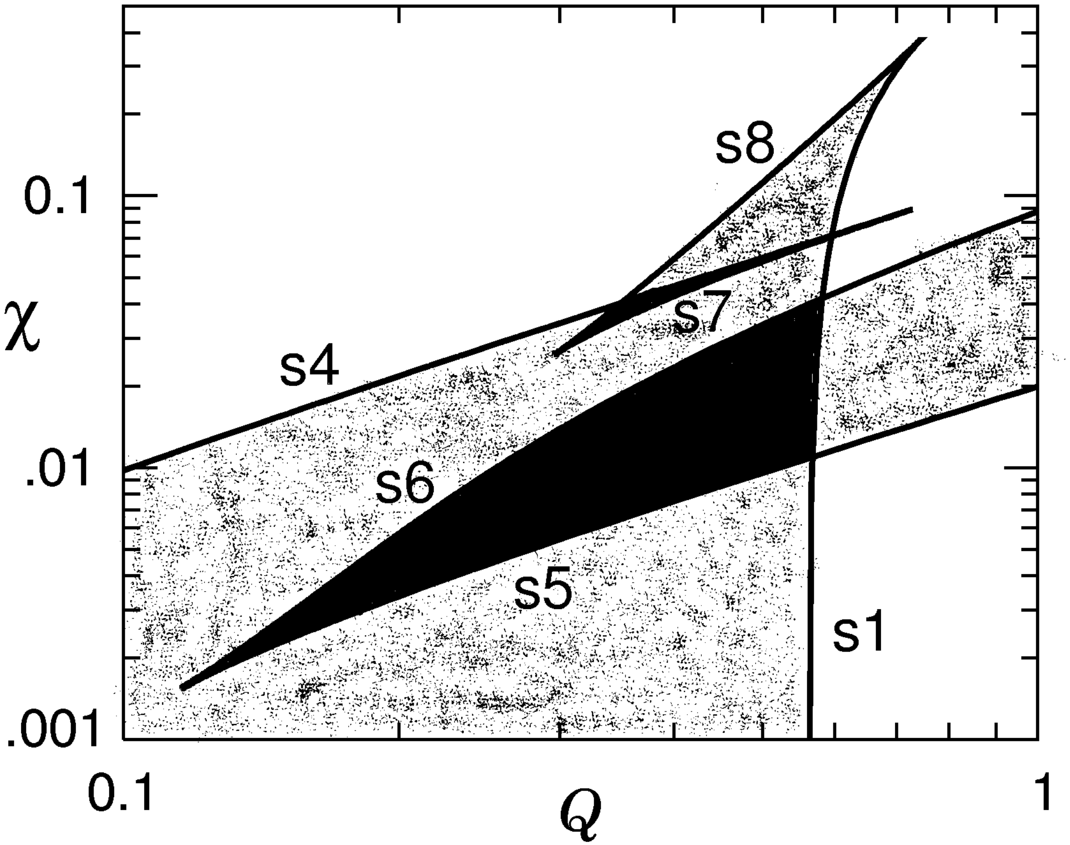




- As you increase χ a different fi vefold domain appears through the creation of s7 and s8 at another local cusp singularity!

An amusing and instructive puzzle

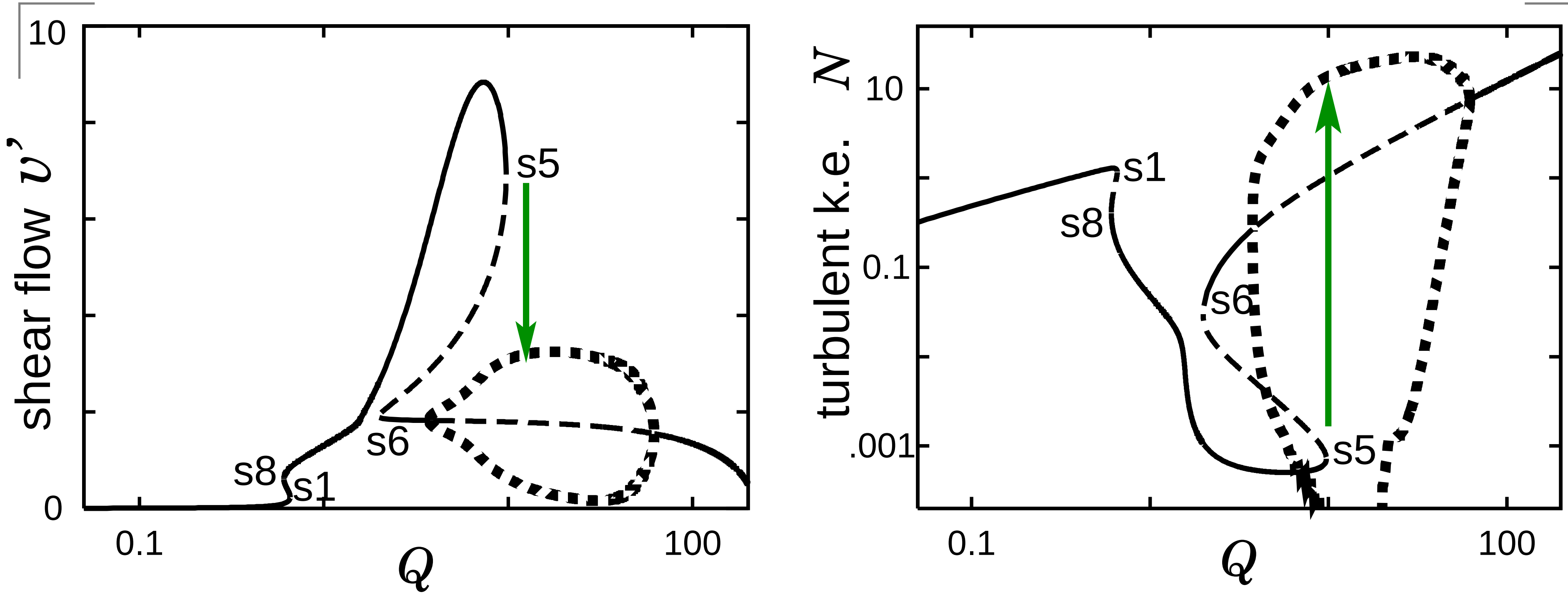
- The turning points s1, s4, s5, s6, s7, and s8 are computed over the dissipation coeffi cent χ and the power input Q.
- The resulting lines of turning points are projected onto χ–Q plane.



Zone	Multiplicity
Black	fi ve
Dusted	three
White	one

Since the two black zones do not over-lap, there is no do-main of sevenfold multiplicity!

The rise and fall of ~~the Roman Empire~~ a turbulent plasma



- The system transits from s5 to a limit cycle, rather than to a stable intermediate steady state.
- The turbulence is enormously suppressed due to uptake of energy by the shear flow, but rises again dramatically with this hard onset of oscillations.

A nonlinear shear flow drive unifi es the model

$$\varepsilon \frac{dP}{dt} = Q - \gamma N P - v'^2 r(P)$$

$$\frac{dN}{dt} = \gamma N P - \alpha v'^2 N - \beta N^2 + \kappa v'^2$$

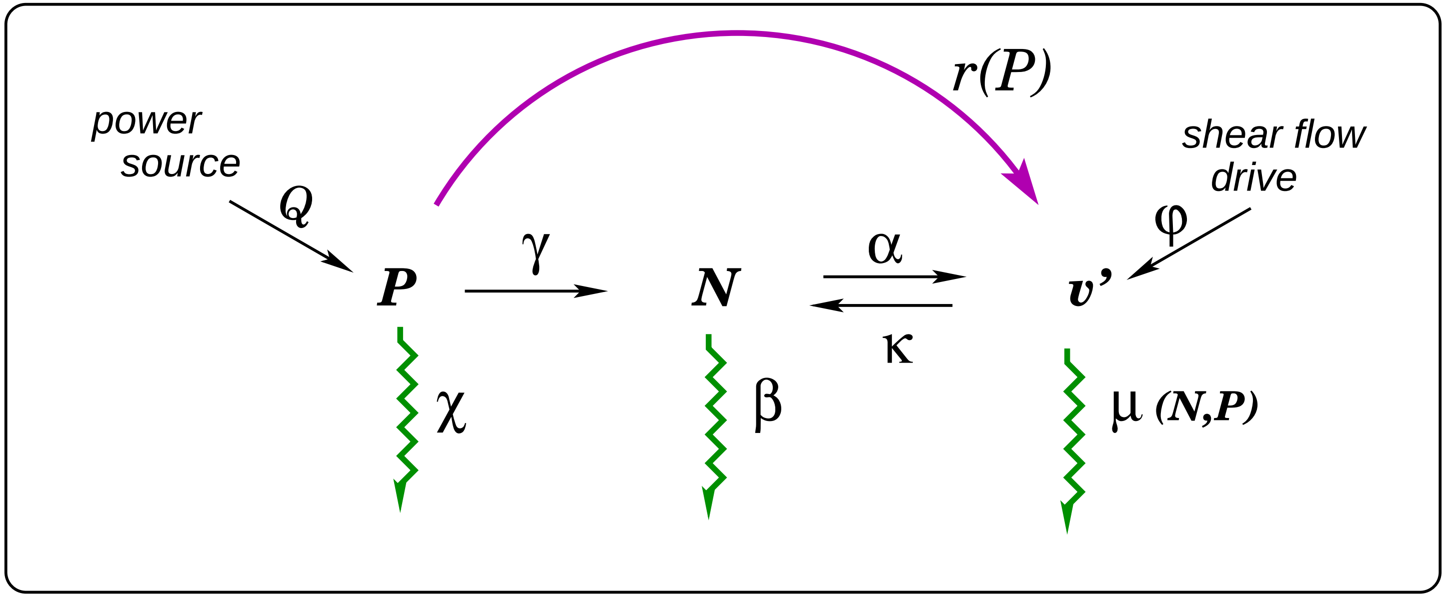
$$2 \frac{dv'}{dt} = \alpha v' N - \mu(P, N) v' + v' r(P) - \kappa v' + \varphi$$

$$r(P) = \nu \exp\left(- \left(w^2 / P \right)^2 \right) \quad \dagger$$

$$\mu(P, N) = \mu_{ne} P^{-3/2} + \mu_{an} P N$$

† Shaing, K.C. and Crume, E.C. *Phys. Rev. Lett.* 63, 2369, 1989; Itoh, S.-I. and Itoh, K. *Phys. Rev. Lett.* 60, 2276, 1988, ++ *100s of similar papers since*

Competitive potential energy distribution channels

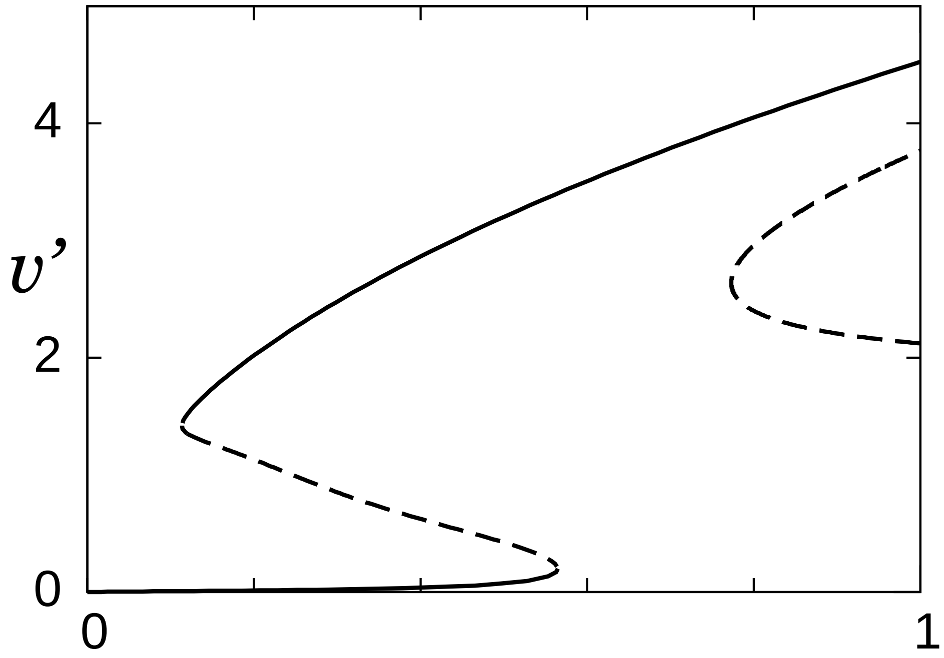


- $r(P) = \nu \exp\left(-\left(w^2/P\right)^2\right)$ is a **competing** potential energy conversion channel —
  - can dominate the dynamics when the critical escape velocity w is low or the pressure is high.
- What effects does it have on the bifurcation structure?

Bifurcation diagrams for the unifi ed model

As $r(P)$ begins to take over:

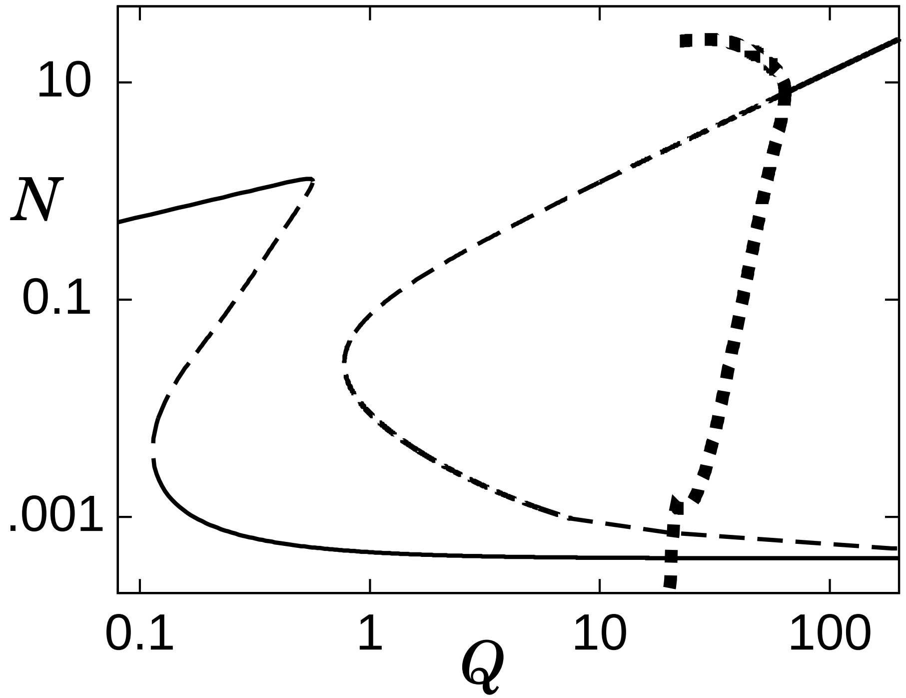
- The high shear fbw peninsula is elongated and fbattened.
- Fivefold régime disappears.
- No practicably accessible intermediate branch in the transition region.



- **Locally, the bifurcation diagram begins to look more like the simple S-shaped, cubic normal form schematics featured in numerous papers by the "electric fi eld bifurcation" school . . .**

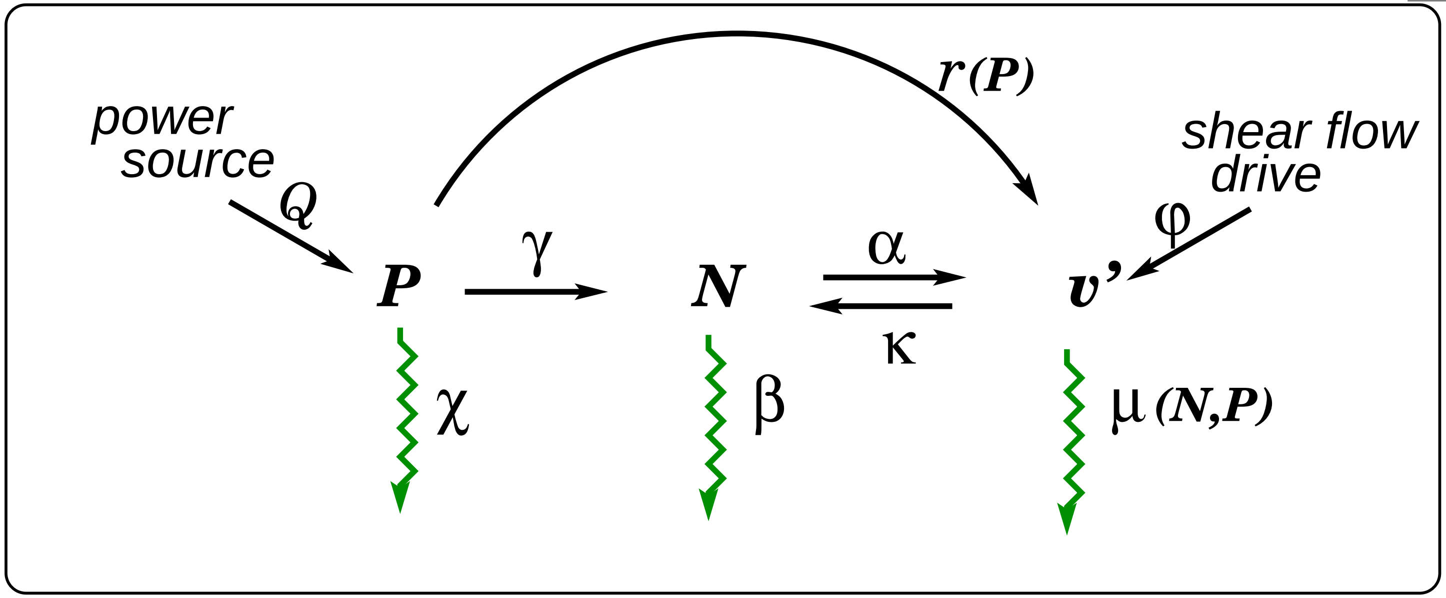
Bifurcation diagrams for the unified model

... but this model accounts for shear flow suppression of the turbulence, whereas their models could not ...

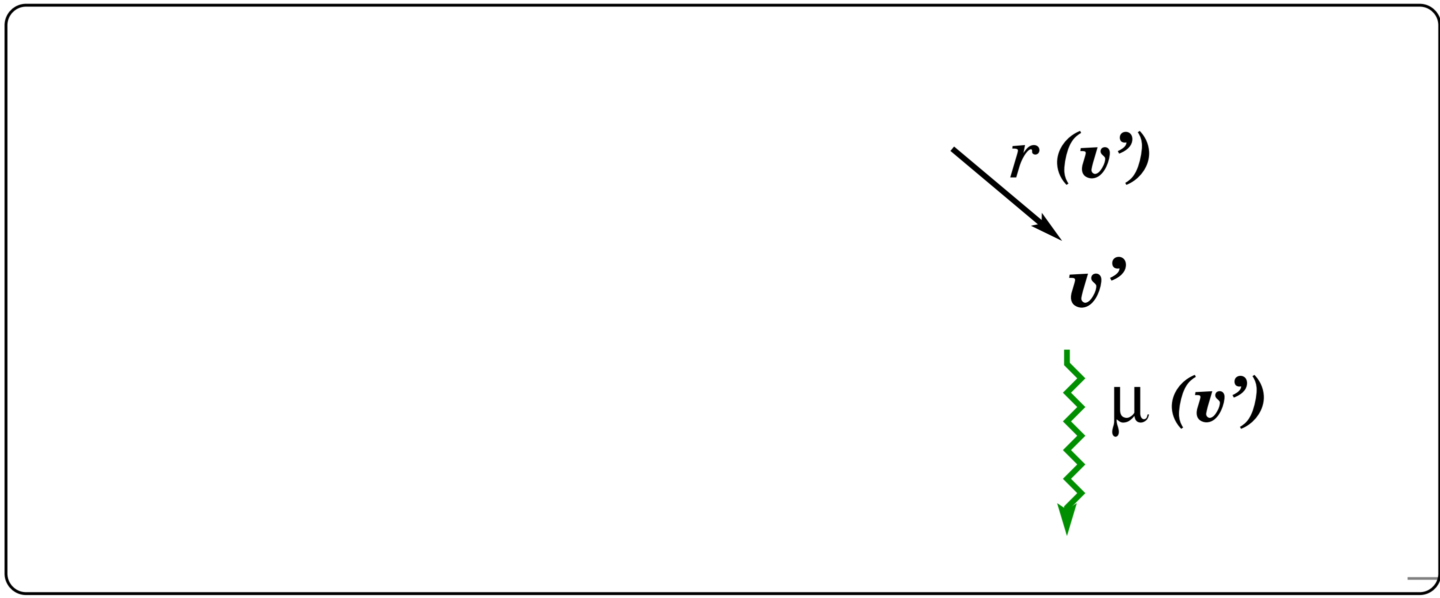


... because they were not coupled to the potential energy and turbulent kinetic energy subsystems.

Ours

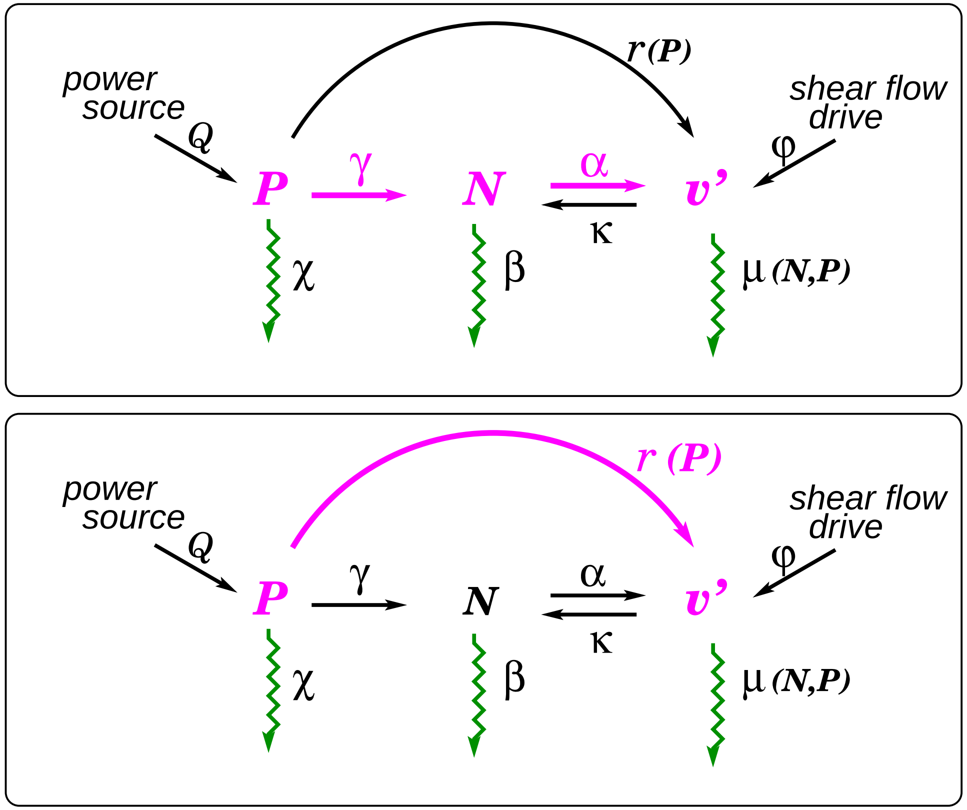


Theirs



Summary

Generation of stable shear flows in fusion plasmas and associated confinement transitions are governed by Reynolds stress decorrelation of turbulence and/or by an induced bistable electric field.



These two mechanisms are seamlessly unified by the first smooth path through the singularity and bifurcation structure of a reduced dynamical model for the system.

Summary

Results in particular:

∗ New strategies for controlling confinement and reducing turbulent transport in new-generation fusion experiments.

Results in general:

∗ Low-dimensional dynamical models have a useful role to play in the study of one of the most formidable of complex systems, a strongly driven turbulent plasma.