The Role of Damping in Stable and Unstable Alfvén Eigenmodes

S. D. Pinches[1], A. Könies[2], Ph. Lauber[1]
H.L.Berk[3], S.E.Sharapov[4] and M.Gryaznavich[4]

[1]Max-Planck-Institut für Plasmaphysik, EURATOM Assoziation, Garching, Germany
[2]Max-Planck-Institut für Plasmaphysik, EURATOM Assoziation, Greifswald, Germany
[3]Institute for Fusion Studies, University of Texas at Austin, Austin, Texas, USA
[4]EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, UK

Motivation

- **In a burning plasma α-particles significantly contribute to pressure**
- **Large scale α-particle driven instabilities**
  - Loss of α-particles and thus self-heating
  - Damage to device
- **Stability boundary determined by damping mechanisms**
  - Disagreement over understanding of dominant physical damping mechanisms

Physics to Capture

- **Multiple scale-lengths**
  - Minor radius (global modes) → orbit width → Larmor radius
- **Realistic geometries**
  - Tokamak and stellarator
- **Self-consistency**
  - Particle distribution ↔ Mode structure
  - Energy and momentum conservation
- **Nonlinear evolution**
  - Saturated mode amplitudes, pitchfork splitting, frequency sweeping

Numerical Tools

- **LIGKA**
  - Linear gyrokinetic non-perturbative tokamak model
  - [Ph. Lauber, Ph.D. Thesis, T.U. München 2003]
- **CAS3D-K**
  - Perturbative drift-kinetic approach for stellarators
  - [A. Könies, Phys. Plas. **7** 1139 (2000)]
- **HAGIS**
  - Initial value nonlinear drift-kinetic δf model
  - [S. D. Pinches et al., Comput. Phys. Commun. **111**, 131 (1998)]

LIGKA: Gyrokinetic Model

Based on model by H. Qin, W. M. Tang and G. Rewoldt

- **Linear shear Alfven perturbations**
  - Calculates mode frequency, growth rate and mode structure
- **Gyrokinetic**
  - Particles feel perturbation around gyro-orbit
- **Non-pertubative**
  - Solves Ampere's law and quasi-neutrality equation simultaneously
  - Allows change from MHD eigenmode structure
  - Nonlinear eigenvalue problem (Nyquist solver)
- **Accurate treatment of unperturbed particle orbits**
  - Numerical integration of full drift orbit effects (HAGIS)
- **General tokamak geometry**
  - From numerical equilibrium code (e.g. HELENA)

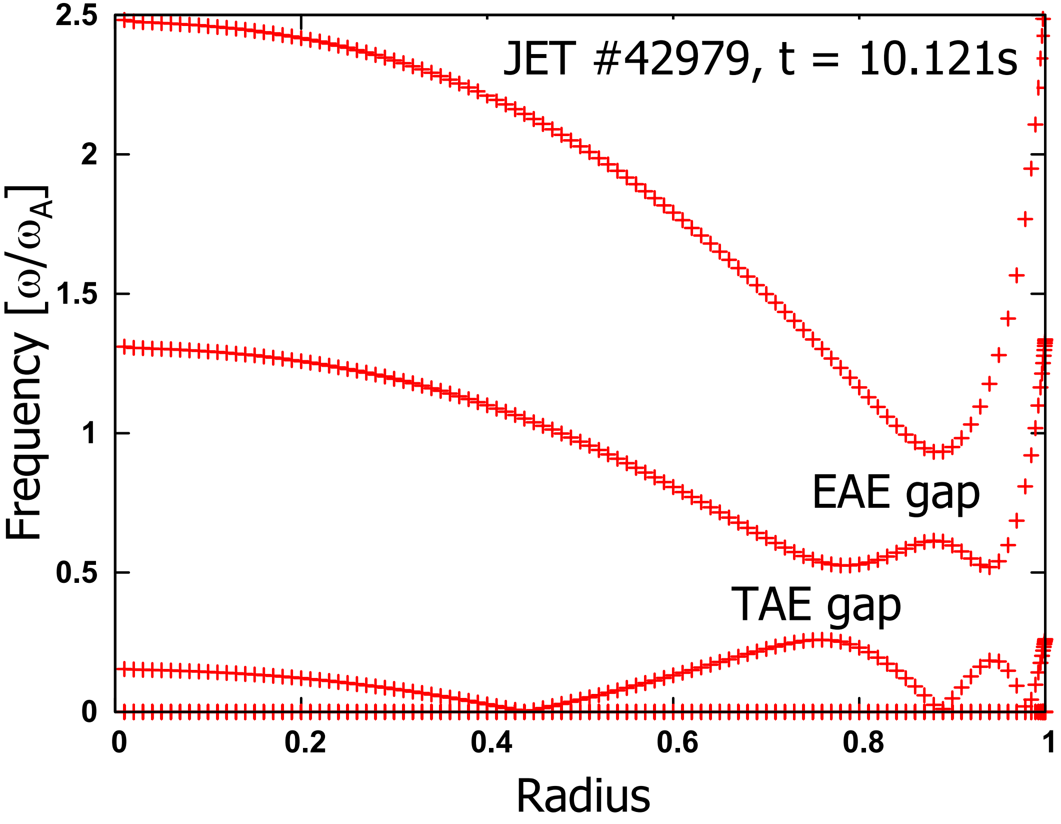
[H. Qin, W. M. Tang, G. Rewoldt, Phys. Plas. **6** 2544 (1999)

Shear Alfvén Continuum

- **Described by local dispersion relation:**

$$\omega^2 = k_{\|}^2 v_A^2$$

- **Mode coupling creates frequency gaps and global modes**
  - Large scale interaction with fast particles

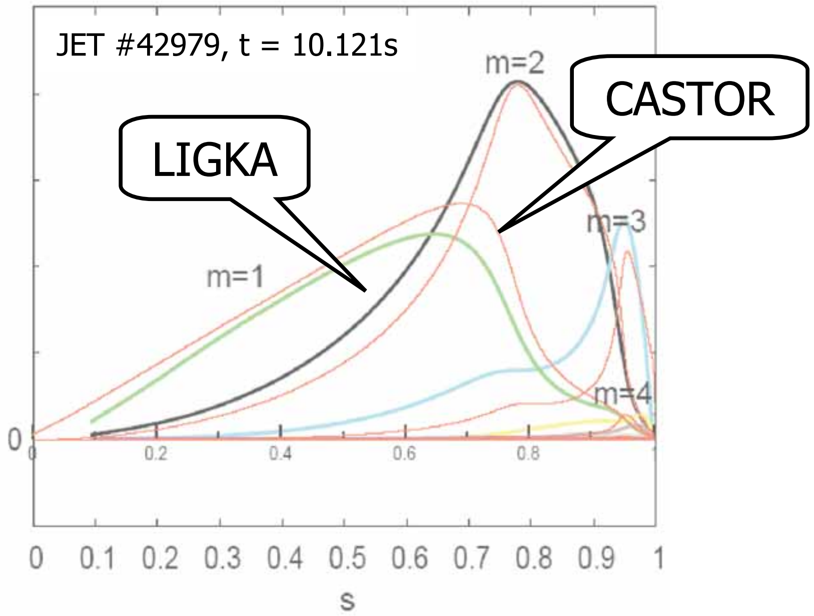


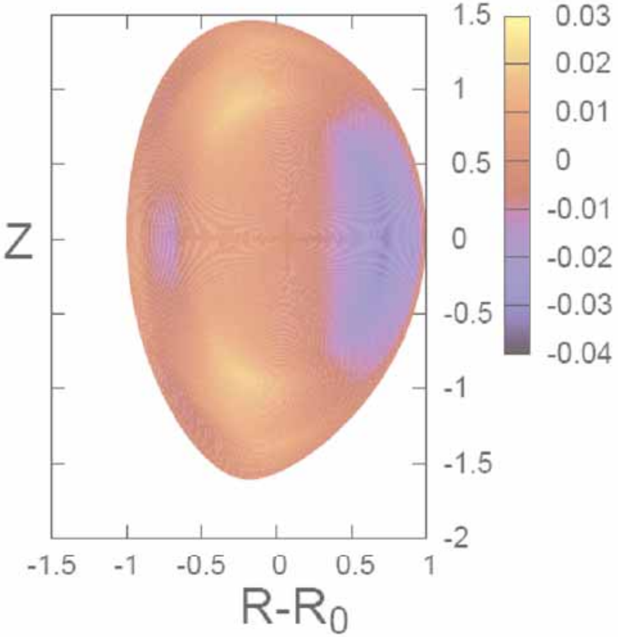
(Equilibrium from A. Jaun)

TAE

- **Global mode formed in toroidicity induced gap**
- **Ballooning character**
- **Principle damping mechanisms**
  - Electron/(ion) Landau damping, continuum/radiative damping

$$\omega \approx \frac{v_A}{2qR}$$





[C.Z. Cheng and M.S. Chance, Phys. Fluids **29** 3695 (1986)]
[D. Borba and W. Kerner, J. Comp. Phys. **153** 101 (1999)]

FLR Effects

- **FLR effects introduce kinetic Alfvén waves (KAW)**

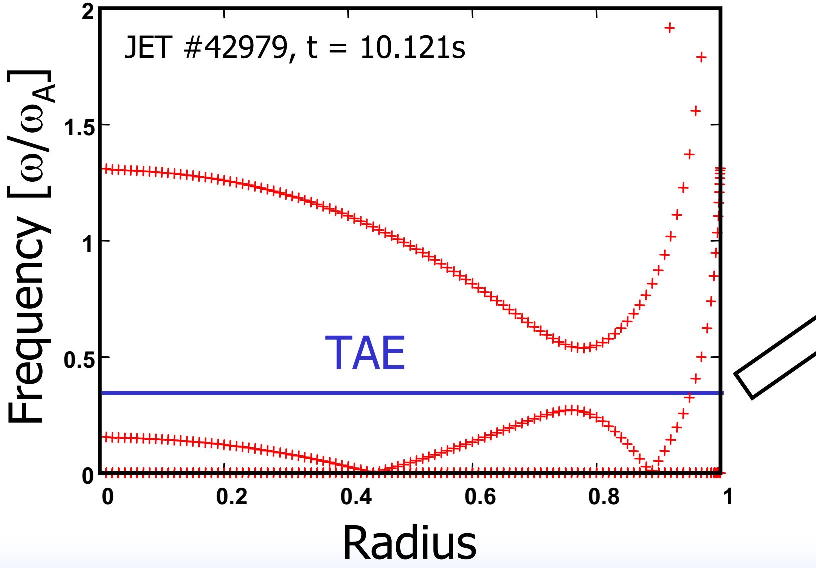
$$\omega^2 = \underbrace{k_\parallel^2 v_A^2}_{\partial\omega/\partial k_\perp=0}\underbrace{\left[1+k_\perp^2\rho_i^2\left(\frac{3}{4}+\frac{T_e}{T_i}\right)\right]}_{\partial\omega/\partial k_\perp\neq 0}$$

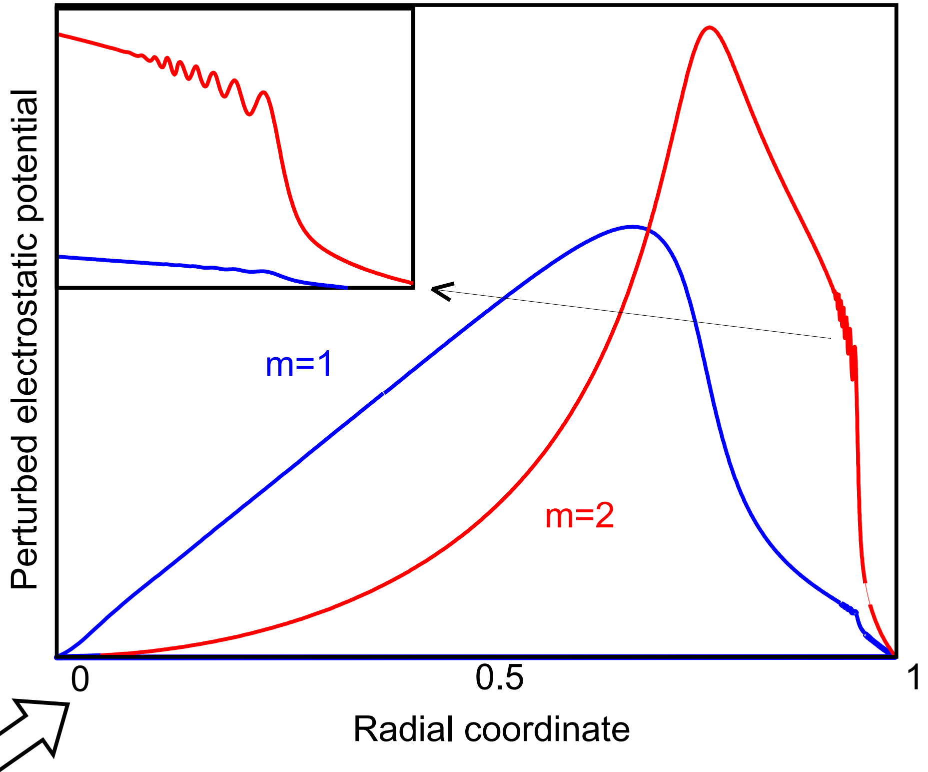
- **Alfvén continuum resolved into discrete spectrum**
  - MHD singularity resolved by higher order equation
- **Coupling of TAE to KAW leads to radiative damping**
  - Energy carried away from gap region
- **Coupling of KAW leads to formation of KTAE**
  - Global modes existing just above top of TAE frequency gap

[Hasegawa & Chen Phys. Fluids **19** 1924 (1976)]
[Mett and Mahajan, Phys. Fluids B **4** 2885 (1992)]
[Conner *et al*, Proc. 21st EPS Conf., **18B** 616 (1996)]

Radiative Damping in LIGKA

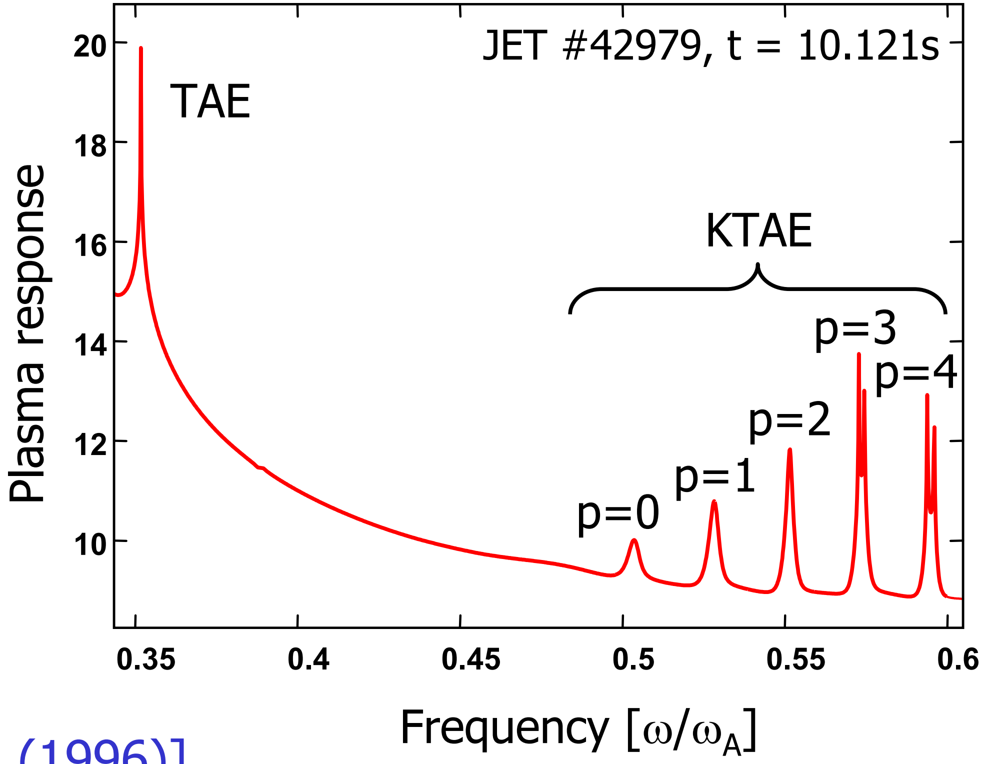
- **With only two harmonics, TAE intersects “continuum”**
- **In this JET case, mode conversion dominantly occurs at edge**





External Antenna Drive

- **Modelled via change in LIGKA code boundary conditions**
  - No vacuum region
- **Systematically find all stable modes**
  - Including damping rates
- **Analogous to TAE antenna experiments**
  - [Fasoli et al., PRL 76 1067 (1996)]

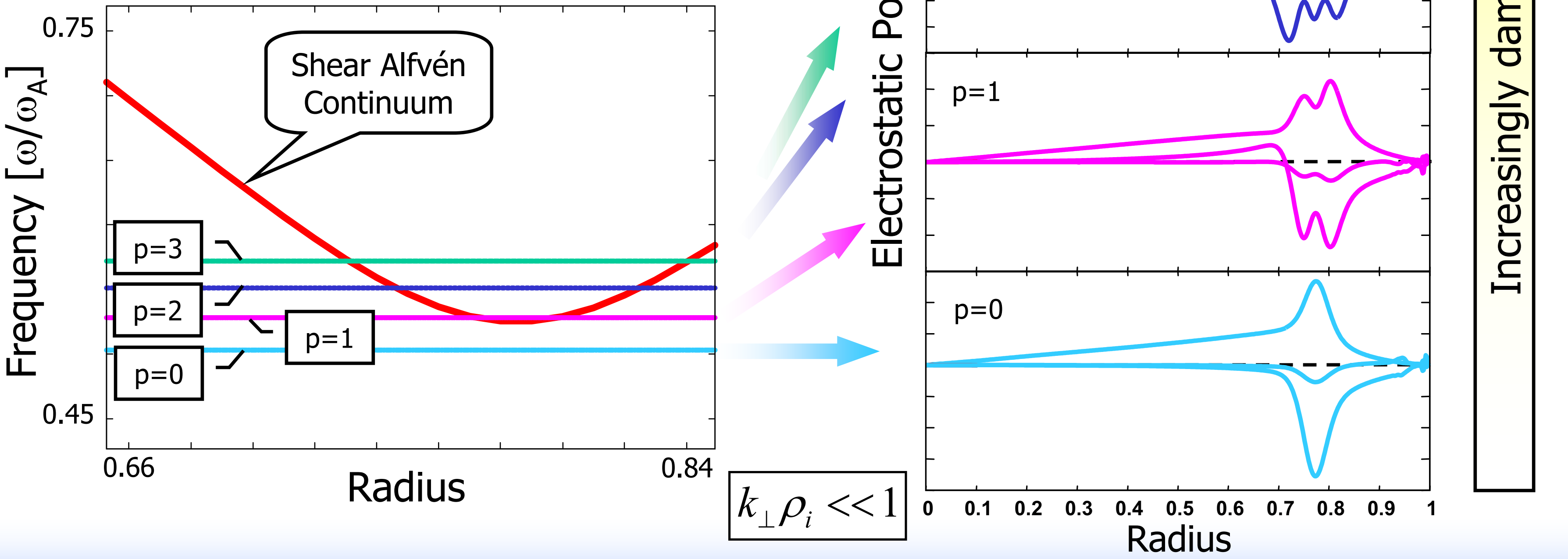


[Conner *et al*, Proc. 21st EPS Conf., Montpellier, **18B** 616 (1996)]

KTAEs in LIGKA

- **Global modes**
- **Anti-ballooning character**
- **Formed at top of TAE gap**
- **Stronger damping than TAE**

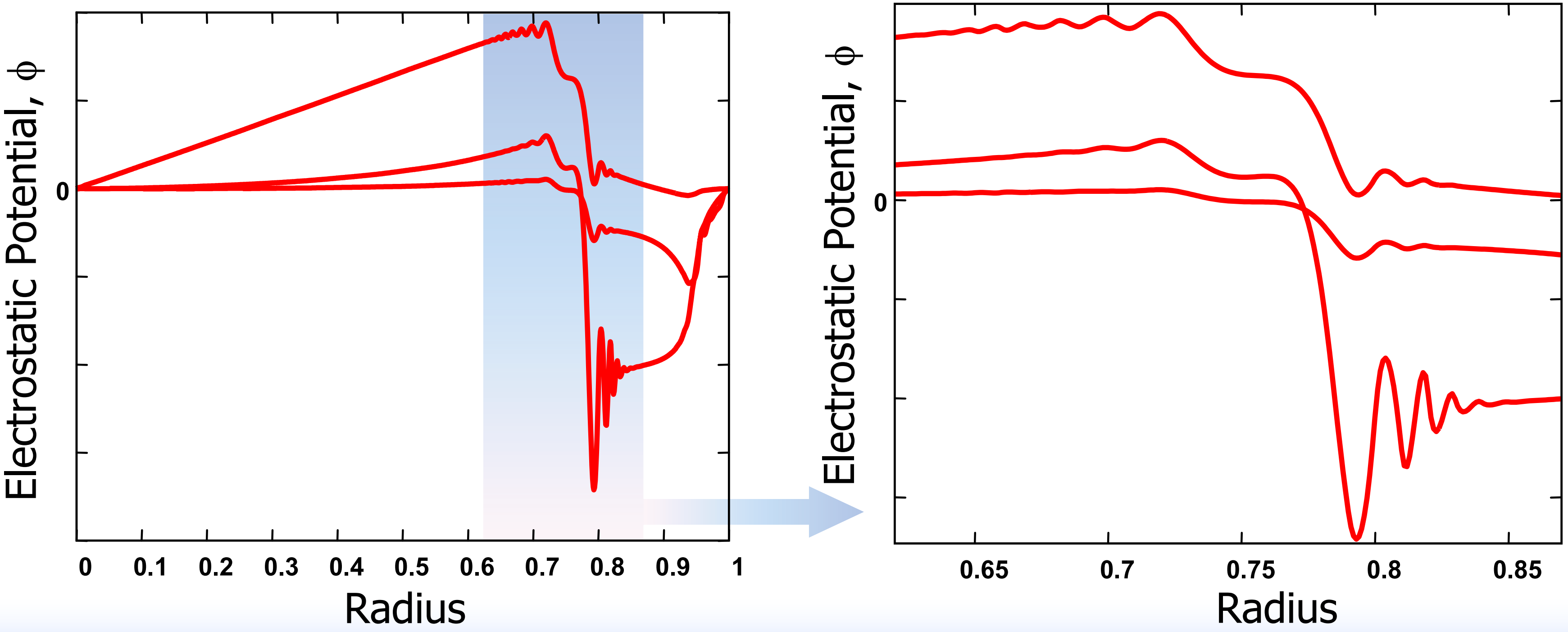
JET #42979, t = 10.121s



$k_\perp \rho_i << 1$

Radiative Damping

- Out-going KAWs at bottom of gap
- Oscillation scale-length ~ O($20\rho_i$)
- Mode heavily damped compared with TAE



Comparison to Experiment

- **Once LIGKA fully benchmarked…**
- **Investigate sensitivity to edge density**
  - TAE gap closes as edge density falls
  - Damping rate increases
  - Better match to experimental measurements…

Perturbative Approach

- **Restricted to MHD-like perturbations**

$$\vec{B}^{(1)} = \vec{\nabla} \times \left(\vec{\xi} \times \vec{B}\right)$$

- **Energy functional derived from MHD moment equation:**

$$\vec{\nabla} \cdot \vec{P} = -\vec{B} \times \left(\vec{\nabla} \times \vec{B}\right)$$

- $\vec{P}$ **is replaced with a kinetic expression**
  - I.e. integrals over distribution function
- **Analogous to calculating growth rate from wave-particle energy transfer rate**

CAS3D-K

- **Perturbative stability code based on hybrid MHD drift-kinetic model**
- **General mode structure and 3D equilibrium**
  - e.g. AE in W7-AS and W7-X
- **Particle drifts approximated as bounce averages**
- **Presently zero radial orbit width**
- **Perturbative growth/damping rate**

$$\Delta\omega_s + i\gamma_s \approx \frac{1}{2}\frac{\delta W_s(\omega_0)}{\delta W_{\mathrm{mag}}}\omega_0$$

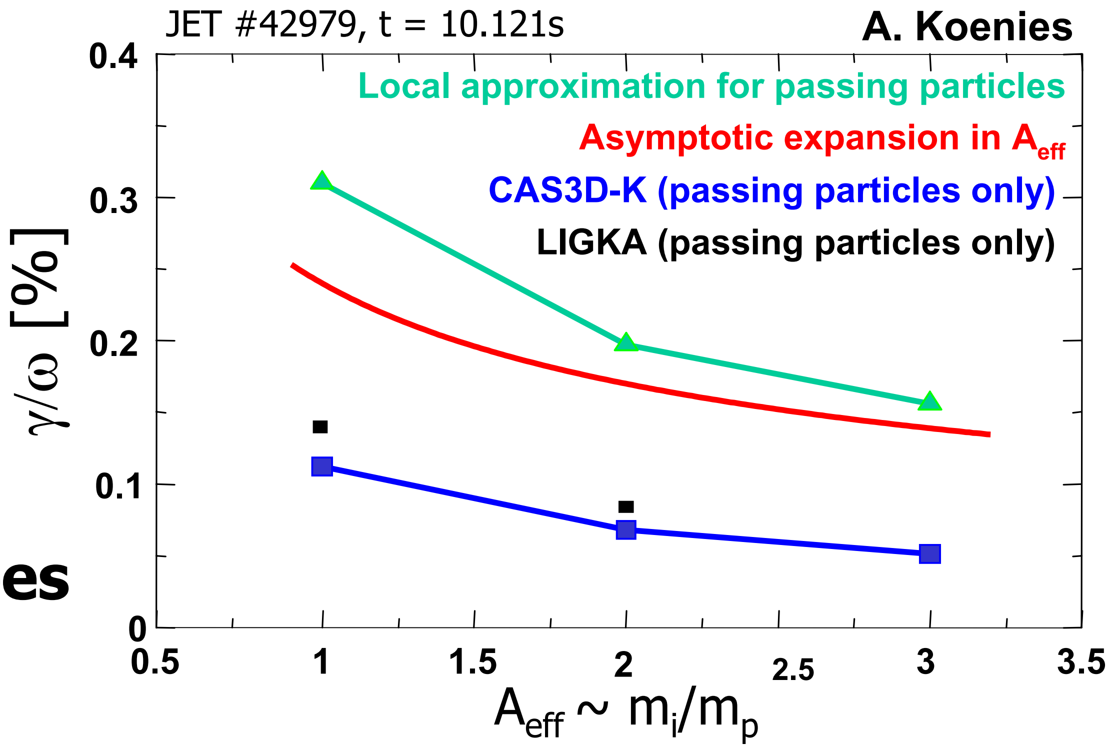
using MHD eigenfunctions and eigenfrequency

  - δW_{mag} from the ideal MHD stability code CAS3D
    [C. Nührenberg Phys. Plas. **6** 137 (1998)]

[A. Könies, Phys. Plasmas **7** 1139 (2000)]

Global n=1 TAE Damping

- **Mass scaling of electron Landau damping rate**
  - [A. Könies 2004]
- **Local fluid approximation**
  - $\gamma/\omega \sim A_{eff}^{-1/2}$
- **Kinetic model agrees with hybrid model**
  - LIGKA and CAS3D-K
- **Trend agrees with experimental results**
  - But factor 10 too small, $\gamma_d/\omega \sim 1\%$
  - [A.Fasoli et al, Phys. Lett. A **265** (2000) 288]



$$A_{eff} = \left(\sum_i n_i m_i \right) / \sum_i n_i$$

[Fu & Van Dam, Phys. Fluids B **1** 2404 (1989)]

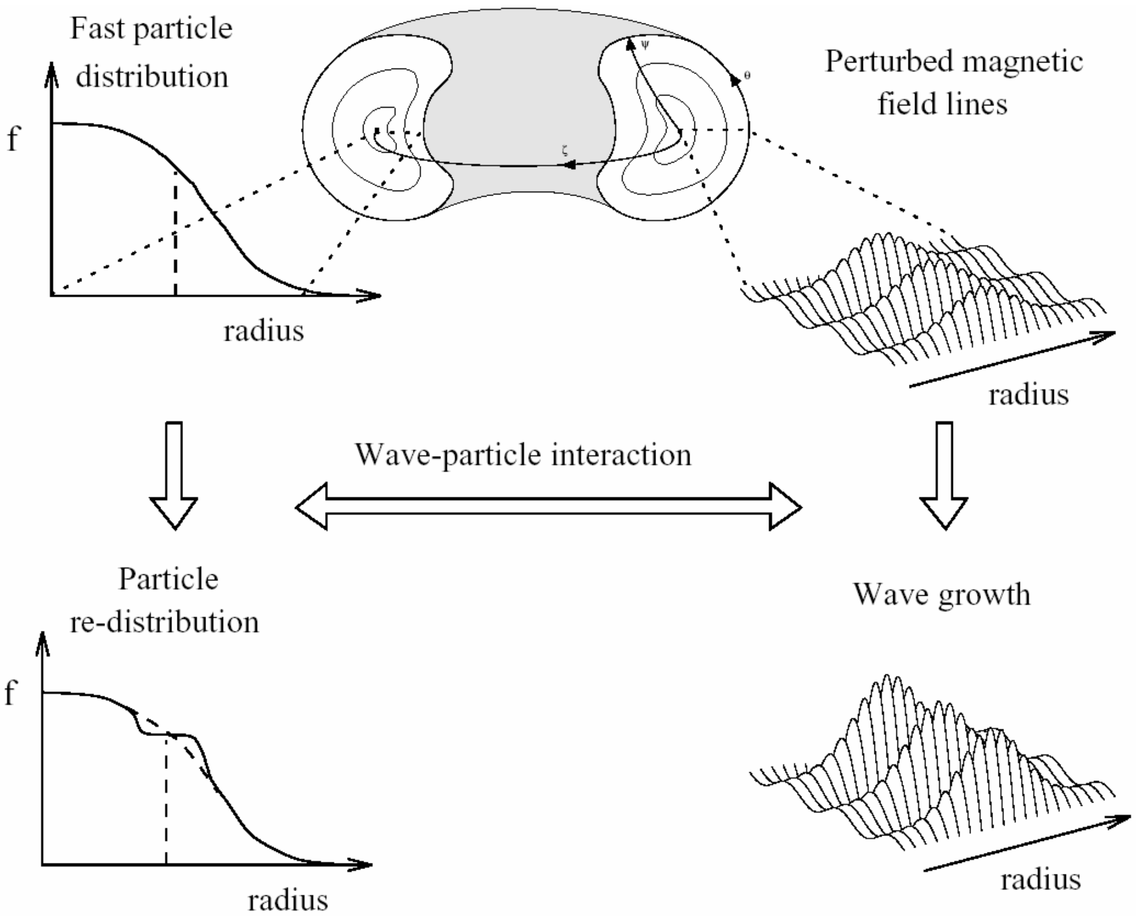
Nonlinear Effects

- **Near-threshold phenomena**
  - System can’t go far beyond threshold
  - Weak source of fast ions ⇒ Population builds up on much longer timescale than characteristic growth time
- **Observed behaviour:**
  - Mode saturation, pitchfork splitting, frequency sweeping
  - [Recall talk by Sharapov]
- **Model with nonlinear HAGIS code**
  - Evolves ensemble of Hamiltonian drift-kinetic markers
  - Delta-f representation
  - Fixed mode structure

Nonlinear Evolution

- **Linearly unstable TAE grows and saturates**
  - Nonlinear wave-particle interaction
  - Wave redistributes fast ions and removes drive

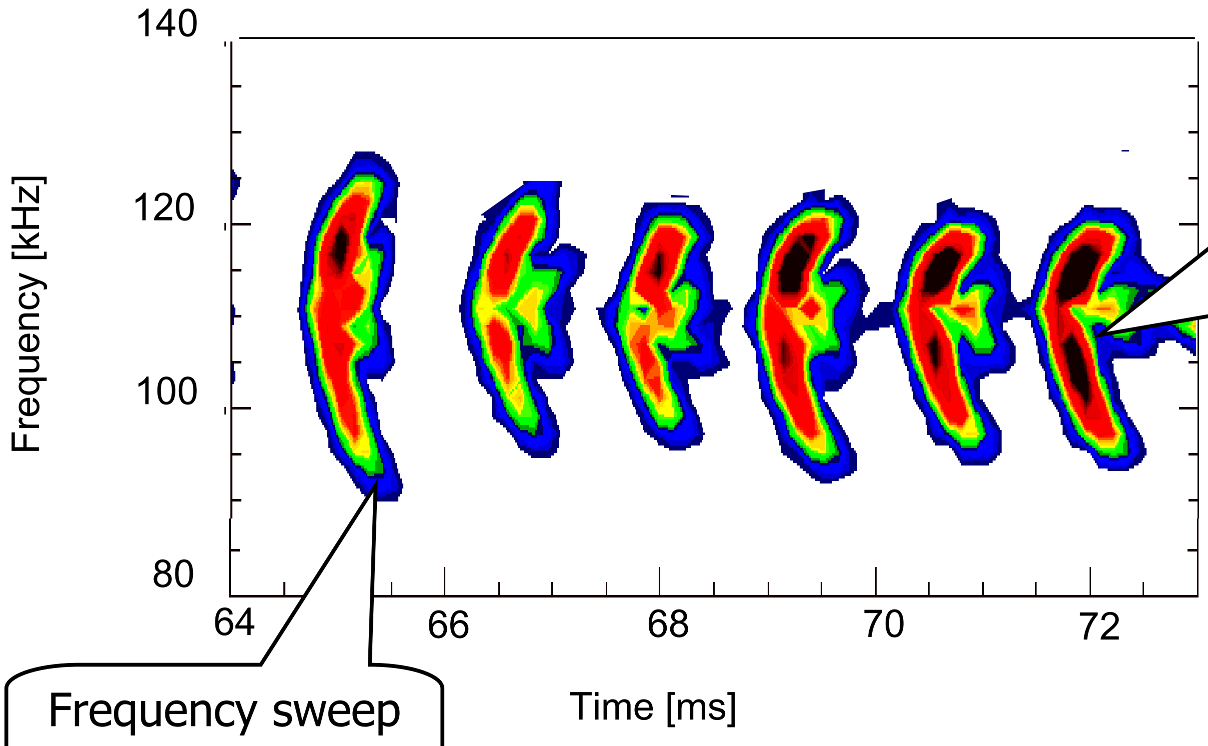
HAGIS code



S. D. Pinches *et al.*, CPC **111** 131 (1998)

Experimental Observations

- **Frequency sweeping in MAST #5568**

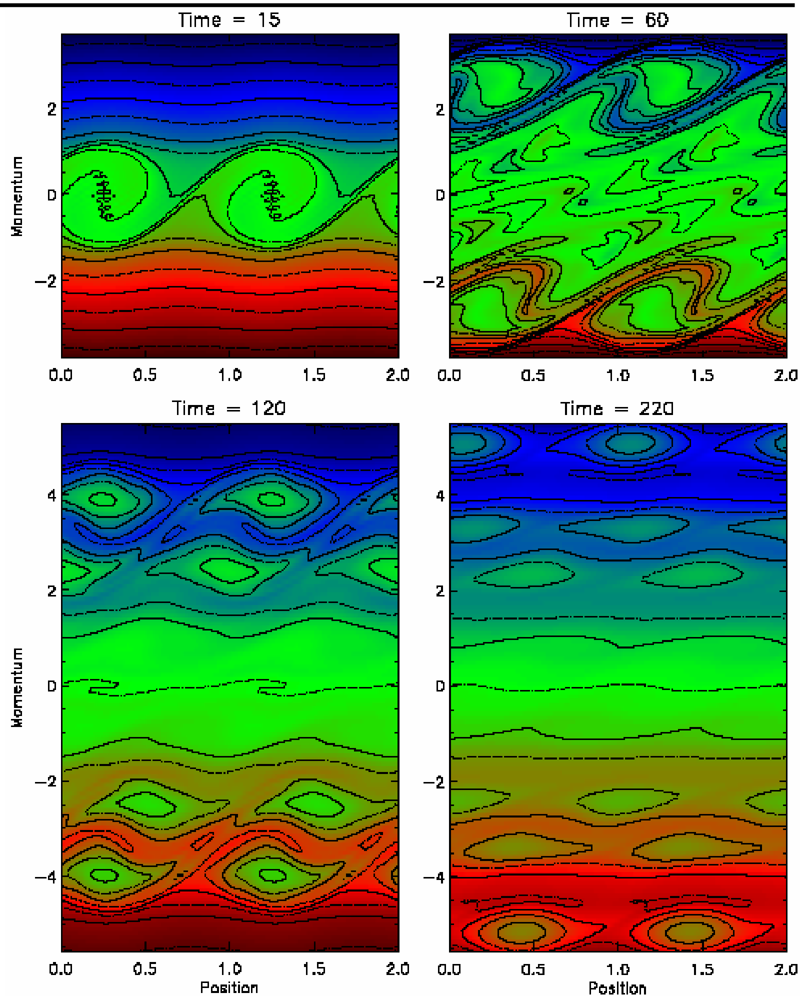


Chirping modes exhibits simultaneous upwards and downwards frequency sweeping

Frequency sweep $\delta\omega/\omega_0 \sim 20\%$

Frequency Sweeping

- **Occurs when mode is close to marginality**
  - Damping balancing drive
- **Structures form in fast particle distribution function**
  - Holes and clumps
- **These support long-lived nonlinear BGK waves**
- **Background dissipation is balanced by frequency sweeping**



[H.L. Berk, B.N. Breizman & N.V. Petviashvili, *Phys. Lett. A* **234** 213 (1997)]
[Errata *Phys. Lett. A* **238** 408 (1998)]

Wave-Particle Interaction

- **Evolution governed by wave-particle interaction**
  - Principle mechanism wave-particle trapping
- **Constants of motion for wave**
  $E(r,t) = C(t)\, E(r,\theta,n\phi - \omega_0 t)$
  - Magnetic moment, μ (if $\omega_0 \ll \omega_c$ and $L_\omega > \rho_i$)
  - Energy in rotating frame, $H' = H - (\omega_0/n)\, P_\zeta$ (if $1/C\; dC/dt \ll \omega_0$)
- **Motion of particles trapped in wave described by "pendulum equation"**

$$\frac{d^2\xi}{dt^2} + \omega_{bl}^2(t) \sin\xi = 0$$

F is a phase space dependent form factor

- **Trapping frequency,** $\omega_{bl}(t) \propto |E|^{1/2} F(H', \mu)$

Diagnostic Information

- **Trapping frequency is related to TAE amplitude**

$$\omega_{b,l}(t) \propto |\delta B|^{1/2}$$

- **Frequency sweep is related to trapping frequency**

$$\delta\omega \propto \omega_b^{3/2} t^{1/2}$$

[Berk, Breizman & Petviashvili, Phys. Lett. A **234** 213 (1997)]

- **Amplitude related to frequency sweep**

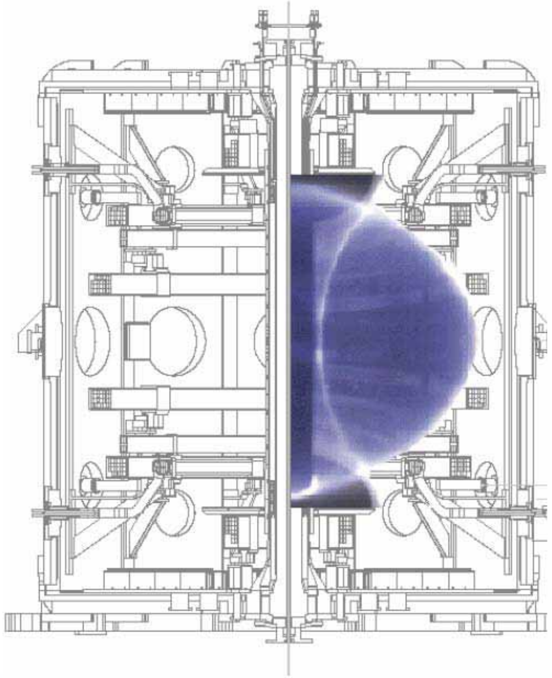
$$\frac{\delta B}{B} = \frac{1}{C_1^2}\left(\frac{\delta\omega^2}{C_2^2 t}\right)^{2/3}$$

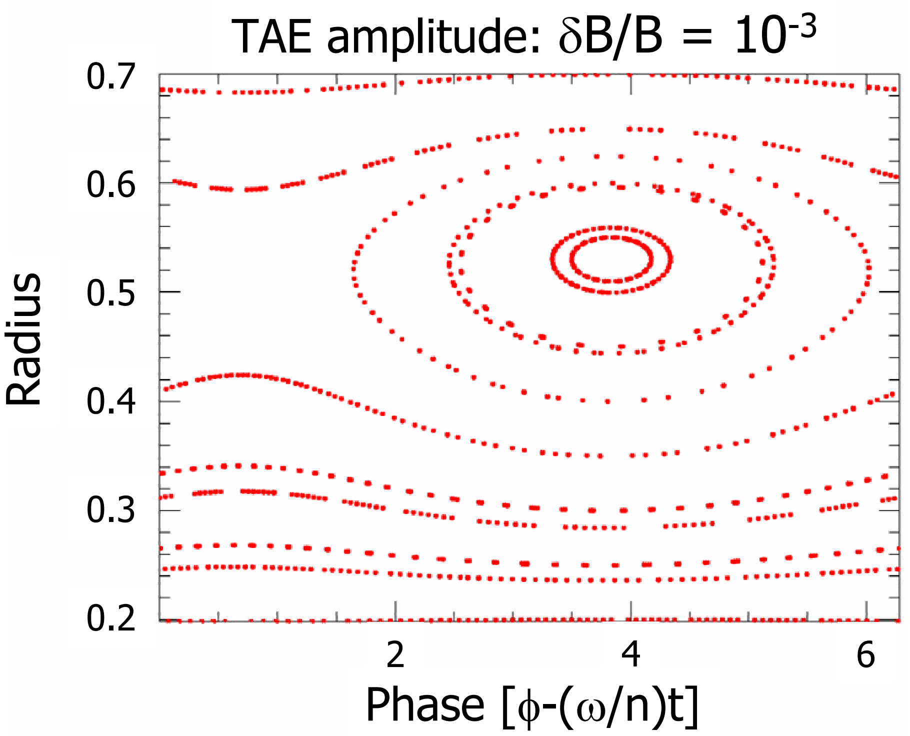
Use numerical simulation to obtain coefficients.

[S D Pinches *et al., Plasma Phys. Control. Fusion* **46** S47-S57 (2004)]

Wave-particle Interaction

- **Employ HAGIS to establish δB from observed frequency sweeping**
  - General geometry



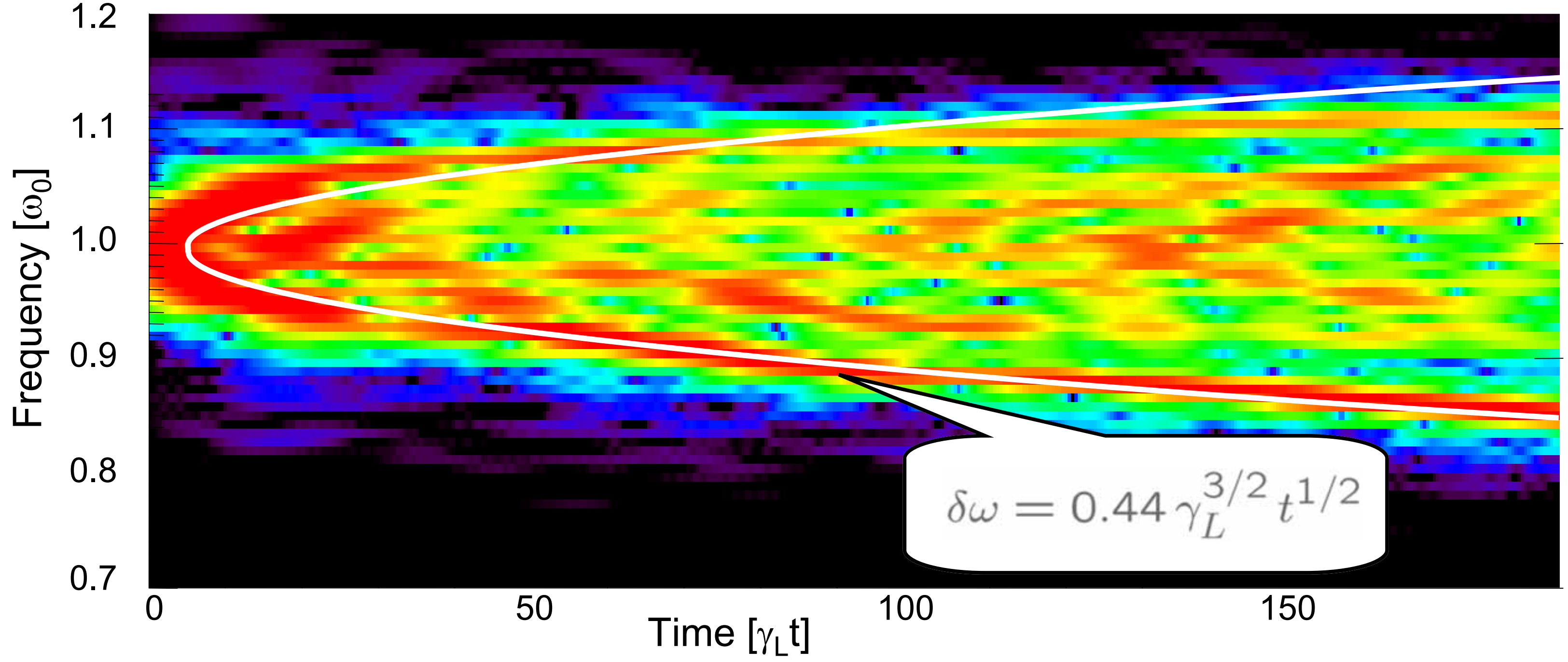


- **TAE trapped fast ions in MAST**
  - All particles have same $H' = E - \omega/n\ P_\zeta = 20$ keV
  - Determine scaling of bounce frequency with mode amplitude:

$$\boxed{\omega_b = 1.156 \times 10^6 \sqrt{(\delta B/B)}}$$

Nonlinear Simulation

- **Fourier spectrum of evolving mode**
  - Sweeping behaviour agrees with analytic theory



$\delta\omega = 0.44\,\gamma_L^{3/2}\,t^{1/2}$

- **Peak internal amplitude calculated agrees with Mirnov measurements,** $\frac{\delta B}{B} = 4 \times 10^{-4}$

[S D Pinches *et al., Plasma Phys. Control. Fusion* **46** S47-S57 (2004)]

Summary & Outlook

- **Suite of codes developed**
- **Capture principle tokamak damping effects with linear gyro-kinetic code (LIGKA)**
  - Describes electron/ion Landau damping, FLR effects, radiative damping
  - Recently extended to antenna version
- **Perturbative hybrid code CAS3D-K developed to address stability boundaries in stellarator**
  - Benchmarked against LIGKA, NOVA-K and theory
- **Mass scaling of electron Landau damping investigated**
  - Trend agrees with experiment, although magnitude too small
- **Model nonlinear frequency sweeping AE with HAGIS**
  - Infer information about internal mode amplitude