Kinetic & Fluid descriptions of interchange turbulence

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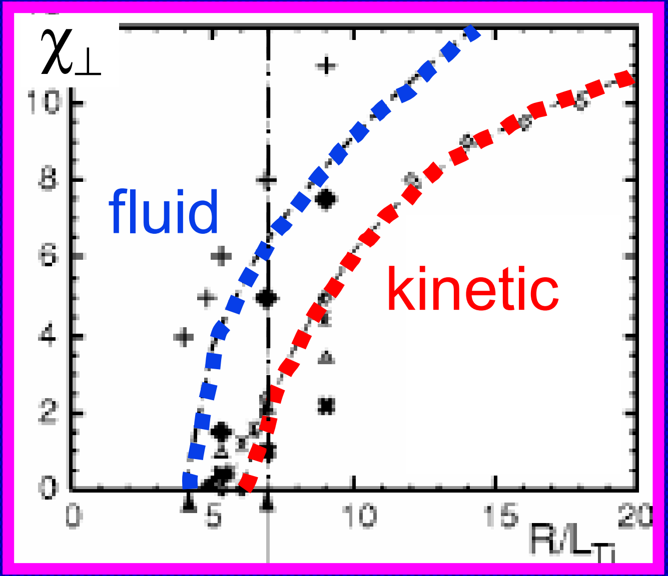
Motivations

- Strong **discrepancies** between **kinetic & fluid descriptions** of turbulence**:**

  Linear thresholds

  Non linear fluxes [Beer '95, Dimits '00]



- **New type of non collisionnal closures:**

  Non local [Hammet-Perkins '90, Snyder-Hammet-Dorland '97, Passot-Sulem '03]

  Non dissipative [Sugama-Watanabe-Horton '01,'04]

Outline of the talk

Standard closure assumes weak departure from local thermodynamical equilibrium (F Maxwellian)

$\Rightarrow$ small number of moments required

Aim:

Compare kinetic & fluid approaches (linear & non-linear)

in a simple turbulence problem:

- Same instability (2D interchange)
- Same numerical tool
- Closure based on entropy production rate

2D+1D interchange instability

Constant curvature drift: $E v_d \vec{e}_y$

Slab geometry (x,y)

Limit $k_\perp \rho_i \rightarrow 0$

$v_{//}=0$ ions $\rightarrow$ $E \approx v_\perp^2$

Adiabatic electrons

Hamiltonian: $H = v_d E x + \phi$

$$\partial_t f + [\phi, f] + v_d E \, \partial_y f = 0$$

Drift kinetic eq.

$$\phi - \langle \phi \rangle - \nabla_\perp^2 \phi = \frac{1}{n_{eq}} \int_0^\infty f \, \mathrm{d}E - 1$$

Quasi-neutrality

Fluid description

2 first moments of Vlasov $\Rightarrow$ evolution of density & pressure

$$n = \int f \, \mathrm{d}E = n_{eq} \left(1 + \phi - \langle \phi \rangle - \nabla_\perp^2 \phi\right)$$

$$P = \int f \, E \mathrm{d}E$$

$$\begin{cases} \partial_t n + [\phi, n] + v_d \, \partial_y P = 0 \\ \partial_t P + [\phi, P] + v_d \, \partial_y Q = 0 \end{cases}$$

Closure: $Q \equiv \int f \, E^2 \mathrm{d}E = \Upsilon P T$

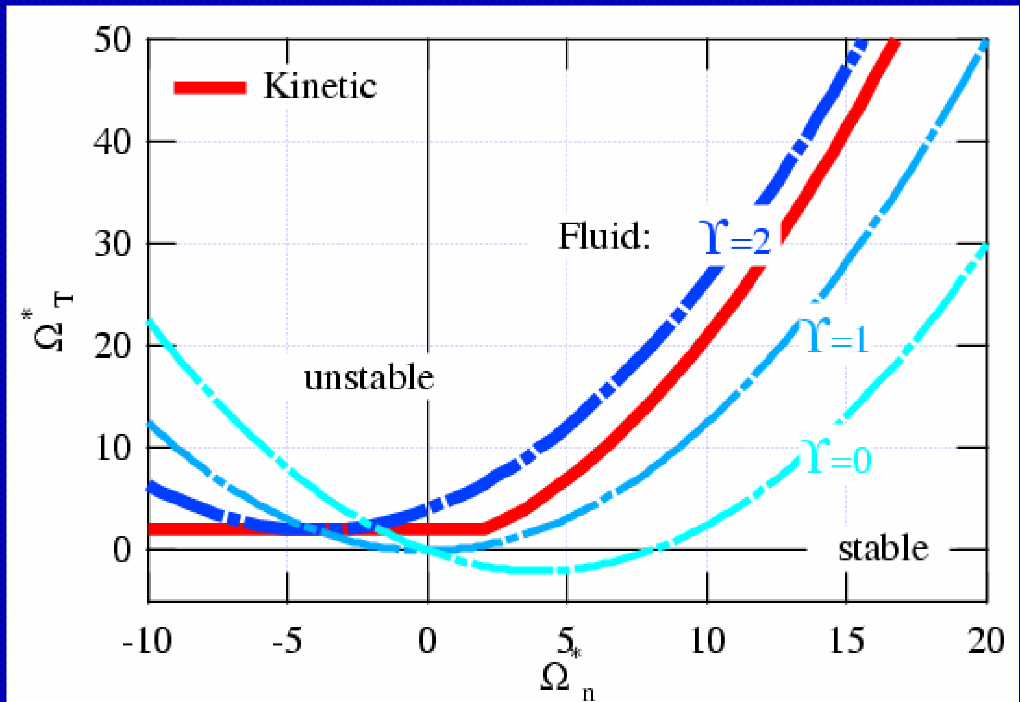
$\Upsilon = 2 \Leftrightarrow \int (f - f_M) \, E^2 \mathrm{d}E$ neglected

$\left(f_M = \frac{n}{T} \, \mathrm{e}^{-\frac{E}{T}} \right)$

Different linear stability diagrammes

- Threshold instability: $\Omega^*_{Tc}\Big|_{\text{at }\Omega^*_n=0} = \begin{cases} (1 + k_\perp^2)\,\omega_d & \text{kinetic} \\ \Upsilon(\Upsilon-1)\,(1+k_\perp^2)\,\omega_d & \text{fluid} \end{cases}$

- Vanishing relative discrepancy for large density gradients ($\Omega^*_n \to \infty$)



Fluid ≈ F at 2 energies

Constraint: *same numerics to treat fluid & kinetic descriptions*

cf [V. Grandgirard, 2004 & this conference]

2 distributions $f_\pm(x,y)$ at energies $E_\pm = T_0 \pm \varepsilon$

$$\partial_t f_\pm + [\phi, f_\pm] + v_d(T_0 \pm \varepsilon)\,\partial_y f_\pm = \underbrace{D\,\Delta_\perp f_\pm}$$

Ensures | dissipation at small scales
stability of

$$\begin{cases} \partial_t n + [\phi, n] + v_d\,\partial_y P = D\,\Delta_\perp n \\ \partial_t P + [\phi, P] + v_d\,\partial_y Q = D\,\Delta_\perp P \end{cases}$$

$$\begin{aligned} n &= f_- + f_+ \\ P &= E_- f_- + E_+ f_+ \\ &= nT_0 + \varepsilon(f_+ - f_-) \\ T &= P/n \end{aligned}$$

$$Q = E_-^2 f_- + E_+^2 f_+ = nT^2 + 4\varepsilon^2 \frac{f_- f_+}{n}$$

Equivalent to ϒ=1 closure
in the limit ε <<1

Adjustable linear properties

3 degrees of freedom in fluid: T_0, ε and D

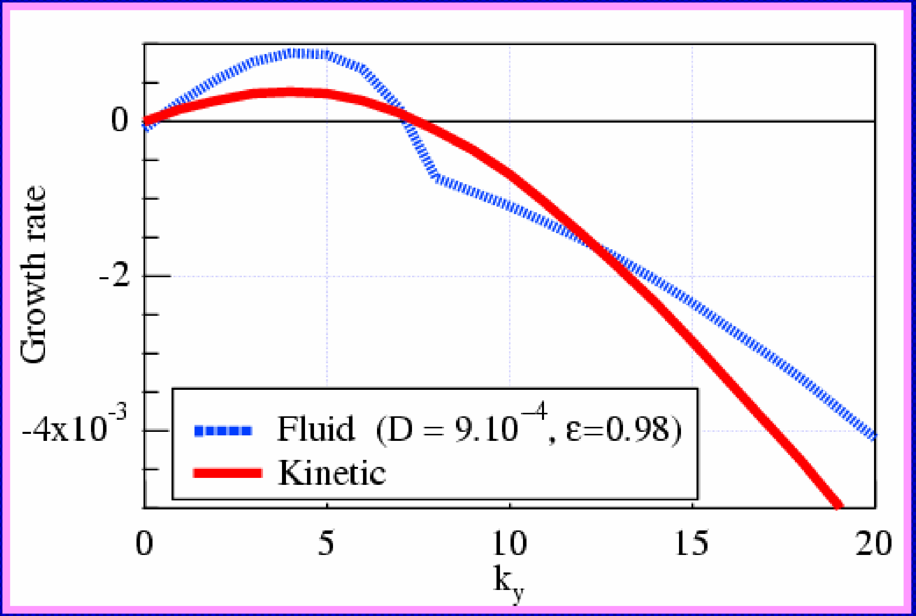
Linear fluid properties can mimic kinetic ones:

1. Linear threshold:

$$\Omega^*_{Tc}\Big|_{\text{at }\Omega^*_n=0} = \omega_d(1+k_\perp^2) \times F(T_0, \varepsilon, D)$$

adequate choice

2. Unstable spectrum width (or maximum growth rate)

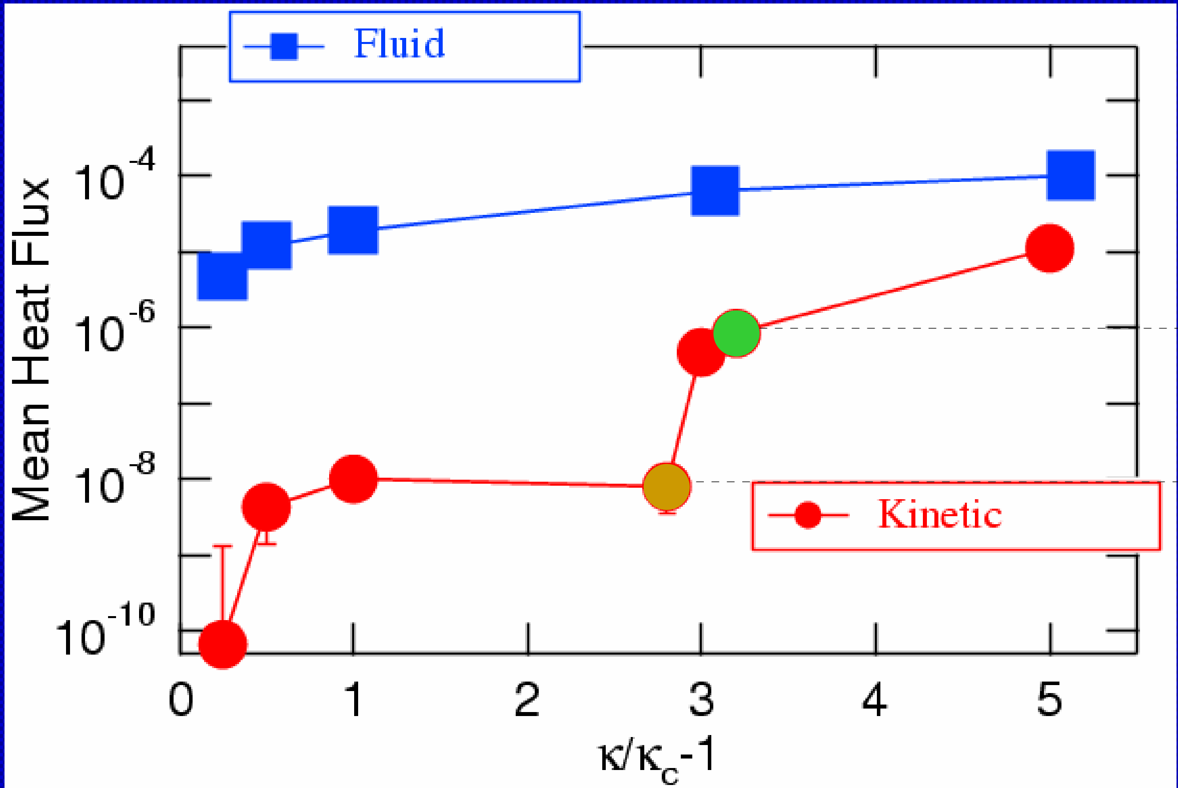


Non linear discrepancy: $Q^{fl} >> Q^{kin}$

Heat turbulent transport larger in fluid than kinetic

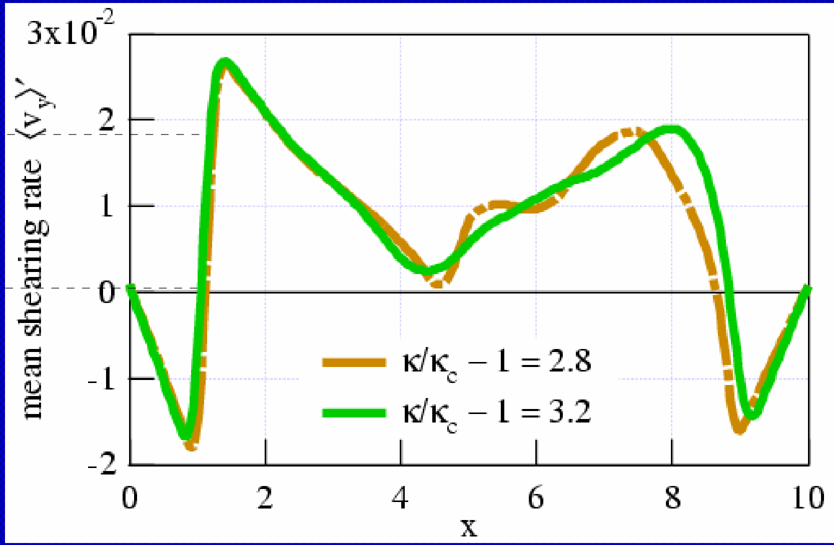
by orders of magnitude

Suggest non linear threshold (Dimits upshift ?)



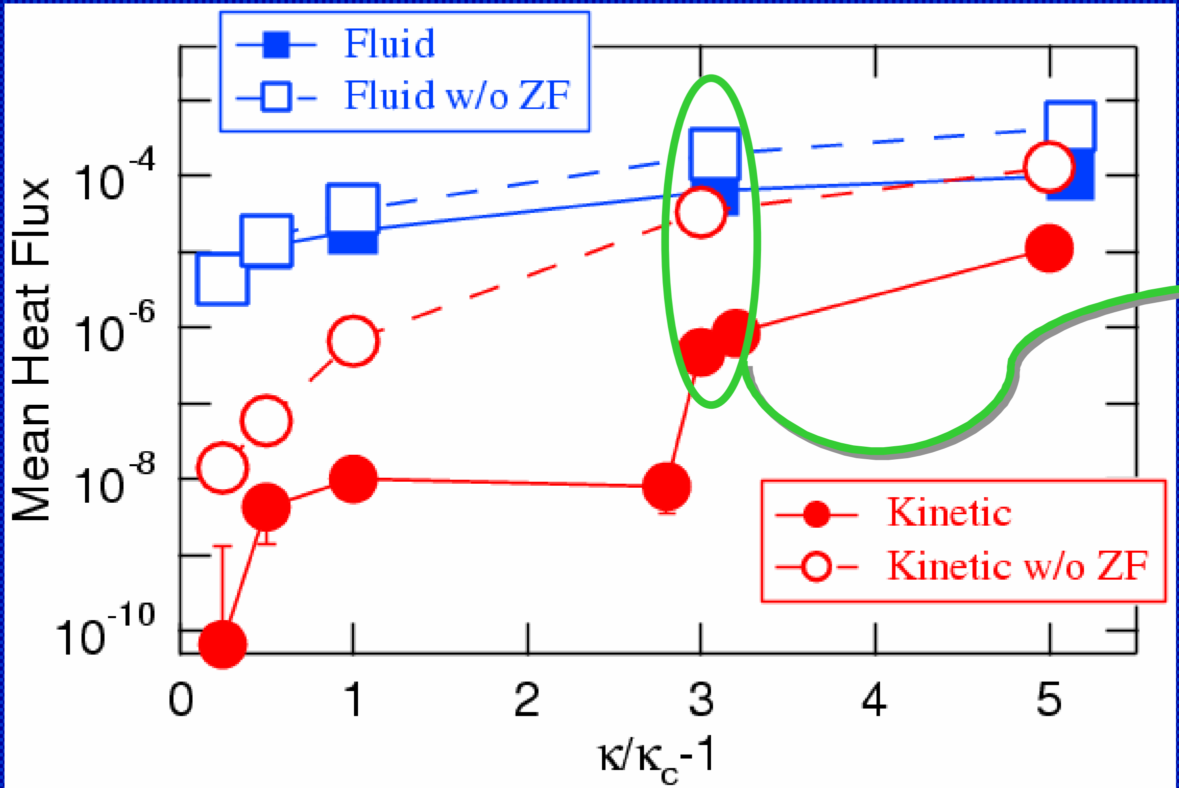
(analogous to "Dimits graph" with the same code)

Transition not understood: ZF unchanged (amplitude & dynamics)



Zonal Flows DO NOT explain the whole difference

- Larger turbulent flux when ZF artificially suppressed
- **Difference still present between kinetic & fluid** (orders of magnitude)



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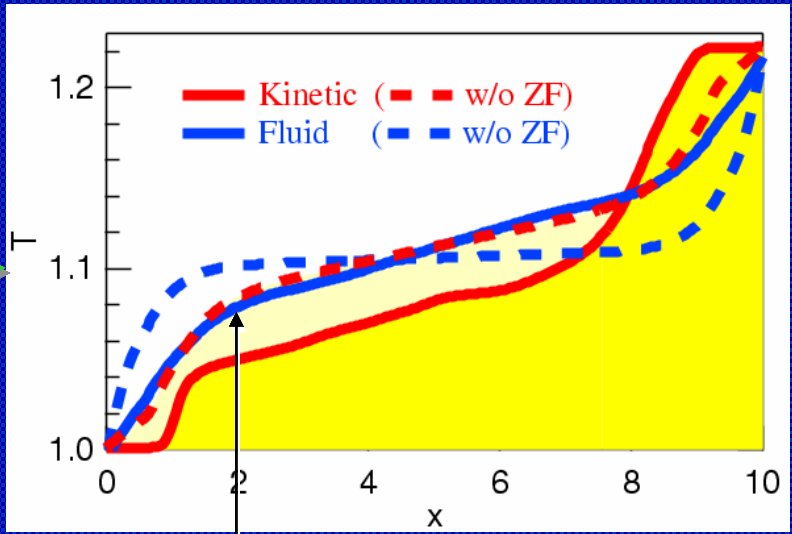


Note similar T profiles for similar fluxes

Quantifying the departure from $F_{Maxwell}$

Projection on the basis of Laguerre polynomials L_p

(standard approach for neoclassical transport)

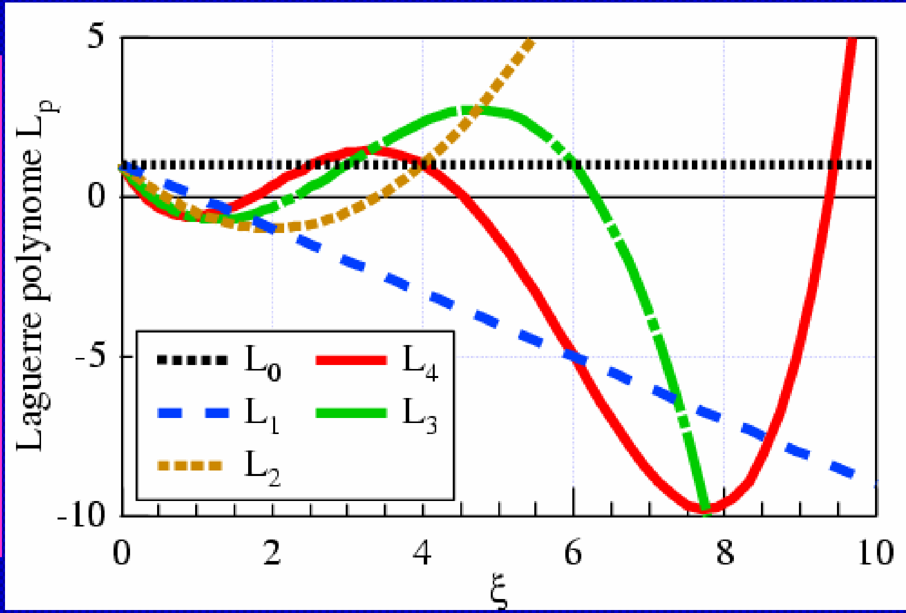
$$f(x,y,E,t) = \sum_{p=0}^{\infty} \hat{f}_p(x,y,t)\, L_p(\xi)\, \mathrm{e}^{-\xi} \quad \textit{with} \quad \xi \equiv E/T_{eq}$$

Correspondance

k^{th} Fluid moment $\leftrightarrow$ Polynomes $L_1 \ldots L_k$

$$M_k = \sum_{j=0}^{k} c_j \int_0^{\infty} L_j(\xi)\, \mathrm{e}^{-\xi} \mathrm{d}\xi$$

$$L_0 = 1$$
$$L_1 = 1 - \xi$$
$$\ldots$$



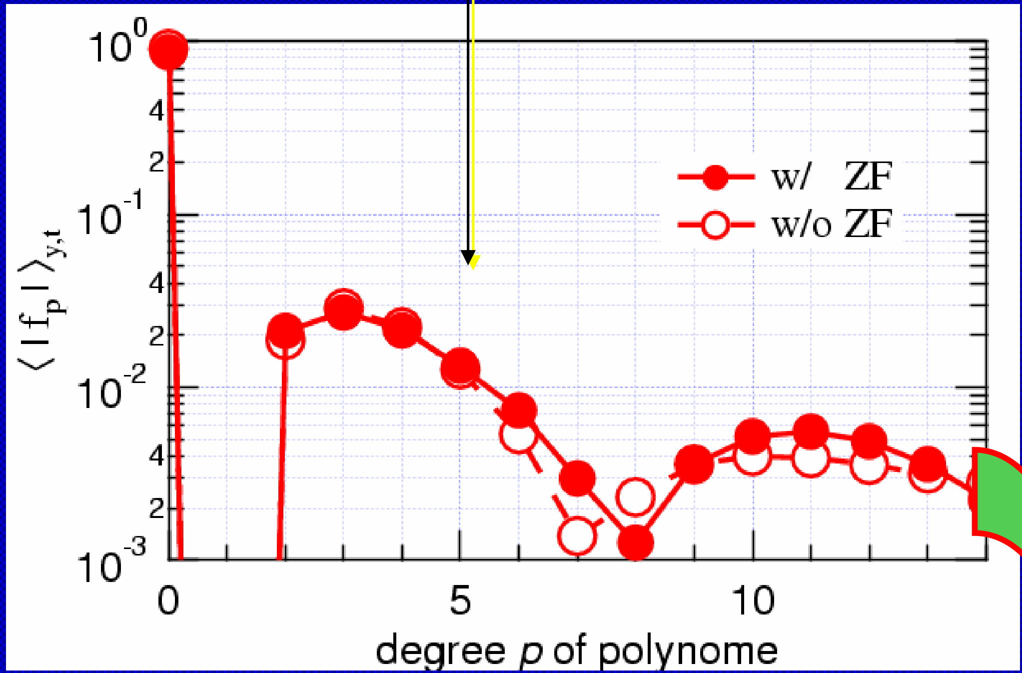
2 fluid moments are not enough

Ortho-normal basis $L_p(\xi)$ $\Rightarrow$

$$\hat{f}_p \equiv \int_0^{\infty} L_p f \, \mathrm{d}\xi$$

Slow convergence towards 0

Suggest any fluid description of the problem should account for high order moments M_k $(k>2)$



May explain why fluid & kinetic results are still different w/o ZF

Alternative closure: entropy production rates

Main ideas:
- $\dot{S}^{QL}$ governed by QL transport
- Closure fulfils 2nd principle

Fluid closure: $Q = \Upsilon \, (P + P_{eq} \, \sigma) \, T$

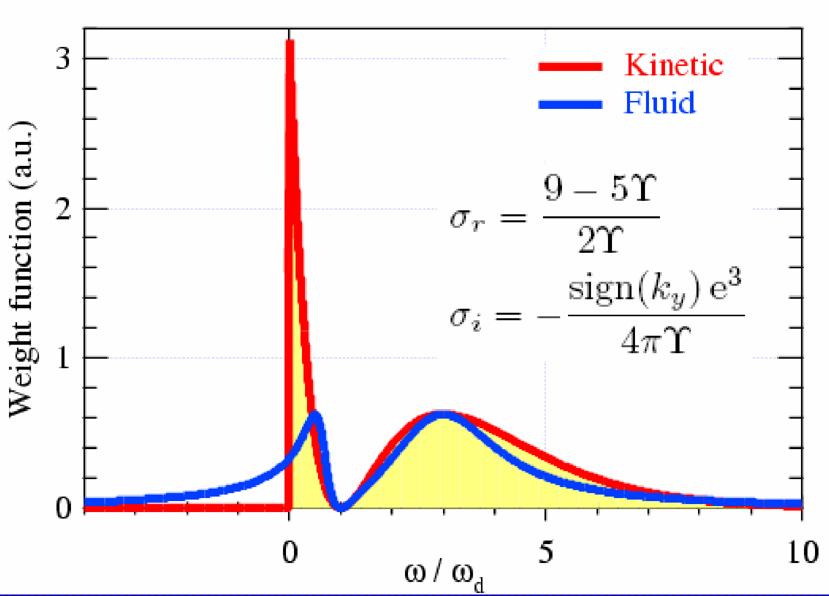
↳ **operator**: $\sigma_r(k_y) + i \, \sigma_i(k_y)$

$$\frac{dS^{QL}}{dt} = \int dx \, n_{eq} \left(\frac{T'_{eq}}{T_{eq}} \right)^2 \sum_{k,\omega} |k \hat{\phi}_{k,\omega}|^2 \, W^{QL}$$

Weights W^{QL} :

- Kinetic: infinity of resonances
- Fluid: only 2

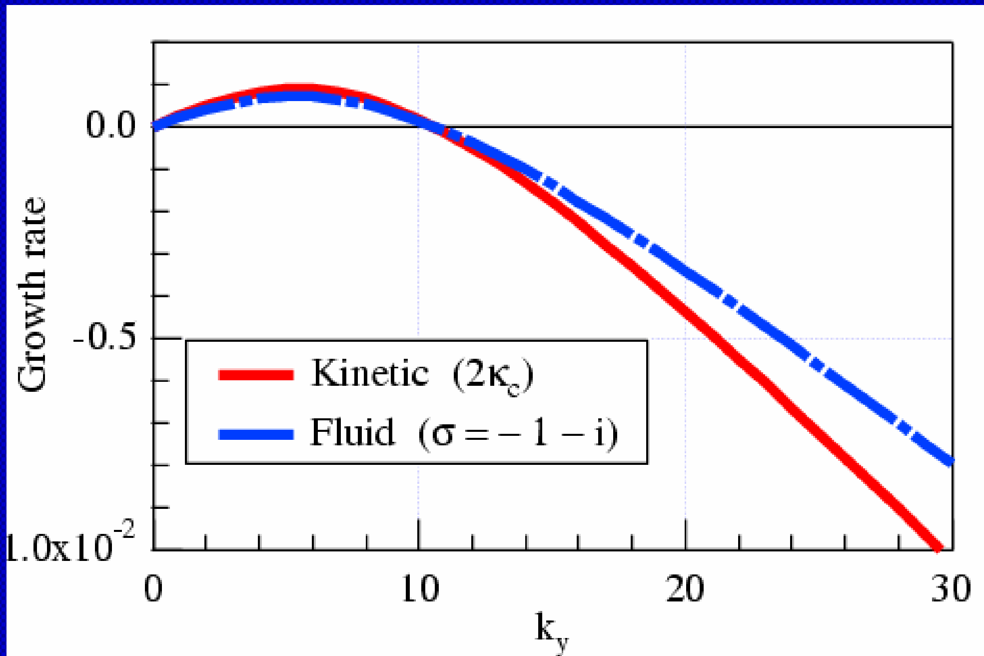
$$W^{QL}_{kin} = \frac{\pi}{|\omega_d|} \left(\frac{\omega}{\omega_d} - 1 \right)^2 \exp\left(- \frac{\omega}{\omega_d} \right)$$



$$W^{QL}_{fl} = \frac{-\Upsilon \omega_d \sigma_i (\omega - \omega_d)^2}{[\omega^2 - \Upsilon(\sigma_r + 2)\omega_d \omega + \Upsilon(\sigma_r + 1)\omega_d^2]^2 + [\Upsilon \omega_d \sigma_i (\omega - \omega_d)]^2}$$

Similar linear behaviour w/o ad-hoc dissipation

- **Same threshold** as in kinetic: $\Omega^*_{Tc}{}^{fl} = k_y v_d(1+k_\perp{}^2) = \Omega^*_{Tc}{}^{kin}$
- **Stability of small scales:** implies $\sigma_i / k_y < 0$



Similar linear spectra → what about non linear behaviour ?

Conclusions

- Looking for adequate fluid closures: what degree of convergence kinetic-fluid is requested? ($\chi_\perp$, spectrum, dynamics, …)

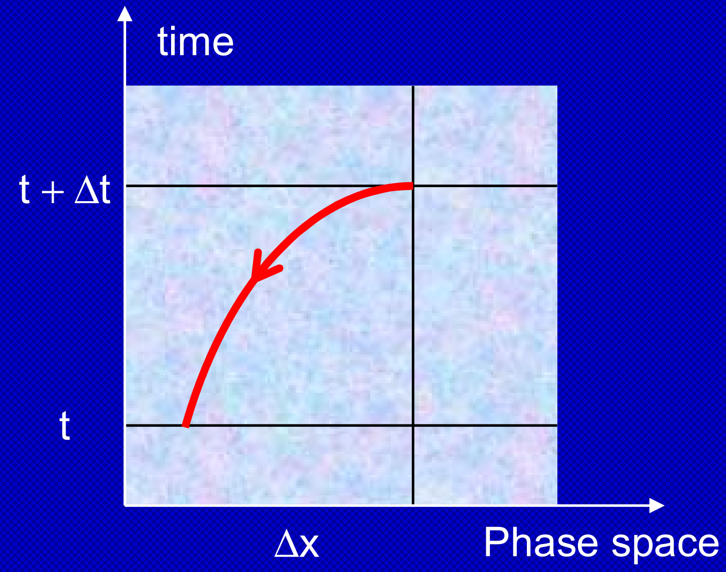
- Same numerical tool applied to 2D interchange model

- 1st closure: weak departure from F_M ($Q = \Upsilon P T$)
  - Linear properties can be made comparable (D required)
  - Fluid transport >> Kinetic transport
  - **Non linear upshift not captured by ZF only (≠ Dimits)**
  - Possible explanation: large number of fluid moments required

- **2nd closure: $Q = \Upsilon (P + P_{eq}\,\sigma)\,T$**
  - **Target: balance entropy production rates $\Rightarrow \sigma$**
  - Linear properties are similar

Semi-Lagrangian numerical scheme

- **Semi-Lagrangian scheme:**
  Fixed grid in phase space
  Follow the characteristics backward
  in time



- **Total distribution function F**
- **Global code**

- **Damping at radial ends** to prevent numerical instabilities at boundaries

- **Good conservation properties** (*e.g.* Error on energy < 1%)

[Grandgirard et al. 2004]