Theory and Simulation of Turbulence Spreading in Magnetized Plasmas

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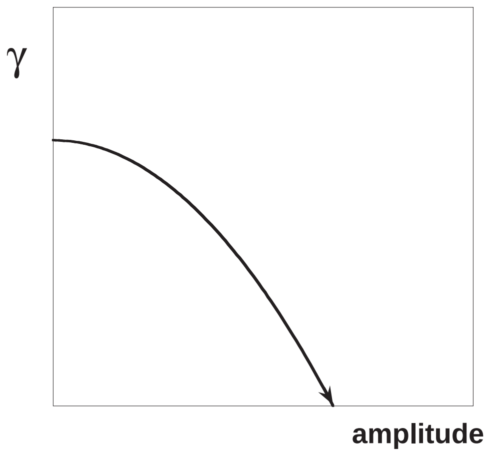
Outline and Conclusions

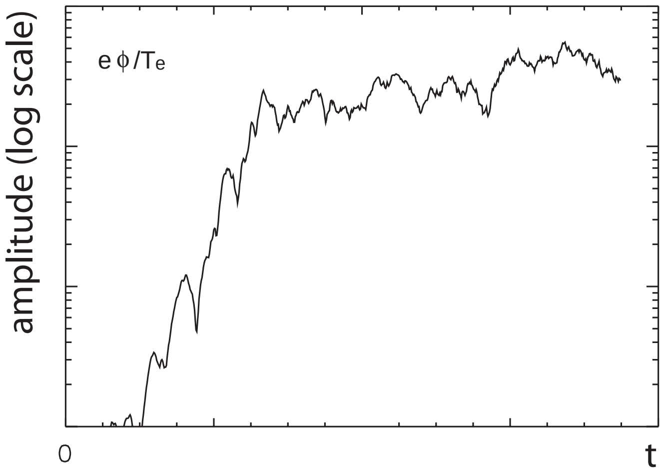
- Turbulence spreading into linearly stable zone is studied using global gyrokinetic particle simulations and theory.
- Motivation from experiments to study spreading of Edge Turbulence into Core.
- Results
  - Fluctuation amplitude in the linearly stable zone can be significant due to turbulence spreading.
  - Sometimes **Spreading of Edge Turbulence** into Core can exceed local turbulence in connection region.
  - It is likely to affect "the edge boundary conditions" used in core modeling, and predictions of pedestal extent.

Determination of Fluctuation Amplitude

$$\gamma = \gamma_{lin} - k_\perp^2 D_{turb} \to 0$$

- Nonlinear coupling induced dissipation leads to saturation (B. Kadomtsev '65)



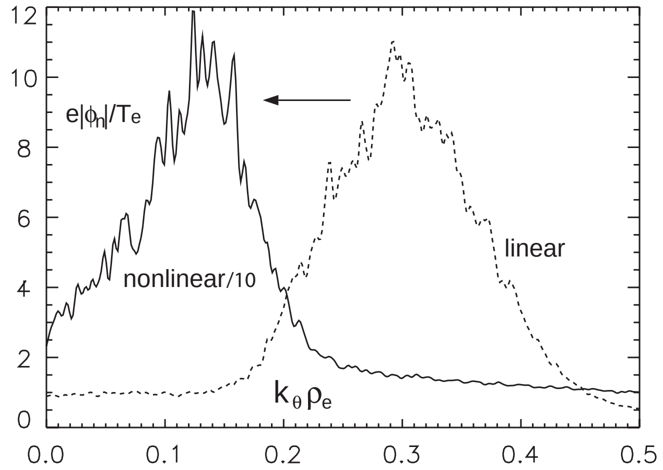


- "Local Balance in Space" for a mode $\mathbf{k}$
- "Conceptual Foundation of Most Transport Models"
- **Missing:**
  - Meso-scale Phenomena: Barrier Dynamics, Avalanches,...
  - Anomalous transport in the region $\gamma_{lin} < 0$
  - **Turbulence Spreading into Less Unstable Zone**

Excitation of Linearly Damped Modes

- Nonlinear Saturation from Balance between:

  γ_{lin} vs. **Spectral Transfer** from Nonlinear Mode Coupling



→ **Non-zero Amplitude for Linearly Damped Modes**

Sagdeev and Galeev, *Nonlinear Plasma Theory* (1969)

Gang-Diamond-Rosenbluth, Phys. Fluids B **3**, 68 (1991)

Hahm-Tang, Phys. Fluids B **3**, 989 (1991)

Horton, Rev. Mod. Phys. (2000) *for more references*

Nonlinear Coupling Leads To Radial Diffusion

- Nonlinear interactions of modes must spread fluctuation energy in radius due to:

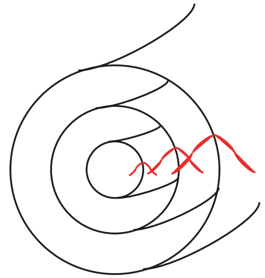
  i) $ik_x \rightarrow \frac{\partial}{\partial x}$

  ii) poloidal harmonics at $q(r) = m/n$

  iii) with different radial extents

  iv) Numerical Studies with both Linear Toroidal Coupling and Nonlinear Coupling

  [Garbet-Laurent-Samain-Chinardet, NF 1994]



- $\mathbf{E} \times \mathbf{B}$ nonlinearity $\rightarrow$ "local turbulent damping" and "radial diffusion" :

$$(\mathbf{k} \times \mathbf{k}' \cdot \mathrm{b})^2 R_{k,k'} I_k I_{k'} \rightarrow -\frac{\partial}{\partial x} D_r(I) \frac{\partial}{\partial x} I + k_\theta^2 D_\theta(I) I.$$

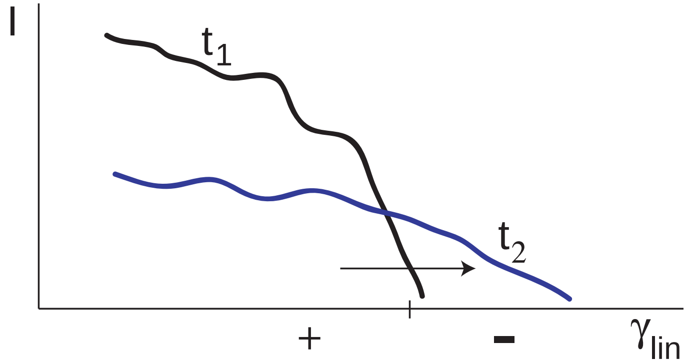
[eg., Kim-Diamond-Malkov-Hahm *et al.*, NF 2003]

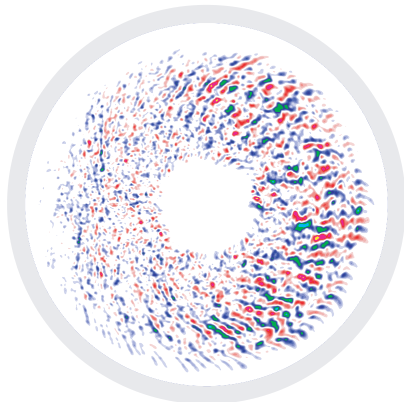
Simple Model of Turbulence Spreading

[**Hahm, Diamond, Lin, Itoh, Itoh**, PPCF **46**, A323 '04]

$$\frac{\partial}{\partial t} I = \gamma(x) I - \alpha I^2 + \chi_0 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I\right)$$

- $\gamma(x)$ is "local" growth rate, α: a local nonlinear coupling
- $\chi_0 I = \chi_i$ is a turbulent diffusivity
- I: turbulence intensity, $\Sigma_{\mathbf{k}}$ *Modes* $\sim \Sigma$ *Eddys*





$$\frac{\partial}{\partial t} \int_{x-\Delta}^{x+\Delta} dx' I(x', t) \sim \chi_0 I \frac{\partial}{\partial x} I \Big]_{x-\Delta}^{x+\Delta} + \ldots$$

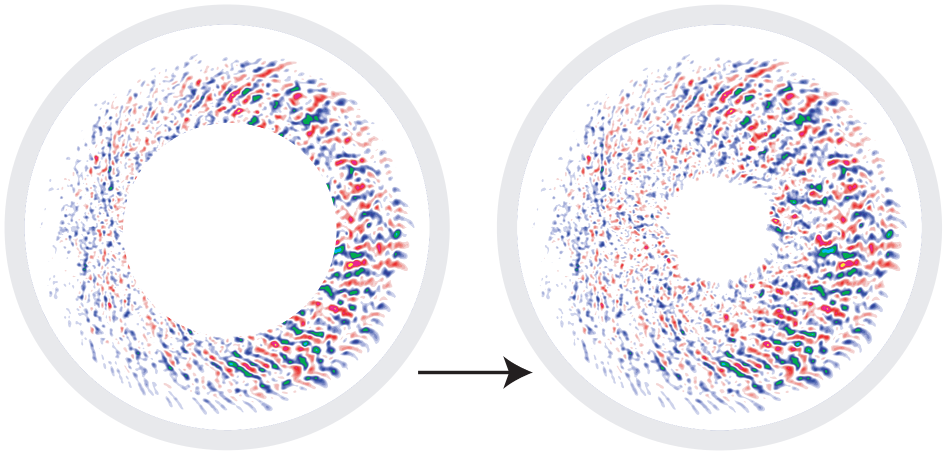
- **Profile of Fluctuation Intensity** crucial to its Spatio-temporal Evolution

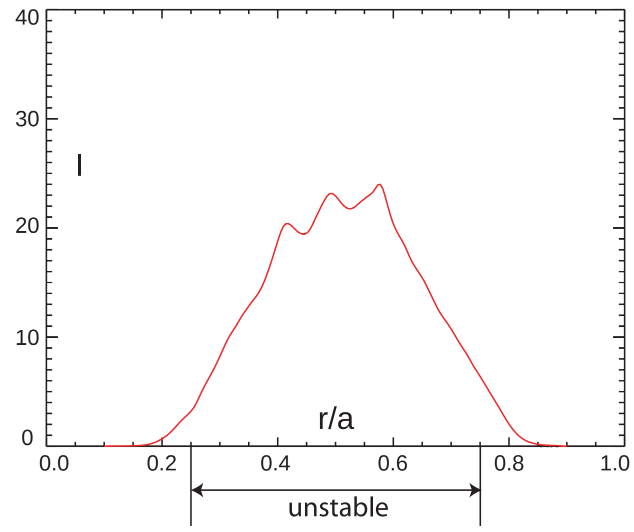
Energetics of Turbulence Spreading

- Look for Radial Energy Flow from:
$\frac{\partial}{\partial t}\int_{x-\Delta x}^{x+\Delta x} dx' I(x',t) = \Delta'(I) I(x,t) + \int_{x-\Delta x}^{x+\Delta x} dx'(\gamma(x')I - \alpha I^2)$

- Here, $\mathbf{\Delta'(I)} \equiv \chi_0 \frac{\partial}{\partial x} I]_{x-\Delta x}^{x+\Delta x}$
[*Gurcan, Diamond, Hahm, Lin*, Phys. Plasmas, March '05]
characterizes the net flux of turbulence into (out of) $[x-\Delta x, x+\Delta x]$ via a jump in the slope *a la* **FKR**
- The **sign** of $\Delta'(I)$ determines the condition for growth!
- $\gamma \sim \Delta'(I)/\Delta x$ and $\gamma_{prop} \sim U_x/\Delta x$ illustrate its physical meaning and significance.
- Note that **FKR** predicts $\gamma \propto \eta^{3/5}\Delta'^{4/5}$, $\Delta x \propto \eta^{2/5}\Delta'^{1/5}$ satisfying $\gamma \propto \Delta' \eta/\Delta x$
cf. For Tearing Modes: presentations on Friday

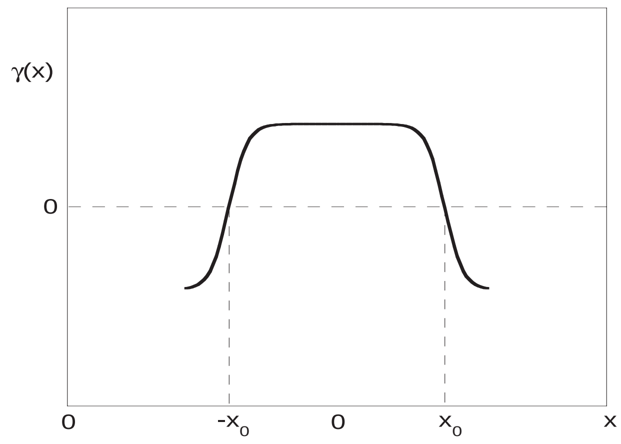
Turbulence Spreading after Local Saturation





From Gyrokinetic (GTC) simulations, turbulence spreads radially ($\sim 25\rho_i$) into the linearly stable zone, causing deviation from GyroBohm scaling.

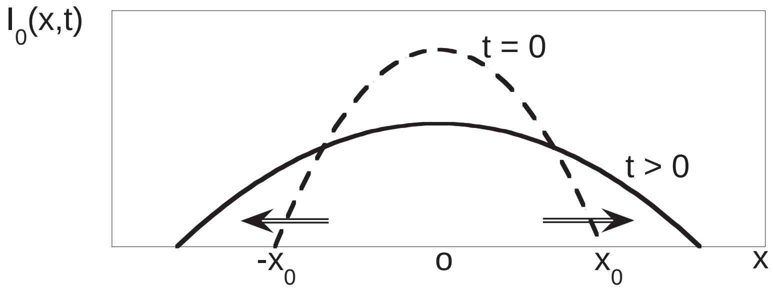
[Lin *et al.*, Phys. Rev. Lett. (2002)]



Local excitation rate γ(x) as a function of radius

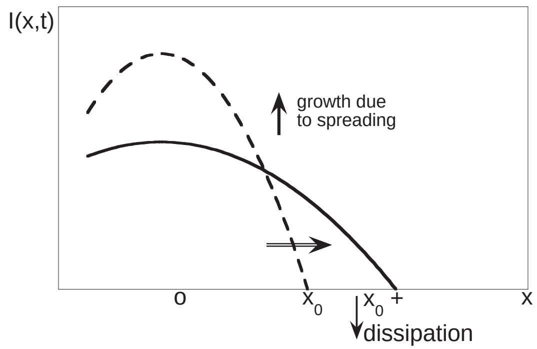
Propagation and Saturation of Fluctuation Front

- The **nonlinear** diffusion, in the absence of dissipation, will make the front propagate beyond x_0 indefinitely.



Front propagation stops when radial flux due to propagation is balanced by dissipation:

$$T_{prop} \simeq \Delta / U_x \Longleftrightarrow T_{damp} \sim (|\gamma'|\Delta)^{-1}$$

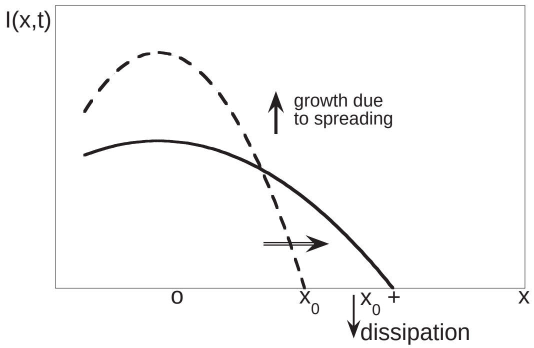


$\Delta^2 \simeq \frac{12\chi_0 I_0}{|\gamma'|x_0}$, using the values from simulation $\rightarrow \Delta \simeq 18\rho_i$

From GK simulation for a profile considered: $\Delta \simeq 25\rho_i$

On the Scaling of Turbulence Spreading

From the balance between
Radial Flux due to Propagation
and Dissipation:

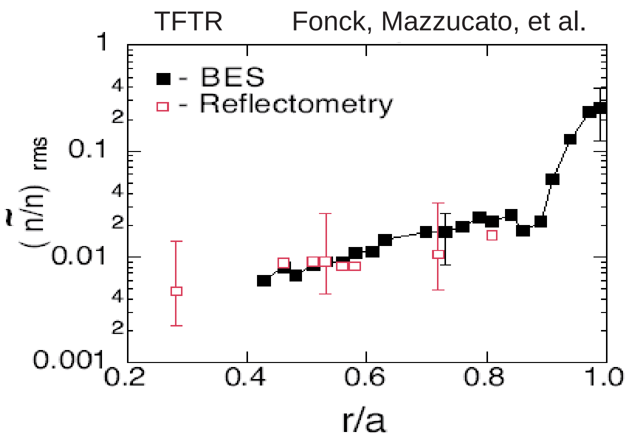


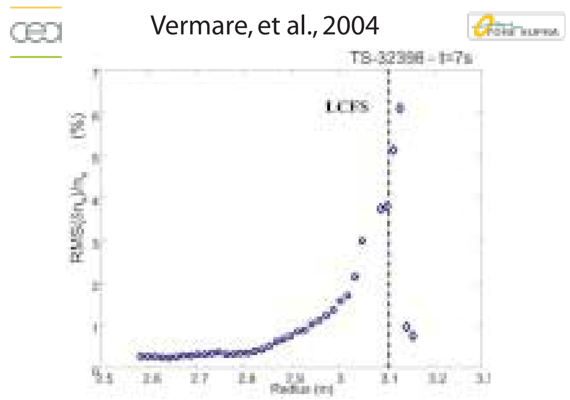
$$\Delta \simeq \left(\frac{12\chi_0 I_0}{|\gamma'|x_0}\right)^{1/2}$$

$\sim$ **Kolmogorov Dissipation Scale:** $k_d = \left(\frac{\epsilon}{\nu^3}\right)^{1/4}$

from the balance between
Energy Input from Nonlinear Interaction
and Energy Drain from Viscous Dissipation

Connection Region between Edge and Core





- Profile of Turbulence Intensity crucial in turbulence spreading: $\Gamma_I = -\chi(I)\frac{\partial}{\partial x}I$

- Core confinement improvement
  after L-H transition:
  JET, ASDEX, DIII-D, C-mod,...

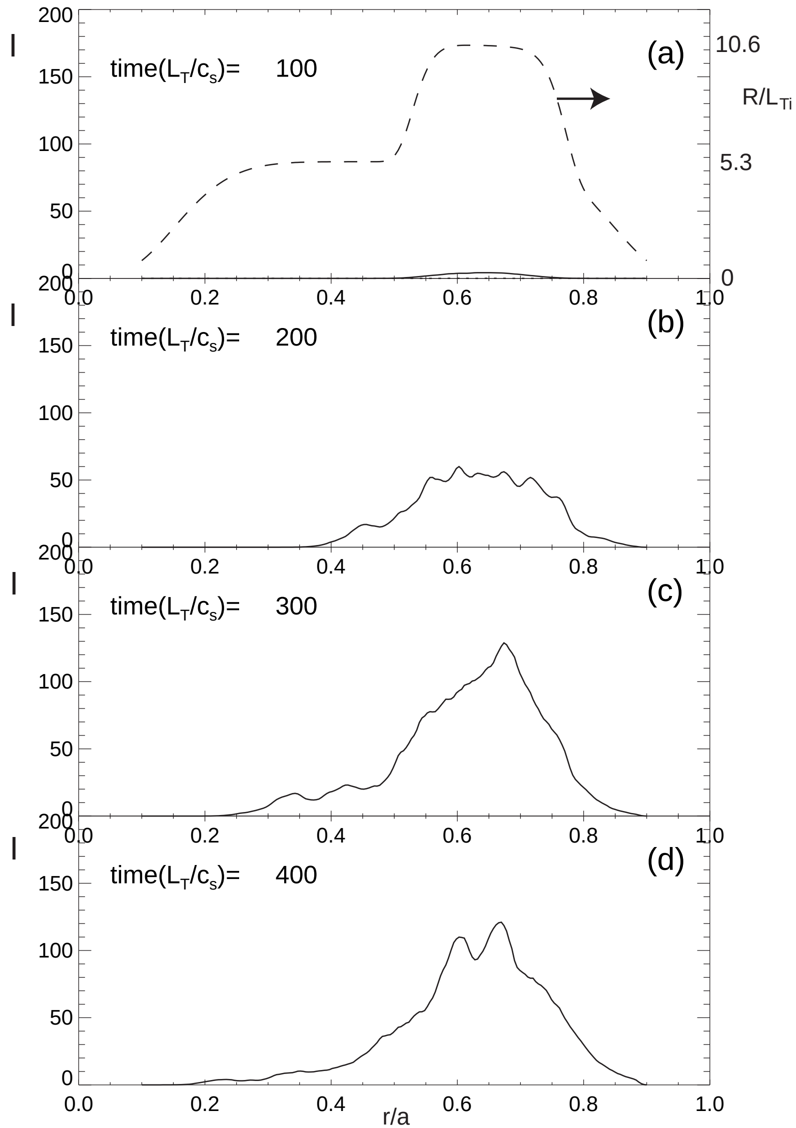
- **Connection Region:**
  Local Turbulence + **Incoming Edge Turbulence**

Turbulence Spreading from Edge to Stable Core

- **Nonlinear GTC Simulations of Ion Temperature Gradient Turbulence:**
  $\frac{R}{L_T} = 5.3$ at core
  (within Dimits shift regime)
  $\frac{R}{L_T} = 10.6$ at edge:

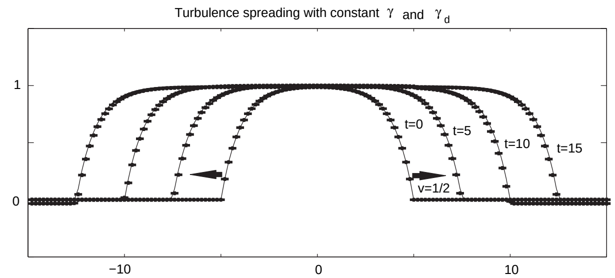
- Initial Growth at Edge
  $\rightarrow$ Penetration into stable Core
  (Hahm-Lin-Diamond,
  PPCF '04, PoP '04)

- Saturation Level at Core:
  $\frac{e\delta\phi}{T_e} \sim 3.6\frac{\rho_i}{a}$
  $\rightarrow \nabla \cdot \mathbf{\Gamma_I} >> \gamma_{local} I$



Spreading in Unstable Zone

[**Gurcan, Diamond, Hahm, and Lin**, *Phys. Plasmas, March '05*]



$$\frac{\partial}{\partial t}I = \gamma(x)I - \alpha I^2 + \chi_0\frac{\partial}{\partial x}(I\frac{\partial}{\partial x}I)$$

- When $\gamma(x)$, α, χ_0 are constant in radius, the Fisher-Kolmogorov equation with nonlinear diffusion exhibits **"propagating front"** solutions.
- The spreading can beat local growth and a solution exhibits ballistic propagation $d(t) = U_x t$ with

$$U_x = \gamma^{1/2} \times (\frac{\chi_0 I}{2})^{1/2}$$

- $U_x \sim$ geometric mean of "local growth" and "turbulent diffusion", faster than transport time scale.

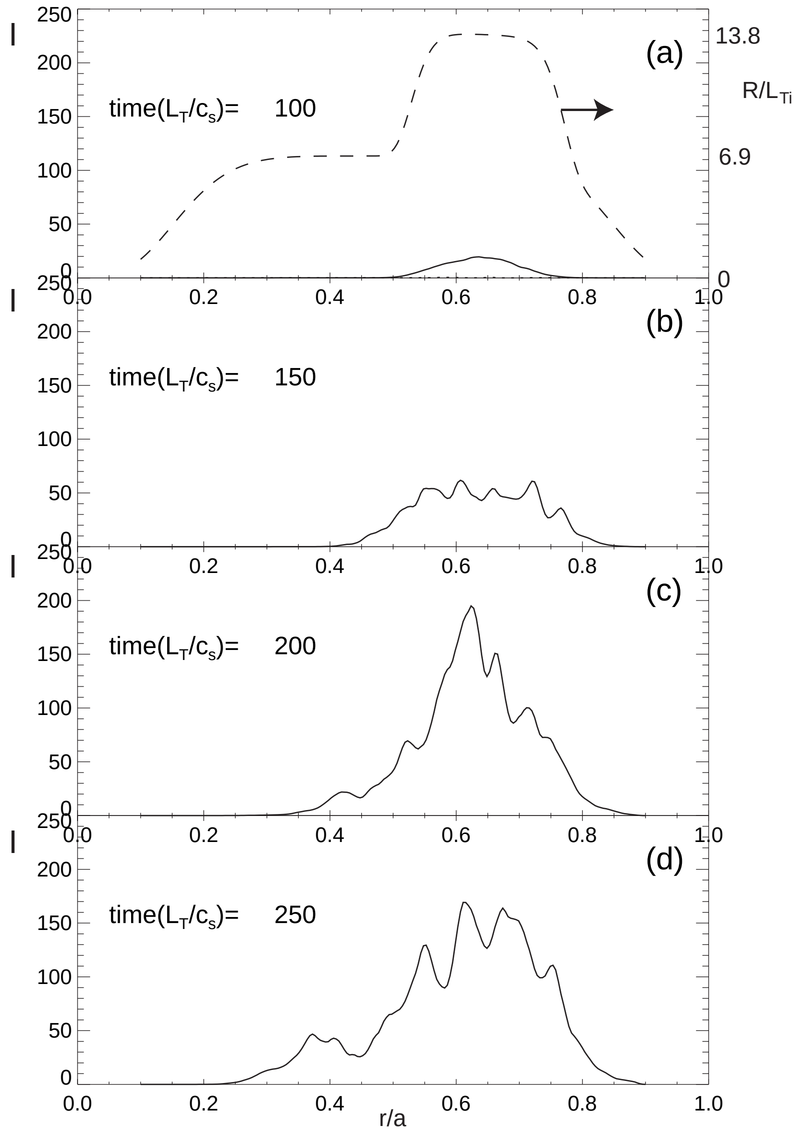
Edge Turbulence Spreading to Unstable Core

- **Nonlinear Gyrokinetic Simulations of Ion Temperature Gradient Turbulence:**
  $\frac{R}{L_T} = 6.9$ at core (Cyclone value)
  $\frac{R}{L_T} = 13.8$ at edge

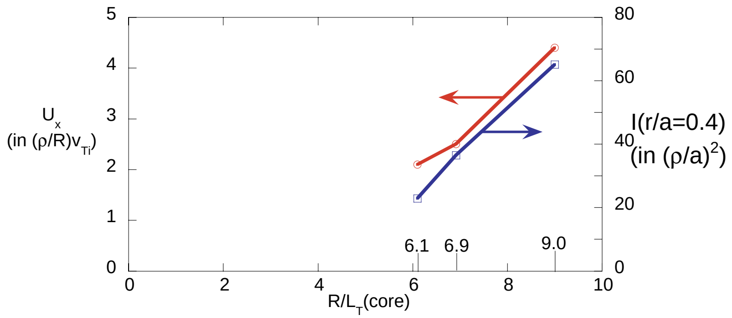
- Initial Growth at Edge followed by **Ballistic Front Propagation** into Core

- Saturation Level at Core $\sim 2\times$ Core (only) Result

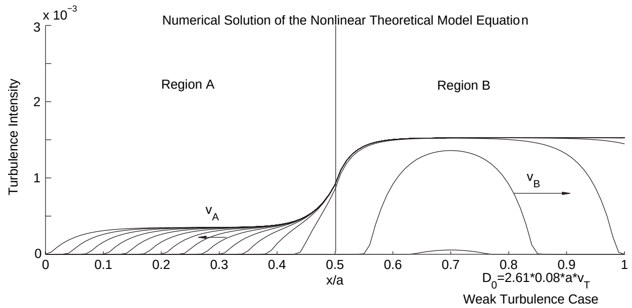
  $\nabla \cdot \Gamma_{\mathbf{I}} \sim \gamma_{local} I$



Front Propagation Speed Increases with R/L_T



- From Simulation, U_x and I increase with $(\frac{R}{L_T})$
- Nonlinear Diffusion Model: $U_x \propto (\gamma I)^{1/2}$
  by [Gurcan-Diamond-Hahm-Lin, *PoP* '05]



- Toroidal Linear Coupling dominant Regime: $U_x \sim \frac{\rho_i}{R} v_{Ti}$
  by [Garbet-Laurent-Samain-Chinardet, NF '94]
- Four Wave Model: Complex Bursty Spreading
  by [Zonca-White-Chen, PoP '04]

Summary

- Turbulence spreading has been widely observed in global gyrokinetic particle simulations: It can be responsible for deviation of transport scaling from GyroBohm.

- Fluctuation Intensity in the linearly stable region can be significant due to turbulence spreading.

- Sometimes **Spreading of Edge Turbulence** into Core can exceed local turbulence in connection region.

- It is likely to affect "the edge boundary conditions" used in core modeling, and predictions of pedestal extent.

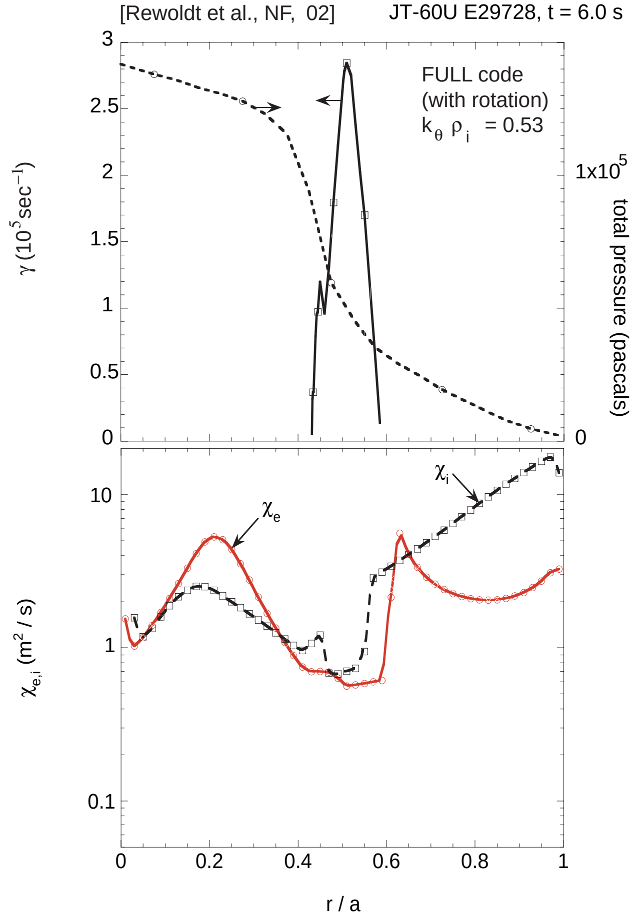
Turbulence Spreading has been widely observed

- From Most Global Gyrokinetic/Gyrofluid Simulations:
  - X. Garbet *et al.*, NF '94 (Mode-coupling in Torus)
  - R. Sydora *et al.*, PPCF '96 (Torus with Zonal Flows)
  - Y. Kishimoto *et al.*, PoP '96 (Torus with Zonal Flows)
  - S. Parker *et al.*, PoP '96 (Torus without Zonal Flows)
  - W.W. Lee *et al.*, PoP '97 (Torus without Zonal Flows)
  - Y. Idomura *et al.*, PoP '00 (Sheared Slab with Zonal Flows)
  - **Z. Lin** *et al.*, **PRL '02** (Torus with Zonal Flows)
  - L. Villard *et al.*, IAEA '02 (Cylinder with Zonal Flows)
  - R. Waltz *et al.*, PoP '02 (Torus with Zonal Flows)
  - Y. Kishimoto *et al.*, H-mode '03 (Sheared Slab with ZF)

- Neither Zonal Flows nor Toroidal Coupling necessary for Turbulence Spreading.

Anomalous Transport where $\gamma_{lin} < 0$

Core of Reversed Shear Plasmas where profiles are nearly flat (JT-60U, TFTR, DIII-D,...)



→ Nonlinearly Unstable?

(CDBM (Itoh, Itoh, yagi, Fukuyama)

Self-sustained Turbulence (B. Scott))

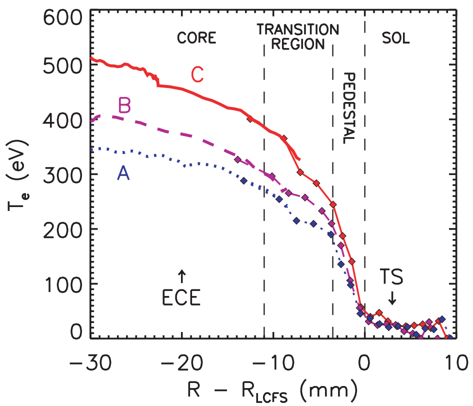
→ **Spreading from the Linearly Unstable Zone**

Distinction between "Core" and "Edge" blurred

- *Researchers have frequently divided the tokamak into three zones — a central sawtoothing zone, a middle 'confinement zone', and an edge zone...*
  Goldston-U.S.A. *Kyoto IAEA (1986)*

- *the edge..., often used as a boundary condition for core transport modeling*
  V. Parail, *Plasma Phys. Control. Fusion*, **44**, A63 (2002)

- $\frac{\partial}{\partial x}\gamma(x) \sim \frac{\partial^2}{\partial x^2}P$: large at the top of pedestal



C-mod, Hughes et al., 2002

Long Term Behavior: Sub-Diffusion

- Self-similar Variable: $\ell(t)^2 \sim \chi_0 I^\beta t$

- $I(t)\ell(t) = I(0)\ell(0) \equiv \epsilon$, up to dissipation

- $\ell(t) \sim [\chi_0 \epsilon^\beta t]^{\frac{1}{2+\beta}}$
  $\sim t^{1/3}$: Weak Turbulence
  $\sim t^{2/5}$: Strong Turbulence

- Previous numerical mode coupling study:
  X. Garbet *et al.*, NF 1994
  Linear toroidal coupling usually dominates $\sim t^1$: convective
  Without linear toroidal mode coupling $\sim t^{1/2}$: **diffusive**

Short Term Behavior: Ballistic Propagation

- $x_{front} = (x_0^3 + 6\epsilon\chi_0 t)^{1/3}$

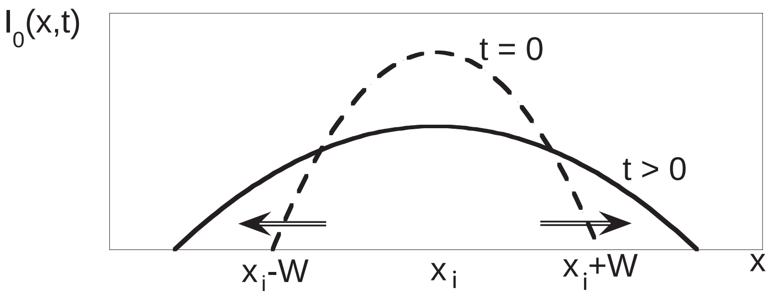
- $U_x = \frac{d}{dt} x_{front}$
  $\sim 2\epsilon\chi_0 / x_0^2$: for small t (consequence of $\Delta << x_0$)
  $\sim t^{-2/3}$: for large t (sub-diffusion)
  Note: $\epsilon \propto I$, turbulence intensity

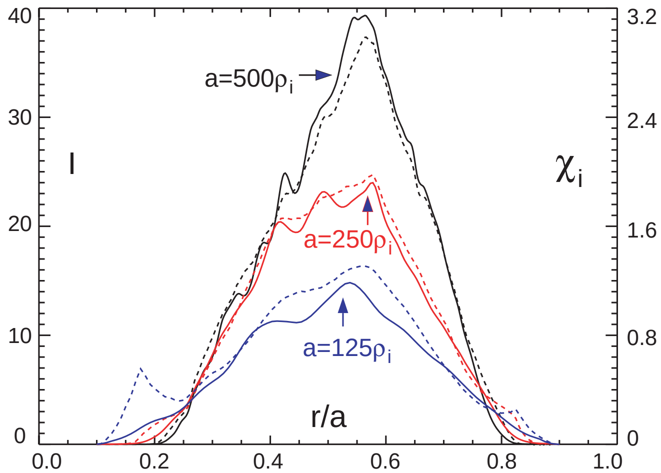
- Scaling of U_x drastically different from $\mathbf{V}_{gr}$ of linear drift (ITG) wave

  $\rightarrow$ contrast our theory from others relying on linear dispersion
  [eg., Garbet *et al.*, PoP '96; Zonca *et al.*, PoP '04]

Simple theory captures ρ^* dependence of spreading

$$I_0(x,t) = \frac{\epsilon}{(6\epsilon\chi_0 t + W^3)^{1/3}}\left(1 - \frac{(x-x_i)^2}{(6\epsilon\chi_0 t + W^3)^{2/3}}\right) \times H\left((6\epsilon\chi_0 t + W^3)^{1/3} - |x - x_i|\right)$$





Spreading of Sub-Critical Turbulence

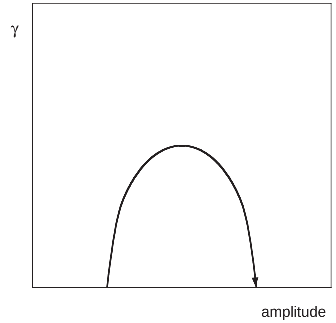
[**Itoh, Itoh, Hahm, and Diamond**, *submitted to J. Phys. Soc. Jpn. '04*]

$$\frac{\partial}{\partial t} I = \Gamma_{NL}(I, x) I + \chi_0 \frac{\partial}{\partial x}(I \frac{\partial}{\partial x} I)$$

- Model Sub-critical Turbulence [*eg., K. Itoh, PPCF* **36**, *A307*]:

  $\Gamma_{NL}(I, x) > 0$ for $I_{crit} < I < \frac{\gamma_0}{\alpha}$,
  $\Gamma_{NL}(I, x) = 0$ for $I < I_{crit}, I > \frac{\gamma_0}{\alpha}$, at $|x| < L$

  $\Gamma_{NL}(I, x) < 0$ at $|x| > L$
  according to local damping



- Due to turbulence spreading, there exists a minimum size system (L) that can sustain the self-sustained turbulence

Suocola Grande di San Ae

Structural Formation and Selection Rules in Turbulent Plasmas

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