Internal Kink Stabilisation and the Properties of Auxiliary Heated Ions and Alpha Particles

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Outline

- Sawtooth model - importance of macroscopic drive δW.
- Important kinetic effects often neglected in hybrid stability codes:
  - ▹ Plasma rotation: effect on internal kink mode
  - ▹ Finite orbit effects in adiabatic response: unbalanced NNBI and asymmetric distributions of highly energetic ions.
  - ▹ Anisotropy: degree to which auxiliary ions can represent role of alpha particles
  - ▹ Anisotropy: effect on the equilibrium
- Conclusions

Model for Sawtooth Trigger

Three distinct components to ITER sawtooth model: [F. Porcelli *et al* PPCF 38, 2163 (1996)].

1) Current and pressure profiles evolve during sawtooth quiescence.

2) Sawtooth Trigger. In JET it is argued that instability could be governed by threshold against $m = 1$ reconnection with two-fluid effects [L. Zackharov *et al* Phys. Fluids B 5, 2498 (1993)]:

- $\tau_{saw} \lesssim$ resistive diffusion time:

$$\frac{\delta \hat{W}}{s_1} > \hat{\rho}_i \longrightarrow \frac{\delta \hat{W}}{s_1} < \hat{\rho}_i \quad \text{AND} \quad s_1 < s_c(\beta) \longrightarrow s_1 > s_c(\beta).$$

where Larmor radius ρ_i implies ion kinetic regime, and macroscopic drive:

$$\delta W = -\frac{1}{2} \int d^3x \, \boldsymbol{\xi}^* \cdot (\boldsymbol{\delta j} \times \boldsymbol{B} + \boldsymbol{j} \times \boldsymbol{\delta B} - \boldsymbol{\nabla} \cdot \underline{\underline{\delta P}}).$$

3) Profiles are relaxed at sawtooth crash. s_1 and β_p return to smaller values.

Macroscopic Drive Terms employed in 96 ITER Model

Bussac Toroidal term:

$$\delta\hat{W}_{\mathsf{MHD}} \approx (1-q_0)\left[\left(\beta_p^c\right)^2 - \beta_p^2\right].$$

The following assumes $P_{i,h} = P_0(1-(r/a)^2)$ and $\epsilon_1 \equiv r_1/R \sim (r_1/a)^2$:

Trapped thermal ion term [M. D. Kruskal and C. R. Oberman, Phys. Fluids 1, 275 (1958)] assumes $\omega \gg \{\omega_{*i}, \langle\omega_{mdi}\rangle\}$:

$$\delta\hat{W}_{\mathsf{KO}} \approx \frac{1}{4\pi\sqrt{2\epsilon_1}}\beta_{pi}.$$

Isotropic ion population (e.g. alpha particles, balanced NBI ions, thermal ions) and assumption $\omega \ll \{\omega_{*h}, \langle\omega_{mdh}\rangle\}$ [B. Coppi *et al* Phys. Fluids B 2, 927 (1990)]:

$$\delta\hat{W}_{kh} \approx \frac{1}{3\pi\sqrt{2\epsilon_1}}\beta_{ph}.$$

Trapped thermal ions could be more stabilising than isotropic fast ions if $\beta_i > \beta_h$.

Effect of Toroidal Rotation in Dispersion Relation

- Perturbed inertia evaluated in frame absent of electrostatic potential Φ. e.g. without two fluid effects:

$$\delta K \equiv -\frac{1}{2}\int d^3x\rho_m\,|\delta V - V_\Phi|^2 = -\frac{1}{2}\int_s d^3x\rho_m\,\xi_\theta^2(\omega-\Omega_\Phi)^2.$$

where $\Omega_\Phi = -q\Phi'/B_0 r$ and $\xi_\theta = i(r\xi_r)' \gg \xi_r$ near $q = 1$.

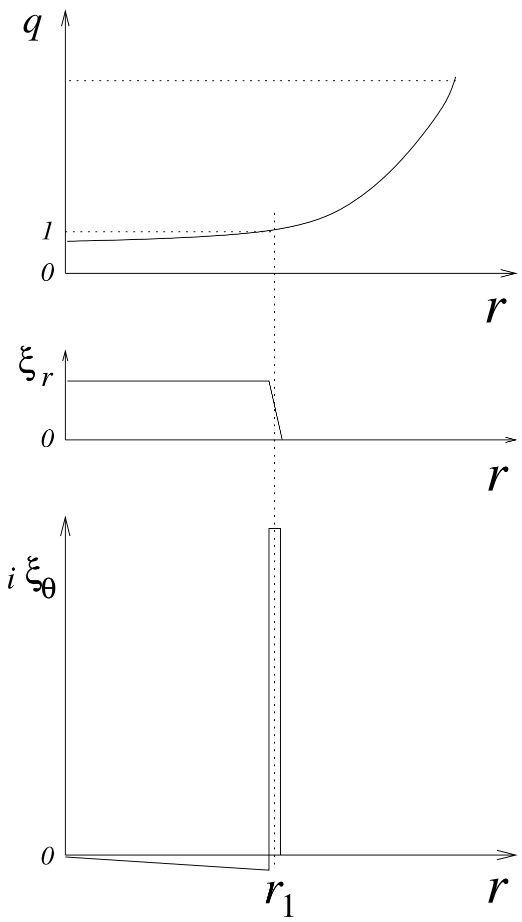
- Magnetic precession is dominated by $E \times B$ drift:

$$\langle\omega_d\rangle = \omega_{md} + \Omega_\Phi(r).$$

- In plasma frame, normal mode and precession drift frequencies are:

$$\tilde{\omega} = \omega - \Omega_\Phi(r_1) \quad \text{and} \quad \langle\omega_{md}\rangle + \Delta\Omega_\Phi(r)$$

where $\Delta\Omega_\Phi(r) = \Omega_\Phi(r) - \Omega_\Phi(r_1)$.



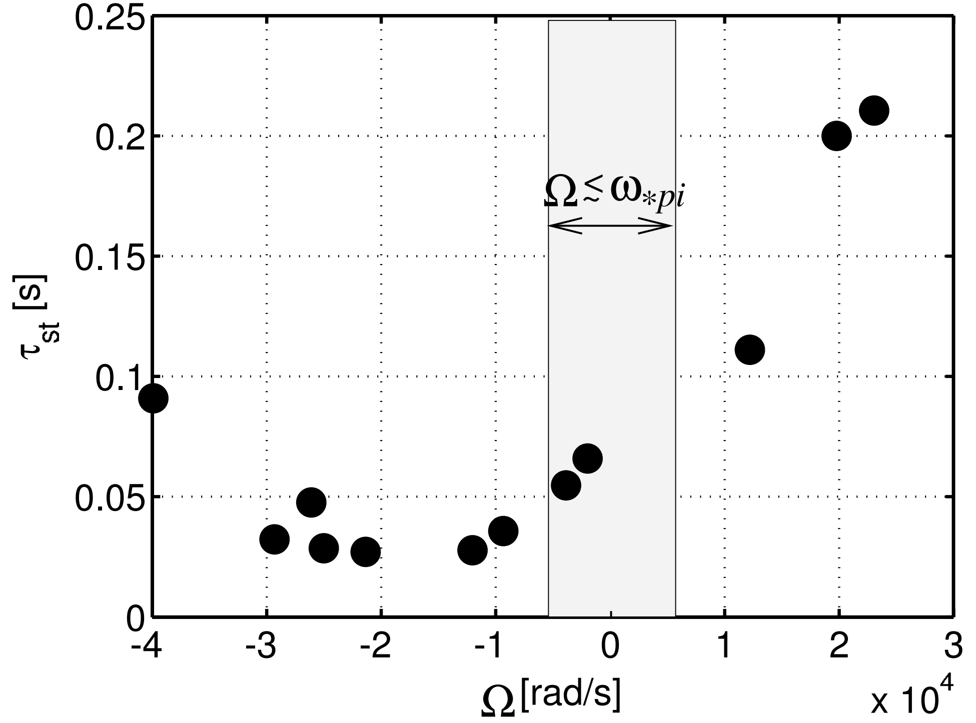
Effect of Toroidal Rotation in Dispersion Relation

The dispersion relation $\delta K + \delta W = 0$ can then be solved for $\tilde{\omega} = \omega - \Omega_\Phi(r_1)$:

$$i \left. \frac{s3^{1/2}[\tilde{\omega}(\tilde{\omega} - \omega_{*i})]^{1/2}}{3\pi\epsilon^2\omega_A} \right|_{r_1} = \delta\hat{W}_{\mathsf{MHD}} + \beta_i \int d^3x d^3v \, \langle \boldsymbol{\xi} \cdot \boldsymbol{\kappa} \rangle^2 \left[\frac{\tilde{\omega} - \Delta\Omega_\Phi(r) - \omega_{*i}}{\tilde{\omega} + i\nu_{eff} - \Delta\Omega_\Phi(r) - \langle \omega_{mdi} \rangle} \right].$$

- Rigid rotation only Doppler shifts mode $\omega \rightarrow \tilde{\omega} = \omega - \Omega_\Phi(r_1)$.
- Solution for sawtooth mode typically $\tilde{\omega} \sim \omega_{*i}$.
- Third adiabatic invariant conserved for $\tilde{\omega} \sim \omega_{*i} \ll \langle \omega_{mdi} \rangle + \Delta\Omega_\Phi(r)$.
  - ▹ implies improved stabilisation for $\Delta\Omega_\Phi(r) > 0$ (co-rotation). Impaired stabilisation for $\Delta\Omega_\Phi(r) < 0$ (counter-rotation).
- Fast ion response modified much less than thermal ion because typically $|\Delta\Omega_\Phi| \lesssim \langle \omega_{mdh} \rangle$.
- Large co-rotation will yield KO stabilisation for thermal ions. Counter-rotation?

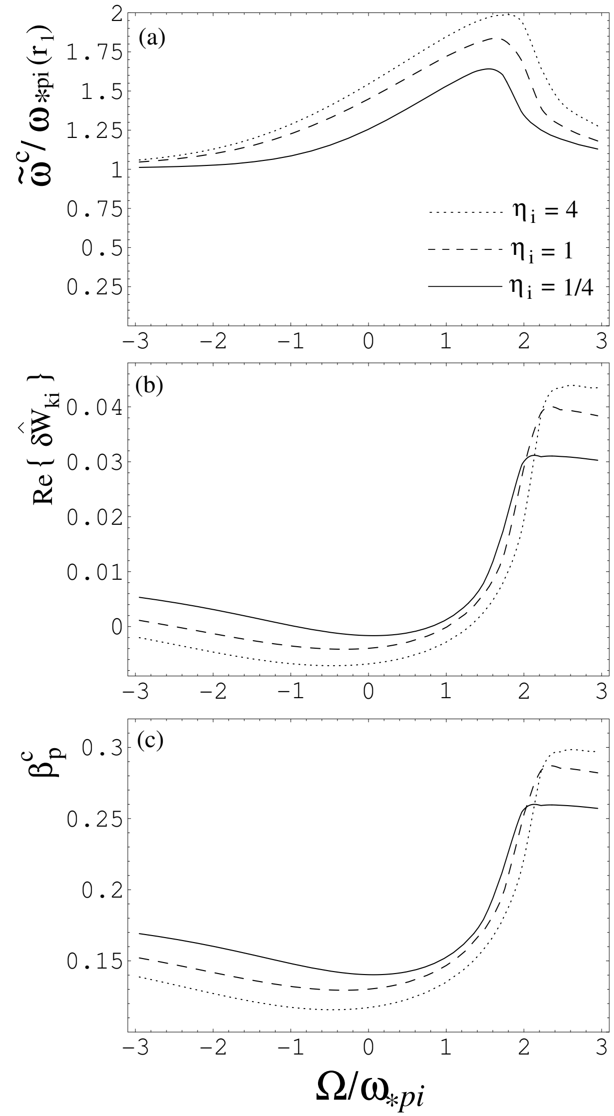
Sawtooth Period and Toroidal Rotation



F. Nave *et al*, Submitted to Nuc. Fusion (2005)

Negative ion neutral beam (NNBI) in ITER-FEAT is predicted to induce central toroidal flows of $\Omega \sim 1.5 \times 10^4$ rad/s [R. Budny, 4th IAEA, Gandinhagar (2005)] for which $\Omega \sim \omega_{*pi}$.

Since normalised rotation Ω/ω_{*pi} is the important quantity, these predictions indicate that toroidal rotation will not have such a large effect on sawteeth in ITER.



J. P. Graves: PPCF 42, 1049 (2000)

Asymmetric Negative Ion Based NBI

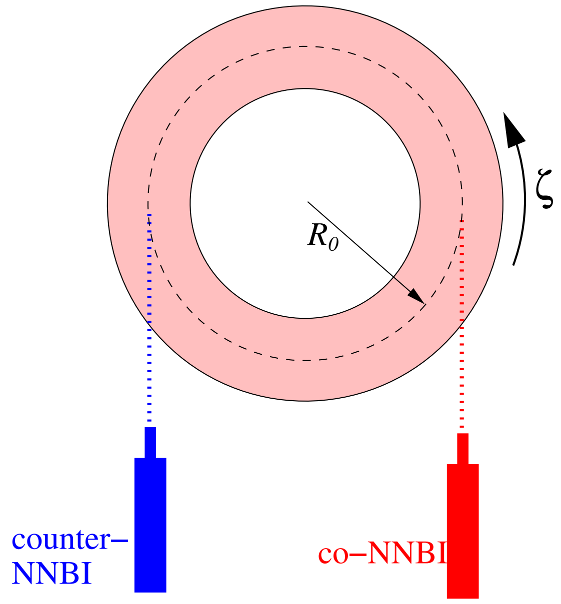
- Highly tangential injection (350 keV in JT-60U) is employed. Passing fraction of particles is very large.

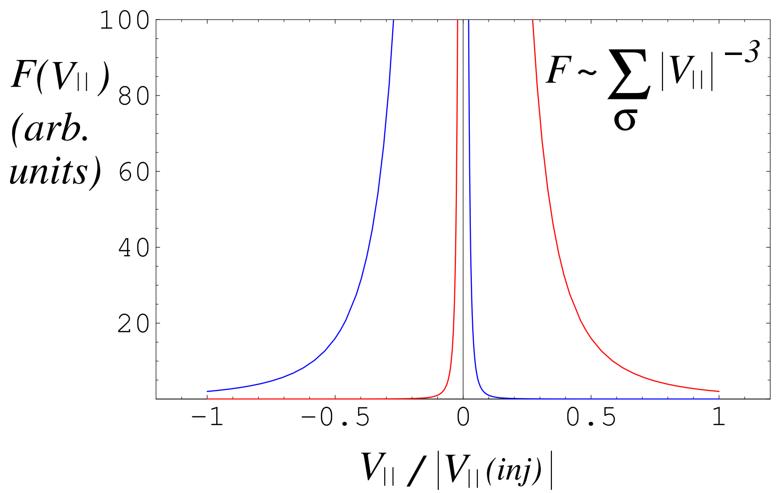
- Unbalanced NNBI produces asymmetric distribution. Choose to approximate with one sided slowing down distributions:

$$F_h^\sigma = \frac{P_\parallel^\sigma}{2^{3/2}\pi m_h B_0 \mathcal{E}^{\mathrm{inj}}} \mathcal{E}^{-3/2}\delta(\lambda)$$

where $P_\parallel = \sum_\sigma P_\parallel^\sigma$ and we define angle of asymmetry

$$A \equiv \frac{\sum_\sigma \sigma P_\parallel^\sigma}{P_\parallel} = \frac{P_\parallel^+ - P_\parallel^-}{P_\parallel^{+1} + P_\parallel^{-1}}$$





Solution to Fast Ion Response

- The perturbed distribution function describing the energetic ion response to stability is [P. Helander *et al Phys. Plasmas* 4, 2182 (1997)]:

$$\delta F_h = \delta F_{hf} + \delta F_{hk}, \quad \text{where} \quad \delta F_{hf} = -(Ze/m_h)(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}\psi_p)\frac{\partial F_h}{\partial \mathcal{P}_\phi}$$

is the adiabatic (fluid) contribution, with $\boldsymbol{\xi} \sim \exp(-im\theta - in\phi - i\omega t)$ the MHD displacement, and the non-adiabatic (kinetic) contribution δF_{hk} can be approximately written as 'bounce time' τ_b periodic function of time:

$$\delta f_{hk} = \sum_{l=-\infty}^{\infty} \delta F_{hk}^{(l)} \exp\left[-i\left(\omega + l\omega_b + n\left\langle\dot{\phi}\right\rangle\right)t\right]$$

$$\text{where} \quad \delta F_{hk}^{(l)} = -\frac{\omega - n\omega_{*h}}{\omega + n\left\langle\dot{\phi}\right\rangle + l\omega_b}\frac{\partial F_h}{\partial \mathcal{E}} \times$$

$$\left\langle\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\boldsymbol{\kappa}\cdot\hat{\boldsymbol{\xi}}_\perp \exp\left[i\left(\omega + l\omega_b + n\left\langle\dot{\phi}\right\rangle\right)t\right]\right\rangle \tag{1}$$

Solution to Fast Ion Response

Make progress by writing δF_h as a sum of MHD and non-MHD terms. Hence we expand about orbit centres:

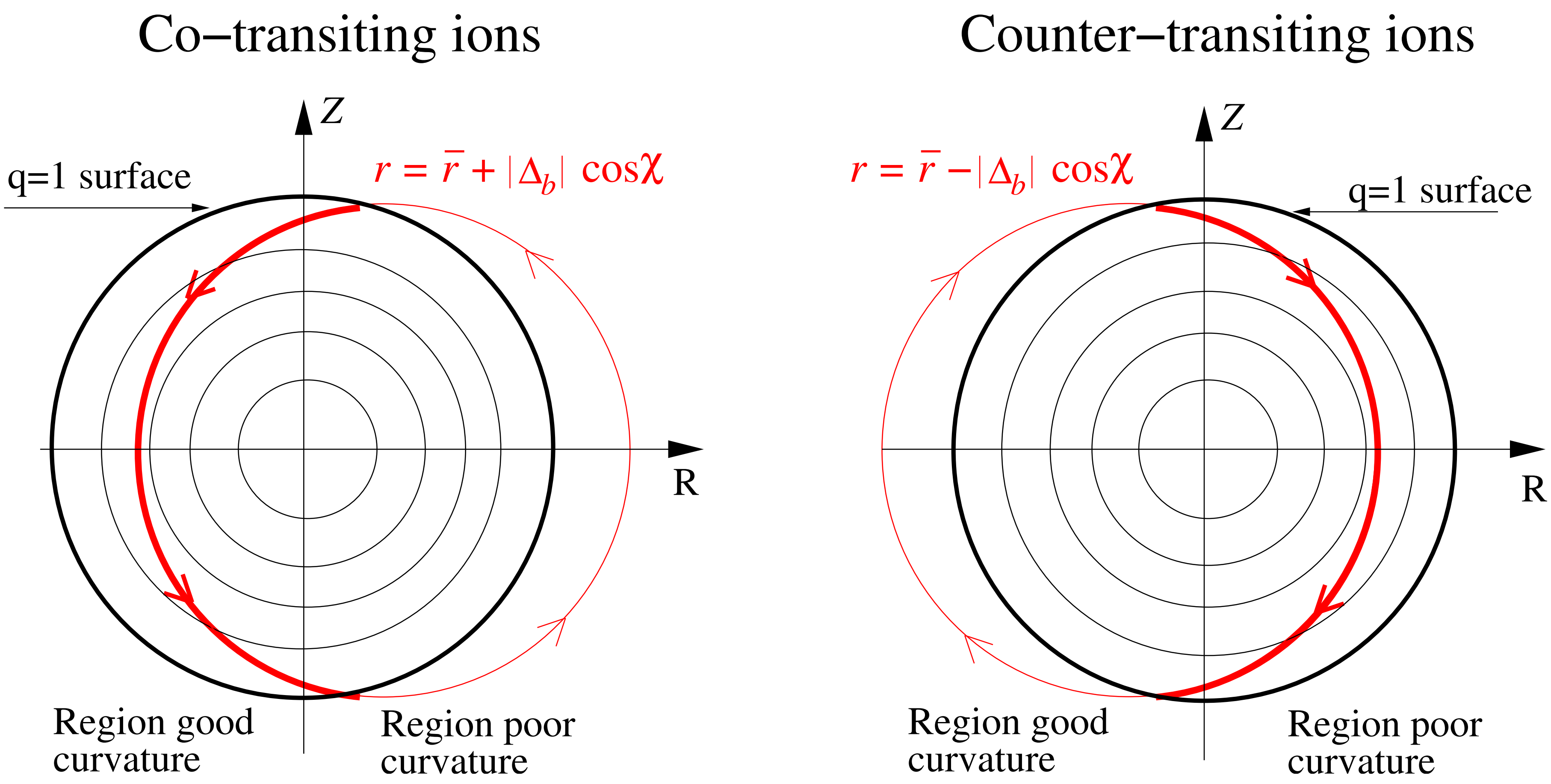
$$\begin{aligned} r &= \overline{r} + \Delta_b \cos\theta \quad \text{with} \quad \Delta_b = \frac{q(\overline{r})v_\parallel}{\omega_c} \\ \theta &= \chi + (1+s)\frac{\Delta b}{\overline{r}}\sin\chi \quad \text{with} \quad \chi = \frac{v_\parallel}{q(\overline{r})R}(t-t_0) \\ \zeta &= q(\overline{r})\,\chi. \end{aligned}$$

adiabatic response can then be written as:

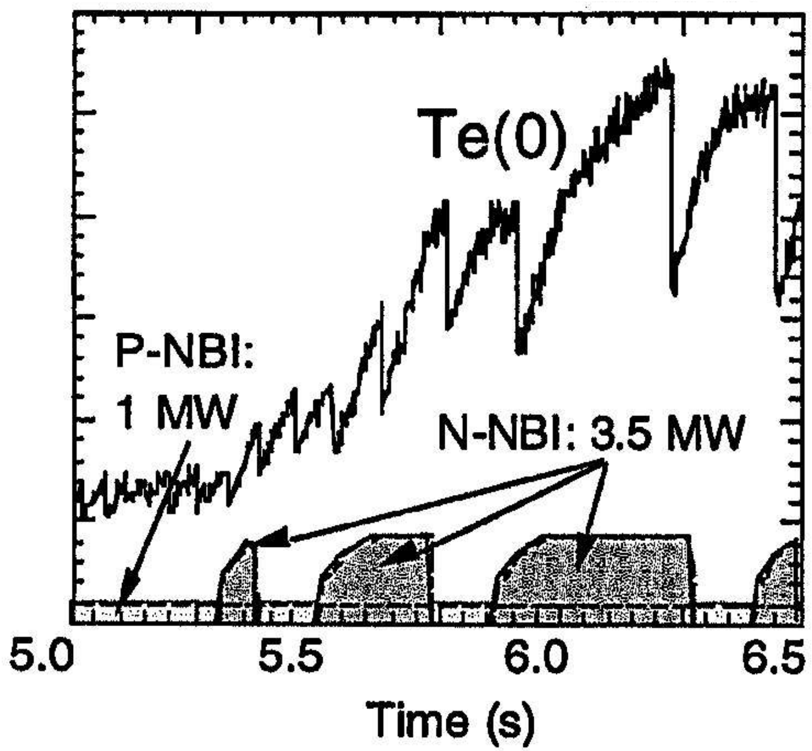
$$\delta\hat{F}_{hf} = -\xi_0 H[r_1 - r]\left(\exp(-i\theta) + \sigma\left\{1+\exp(-i2\theta)\right\}\left[\frac{\Delta_b}{r} - \frac{1}{2r}\frac{\partial}{\partial r} r\Delta_b\right]\right)\frac{\partial F_h(r)}{\partial r}$$

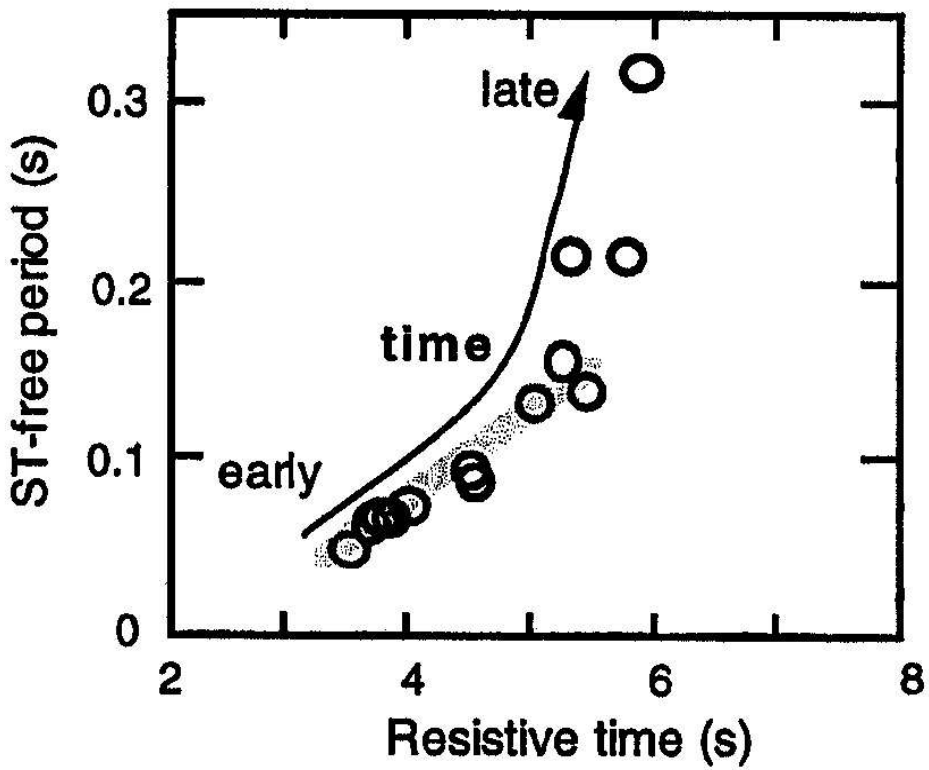
where $\exp(-i\theta)$ is the fluid term, the finite orbit term $\frac{\Delta_b}{r}$ cancels the non-adiabatic term of [S. Wang, *et al*, Phys. Rev. Lett. 88, 105004 (2002)], but the finite orbit term $\frac{1}{2r}\frac{\partial}{\partial r} r\Delta_b$ remains [J. P. Graves, Phys. Rev. Lett. 92, 185003 (2004)].

Finite Orbits Intersecting $q = 1$ Radius



Sawteeth in JT-60U





K. Tobita *et al*, Proc. 6th IAEA Technical Committee Meeting on Energetic Particles in Magnetic Confinement Systems, JAERI, Naka, Japan, p. 73

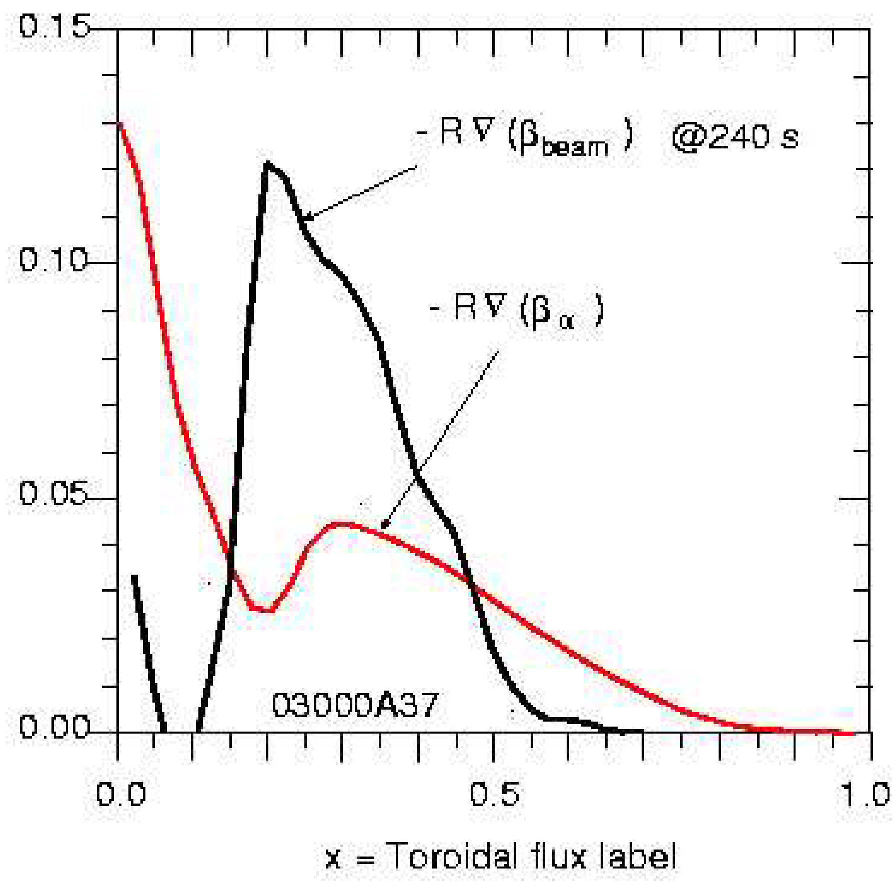
The sawtooth period does not simply increase linearly with the resistive diffusion time.

NNBI in ITER-FEAT

- TRANSP simulations of 1MeV NNBI demonstrates that locally pressure gradient of NNBI could exceed that of the alpha particles.
- In particular could expect the response of asymmetic NNBI passing to compete with the stabilising response of trapped alpha particles if

$$r\frac{dP_{\mathrm{NNBI}}}{dr}\bigg|_{r_1} \approx \frac{1}{\epsilon_1^{1/2}}\int_0^{r_1} dr \left(\frac{r}{r_1}\right)^2 \frac{dP_\alpha}{dr}$$

- Nevertheless, these TRANSP simulations also demonstrate a very large current drive effect which could sustain $q > 1$ for hundreds of seconds.



R. Budny, 8th IAEA Technical Meeting on Particles General Atomics, San Diego, CA, Oct 6-8, 2003

Anisotropy

Recipe for separating isotropic, anisotropic and finite orbit terms:

- Separate $\delta F_h = \delta F_{hf} + \delta F_{hk}$ where $\delta F_{hf} = -(Ze/m_h)(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}\psi_p)\partial F_h(\mathcal{P}_\phi)/\partial\mathcal{P}_\phi$
- Expand δF_{hf} around orbit centre:

$$\delta F_{hf} = -\boldsymbol{\xi}\cdot\nabla\partial F_h + \text{adiabatic finite orbit terms}$$

- Choosing diagonal pressure tensor yields fluid potential energy δW_{hf}:

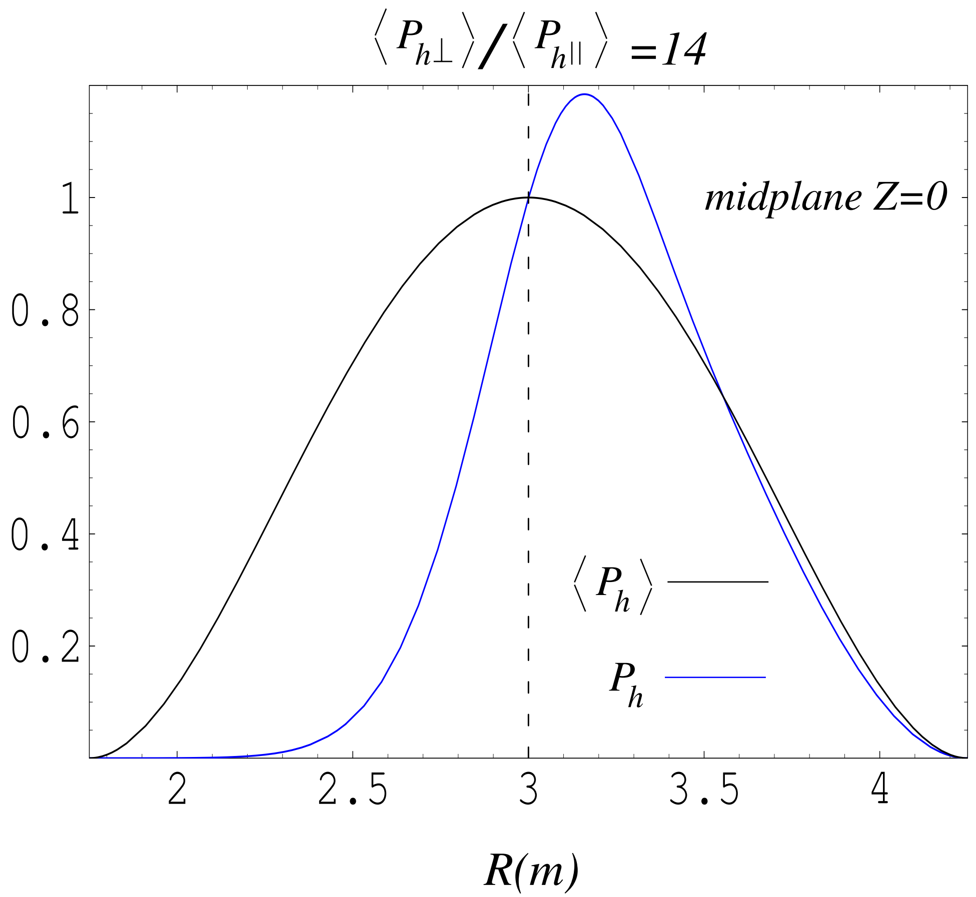
$$\delta W_{hf} = -\frac{1}{2}\int d^3x\,\left(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}(P_{h\|}+P_{h\perp}) - (P_{h\|}+P_{h\perp}+C_h)\frac{\boldsymbol{\xi}\cdot\boldsymbol{\nabla}B}{B}\right)\frac{\boldsymbol{\xi}^*\cdot\boldsymbol{\nabla}B}{B}$$

  where $C_h = 2m_h\int dv^3\,(\mu B)^2\partial F_h/\partial v$.

- We wish to combine the isotropic part of δW_{hf} with the core plasma MHD contribution. Hence we need to identify anisotropic corrections.
- For isotropic plasma $C_h + P_{h\|} + P_{h\perp} = 0$ and $(P_{h\|}+P_{h\perp})/2 = P_h(\psi)$
- For strongly anisotropic plasma for which $P_{h\|}/P_{h\perp} \sim \epsilon$ we have:

$$C_h \sim \epsilon^{-1}P_{h\perp} \quad \text{and} \quad \frac{\partial P_{h\perp}}{\partial\theta} \sim P_{h\perp}$$

Anisotropy



For a distribution function of the form

$$F_h(\mathcal{E},\mu,r)=\frac{\langle P(r)\rangle\ c(r)}{\mathcal{E}^{3/2}}\exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2]$$

where we choose $\lambda\equiv B_0\mu/\mathcal{E}=1$ and $\Delta\lambda=0.1$ to give $\langle P_\perp\rangle\,/\,\langle P_\parallel\rangle\big|_{r=0}=14$.

Separating MHD and anisotropic terms

We can calculate fluid response by either

(1) Using e.g. TERPSICHORE [Cooper, Varenna 1992] - capable of evaluating growth rates for anisotropic fluid plasma

(2) Analytically separating MHD isotropic and anisotropic contributions:

- Given a distribution $F(\mathcal{E},\mu,\psi)$, conservation of $\mathcal{E}$ and μ gives:

$$\frac{\partial}{\partial\theta}(P_{h\perp}+P_{h\|}) = (P_{h\perp}+P_{h\|}+C_h)\frac{1}{B}\frac{\partial B}{\partial\theta}$$

- This differential equation can be solved for $B = B_0(1-\epsilon\cos\theta)$ to give

$$P_{h\perp}+P_{h\|} = \left\langle P_{h\perp}+P_{h\|}\right\rangle + P^A_{h\perp}(\theta) + P^A_{h\|}(\theta)$$

- Hence we can write

$$\delta W_f = \delta W_{MHD}\left(P_{\text{core}} + \left\langle P_{h\|}+P_{h\perp}\right\rangle/2\right) + \delta W_f^A$$

The Internal Kink Mode with Anisotropy

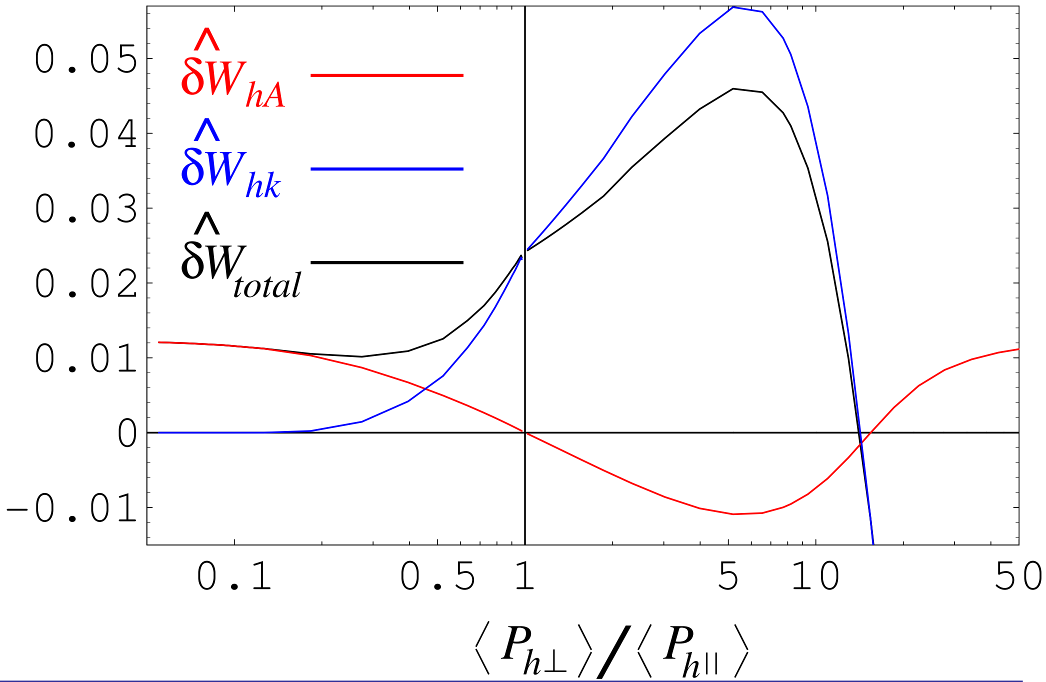
Assume symetric but anisotropic distibution. Include effects of toroidicity, anisotropy and trapped kinetic effects:

$$\delta\hat{W} = 3(1-q_0)\left[\{\beta_p(\text{core}) + \beta_p(\text{hot})\}^2 - 0.3^2\right] + \delta\hat{W}_{hA} + \delta\hat{W}_{hk}$$

- $\delta\hat{W}_{hA}$ provides stabilisation mechanism for parallel anisotropy $\langle P_{h\perp}\rangle / \langle P_{h\|}\rangle \ll 1$
- Anisotropy of $\langle P_{h\perp}\rangle / \langle P_{h\|}\rangle \sim 10$ is most stabilising, for which $\delta\hat{W}_{hk}$ dominates.

Effect of anisotropy on hot ion stabilisation

- Without self-consistently separating toroidal and anisotropic effects due to fast ions, we incorrectly find that δW_h is insensitive to $\langle P_{h\perp}\rangle / \langle P_{h\|}\rangle$.



Effect of Anisotropy on Equilibrium

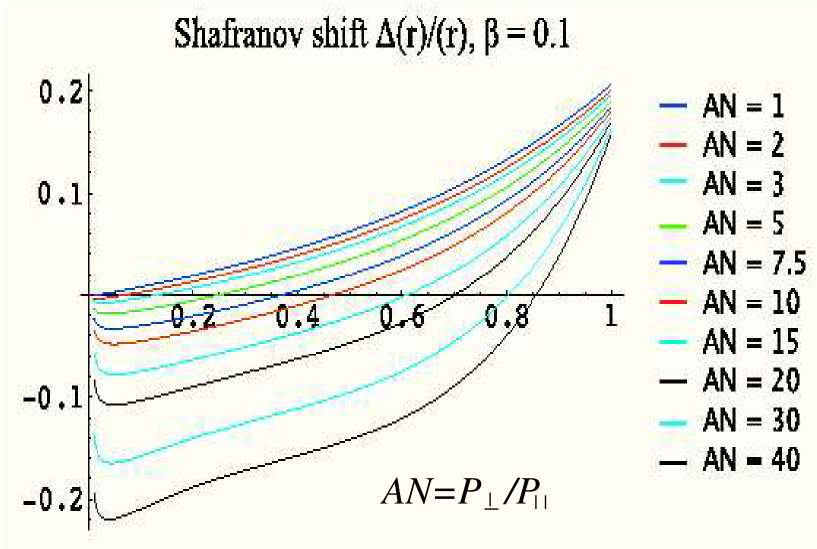
- Equilibrium reconstruction should generally be able to account for anisotropy. This is possible with e.g VMEC [Cooper, PPCF 47, 561 (2005)].
- This would be particularly important in spherical tokamaks having large β_h, and $\langle P_{\perp h}\rangle / \langle P_{\| h}\rangle \gg 1$.
- The effect on cross section shaping has been reported in [Madden and Hastie, Nucl. Fusion 34, 519 (1994)]. Shafranov shift modified as:

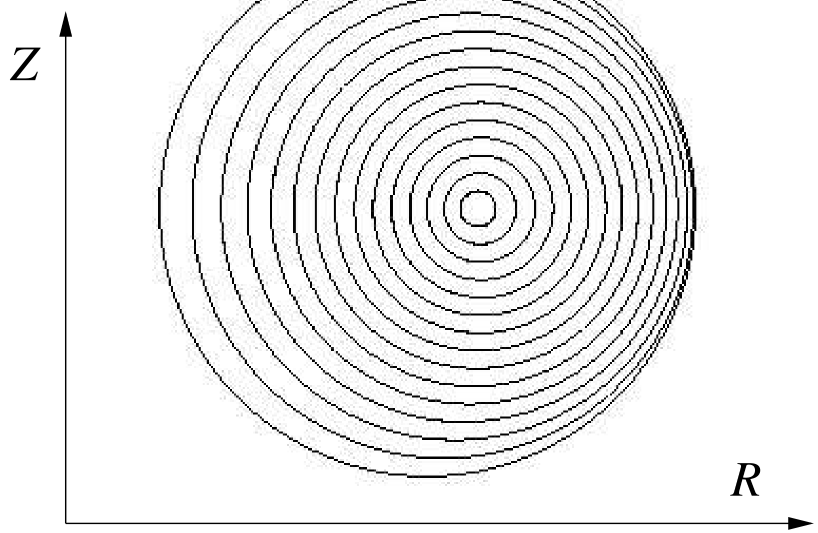
$$\frac{d\Delta}{dr} = \epsilon\left[\frac{l_i}{2} + \langle\beta_p\rangle + \beta_{ph}^A\right]$$

where

$$\beta_{ph}^A = \left(\frac{2\mu_0}{B_p^2}\right)\left\langle\frac{(P_{h\perp}+P_{h\|})}{2}\cos 2\theta\right\rangle$$

- Δ can change sign in the core! i.e. obtain reverse shift. Purely toroidal MHD modes (e.g. internal kink) are expected to be strongly modified.





Conclusions

- Investigated are effects of auxiliary heated ions on the internal kink mode. Special attention given to important effects typically ignored in kinetic-MHD hybrid codes.
- NBI induced toroidal rotation is analysed. Sheared flow significantly modifies the collisionless thermal ion response.
  - ▷ Predictions of relatively small flows in ITER indicate that NNBI induced rotation is not likely to have a large impact on sawteeth.
- A mechanism has been identified where unbalanced injection of NNBI can stabilise the internal kink mode.
  - ▷ Predictions of large local NNBI pressure gradients in ITER indicate that NNBI stabilisation could compete with stabilisation from alpha particles.
- Anisotropy is found to significantly modify the stability of the internal kink mode. All except very strongly trapped hot ion distributions are found to be stabilising.
  - ▷ The damping rate of RF distributions with $\langle P_{h\perp} \rangle / \langle P_{h\parallel} \rangle \lesssim 10$ is found to be around twice as large as for an isotropic distribution (i.e. alphas) with the same energy content.

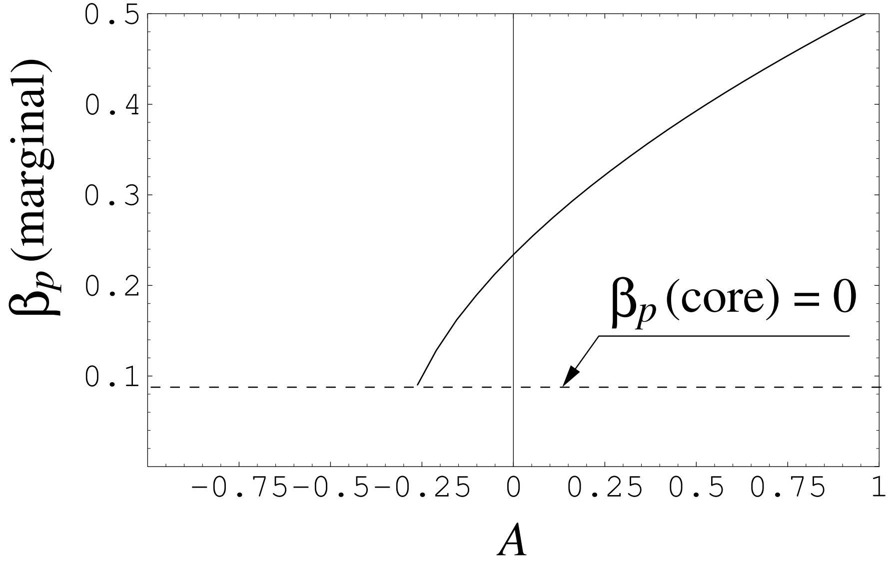
Ideal Internal Kink Stability with NNBI

- The finite orbit, MHD anisotropic, and MHD isotropic contributions combine as:

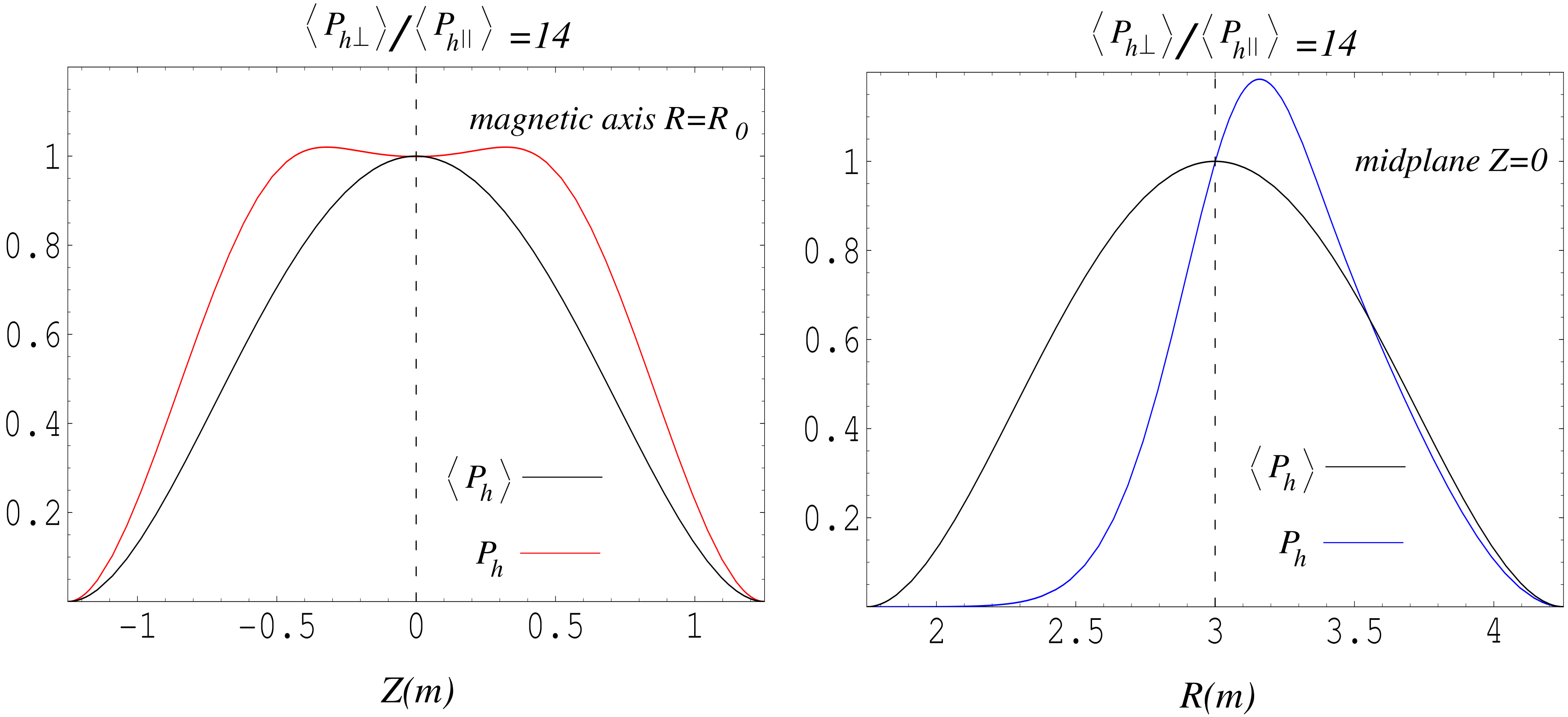
$$\delta\hat{W} = -\epsilon_1^{-1}\left|\frac{\Delta_b}{r_1}\right|\left(\frac{2\mu_0}{B_0}\right)\left[\left(A-\frac{2F(\omega)}{\pi s_1}\right)\sigma r\left.\frac{dP_h}{dr}\right|_{r_1}\right]$$
$$-\frac{1}{2}\left(\frac{2\mu_0}{B_0}\right)\int_0^{r_1}dr\left(\frac{r}{r_1}\right)^2\frac{dP}{dr}$$
$$+3\epsilon_1^2(1-q_0)\left[\beta_{\text{crit}}^2-\beta_p^2(\langle P_\parallel\rangle/2+P_c)\right].$$

A describes asymmetric adiabatic response. $F(\omega)$ describes the non-adiabatic response which is smaller and depends sensitively on $\omega + q\omega_p - \left\langle\dot{\phi}\right\rangle$ [J. P. Graves, Varenna conf. proc. 2004].

Employ typical JT-60U parameters and solve ideal dispersion relation at marginal stability J. P. Graves, Phys. Rev. Lett. 92, 185003 (2004)



Anisotropy



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- This differential equation can be solved for $B = B_0(1 - \epsilon\cos\theta)$ to give $P_{h\perp} + P_{h\|} = \left\langle P_{h\perp} + P_{h\|}\right\rangle + P_{h\perp}^A + P_{h\|}^A$ where

$$P_{h\perp}^A + P_{h\|}^A = -\epsilon\left[1 - \frac{1}{2\pi}\int_0^{2\pi} d\theta\right]\left[(P_{h\perp} + P_{h\|} + C_h)\cos\theta - \int_\theta d\theta\,\cos\theta\frac{\partial}{\partial\theta}(P_{h\perp} + P_{h\|} + C_h)\right]$$

- Hence we can write $\delta W_f = \delta W_{MHD}\left(\left\langle P_\| + P_\perp\right\rangle/2\right) + \delta W_f^A$ where

$$\delta W_f^A = -\frac{1}{2}\int d^3x\left(\boldsymbol{\xi}\cdot\boldsymbol{\nabla}(P_{h\|}^A + P_{h\perp}^A) - (P_{h\|} + P_{h\perp} + C_h)\frac{\boldsymbol{\xi}\cdot\boldsymbol{\nabla}B}{B}\right)\frac{\boldsymbol{\xi}^*\cdot\boldsymbol{\nabla}B}{B}$$