

**2nd TM on Theory of Plasma Instabilities:
Transport, Stability and their interaction
Trieste, Italy, 2-4 March 2005**



**Two-dimensional Structure and Particle Pinch
in a Tokamak H-mode**

N. Kasuya and K. Itoh (NIFS)

Outline

1. Motivation

H-mode, poloidal shock

2. 2-D Structure

model

weak E_r : homogeneous

strong E_r : inhomogeneous

3. Impact on Transport

particle pinch,

ETB pedestal formation

4. Summary

Motivation

H-mode

Improve confinement

Radial structure – studied in detail

Bifurcation phenomena **transition (jump)**

Turbulence suppression **$E \times B$ flow shear**

$$D = D_c + \frac{D_{t0}}{1 + (h \, dE_r/dr)^2}$$

K. Itoh, et al.,
PPCF **38** (1996) 1

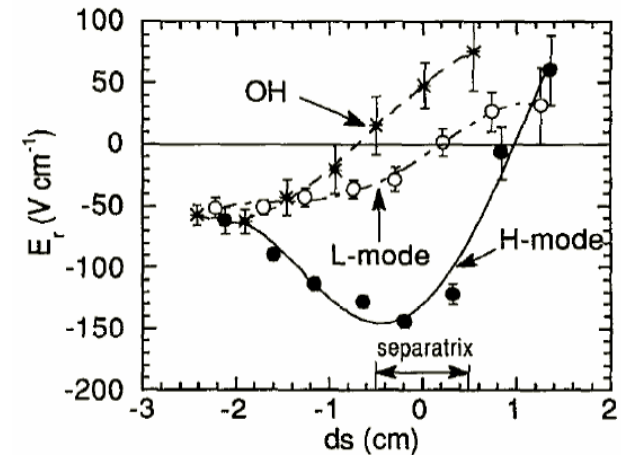
Still remain questions.

Q: Fast pedestal formation mechanism?

Particle Pinch effect ?

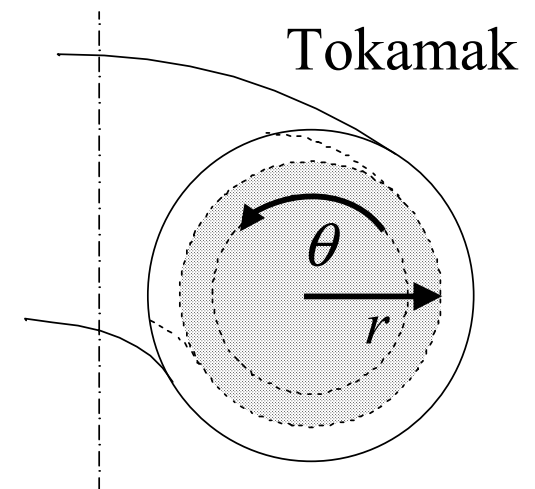
Q: How is two-dimensional (2-D) structure?

e.g.)



Radial profile of edge electric field
in JFT-2M

K. Ida et al., Phys. Fluids B **4** (1992) 2552



Poloidal Shock

Steady **jump** structure of density and potential when poloidal Mach number $M_p \sim 1$

K. C. Shaing, et al., Phys. Fluids B 4 (1992) 404
T. Taniuti, et al., J. Phys. Soc. Jpn. 61 (1992) 568

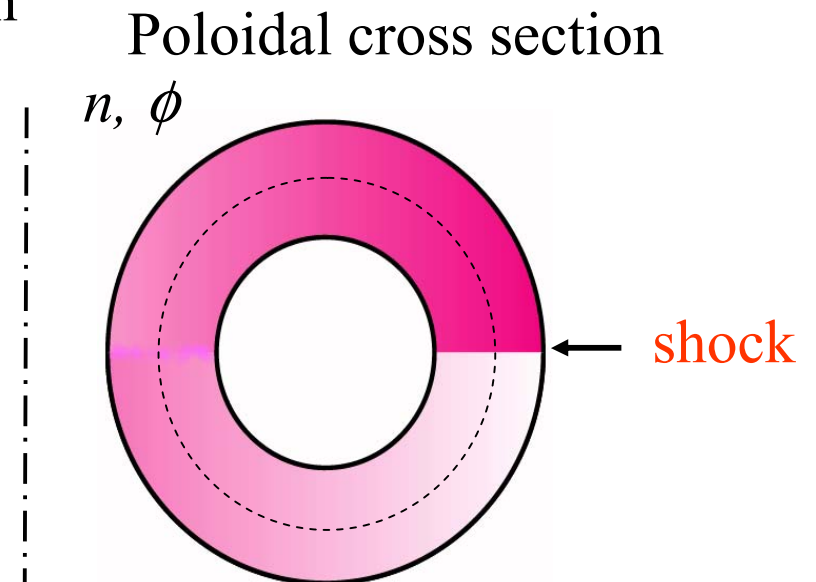
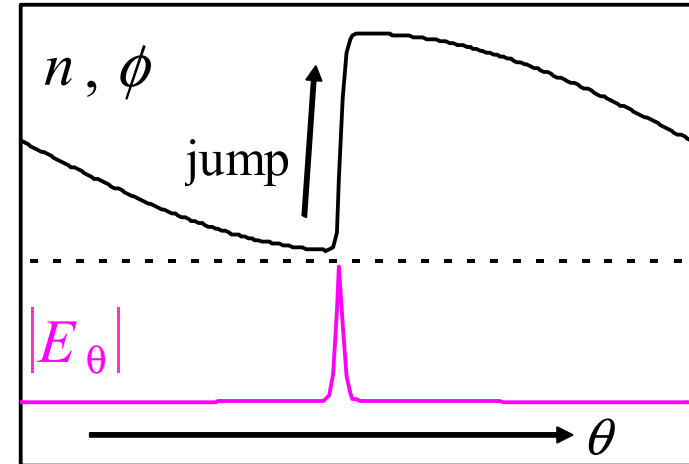
H-mode

Large $E \times B$ flow in the poloidal direction

Prediction of appearance of a **shock structure**

Not much paid attention

Consideration of 2-D structure



Approach

In this research

Density and potential profiles in a tokamak H-mode

Solved as two-dimensional (radial and poloidal) problem

radial structural bifurcation from plasma nonlinear response

+

poloidal shock structure

Both mechanisms are included

Poloidal inhomogeneity \rightarrow radial convective transport

Effect on the density profile formation

2-D Structure

Model

variables $V_p(r), \Phi(r, \theta), n(r, \theta)$

Shear viscosity coupling model

momentum conservation

$$m_i n \frac{d}{dt} \vec{V}_i = \vec{J} \times \vec{B} - \vec{\nabla}(p_i + p_e) - \left(\vec{\nabla} \cdot \vec{\pi}_i \right)_{\text{bulk}} - \left(\vec{\nabla} \cdot \vec{\pi}_i \right)_{\text{shear}}$$

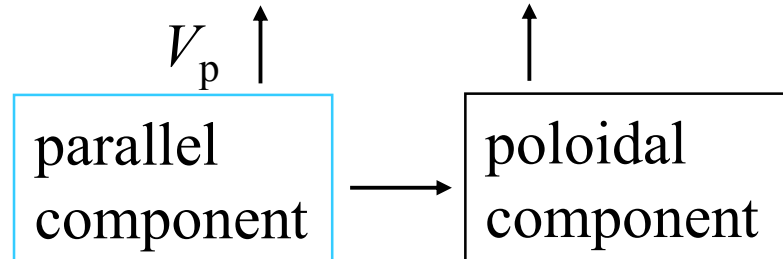
(a)

(b)

(c)

n : density
 \vec{J} : current
 p : pressure
 $\vec{\pi}$: viscosity

$\Phi(r, \theta) = \Phi_0(r) + \Delta\Phi(r, \theta)$ poloidal structure $(V \cdot \nabla)V \dots$ (a)



$$\Delta\Phi \leftrightarrow \Delta n$$

Boltzmann relation

solved iteratively in shock ordering $\frac{e\Delta\Phi}{T} = O(\epsilon^{1/2})$

Previous L/H transition model bifurcation nonlinearity $\nabla \cdot \pi \dots$ (b)

$\left(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i \right)_{\text{shear}} = -m_i n \mu B \nabla_{\perp}^2 V_{\parallel}$ radial and poloidal coupling \dots (c)

V_p, Φ_0 Poloidal component (flux surface average) (i)

$$\left\langle -\frac{nI}{KB^2 r R} \frac{\partial \Phi}{\partial \theta} \frac{\partial}{\partial r} \left[\frac{1}{2} \left(\frac{KB_p}{n} \right)^2 \right] + \frac{B_p}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{2} \left(\frac{KB_p}{n} \right)^2 \right] \right\rangle$$

$$= \frac{1}{m_i} \left\langle \frac{JB_p B_\zeta}{n} \right\rangle - \frac{1}{m_i} \left\langle \frac{\vec{B}_p \cdot \vec{\nabla} \cdot \vec{\pi}_i}{n} \right\rangle_{\text{bulk}} - \frac{1}{m_i} \left\langle \frac{\vec{B}_p \cdot \vec{\nabla} \cdot \vec{\pi}_i}{n} \right\rangle_{\text{shear}}$$

$\Delta \Phi$ Parallel component (ii)

Substitution of obtained $V_p(r)$

$$-\frac{nI}{KB^2 r R} \frac{\partial \Phi}{\partial \theta} \frac{\partial}{\partial r} \left[\frac{1}{2} \left(\frac{KB}{n} \right)^2 \right] + \frac{B_p}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{2} \left(\frac{KB}{n} \right)^2 \right] + \frac{IB_\zeta}{B^2 r R} \frac{\partial \Phi}{\partial \theta} \frac{\partial}{\partial r} \left[\frac{I}{RB_p B_\zeta} \frac{\partial \Phi}{\partial r} \right] - \frac{KB_p B_\zeta}{nr} \frac{\partial}{\partial \theta} \left[\frac{I}{RB_p B_\zeta} \frac{\partial \Phi}{\partial r} \right]$$

$$= -\frac{B_p}{m_i r} \frac{\partial}{\partial \theta} \left(\frac{\langle P_e \rangle}{\langle n \rangle} \ln n + \frac{5}{2} \frac{\langle P_i \rangle}{\langle n^{5/3} \rangle} n^{2/3} \right) - \frac{1}{m_i n} (\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i)_{\text{bulk}} - \frac{1}{m_i n} (\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i)_{\text{shear}}$$

Response between n and Φ Boltzmann relation $n = \bar{n} \exp \frac{e\Delta\Phi}{T}$

Variables $K(r), \Phi(r, \theta), n(r, \theta)$

$$K = \frac{nV_p}{B_p}$$

(i) Radial Solitary Structure

Electrode Biasing

Charge conservation law

$$\frac{\partial}{\partial t} E_r = -\frac{1}{\epsilon_0 \epsilon_{\perp}} (J_{\text{visc}} + J_r - J_{\text{ext}})$$

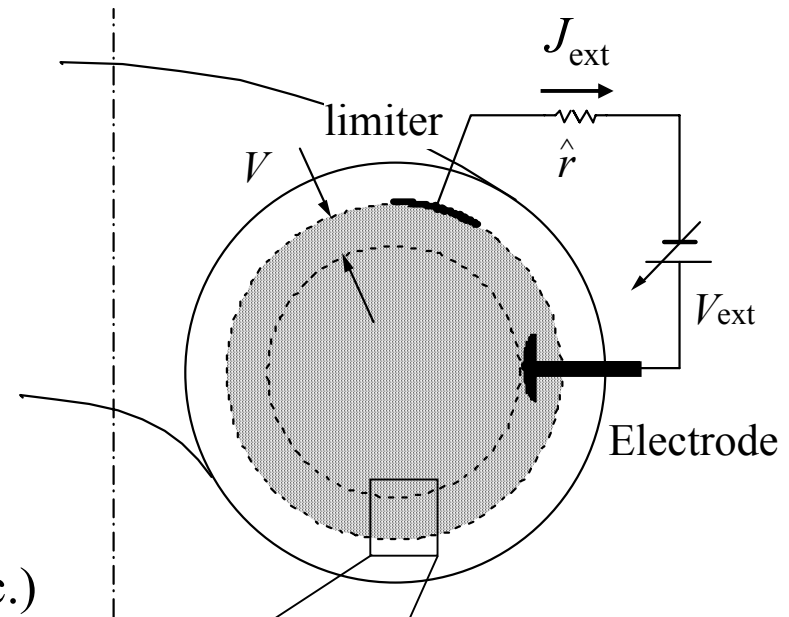
J_{visc} : shear viscosity of ions (anomalous)

J_r : bulk viscosity (neoclassical)

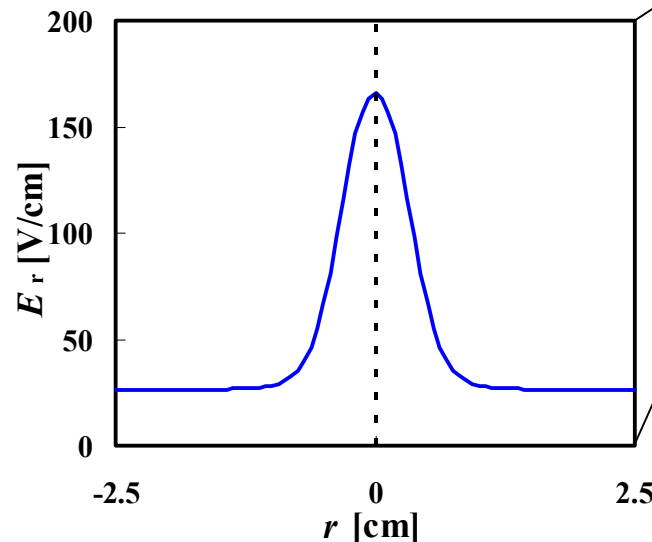
J_{ext} : external current (electrode, orbit loss, etc.)

stable solitary solutions

N. Kasuya, et al., Nucl. Fusion **43** (2003) 244



R. R. Weynants et al., Nucl. Fusion **32** (1992) 837.



radial structure

Flux surface averaged quantities

(ii) Poloidal Variation

$$V_r/V_p \ll 1 \quad (K(\psi)) \quad , \quad (\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i)_{\text{shear}} = -m_i n \mu B \nabla_{\perp}^2 V_{\parallel}$$

$$-\hat{\mu} r^2 \frac{B_0}{B_p} \frac{\partial^2}{\partial r^2} \left\{ M_p [\exp(-\chi) - 1] \right\} + \frac{2}{3} D \exp(-\chi) \frac{\partial^2 \chi}{\partial \theta^2} + (1 - M_p^2) \frac{\partial \chi}{\partial \theta} + 2A \frac{\partial \chi^2}{\partial \theta}$$

$$= \varepsilon \left(\left\{ D - \hat{\mu} \frac{B_0}{B_p} \left[2r^2 \frac{\partial^2 M_p}{\partial r^2} + 4r \frac{\partial}{\partial r} M_p - 2M_p \right] \right\} \cos \theta - 2M_p^2 \sin \theta \right)$$

$\hat{\mu} = 0 \rightarrow$ Previous works (Shaing, Taniuchi)

$\chi = \ln(n/\bar{n})$: density (to be obtained)

$M_p = \frac{KB_0}{\bar{n} v_{ti} C_r}$: poloidal Mach number (from Eq. in (i))

Solve this equation to obtain 2-D χ profile

$$\hat{\mu} = \frac{\mu}{r v_{ti} C_r} \quad K = \frac{n V_p}{B_p} \quad A = \frac{M_p^2}{2} + \frac{5}{36} \frac{1}{C_r^2} \quad C_r^2 = \left[\frac{5}{3} + \frac{T_e}{T_i} \right] / 2 \quad D = \frac{4\sqrt{\pi}}{3} \frac{I_{ps} K B_0}{\bar{n} v_{ti} C_r^2}$$

Simplified case M_p : giving a solitary profile
 strong toroidal damping
 boundary condition : $\chi = 0$

L-mode

Weak flow, homogeneous E_r case

$M_p = 0.33$ (spatially constant)

Boundary condition

$$\Delta\Phi = 0 \text{ at } r - a = 0, -5[\text{cm}]$$

$$R = 1.75[\text{m}], a = 0.46[\text{m}],$$

$$B_0 = 2.35[\text{T}], T_i = 40[\text{eV}],$$

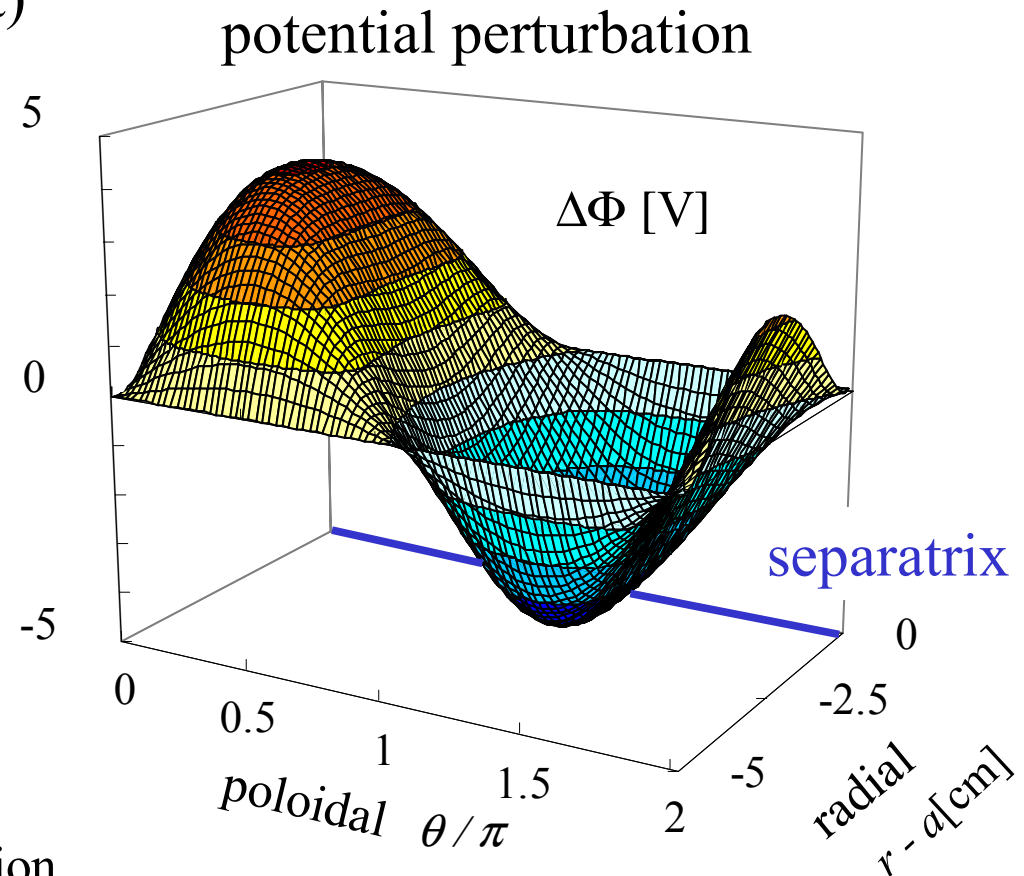
$$I_p = 200[\text{kA}]$$

$$\mu = 1.0[\text{m}^2/\text{s}]$$

gradual spatial variation
no shock

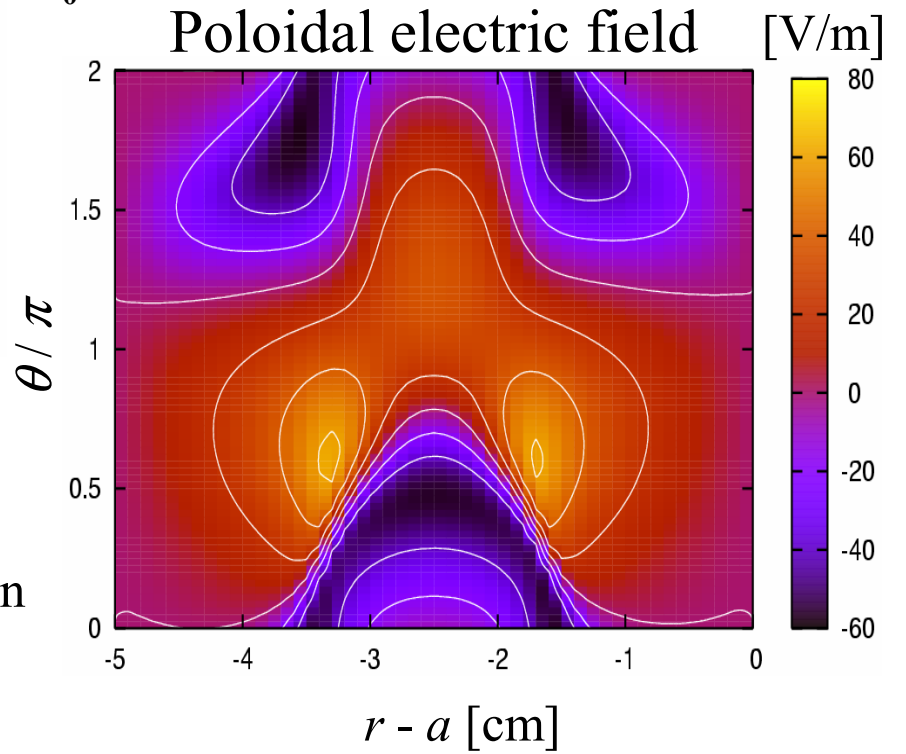
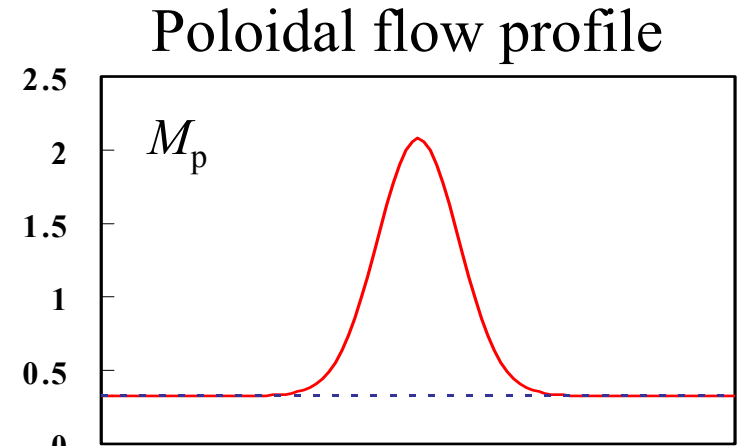
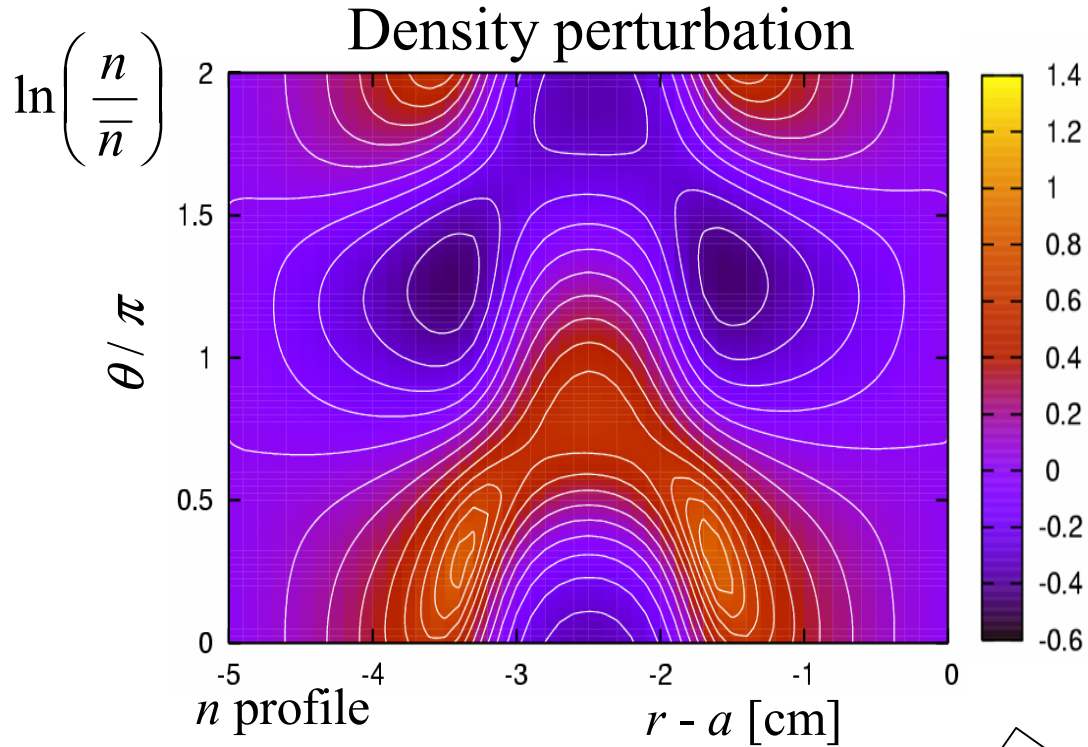
μ : relative strength of radial diffusion
to poloidal structure formation

N. Kasuya et al., submitted to J. Plasma Fusion Res.

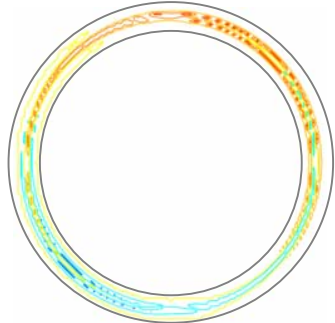


Strong E_r

$\mu = 1$ [m²/s] (experimentally, intermediate case)



(poloidal cross section)



Boltzmann relation

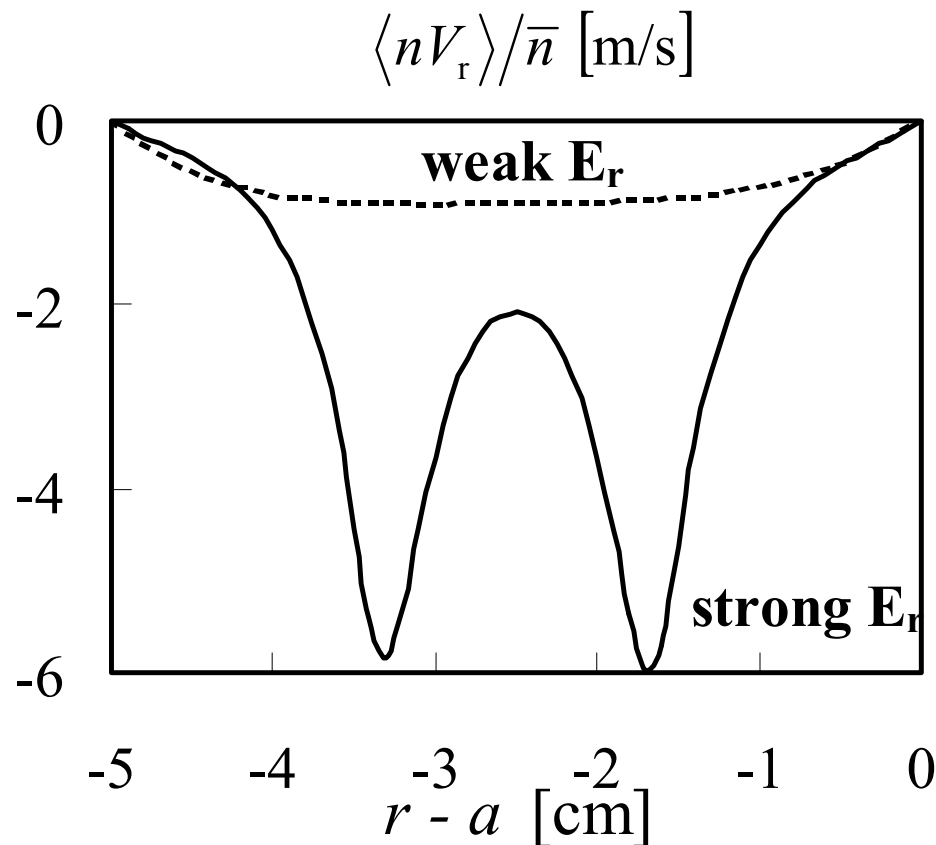
$$n = \bar{n} \exp\left(\frac{e\Delta\Phi}{T}\right)$$

Effect on Transport

Radial Flux

$$\langle nV_r \rangle = \left\langle n \frac{E_\theta}{B} \right\rangle = \frac{1}{2\pi B_0} \int n E_\theta (1 + \varepsilon \cos \theta)^2 d\theta$$

Inward flux arises from poloidal asymmetry.



Inward flux is larger in the shear region.

shear viscosity term

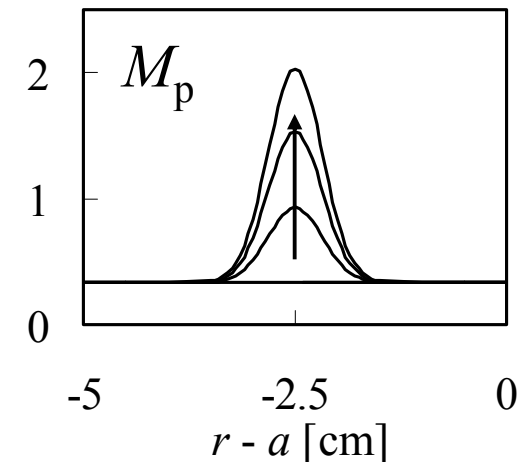
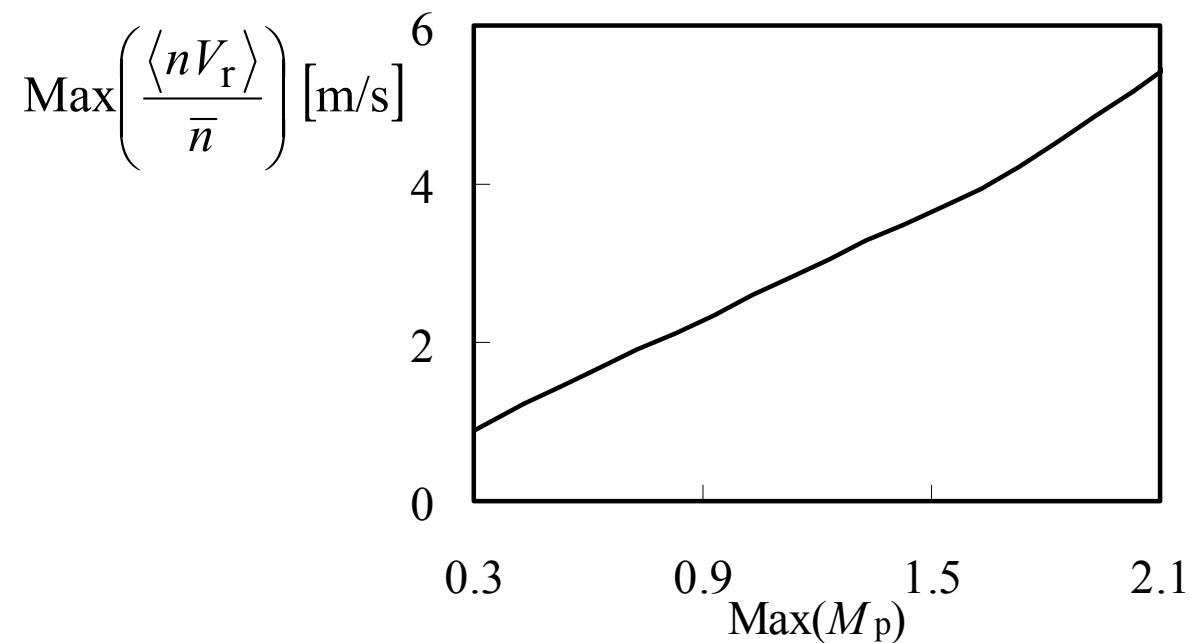
$$(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i)_{\text{shear}} = -m_i n \mu B \underbrace{\nabla_\perp^2}_{\substack{\text{poloidal} \\ \text{asymmetry}}} \underbrace{V_{\parallel}}_{\substack{\text{gradient} \\ \text{and} \\ \text{curvature}}}$$

Impact on Transport (1)

Inward Pinch

If poloidal asymmetry exists, it brings particle flux that can determine the density profile.

Asymmetry coming from toroidicity gives $V_r \sim O(1)[\text{m/s}]$



increase of convective transport

Impact on Transport (2)

L/H Transition

Inside the shear region

local poloidal flow

→ 2-D shock structure

→ averaged inward flux

Transition

suppression of turbulence
and reduction of diffusive transport

(Well known)

+

sudden increase of convective transport

(New finding)

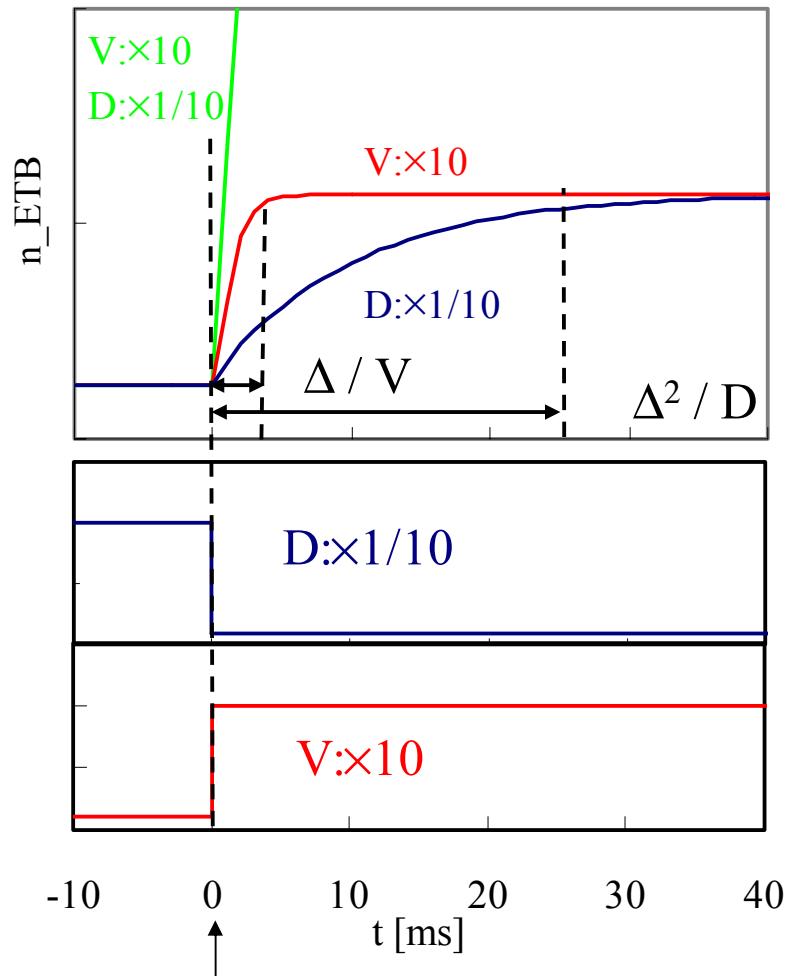
continuity equation

$$\frac{\partial n}{\partial t} = -\nabla \left(\underbrace{nV}_{\text{convective}} - \underbrace{D\nabla n}_{\text{diffusive}} \right)$$

$V_r \nearrow$ $D \searrow$

Rapid Formation of ETB Pedestal

Density profile

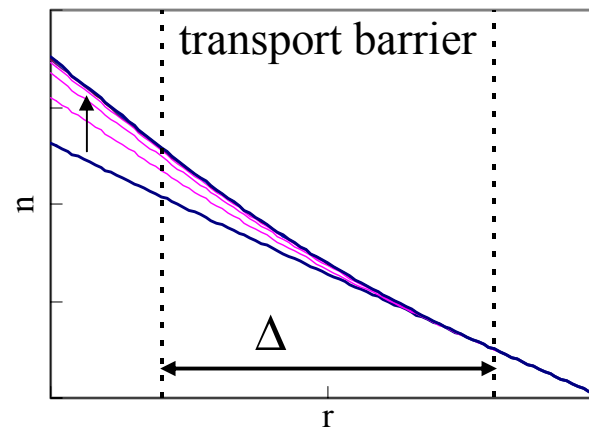


L-H transition

Influence of the jump in convection

Transport suppression only gives slow ETB pedestal formation.

Sudden increase of the convective flux induces the rapid pedestal formation.



Direction of Convective Velocity

Direction of particle flux can be changed by inversion of

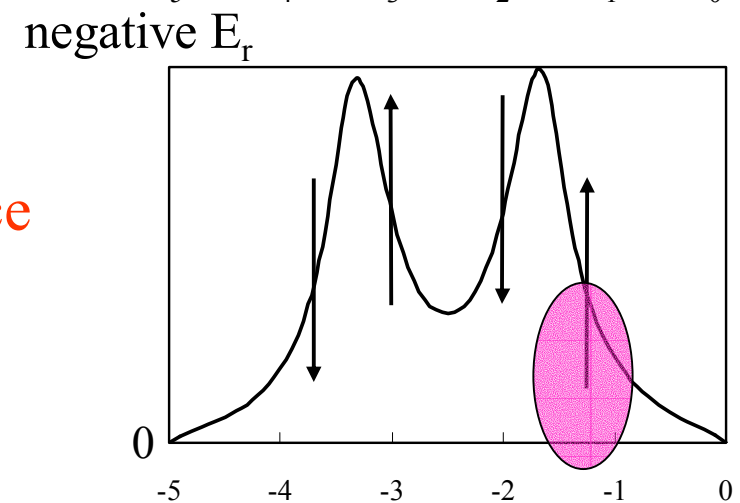
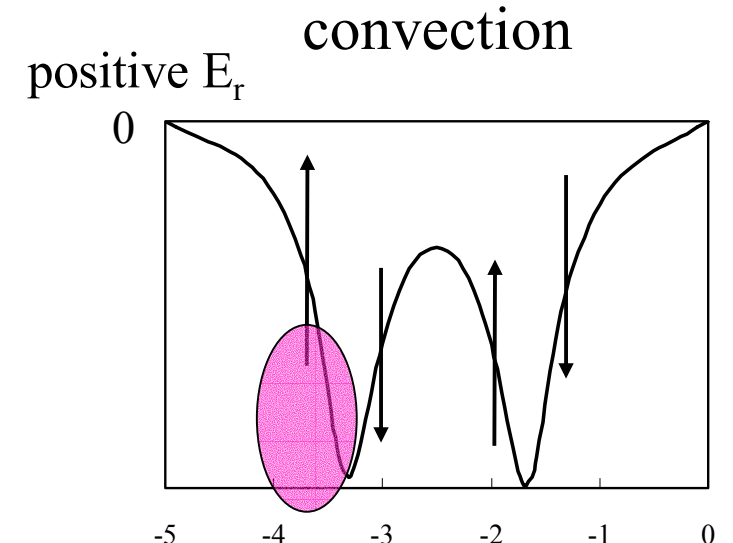
$$M_p(E_r), B_t, I_p$$

Divergence of particle flux leads the density to change.

$$\frac{\partial n}{\partial t} = -\nabla(nV - D\nabla n) + S$$

Sign of the electric field makes a difference in the position of the pedestal.

The particle source and the boundary condition are important to determine the steady state.



increase of density

Summary

Multidimensionality is introduced into H-mode barrier physics in tokamaks.

radial steep structure in H-mode + poloidal shock structure

Shear viscosity coupling model shock ordering

structural bifurcation from nonlinearity

Poloidal flow makes poloidal **asymmetry** and generates **non-uniform particle flux**.

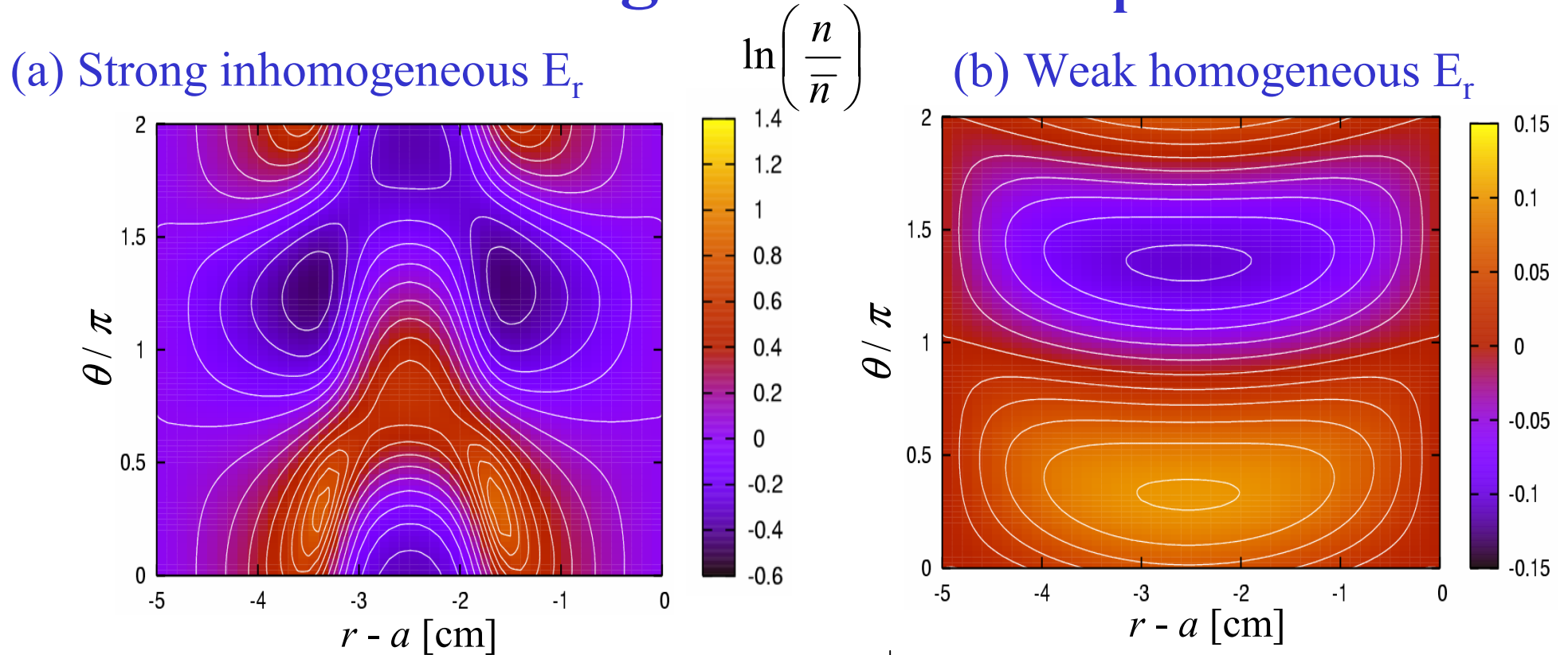
inward pinch $V_r \sim \text{O}(1-10)[\text{m/s}]$

Sudden **increase of convective transport** in the shear region.

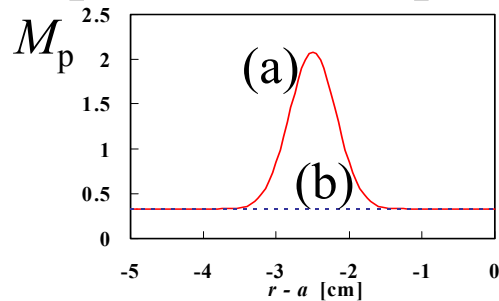
This gives new explanation of fast H-mode pedestal formation.

The steepest density position in ETB changes in accordance with the direction of E_r , B_t and I_p .

Strong and Weak E_r



poloidal flow profile



Strong	Weak
$\Delta\Phi_{\max} \sim 50[\text{V}]$	$\Delta\Phi_{\max} \sim 4[\text{V}]$
$E_{\theta\max} \sim 63[\text{V/m}]$	$E_{\theta\max} \sim 9[\text{V/m}]$
$V_{r\max} \sim 28[\text{m/s}]$	$V_{r\max} \sim 4[\text{m/s}]$

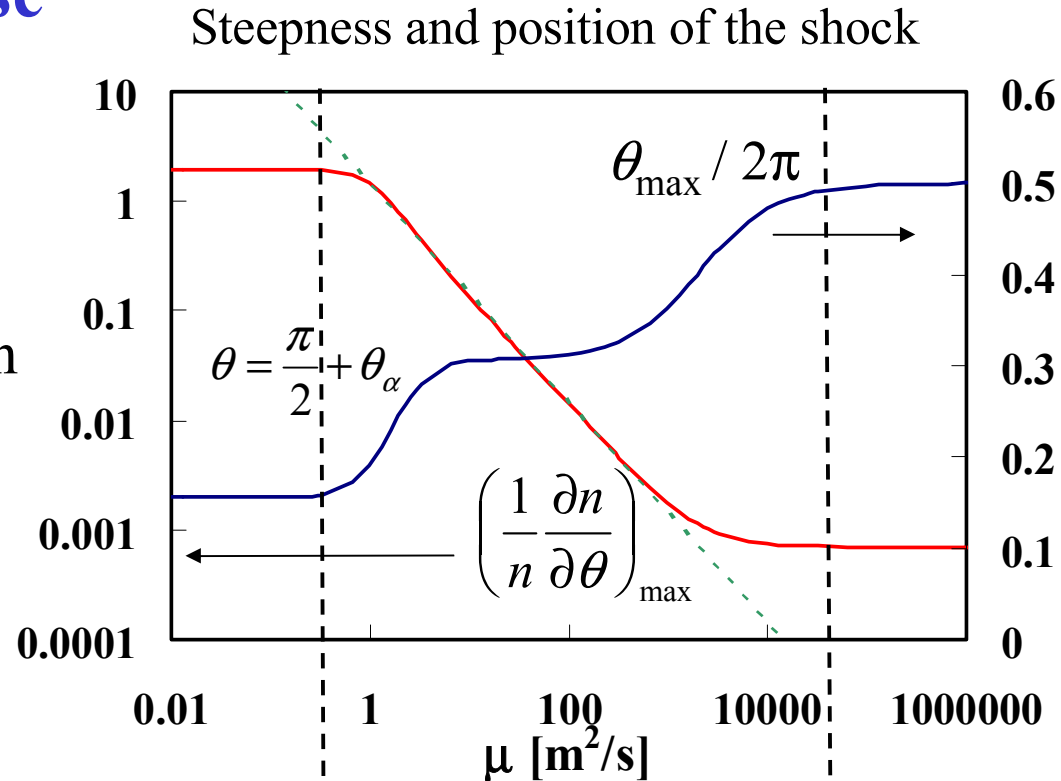
$\mu = 1[\text{m}^2/\text{s}]$

Radial and Poloidal Coupling

Fast rotating case

$$M_p = 1.2 : \text{const}$$

Shear viscosity μ controls the strength of coupling.



$$\left(\frac{1}{n} \frac{\partial n}{\partial \theta} \right)_{\max}$$

$$= \frac{3\varepsilon}{2D} \sqrt{2M_p^2 + D^2} [\cos(\theta_{\text{shock}} + \theta_\alpha) + 1]$$

Shock region

Intermediate region

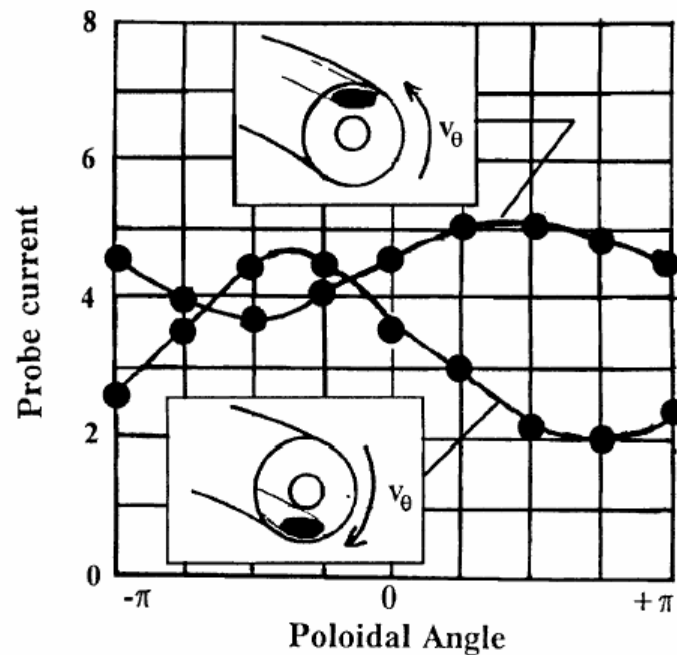
Viscosity region

$$\left(\frac{1}{n} \frac{\partial n}{\partial \theta} \right)_{\max} \propto \frac{1}{\mu}$$

$$\left(\frac{1}{n} \frac{\partial n}{\partial \theta} \right)_{\max} \sim \frac{\varepsilon d^2}{4a^2}$$

Remark on Experiment

Poloidal density profile in electrode biasing H-mode in CCT tokamak

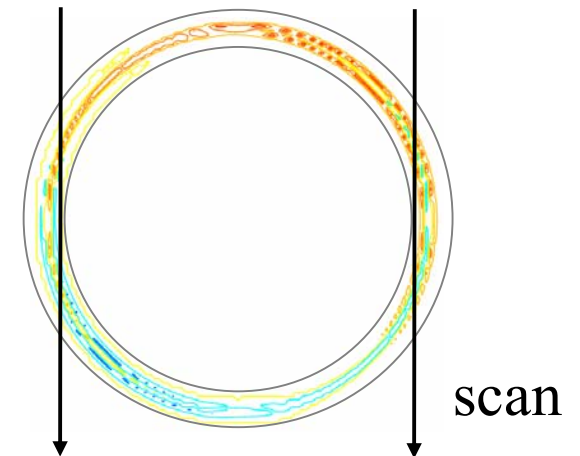


G. R. Tynan, et al., PPCF **38** (1996) 1301

2D structure!

To observe the poloidal structure, identification of measuring points on the **same magnetic surface** is necessary.

Alternative way: measurement of up-down asymmetry in various locations



The shock position differs in accordance with M_p , so controlling the flow velocity by electrode biasing will be illuminating.

Inversion of E_r , B_t and I_p

Model equation

$$\begin{aligned}
 & \underbrace{-\mu' r^2 \frac{B_0}{B_p} \frac{\partial^2}{\partial r^2} \{M_p [\exp(-\chi) - 1]\}}_{\text{blue}} + \underbrace{\frac{2}{3} D \exp(-\chi) \frac{\partial^2 \chi}{\partial \theta^2} + (1 - M_p^2) \frac{\partial \chi}{\partial \theta} + 2A' \frac{\partial \chi^2}{\partial \theta}}_{\text{pink}} \\
 & = \varepsilon \left(\underbrace{\left\{ D - \mu' \frac{B_0}{B_p} \left[2r^2 \frac{\partial^2 M_p}{\partial r^2} + 4r \frac{\partial}{\partial r} M_p - 2M_p \right] \right\}}_{\text{blue}} \cos \theta - 2M_p^2 \sin \theta \right)
 \end{aligned}$$

- : shear viscosity direction of the flux $\left(\begin{array}{l} \text{not change} \\ \text{change} \end{array} \right)$ by inversion of B_t or I_p
- : poloidal shock

L-mode – shear viscosity dominant, H-mode – shock dominant

In spontaneous H-mode

B_t and I_p are co-direction \rightarrow outward flux
 counter-direction \rightarrow inward flux

$$M_p: \times -1 \rightarrow \Phi_-(\theta) = \Phi_+(-\theta)$$

Basic Equations

Momentum conservation ion + electron

$$m_i n \frac{d}{dt} \vec{V}_i = \vec{J} \times \vec{B} - \vec{\nabla}(p_i + p_e) - \vec{\nabla} \cdot \vec{\pi}_i \quad (1)$$

\vec{J} : current p : pressure $\vec{\pi}$: viscosity n : density

$$\vec{V} = \vec{V}_{||} + \frac{\vec{E} \times \vec{B}}{B^2} = \begin{pmatrix} -\frac{I}{rRB^2} \frac{\partial \Phi}{\partial \theta} \\ \frac{KB_p}{n} \\ \frac{KB_\zeta}{n} - R \frac{\partial \Phi}{\partial \psi} \end{pmatrix} \leftarrow \text{radial flow}$$

$$K \equiv \frac{nV_p}{B_p}$$

$$I = R^2 \vec{B} \cdot \nabla \zeta \quad \Phi : \text{potential}$$

toroidal symmetry

Radial and poloidal components are coupled
with **radial flow** and **shear viscosity**

strong poloidal shock case Eq. (2) poloidal structure

Eq. (3) radial structure

$$\left(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i\right)_{\text{shear}} = -m_i n \mu \vec{B} \cdot \nabla_{\perp}^2 \vec{V} \quad \mu : \text{shear viscosity}$$

$$\left(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi}_i\right)_{\text{bulk}} = \frac{2}{3} \frac{B_p}{r} \frac{\partial}{\partial \theta} (p_{\parallel} - p_{\perp}) - (p_{\parallel} - p_{\perp}) \frac{B_p}{B} \frac{1}{r} \frac{\partial B}{\partial \theta}$$

Nonlinearity with the electric field of bulk viscosity

→ structural bifurcation

solitary structure

N. Kasuya, et al., Nucl. Fusion **43** (2003) 244

Transport

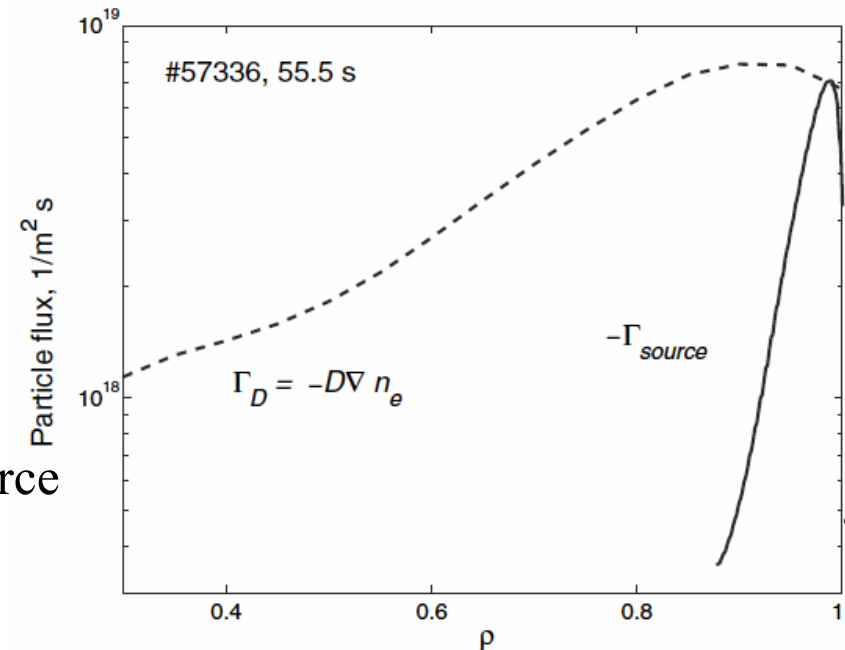
continuity equation

$$\frac{\partial n}{\partial t} = -\nabla (\underbrace{nV}_{\text{convective}} - \underbrace{D\nabla n}_{\text{diffusive}}) + S$$

n : density, V : flow velocity,
 D : diffusion coefficient, S : particle source

peaked profile ← inward pinch

$$V_r \sim 1 [\text{m/s}]$$



Radial profiles of particle source and diffusive particle flux in JET

H. Weisen, et al., PPCF **46** (2004) 751

Origin of inward pinch has not been clarified yet.

Ware pinch (toroidal electric field)

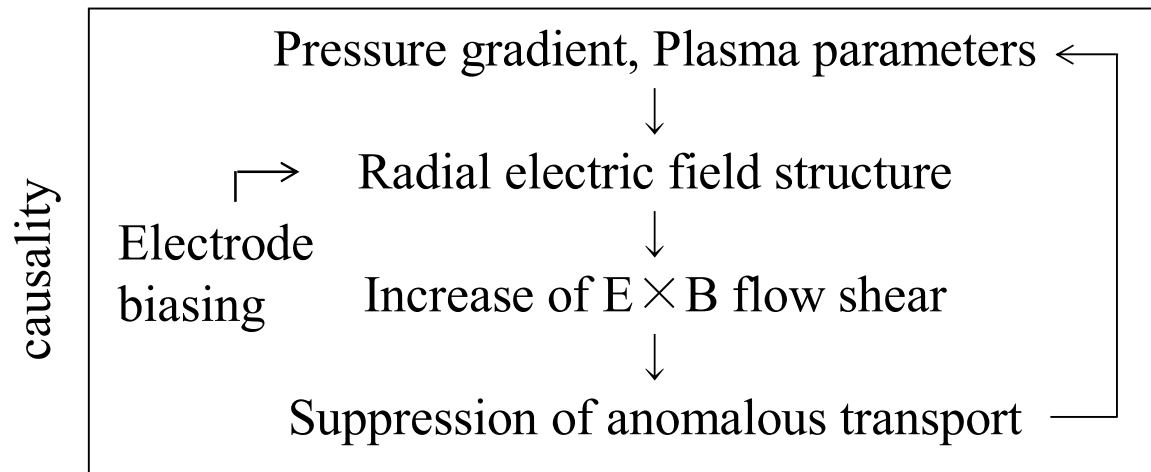
← inward pinch exists in helical systems U. Stroth, et al., PRL **82** (1999) 928

anomalous inward pinch (turbulence) X. Garbet, et al., PRL **91** (2003) 035001

H-mode

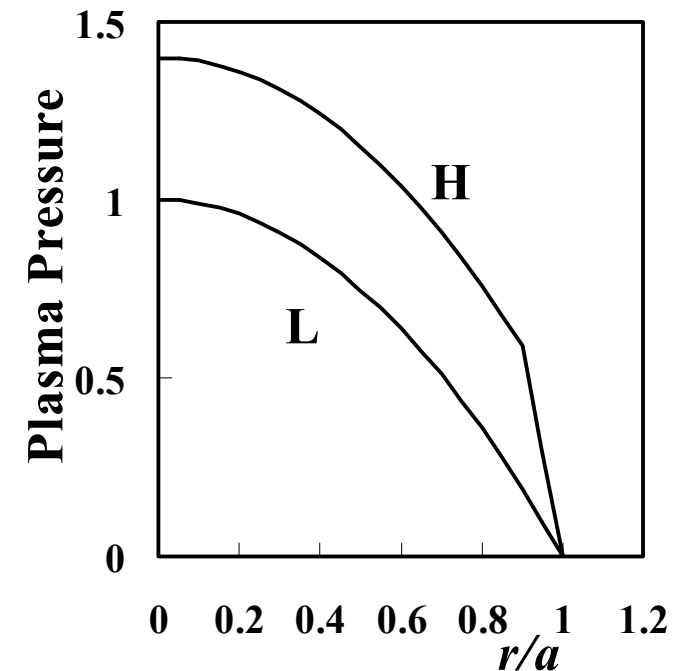
Formation of edge transport barrier (ETB)

Self-sustaining loop of plasma confinement



steep radial electric field structure

K. Ida, PPCF 40 (1998) 1429

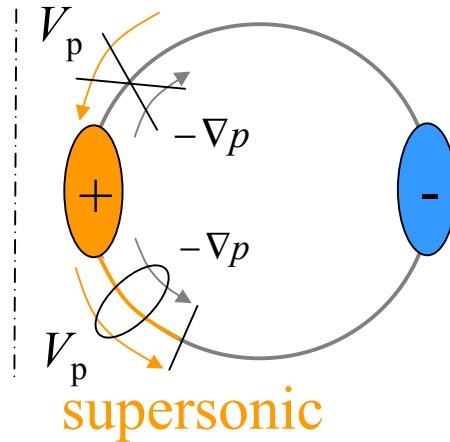


Understanding the structural formation mechanism is important.

Large $E \times B$ flow in the poloidal direction
poloidal Mach number $M_p \sim O(1)$

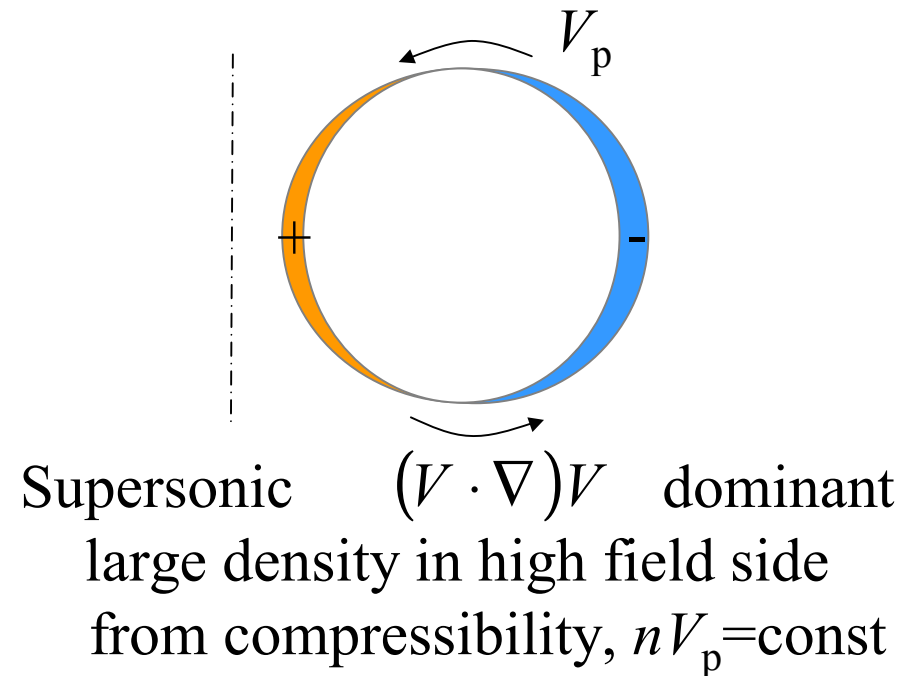
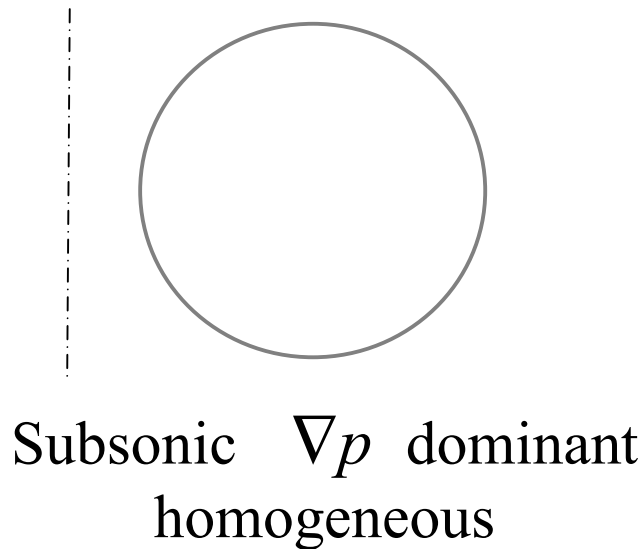
Shock Formation

when $M_p \sim 1$



A shock structure appears at the boundary between the supersonic and subsonic region

Effect of the higher order term appears, and the poloidal shock is formed.



H-mode Pedestal

Steep density profile is formed near the plasma edge just after L/H transition.

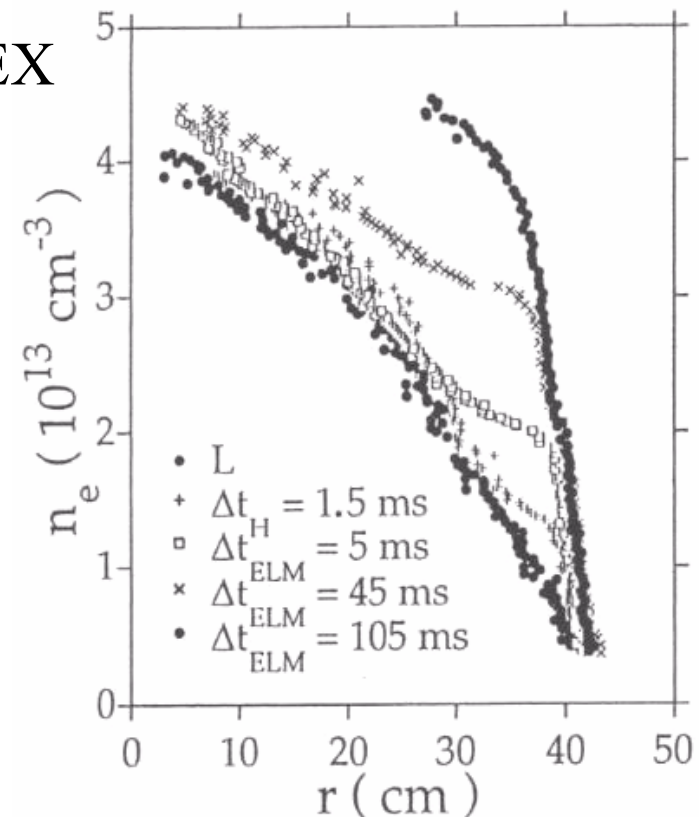
rapid formation

$$\Delta t \ll 10[\text{ms}]$$

Reduction of diffusive transport only cannot explain this short duration.

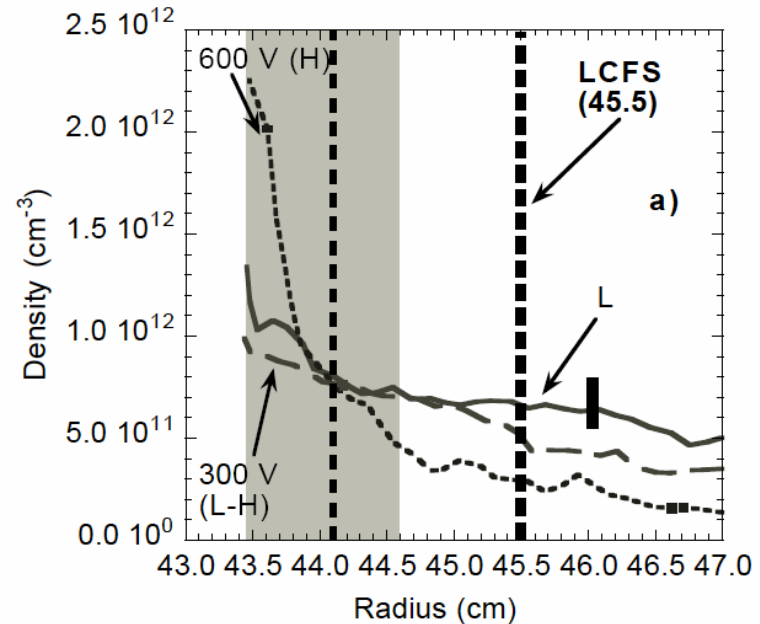
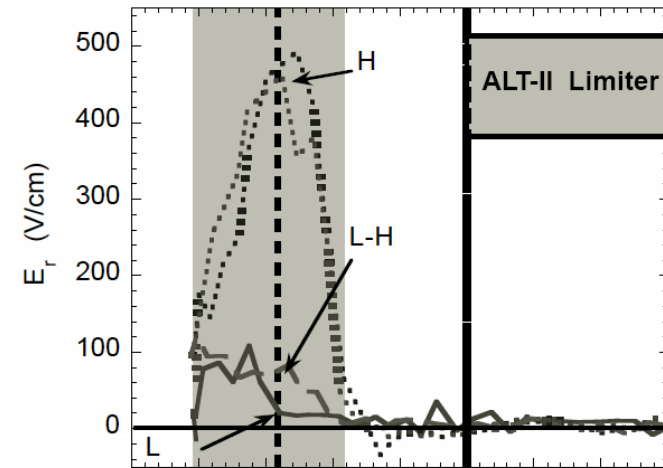
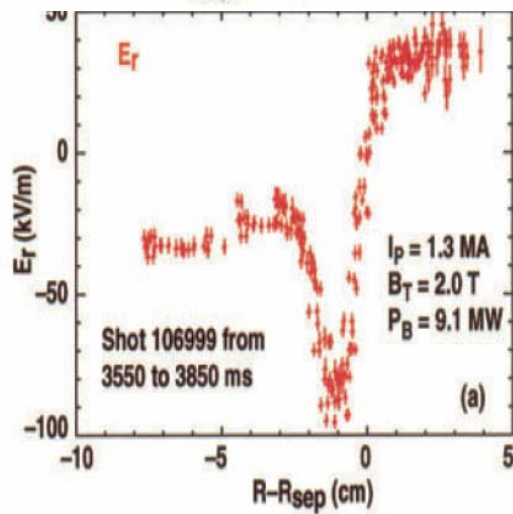
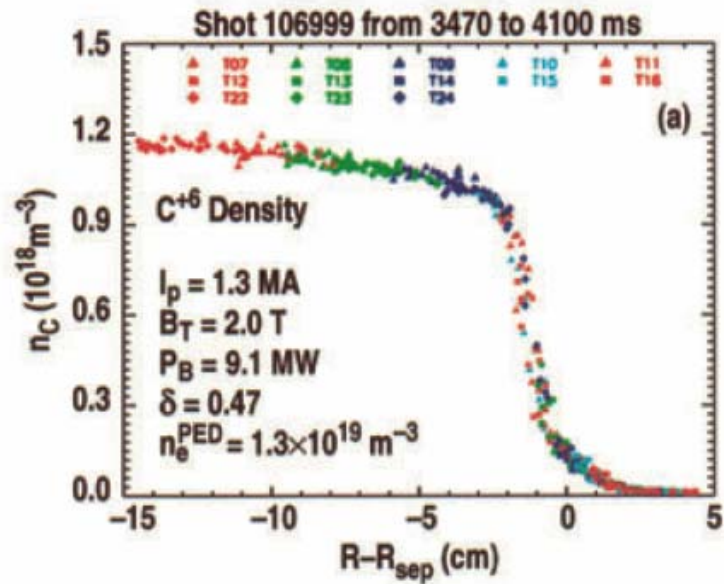
Pedestal formation in H-mode

ASDEX



F. Wagner, et al., Proc. 11th Int. Conf., Washington, 1990, IAEA 277

Profile



Poloidal Shock

$\mu = 0$ (no radial coupling, Shaing model) μ : shear viscosity

$$\frac{2}{3} D \frac{\partial \chi}{\partial \theta} + (1 - M_p^2) \chi + 2A'(\chi^2 - \langle \chi^2 \rangle) = \varepsilon [2M_p^2 \cos \theta + D \sin \theta]$$

LHS 1st term : viscosity (pressure anisotropy)

2nd term : difference between convective derivative $(V \cdot \nabla)V$
and pressure ∇p

3rd term : nonlinear term

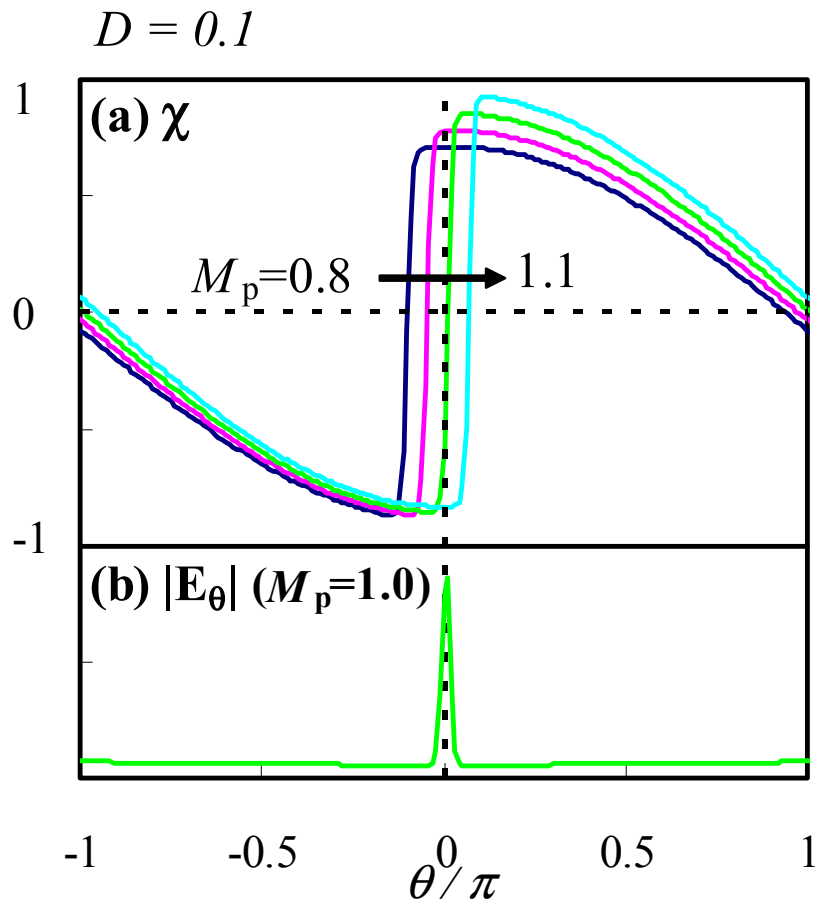
RHS $\chi = \frac{e \Delta \Phi}{T_e}$: toroidicity
potential perturbation (Boltzmann relation)

$M_p \ll 1$ ∇p dominant homogeneous structure

$M_p \gg 1$ $(V \cdot \nabla)V$ dominant larger density in the high field side

$M_p \sim 1$ competitive, shock formation affected by nonlinearity of
the higher order

Shock solutions



sharpness of shock

$$\left. \frac{\partial \chi'}{\partial \theta'} \right|_{\max} = \frac{3\varepsilon}{2D} \sqrt{(M_p^2 + 2C) + D^2} [\cos(\theta_{\text{shock}} + \theta_\alpha) + 1]$$

$$\propto \frac{1}{D} \quad (D \ll 1) \quad (4)$$

position of shock

$$\theta_{\text{shock}} = -\theta_\alpha + 2 \arcsin \frac{\pi |1 - M_p^2|}{8 \sqrt{A' \varepsilon \sqrt{(M_p^2 + 2C) + D^2}}} \quad (5)$$

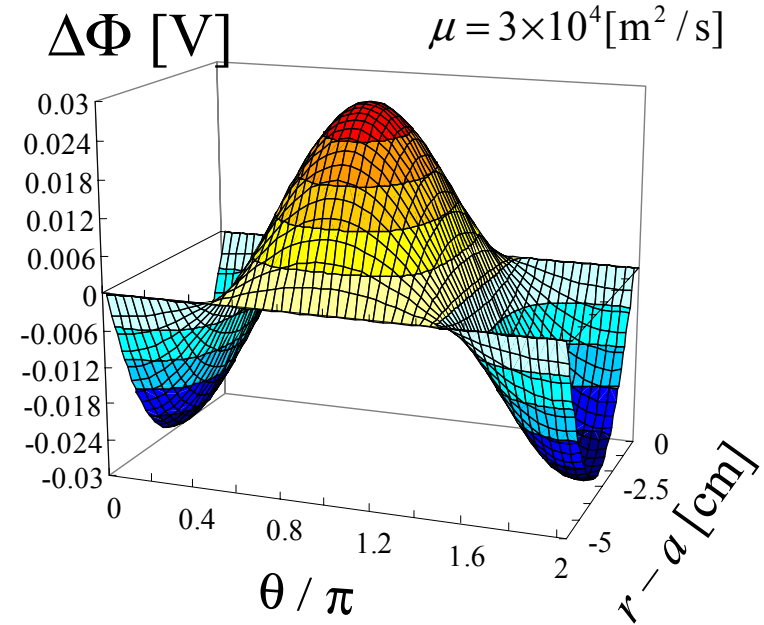
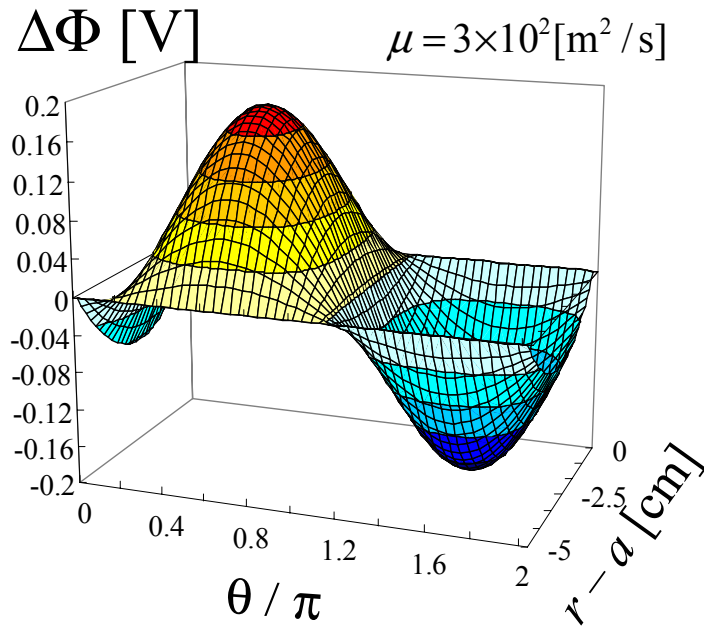
$$\tan \theta_\alpha = -\frac{D}{M_p^2 + 2C}$$

dependence on M_p

Potential Profile

$$1 \gg \mu' \frac{B_0}{B_p} \gg \frac{Dd^2}{12a^2 M_p}$$

$$\mu' \frac{B_0}{B_p} \gg 1$$



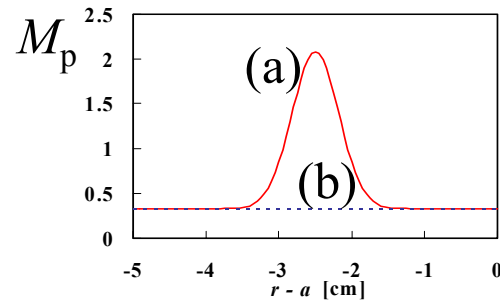
$$\chi(r, \theta) \sim \frac{\epsilon \sqrt{D^2 + 4M_p^4}}{2\mu'a^2 M_p} \frac{B_p}{B_0}$$

$$\chi(r, \theta) \sim \frac{\epsilon}{a^2} (r-a)(r-a+d) \cos \theta$$

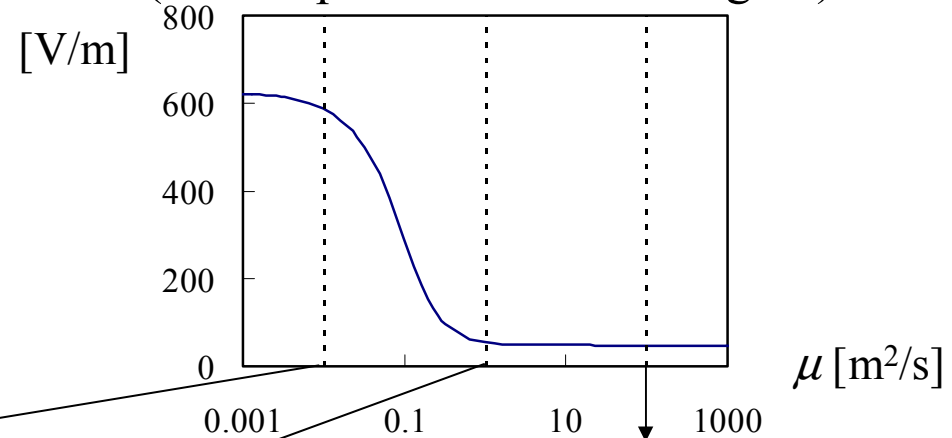
$$\times (r-a)(r-a+d) \sin(\theta + \theta_\alpha) \propto \frac{1}{\mu}$$

2-D Structure

Poloidal flow profile



Maximum of the poloidal electric field
(middle point of the shear region)



Shock region

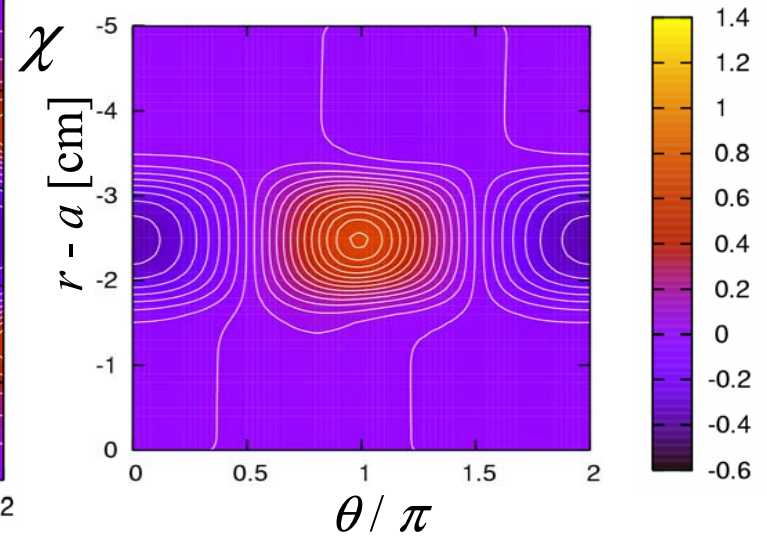
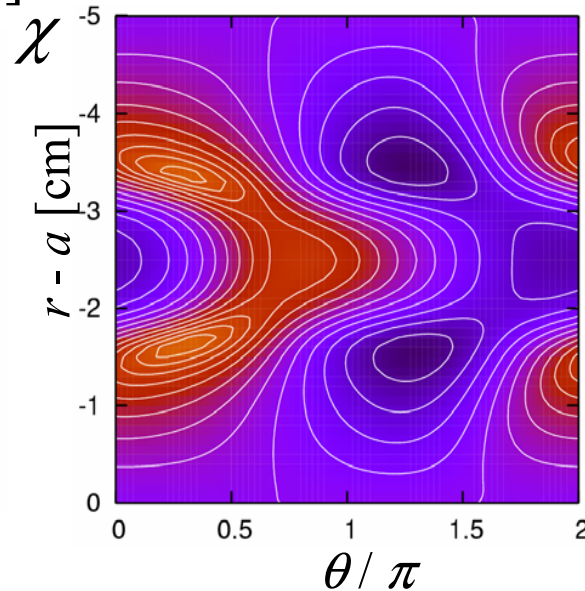
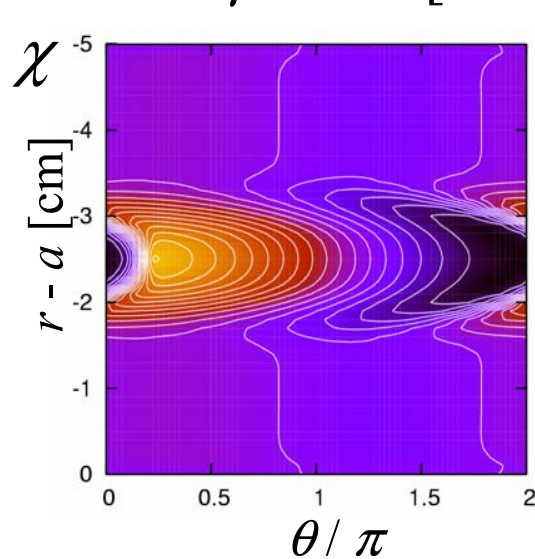
$\mu = 0.01 [\text{m}^2/\text{s}]$

Intermediate region

$\mu = 1 [\text{m}^2/\text{s}]$

Viscosity region

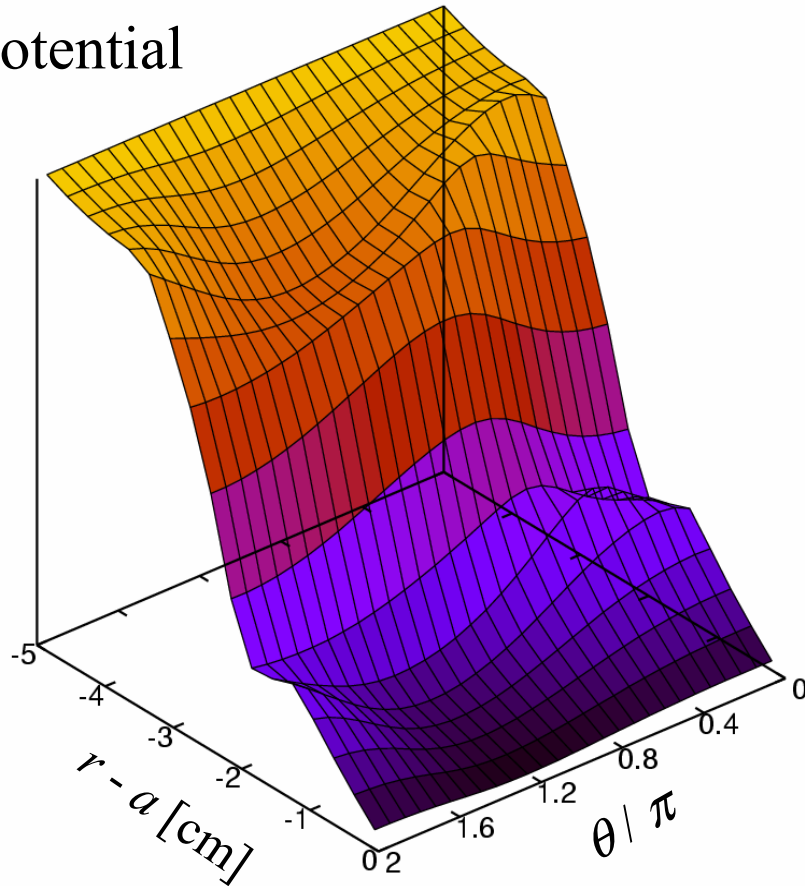
$\mu = 100 [\text{m}^2/\text{s}]$



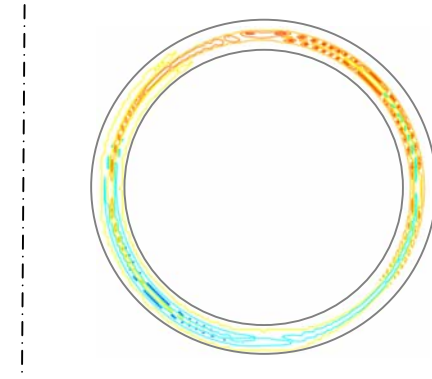
Intermediate case

$$\mu = 1 [\text{m}^2/\text{s}]$$

potential



χ profile (poloidal cross section)



poloidal electric field

