Extension of Geodesic Acoustic Mode to Helical Systems:

(Is there a Geodesic Acoustic Mode Oscillation in the low frequency range?)

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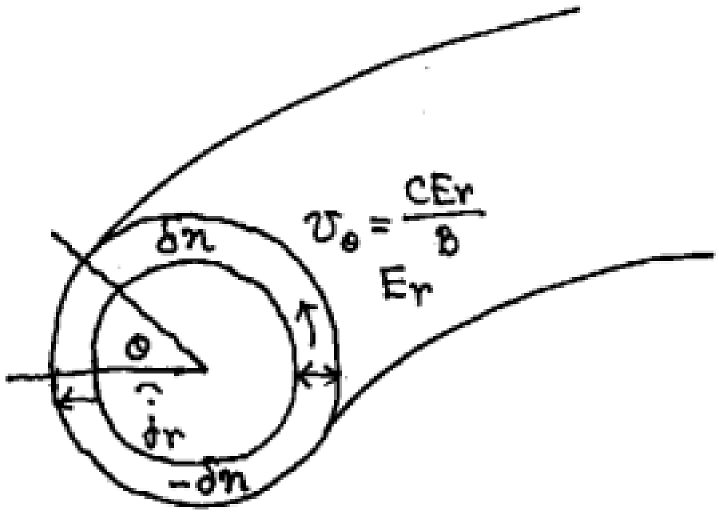
Abstracts:
In this paper Geodesic Acoustic Mode theory is re-considered based on the linear dispersion relation obtained previously by use of drift kinetic equation. Specifically, kinetics in the low frequency range is improved and attention is paid to the question if GAM has only single frequency of oscillation. In the text, it is found that a high aspect ratio tokamak with circular cross-section may have two oscillating frequencies. The new solution with lower frequency is attributed to the unique feature of the geodesic response functions defined in this paper. In a similar consideration, it is shown that a straight helical system of single helicity will have two modes. However, such solutions exist in limited configuration space and in plasma parameter range.
It is also shown that a system of two mixed helicity (consisting of helical and toroidal ripples) have two modes in wide range of plasma parameters: The lower frequency mode is a small modification from tokamak type the low frequency GAM and the higher frequency mode transits between tokamak GAM to Helical GAM depending on the size of corresponding curvature of magnetic lines of force relative to the other. In the transitional parameter space, there may be four modes. The three modes are from the top helical GAM, hybrid GAM, and tokamak GAM.

Introduction 1

GAM oscillation



$$\tilde{n}_i = n_{i0} \int dt \frac{2}{R_0} \sin\theta \frac{E_r}{B_0}$$

$$\tilde{P}_i = \gamma(\tilde{n}_e T_{e0} + \tilde{n}_i T_{i0}) \propto \sin\theta$$

$$\tilde{j}_r = \frac{1}{B}\frac{1}{r}\frac{\partial P}{\partial \theta}$$

$$\sim \gamma(T_{e0}+T_{i0}) n_{i0} \frac{2}{B B_0 R_0 r} \cos\theta \int dt\, E_r$$

$$\int \tilde{j}_r\, ds \sim \gamma(T_{e0}+T_{i0}) n_{i0} (2\pi)^2 \frac{2r}{B_0^2 R_0} \int dt\, E_r$$

Polarization Current

$$\hat{j}_{\perp} \sim \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{4\pi} \frac{dE}{dt}$$

$$\int \hat{j}_r \, dS \sim \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{4\pi} (2\pi)^2 R_0 r \frac{dE}{dt}$$

Charge Neutrality → Dispersion Relation

$$\frac{d^2 E_r}{dt^2} + \omega_{GA}^2 E_r = 0$$

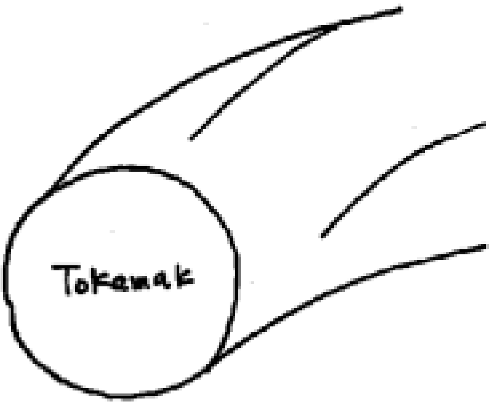
$$\omega_{GA}^2 = \frac{2\gamma}{R^2} \frac{T_{e0} + T_{i0}}{M_i} = \frac{2 C_s^2}{R^2} \gamma$$

Introduction II

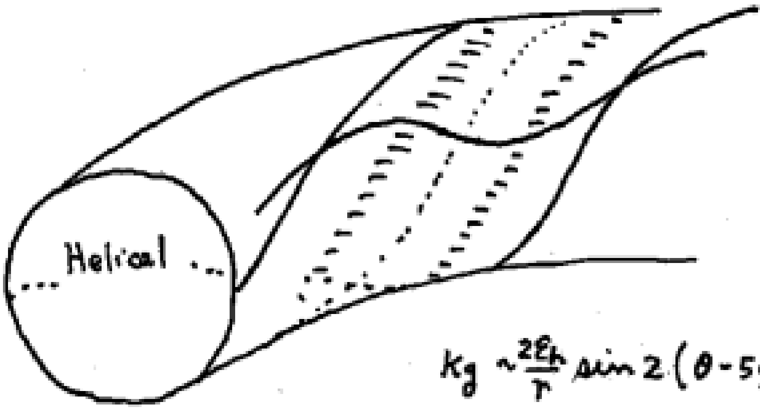
Geodesic Curvature

G. Curvature is large at $\theta = \pm \frac{\pi}{2}$

$k_g \sim \frac{1}{R} \sin\theta$



$\theta - 5\phi = 0 \quad \theta - 5\phi = \pi$



$k_g \sim \frac{2\varepsilon_h}{r} \sin 2(\theta - 5\phi$

CHS, LHD mixture of

toroidal and helical ripples

Decompose the magnetic field in Furrier spectrum:

$$B^2 = B_0^2(1+\sum_{m,n} \delta_{m,n}(\psi)\cos(m\theta - n\zeta)) \quad (1)$$

Wright the polarization current and geodesic current using conductivity constants:

$$j_{geo}(\psi) = -\sigma_{geo}(\psi)\frac{d\phi}{d\psi} \qquad (2)$$

$$j_{pol}(\psi) = -\sigma_{pol}(\psi)\frac{d\phi}{d\psi}, \qquad (3)$$

With

$$\sigma_{pol} \equiv -\frac{\omega^2{}_{p,i}}{\omega^2{}_{c,i}}\frac{\omega}{4\pi i}\frac{qB_\zeta + B_\theta}{2\pi B^2}\int \frac{1}{B^2}\left|\vec{\nabla}\psi\right|^2 d\theta d\zeta = -\frac{\omega}{i}\tilde{\sigma}_{pol} \qquad (4)$$

$$\begin{aligned}\sigma_{geo} &\equiv (\frac{e^2c^2}{2}n_0T(\frac{1}{\sqrt{g}B^2})^2(\frac{1}{2}\tilde{V}')4B_t^2l_\psi^2)\sum_{m,n} \eta^2{}_{m,n}(\psi)F(\zeta_{m,n}) \\ &\equiv \tilde{\sigma}_{geo}\sum_{m,n} \eta^2{}_{m,n}(\psi)F(\zeta_{m,n})\end{aligned} \qquad (5)$$

Continued:

Related quantities are defined as follows:

$$\eta^2{}_{m,n} \equiv \frac{(mB_\zeta + nB_\theta)^2 \delta^2{}_{m,n}(\psi)}{4B_t^2 l_\psi^2} \qquad (6)$$

$$l_\psi^2 \equiv q^2 \int \frac{1}{B^2} |\nabla \psi|^2 \, d\theta d\zeta \qquad (7)$$

$$(\frac{1}{2}\tilde{V}') \equiv \int \sin^2(m\theta - n\zeta)\sqrt{g}\, d\theta d\zeta \qquad (8)$$

Equating the two currents,

$$j_{total} = \tilde{\sigma}_{pol}(\frac{\omega}{i} - \tilde{\tilde{\omega}}_G^2 \sum_{m,n} \eta_{m,n}^2 F_{m,n})\frac{\partial \phi}{\partial \psi} = 0 \qquad (9)$$

where,

$$\tilde{\tilde{\omega}}_G^2 = \frac{\tilde{\sigma}_{geo}}{\tilde{\sigma}_{pol}} = \frac{T_i}{M_i}\frac{1}{R^2} \qquad (10)$$

$F_{m,n} = F_i(\zeta_{m,n}) + F_e(\zeta_{m,n})$ were calculated in the previous work using drift kinetic equation.

$$F_i =_i \int \left(\frac{1}{i(\omega - k_{\|}\upsilon_{\|})} + \frac{1}{i(\omega + k_{\|}\upsilon_{\|})}\right)\left(\frac{(m\upsilon_{\|}^2 + \frac{1}{2}m\upsilon_{\perp}^2)}{2T_i}\right)^2 f_{i,0} d\vec{\upsilon}$$

$$F_e(\zeta) = \frac{q_i}{q_e}\frac{n_i}{n_{e,0}} T_e \int \left(\frac{(m\upsilon_{\|,e}^2 + \frac{1}{2}m\upsilon_{\perp,e}^2)}{2T_e}\right) f_{e,0} d\vec{\upsilon}$$

$$= T_e \frac{q_i}{q_e}\frac{n_i}{n_{e,0}}\frac{T_e}{T_i} \int \left(\frac{1}{i\omega - ik_{\|}\upsilon_{\|}} + \frac{1}{i\omega + ik_{\|}\upsilon_{\|}}\right)\left(\frac{(m\upsilon_{\|}^2 + \frac{1}{2}m\upsilon_{\perp}^2)}{2T_i}\right) f_{i,0} d\vec{\upsilon}$$

$$\zeta_{m,n} = \omega / k_{\|,m,n}\upsilon_T$$

$$k_{\|,m,n} = \frac{1}{B_0}(mB^{\theta} - nB^{\zeta})$$

We define two dispersion functions(geodesic dispersion functions) :

$Z_{geo,1}(\zeta)$ and $Z_{geo,2}(\zeta)$.

$$Z_{geo,1} = \frac{1}{\sqrt{\pi}} \int \frac{1}{x-\zeta} ((x)^4 + (x)^2 + \frac{1}{2}) \exp(-x^2) dx \qquad (14)$$

$$Z_{geo,2} = \frac{1}{\sqrt{\pi}} \int \frac{1}{x-\zeta} (x^2 + \frac{1}{2}) \exp(-x^2) dx \qquad (15)$$

For reference, well-known plasma dispersion function has a form.

$$Z_p = \frac{1}{\sqrt{\pi}} \int \frac{1}{x-\zeta} \exp(-x^2) dx \qquad (16)$$

And write

$$\begin{aligned} F_{m,n} &= F_i(\zeta_{m,n}) + F_e(\zeta_{m,n}) \\ &= -2 \frac{1}{ik_{\parallel,m,n} \upsilon_T} (Z_{geo,1}(\zeta_{m,n}) + \frac{T_e}{T_i} Z_{geo,2}(\zeta_{m,n})) \end{aligned}$$

Properties of the $Z_{GAM,1}$ and $Z_{GAM,2}$ functions:

2-1. Asymptotic expansion.

$$Z_{geo,1} \approx -(\frac{7}{4}\frac{1}{\zeta} + \frac{23}{8}\frac{1}{\zeta^3}) + i\sqrt{\pi}((\zeta)^4 + (\zeta)^2 + \frac{1}{2})\exp(-\zeta^2)$$

$$Z_{geo,2} = -(\frac{1}{\zeta} + (\frac{1}{\zeta})^3) + i\sqrt{\pi}(\zeta^2 + \frac{1}{2})\exp(-\zeta^2)$$

$$Z_p \approx -(\frac{1}{\zeta} + \frac{1}{2}\zeta^3) + i\sqrt{\pi}\exp(-x^2)$$

2-2. Series expansion

$$Z_{geo,1} \approx \frac{1}{2}\zeta - \frac{1}{3}\zeta^3 + i\sqrt{\pi}\left(\frac{1}{2} + \zeta^2 + \zeta^4\right)\exp(-\zeta^2)$$

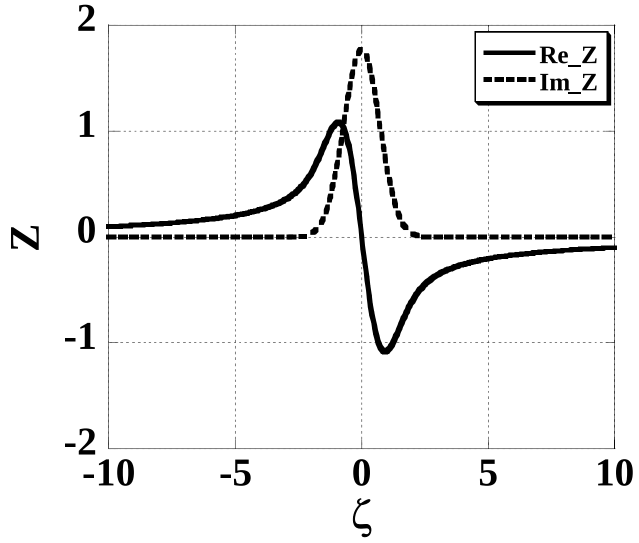
$$Z_{geo,2} \approx -\frac{4}{3}\zeta^3 + i\sqrt{\pi}(\frac{1}{2} + \zeta^2)\exp(-\zeta^2)$$

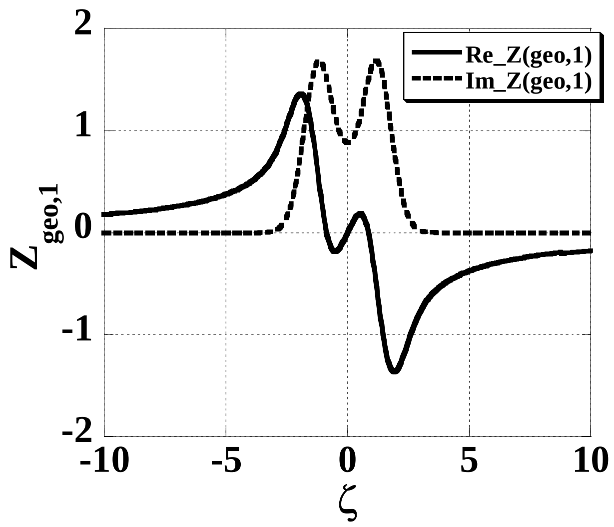
$$Z_p \approx -(2\zeta - \frac{4}{3}\frac{1}{\zeta^3}) + i\sqrt{\pi}\exp(-x^2)$$

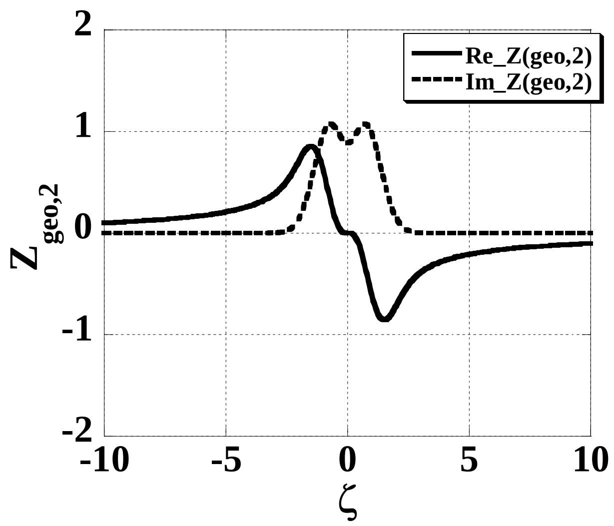
The Geodesic Dispersion Functions have peculiar features around $\zeta = 0$:

The coefficients to ζ has different signs:

New kind of GAM oscillation frequency can be found.


Re_Z
Im_Z
Z
2
1
0
-1
-2
-10
-5
0
5
10
ζ


Re_Z(geo,1)
Im_Z(geo,1)
$Z_{geo,1}$
2
1
0
-1
-2
-10
-5
0
5
10
ζ


Re_Z(geo,2)
Im_Z(geo,2)
$Z_{geo,2}$
2
1
0
-1
-2
-10
-5
0
5
10
ζ

Now, it is easy to calculate the $F_{m,n}$.

In asymptotic expansion, we have:

$$Z_{geo,1}(\zeta_{m,n})+\frac{T_e}{T_i}Z_{geo,2}(\zeta_{m,n})=\left\{\ -(\frac{7}{4}+\frac{T_e}{T_i})\frac{1}{\zeta}-(\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})\frac{1}{\zeta^3}\ \right\}$$

$$+i\sqrt{\pi}\left\{\ (\zeta_{m,n}^4+\zeta_{m,n}^2+\frac{1}{2})+\frac{T_e}{T_i}(\zeta_{m,n}^2+\frac{1}{2})\ \right\}\exp(-\zeta_{m,n}^2)$$

In series expansion, we have:

$$Z_{geo,1}(\zeta_{m,n})+\frac{T_e}{T_i}Z_{geo,2}(\zeta_{m,n})=\left\{\ (\frac{1}{2}\zeta_{m,n}-(\frac{1}{3}+\frac{4}{3}\frac{T_e}{T_i})\zeta_{m,n}{}^3)\ \right\}$$

$$+i\sqrt{\pi}\left\{\ \frac{T_e}{T_i}(\frac{1}{2}+\zeta_{m,n}{}^2)+\left(\frac{1}{2}+\zeta_{m,n}{}^2+\zeta_{m,n}{}^4\right)\ \right\}\exp(-\zeta_{m,n}{}^2)$$

The GAM frequency obtained in the previous paper is correct in the frequency domain and regarded as the highest GAM frequency among the existing ones.

3. Single Helicity Problem:

3-1. High frequency range: $k_{\parallel,m,n}\upsilon_T < \omega$

(Asymptotic Expansion is applied)

For a single helicity magnetic configuration, Eq.() assumes a form.

$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}\left[Z_{GAM,1}\left(\zeta_{m,n}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{m,n}\right)\right]\right) \quad \textbf{(25)}$$

The dispersion relation given in (25) is readily converted to following non-dimensional form:

$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}\left\{ -(\frac{7}{4}+\frac{T_e}{T_i})\frac{1}{\zeta}-(\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})\frac{1}{\zeta^3} \right\}\right) \quad \textbf{(25)}$$

, which is characterized by 3 parameters.
The solution to it is approximately

$$\zeta^2{}_{m,n} = 2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right) \quad \textbf{(26)}$$

$$\xi = (\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})/(\frac{7}{4}+\frac{T_e}{T_i}) \quad \textbf{(27)}$$

For a simple tokamak, only the toroidal ripple (m,n)=1,0) is dominant and $\eta^2_{m,n}=1$ by definition.

$$\zeta^2{}_{m,n} = 2\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(2(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right)$$

i.e.,

$$\omega^2 = \frac{2}{M}\frac{1}{R^2}\left((\frac{7}{4}T_i+T_e)(1+\xi\frac{1}{\zeta_0{}^2})\right)$$

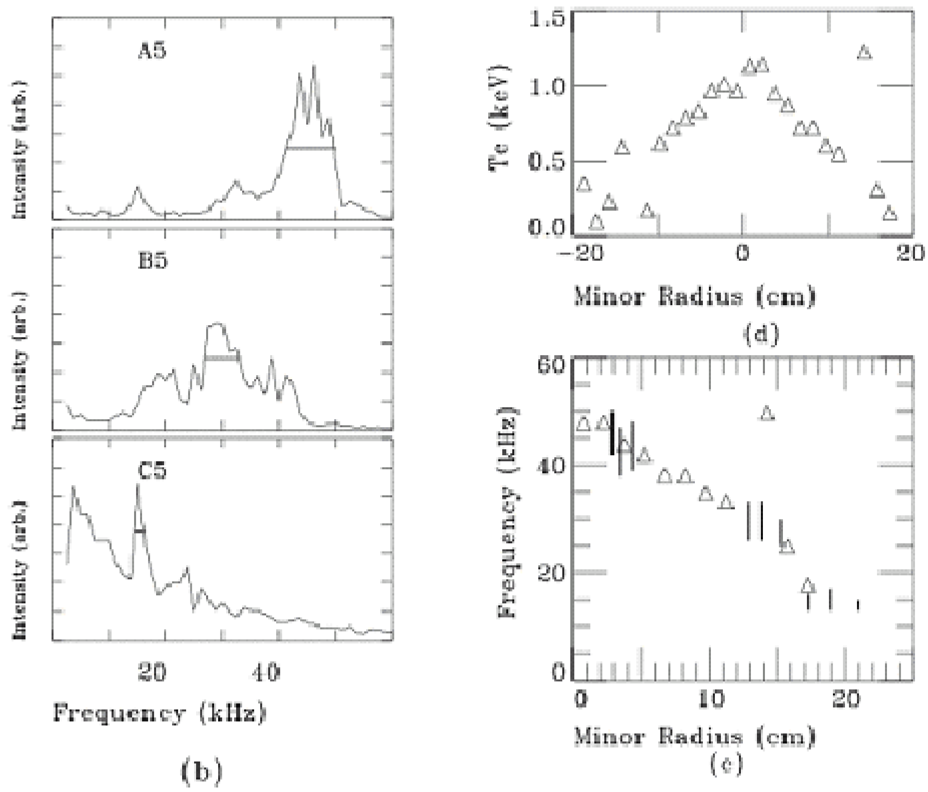
For a straight helical system, only single helicity (m,n)=(M,N) is assumed dominant. The amplitude of the helicity $\eta^2_{M,N}$ is obtained from Eq.()

$$\zeta^2{}_{M,N} = 2\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,M,N}\upsilon_T\right)^2}\left(\eta_{M,N}(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right) \qquad \textbf{(28)}$$

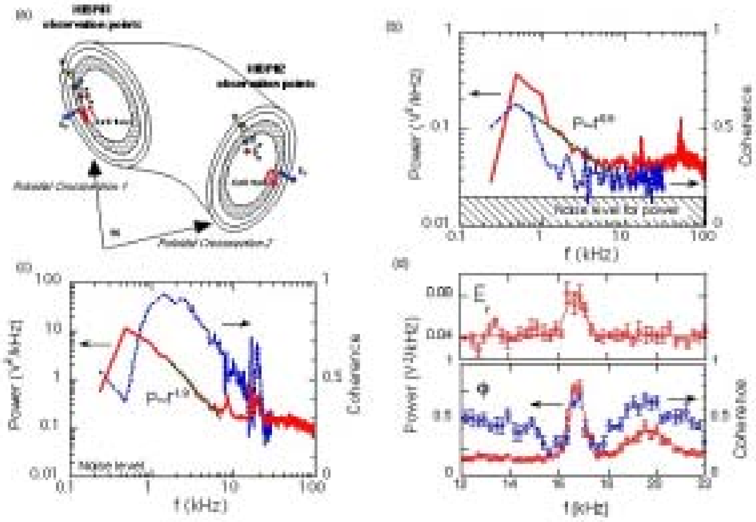
These result is the reproduction of the previous work (though it is written in a non-dimensional form here).

Only difference between the helical GAM and tokamak GAM is in the difference of the factor η^2, which is usually larger in the former.

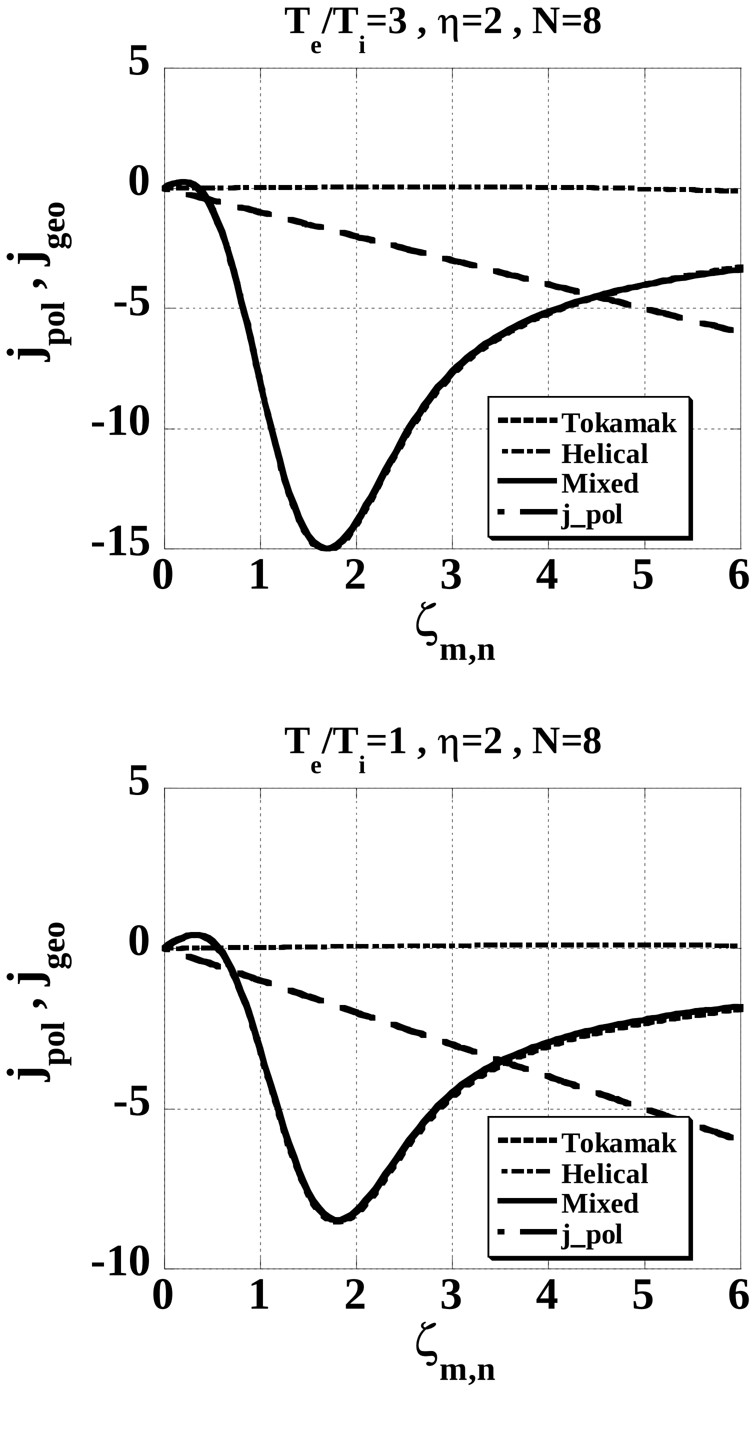
Frequency spectrum of potential fluctuation, showing GAM.
(Y.Hamada)



Potential oscillation spectrum in CHS (Fujisawa)



Tokamak has an additional low frequency GAM branch:



3.2 Low frequency range: $\omega < k_{\|,m,n}\upsilon_T$

In this frequency range, series expansion, eq(), is applied to obtain the following dispersion relation. By substituting Eq.() into Eq.(), following dispersion relation is obtained.

$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}(\frac{1}{2}\zeta_{m,n} - (\frac{1}{3}+\frac{4}{3}\frac{T_e}{T_i})\zeta_{m,n}{}^3)\right)$$

Eq.() has two solusions.

$$\zeta^2{}_{m,n} = (1+\eta_{m,n}\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2})\left(\eta^2{}_{m,n}\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}(\frac{2}{3}+\frac{8}{3}\frac{T_e}{T_i})\right)^{-1}$$

$$\zeta_{m,n} = 0$$

$$\omega^2 = \left\{\ 1+(q^2/2)(1+\frac{T_e}{T_i})\ \right\}\left\{\ (q^2/2)(\frac{2}{3}+\frac{8}{3}\frac{T_e}{T_i})\ \right\}^{-1}\frac{1}{q^2}(\frac{2T/M}{R^2})$$

It means that there will be a branch of GAM oscillation in the frequency range lower than GAM by approximately 1/q.

Straight helical system has also two solutions:
high and low frequency GAMs (similar to tokamaks):

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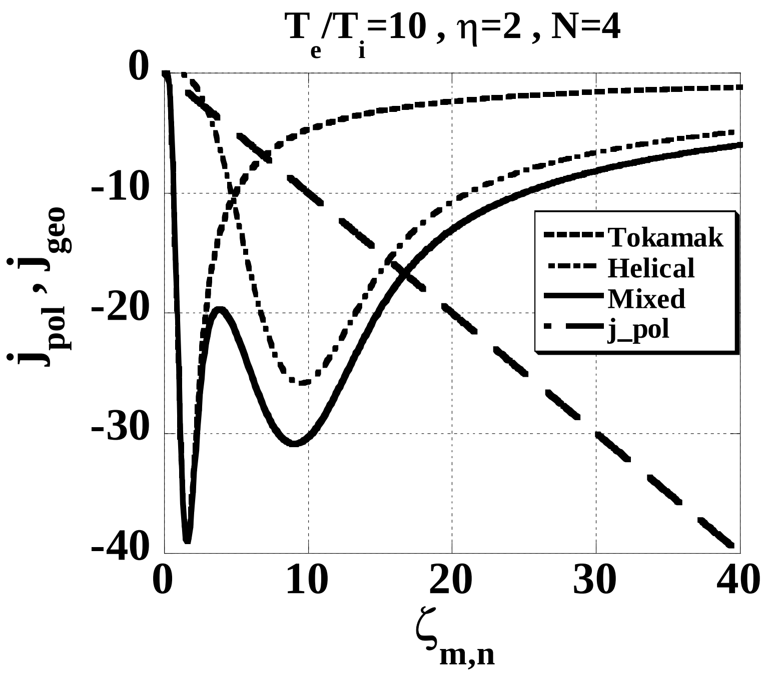
high and low frequency GAMs (similar to tokamaks):



Helical GAM appears only in limmited conditions.
If there exists a GAM, they are two: high and low frequency GAMs.

Helical GAM occurs under following conditions:

Large $\eta_{M,N}$

Small toroidal pitch N, Small q-value,
Large Te/Ti

$$\left(\frac{\tilde{\tilde{\omega}}_G}{k_{\|,M,N}\upsilon_{T,i}}\right)^2 \approx \left(\frac{\tilde{\tilde{\omega}}_G}{\upsilon_{T,i}}\frac{B_0}{(MB^\theta - NB^\zeta)}\right)^2 \approx \frac{1}{2}\frac{1}{(q-M/N)^2}\left(\frac{1}{N}\right)^2$$

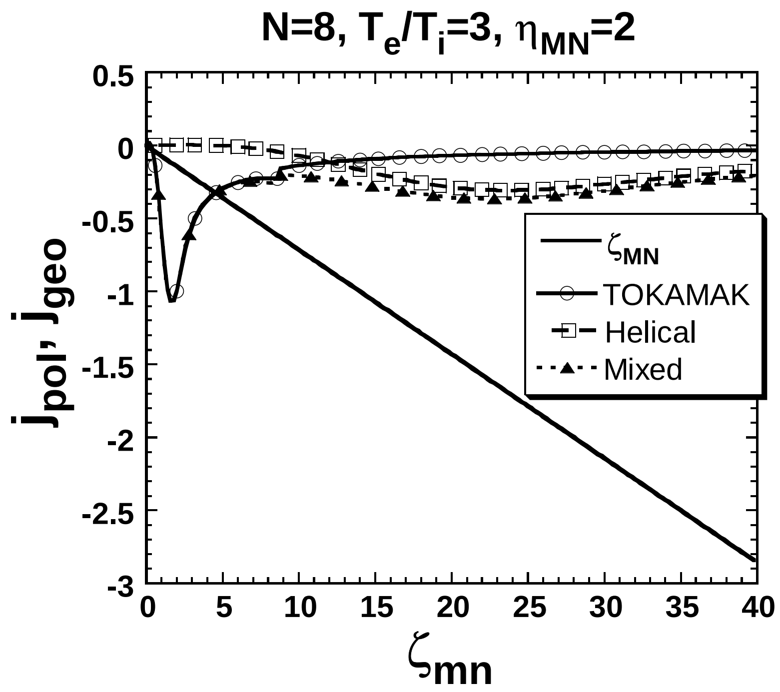
$$(k_{\|,M,N}\upsilon_{T,i})^2 \approx \frac{1}{B_0}(MB^\theta - NB^\zeta)\frac{2T_i}{M} \approx \frac{2T_i}{MR^2}N^2$$

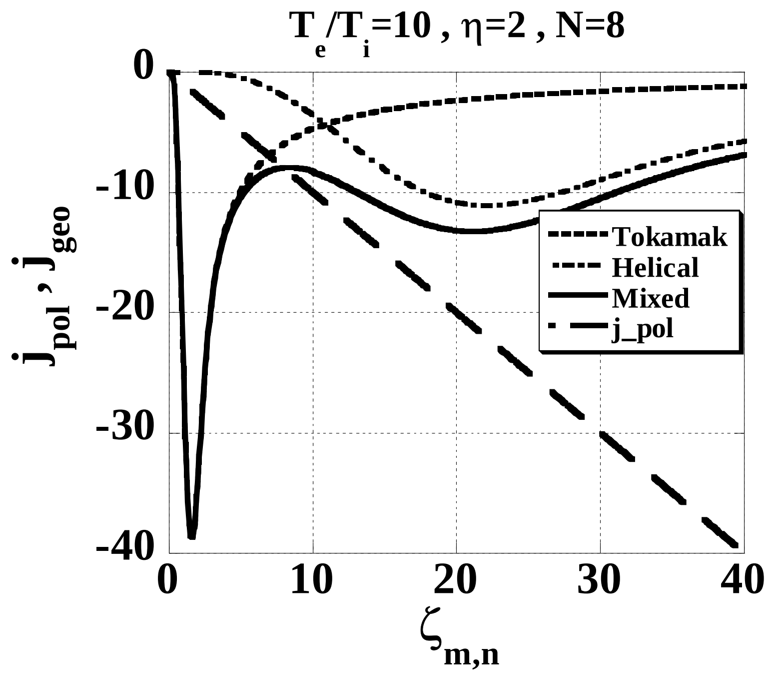
Two helicity problem:

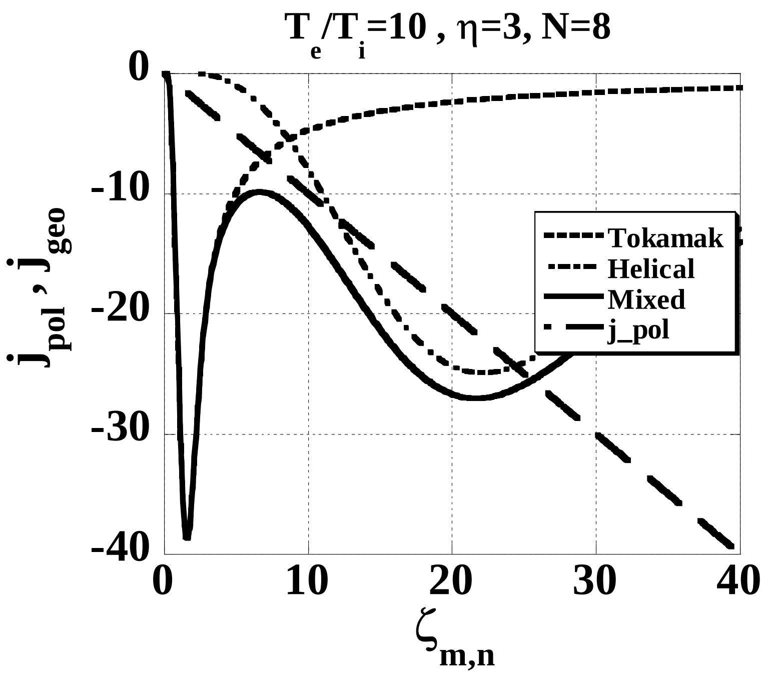
For standard helical torus like CHS and LHD, two helicity dominates. The dispersion relation is obtained keeping these two terms.

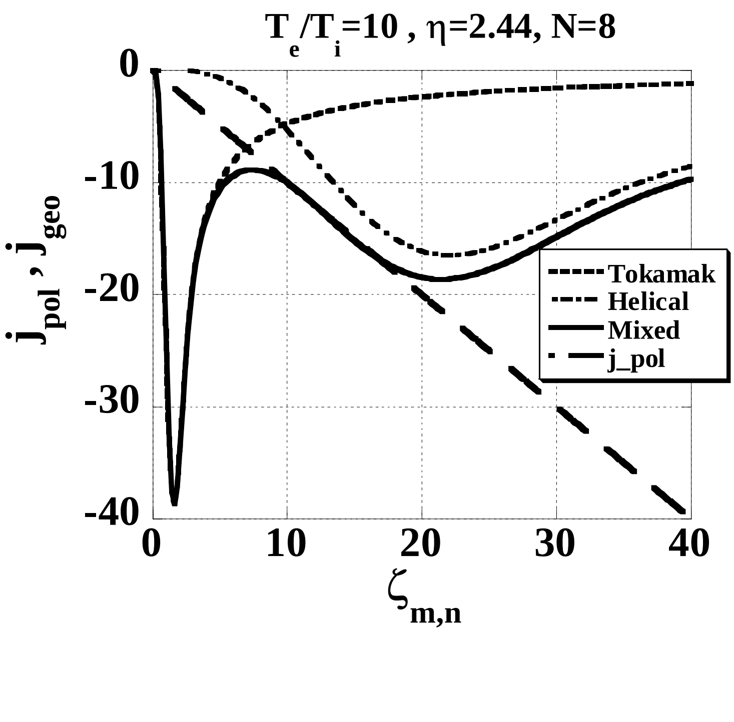
$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\begin{array}{l}\eta^2{}_{m,n}\left[Z_{GAM,1}\left(\zeta_{m,n}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{m,n}\right)\right] \\ +\eta^2{}_{M,N}\frac{k_{//,m,n}}{k_{//,M,N}}\left[Z_{GAM,1}\left(\zeta_{M,N}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{M,N}\right)\right]\end{array}\right)$$

(m,n)=(1,0) standing for the tokamak type ripple
(M,N)=(2,8) stands for helical ripple for example in CHS device.


N=8, T_e/T_i=3, η_{MN}=2
j_{pol}, j_{geo}
ζ_{mn}
ζ_{MN}
TOKAMAK
Helical
Mixed


T_e/T_i=10 , η=2 , N=8
j_{pol}, j_{geo}
$\zeta_{m,n}$
Tokamak
Helical
Mixed
j_pol


T_e/T_i=10 , η=3, N=8
0
-10
-20
-30
-40
0
10
20
30
40
j_pol , j_geo
ζ_m,n
Tokamak
Helical
Mixed
j_pol


T_e/T_i=10 , η=2.44, N=8
0
-10
-20
-30
-40
0
10
20
30
40
j_pol , j_geo
ζ_m,n
Tokamak
Helical
Mixed
j_pol

Conclusions:

Geodesic acoustic mode theory was extended to helical systems.

Drift kinetic Equation was used to evaluate kinetics properly.

Two kinds of dispersion functions were defined and their properties were investigated:

In the low frequency regime, the direction of the geodesic current is reversed.

A new low frequency GAM was found in a simple tokamak due to this mechanism.

The same dispersion relation suggests presence of helical GAM and low frequency helical GAM.

However, the helical GAM occurs only under limited conditions.

Mixed helicity problem:

Depending on the relative intensity of the geodesic curvatures, Tokamak type GAM and Helical like GAM is switched.

Since this value varies in a CHS type helical configuration, the nature of GAM will shift from tokamak type to helical type according to the minor radius.

Under certain conditions four GAM frequencies can appear.