Linear gyro-kinetic simulations of plasma discharges

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CONTENTS

- Trapped electron modes (electron heat flux in discharges with dominant electron heating)
- Influence of collisionality on the TEM (transition to a dominant ITG))
- Stabilization of the ITG due to a uniform electric field
- Toroidal momentum transport

LINEAR THEORY

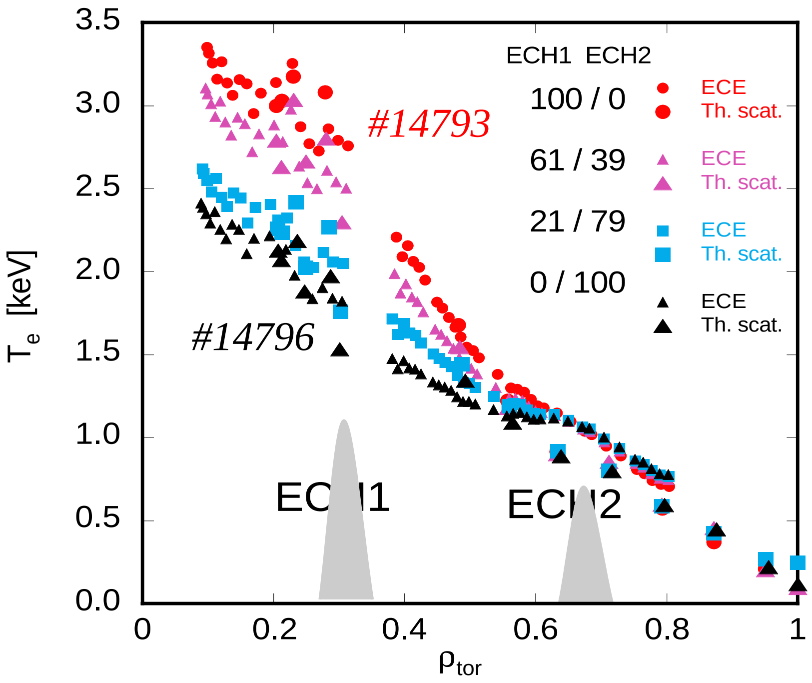
- Core turbulence often weakly non-linear (i.e. mode structure is close to the one of linear theory)

- Saturation is hard to predict but relation between the channels can follow the predictions of linear theory

- Many other problems like the non-linear shift of the spectrum, Dimits shift, ...

- Still, linear theory, although incomplete, reflects some of the behavior of turbulent transport and can be used together with non-linear studies to increase our understanding

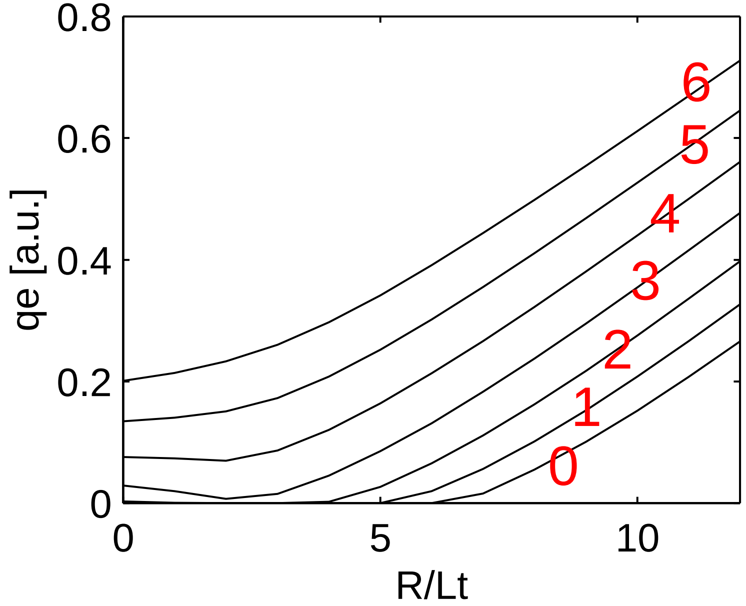
- Best documented transport channel
- The heat flux is not linear in the temperature gradient
- There is evidence of a threshold behavior for the electron temperature gradient (AUG, Tore Supra, not DIII-D)
- Stiffness is not absolute



DOMINANT ELECTRON HEATING

- Studied with the GS2 code
- Dominant mode is found to be a TEM
- The growth rate is sensitive to the electron, density gradient, magnetic shear and collisionality
- Heat flux is found to be roughly offset linear in the electron temperature gradient
- Threshold scales roughly as

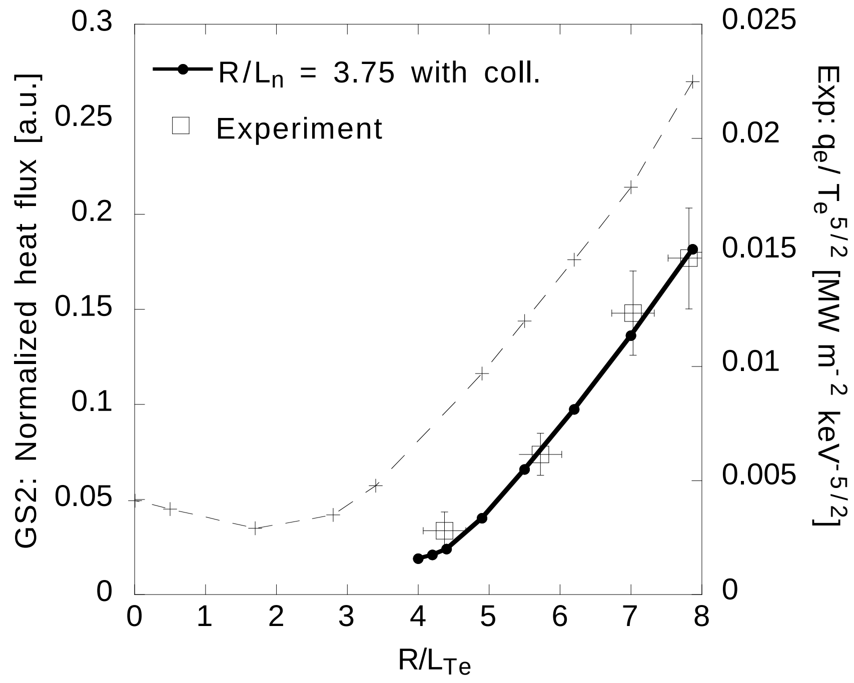
Eflectron heat flux versus
∇Te / Te for different ∇n / n



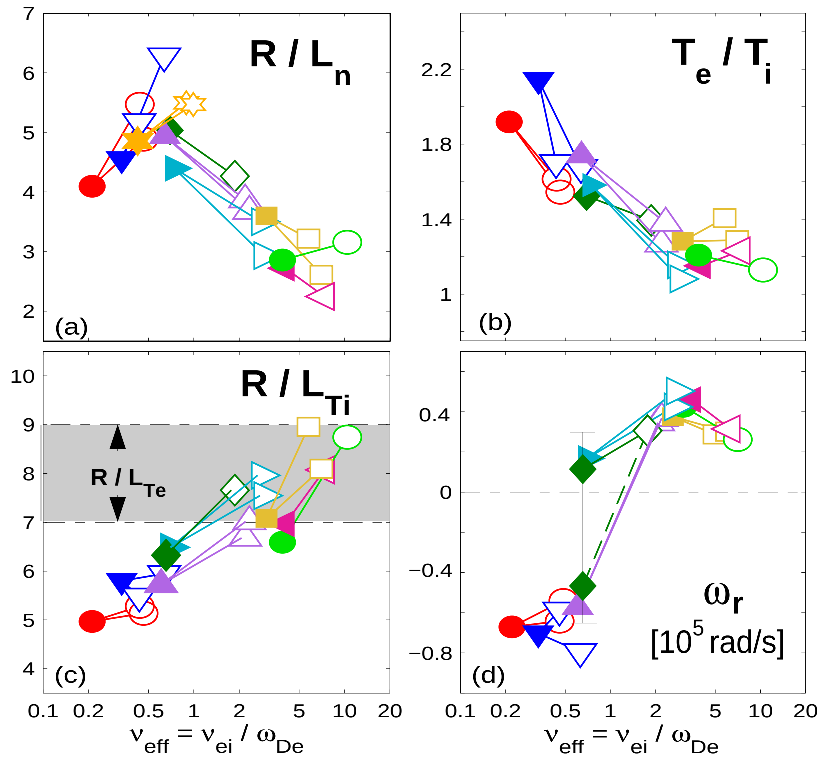
$$\frac{R}{L_{Tcrit}} = \frac{0.357\sqrt{\epsilon}+0.271}{\sqrt{\epsilon}}[4.90-1.31\frac{R}{L_N}+2.68\hat{s}+\ln(1+20\nu_{\rm eff})]$$

DOMINANT ELECTRON HEATING

- Comparison between linear results and experiment
- Arbitrary scaling factor used for the calculations
- Experiments normalized with gyro-Bohm and q
- Nicely explains the threshold and not linear behavior
- Collisions have a non negligible influence



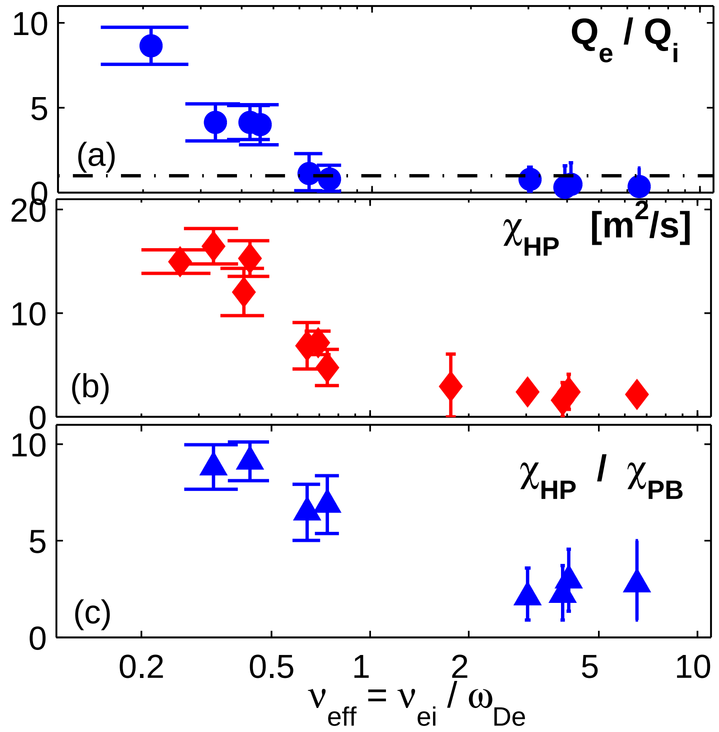
- Electron collision frequency not small compared to ion transit time
- Collisions weaken the TEM (not the ITG)
- Even for dominant electron heating a transition to a dominant ITG is generated at higher collisionality density
- Also controlled over the collisional coupling between electrons and ions



- Transition is observed in the transport behavior

- The propagation speed of the electron temperature perturbations (which is a direct measure of the increase of the electron heat flux with the temperature gradient) decreases

- Also with respect to the steady state heat conductivity



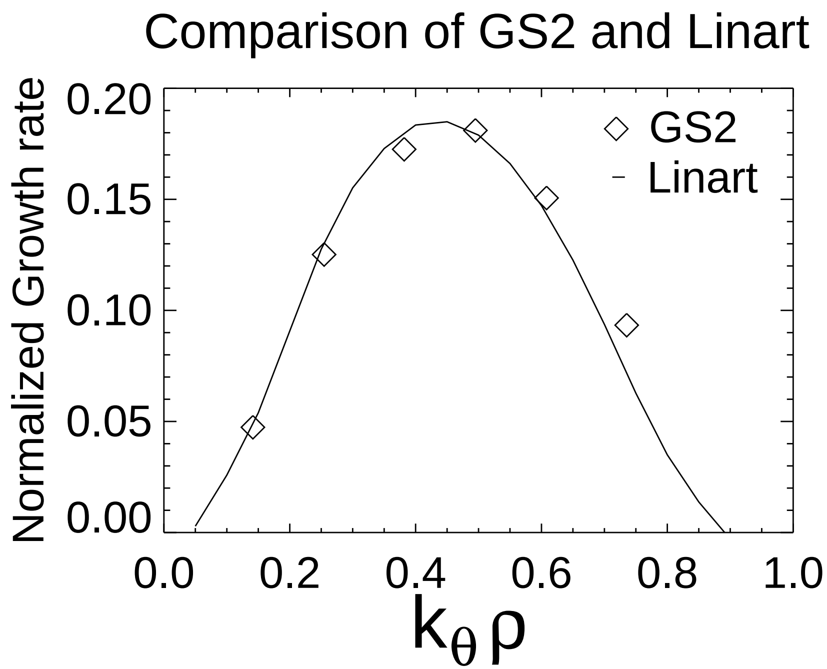
From now on done with LINART: Solves the equation

$$
\begin{aligned}
\frac{\partial f}{\partial t} + \quad & (v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{u}_E) \cdot \nabla f + \frac{1}{2}(v_\parallel \mathbf{b} + \mathbf{u}_E) \cdot \frac{\nabla B}{B} \frac{v_\perp}{v_\parallel} \frac{\partial f}{\partial \theta} \\
& - \frac{e}{m v^2} (v_\parallel \mathbf{b} + \mathbf{v}_d) \cdot [\nabla \langle \phi_0 \rangle + \frac{v_\perp \nabla B}{2B} \frac{\partial \langle \phi_0 \rangle}{\partial v_\perp}] \times \\
& [v \frac{\partial f}{\partial v} - \frac{v_\perp}{v_\parallel} \frac{\partial f}{\partial \theta}] = -\mathbf{v}_E \cdot \nabla F + \frac{e}{m v^2} (v_\parallel \mathbf{b} + \mathbf{v}_d) \cdot \\
& [\nabla \langle \phi \rangle + \frac{v_\perp \nabla B}{2B} \frac{\partial \langle \phi \rangle}{\partial v_\perp}] \times [v \frac{\partial F}{\partial v} - \frac{v_\perp}{v_\parallel} \frac{\partial F}{\partial \theta}],
\end{aligned} \tag{1}
$$

Note that there exists a background electric field potential in this equation, which, however, is not used in these studies.

Equation are solved within the ballooning transform

- Uniform electric field has been reported to stabilize the ITG through rotation of the mode into the stable region [Maccio, PoP]
- The calculations were done with a Maxwellian background as well as a background ExB velocity
- However the Maxwell is not a solution of the gyro-kinetic equation in the presence of a background potential



- Proper solution

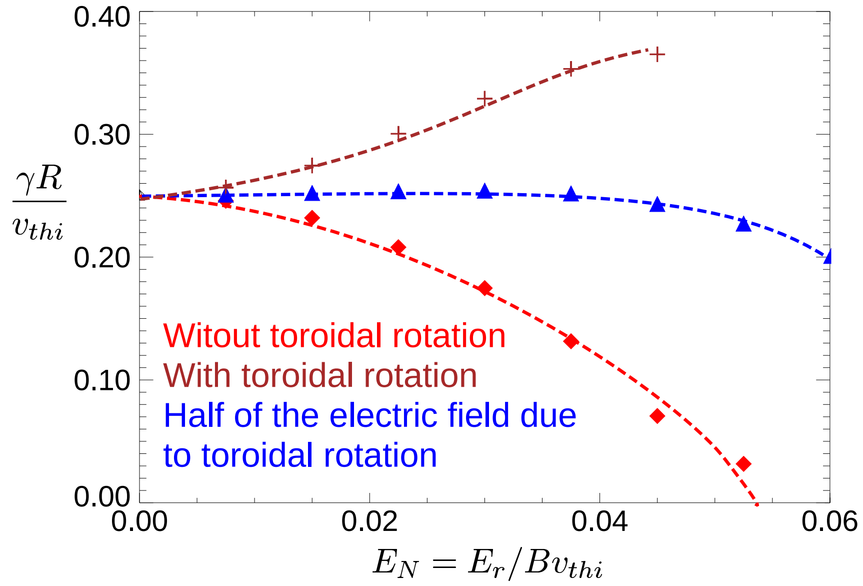
$$F = \frac{n}{\pi^{3/2} v_{thi}^3} \left[1 - (1-\alpha) \frac{e\phi}{T} + \alpha \frac{mc\langle RB_\phi \rangle}{BT} v_\parallel \frac{\partial \phi}{\partial \psi} \right] \exp\left[- \frac{v^2}{v_{thi}^3} \right]$$

- α is an arbitrary parameter

- Agrees with radial force balance, the electric field is balanced by $\alpha = 1$ Toroidal rotation $\alpha = 0$ Pressure gradient

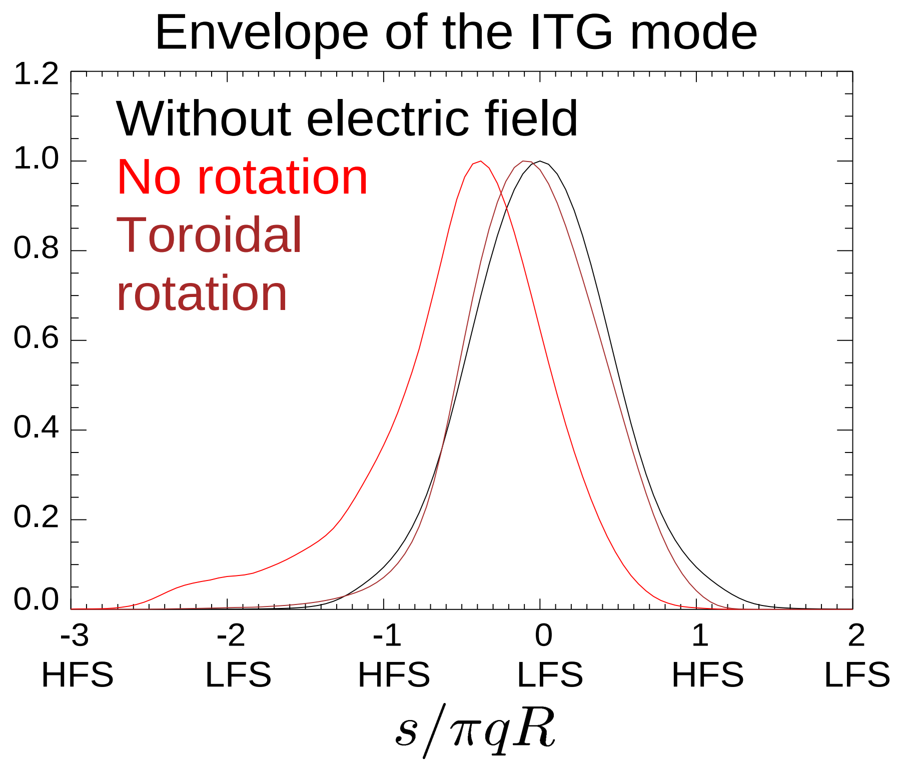
- With toroidal rotation no stabilization of the mode

Normalized growth rate as a function of the normalized electric field



Structure of the potential along the magnetic field

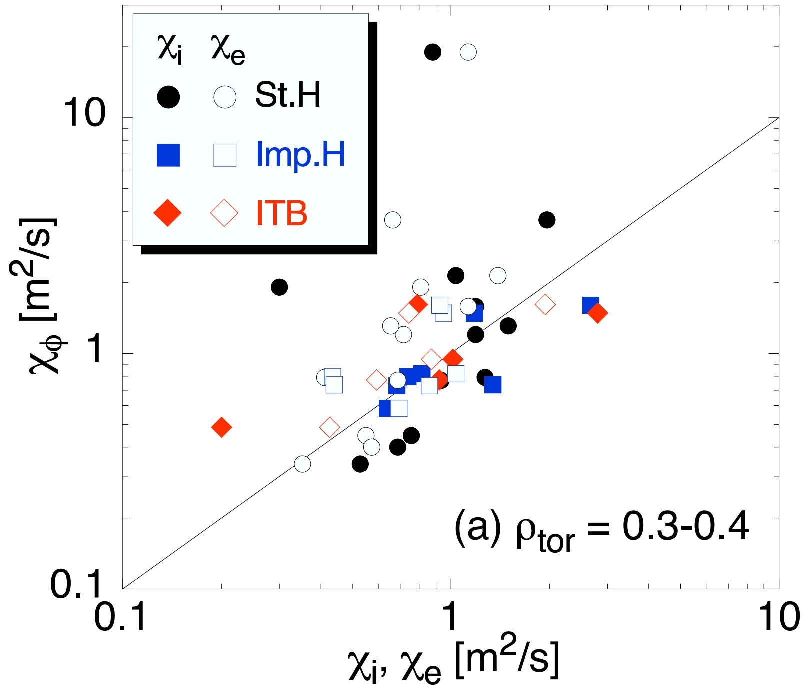
- Without electric field Gaussian shape
- With electric field but no toroidal rotation the maximum is shifted towards the low field side, the mode is more stable
- With electric field and toroidal rotation almost no change in the shape of the potential, the mode is not stabilized



In many experiments electric field is dominantly balanced by toroidal rotation, i.e. no stabilization is expected.

- Transport coefficients for toroidal momentum transport and ion heat transport are similar $\chi_\phi \approx \chi_i$

- Adding ICRH to a NBI heated discharges leads to a reduction in the toroidal rotation which is consistent with the power degradation of transport



From D. Nishijima

GENERAL ASSUMPTIONS ON TOROIDAL MOMENTUM TRANSPORT

- Splitting in toroidal and poloidal: Poloidal flows are damped, (total, sum over species !) toroidal angular momentum is conserved, but can be transported (and absorbed by the scrape-off layer).

- Dynamics however is split in parallel and perpendicular. The radial electric field (which is closely connected with toroidal rotation) is connected with perpendicular dynamics. Toroidal rotation is largely dominated by parallel flows.

- Transport can be generated over both perpendicular dynamics as well as transport of parallel flows. The latter one is the one discussed here (is the only one that can be discussed in linear theory). We assume that the flux of momentum is given by

$$\Gamma_\phi = m \langle \tilde{v}_E \tilde{u}_\parallel \rangle$$

- The electric field quickly adjusts to match changes in the parallel flow.

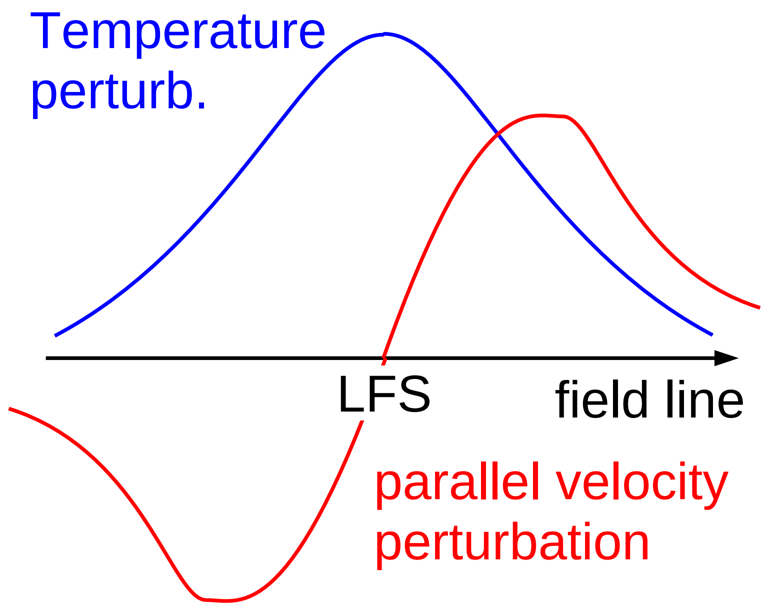
Without parallel velocity shear or background parallel velocity

$$-\mathrm{i}\omega n_i = -i\omega_* \phi + \mathrm{i}\omega_D \phi - \nabla_\parallel U_{\parallel i} + \mathrm{i}\frac{\omega_D}{2\tau}(P_{\parallel i} + P_{\perp i})$$

$$-\mathrm{i}m\omega U_{\parallel i} = \nabla_\parallel (P_{\parallel i}/\tau + Z\phi) + \mathrm{i}\omega_D (2U_{\parallel i} + \mathrm{i}\sigma_t U_{\parallel i})$$

Note that in the equation for the density only the parallel derivative of the velocity occurs, whereas in the equation for the parallel velocity only the parallel derivative of the density and pressure appears

The solution for density, temperature and potential will be even in the poloidal angle whereas the solution for the parallel flow will be odd. $\longrightarrow$ Integration of $\langle \mathbf{v_E u_\parallel} \rangle$ yields zero transport



A transport of toroidal momentum occurs therefore only if $U_\parallel \neq 0$, or $\nabla U_\parallel \neq 0$

$$\Gamma_\phi = \mathcal{L}(\frac{\mathrm{d}U_\parallel}{\mathrm{d}r})$$

The algebraic equations yields the dispersion relation, and from the growth rate and phase relations follows the quasi-linear flux:

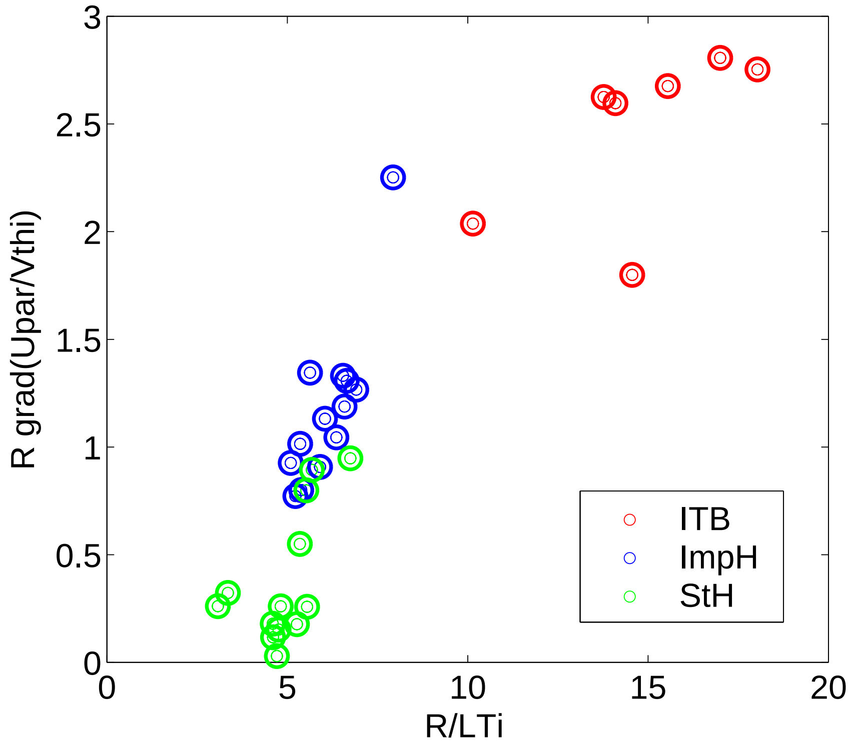
$$M_i n_i \frac{\mathrm{d}U_\phi}{\mathrm{d}t} = \frac{1}{V'}\frac{\mathrm{d}}{\mathrm{d}\rho}\left[V' \langle |\nabla \rho| \rangle \left(M n_i \chi_\phi \frac{\mathrm{d}U_\phi}{\mathrm{d}r} + M_i U_\phi \Gamma \right)\right]$$

Almost perfect diffusion (small correction due to particle flux Γ) Of course

$$\chi_\phi = f(\nabla U_\parallel, \nabla T, \nabla n, q, \hat{s}, \ldots)$$

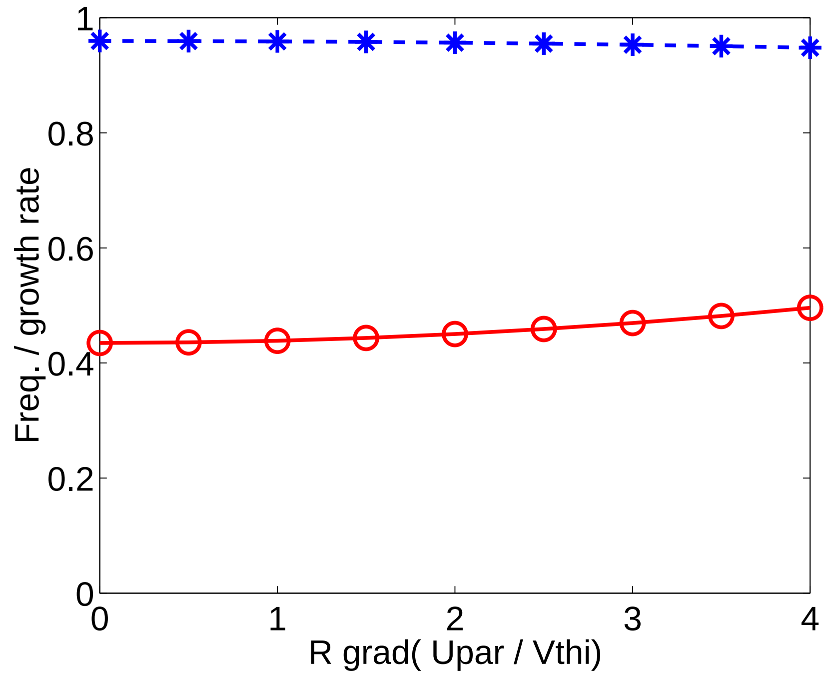
WHAT IS THE EXPERIMENTAL VALUE OF THE PARALLEL FLOW SHEAR

- For standard H-modes this value is usually below 1
- It is around 1 for the Improved/low density H-mode
- It reaches values up to 3 for the internal transport barriers



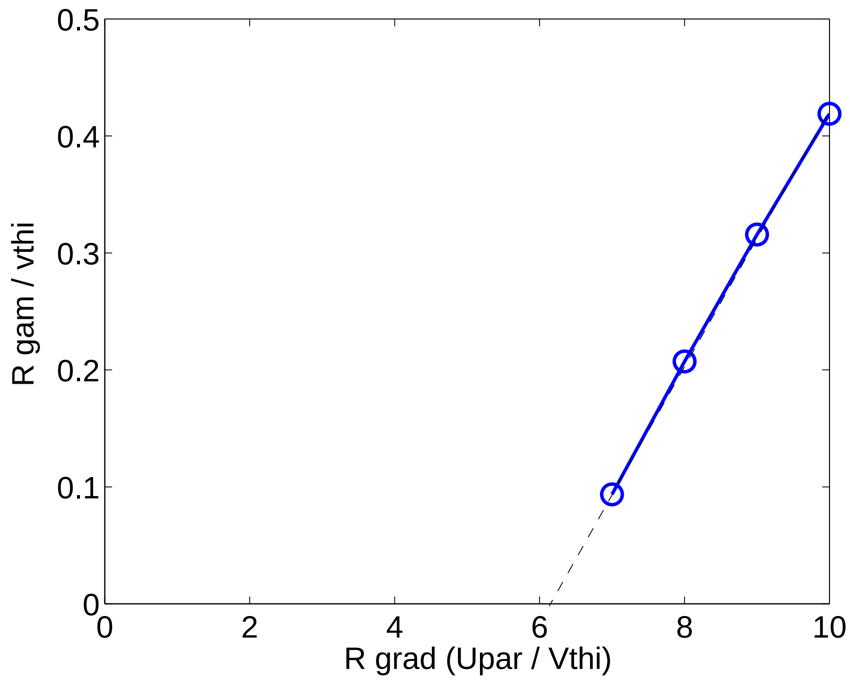
Note that the discharges with higher parallel velocity shear in general have a higher value of the ion temperature gradient length.

- The influence of the parallel velocity gradient on the instability is small
- The parallel velocity gradient can drive instabilities at high values of the gradient
- The influence of the density gradient is small
- The influence of the inverse aspect ratio is small
- The influence of q is small
- The influence of shear is sizeable



MAIN RESULTS

- The influence of the parallel velocity gradient on the instability is small
- The parallel velocity gradient can drive instabilities at high values of the gradient
- The influence of the density gradient is small
- The influence of the inverse aspect ratio is small
- The influence of q is small and in the wrong direction
- The influence of shear is sizeable and in the wrong direction



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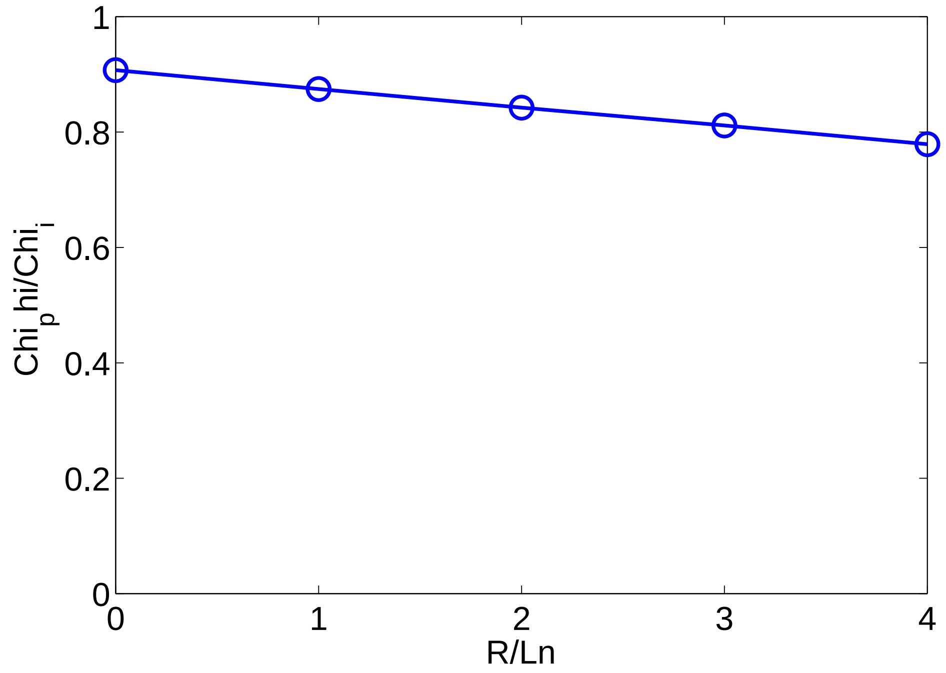

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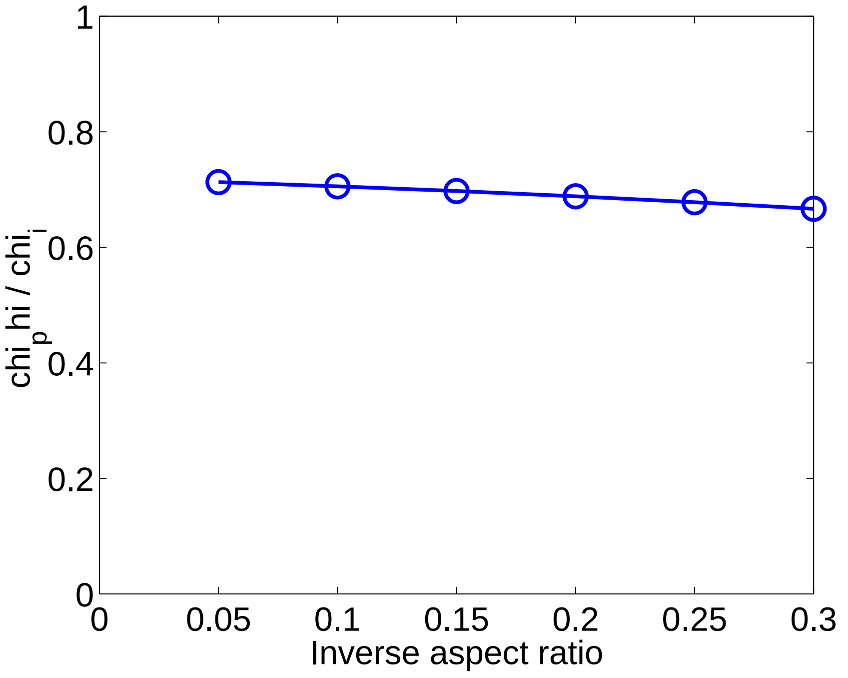

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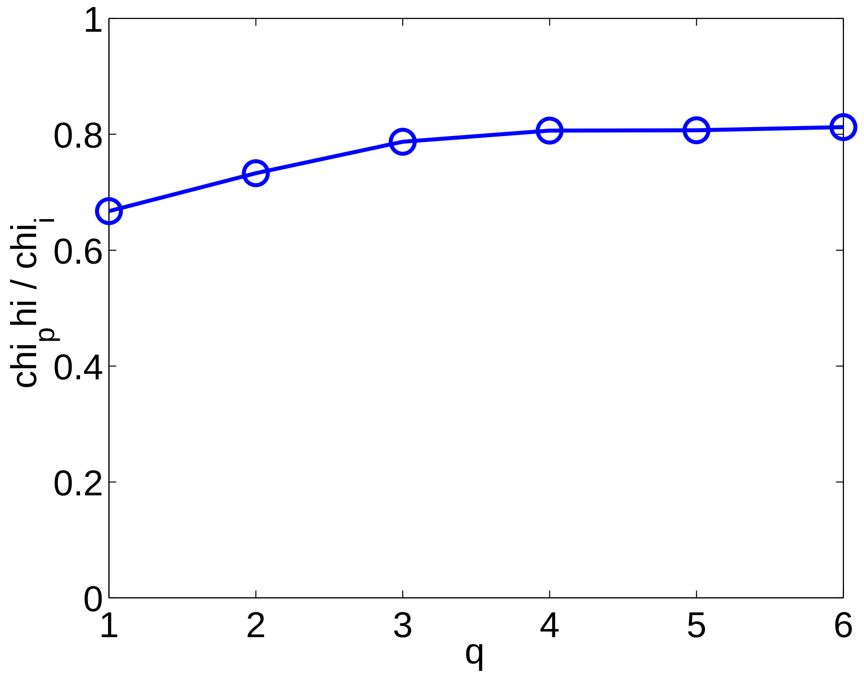

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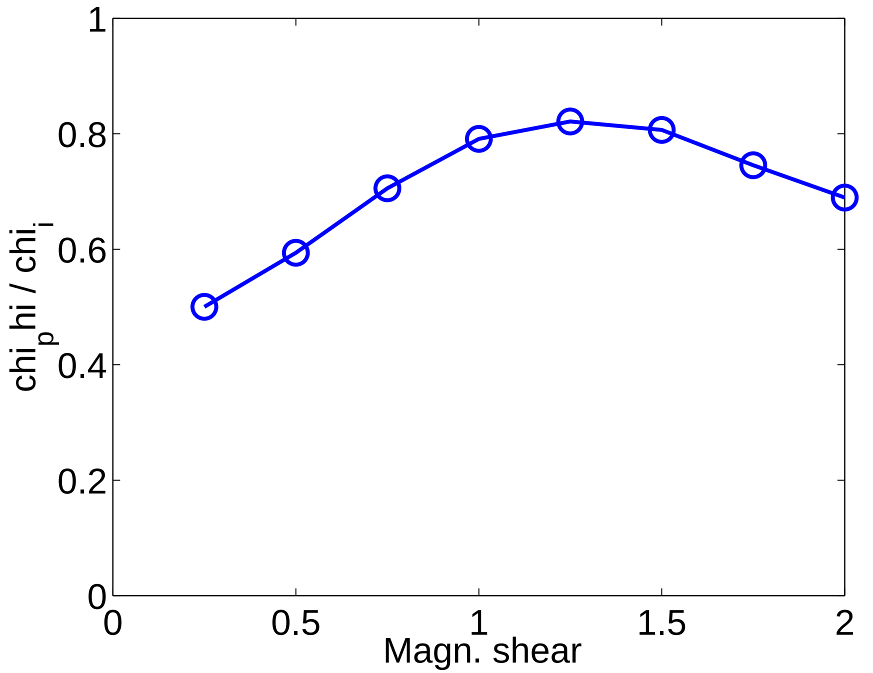

SHEARING RATE / TRANSITION TO ITB

- Ion heat and momentum transport are both dominated by the ITG
- Suppose there is a mechanism that makes the ITG more stable
- Then in the presence of a constant source the toroidal velocity gradient will increase
- This lead to a larger ω_{ExB} shearing rate
- Which in turn leads to a stronger stabilization of the ITG
- Etc. Mechanism for the transition into an ITB?

It is clear that the above feedback loop exists. But one has to take into account that

- A more stable ITG leads in the presence of a constant heat source to a larger ion temperature gradient and therefore more drive of the ITG
- There is a higher inverse gradient length but for a transition one needs a runaway effect

This problem can be discussed best using the transport fluxes

$$q_i = n\chi_i \nabla T_i \qquad \Gamma_\phi = n\chi_\phi \nabla V_\parallel$$

Assume that

$$\chi_i = g\chi_\phi$$

Then the ratio of the fluxes yields

$$g\frac{\nabla T}{\nabla V_\parallel} = \frac{q_i}{\Gamma_\phi} = \frac{m v_{\rm fast}^2}{2\alpha v_{\rm fast}} = \frac{1}{2\alpha} m v_{\rm fast}$$

Then using

$$E_r \approx B_p V_\parallel \qquad \omega_{ExB} \approx \frac{{\rm d}v_E}{{\rm d}r}$$

We obtain

$$g\frac{B_p}{B}\frac{\nabla T}{\omega_{ExB}} = \frac{m v_{\rm fast}}{2\alpha}$$

Now we normalize the temperature gradient to obtain $R/L_T = R\nabla T/T$, and use the approximated formula

$$\frac{R\gamma}{v_{thi}} = f(\ldots)[\frac{R}{L_T} - \frac{R}{L_{Tcrit}}]$$

This allows us to write the formula above in the form

$$\frac{\gamma}{\omega_{ExB}} = \frac{f}{g\alpha}\frac{v_{\rm fast}}{v_{thi}}\frac{B}{B_p} - \frac{f v_{thi}}{\omega_{ExB} L_{Tcrit}}$$

Alternatively one can work out the change in the gradient length $\Delta(R/L_T)$ if one uses $\gamma \approx \omega_{ExB}$ as well as the equation for γ as a function of the gradient length

$$\Delta(\frac{R}{L_T}) = \frac{1}{C-1}\frac{R}{L_{Tcrit}}$$

with

$$C = \frac{f}{g\alpha}\frac{v_{\rm fast}}{v_{thi}}\frac{B}{B_p}$$

$$\Delta(\frac{R}{L_T}) = \frac{1}{C-1}\frac{R}{L_{Tcrit}}$$

For a reasonably strong effect

$$C = \frac{f}{g\alpha}\frac{v_{\rm fast}}{v_{thi}}\frac{B}{B_p} \leq 2$$

This is hard. Very rough estimates yield

$$C = \frac{15}{g}$$

Essentially for this process to work one has to break the relation between the momentum and heat transport through a smaller g

CONCLUSIONS

- In plasmas with dominant electron heating the dominant mode is the TEM. This mode has a threshold and an offset linear relation of the heat flux with respect to the electron temperature gradient, in agreement with the experiment

- Collisionality strongly affects the TEM. Even for dominant electron heating a transition occurs to a dominant ITG

- Uniform electric field are not expected to stabilize the ITG under experimentally relevant conditions

- Toroidal momentum is effectively transported by the ITG. The ratio of transport coefficients for the momentum and ion heat is close to 1 and relatively insensitive to plasma parameters