Density Dependent Hartree-Fock (DDHF)

The 3-body zero-range Density Dependent Skyrme Int.

$$
\begin{aligned}
V_{Sk} &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}_i-\mathbf{r}_j) \\
&+ \tfrac{1}{2}t_1(1+x_1P_\sigma)\{\mathbf{p}_{12}^2\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{12}^2\} \\
&+ t_2(1+x_2P_\sigma)\mathbf{p}_{12}\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{p}_{12} \\
&+ \tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\bar{\mathbf{r}})\delta(\mathbf{r}_i-\mathbf{r}_j) \\
&+ iW_0(\sigma_1+\sigma_j)\cdot\mathbf{p}_{12}\times\delta(\mathbf{r}_1-\mathbf{r}_j)\mathbf{p}_{12}
\end{aligned}
$$

$p_{12} = p_i - p_j$ is the relative momentum

P_σ → Spin Exchange Operator $\sigma_i \leftrightarrow \sigma_j$

σ → Spin Pauli Matrices

$\bar{r} = \frac{1}{2}(r_i + r_j)$

t_3 → 3 – Body Contact Interaction Term

W_o → Spin – Orbit Term

Advantage: Expectation value of E wrt Slater Determinant (Mean Field) WF
→ In Analytical Form

The Total Energy Functional E → Skyrme, Coulomb, Pairing Parts + Spurious CM Motion

$$E = E_{Sk} + E_{Coul} + E_{Pair} - E_{c.m.}$$

Spherical Nuclei → s.p. WF

$$\varphi_\beta(\mathbf{r}) = \frac{R_\beta(r)}{r} \mathcal{Y}_{j_\beta l_\beta m_\beta}(\theta, \phi)$$

and

$$\mathcal{Y}_{j_\beta l_\beta m_\beta} (= [Y_{l_\beta} \times \chi_{1/2}]_{j_\beta m_\beta})$$

Are Spinor Spherical Harmonics

$$
\begin{aligned}
E_{Sk} \quad = \\
& 4\pi \int drr^2 \{ \frac{\hbar^2}{2m}\tau + \tfrac{1}{2}t_0(1+\tfrac{1}{2}x_0)\rho^2 - \tfrac{1}{2}t_0(\tfrac{1}{2}+x_0)\sum_q \rho_q^2 \\
& + \tfrac{1}{4}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\rho\tau \\
& - \tfrac{1}{4}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\sum_q \rho_q\tau_q \\
& + \tfrac{1}{16}[3t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\sum_q \rho_q\nabla^2\rho_q \\
& - \tfrac{1}{16}[3t_1(1+\tfrac{1}{2}x_1) - t_2(1+\tfrac{1}{2}x_2)]\rho\nabla^2\rho \\
& - \tfrac{1}{2}W_0[\rho\nabla\cdot\mathbf{J} + \sum_q \rho_q\nabla\cdot\mathbf{J}_q]\}
\end{aligned}
$$

$$\nabla^2 = \partial_r^2 + \frac{2}{r}\partial_r \quad , \qquad \partial_r \to \frac{\partial}{\partial r}$$

The Spherical Densities and Currents are

$$\rho_q(r) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} \left(\frac{R_\beta}{r}\right)^2$$

$$\tau_q(r) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} [\left(\partial_r \frac{R_\beta}{r}\right)^2 + \frac{l(l+1)}{r^2}\left(\frac{R_\beta}{r}\right)^2]$$

$$\nabla \mathbf{J}_q(r) = (\partial_r + \frac{2}{r}) J_q(r)$$

$$J_q(\mathbf{r}) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} (j_\beta(j_\beta+1) - l_\beta(l_\beta+1) - \tfrac{3}{4}) \frac{2}{r} \left(\frac{R_\beta}{r}\right)^2$$

The Occupation Probabilities n_β^q → **Independent of** m_β **.** q **Runs over n and p**

$$\rho = \rho_p + \rho_n \qquad \tau = \tau_p + \tau_n$$

$$\nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J}_p + \nabla \cdot \mathbf{J}_n$$

The Variation of E wrt R_β → **HF Equations**

$$h_q R_\beta = \epsilon_\beta R_\beta$$

The Mean Field Hamiltonian

$$h_q = \partial_r \mathcal{B}_q \partial_r + U_q + U_{ls,q} \mathbf{l}\sigma$$

$$\mathcal{B}_q = \frac{\hbar^2}{2m_q} + \frac{1}{8}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\rho - \frac{1}{8}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\rho_q$$

$$U_{ls,q} = \frac{1}{4}W_o(\rho+\rho_q) + \frac{1}{8}(t_1 - t_2)J_q - \frac{1}{8}(x_1 t_1 + x_2 t_2)J$$

$$
\begin{aligned}
U_q = \; & t_0(1+\tfrac{1}{2}x_0)\rho - t_0(\tfrac{1}{2}+x_0)\rho_q \\
& + \tfrac{1}{12}t_3\rho^\alpha[(2+\alpha)(1+\tfrac{1}{2}x_3)\rho \\
& - 2(\tfrac{1}{2}+x_3)\rho_q - \alpha(\tfrac{1}{2}+x_3)\frac{\rho_p^2+\rho_n^2}{\rho}] \\
& + \tfrac{1}{4}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\tau \\
& - \tfrac{1}{4}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\tau_q \\
& - \tfrac{1}{8}[3t_1(1+\tfrac{1}{2}x_1) - t_2(1+\tfrac{1}{2}x_2)]\nabla^2\rho \\
& + \tfrac{1}{8}[3t_1(\tfrac{1}{2}+x_1) + t_2(\tfrac{1}{2}+x_2)]\nabla^2\rho_q \\
& - \tfrac{1}{2}W_o(\nabla\cdot\mathbf{J} + \nabla\cdot\mathbf{J}_q) + \delta_{q1/2}\, U_{Coul}
\end{aligned}
$$

U_{Coul} → Direct + Exchange Terms.

Direct Term is Trivial, Exchange Term is Taken As

$$U_{coul}(exchange) = \left(-\frac{3}{\pi}\right)^{1/3} 4\pi \int drr^2 \rho_p^{4/3}$$

$$E_{c.m.} = \frac{\langle P_{c.m.}^2 \rangle}{2AM}$$

For H.O. s.p. Basis $E_{c.m} = \frac{3}{4}\hbar\omega$

BCS Type Occupation Probabilities are obtained Through Gap and Number Equations. Then

$$E_{Pair} = -\sum_q \mathcal{G}_q \left[\sum_{\beta\in q} \sqrt{n_\beta^q(1-n_\beta^q)}\right]^2$$

Illustration:

Skyrme Int. Parameters

t_o = -1057, t_1 = 235.9, t_2 = -100, t_3 = 14463.5

W_o = 120, x_o = 0.56 , $x_1 = x_2$ = 0.0, x_3 = 1.0

$\alpha = 1$

Gap Parameter:

$$\Delta_q = 11.2 MeV/\sqrt{A} \qquad A = A_p + A_n$$

$$r_c = \sqrt{r_p^2 + 0.64} \quad , \quad r_m = \left[\left(Z r_p^2 + N r_n^2\right) / \left(Z + N\right)\right]^{1/2}$$

	$^{16}_{8}O$	$^{40}_{20}Ca$	$^{48}_{20}Ca$	$^{90}_{40}Zr$	$^{208}_{82}Pb$
BE/A	-8.22 (-7.98)	-8.64 (-8.55)	-8.93 (-8.67)	-8.81 (-8.71)	-7.89 (-7.87)
r_c	2.68 (2.73)	3.41 (3.49)	3.46 (3.48)	4.22 (4.27)	5.44 (5.50)
r_m	2.55	3.29	3.43	4.17	5.45

Calculations Reproduce Expt. Well.

Relativistic Mean Field (RMF) Approach

Non-Relativistic Analysis Indicates That

$$U \sim 50 \text{ MeV} \ll mc^2 \ (\sim 1000 \text{ MeV}).$$

Question: Why Relativistic Formulation?

Reasons::
**** Nuclear l.s splitting is 30 Times Larger and is of Opposite Sign as That of Atomic Case|**

**** Convential Optical Model (OPM) Fails to Describe Spin Observables in the Intermediate Energy Polarized Proton – Nucleus (p-A) Scattering.**

Dirac Phenomenology: Use of Dirac Eq. With Lorentz Scalar and Vector Potentials in Place of Schrodinger Eq. Remarkably Successful. Scalar Pot. U and Vector Pot. V are of the Order of -400 and 350 MeV – Their Diff. Yields Required 50 MeV. This Success Triggered the Application of RMF to Nuclear Structure.

RMF-Formulation:

Nucleon Interacts With the Meson (σ,ω and ρ) and e.m. (γ) Fields. The Lagrangian : Free Baryon (L_B) , Mesons (L_M) and the INT. (L_{BM}) Terms.

$$L_B = \bar{\psi}_i \left(\iota \gamma^{\mu} \partial_{\mu} - M \right) \psi_i$$

$$L_M = \frac{1}{2} \partial^{\mu} \sigma \, \partial_{\mu} \sigma - U(\sigma)$$
$$- \frac{1}{4} \Omega^{\mu\nu} \, \Omega_{\mu\nu} + \frac{1}{2} m_{\omega}^2 \, \omega^{\mu} \omega_{\mu}$$
$$- \frac{1}{4} \vec{\mathrm{R}}^{\mu\nu} \, \vec{\mathrm{R}}_{\mu\nu} + \frac{1}{2} m_{\rho}^2 \, \vec{\rho}^{\mu} \, \vec{\rho}_{\mu} \quad - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathrm{L_{MB}} = \begin{aligned} & -g_\sigma \; \bar{\psi}_i \psi_i \; \sigma \\ & -g_\omega \; \bar{\psi}_i \gamma^\mu \psi_i \; \omega_\mu \\ & -g_\rho \; \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i \; \vec{\rho}_\mu \\ & -e \; \bar{\psi}_i \gamma^\mu \frac{(1+\tau_3)}{2} \psi_i \; A_\mu \end{aligned}$$

With

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

The Field Tensors

$$\begin{aligned} \Omega^{\mu\nu} &= \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \\ \vec{\mathrm{R}}^{\mu\nu} &= \partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu - g_\rho (\vec{\rho}^\mu \times \vec{\rho}^\nu) \\ \mathrm{F}^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu \end{aligned}$$

The Classical Variation Principle Gives the EQS. Of Motion. Replacing the Fields By Their Expectation Values →
Dirac EQ. With Pot. Terms for Nucleons and KG Type EQS. With Sources For Meson and the Photon

Non Linear Walecka Model

Relativistic Mean Field: RMF

Ann. Phys. 198 (1990) 132
Gambhir, Ring and Thimet

Nucleons Interact With Meson Fields

σ, ω, ρ and γ

Lagrangian
- Masses: $M, m_\sigma, m_\omega, m_\rho$
- Couplings: $g_\sigma, g_\omega, g_\rho$, g_2, g_3

↓ Classical Variation Principle

Equations of Motion

Approx: Field Operators ⇒ C – Numbers

↓

Dirac Equation: Nucleons
K. G. Equation: Mesons & Photon

For: Static Case + Charge Conserv + Time Reversal

Dirac Eqn: ⟹ Scalar + Time like vector

$M^* = M + \sigma g_\sigma \; ; \omega^o, \rho^{oo}, A^o$

K. G. Eqn: ⟹ Sources: Baryon Densities and Currents

Closed Set of Equations (RMF Equations)

⟹ **To be solved Self – Consistently**

The Dirac Eq.:

$$\left(-\iota\alpha\cdot\nabla+\beta(M+g_\sigma\sigma)+g_\omega\omega^o\cdot\right.$$
$$\left.+g_\rho\tau_3\rho_3^o+e\frac{1+\tau_3}{2}A^o\right)\psi_i=\epsilon_i\,\psi_i.$$

The KG Eqs.:

$$\left\{-\nabla^2+m_\sigma^2\right\}\sigma=-g_\sigma\rho_s-g_2\sigma^2-g_3\sigma^3$$

$$\left\{-\nabla^2+m_\omega^2\right\}\omega^o=g_\omega\rho_v$$
$$\left\{-\nabla^2+m_\rho^2\right\}\rho_3^o=g_\rho\rho_3$$
$$-\nabla^2A^o=e\rho_c$$

$m_\sigma\ (g_\sigma),\ m_\omega\ (g_\omega),\ m_\rho\ (g_\rho)$ **are Meson Masses (Coupling Constants)**

g2 (g3) : Coupling Constants of Cubic (Quartic) Non-Linear Terms

Currents and Densities are:

$$\begin{aligned}
\rho_s &= \sum_i n_i \bar{\psi}_i \psi_i \\
\rho_v &= \sum_i n_i \psi_i^\dagger \psi_i \\
\rho_3 &= \sum_i n_i \psi_i^\dagger \tau_3 \psi_i \\
\rho_c &= \sum_i n_i \psi_i^\dagger \left(\frac{1+\tau_3}{2}\right) \psi_i
\end{aligned}$$

Pairing:

Important

- **Constant Gap Approx.**

Δ_n (Δ_p) **Fixed** $\Rightarrow$

Simple BCS Type

- **Self Consistent:**

Bogoliubov Transform $\Rightarrow$

Relativistic Hartree

Bogoliubov (RHB) Equations

RHB Equations:

$$\begin{pmatrix} h_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -h_D^* + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k$$

h_D: **Dirac Hamiltonian**

$\hat{\Delta}$: **Pairing Field**

U_k, V_k : **Dirac Super-Spinors**

Dirac Hamiltonian:

$$h_D = -\iota\alpha\cdot\nabla + \beta(M + g_\sigma\sigma) + g_\omega\omega^o + g_\rho\tau_3\rho_3^o + e\frac{1-\tau_3}{2}A^o$$

$$\int\left(U_k^\dagger U_{k'} + V_k^\dagger V_{k'}\right) = \delta_{kk'}$$

Kernel of Pairing Field $\hat{\Delta}$ is:

$$\Delta_{ab}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\sum_{c,d} V^{pp}_{abcd}(\mathbf{r},\mathbf{r}')\,\kappa_{cd}(\mathbf{r},\mathbf{r}')$$

κ : **Pairing Tensor** $\rightarrow$

$$\kappa_{cd}(\mathbf{r},\mathbf{r}') = \sum_{E_k>0} U^*_{ck}(\mathbf{r})\,V_{dk}(\mathbf{r}')$$

V^{pp}: **Interaction in** pp **channel**

Constant Gap Approximation:

$\hat{\Delta}$ **Diagonal**

RHB $\rightarrow$ **RMF with FROZEN GAP:**

Occupancies: given by BCS equation:

$$n_k = v_k^2 = \frac{1}{2}\left[1 - \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right]$$

λ $\rightarrow$ **Lagrange Multiplier**

Δ $\rightarrow$ **Pairing Gap**

ϵ_k $\rightarrow$ **Single Particle Energies**

RMF / RHB Calculations

Input:

Lagrangian Parameter Set (NL3)

Δ / Pairing Interaction

Δ: Odd – Even Mass Difference

OR

Determine so as to Reproduce

RHB Proton / Neutron Pairing Energies
With Gogny D1S Interaction

RMF / RHB Calculations

Output:

- Dirac Spinors / Mesonic Fields
- Single Particle Energies
- Binding Energies
- Densities
- Radii

NL3 Parameter Set

Masses (MeV)	m	939	m_σ	508.194
	m_ω	782.501	m_ρ	763.0
Coupling Constants	g_σ	10.217	g_ω	12.868
	g_ρ	4.474	g_2	-10.431
	g_3	-28.885		(fm^{-1})

•Zero Range Density Dependent:

$$V(\mathbf{r_1}, \mathbf{r_2}) = \frac{1}{4} V_o \delta(\mathbf{r_1} - \mathbf{r_2}) (1 - \sigma_1 \sigma_2) \left(1 - \frac{\rho(r)}{\rho_o} \right)$$

ρ_o **=0.152** fm^{-3}

V_o **≈-700 MeV-**fm^{-3}

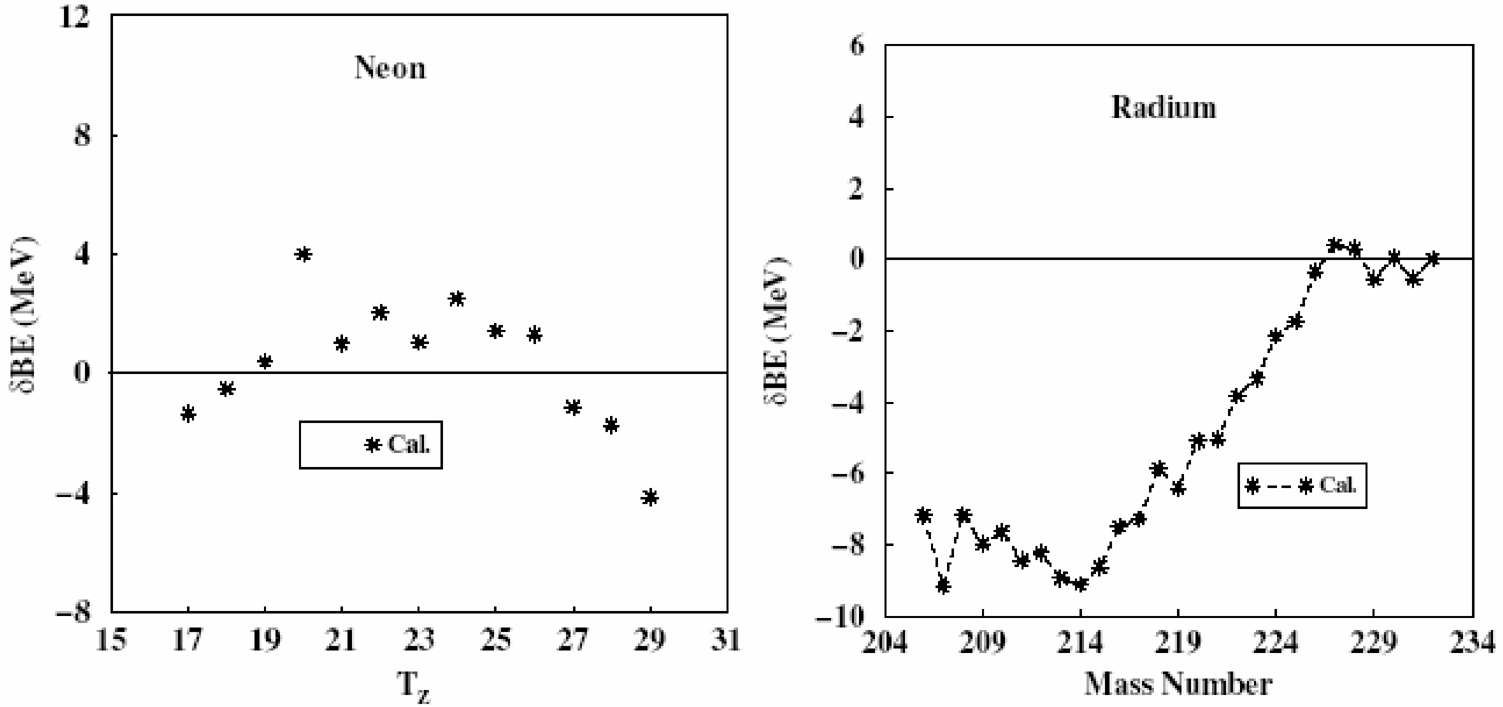
$\mathbf{V^{pp}(r,r')}$**:Non Relativistic:**

- **Gogny D1S:**

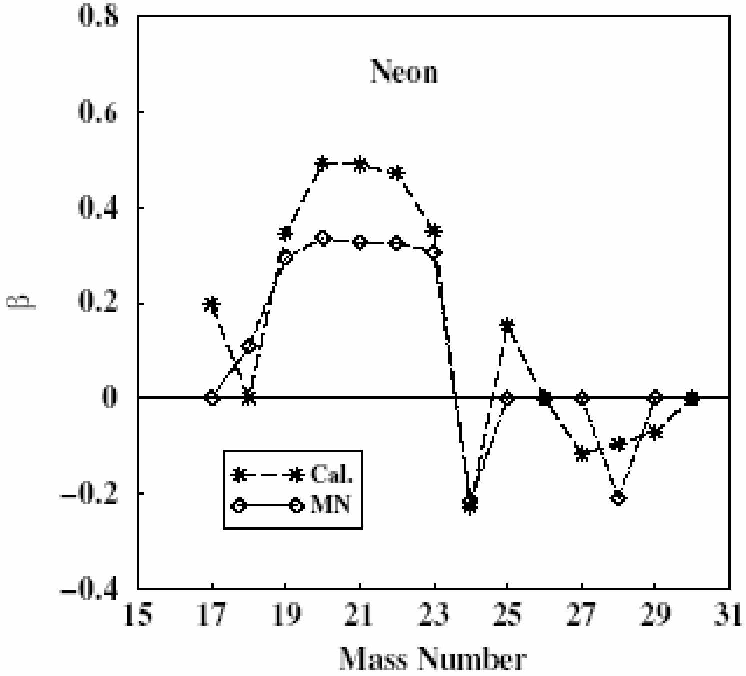
$$V(\mathbf{r_1},\mathbf{r_2}) = \sum_{i=1,2} e^{-\{(\mathbf{r_1}-\mathbf{r_2})/\mu_i\}^2}(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$

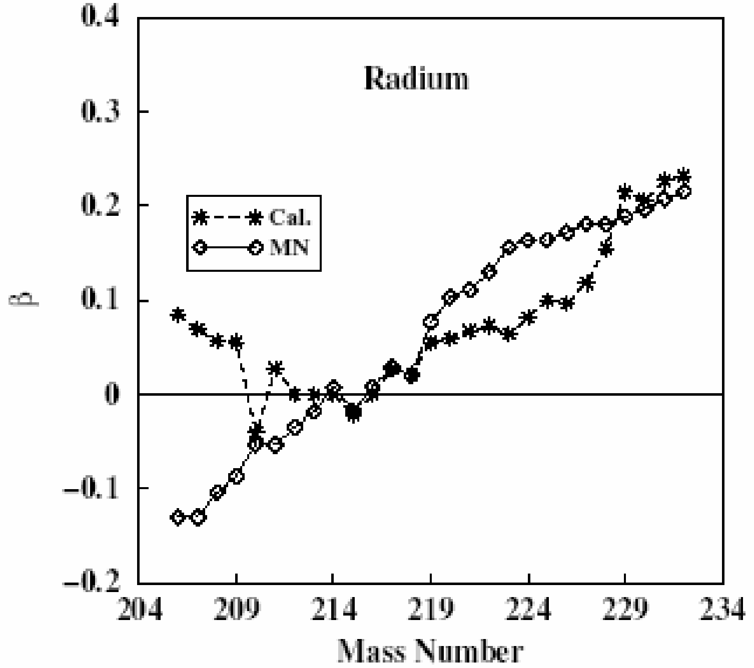
D1S Parameters:

Parameter	$i = 1$	$i = 2$
μ_i	0.7	1.2
W_i	-1720.30	103.64
B_i	1300.00	-163.48
H_i	-1813.53	162.81
M_i	1397.60	-223.93

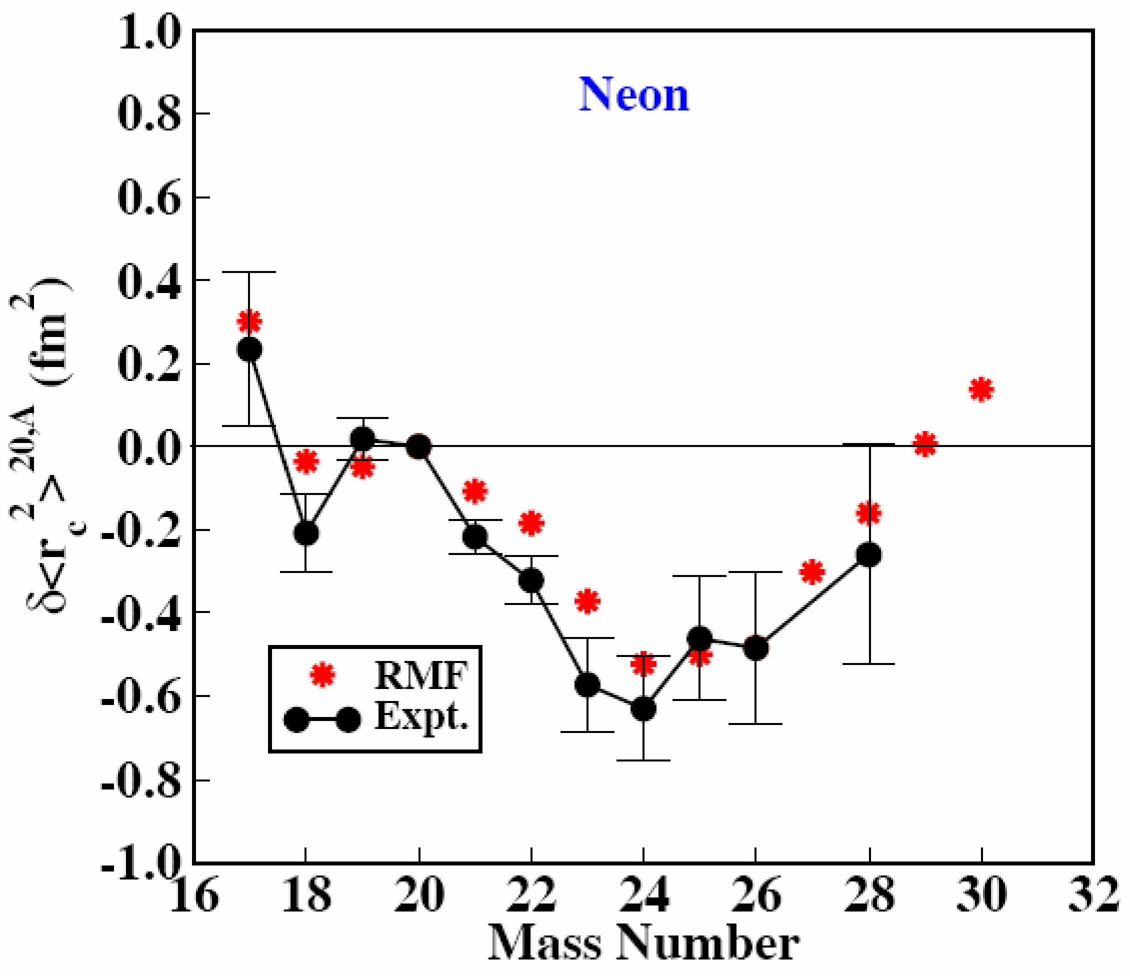


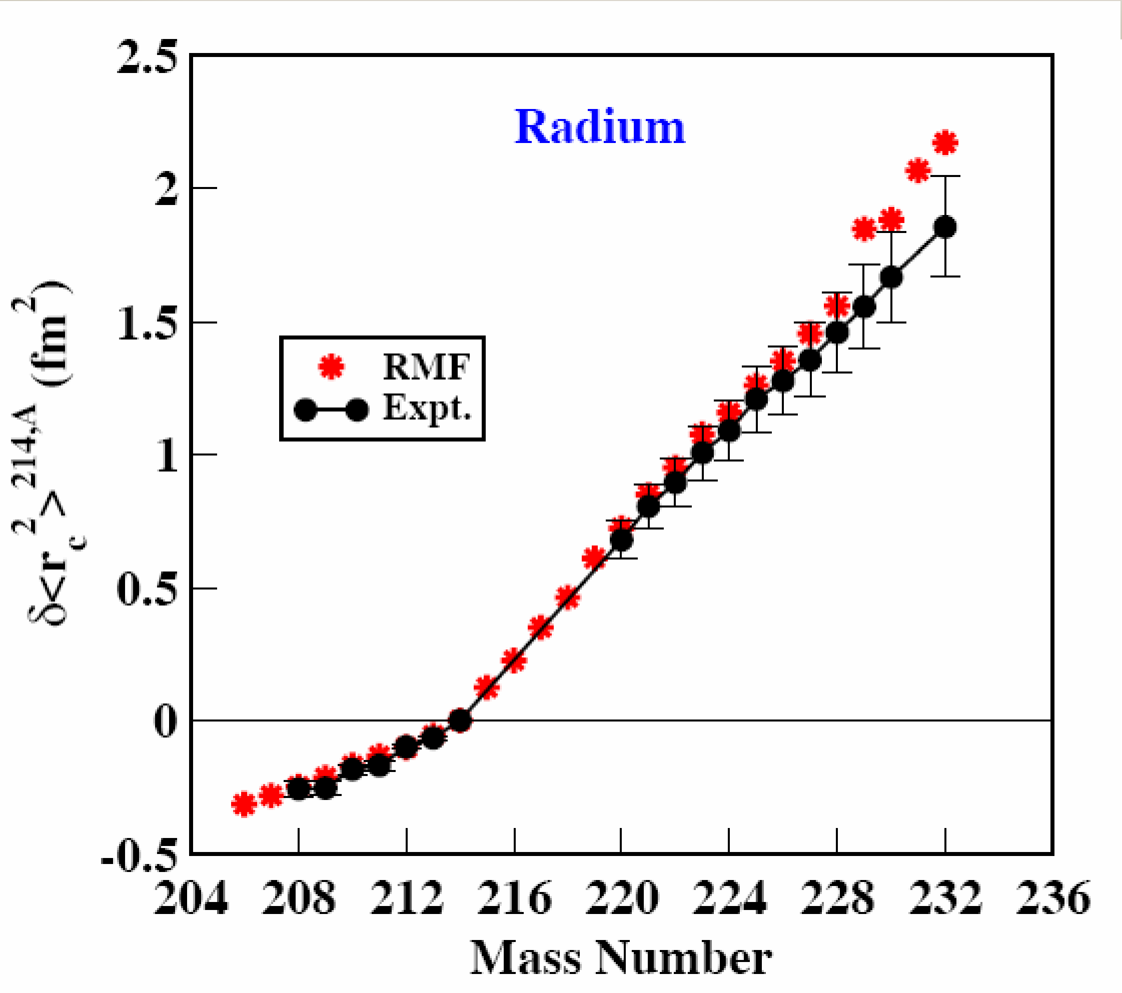
Neon
δBE (MeV)
12
8
4
0
−4
−8
15
17
19
21
23
25
27
29
31
T_z
Cal.
Radium
δBE (MeV)
6
4
2
0
−2
−4
−6
−8
−10
204
209
214
219
224
229
234
Mass Number
Cal.


Neon
β
Mass Number
Cal.
MN
0.8
0.6
0.4
0.2
0
−0.2
−0.4
15
17
19
21
23
25
27
29
31


Radium
β
Mass Number
Cal.
MN
0.4
0.3
0.2
0.1
0
−0.1
−0.2
204
209
214
219
224
229
234

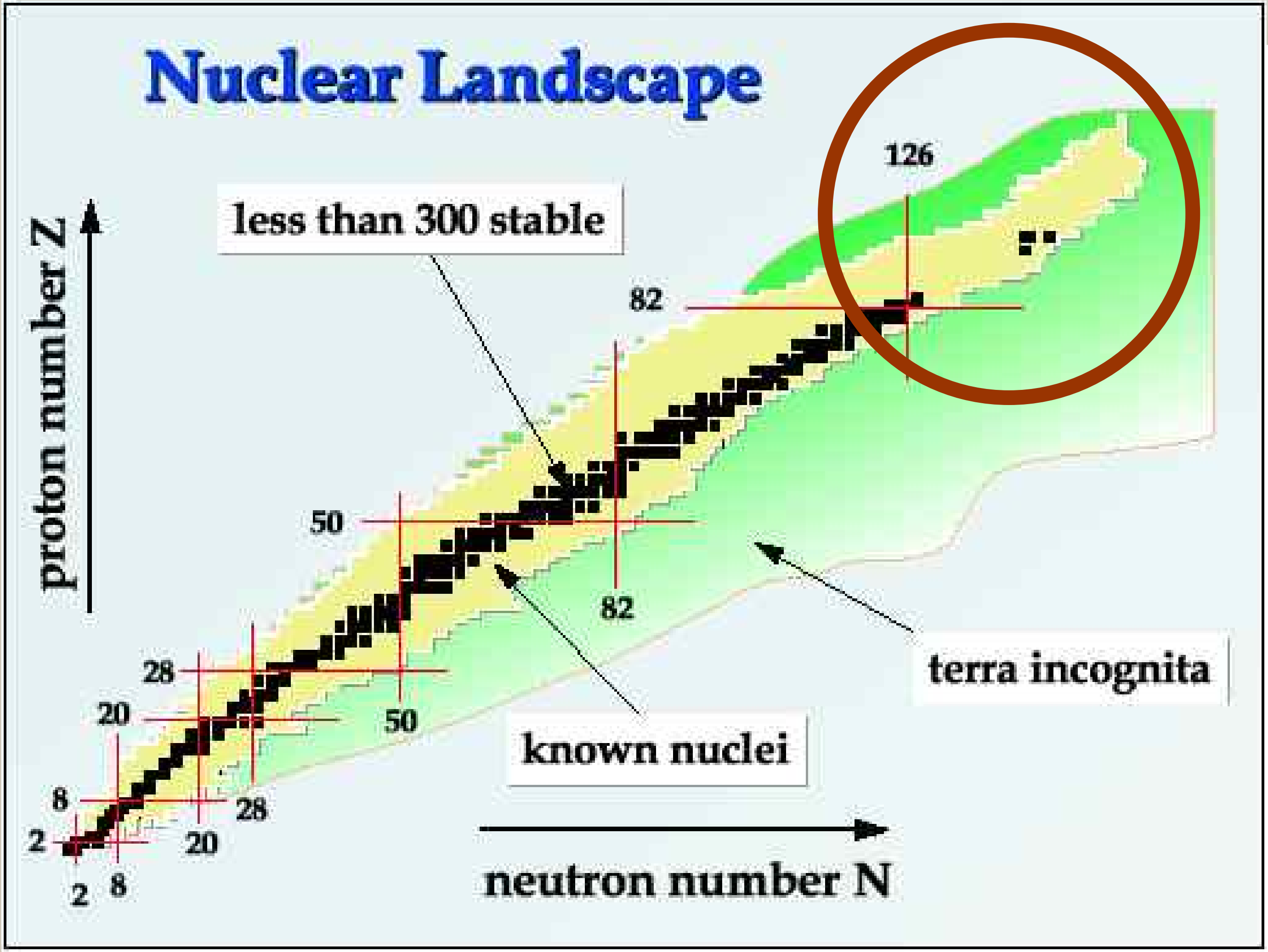
Ground State Properties

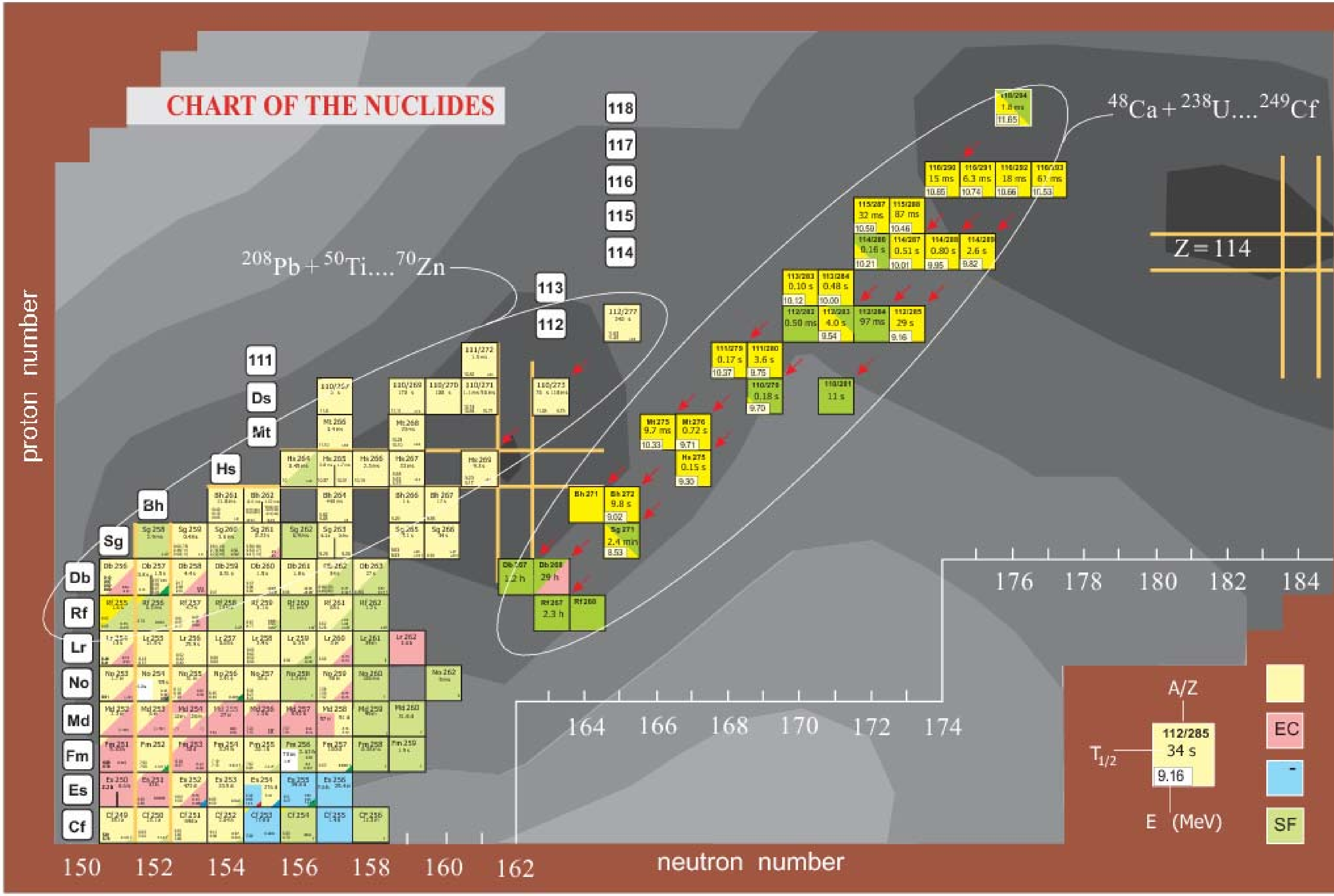



Radium
RMF
Expt.
δ<r_c^2>^{214,A} (fm^2)
Mass Number
2.5
2
1.5
1
0.5
0
-0.5
204
208
212
216
220
224
228
232
236

Ground State Properties

- **Binding Energies: Well Reproduced ~ 0.25% of Expt. (On the Average)**
- **β: Reasonable: Consistent with Moller – Nix Systematics**
- **Charge Radii: Upto 2nd Decimal**
- **Isotopic Shifts: Reproduced**


Nuclear Landscape
126
less than 300 stable
proton number Z
82
50
82
terra incognita
28
20
50
known nuclei
8
28
2
20
2
8
neutron number N



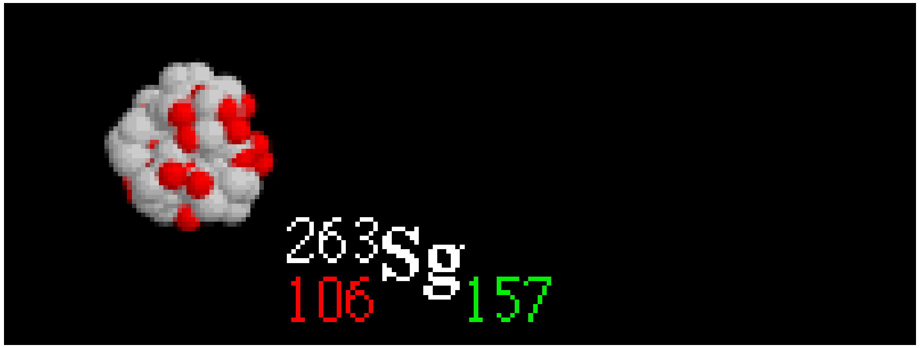
Yu. Ts. Oganessian *et al.*, JINR Preprint E7-2004-160

Production:

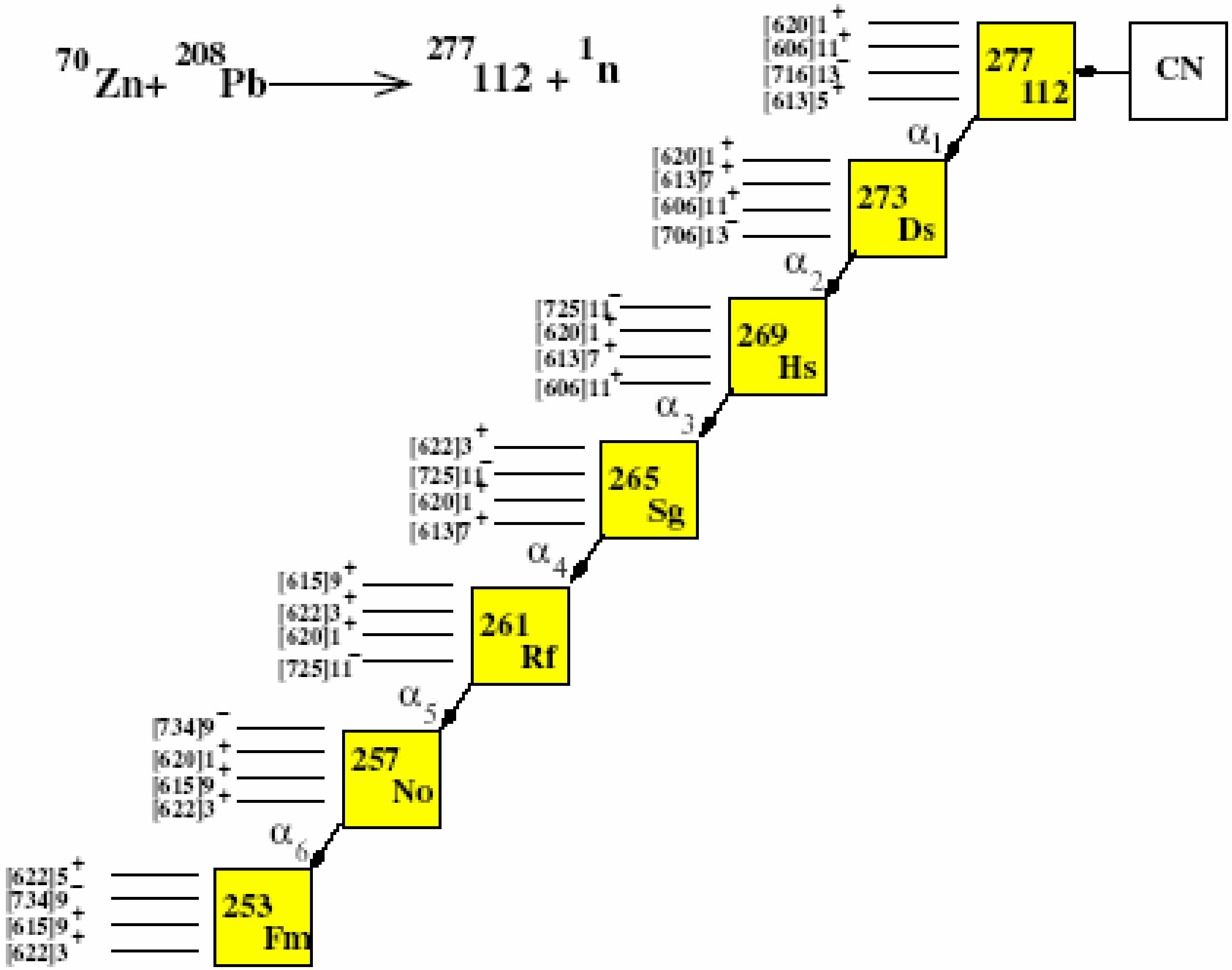
(Cold Fusion)

Decay:

(α Emission)



http://ie.lbl.gov/education/glossary/glossaryf.htm



S. Hofmann et al, Z. Phys. A 354 (1996) 229.

:Superheavy Nuclei:
(Half Lives)

Calculation:

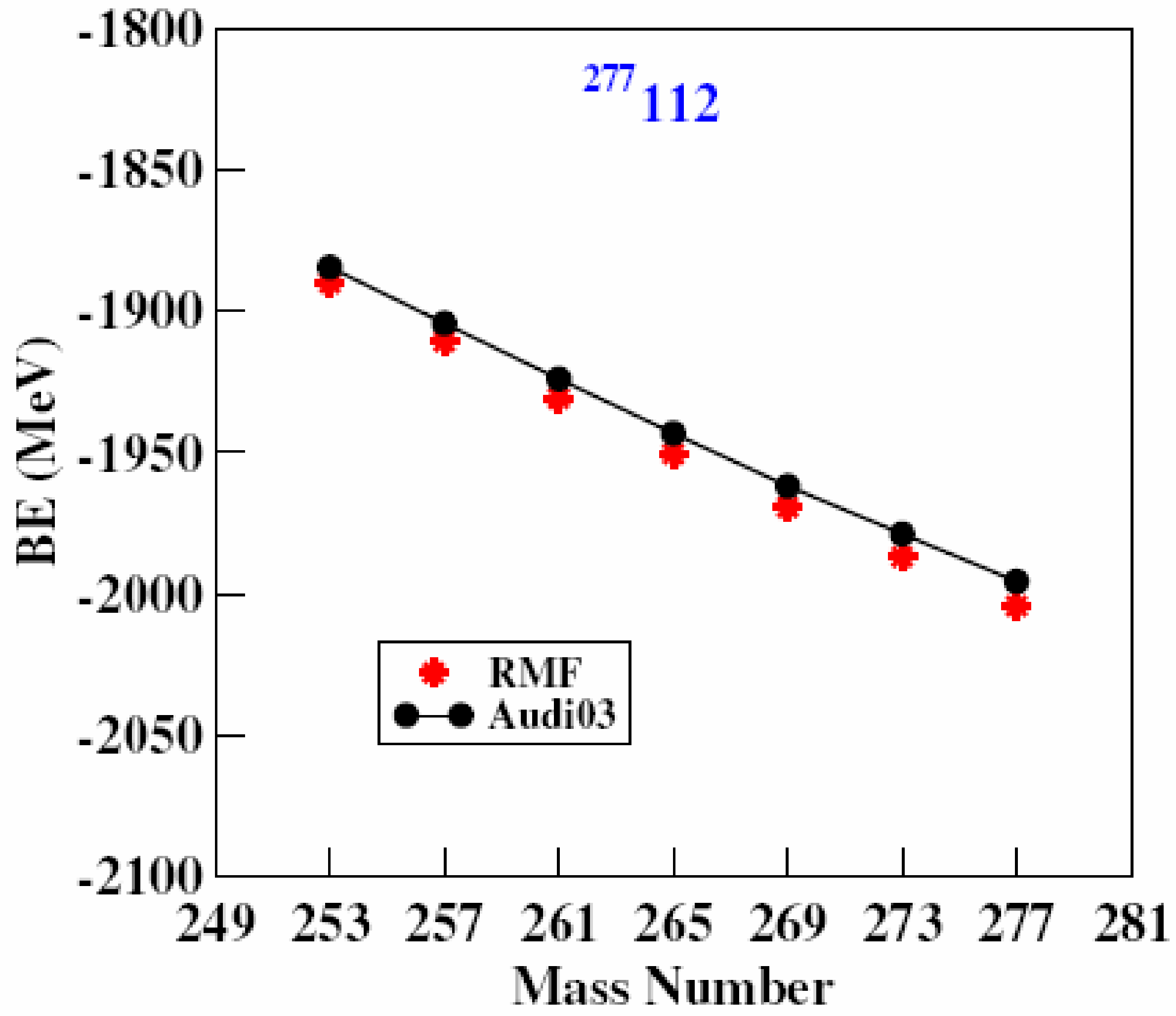
Ground State Properties: RMF

-- Well Reproduced (BE, β, etc.)

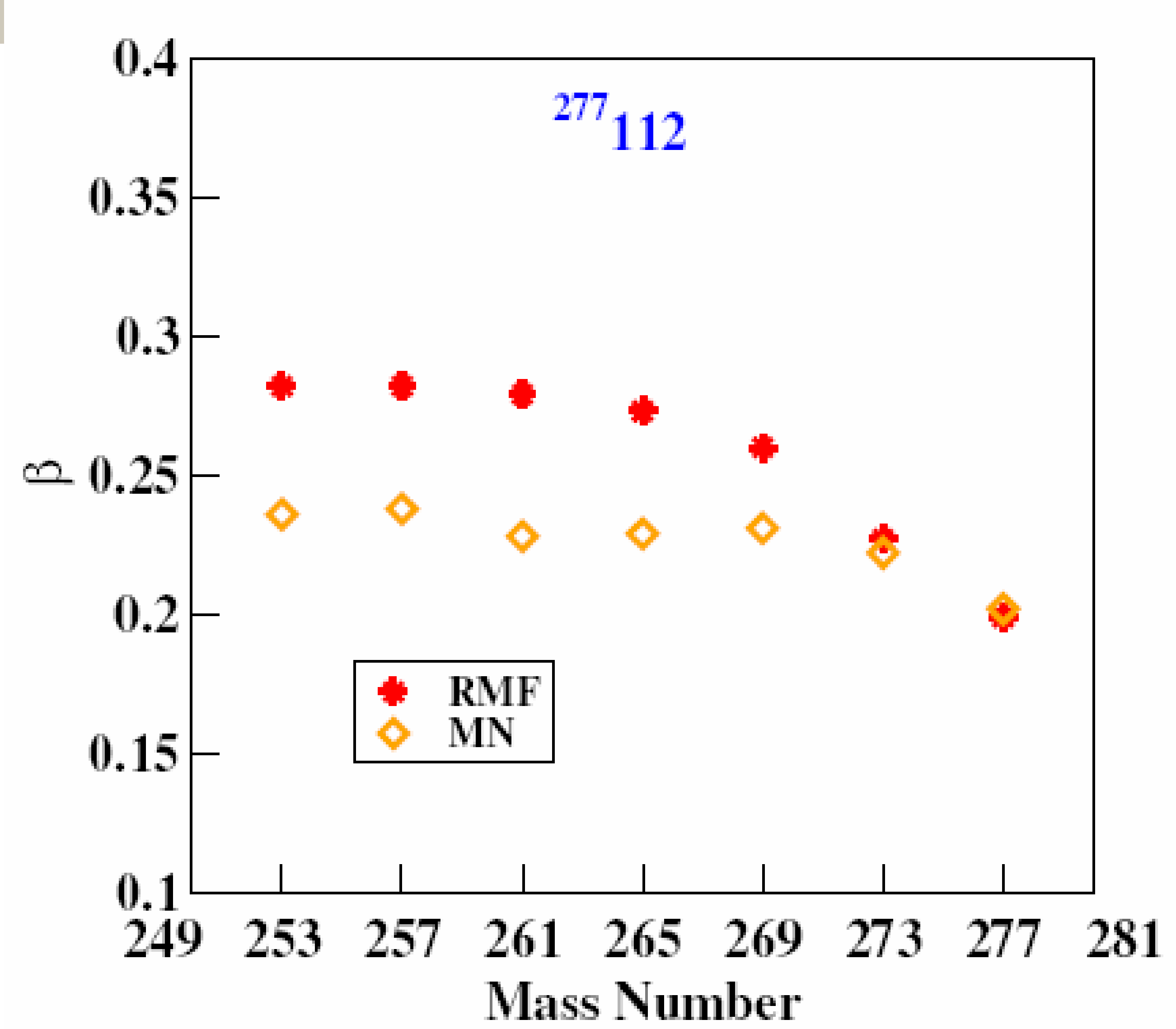
Half Lives: WKB Approximation

-- Requires: Q Values + Potentials

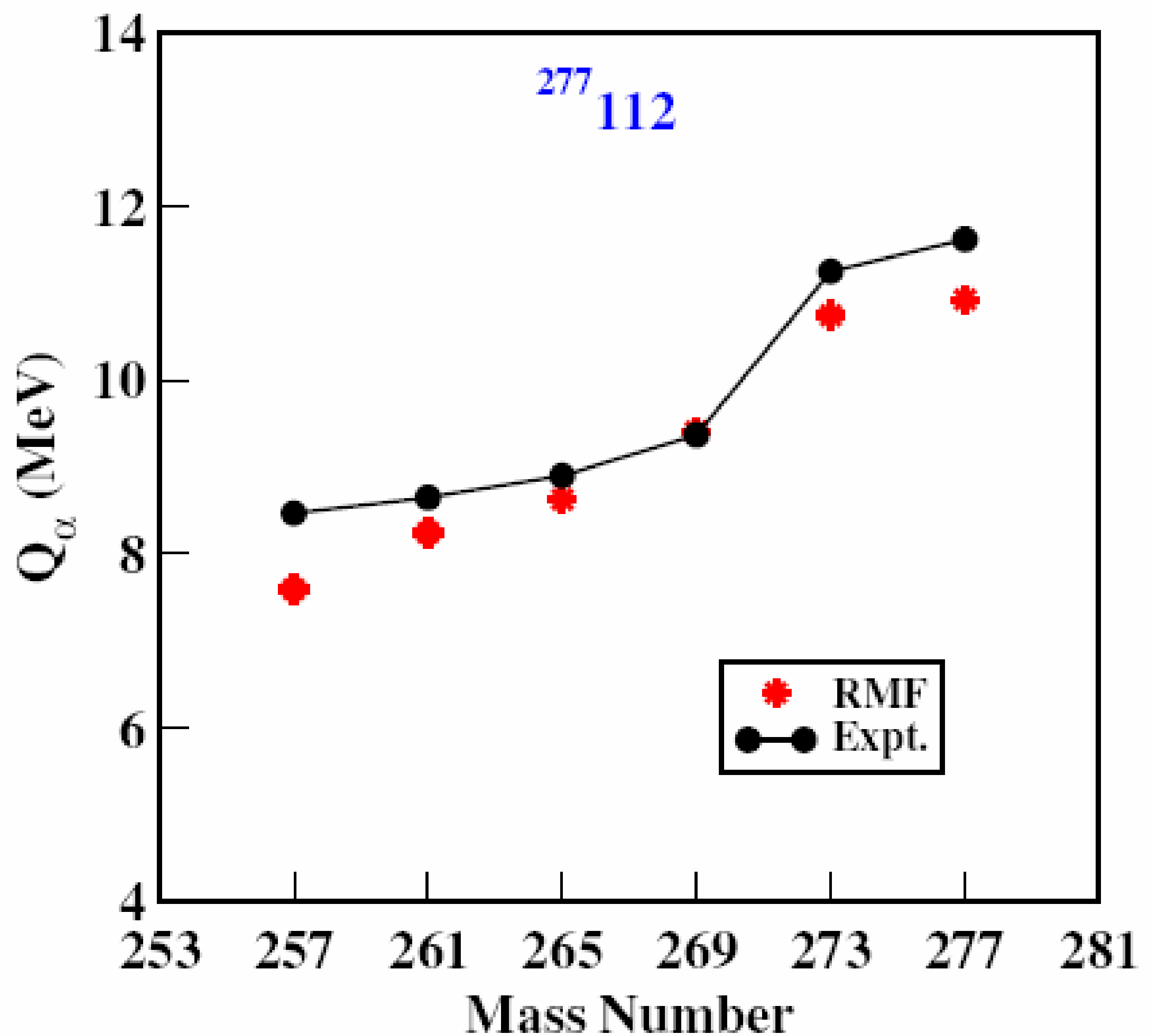
→ α - Daughter Interaction Potentials (Double Folding Model)



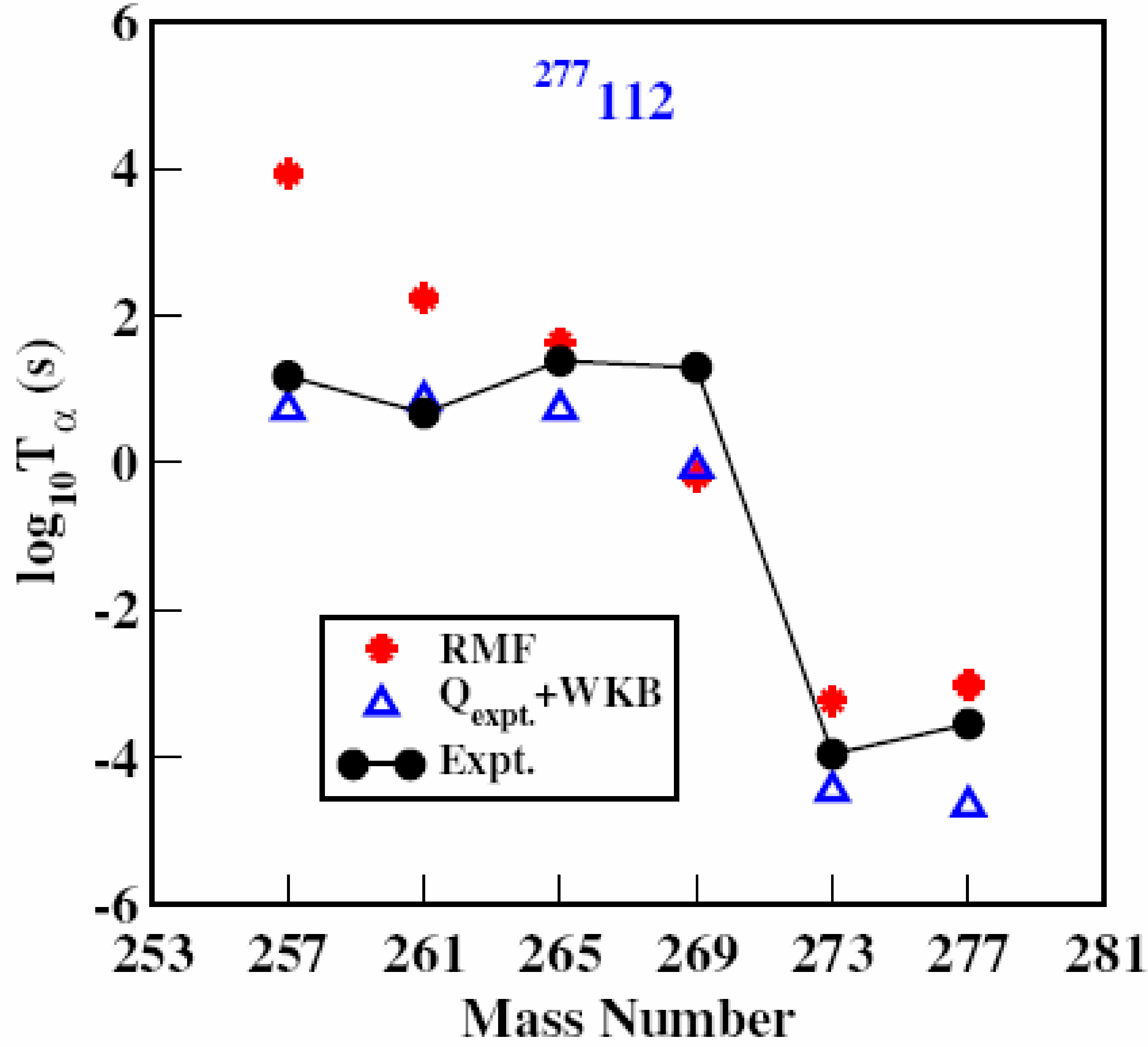
$^{277}112$
BE (MeV)
-1800
-1850
-1900
-1950
-2000
-2050
-2100
249
253
257
261
265
269
273
277
281
Mass Number
RMF
Audi03



277112
β
0.4
0.35
0.3
0.25
0.2
0.15
0.1
249
253
257
261
265
269
273
277
281
Mass Number
RMF
MN



$^{277}112$
Q$_\alpha$ (MeV)
14
12
10
8
6
4
253
257
261
265
269
273
277
281
Mass Number
RMF
Expt.



277 112
6
4
2
0
-2
-4
-6
log10 Tα (s)
RMF
Qexpt.+WKB
Expt.
253
257
261
265
269
273
277
281
Mass Number

Summary & Conclusions

- **Q Values: Well Reproduced**
- **Experimental Q values + WKB →**
  Reproduces half lives well →
  Double Folding Potential Reliable
- **Half Lives Depend Sensitively on**
  Q values

RMF IS SUCCESSFUL

Recent Developments:

Large Shell-Model Calculations:

Large Dimensions
Effective Interaction

ANTOINE (E.Courier, Strasburg, France)

OXBASH (B.A.Brown; Michigan, USA)

DUSM (Vallieres + Novoselsky, Phil., USA)

sd – shell (^{16}O Core) → Good

pf – shell (^{40}Ca Core) →
Yet to be Achieved Fully

Limits: 10^7 – 10^8 Basis States

How Much Dependence on Effective Interaction?

What can we Learn from Eigenvectors with Billions of Components?

Exotic Nuclei: Asymptotics is Important

Continuum Shell Model

Ab – Initio : No Core Shell Model (NCSM) With nn, nnn Interaction

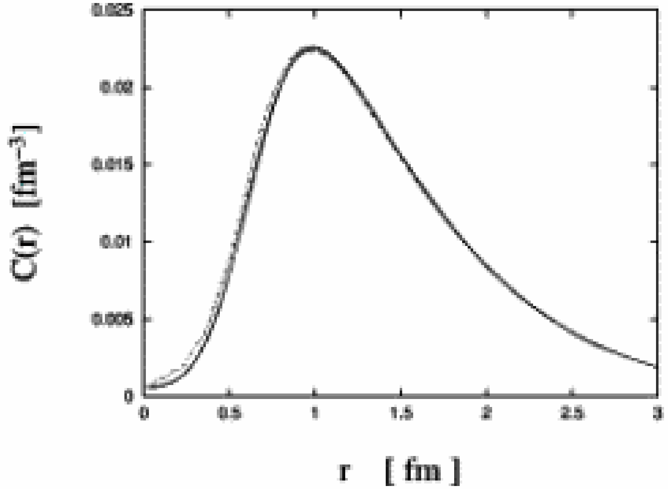
- Hyperspherical harmonic variational:
- Green's function Monte Carlo: $A \leq 7$
- No-core shell model: $A \leq 12$

Benchmark calculation for *A=4*

- Test calculation with realistic interaction: all methods agree.

$$\langle \Psi | \sum_{k<l}^{4} \delta(r - r_{kl}) | \Psi \rangle$$

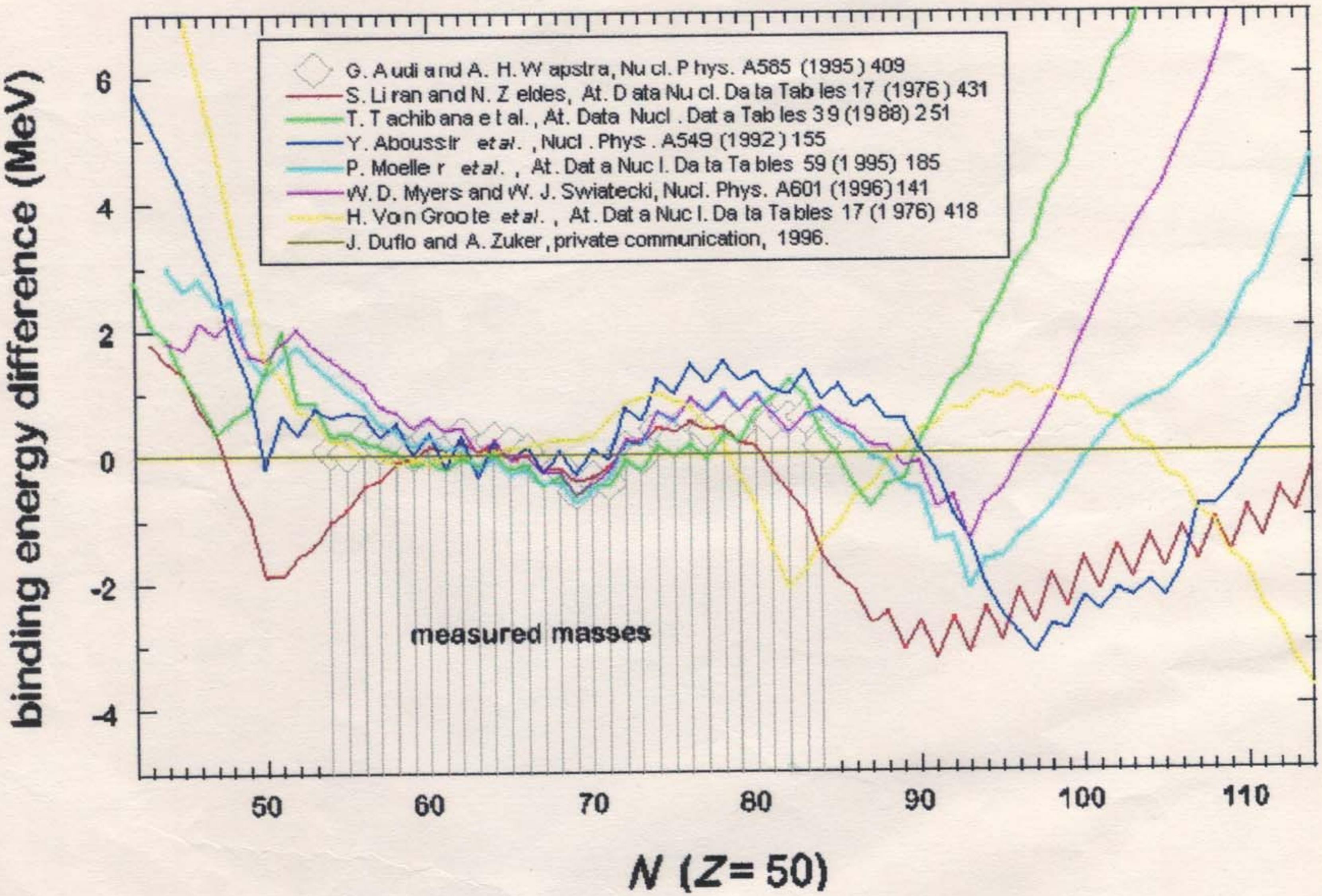
Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486



- But E_{expt}=-28.296 MeV ⇒ need for three-nucleon interaction.

H. Kamada *et al.*, Phys. Rev. C **63** (2001) 034006

^{16}O Calculations in 2010!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measured masses
binding energy difference (MeV)
N (Z = 50)
6
4
2
0
-2
-4
50
60
70
80
90
100
110