Gyrokinetic Edge Turbulence

and the Edge/Core Transition

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Outline

- **The Tokamak Edge**
  — properties
  — computational setup
  — turbulence, mechanisms

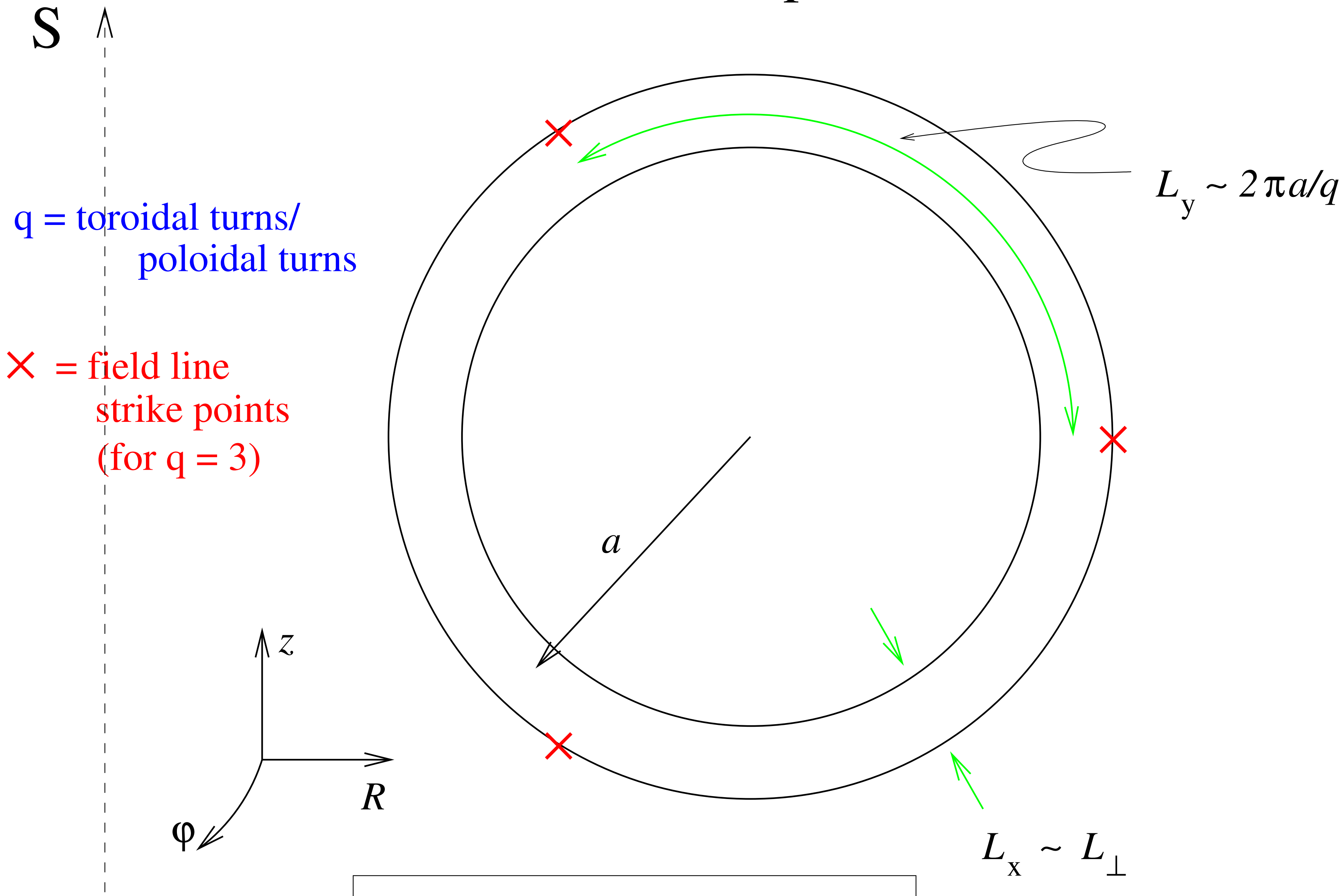
- **Gyrokinetic Edge Turbulence**
  — basic model, add geometry handling and collisions
  — kinetic shear Alfvén behaviour
  — mode structure of edge turbulence
  — velocity space effects

The Tokamak Edge

- coordinates: radial (x), electron drift (y), parallel (s)
- "thin atmosphere" property: $L_\perp/a \ll 1$ hence $L_y \gg L_x$ (in a code, $L_y/L_x \sim 4$)
- field line connection: $L_s = 2\pi qR$, with property that $k_\parallel \neq 0$ for $k_y \neq 0$
- small but moderate drift scale $\delta = \rho_s/L_\perp \ll 1$ but $\delta > 10^{-2}$
- turbulence, small scale isotropy: both $\mathrm{Max}(k_x\rho_s) > 1$ and $\mathrm{Max}(k_y\rho_s) > 1$
- two-fluid adiabatic response: $n_e e E_\parallel \sim \nabla_\parallel p_e$
- NB: if $T_i \sim T_e$ then $\rho_i \sim \rho_s$ hence $k_\perp\rho_i > 1$ hence full FLR

well constructed computations respect all of these
in every run

Computational Domain



large aspect ratio: $L_y >> L_x$

edge parameters

- typical situation: Alfvén/electron transit, collision, and drift frequencies comparable
- drift frequency is $c_s/L_\perp$, spectral range of main interest is $0.1 < k_y\rho_s < 1$
- steep gradient

$$\hat{\mu} \equiv \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{V_e/qR}\right)^2 > 1$$

- collisional

$$C \equiv \frac{0.51\nu_e}{c_s/L_\perp}\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = 0.51\frac{\nu_e\, c_s/L_\perp}{(V_e/qR)^2} > 1$$

- electromagnetic

$$\hat{\beta} \equiv \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{v_A/qR}\right)^2 \gtrsim 1$$

what determines the edge?

- mainly, the first of the conditions: $\hat{\mu} > 1$
- consider the boundary, $\hat{\mu} = 1$

$$\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = 1$$

- solve this for the profile scale length

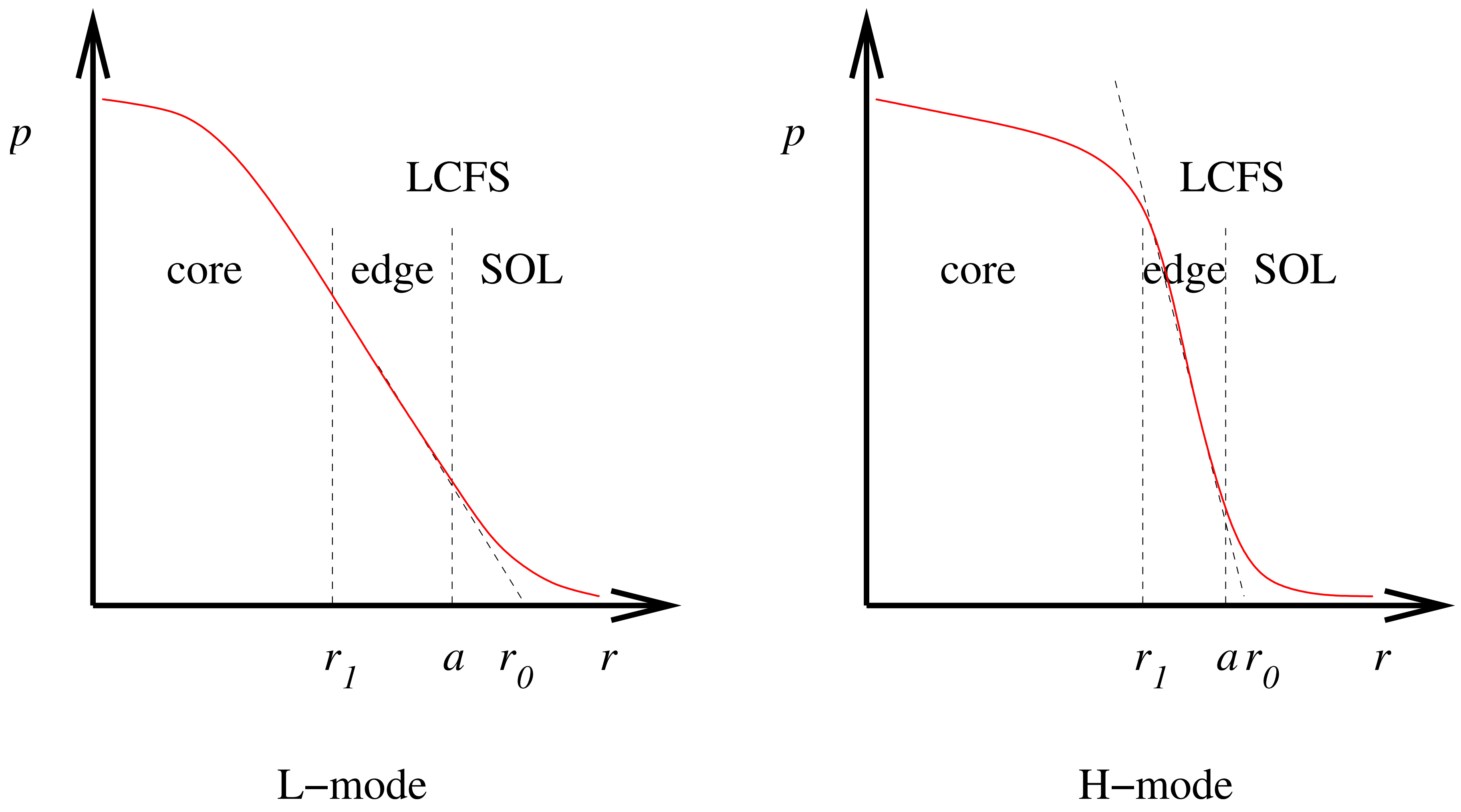
$$L_\perp = \sqrt{m_e/M_D}\,qR$$

- for linear profile gradients this is typically about 8 cm
  - ○ and it holds over about the last 4 cm within the LCFS

if a pedestal exists, the top is the edge/core boundary

Edge Layer Extent

$\hat{\mu} = 1$ at $r = r_1$



Edge Layer Extent

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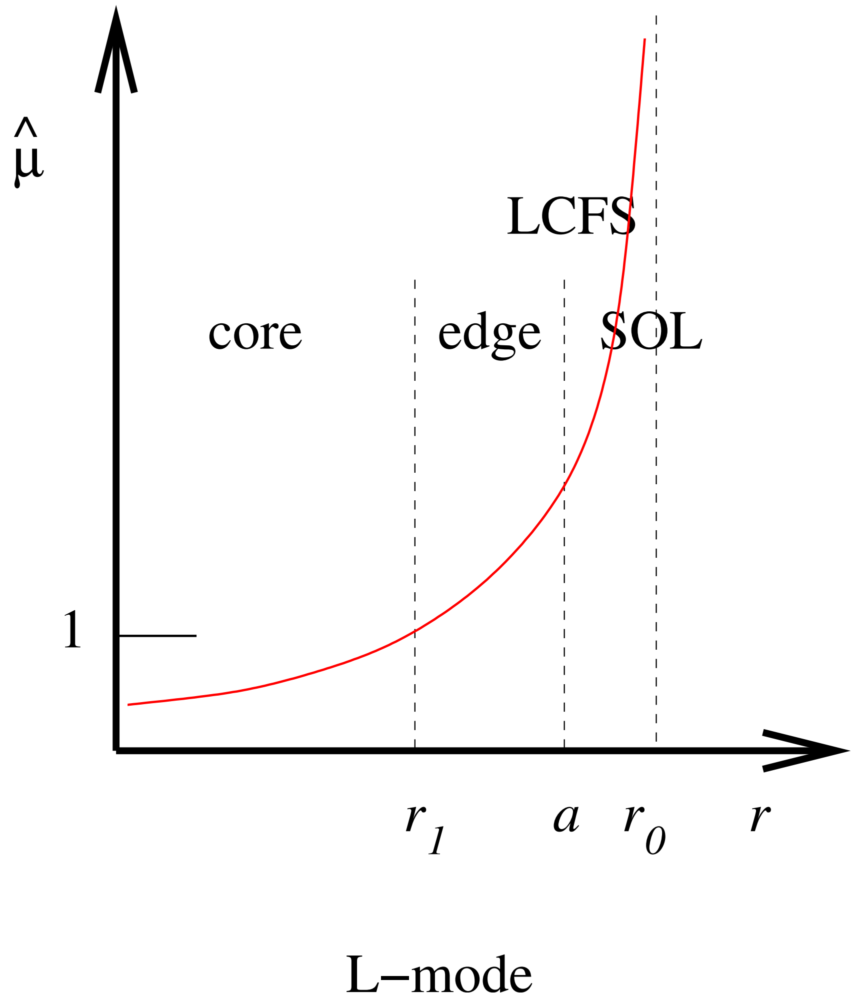
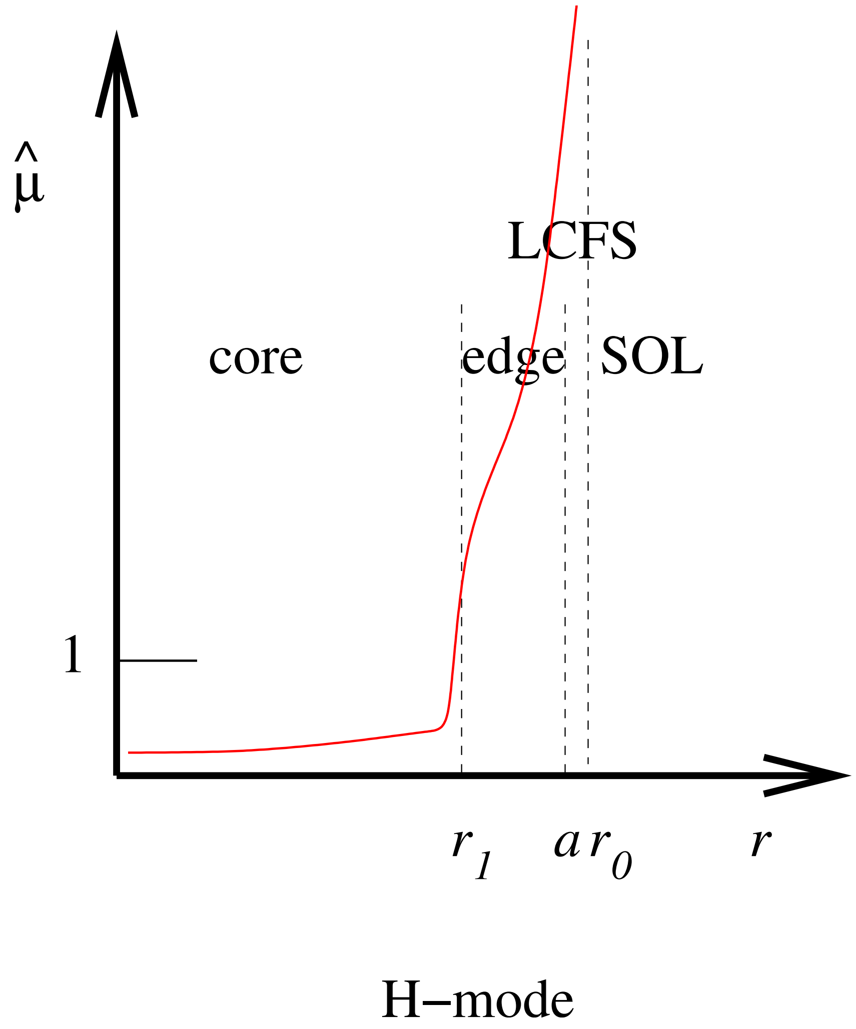
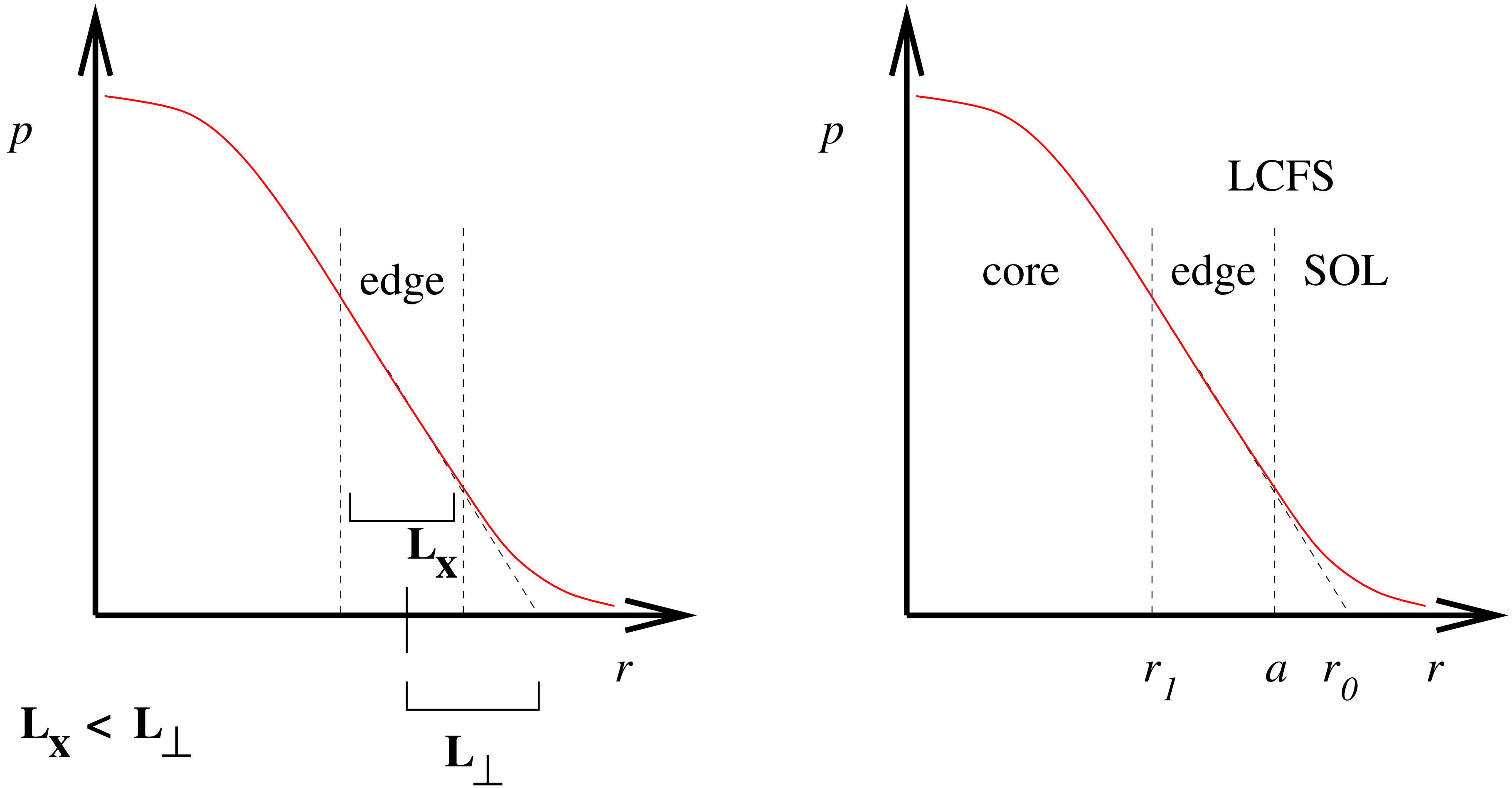




Edge Turbulence Computation Arrangement



Drive/Saturation Mechanisms, Turbulence

- linear modes at these scales are resistive ballooning

  (P Guzdar et al, PoP Oct 1993)

- turbulence: all degrees of freedom participate, including (especially) damped modes
  - self sustained drift wave turbulence

  (B Scott, PRL Dec 1990, PFB Aug 1992)

- turbulence has energetic character of collisional drift waves
  - special: ion temperature, ITG/interchange for $\eta_i > 2$

  (B Scott, PPCF Oct 1997 & New J Phys 2002; V Naulin, PoP Oct 2003)

- observe clear changes in mode structure, linear → saturation → turbulence

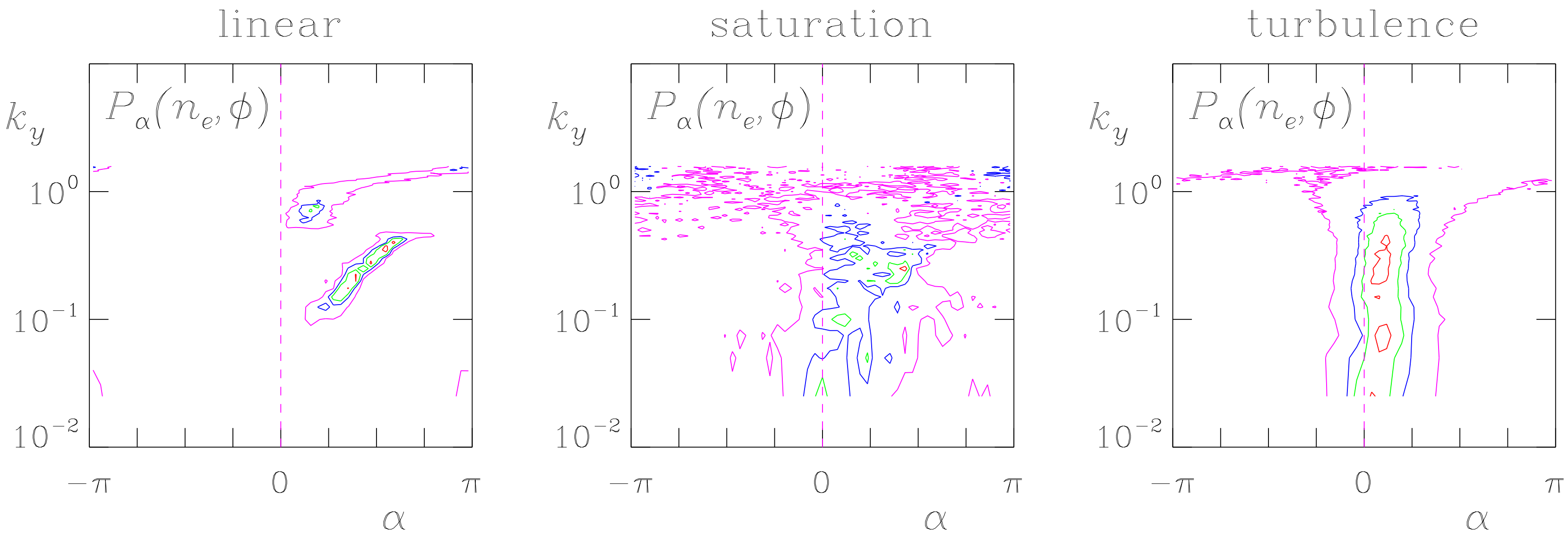
  (B Scott, PPCF Dec 2003)

- presence/absence tests with nonlinearities (NL) show:
  - vorticity NL is a drive mechanisms
  - ExB density NL is the (only) saturation mechanisms

- correlation lengths do not follow linear instability scales
  (see gyrokinetics, below)

Self Sustained Turbulence erases Linear Instabilities

e.g., phase shift distributions for each wavelength

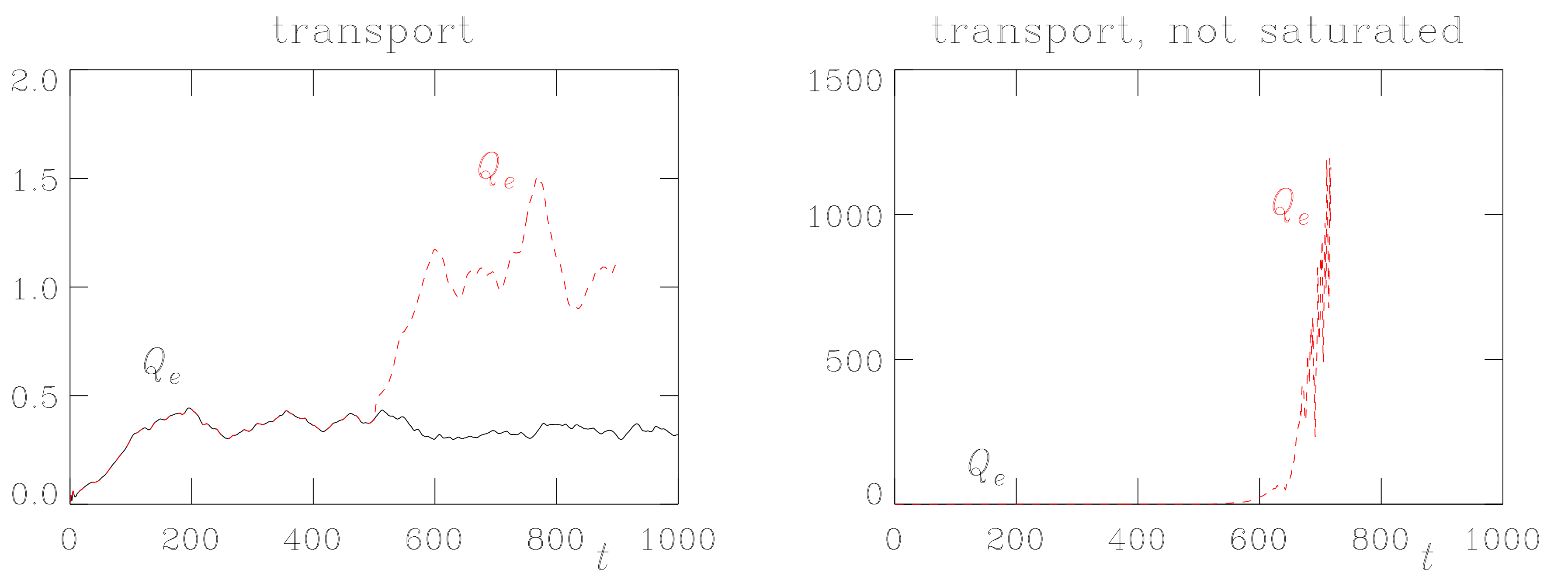


- for linear modes: part of the dispersion relation

- for turbulence: part of the statistical character
  - involves damped or stable, as well as driven, transients
  - this pattern is a clear signature of drift wave mode structure

Nonlinearity Tests — Saturation

- alternatively remove $\mathbf{v}_E \cdot \nabla\widetilde{\Omega}$ … or $\mathbf{v}_E \cdot \nabla\widetilde{n}_e$



- ExB diffusive mixing of electrons is the saturation mechanism
  - ◦ vorticity drive result is same as 3D slab → drive mechanism

(B Scott, IAEA 2000)

Why Kinetic?

- previous argument: electron mean free path

$$\chi_{\|} \sim \frac{T_e}{m_e \nu_e} \sim \lambda_{\rm mfp} V_e$$

- not sufficiently considered: time dependence ...

$$\omega \sim V_e/qR \qquad\qquad \omega \sim \nu_e$$

... especially in the ions, by two orders of magnitude!

$$\omega \gg V_i/qR \qquad\qquad \omega \gg \nu_i$$

- many potential issues, of varying importance

gyrokinetic explorations should be considered very useful

Low-frequency delta-f Gyrokinetic Formulation

- wide spectrum turbulence
- usual ordering: $e\widetilde{\phi}/T_e \sim \rho_s/L_\perp \equiv \delta \ll 1$ and $k_\perp \rho_i \sim 1$
  ◦ but allow for $e\widetilde{\phi}/T_e \sim 1$ at $k_\perp \rho_i \sim \delta$
- assume $\widetilde{f} \ll F^M$ (background Maxwellian), linearise parallel acceleration
- keep ExB advection and "magnetic flutter" as quadratic nonlinearities
- keep curvature and trapping
  ◦ manipulate factors of B to maintain free energy conservation law
- add collisions
- ensure flux surface geometry representation allows arbitrary mode structure
- parameters can be of order unity, except for one:

$$\delta \equiv \frac{\rho_s}{L_\perp} \ll 1$$

collisionless slab (Alfvén) part

(F Jenko and B Scott, PoP Jun 1999)

- terms describing Alfvén dynamics

$$\frac{\partial f}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla f - \frac{e}{m}\left(\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \mathbf{b}\cdot\nabla\phi\right)\frac{\partial f}{\partial v_\parallel} = 0$$

- use F^M for f in $\partial/\partial v_\parallel$
- combine $A_\parallel$ with f under $\partial/\partial t$
- combine ϕ with f under $\mathbf{b}\cdot\nabla$

$$\frac{\partial}{\partial t}\left(f + e\frac{v_\parallel}{c}A_\parallel\frac{F^M}{T}\right) + v_\parallel \mathbf{b}\cdot\nabla\left(f + e\phi\frac{F^M}{T}\right) = 0$$

- note we must have $F^M = F^M(v_\parallel, \mu)$ only!

- now keep nonlinearities, separately

$$\frac{\partial f}{\partial t} \rightarrow \frac{\partial f}{\partial t} + \mathbf{v}_E \cdot \nabla f \qquad\qquad \mathbf{b} \cdot \nabla f \rightarrow \mathbf{b}^{(0)} \cdot \nabla f + \mathbf{b}_\perp \cdot \nabla f$$

- express as brackets — determine coordinates later

$$\mathbf{v}_E \cdot \nabla f = [\phi, f] \qquad\qquad \mathbf{b}_\perp \cdot \nabla f = -\frac{1}{c}[A_\parallel, f]$$

- use $\mathbf{v}_E \cdot \nabla\phi = 0$ to combine f and ϕ

$$\frac{\partial}{\partial t}\left(f + e\frac{v_\parallel}{c}A_\parallel\frac{F^M}{T}\right) + v_\parallel \mathbf{b}^{(0)} \cdot \nabla\left(f + e\phi\frac{F^M}{T}\right)$$

$$+ \left[\phi, \left(f + e\phi\frac{F^M}{T}\right)\right] - \left[\frac{v_\parallel}{c}A_\parallel, \left(f + e\phi\frac{F^M}{T}\right)\right] = 0$$

- define auxiliaries

$$G \equiv f + e\frac{v_\parallel}{c}A_\parallel\frac{F^M}{T} \qquad H \equiv f + e\phi\frac{F^M}{T}$$

- combine potentials

$$\psi \equiv \phi - \frac{v_\parallel}{c}A_\parallel$$

- the equation collapses into a single bracket

$$\frac{\partial G}{\partial t} + v_\parallel \mathbf{b}^{(0)} \cdot \nabla H + [\psi, H] = 0$$

- all dynamics comes from H ("nonadiabatic part")
- induction, magnetic energy, comes from G

- linear term is also a bracket in Hamada coordinates

$$\mathbf{B}\cdot\nabla H = B^s \frac{\partial H}{\partial s} = \frac{\partial \chi}{\partial x}\frac{\partial H}{\partial s} = [\chi, H]_{xs}$$

- curvature terms work through $\log B$ as a potential

$$B\nabla\times\mathbf{b} \equiv -B\nabla\cdot\frac{\mathbf{F}}{B} \approx -\mathbf{F}\cdot\nabla\log B \qquad \text{where} \quad F_{ij} = \epsilon_{ijk}B^k$$

$$\mathbf{b}\times\nabla\mu B = -\frac{\mathbf{F}}{B}\cdot\nabla\mu B = -\mu\mathbf{F}\cdot\nabla\log B$$

- potentials combine (B is constant unless appearing as $\log B$)

$$\psi \to \psi + \frac{mv_\parallel^2 + \mu B}{e}\log B$$

- also add trapping (as bracket) and collisions
  - (usual model scattering both pitch angle and energy)

Gyrokinetic Edge Turbulence

- delta-f gyrokinetic model (as GENE/GS2), with
  - ◦ collisions, "shifted metric" f-tube geometry, non-periodic radial boundaries

$$\frac{\partial G}{\partial t} + \delta\omega_T F^M \frac{\partial \psi_e}{\partial y} + [\delta\psi, H]_{xy} + [\delta_a\psi + v_\parallel \chi, H]_{xs} - (\mu B)\frac{\chi'}{m}[\log B, f]_{sv_\parallel} = C(f)$$

with $\delta = c/B$ and $\delta_a = c/Ba$, and Poisson brackets $[,]$ and potentials

$$G = f + e\frac{v_\parallel}{c}\frac{F^M}{T} J_0 A_\parallel \qquad H = f + e\frac{F^M}{T} J_0 \phi$$

$$\psi_e = J_0\left(\phi - \frac{v_\parallel}{c} A_\parallel\right) \qquad \psi = \psi_e + \frac{m v_\parallel^2 + \mu B}{e}\log B$$

- and with polarisation and induction

$$\sum_{\rm sp}\int d\mathcal{W}\left[e J_0 f + (J_0^2 - 1)\frac{F^M}{T} e^2 \phi\right] = 0 \qquad \nabla_\perp^2 A_\parallel + \frac{4\pi}{c}\sum_{\rm sp}\int d\mathcal{W}\left[e v_\parallel J_0 f\right] = 0$$

Gyrokinetic Energy

- ExB and magnetic energy

$$\mathcal{E}_E = \frac{1}{2}\sum_{\rm sp}\int d\Lambda\,(1-J_0^2)\frac{F^M}{T}e^2\phi^2 \qquad\qquad \mathcal{E}_M = \frac{1}{8\pi}\int d\mathcal{V}\,k_\perp^2 A_\parallel^2$$

- time derivatives (reformulate using polarisation/induction)

$$\frac{\partial\mathcal{E}_E}{\partial t} = \sum_{\rm sp}\int d\Lambda\,eJ_0\phi\,\frac{\partial f}{\partial t} \qquad\qquad \frac{\partial\mathcal{E}_M}{\partial t} = \sum_{\rm sp}\int d\Lambda\,ef\frac{v_\parallel}{c}J_0\,\frac{\partial A_\parallel}{\partial t}$$

- thermal free energy and time derivative (cf.: Lee and Tang 1988)

$$\mathcal{E}_F = \frac{1}{2}\sum_{\rm sp}\int d\Lambda\,\frac{T}{F^M}f^2 \qquad\qquad \frac{\partial\mathcal{E}_F}{\partial t} = \sum_{\rm sp}\int d\Lambda\,\frac{T}{F^M}f\,\frac{\partial f}{\partial t}$$

Gyrokinetic Energy Theorem

- combine three time derivatives for $\mathcal{E} = \mathcal{E}_E + \mathcal{E}_M + \mathcal{E}_F$

$$\frac{\partial \mathcal{E}}{\partial t} = \sum_{\rm sp} \int d\Lambda \, \frac{T}{F^M} H \frac{\partial G}{\partial t}$$

$$= \sum_{\rm sp} \int d\Lambda \left[-v_\parallel \chi' \frac{\partial (\log B)}{\partial s} \frac{\mu B}{F^M} \frac{f^2}{2} - \delta\omega_T T \, f \, \frac{\partial \psi_e}{\partial y} + \frac{T}{F^M} f \, C(f) \right]$$

- entropy term from trapping, linear drive, and collision damping

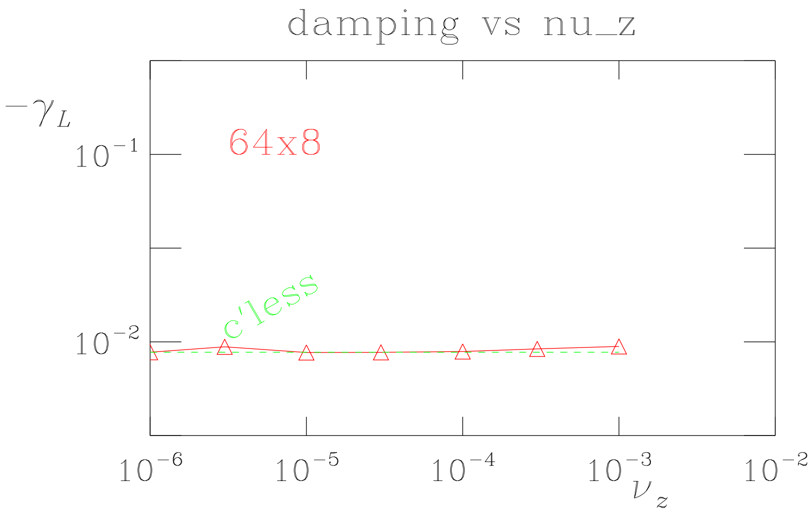
- NB: turbulence *requires* high-$k_\perp$ and high-$k_\parallel$ numerical dissipation
  ◦ collisional scales $\sim \rho_e$ and $k_\parallel q R \sim 100$
  ◦ main cascade process: direct, both $k_\perp$ and $k_\parallel$, through $\mathbf{v}_E \cdot \nabla \widetilde{n}_e$
  (F-Y Gang et al, PFB Jun 1989 & Apr 1991; J Albert et al, PFB Dec 1990)

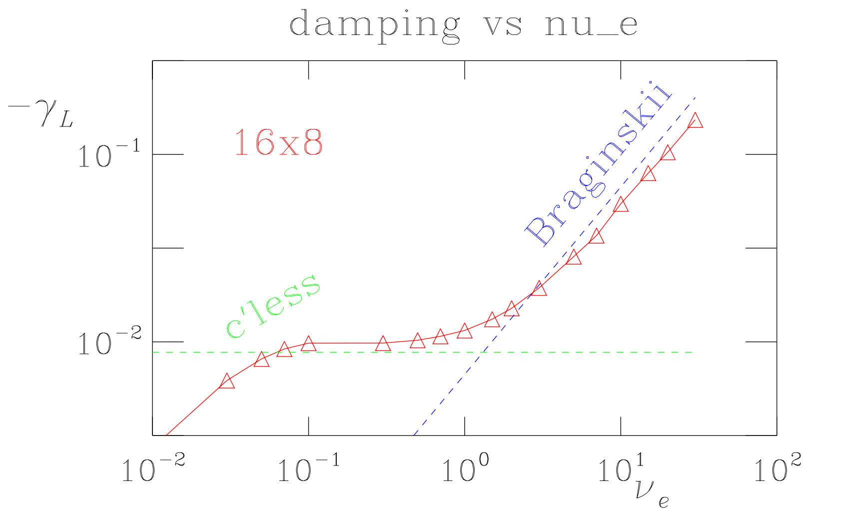
- use energy theorem pieces to diagnose energy transfer, Reynolds stress

Test of KALF Damping Rate, collisionality scaling

FEFI3, $\beta_e = 10^{-4}$ $\mu_e^{-1} = 3670$ $qR/L_\perp = 100$ $k_\perp \rho_s = 0.1$

- collisionless (left) and trans-collisional (right)





- recursion problems for $\nu_z < 10^{-5}$
- non-thermalisation for $\nu_z > 10^{-3}$
- well converged for $N_z = 64$

- collisional regime for $\nu_e > 1$
- $v_\parallel$-space grid problems for $\nu_e < 0.1$
- usual edge turb range is $\nu_e \sim 1$

edge parameters

- typical situation: Alfvén/electron transit, collision, and drift frequencies comparable
- drift frequency is $c_s/L_\perp$, spectral range of main interest is $0.1 < k_y\rho_s < 1$
- steep gradient

$$\hat{\mu} \equiv \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{V_e/qR}\right)^2 > 1 \tag{5}$$

- collisional

$$C \equiv \frac{0.51\nu_e}{c_s/L_\perp}\frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2 = 0.51\frac{\nu_e\, c_s/L_\perp}{(V_e/qR)^2} > 1 \tag{2.55}$$

- electromagnetic

$$\hat{\beta} \equiv \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 = \left(\frac{c_s/L_\perp}{v_A/qR}\right)^2 \gtrsim 1 \tag{1}$$

- NB: $\nu_* = \hat{\mu}^{1/2}\nu\epsilon^{-3/2}$, so for these cases we have $\nu_* = 10\,\nu$ with $\nu = \nu_e L_\perp/c_s$

computational details

- local flux tube model (x, y, s)
  - globally consistent (1 connection length in s), "shifted metric"
  - local version of $x = r^2/a^2$ and $y_k = q(\theta - s_k) - \zeta$ and $s = \theta$
  - s_k is a global constant, $g_k^{xy} = 0$ at $s = s_k$

- perpendicular drift plane domain $L_x = L_y/4 = L_T = 4\,\text{cm} = 67\rho_s$

- velocity space domain $-5 < v_\parallel/V_t < 5$ and $0 < \mu B_0/T_0 < 10$

- spatial grid node count $32 \times 128 \times 16$ in (x, y, s)

- velocity space grid 16×8 in $v_\parallel$ and μB_0

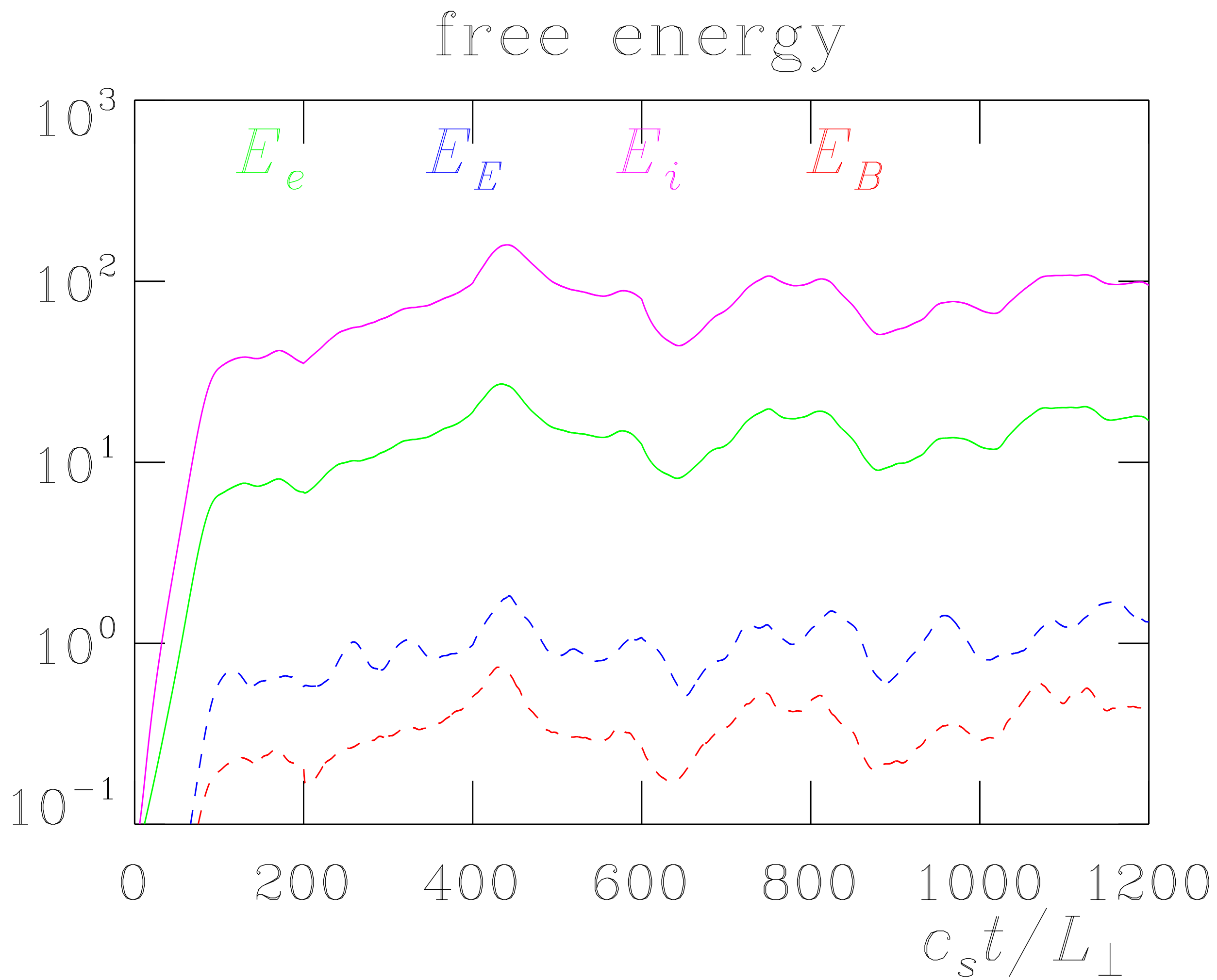
- collisions: pitch angle and energy scattering from fixed background

- conservative part:
  - Arakawa scheme for spatial brackets, Colella MUSCL for trapping
  - time step: Karniadakis 3rd order "stiffly stable" with $\delta t = 0.02 L_T/c_s$

(cf.: V Naulin, PoP Oct 2003)

time traces, showing saturation

nominal case: $\delta = 0.015$, $\hat{\beta} = 1$, $\mu_e = m_e/M_D$, $qR/L_\perp = 100$, $\nu = 1$



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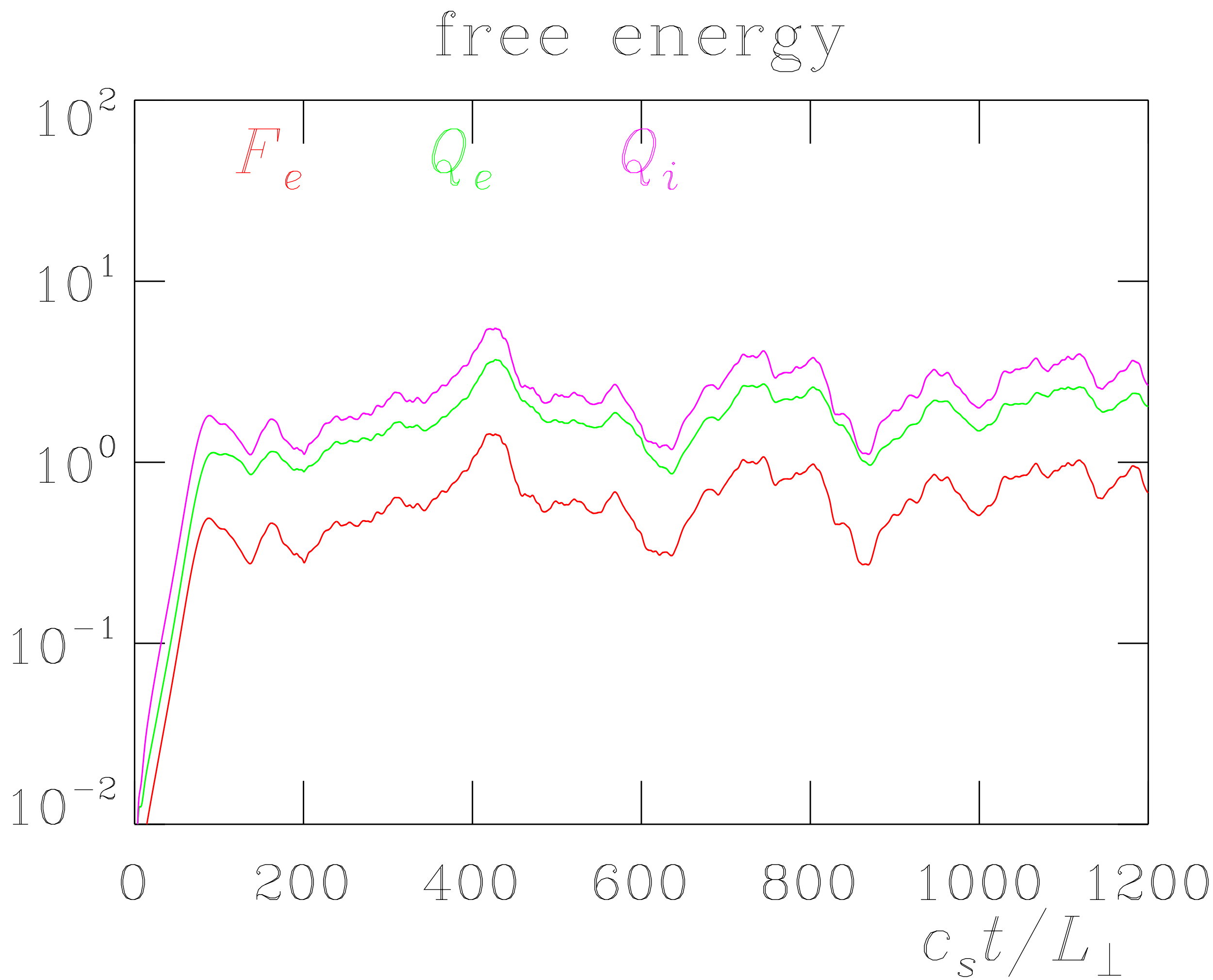
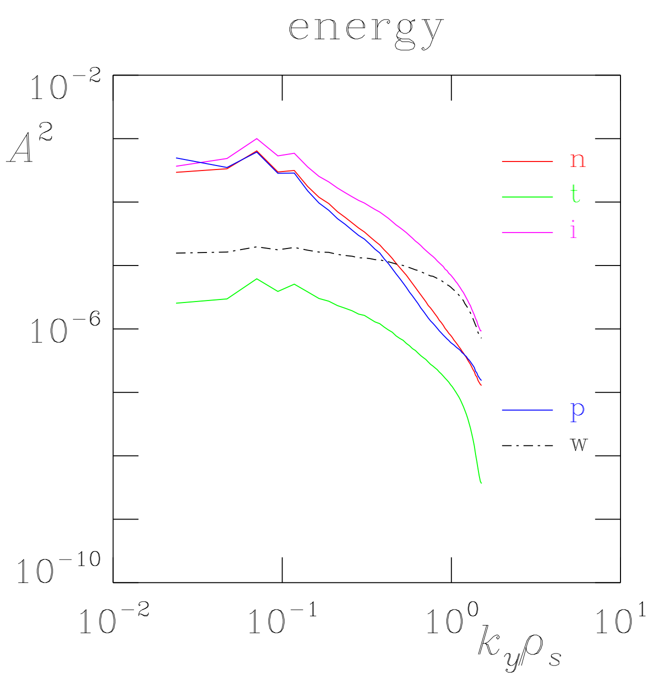
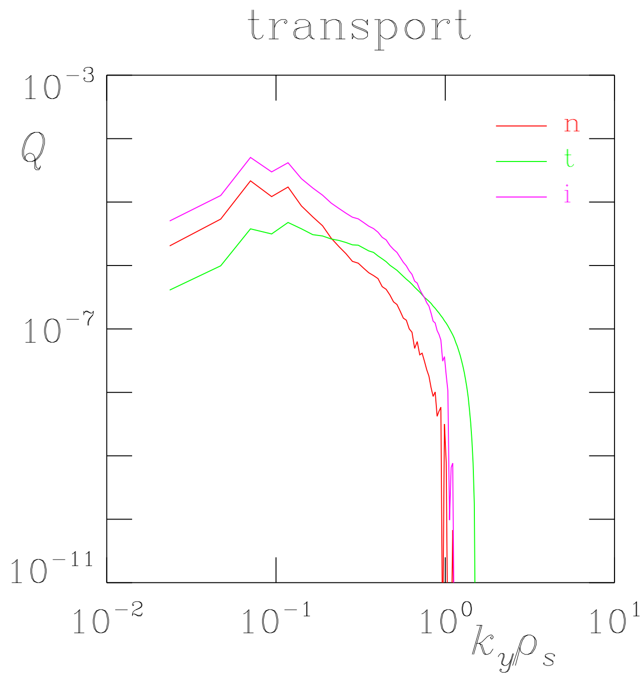


amplitude and flux spectra

nominal case: $\delta = 0.015, \quad \hat{\beta} = 1, \quad \mu_e = m_e/M_D, \quad qR/L_\perp = 100, \quad \nu = 1$

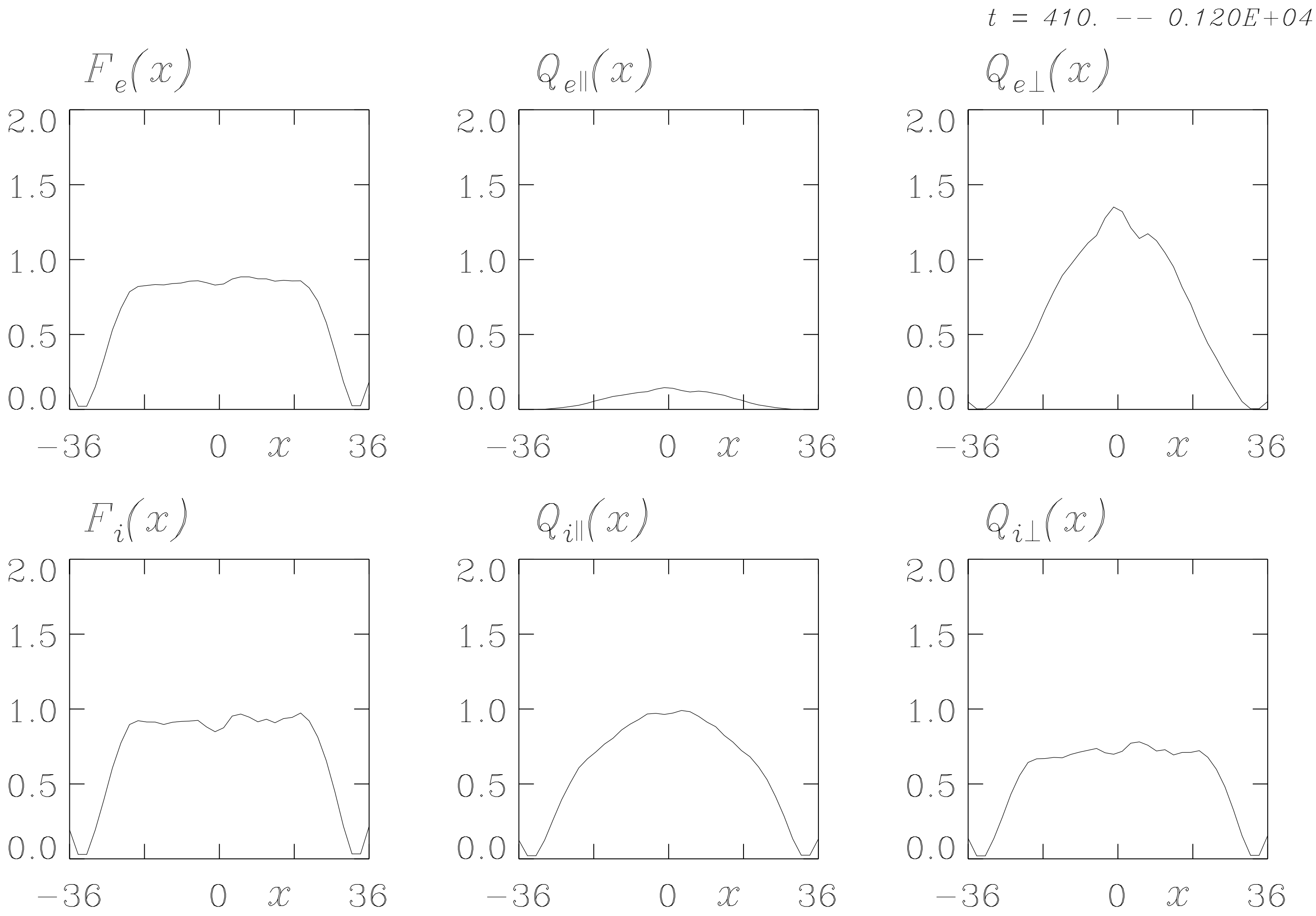




- vorticity spectrum flat all the way to $k_y\rho_s = 1$
- note the differing shape of the electron conduction transport spectrum

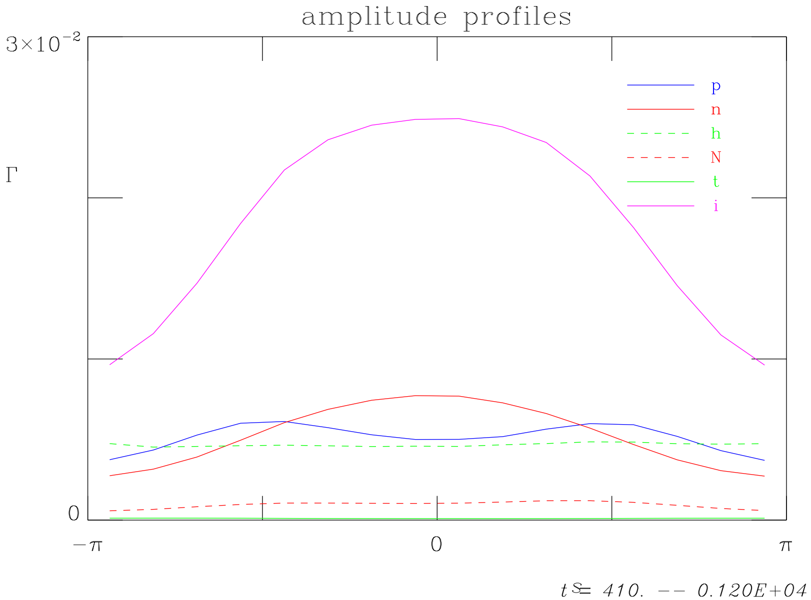
flux profiles

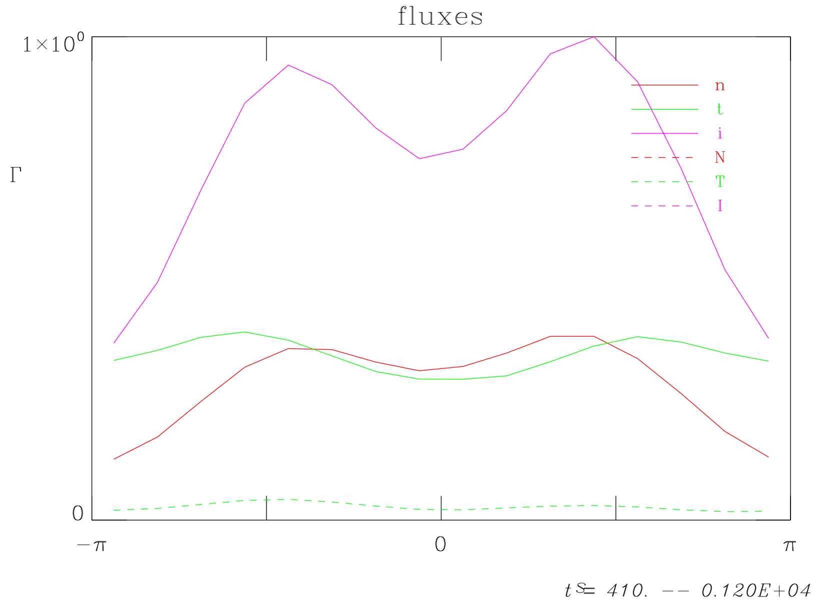
nominal case: $\delta = 0.015$, $\hat{\beta} = 1$, $\mu_e = m_e/M_D$, $qR/L_\perp = 100$, $\nu = 1$



parallel mode structure

nominal case: $\delta = 0.015$, $\hat{\beta} = 1$, $\mu_e = m_e/M_D$, $qR/L_\perp = 100$, $\nu = 1$



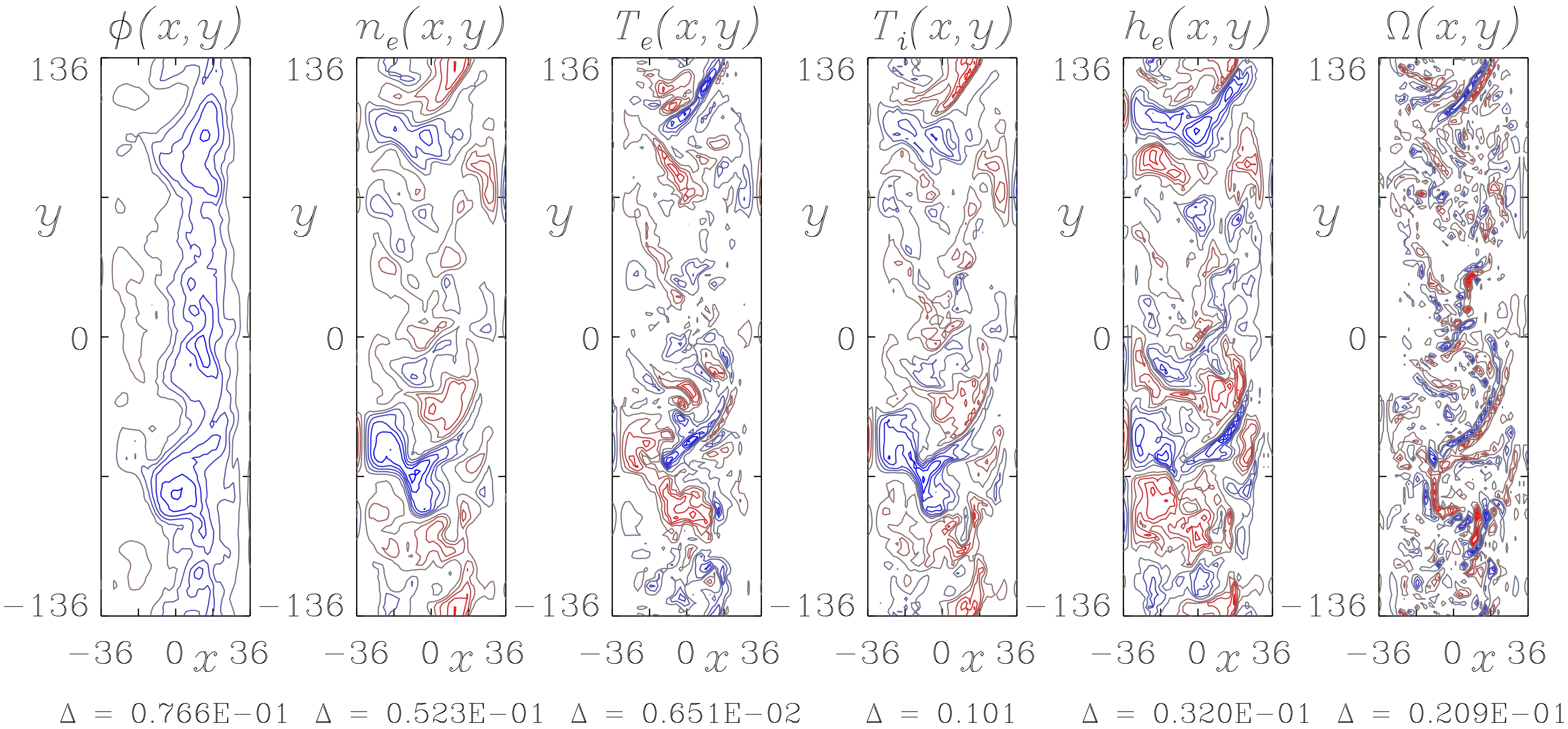


- note that $\widetilde{h}_e = \widetilde{n}_e - \widetilde{\phi}$ is flat though $\widetilde{n}_e$ and $\widetilde{T}_i$ are especially ballooned
- top/bottom enhancements to flux are due to $\widetilde{\phi}$

drift plane morphology

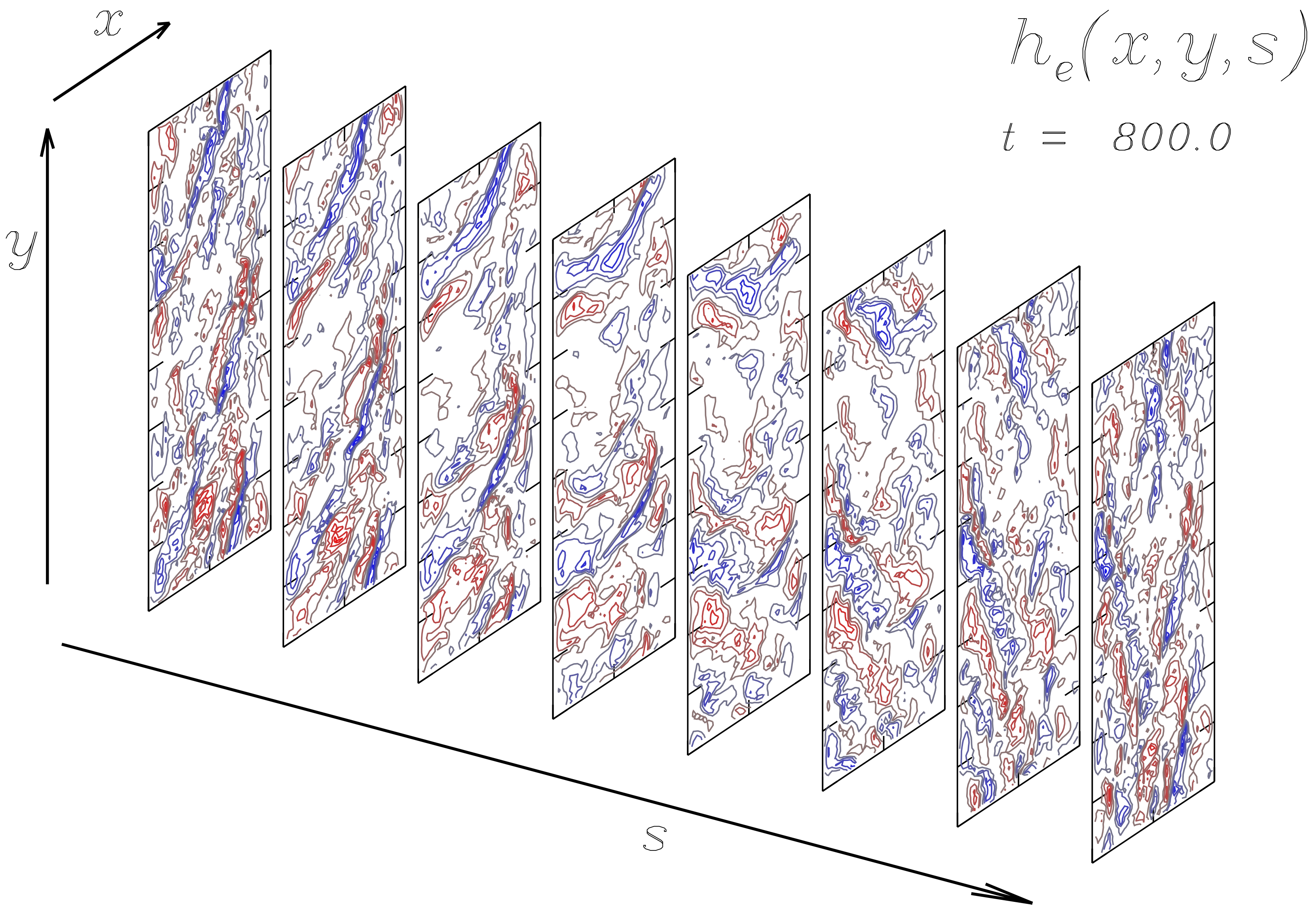
nominal case: $\delta = 0.015$, $\hat{\beta} = 1$, $\mu_e = m_e/M_D$, $qR/L_\perp = 100$, $\nu = 1$

t = 800.

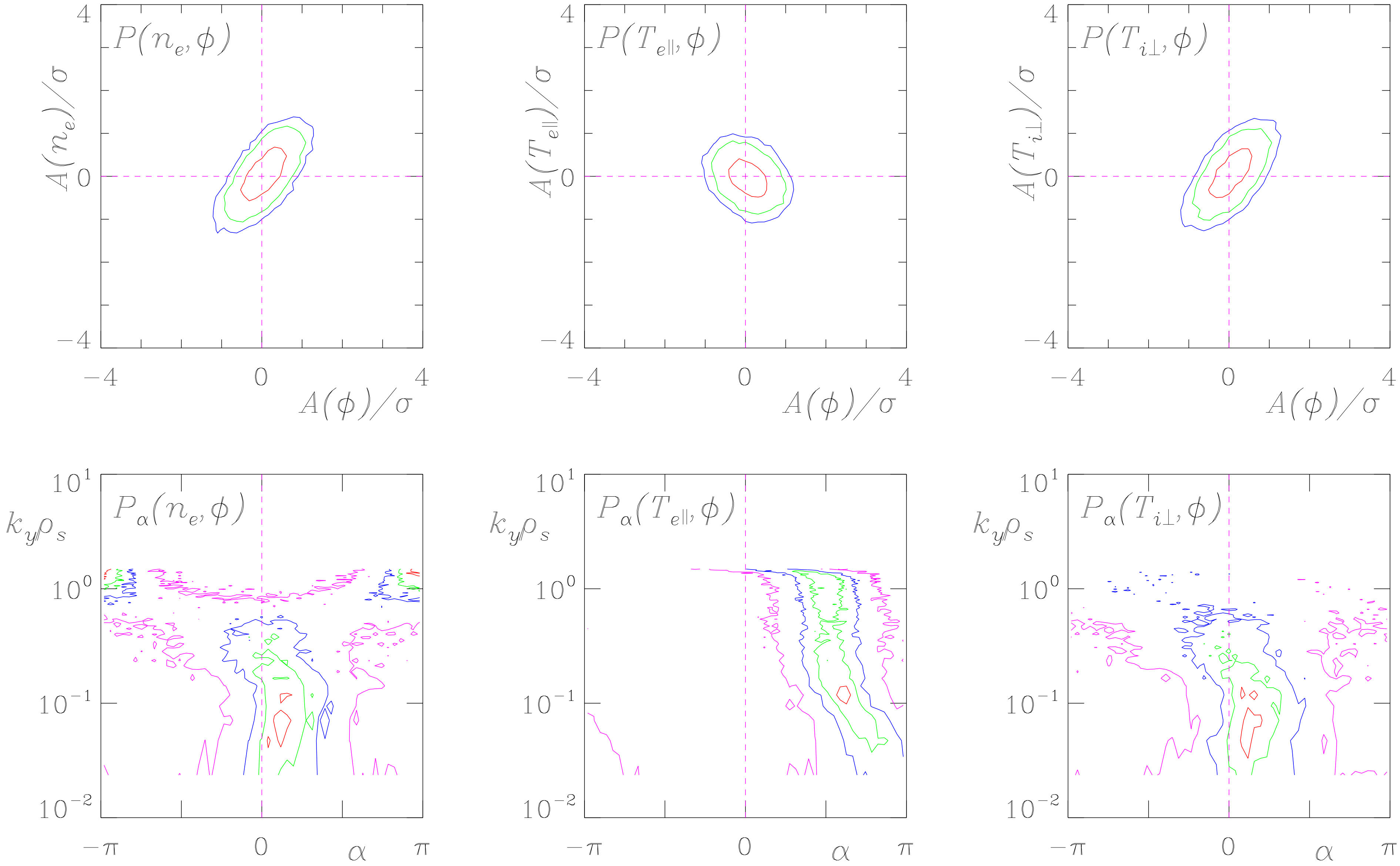


three dimensional morphology

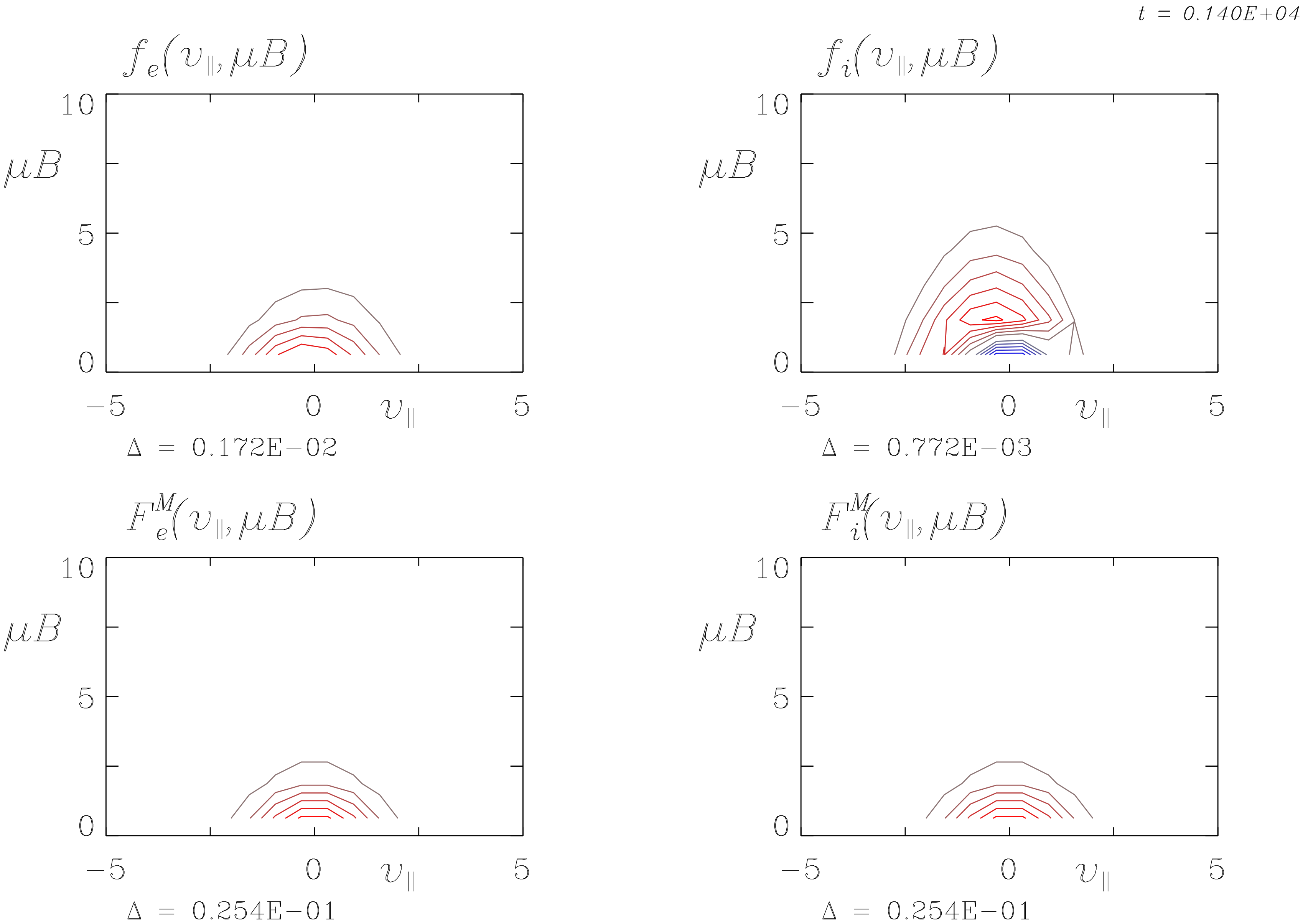
nominal case: $\delta = 0.015$, $\hat{\beta} = 1$, $\mu_e = m_e/M_D$, $qR/L_\perp = 100$, $\nu = 1$



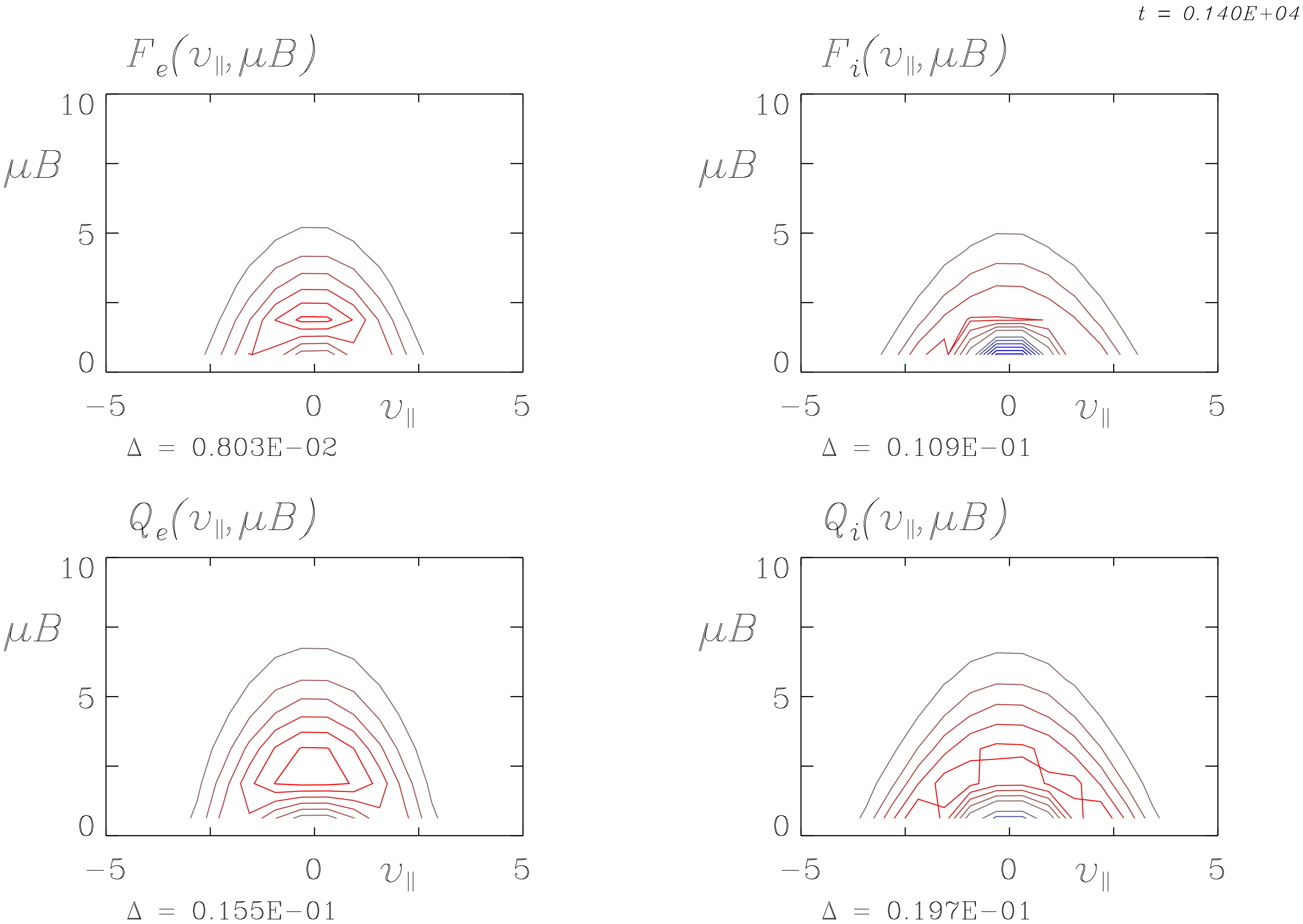
cross correlation and phase shifts



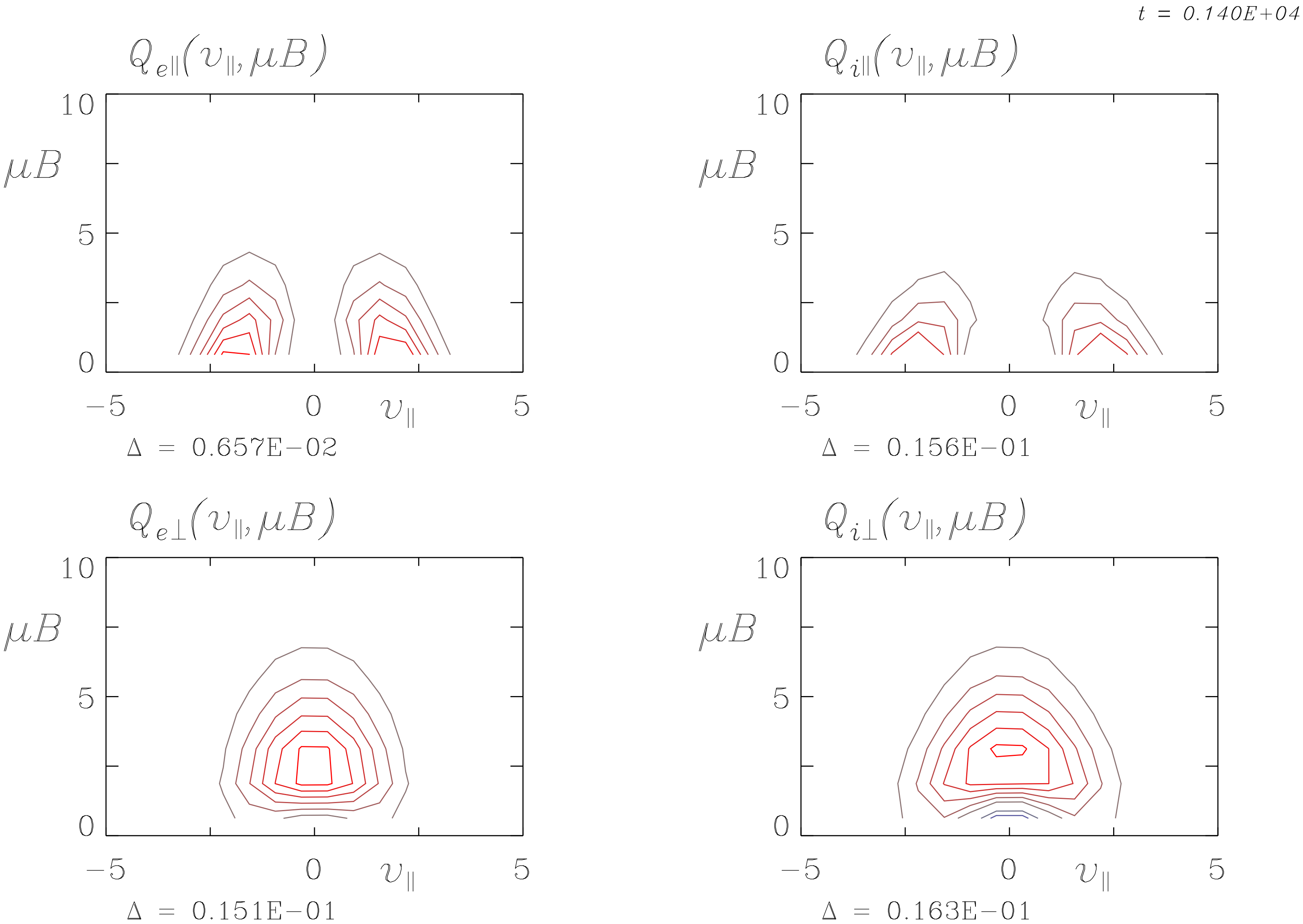
velocity space dependence of fluctuations



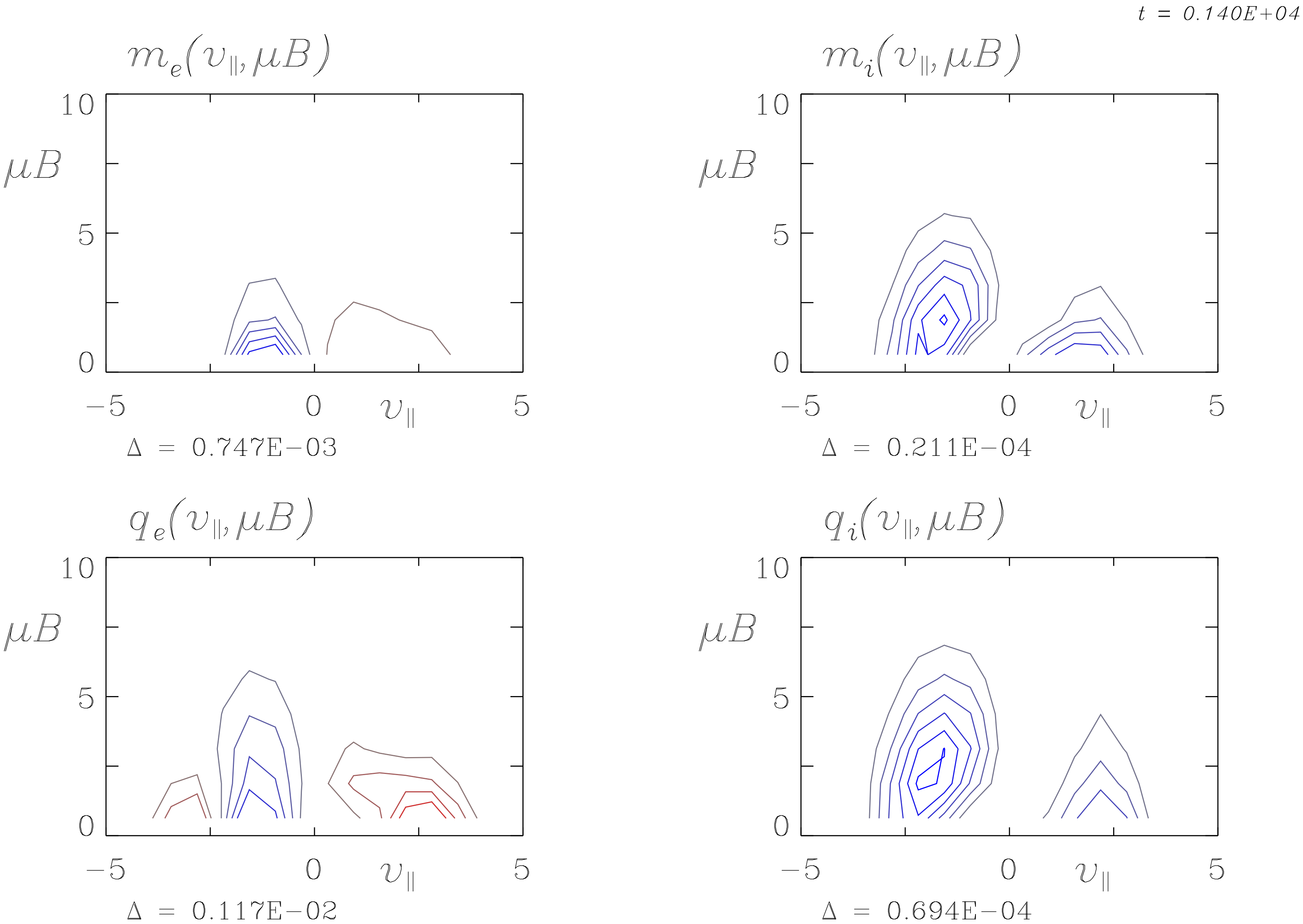
velocity space dependence of ExB thermal fluxes



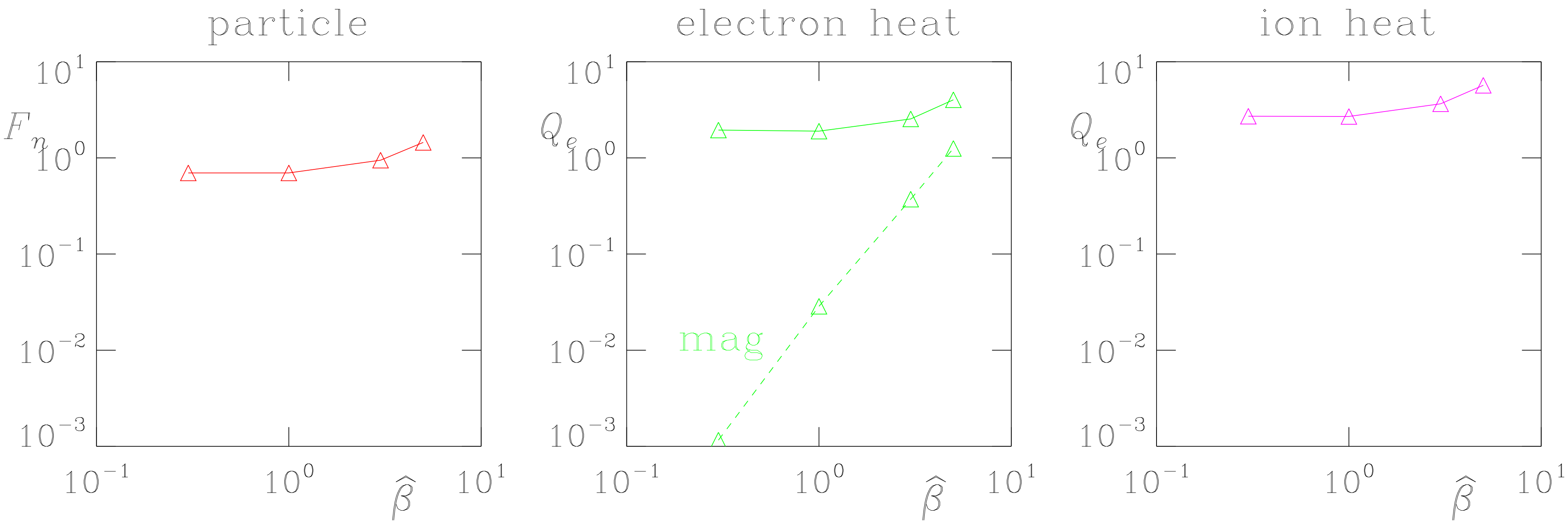
velocity space dependence of thermal flux components



velocity space dependence of magnetic "flutter" fluxes

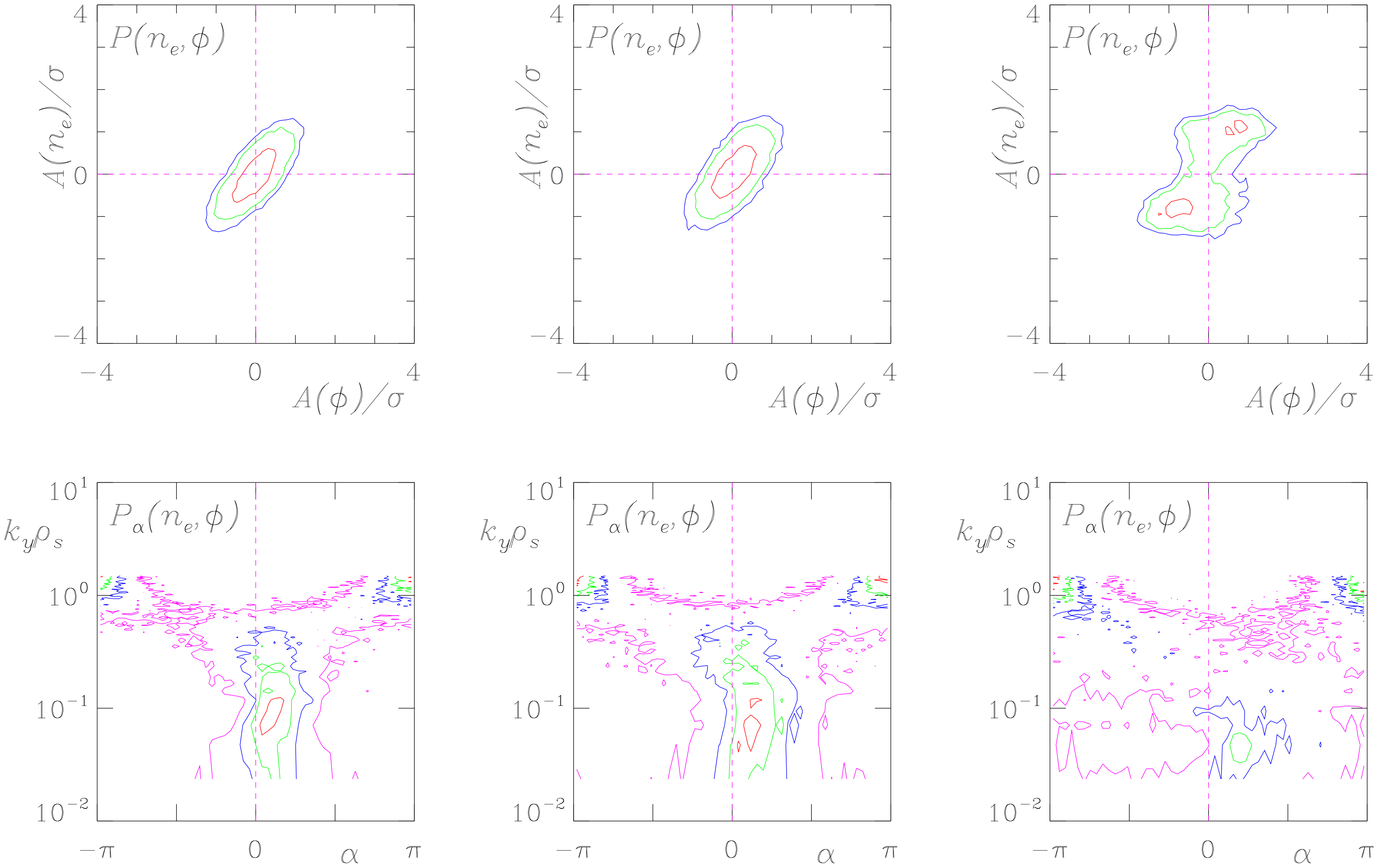


Transport Scaling versus Beta

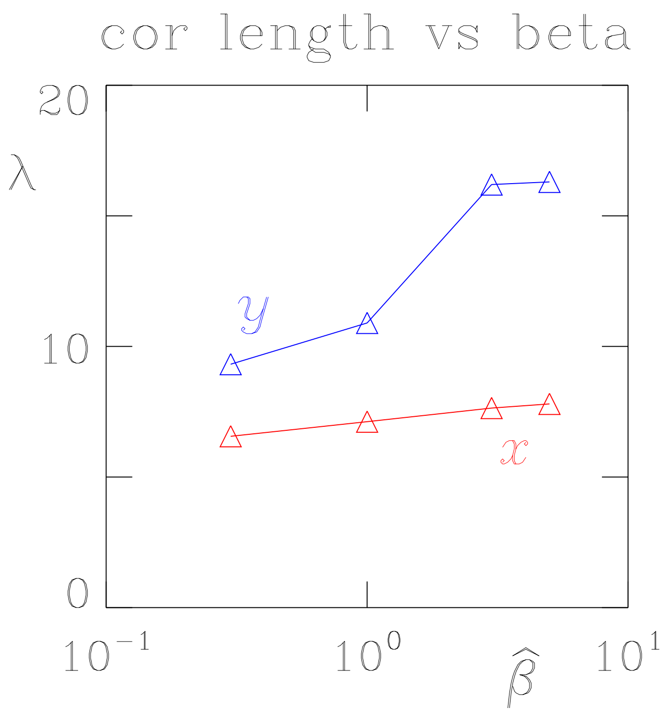


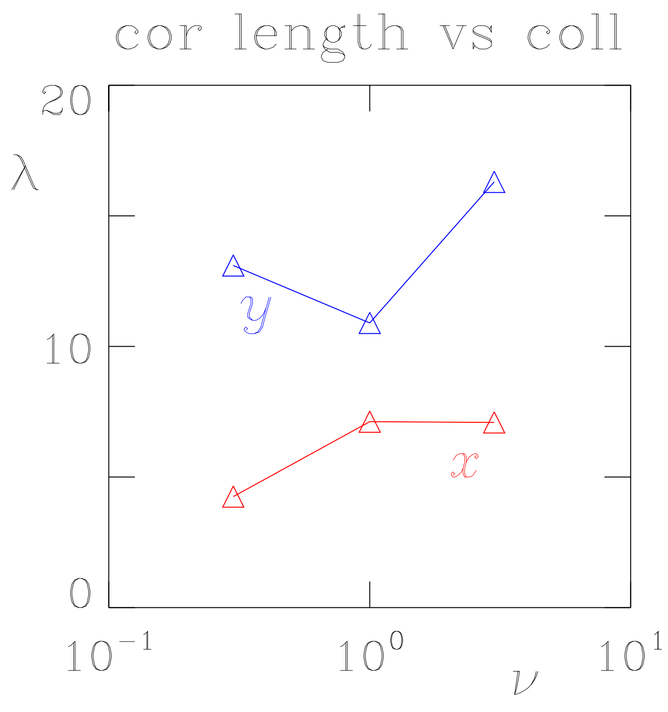
- shallow rise begins for $\hat{\beta} > 1$
- "magnetic flutter" positive for all $\hat{\beta}$ (unique to gyrokinetics)

cross correlation and phase shifts, various beta



Correlation Length Scalings

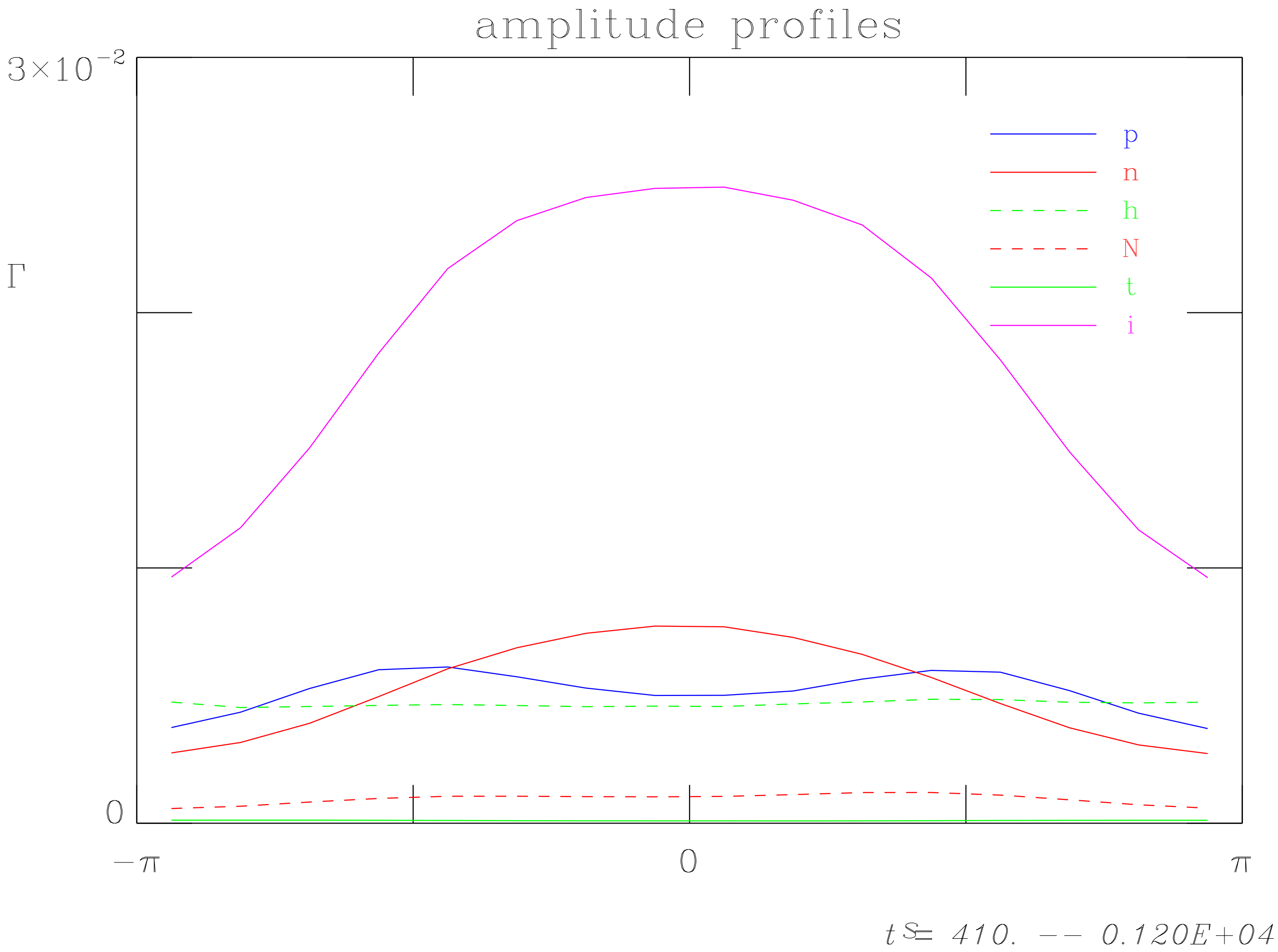




- radial correlation length $\lambda_x \sim 5$ to $7\rho_s$ consistent with observations
- λ_x does not follow linear scales ($\propto \nu^{1/2}$)
- for this parameter choice, $\alpha_M = 0.15\hat{\beta}$ and $\nu_* = 10\,\nu$

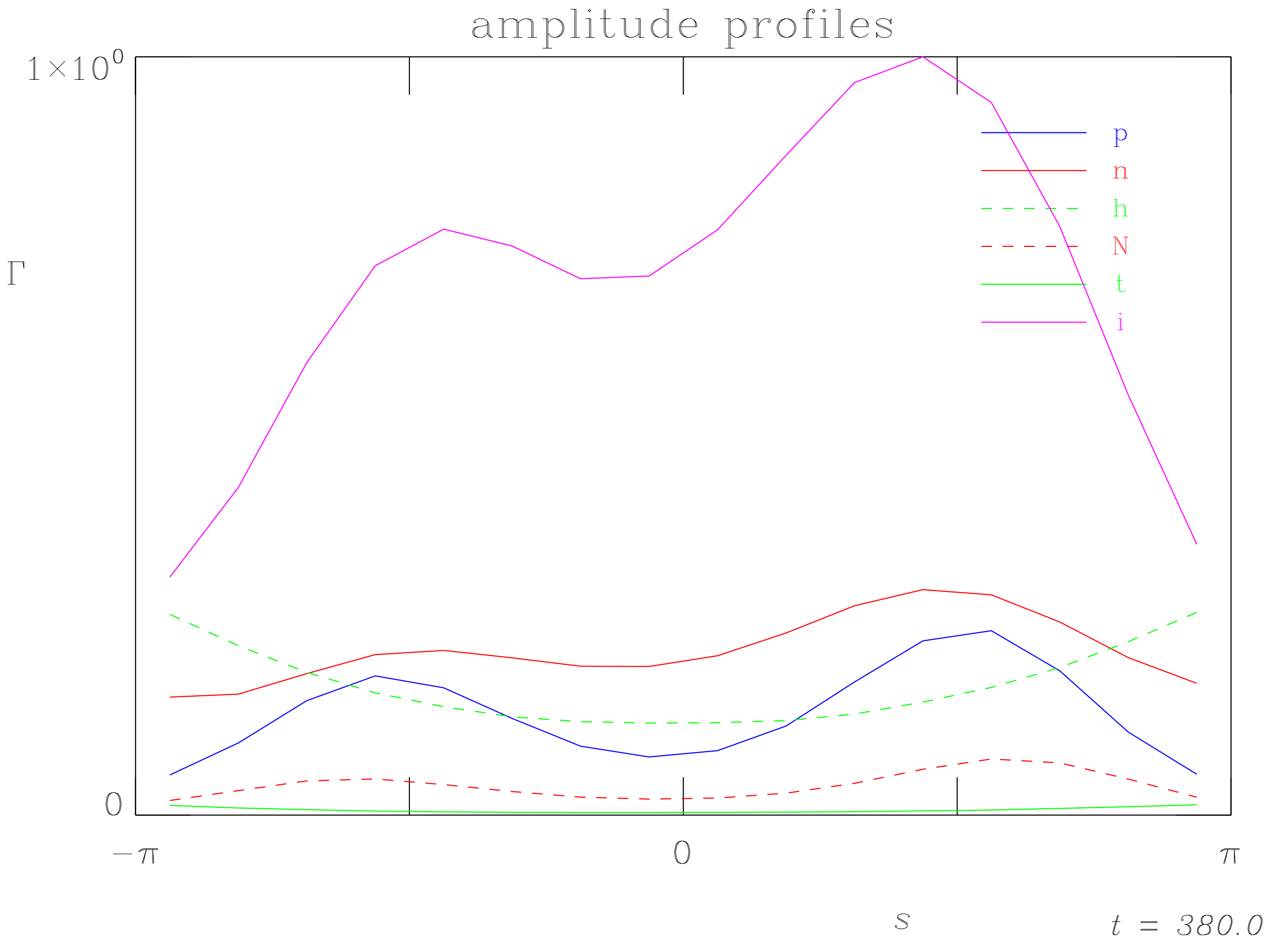
edge/core transition — parallel structure

position 1: $T_e = 100\,\mathrm{eV}$ $L_\perp = 4\,\mathrm{cm}$



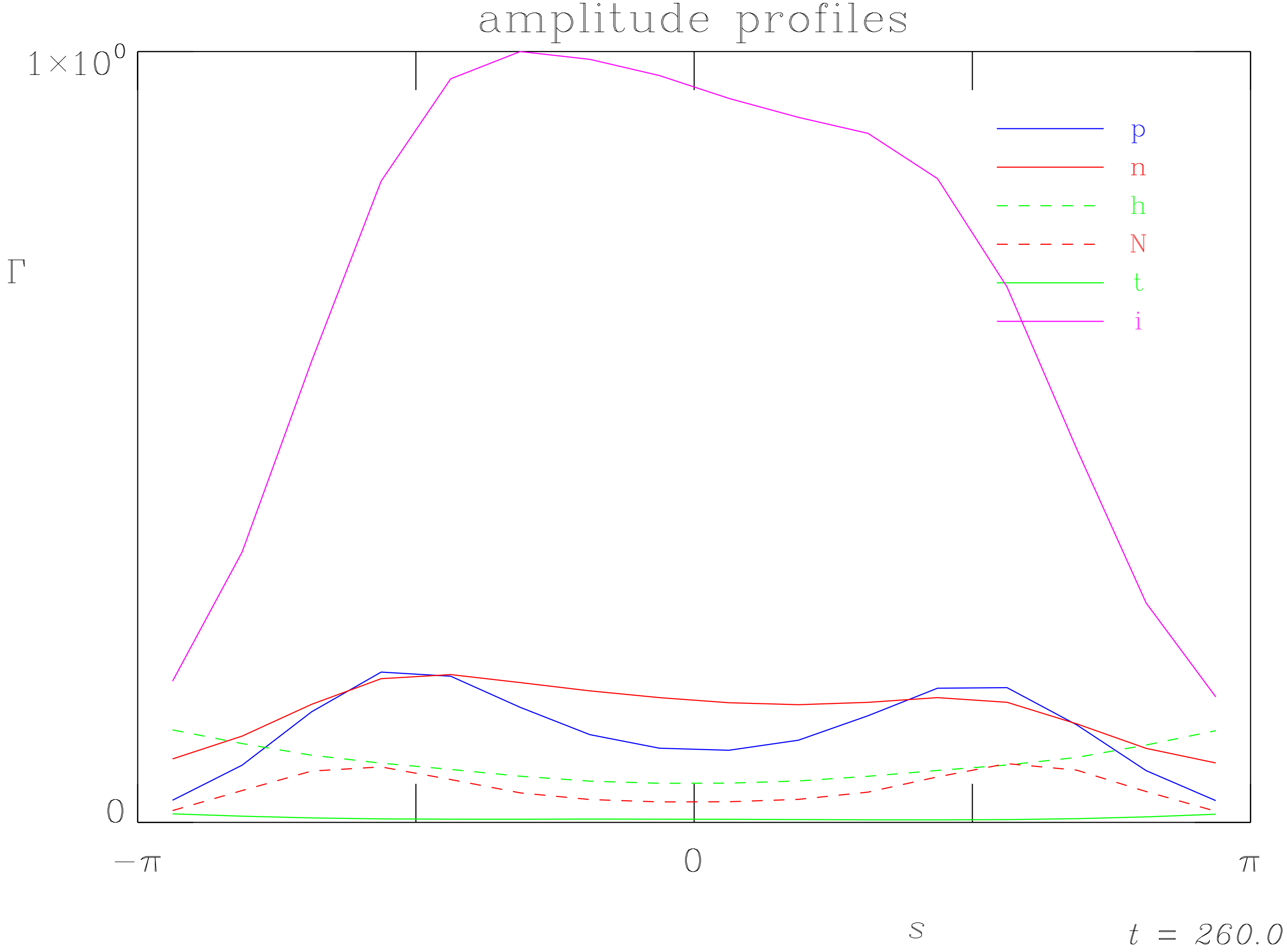
edge/core transition — parallel structure

position 2: $T_e = 200\,\mathrm{eV}$ $L_\perp = 8\,\mathrm{cm}$



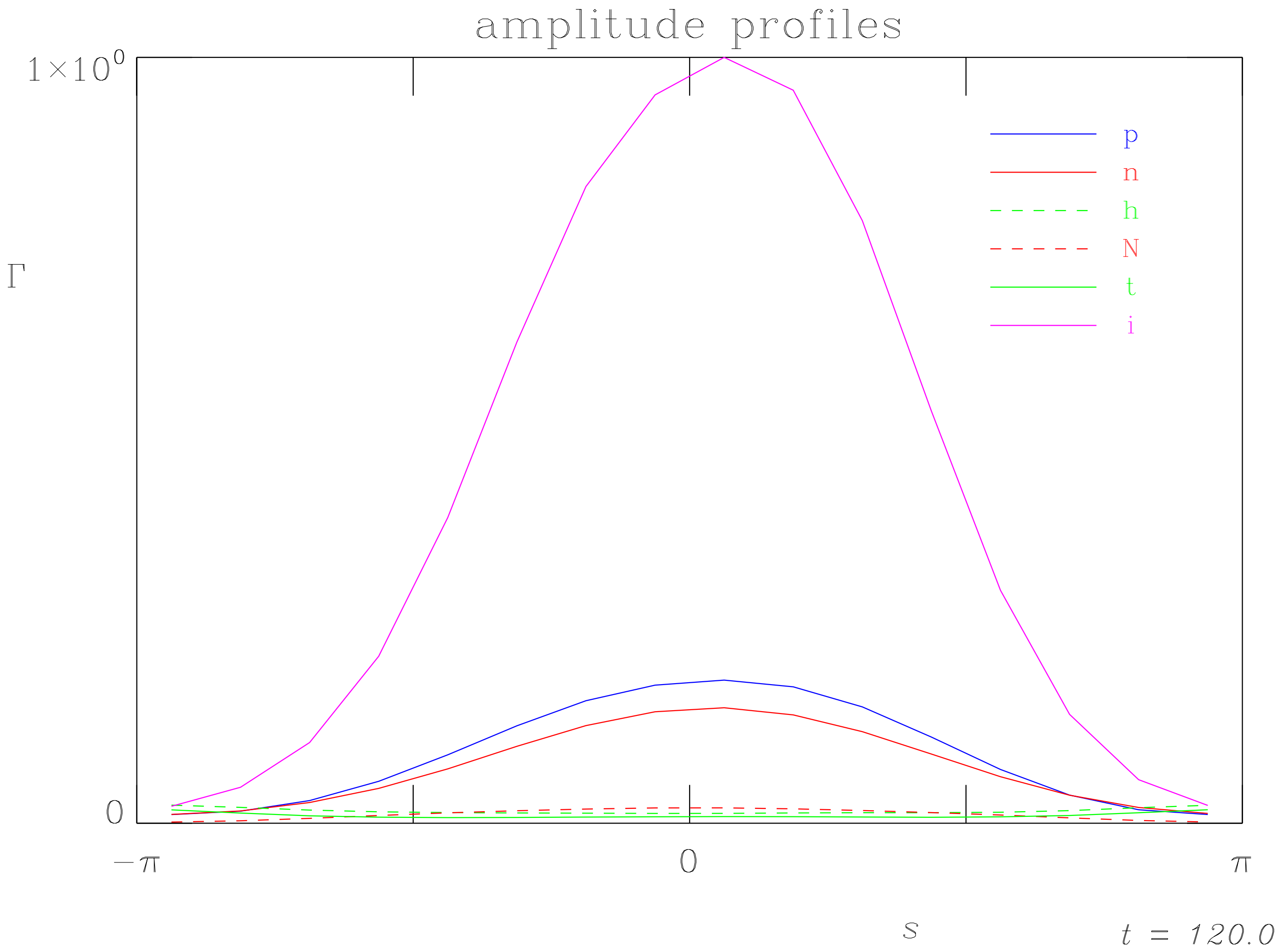
edge/core transition — parallel structure

position 3: $T_e = 300\,\mathrm{eV}$ $L_\perp = 12\,\mathrm{cm}$



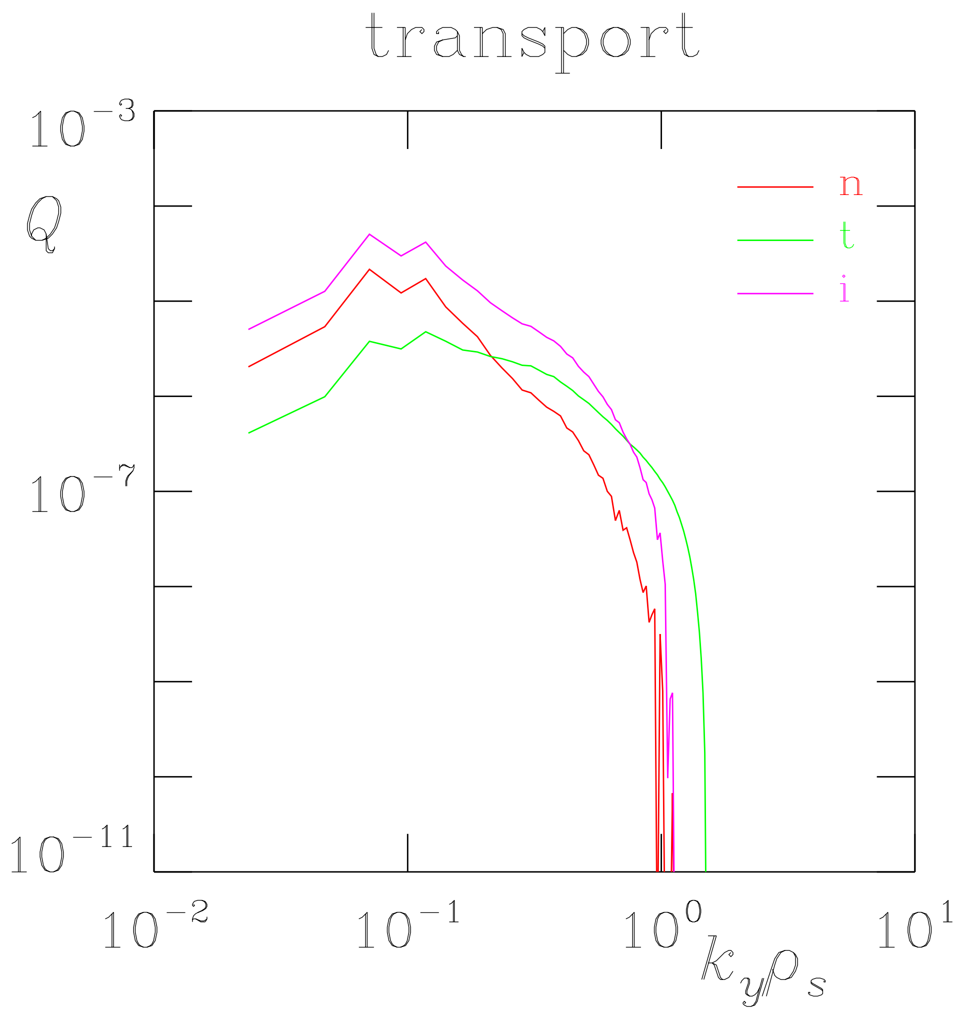
edge/core transition — parallel structure

position 4: $T_e = 400\,\mathrm{eV}$ $L_\perp = 16\,\mathrm{cm}$



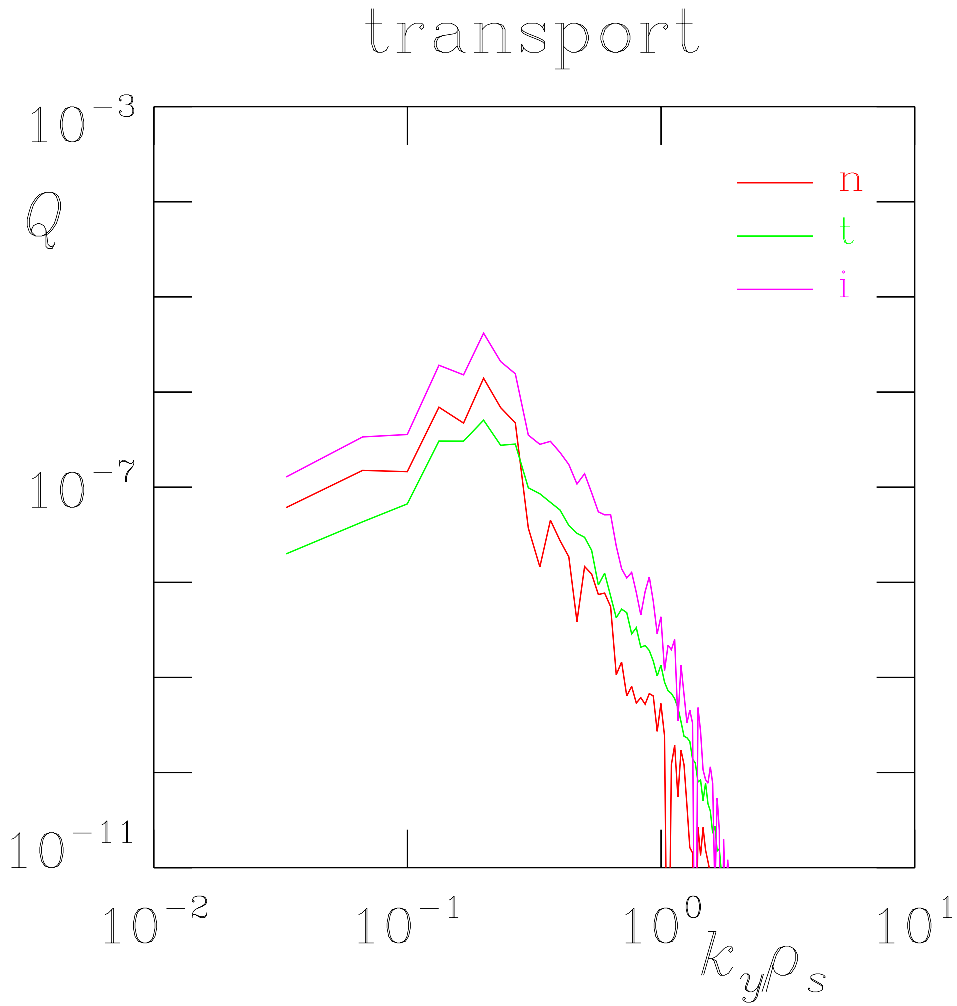
edge/core transition — flux spectrum

position 1: $T_e = 100\,\mathrm{eV}$ $L_\perp = 4\,\mathrm{cm}$



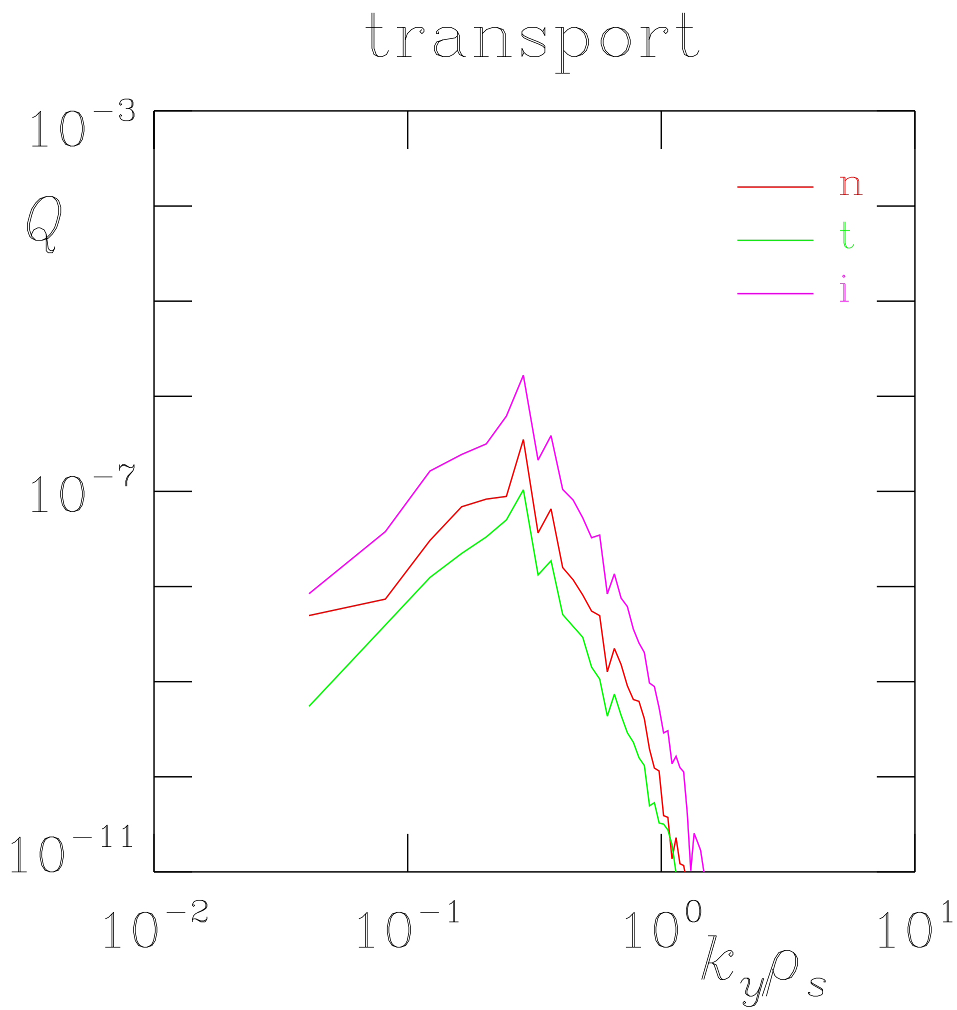
edge/core transition — flux spectrum

position 2: $T_e = 200\,\mathrm{eV}$ $L_\perp = 8\,\mathrm{cm}$



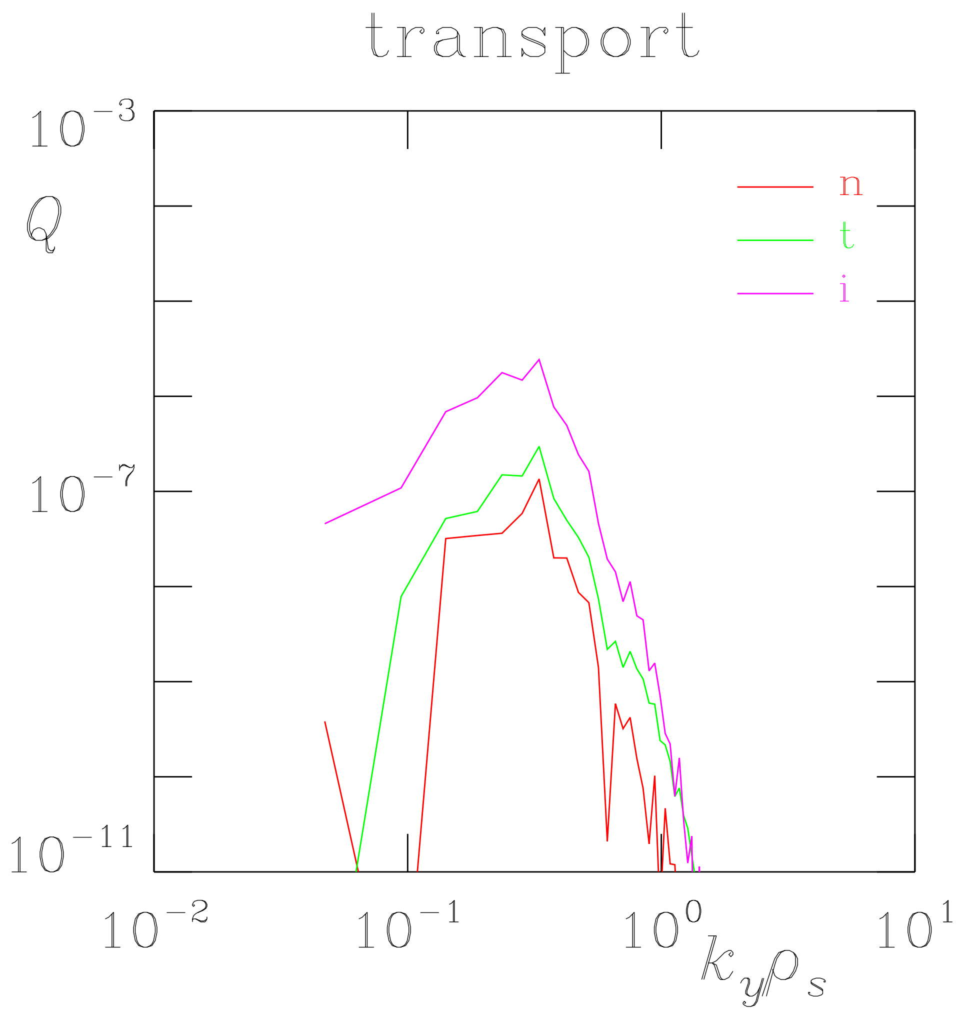
edge/core transition — flux spectrum

position 3: $T_e = 300\,\mathrm{eV}$ $L_\perp = 12\,\mathrm{cm}$



edge/core transition — flux spectrum

position 4: $T_e = 400\,\mathrm{eV}$ $L_\perp = 16\,\mathrm{cm}$



Main Points

- **Turbulence (turb) Drive and Saturation**

  — mechanisms **nonlinear**, do not follow linear scales

  — drive: vorticity nonlinearity, plus linear ITG mechanism

  — saturation: ExB diffusive mixing of electrons

- **Gyrokinetic Edge Turbulence**

  — now feasible

  — basic character same as electromagnetic, collisional, gyrofluid models

  — important differences in T_e effects (anisotropy, also magnetic flutter)

- **Future Generalisation — total-f model (FEFI) almost complete**

  — total-f model (FEFI) in late development stages

  — required for full inhomogeneity, even radial parameter variation

  — apparently minimal necessity to treat pedestal