NEOCLASSICAL TOROIDAL ANGULAR MOMENTUM TRANSPORT IN A ROTATING IMPURE PLASMA

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Overview

- Motivation
- Current observations
- Current predictions
- Impure, rotating plasma
- Calculating the transport
- Results
- Summary

MOTIVATION

- Internal Transport Barriers believed to be caused by sheared electric field, $E_r' \neq 0$

- Toroidal velocity $v_\phi = \frac{E_r}{B_p} + \frac{p_i'}{neB_p} + \ldots$

  $\Rightarrow$ E_r (r) determined by angular momentum transport

- If turbulence is suppressed in an ITB

  $\Rightarrow$ **neoclassical angular momentum transport** should play key role in formation and sustainment of ITBs

CURRENT OBSERVATIONS

- Bulk ion **thermal diffusivity** observed at neoclassical level

  eg ITB core plasma

  Prediction assumes bulk ions low collisionality, banana regime

  - transport scales as $\nu_{ii} \dfrac{q^2 \rho^2}{\varepsilon^{3/2}}$

- Bulk ion **viscosity** order of magnitude higher than prediction

  Bulk ion viscosity determines angular momentum confinement

  $\Rightarrow$ angular momentum transport anomalous

CURRENT PREDICTIONS

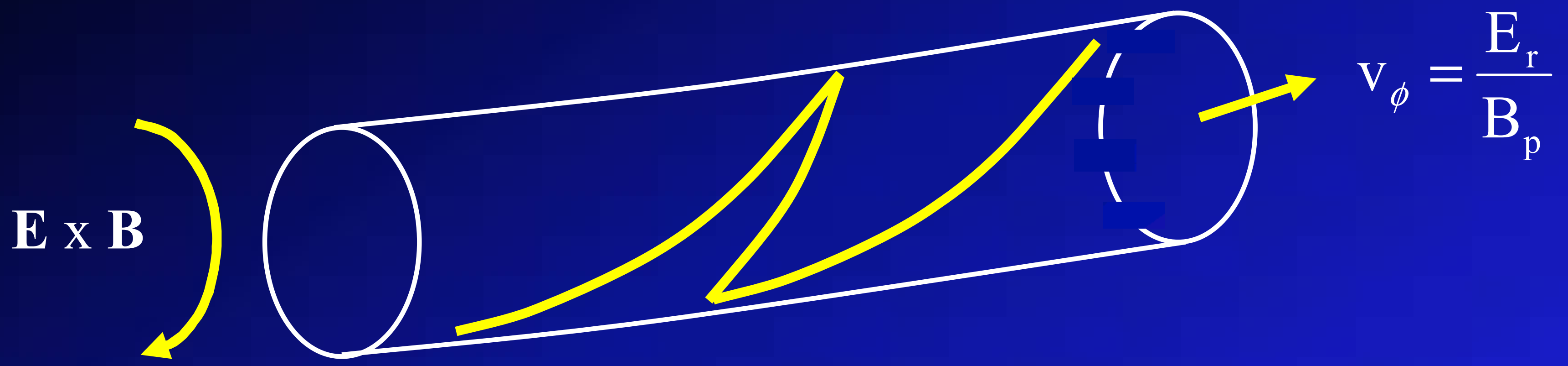
1971, Rosenbluth *et al* calculate viscosity in pure plasma:

- bulk ions in banana regime, slow rotation
- scales as $\nu_{ii} q^2 \rho^2$ - expected for Pfirsch-Schlüter regime

1985, Hinton and Wong extended to sonic plasma rotation:

- still no enhancement characteristic of banana regime
- transport is diffusive, driven by gradient of toroidal velocity
- plasma on a flux surface rotates as a rigid body
- angular velocity determined by local radial electric field

INTERPRETATION



- Trapped particles collide:
  - change position, toroidal velocity determined by **local field**
  - no net transfer of angular momentum

- Angular momentum transported by passing particles:
  - same toroidal velocity as trapped particles due to **friction**
  - typical excursion from flux surface ~ $q\,\rho$

  $\Rightarrow$ momentum diffusivity ~ $\nu_{ii}\, q^2 \rho^2$

IMPURE ROTATING PLASMA

Typically $Z_{eff} > 1$

- plasma contains heavy, highly charged impurity species
- **mixed collisionality plasma**

1976, Hirshman: particle flux ~ Pfirsch-Schlüter regime
heat flux ~ banana regime

Rotating Plasma

- centrifugal force pushes particles to outboard side of flux surface
- **impurity ions undergo significant poloidal redistribution**
- **variation in collision frequency** around flux surface

1999, Fülöp & Helander: particle flux typical of banana regime

Calculating The Transport

Hinton & Wong: expansion of ion kinetic equation in $\delta = \rho_i / L_r$

$$f = f_0 + f_1 + f_2 + \ldots \qquad f_1 \sim \delta f_0$$

- Cross-field transport second order in δ - use flux-friction relations:

Angular momentum flux: $$\Pi = -\frac{m_i}{2e}\left\langle \int d^3v\, m_i R^2 v_\phi^2 C_i(f_1) \right\rangle$$

- Separate classical and neoclassical contributions: $f_1 = \tilde{f}_1 + \bar{f}_1$

$\tilde{f}_1$ determined by Hinton & Wong: - valid for any species

- independent of form of C_i

$\bar{f}_1$ obtained from the drift kinetic equation

- subsidary expansion in ratio of ion collision to bounce frequency

COLLISION OPERATOR

- Impurity concentration typically $\Rightarrow \nu_{ii} \sim \nu_{iz}$
- Explicit form of collision operator: $C_i = C_{ii} + C_{iz}$

  C_{ii} : Kovrizhnykh model operator for self collisions

  C_{iz} : disparate masses $\Rightarrow$ analogous to electron - ion collisions

$$C_{iz} = \nu_{iz}(\Psi,\theta)\left(L(f_1) + \frac{m_i \mathrm{v}_{\|}}{T_i}\mathrm{V}_{\mathrm{z}\|}(\Psi,\theta) f_0\right)$$

- Parallel impurity momentum equation used to determine $\mathrm{V}_{\mathrm{z}\|}$

$$m_z n_z (\mathbf{V}_\mathrm{z} \cdot \nabla)\mathbf{V}_\mathrm{z} = -n_z z e \nabla\Phi - \nabla p + \mathbf{R} + n_z z e \mathbf{V}_\mathrm{z} \times \mathbf{B}$$

  - cross product with **B** gives $\mathrm{V}_{\mathrm{z}\perp}$
  - flow divergence free to first order $\Rightarrow \mathrm{V}_{\mathrm{z}\|}$

TRANSPORT MATRIX

Represent the fluxes in matrix form

$$\begin{pmatrix} \Gamma \\ q \\ \Pi \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} d(\ln p_i)/dr \\ d(\ln T_i)/dr \\ d(\ln \omega)/dr \end{pmatrix}$$

- L_{33} usual measure of viscosity
- Each L is sum of classical, $\tilde{L}$, and neoclassical, $\bar{L}$, contribution

- Restricted to subsonic rotation to calculate neoclassical terms
- $Z_{eff} = 1$ - recover Braginskii, Hinton & Wong results

- Classical contribution $\sim \left\langle n_z \dfrac{R^2}{B^2} \right\rangle$

Enhanced transport
- larger outboard step size $\rho_i \sim 1/B$
- larger angular momentum $m_i \omega R^2$

NEOCLASSICAL COEFFICIENTS

Most experimentally relevant limit:

- conventional aspect ratio, $\varepsilon(\theta) << 1$
- strong impurity redistribution, $\langle n \cos\theta \rangle, \sim 1 \qquad n = n_z / \langle n_z \rangle$

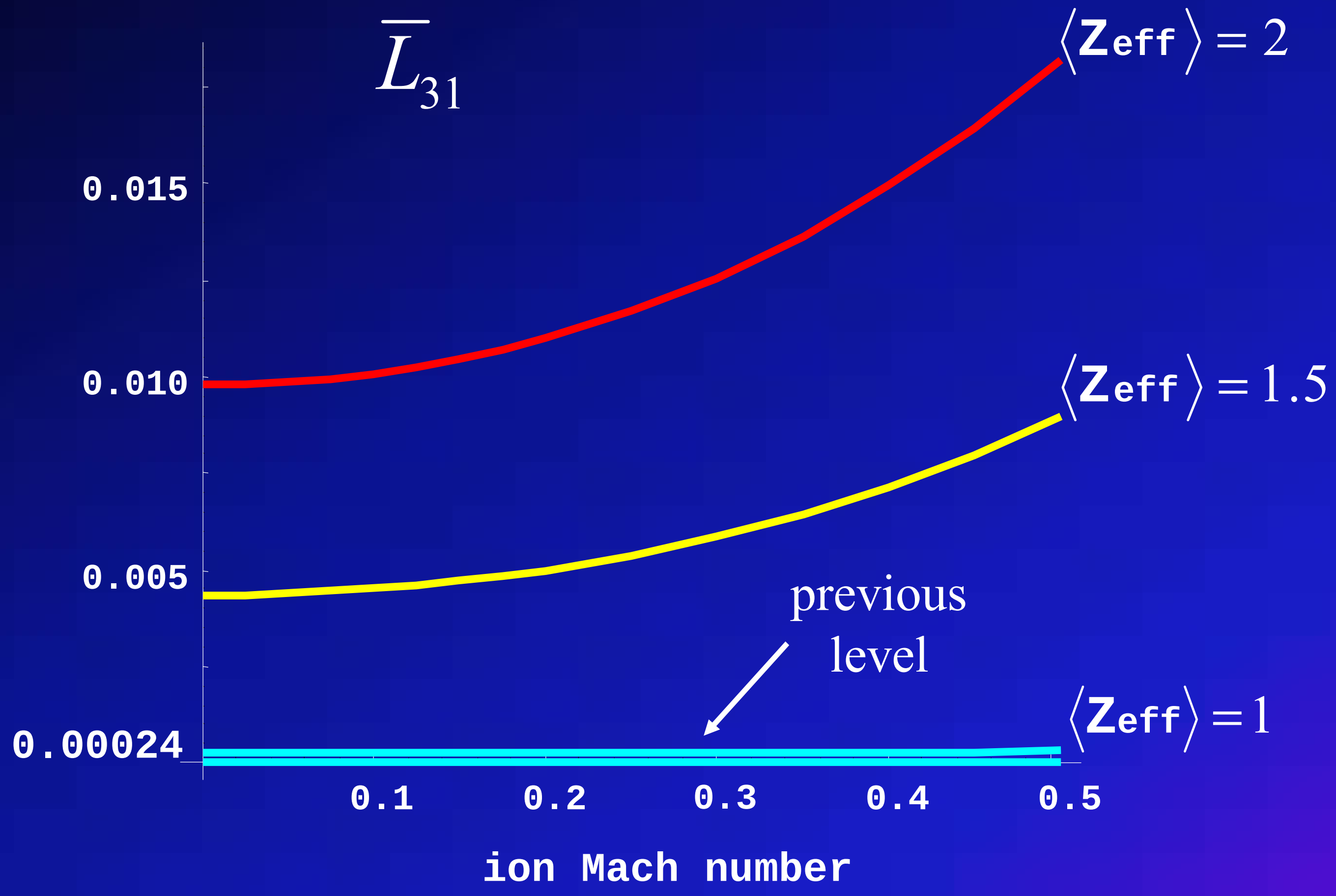
$$\overline{L}_{31} \sim \overline{L}_{32} \sim \nu_{iz} \frac{q^2 \rho^2}{\varepsilon^{3/2}} \qquad \overline{L}_{33} \sim M_i^2 \nu_{iz} \frac{q^2 \rho^2}{\varepsilon^{3/2}}$$

- **Enhancement of $\varepsilon^{-3/2}$ over previous predictions**
  - effectiveness of rotation shear as a drive increased by small factor
  - **radial pressure and temperature gradients dominate**

  $\Rightarrow$ strong density and temperature gradients sustain strong E_r shear

NEOCLASSICAL COEFFICIENTS

Numerical evaluation using magnetic surfaces of MAST - $\varepsilon = 0.14$



- increase with impurity content
- increase with Mach number as impurity redistribution increases

- **Transport ~ 10 times previous predictions**

NEOCLASSICAL COEFFICIENTS

- Large $\bar{L}_{31}$, $\bar{L}_{32}$ $\Rightarrow$ **spontaneous toroidal rotation** may arise:

$$\frac{\omega'}{\omega} = -\frac{1}{\bar{L}_{33}}\left(\bar{L}_{31}\frac{p'}{p} + \bar{L}_{32}\frac{T'}{T}\right)$$

- Rotation direction depends on edge boundary condition

- $\bar{L}_{23}$ relates heat flux to toroidal rotation shear:

$$\begin{pmatrix} \Gamma \\ q \\ \Pi \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} d\left(\ln p_i\right)/dr \\ d\left(\ln T_i\right)/dr \\ d\left(\ln \omega\right)/dr \end{pmatrix}$$

Co-NBI $\Rightarrow$ shear

$\Rightarrow$ **heat pinch**

Sub-neoclassical heat transport…

SUMMARY

- Experimentally, angular momentum transport in regions of neoclassical ion thermal transport has remained anomalous
- In a rotating plasma impurities will undergo poloidal redistribution
- Including this effect a general form for the flux has been derived for mixed collisionality plasma
- At conventional aspect ratio, with impurities pushed towards outboard side, angular momentum flux seen to increase by a factor of $\varepsilon^{-3/2}$ $\Rightarrow$ now typical of banana regime
- Radial bulk ion pressure and temperature gradients are the primary driving forces, not rotation shear $\Rightarrow$ strong density and temperature gradients sustain strongly sheared E_r
- Spontaneous toroidal rotation may arise in plasmas with no external angular momentum source

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