Modenumber dependence

We may rewrite the ion temperature perturbation as

$$\frac{\delta T_i}{T_i} = \frac{\omega}{\omega - \frac{5}{3}\omega_{Di}} \left[\frac{2}{3} + \delta(\omega)\right] \frac{e\phi}{Te}$$

$$\delta(\omega) = \frac{\omega_{*e}(\eta_i - \eta_{ith})}{\omega - \frac{5}{3}\omega_{Di}}$$

Where $\eta_{ith} = \frac{2}{3} + \frac{10}{9\tau} \varepsilon_n$ is the nonlocal threshold in the strong ballooning limit

Since, at this threshold $\omega_r = \frac{5}{3} \omega_{Di}$ we realize that both numerator and denominator tend to zero at this resonance.

Just as in previous derivations the k_{θ} derivative of $\delta(\omega)$ will enter the coupling factor for the zonal flow. It may be written:

$$D = \frac{\eta_i - \eta_{ith}}{\left(\overset{\mathrm{V}}{\omega} + \frac{5}{3\iota}\varepsilon_n\right)^2} \frac{\partial \overset{\mathrm{J}}{\omega}}{\partial k_{\theta}} \qquad \text{where} \qquad \overset{\mathrm{V}}{\omega} = \frac{\omega}{\omega_{*e}}$$

The details of the resonance depend on the group velocity.

The nolocal, strong ballooning dispersion relation is:

$$\Omega(\Omega + a + i\alpha(1 + \frac{5}{3}\varepsilon_n)) + (b + i\alpha)(\eta_i - \eta_{ith}) = 0$$

where

$$a = \frac{1}{1+k^2\rho^2} \left[\varepsilon_n - 1 - k^2\rho^2 \left(\frac{5}{3\tau}\varepsilon_n - \frac{1+\eta_i}{\tau}\right) \right]$$
$$b = \frac{1}{1+k^2\rho^2} \frac{\varepsilon_n}{\tau}$$

$$\alpha = \frac{\varepsilon_n}{4} \frac{|\mathbf{s}|}{q}$$

and $\Omega = \omega + \frac{5}{3\tau} \varepsilon_n$

Now

$$\frac{\partial \Omega}{\partial k} = -\frac{\Omega \frac{\partial a}{\partial k} + (\eta_i - \eta_{ith}) \frac{\partial b}{\partial k}}{2\Omega + a}$$

and

$$2\Omega + a = -i\alpha(1 + \frac{5}{3\tau}\varepsilon_n) + \sqrt{-4\frac{\varepsilon_n}{\tau}(\eta_i - \eta_{ithl}) - \alpha^2(1 + \frac{5}{3\tau})^2 + i\alpha(a + \eta_{ith} - \eta_i)}$$

Although formally of order 1 it is usually small for typical tokamak equilibria. For the Cyclone base case we have s=0.78, q=1.4 and $\varepsilon_n = 0.9$. This gives $\alpha = 0.125$. Thus all terms in the denominator are small. This gives a strong sensitivity to magnetic shear near the local threshold:

$$\eta_{ithl} = \eta_{ith} + \frac{\tau}{4\varepsilon_n} (1 - \varepsilon_n) \left[1 - \varepsilon_n - k^2 \rho^2 (1 - \varepsilon_n + \frac{2}{\tau} (1 + \eta_i - \frac{5}{3} \varepsilon_n)) \right]$$

We notice that the local and nonlocal thresholds coincide for $\varepsilon_n = 1$. Close to this we get a strong dependence on modenumber. The trend is that the excitation of zonal flow becomes stronger for higher modenumber.

The dependence on magnetic shear has already been noted. In nonlinear simulations we obtained (Dastgeer et al 2002) the result



This was obtained for parameters inside the nonlinear upshift.