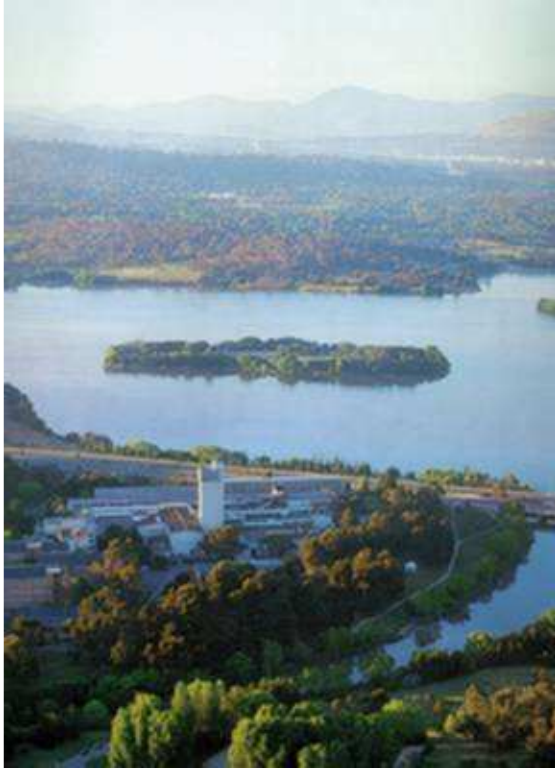


Experimental techniques to deduce J^π



T. Kibédi

*Dept. of Nuclear Physics, Australian National University,
Canberra, Australia*

**Workshop on
“Nuclear Structure and Decay Data:
Theory and Evaluation”
Trieste, Italy, 2006**



Outline:

Lecture I: *Experimental techniques to deduce J^π from*

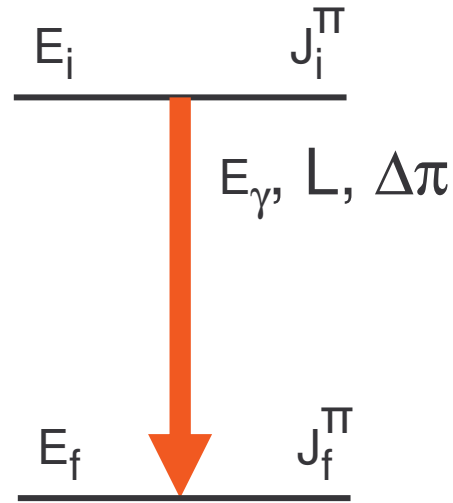
- *Angular distributions and correlations*
- *Directional Correlations from Oriented nuclei (DCO)*
- *Gamma-ray linear polarizations*
- *Internal conversion coefficients*

Lecture II: *New developments in characterizing nuclei using separators*



Electromagnetic Decay and Nuclear Structure

Energetics of γ -decay:



$$E_i = E_f + E_\gamma + T_r$$

$$0 = p_R + p_\gamma$$

where $T_R = (p_R)^2/2M$; usually $T_R/E_\gamma \sim 10^{-5}$

Angular momentum and parity selection rules & multiplicarities

$$|J_i - J_f| \leq L \leq |J_i + J_f| \quad L \neq 0$$

$$\Delta\pi = \text{no}$$

M1, E2, M3, E4, ...

$$\Delta\pi = \text{yes}$$

E1, M2, E3, M4, ...

$$|J_i - J_f| \leq L \leq |J_i + J_f| \quad L \neq 0$$

$$\Delta\pi = \text{unknown}$$

D, Q, O, H, ...

$$J_i = J_f$$

$$L = 0$$

$$\Delta\pi = \text{no};$$

E0

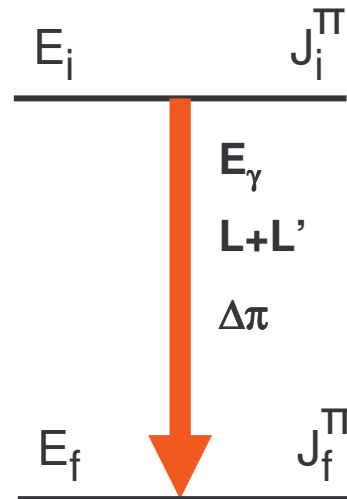
Multipolarity known

ΔJ may not be unique

$\Delta\pi$ unique



More on EM transitions



Mixed multipolarity & Mixing ratio

$$\delta(\pi'L'/\pi L) = I_\gamma(\pi'L') / I_\gamma(\pi L)$$

$$I_\gamma = I_\gamma(\pi L) + I_\gamma(\pi'L')$$

Or in terms of transition probability

$$\lambda_\gamma = \lambda_\gamma(\pi L) + \lambda_\gamma(\pi'L')$$

Total transition probability

$$\lambda_T = \lambda_\gamma + \lambda_{CE} + \lambda_\pi + \lambda_{\gamma\gamma} + \dots$$

λ_{CE} - conversion electrons, K, L1, L2... shells

λ_π - electron-positron pair production; $E_\gamma > 2 m_0 c^2$

$\lambda_{\gamma\gamma}$ - 2 photon emission; very rare (10^{-5})



Determining transition multipolarity

➤ **Gamma-rays**

Angular distribution with spins oriented

Angular correlations

Polarization effects

➤ **Conversion electrons**

Electron conversion coefficients

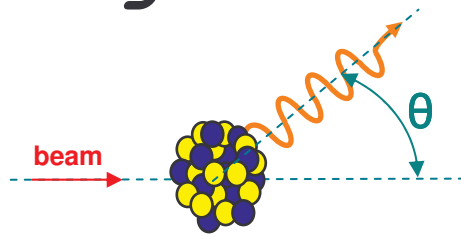
E0 (L=0) transitions

➤ **Electron positron pairs**

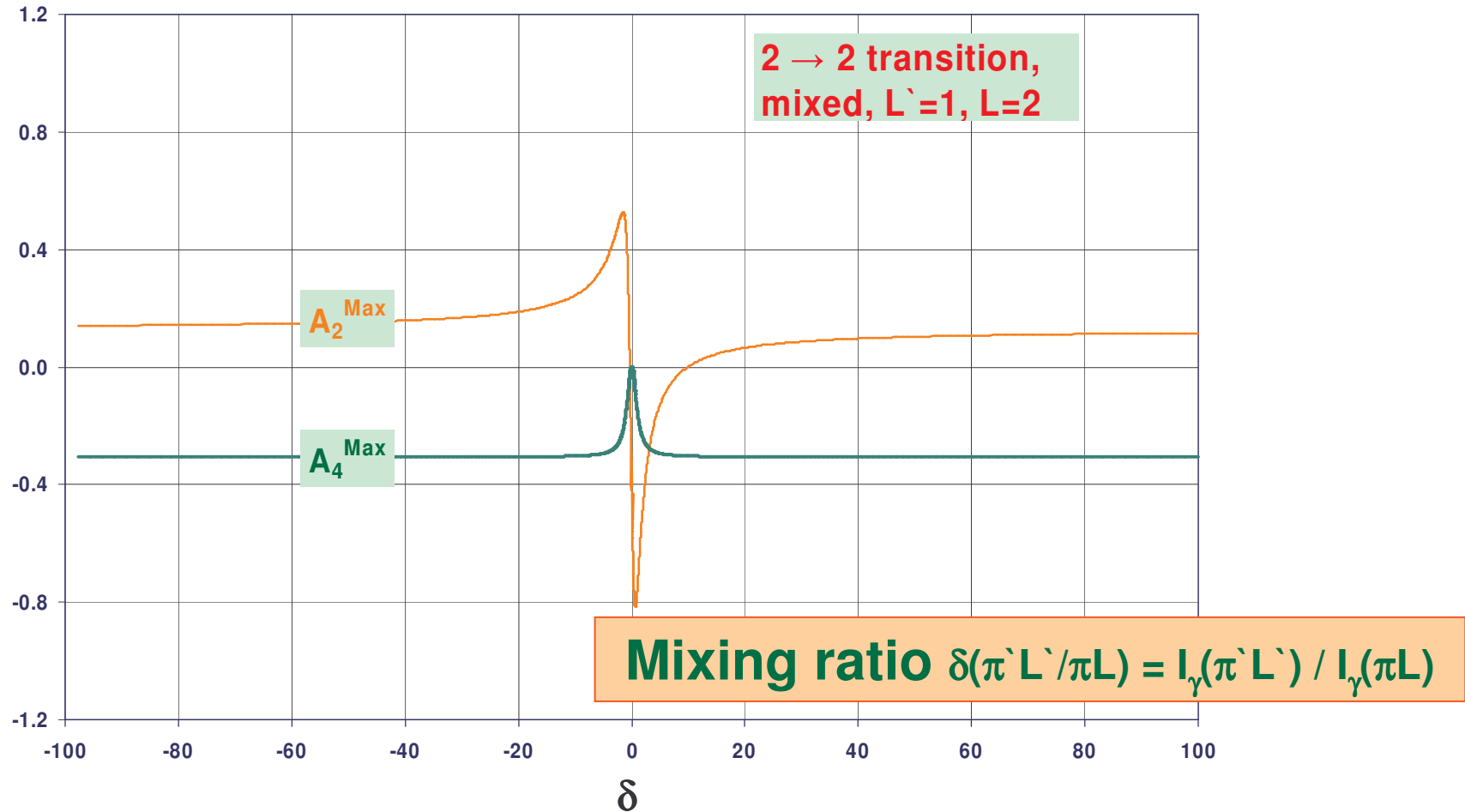
Pair conversion coefficients



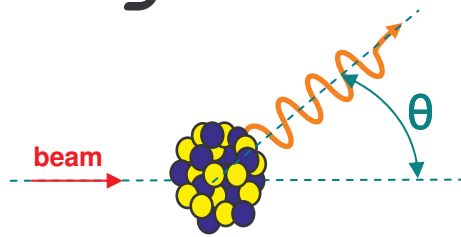
Angular distributions of Gamma-rays



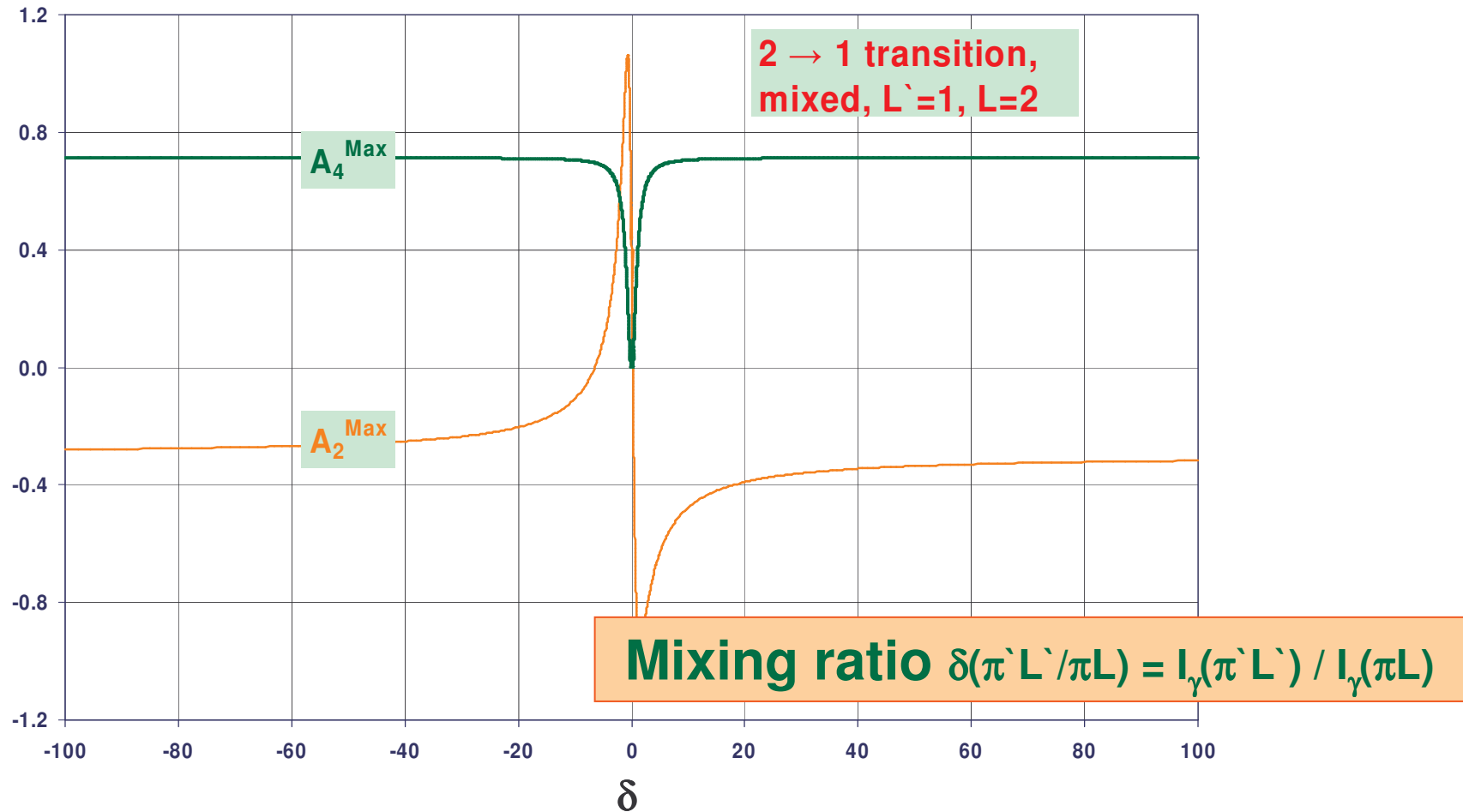
$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$



Angular distributions of Gamma-rays



$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$



Angular distributions of Gamma-rays

$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

Attenuation due to relaxation of nuclear orientation

$$0 \leq A_{kk} \leq A_k^{\max}(J_i, J_f, L); k = 2, 4, \dots$$

$$A_k^{\max}(J_i, J_f, L) = \frac{F_k(LLJ_fJ_i) + 2\delta \times F_k(LL+1J_fJ_i) + \delta^2 \times F_k(L+1L+1J_fJ_i)}{1 + \delta^2}$$

For $F_k(LLJ_fJ_i)$ see E. Der Mateosian and A.W. Sunyar, ADNDT 13 (1974) 407

$$A_{kk} = B_k(J_i) \times A_k^{\max}(J_i, J_f, L)$$

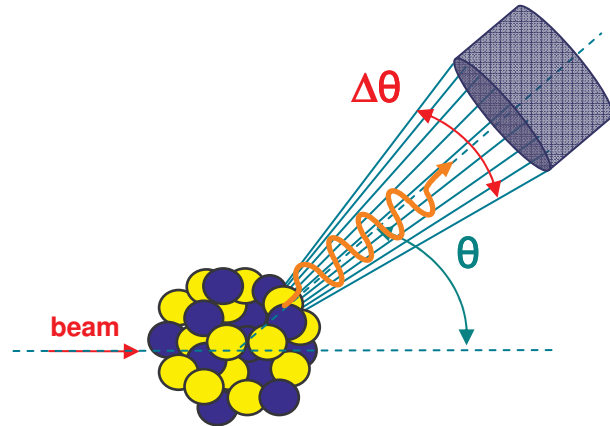
Nuclear orientation can be achieved

- ❖ by interaction of external fields (E,B) with the static moments of the nuclei at low temperatures
- ❖ by nuclear reaction



Attenuation of angular distributions

Detector finite size ($\Delta\theta$):
solid angle attenuation



Beam defines a symmetry axis

$$W(\theta) = 1 + \sum_{k=2,4} Q_k \times B_k(J_i) \times A_k^{\max}(J_i, J_f, L, \delta)$$

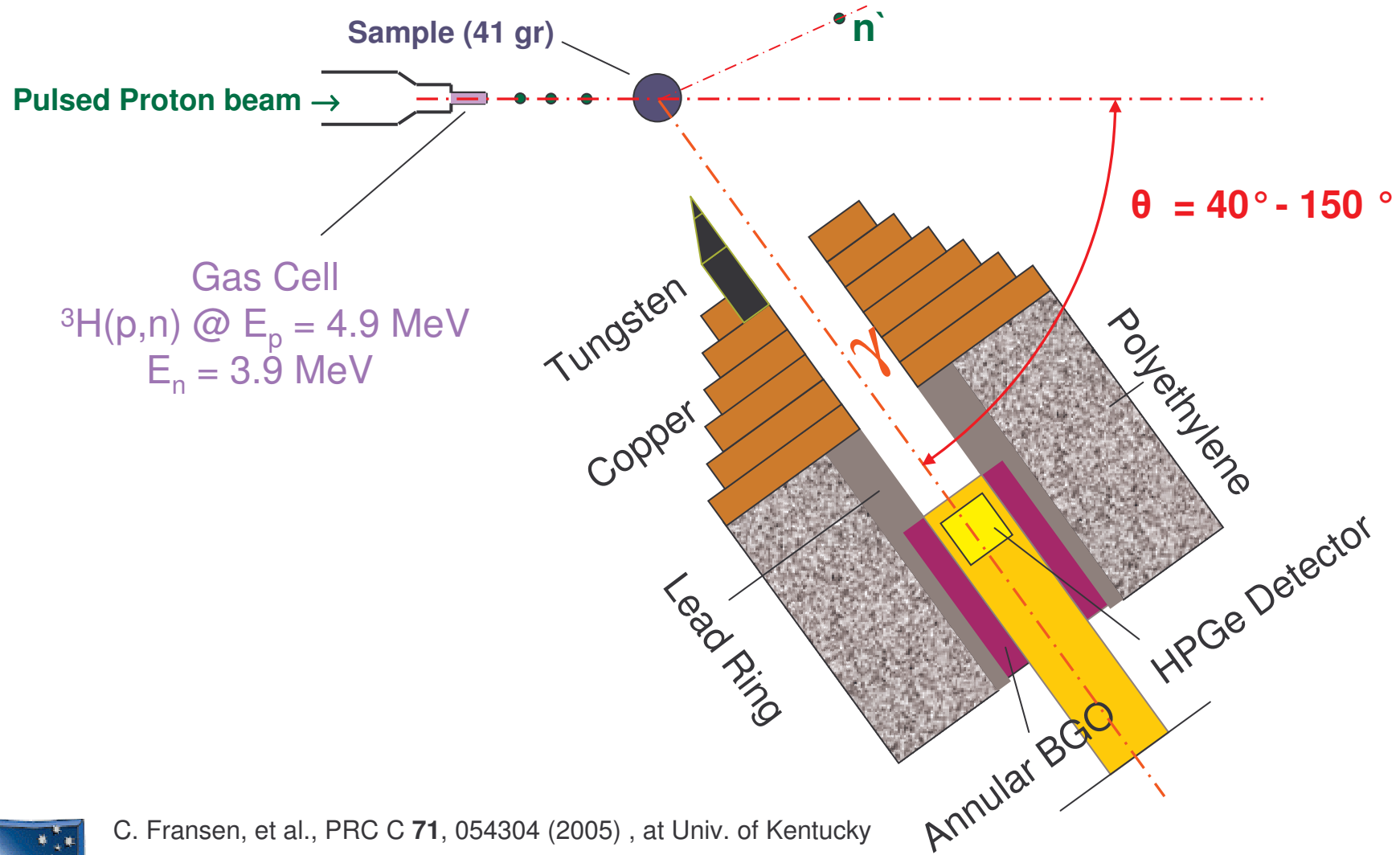
where $B_k(J)$ is the statistical tensor

$$B_k(J_i) = \sum_{m=-J_i}^{+1} (-1)^{J_i+m} \sqrt{(2k+1)(2J_i+1)} \times \left(\begin{matrix} J_i & J_i & k \\ -m & m & 0 \end{matrix} \right) \times \underbrace{\frac{\text{Exp}(-m^2/2\sigma^2)}{\sum_{m=-1}^{J_i} \text{Exp}(-m^2/2\sigma^2)}}_{\text{Approximation with Gaussian distribution}}$$

Approximation with Gaussian distribution



Angular distributions of Gamma-rays (n, n') reaction on ^{92}Zr



C. Fransen, et al., PRC C 71, 054304 (2005), at Univ. of Kentucky

T. Kibèdi, NSDD Workshop, Trieste 2006

Figure courtesy of S.W. Yates

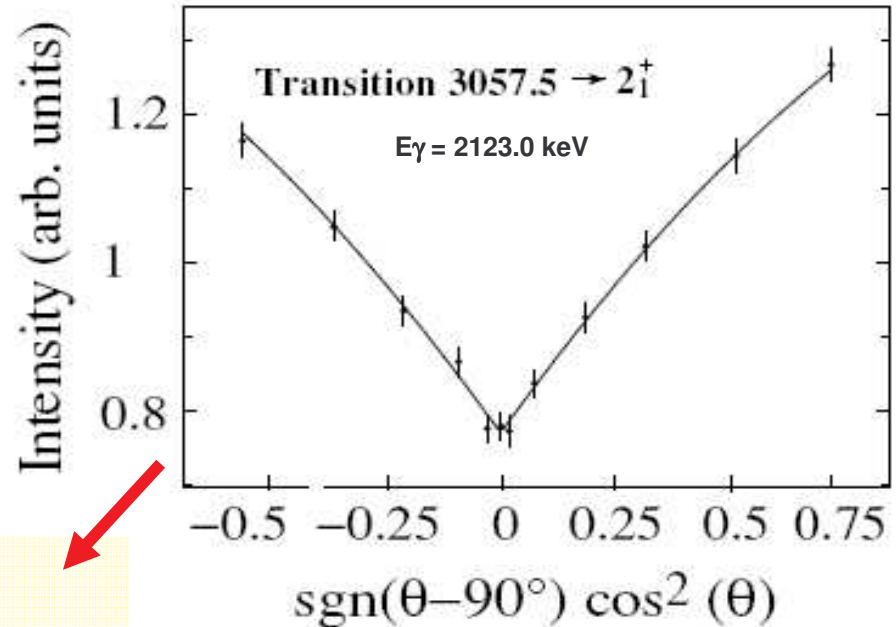
Angular distributions of Gamma-rays

$^{92}\text{Zr}(n,n')$ reaction

12 angles and

12 hours / angle

γ -spectrometer at 1.4 m



C. Fransen, et al., PRC C 71, 054304 (2005), at Univ. of Kentucky

Fit to data

$$W(\theta) = A_0 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

Deduced

$$A_2 = A_{22}/A_0$$

$$A_4 = A_{44}/A_0$$

Typical values

$\Delta J=2$ (stretched quadrupole)

A_2 A_4

+0.3 -0.1

$\Delta J=1$ (stretched dipole)

-0.2 0

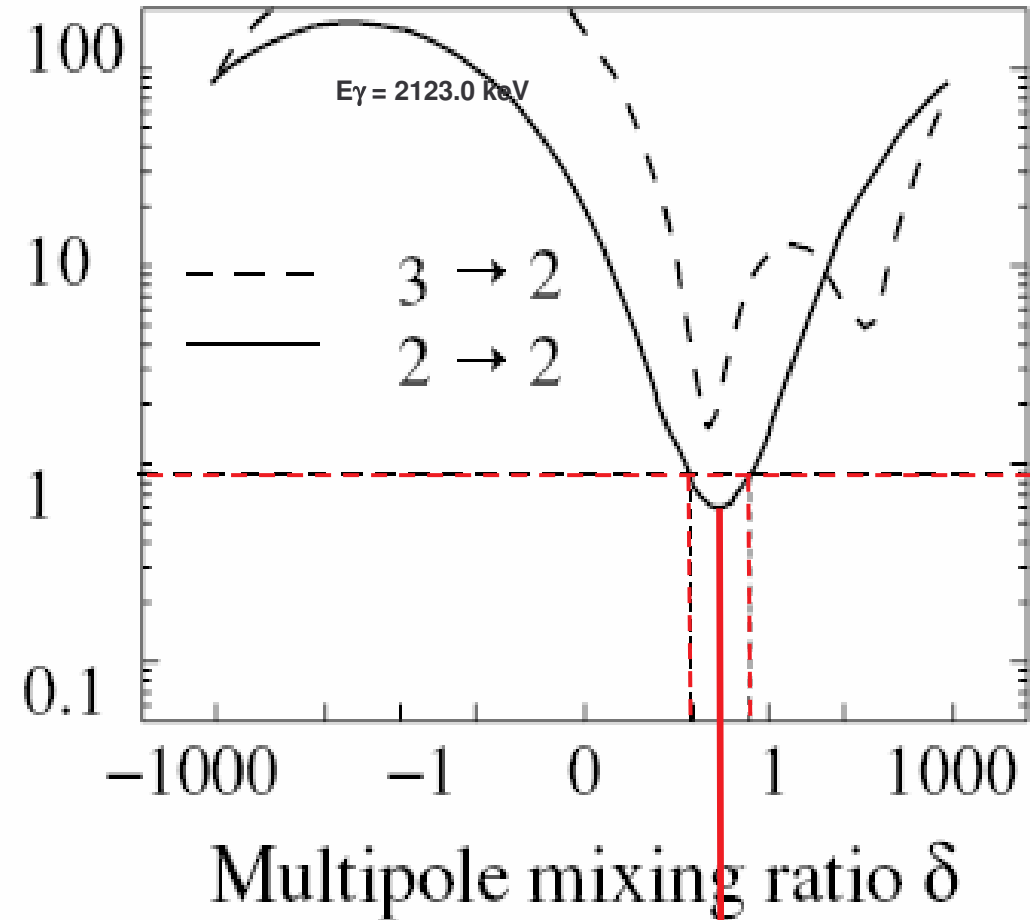
$\Delta J=1, D+Q$

+0.5 to -0.8 >0



Angular distributions - mixing ratio

C. Fransen, et al., PRC C 71, 054304 (2005) , at Univ. of Kentucky



Mixing ratio, δ deduced from χ^2 of

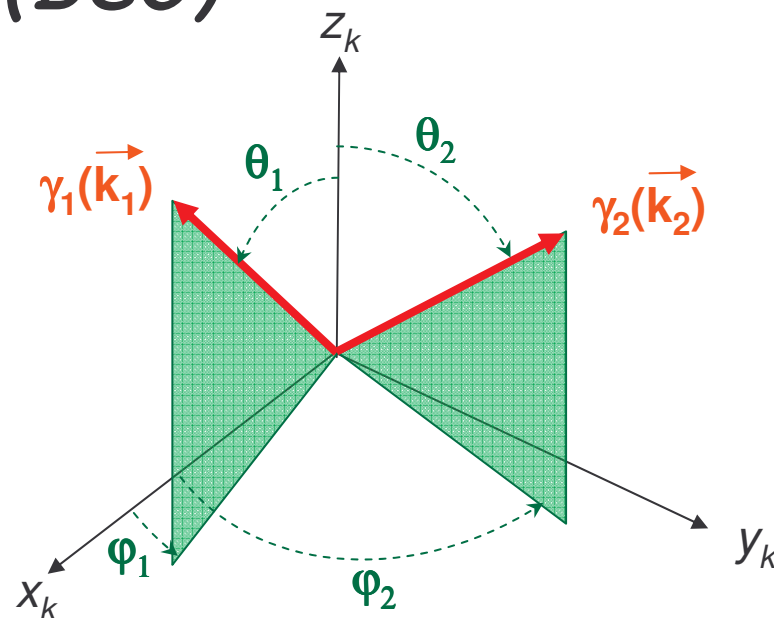
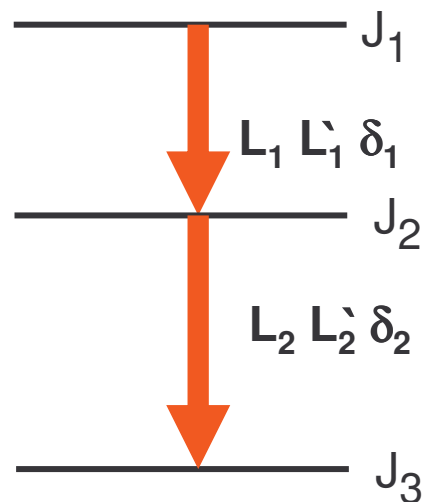
$$\frac{W_{\text{exp}}(\theta)}{W_{\text{calc}}(\theta)} \text{ as a function of } \delta$$

But no information on
Electric or Magnetic character
E1+M2 or M1+E2

$$\delta = 0.69(16) \text{ (D+Q)}$$



Directional Correlations from Oriented nuclei (DCO)



For a $J_1 \rightarrow J_2 \rightarrow J_3$ cascade (see A.E. Stuchbery, Nucl. Phys. **A723** (2003) 69)

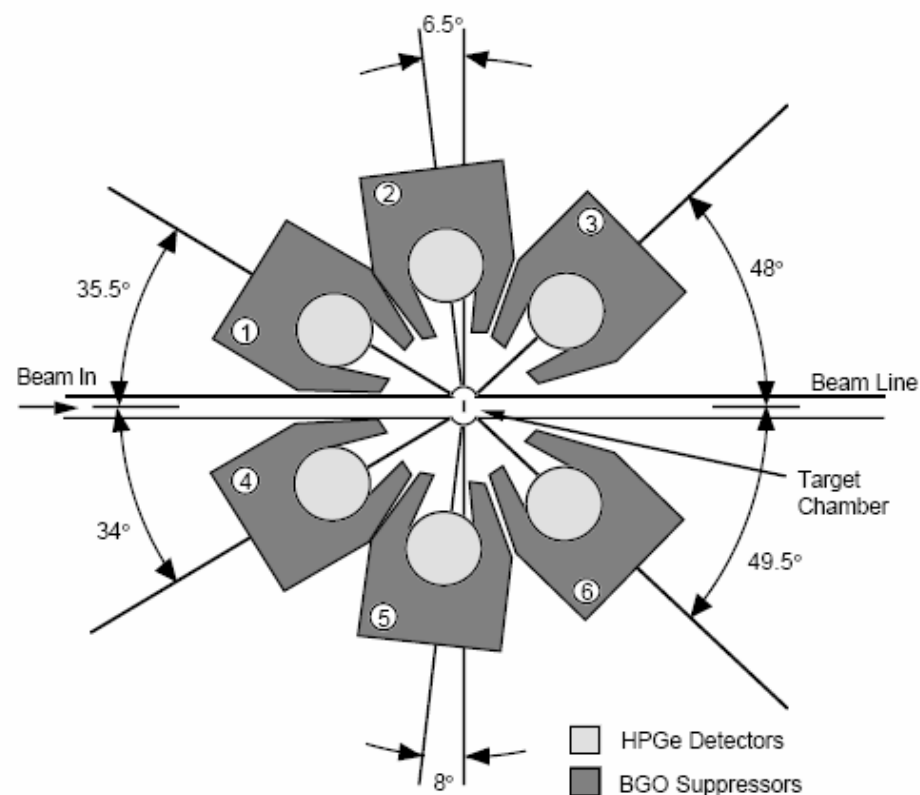
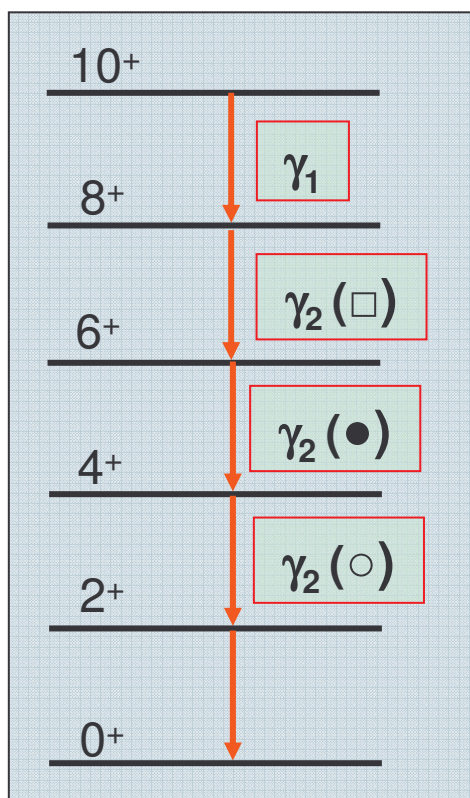
$$\begin{aligned}
 W(\theta_1, \varphi_1, \theta_2, \varphi_2) = & \sum_{k, q, k_1, q_1, k_2, q_2} \rho_{k_1 q_1}(J_1) (-1)^{k_1 + q_1} \sqrt{(2k+1)(2k_1+1)} \begin{pmatrix} k_1 & k & k_2 \\ -q_1 & q & q_2 \end{pmatrix} \\
 & \times A_k^{k_2 k_1} (\delta_{\gamma_{12}} L L J_2 J_1) Q_k(E_{\gamma_{12}}) D_{q_0}^{k*}(\varphi_1, \theta_1, 0) \\
 & \times A_{k_2} (\delta_{\gamma_{23}} L L J_3 J_2) Q_{k_2}(E_{\gamma_{23}}) D_{q_2}^{k_2*}(\varphi_2, \theta_2, 0)
 \end{aligned}$$



Directional Correlations from Oriented nuclei (DCO) - example

^{184}Pt from $^{\text{nat}}\text{Gd} + ^{29}\text{Si}$ @ 145 MeV CAESAR array (ANU)

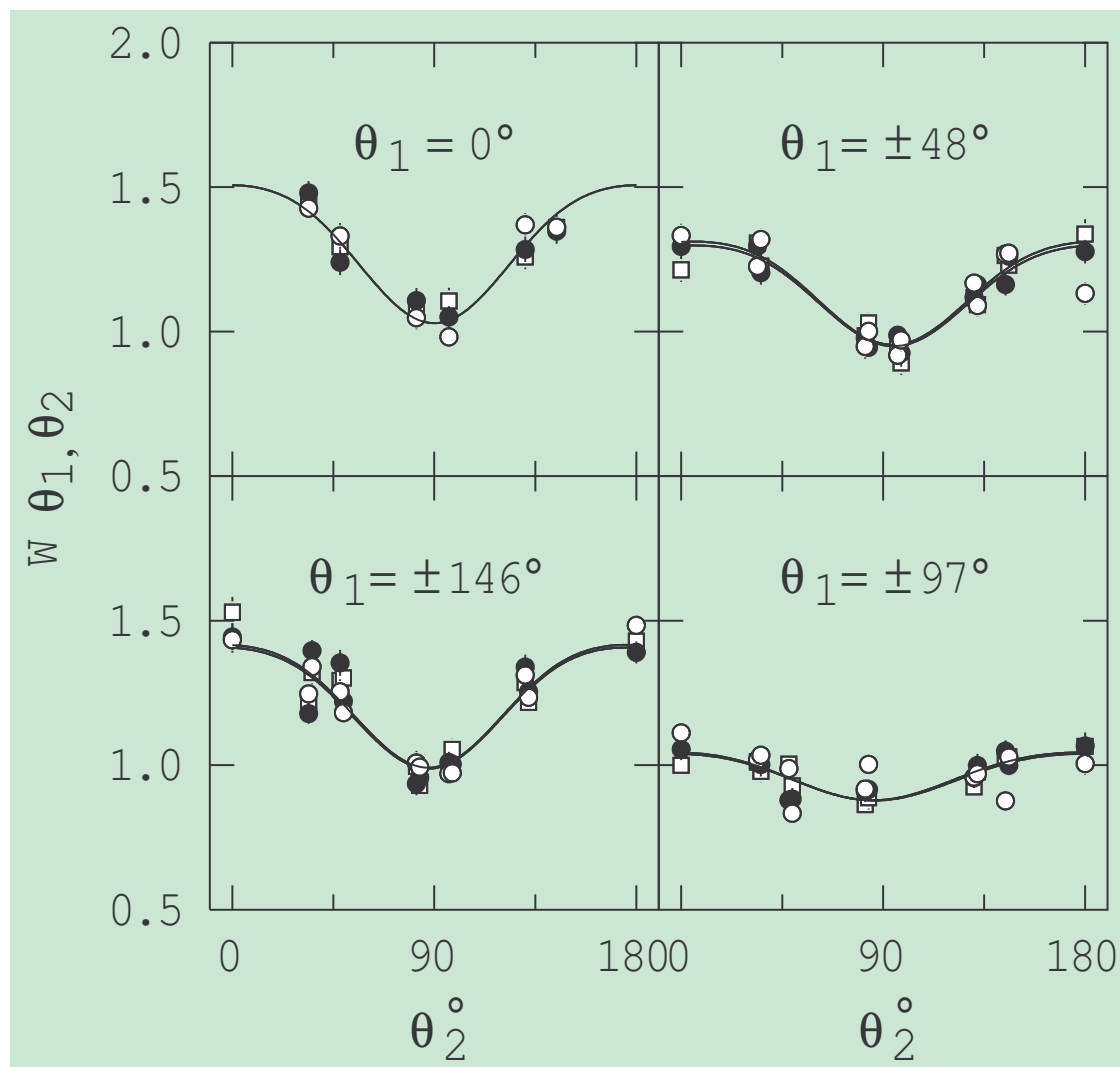
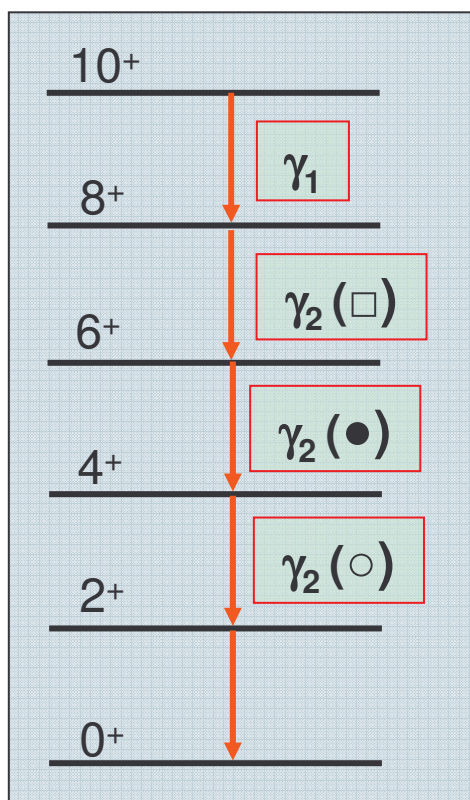
M.P. Robinson et al., *Phys. Lett B* 530 (2002) 74



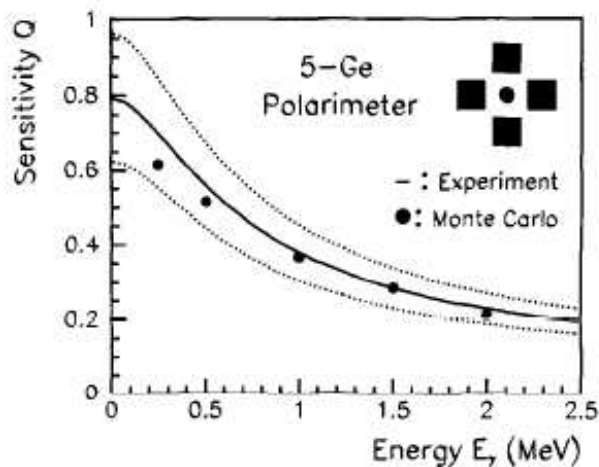
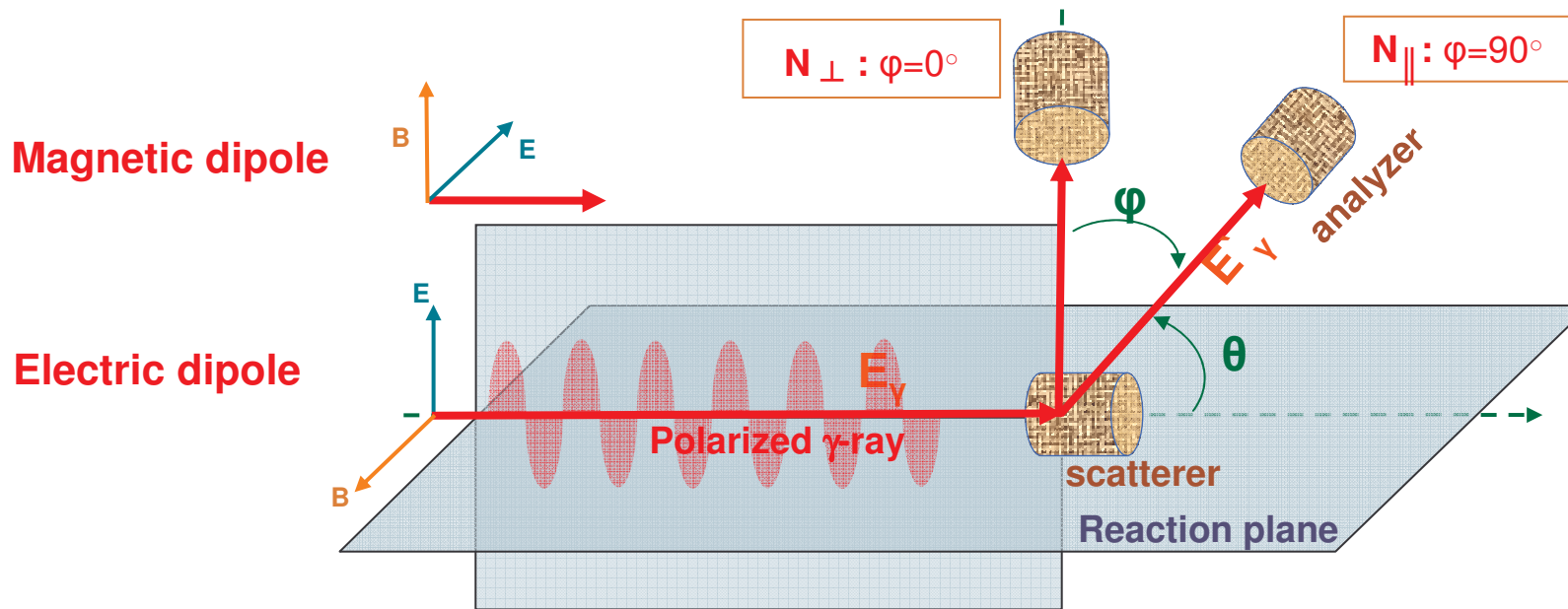
Directional Correlations from Oriented nuclei (DCO) - example

^{184}Pt from $\text{natGd} + ^{29}\text{Si}$ @ 145 MeV CAESAR array (ANU)

M.P. Robinson et al., Phys. Lett B530 (2002) 74



Gamma-ray linear polarization



L.M. Garcia-Raffi, et al. NIM A391 (1997) 461

Compton-scattering

$$E_{\gamma'} = \frac{E_{\gamma}}{1 + \alpha(1 - \cos \theta)} \quad \text{with} \quad \alpha = \frac{E_{\gamma}}{m_0 c^2}$$

Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \frac{E_{\gamma'}^2}{E_{\gamma}^2} \left(\frac{E_{\gamma'}}{E_{\gamma}} + \frac{E_{\gamma}}{E_{\gamma'}} - 2 \sin^2 \theta \cos^2 \varphi \right)$$

Polarization

$$P = A / Q$$

Asymmetry

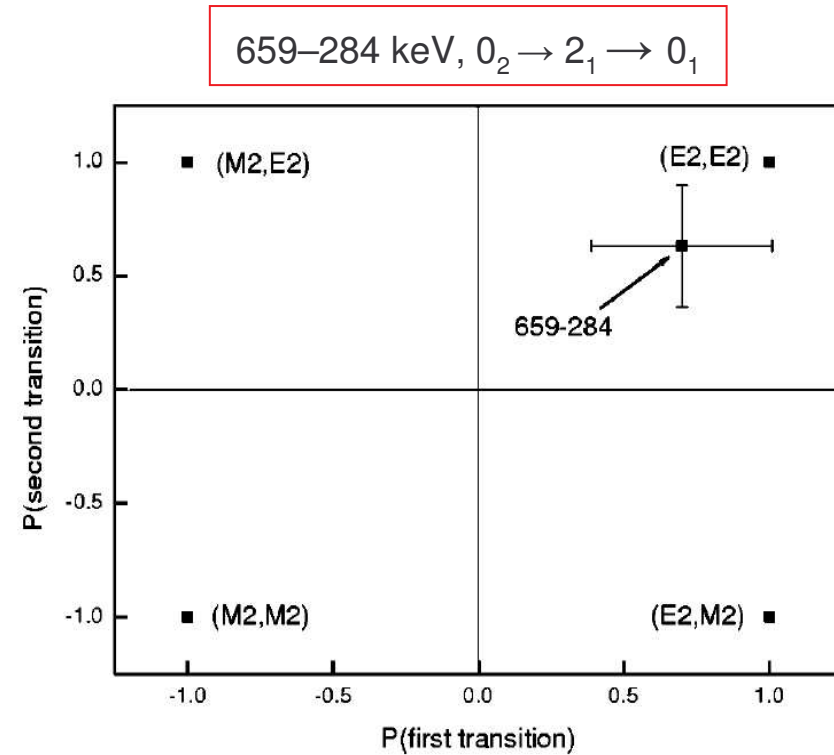
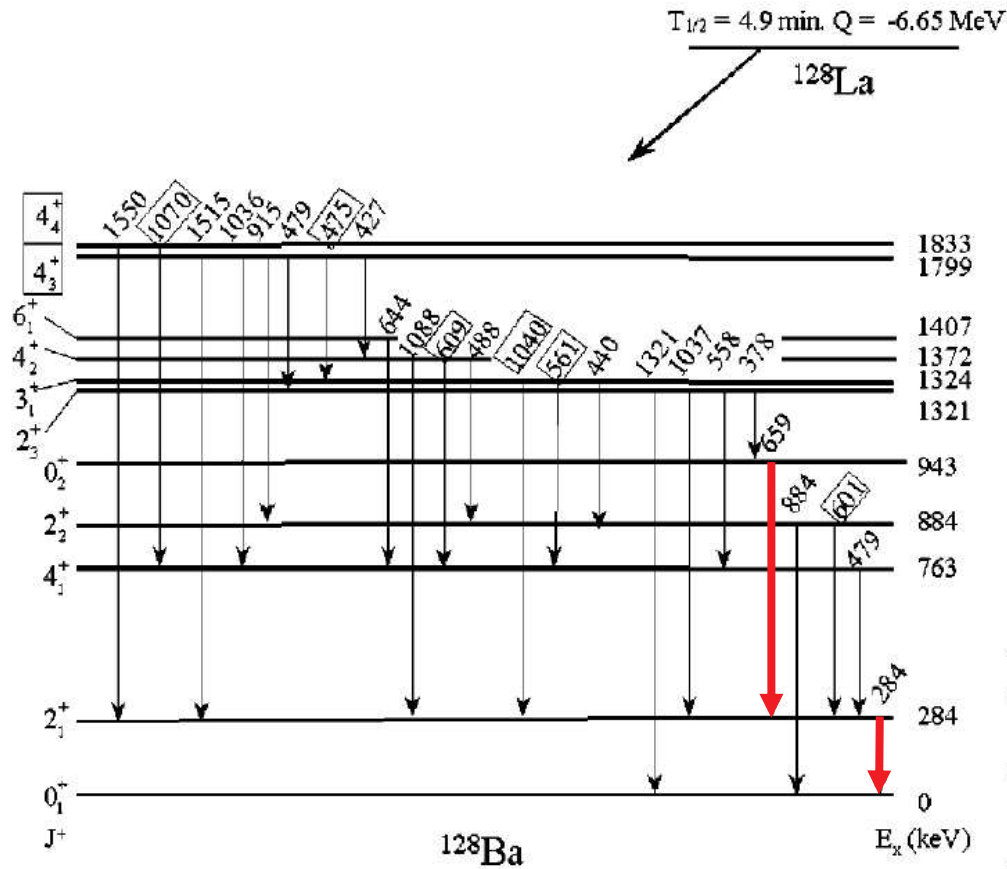
$$A = (N_{\perp} - N_{\parallel}) / (N_{\perp} + N_{\parallel})$$

Polarization sensitivity

$$Q$$



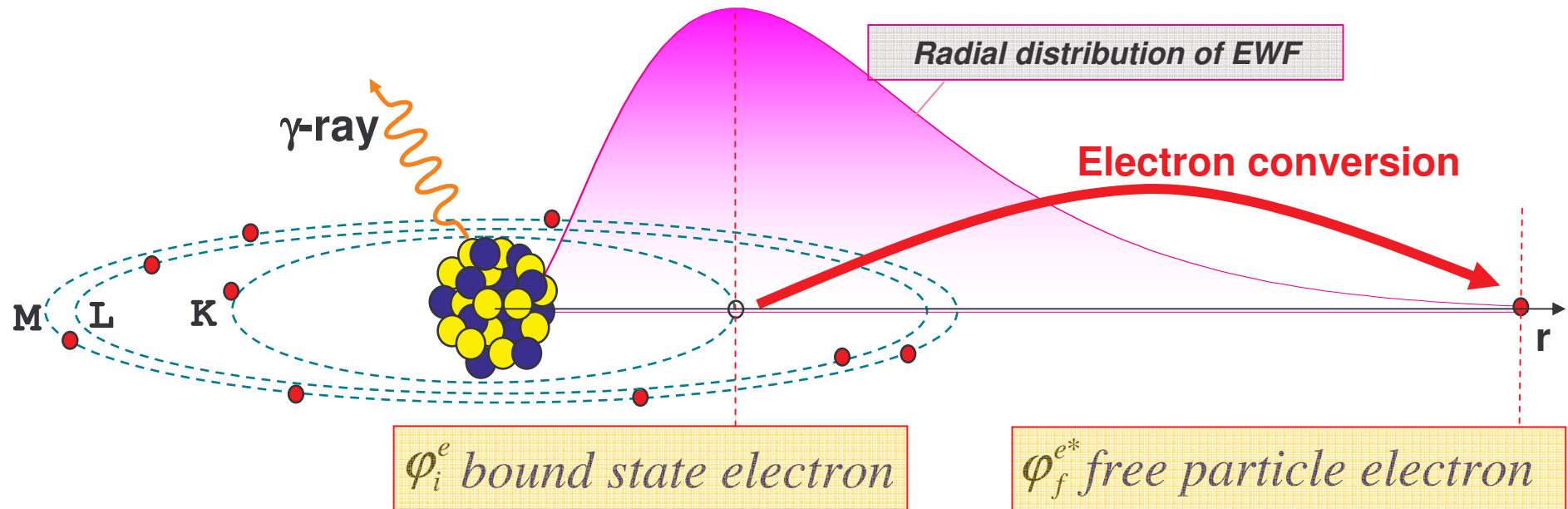
Gamma-ray linear polarization - example



A. Wolf, et al. *Phys. Rev. C* **66**, 024323 (2002)



Conversion electrons (CE)



Energetics of CE-decay ($i=K, L, M, \dots$)

$$E_i = E_f + E_{ce,i} + E_{BE,i} + \cancel{T_\gamma}$$

γ - and CE-decays are independent; transition probability ($\lambda \sim$ Intensity)

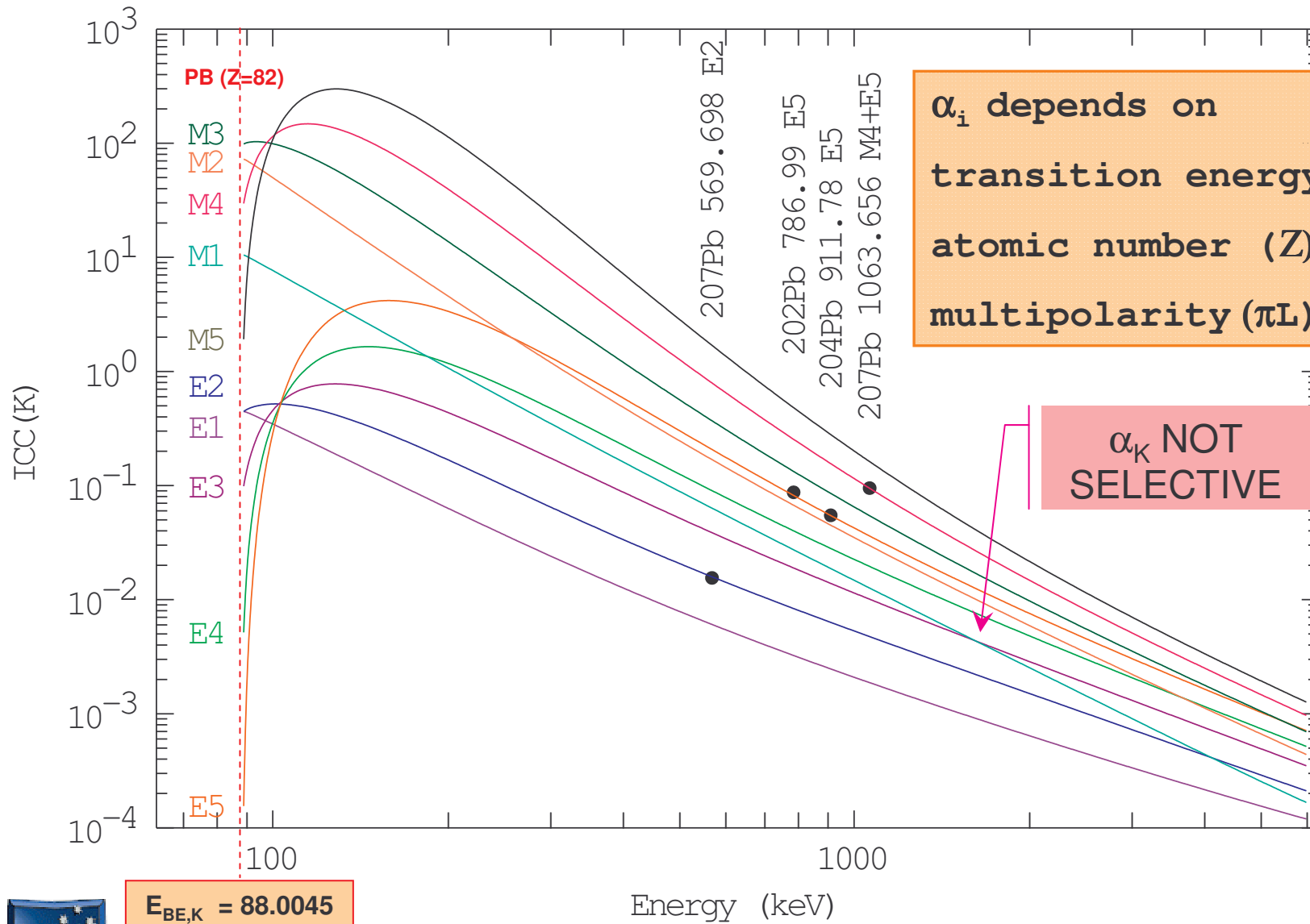
$$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \dots$$

Conversion coefficient

$$\alpha_i = \lambda_{CE,i} / \lambda_\gamma$$



Sensitivity to multipolarity



The physics of conversion coefficients

$$\alpha_K \equiv \frac{\lambda_{e,K}}{\lambda_\gamma}$$

$$\lambda_e = \frac{2\pi}{\hbar} |m_{fi}|^2 \frac{d\rho}{dE} \quad \text{Fermi's golden rule}$$

Density of final electron state
(continuum)

$$m_{fi} = \psi_f^{N*} \varphi_f^{e*} F_{l,m} \psi_i^N \varphi_i^e$$

Nuclear

Electron

Multipolar source

Same for γ and CE

φ_i^e bound state electron
 φ_f^{e*} free particle electron



Theoretical Conversion Coefficients

Current tabulations:

- **Hager and Seltzer** (1968)

Relativistic Hartree-Fock-Slater, WITH Hole, NO dynamic effect

Z=30-103; K, L, M only; limited energy range

- **Rösel-Fries-Pauli** (1978)

Relativistic Hartree-Fock-Slater, NO Hole, NO dynamic effect

Z=30-104; All shells; wider energy range

- **Band-Trzhaskovskaya** (1978)

Relativistic Hartree-Fock-Slater, WITH Hole, WITH dynamic effect

Z=10-104; K, L, M; wider energy range

- **Band-Trzhaskovskaya-Nestor-Tikkanen-Raman** (2002)

Relativistic Dirac-Fock, NO Hole, WITH dynamic effect

Z=10-126; ALL shells; wider energy range

- **Brlcc** (2005)

Relativistic Dirac-Fock, With Hole, WITH dynamic effect

Z=10-95; ALL shells; improved accuracy



Higher order and atomic effects

- **Atomic many body correlations: factor ~ 2 for $E_{\text{kin}}(\text{ce}) < 1 \text{ keV}$ (BrIcc single particle approximation)**
- **Partially filled valence shell: non-spherical atomic field**
- **Shake effect: increases ICC**
- **Resonance Internal conversion: $E_{\text{kin}}(\text{ce}) \approx \text{BE}$**
- **Binding energy unc.: $< 0.5\%$ for $E_{\text{kin}}(\text{ce}) > 10 \text{ keV}$**
- **Chemical effects: $\ll 1\%$**
- **Penetration:
 $n s_{1/2}$ shells (K, L1, M1,..); M1, M2, M3.. multipolarities
For M1 transition:
0.01% (Z=10)
~15% (Z=112)**



Mixed multipolarity and E0 transitions

$$\delta^2 = \frac{I_\gamma(E2)}{I_\gamma(M1)}$$

$$\alpha^{M1/E2} = \frac{\alpha_{M1} + \delta^2 \alpha_{E2}}{1 + \delta^2}$$

In some cases the mixing ratio can be deduced

$$\delta^2 = \frac{\alpha_{M1} - \alpha^{\text{exp}}}{\alpha^{\text{exp}} - \alpha_{E2}}$$

E0 transitions – pure penetration effect; no γ -rays ($I_\gamma=0$)

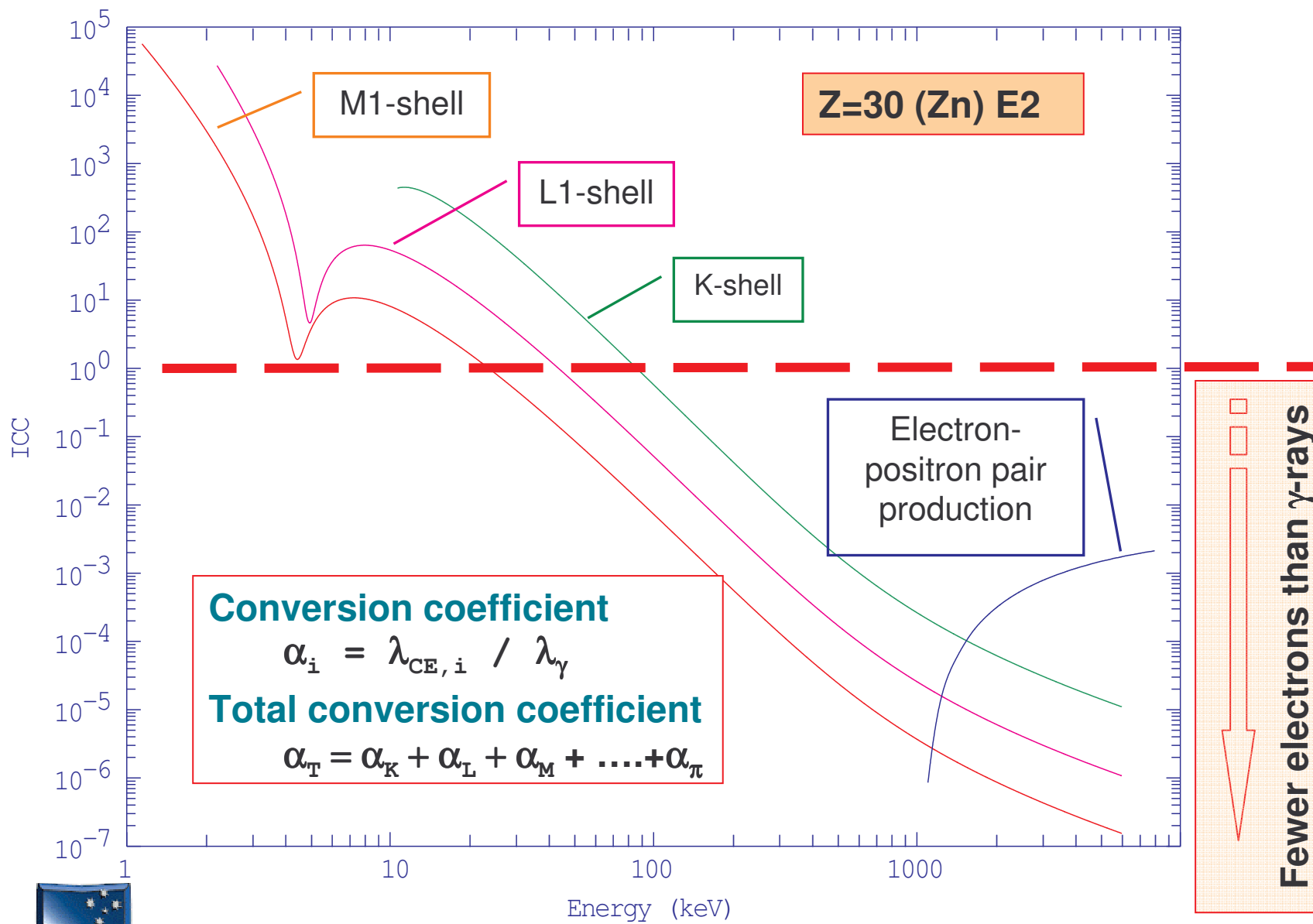
$$\alpha = \frac{I_{CE}}{I_\gamma} = \infty$$

- Pure E0 transition: $0^+ \rightarrow 0^+$ or $0^- \rightarrow 0^-$
- $J \rightarrow J$ ($J \neq 0$) transitions can be mixed E0+E2+M1

$$\alpha = \frac{I_{CE}(E0) + I_{CE}(E2) + I_{CE}(M1)}{I_\gamma(E2) + I_\gamma(M1)}$$



More on conversion coefficients



Measuring conversion coefficients – methods

- NPG: normalization of relative CE ($I_{CE,i}$) and γ (I_γ) intensities via intensities of one (or more) transition with known α

$$\alpha_i = \frac{I_{CE,i}}{I_\gamma} \times \left[\frac{I_\gamma^*}{I_{CE}^*} \times \alpha^* \right]_{KNOWN}$$

- CEL: Coulomb excitation and lifetime measurement

$$\alpha_T = \frac{2.829 \times 10^{11} \times E_\gamma^{-5} (keV)}{B(E2) \uparrow (e^2 b^2) \times T_{1/2} (ns)} - 1$$

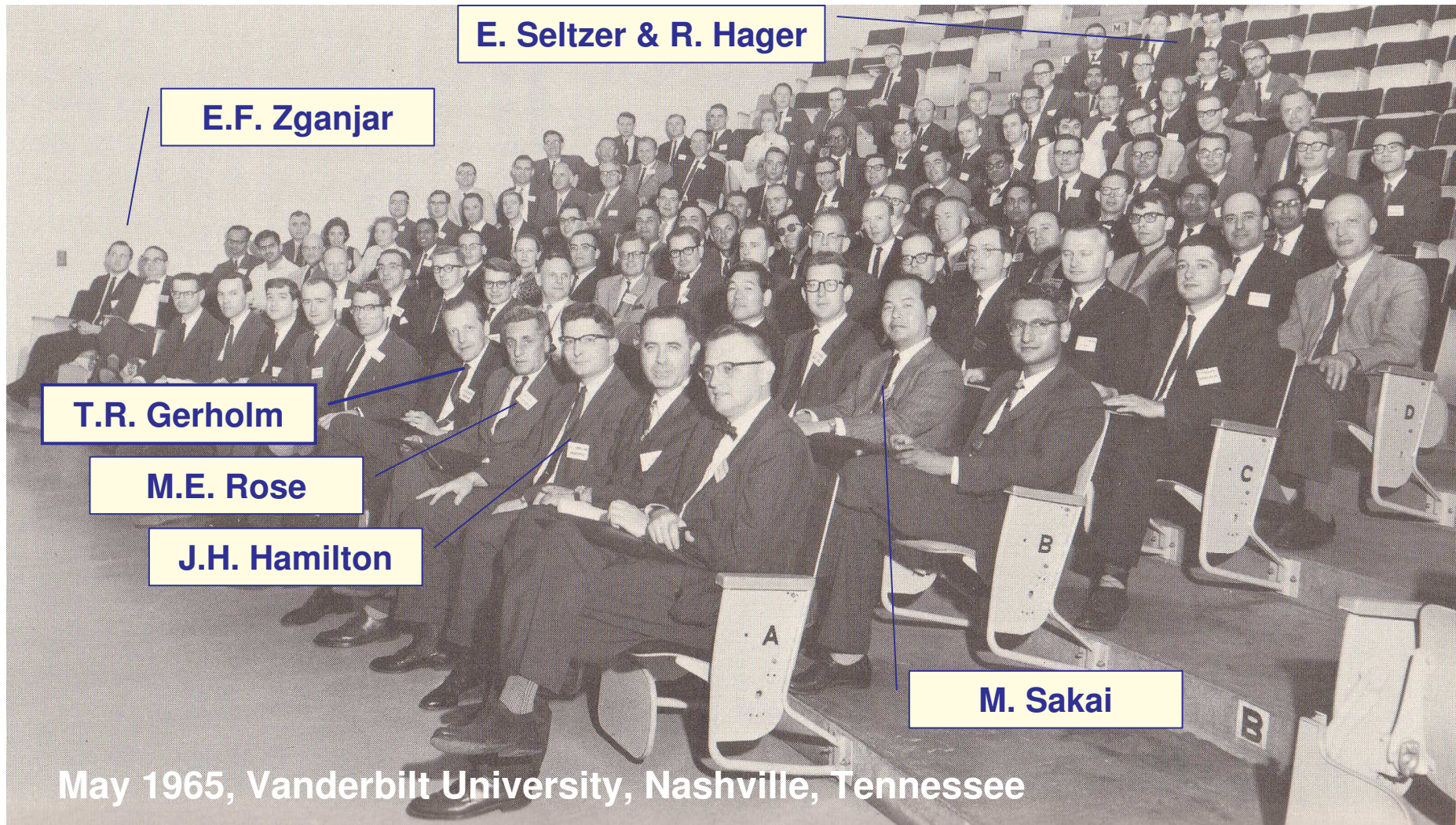
- XPG: intensity ratio of K X-rays to γ -rays with K-fluorescent yield, ω_K

$$\alpha_K = \frac{I_{KX}}{I_\gamma} \times \frac{1}{\omega_K}$$

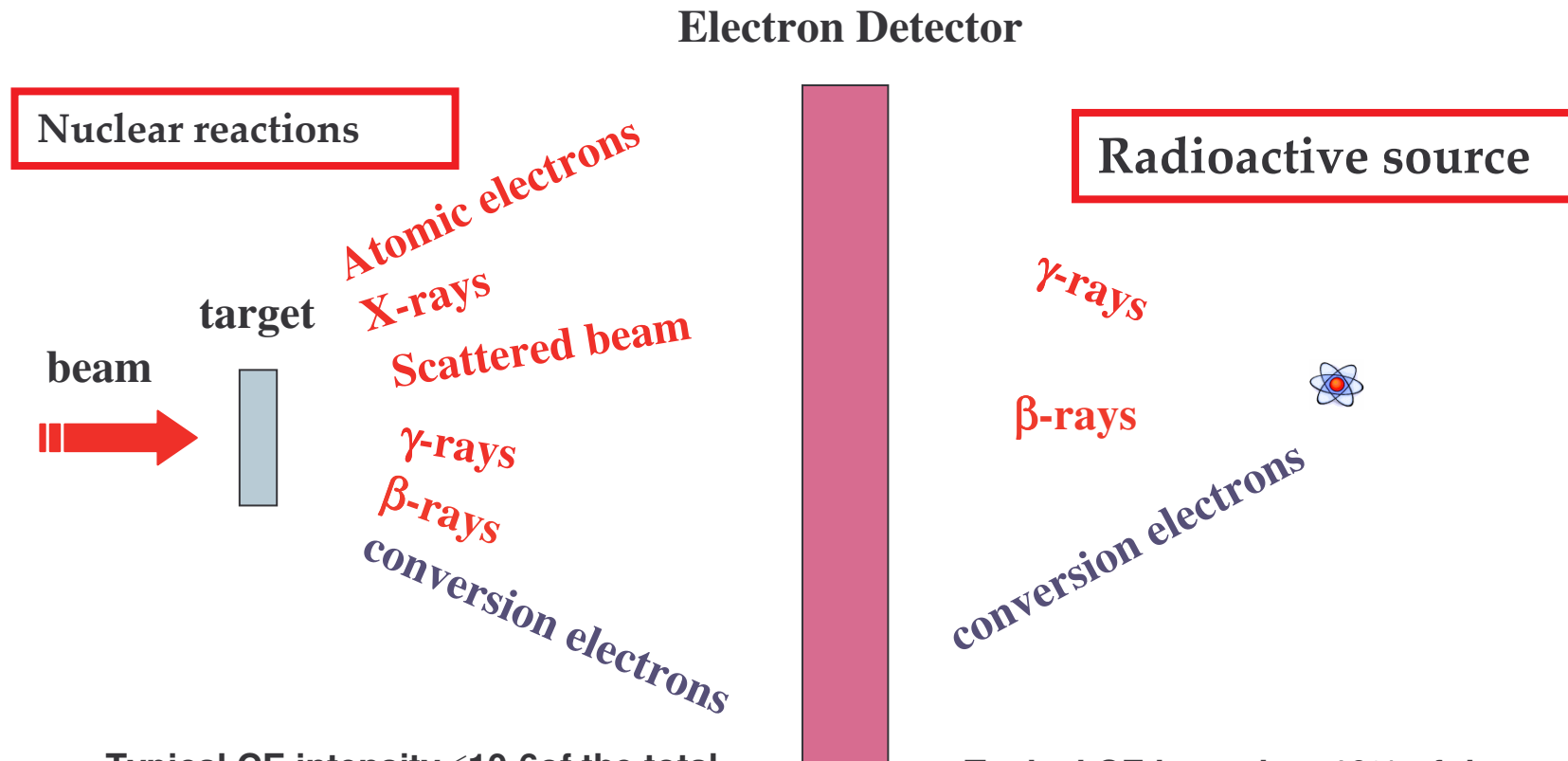
And many more, see Hamilton's book



Internal Conversion Process – The Pioneers



Direct ICC measurements

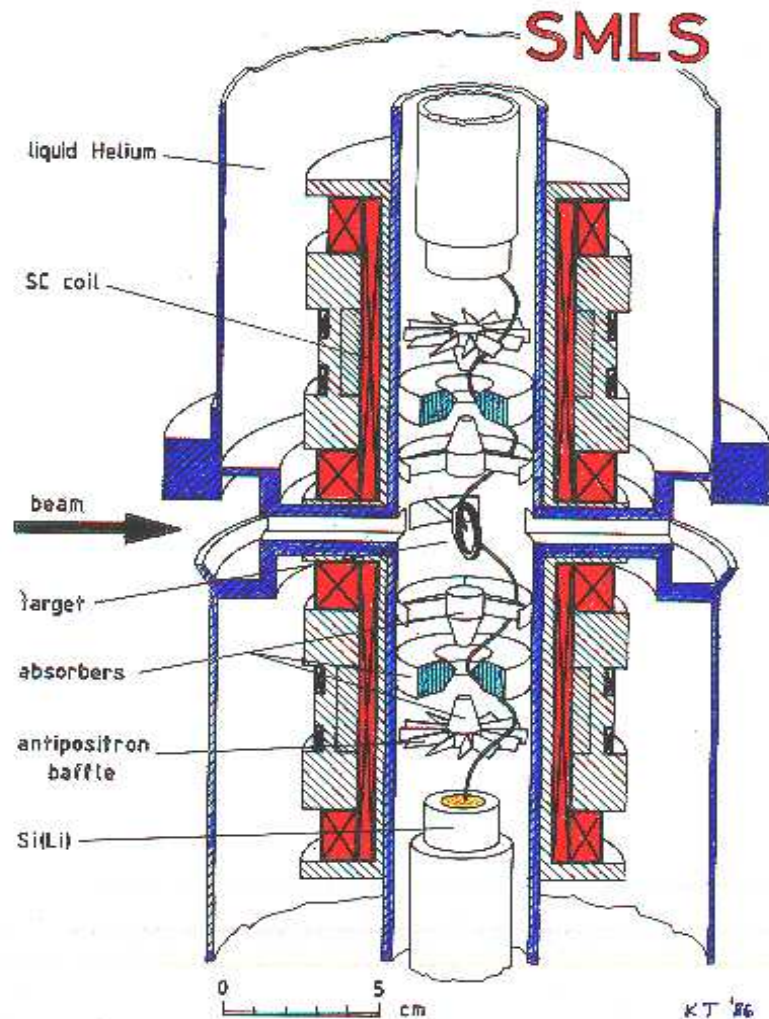


- Typical CE intensity $\leq 10^{-6}$ of the total (*scattered beam excluded*)
- Energy resolution affected by absorption in target; FWHM for a Si(Li): 3-5 keV
- Angular distribution of conversion electrons – usually neglected(?)
- Typical $\Delta\alpha/\alpha \sim 10-20\%$

- Typical CE intensity $\sim 10\%$ of the total
- Thin source, better energy resolution; FWHM for a Si(Li): 1.6-2.5 keV
- No/attenuated angular distribution
- Typical $\Delta\alpha/\alpha \sim 5-10\%$



Basic electron transporters

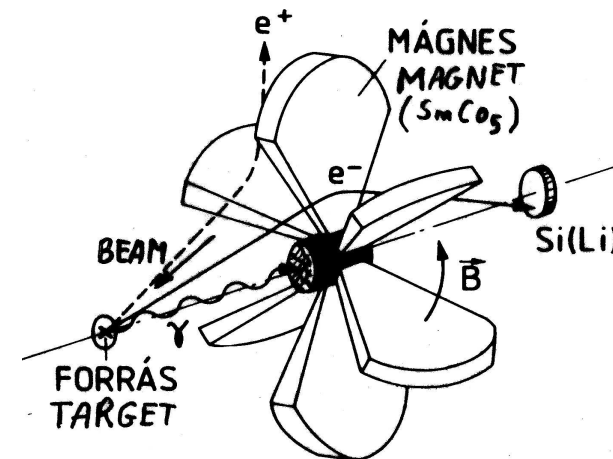


Superconducting solenoid

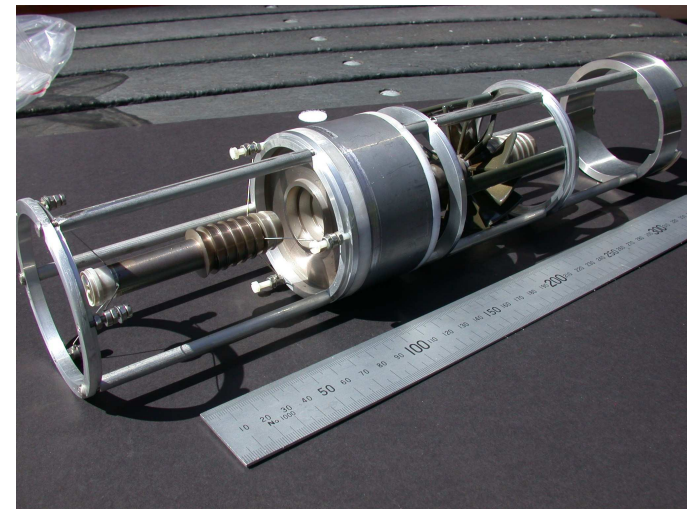
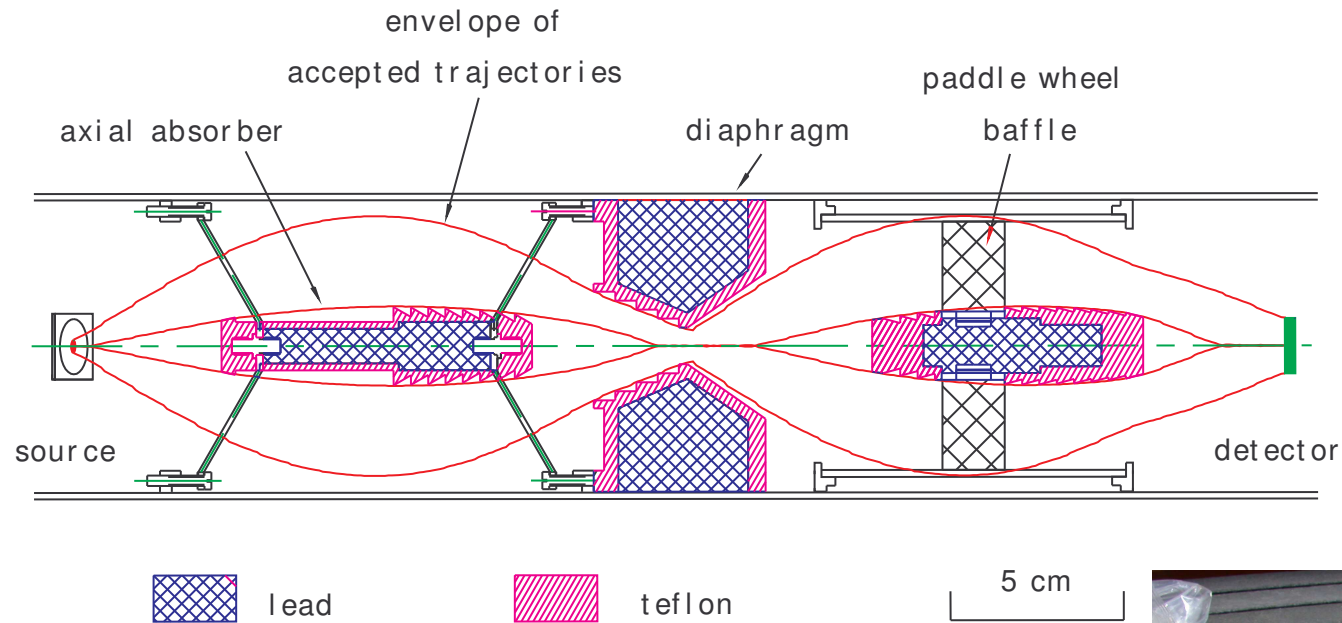
- ❑ Broad-range mode – 100 keV up to a few MeV
- ❑ Lens mode – finite transmitted momentum bandwidth ($\Delta p/p \sim 15-25\%$) – high peak-to-total ratio

Mini-orange (looks like a peeled orange)

- ❑ transmission $> 20\%$
- ❑ small size and portability, but poorer quality



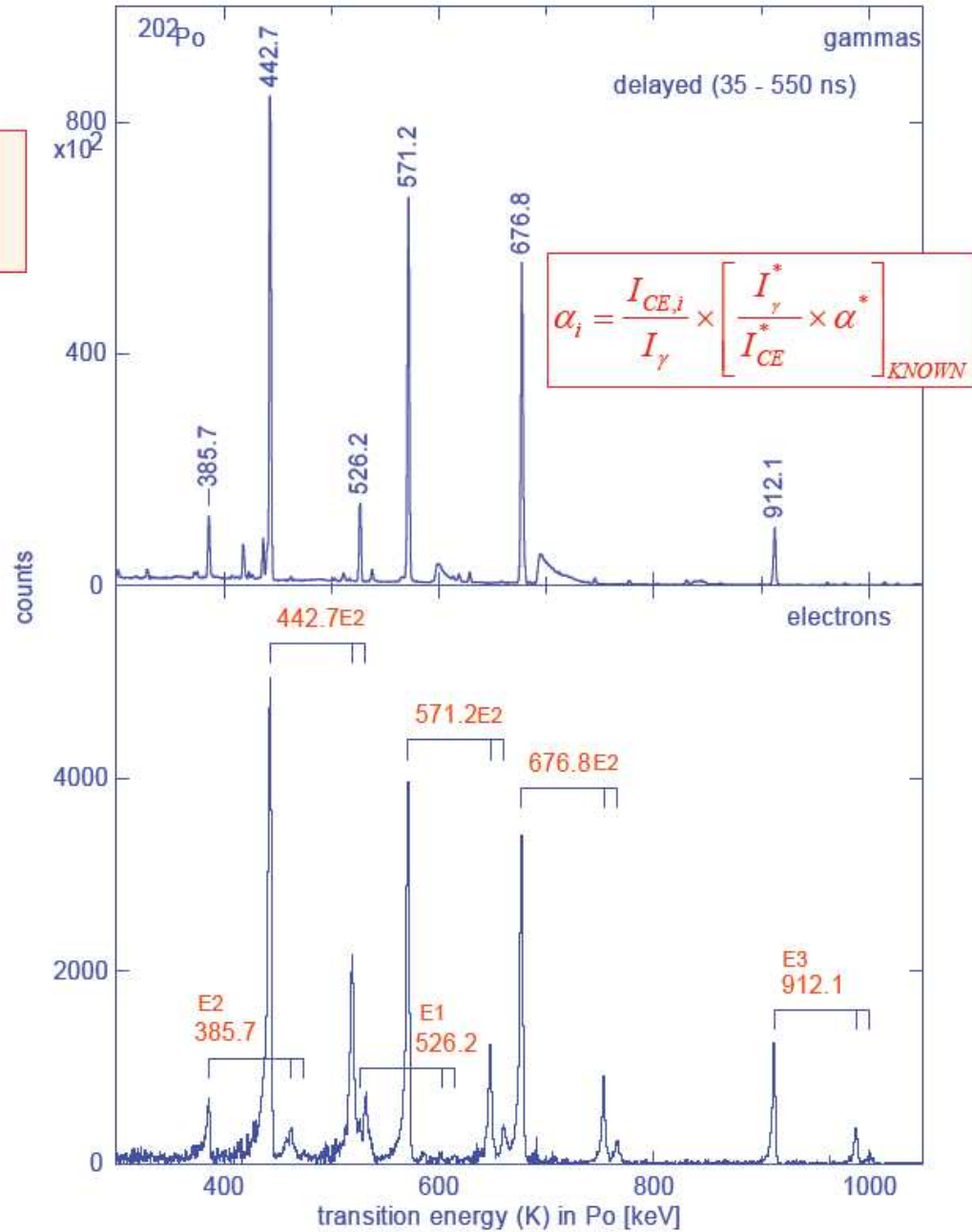
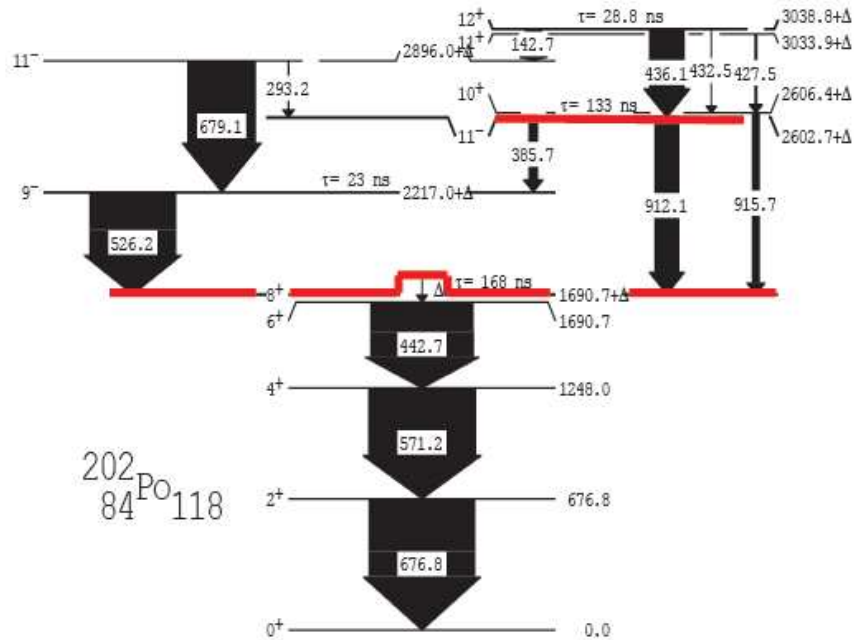
ANU Superconducting Solenoid Spectrometer



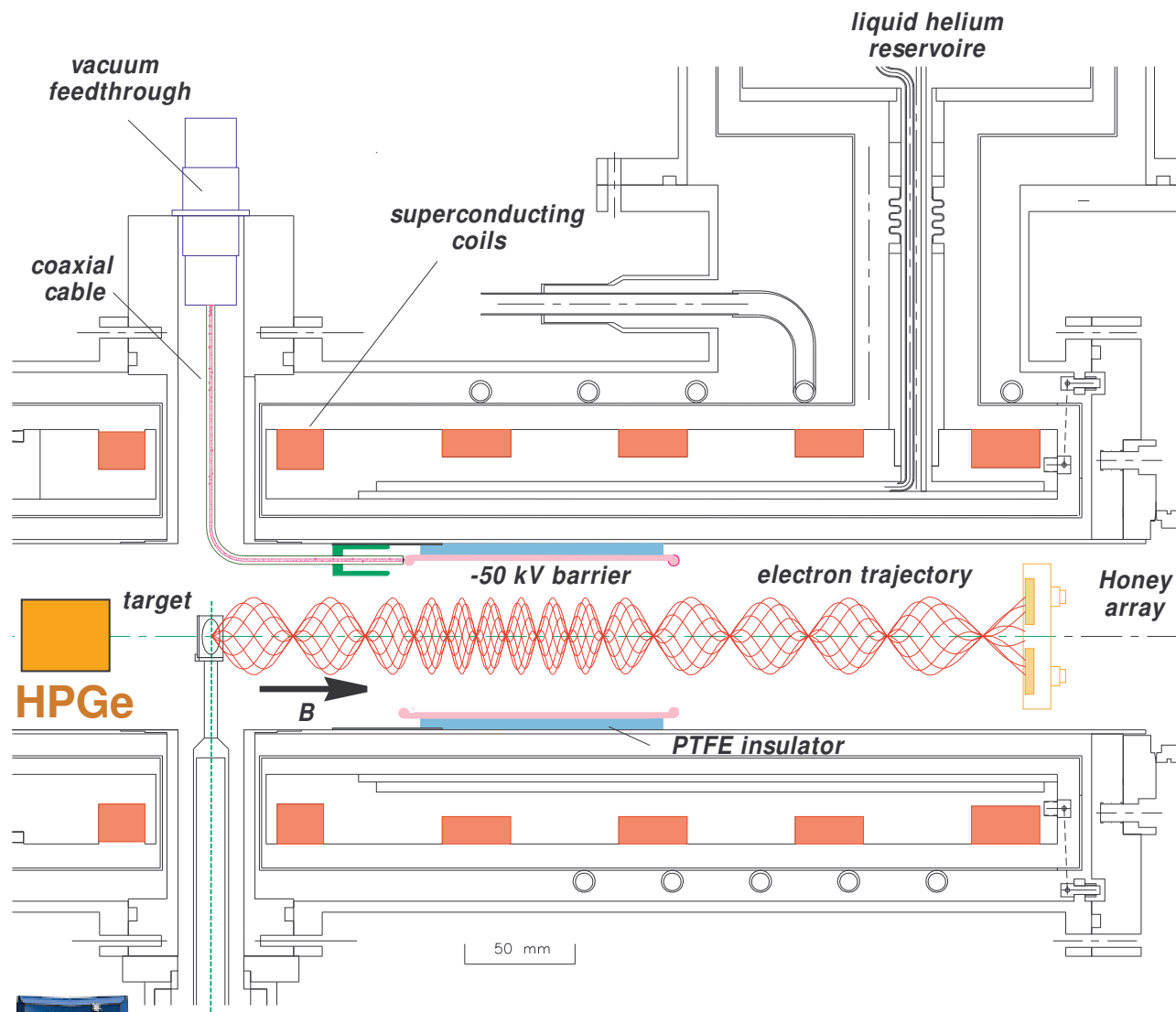
Super-e Lens (ANU)

$^{194}\text{Pt}(^{12}\text{C},4n)^{202}\text{Po}$ @ 76 MeV

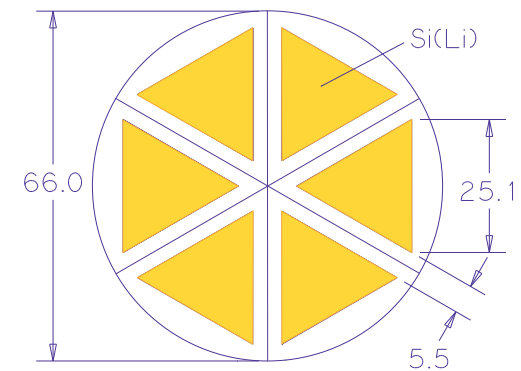
Pulsed beams (~1 ns) with 1.7 μs separation



Super-e Honey (ANU)



Electrons from atomic collisions are the major difficulty in low energy CE spectroscopy using ion induced reactions

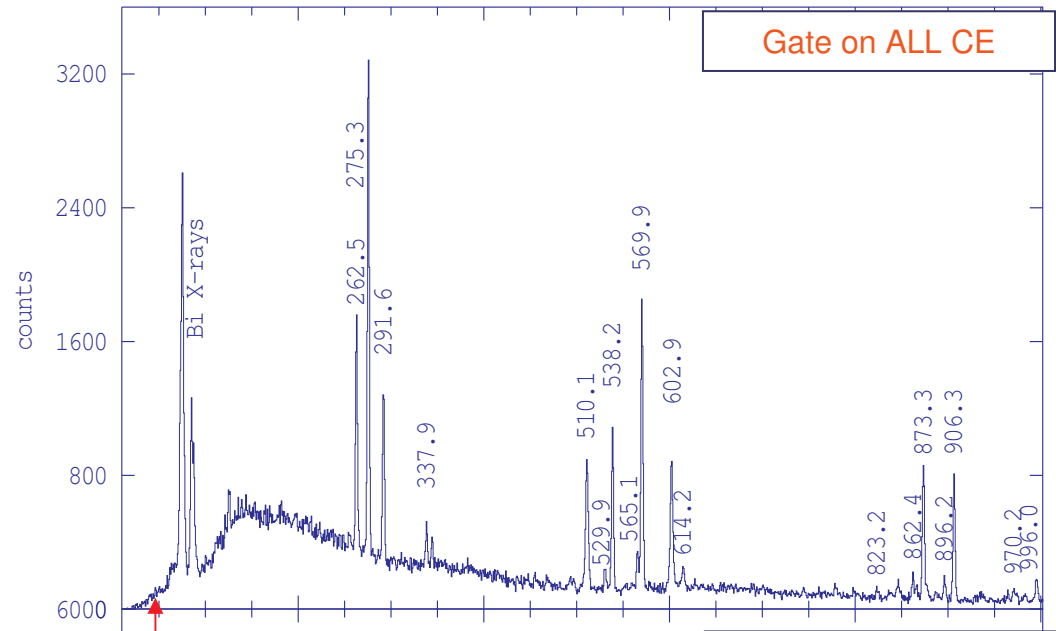


ee γ coincidences



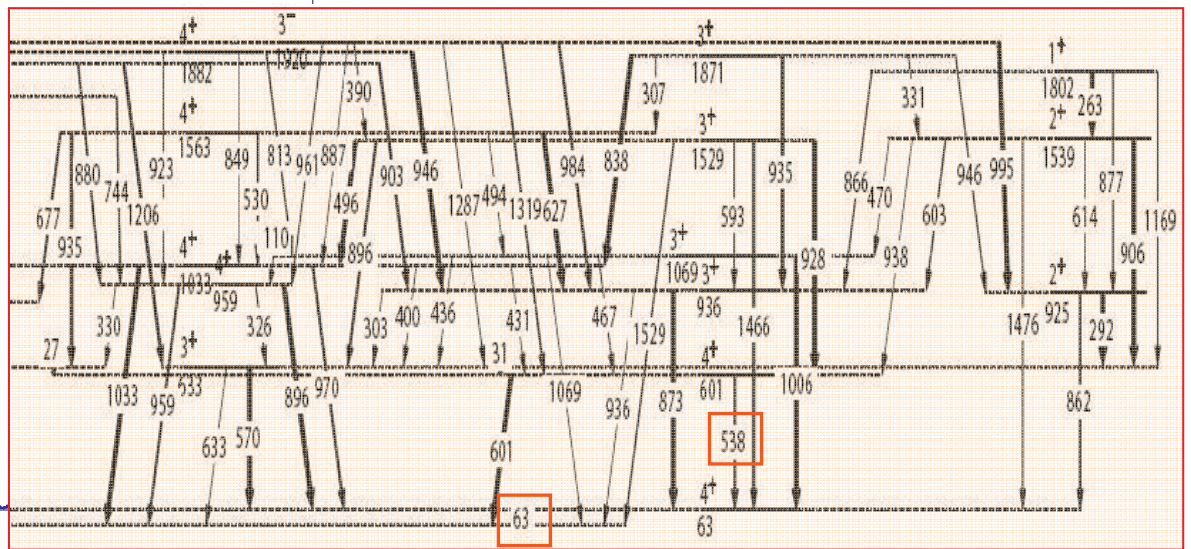
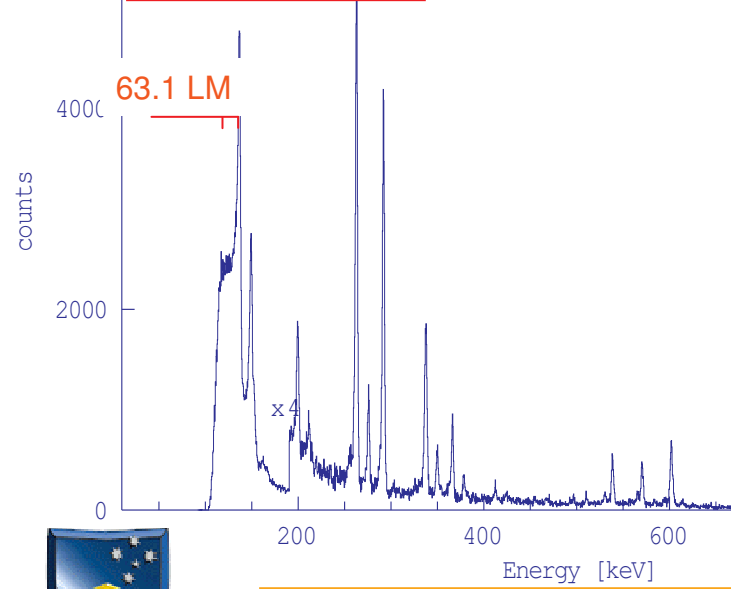
Super-e Honey (ANU) e γ coincidences

$^{208}\text{Pb}(p, n)^{208}\text{Bi}$ @ 9 MeV
 H. Meier et al. (to be published)
 $E_\gamma = 63.1$ keV transition not visible in gamma spectra.
 No K conversion; $BE_K = 90.5$ keV



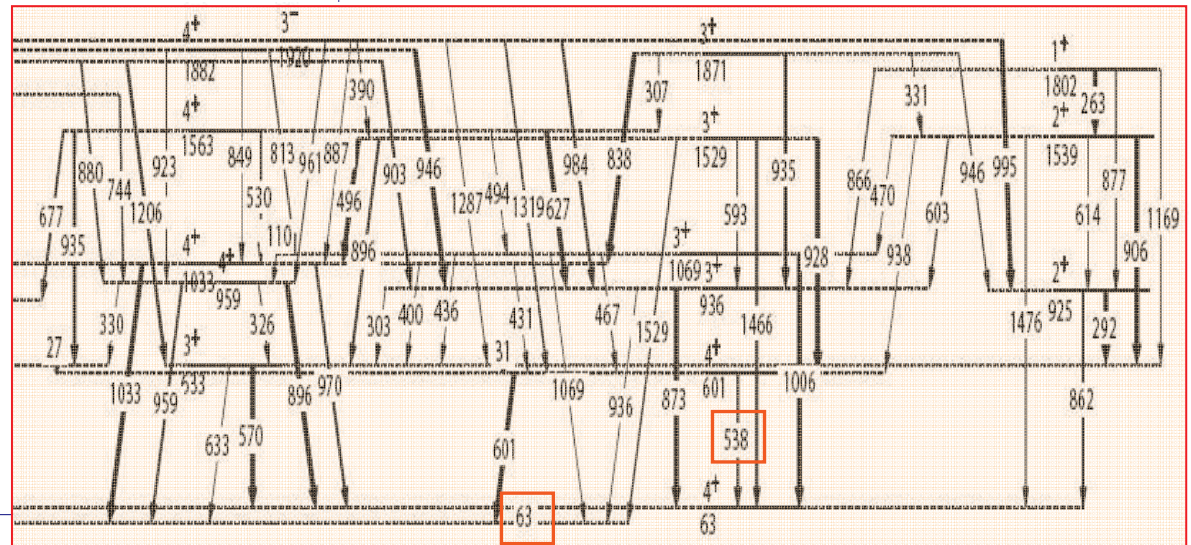
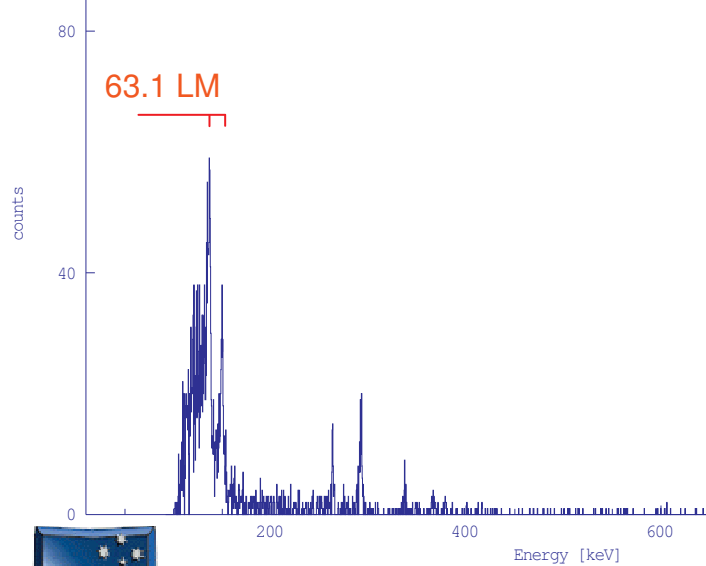
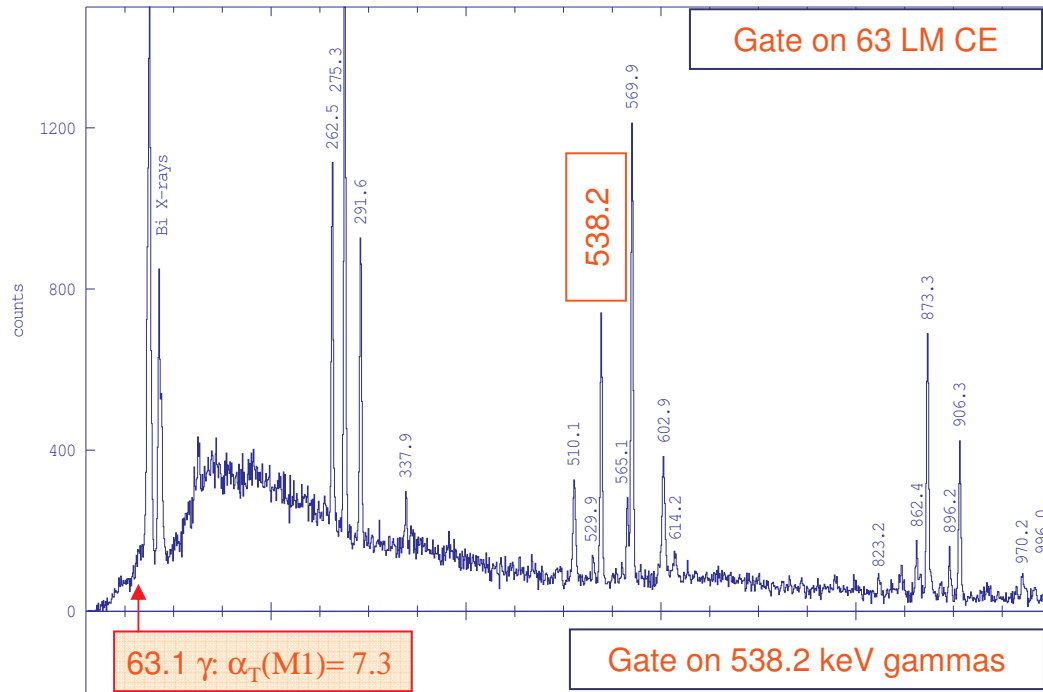
63.1 γ : $\alpha_T(M1) = 7.3$

Gate on ALL gammas



Super-e Honey (ANU) e γ coincidences

$^{208}\text{Pb}(p, n)^{208}\text{Bi}$ @ 9 MeV
 H. Meier et al. (to be published)
 Sample coincidence gates



ICC from total intensity balances – example 1

In out-of-beam (or decay) coincidence data

$$I_{\gamma_{in}}^{tot} = I_{\gamma_{in}} \times (1 + \alpha_{in}^{tot}) \equiv I_{\gamma_{out}}^{tot} = I_{\gamma_{out}} \times (1 + \alpha_{out}^{tot})$$

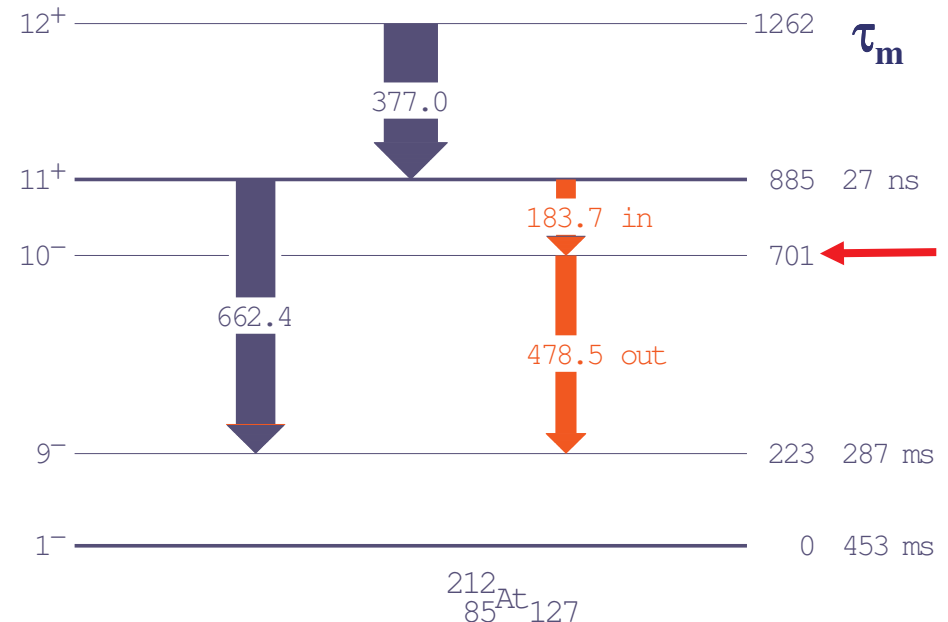
$$\alpha_{in}^{tot} \equiv (I_{\gamma_{out}} / I_{\gamma_{in}}) \times (1 + \alpha_{out}^{tot}) - 1$$

$$\alpha_{out}^{tot}(478.5, M1) = 0.166$$

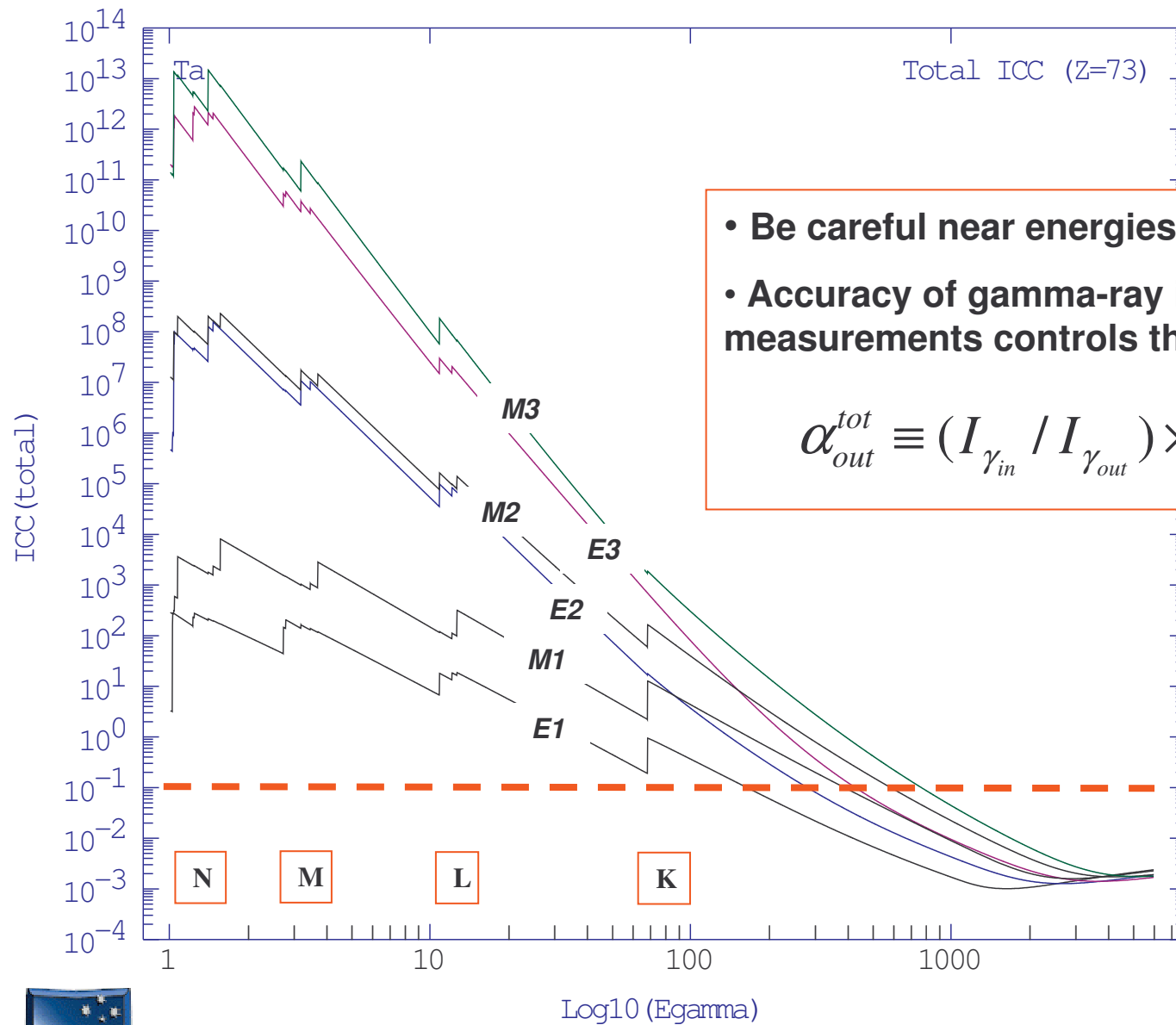
$$\alpha_{in}^{tot}(183.7) = 0.23(5) \quad E1: 0.103 \quad M1: 2.15 \quad E2: 0.674 \quad M2: 10.7$$

E1(+M2)

S. Bayer, et al., Nucl. Phys. A650, 3 (1999)



ICC from total intensity balances – when to use



- Be careful near energies close to shell binding
- Accuracy of gamma-ray intensity measurements controls the range of its use

$$\alpha_{out}^{tot} \equiv (I_{\gamma_{in}} / I_{\gamma_{out}}) \times (1 + \alpha_{in}^{tot}) - 1$$

$$\delta I_{\gamma} / I_{\gamma} \sim 10\%$$

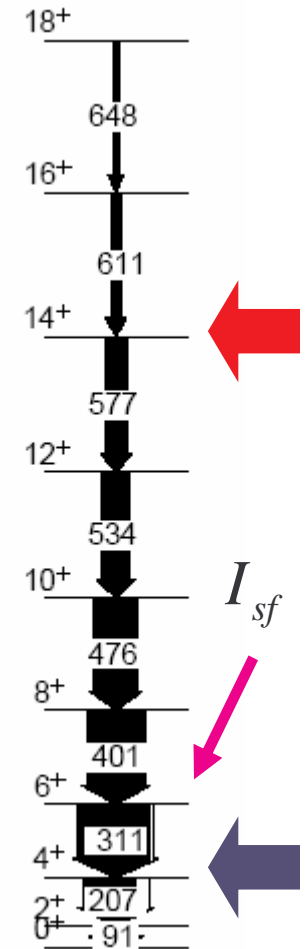
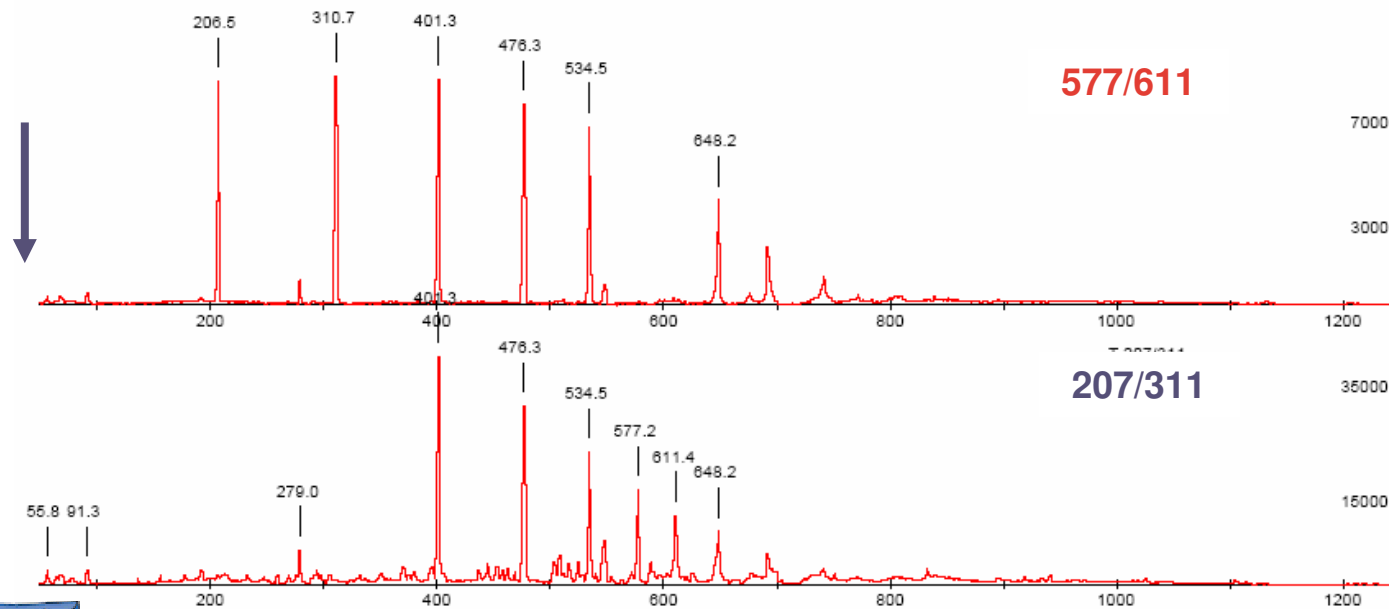


ICC from total intensity balances – example 2

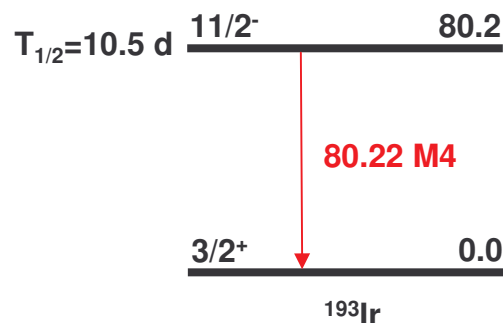
In-beam: only when gating from “above”

$$I_{\gamma_{in}}^{tot} = I_{\gamma_{in}} \times (1 + \alpha_{in}^{tot}) \equiv I_{\gamma_{out}}^{tot} = I_{\gamma_{out}} \times (1 + \alpha_{out}^{tot})$$

$$I_{\gamma_{out}}^{tot} = I_{\gamma_{in}}^{tot} + I_{sf}$$



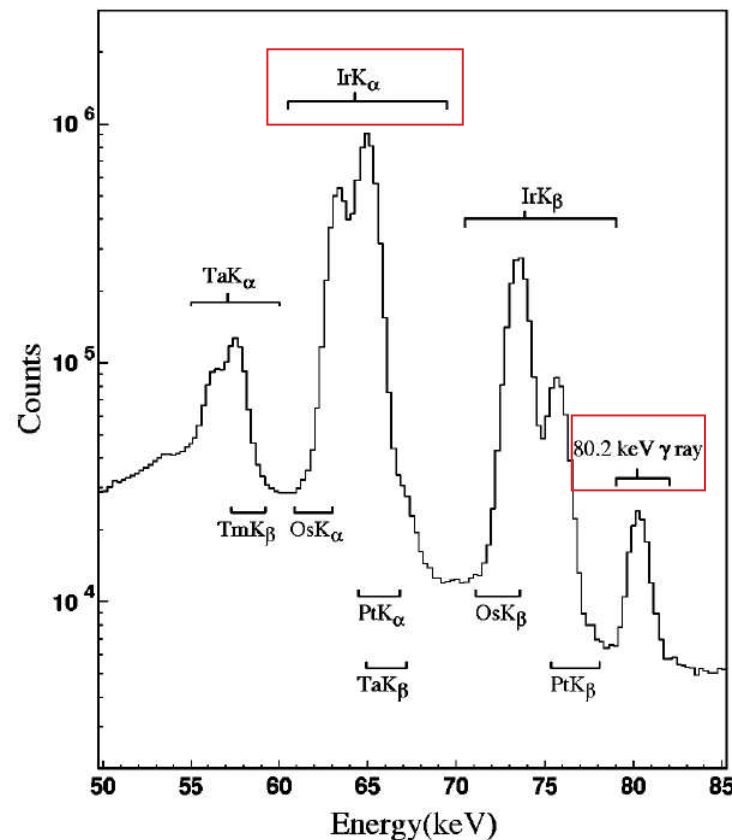
ICC from intensity ratio of K X-rays to γ -rays - ^{193m}Ir



$$\alpha_K = \frac{N_{KX}}{N_\gamma} \times \frac{\epsilon_\gamma}{\epsilon_{KX}} \times \frac{1}{\omega_K}$$

Looks simple but....

- source preparation (purity)
- efficiency (ϵ) calibration
- coincidence summing
- etc.



N. Nica, et al., Phys. Rev. C 70, 054305 (2004)

Determined: $\alpha_K = 103.8(8)$

Note: $\alpha_T = 21333(373)$



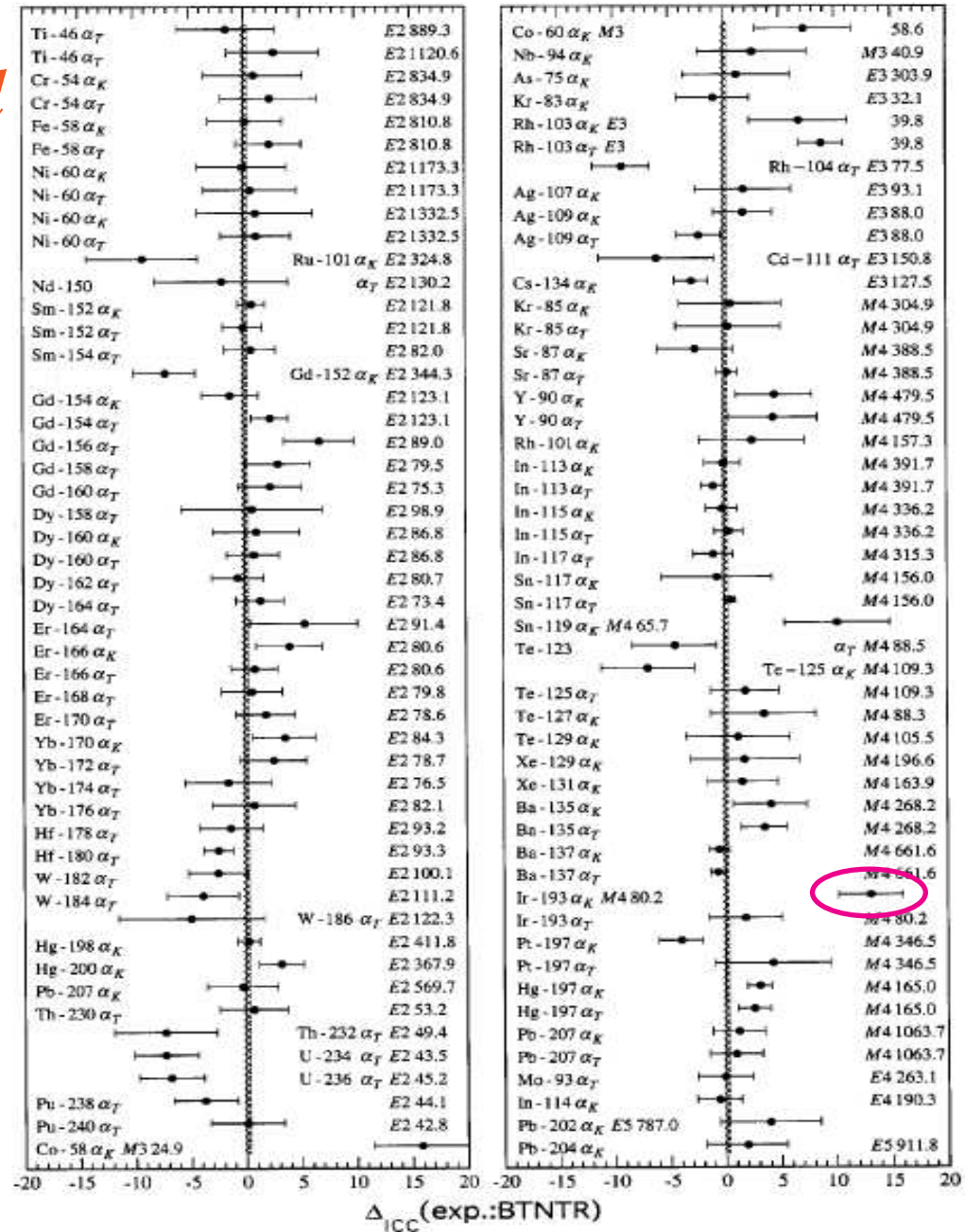
Raman et al. (2002)

“How good are the internal conversion coefficients now?”

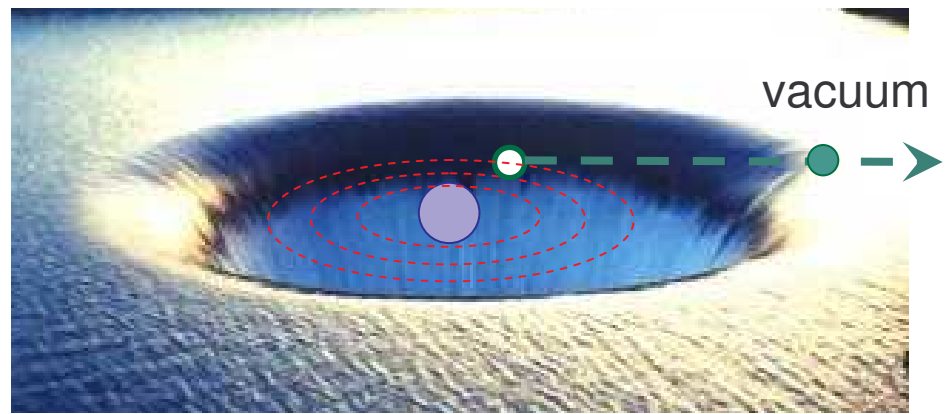
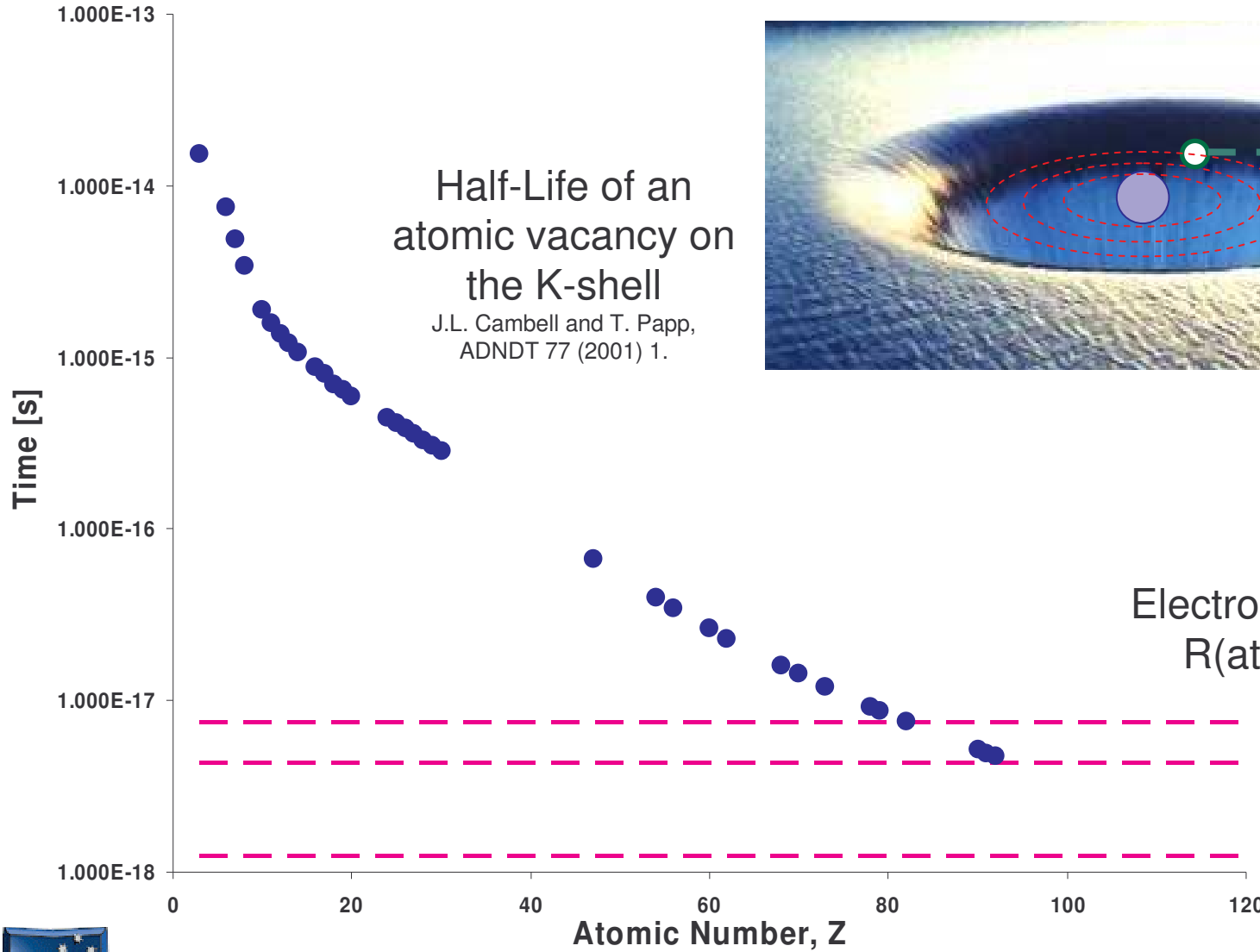
- 100 experimental ICC
- Deviation of ICC

$$\Delta ICC(Exp : Theory) = \frac{[Icc(Exp) - Icc(Theory)]}{Icc(Theory)} \times 100$$

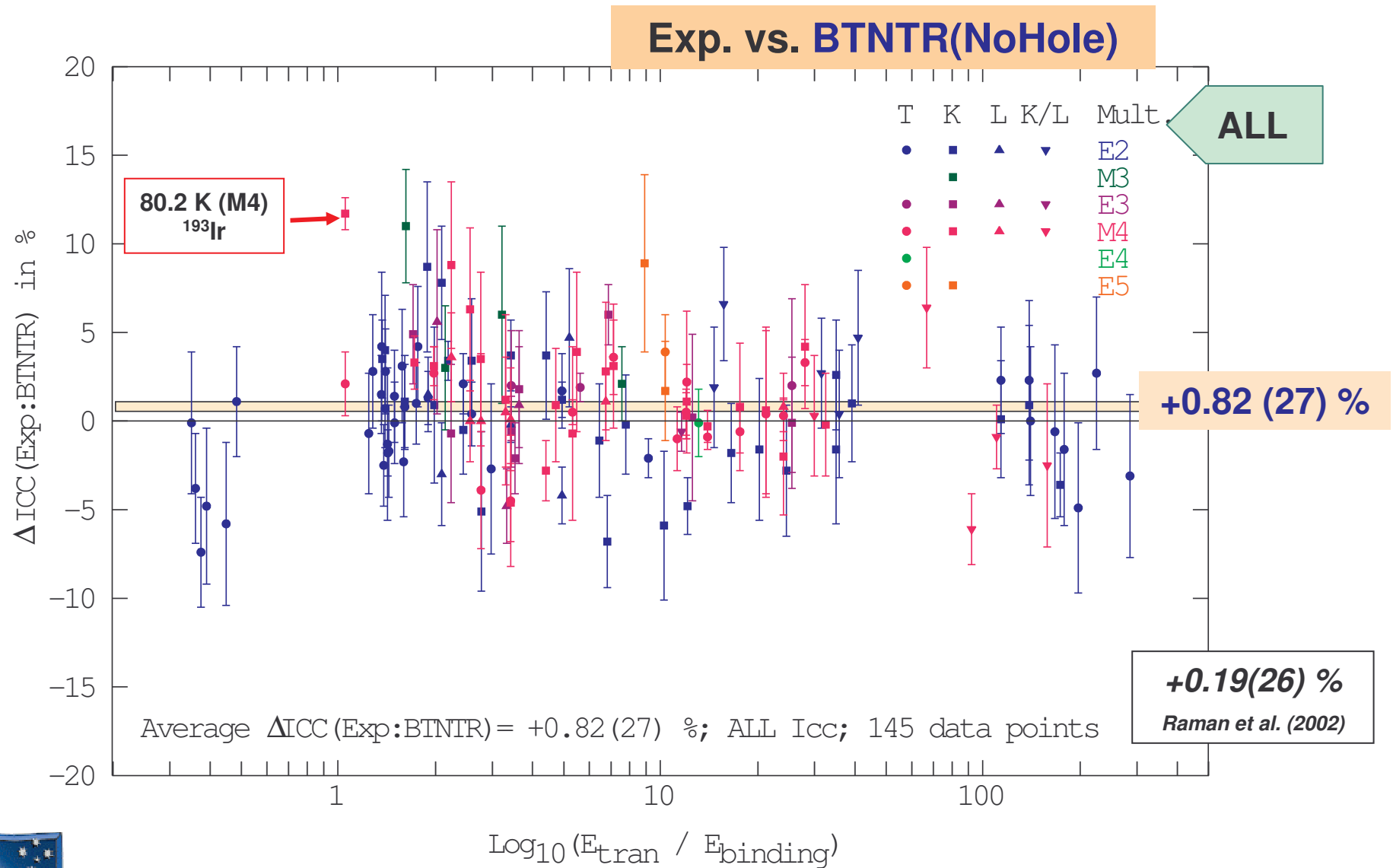
Hager-Seltzer:	-3.01(24)%
Rössel et al:	-2.71(24)%
BTNTR NO Hole	+0.19(26)%
RNIT(1) With Hole	-0.94(24)%
RNIT(2) With Hole	-1.18(24)%



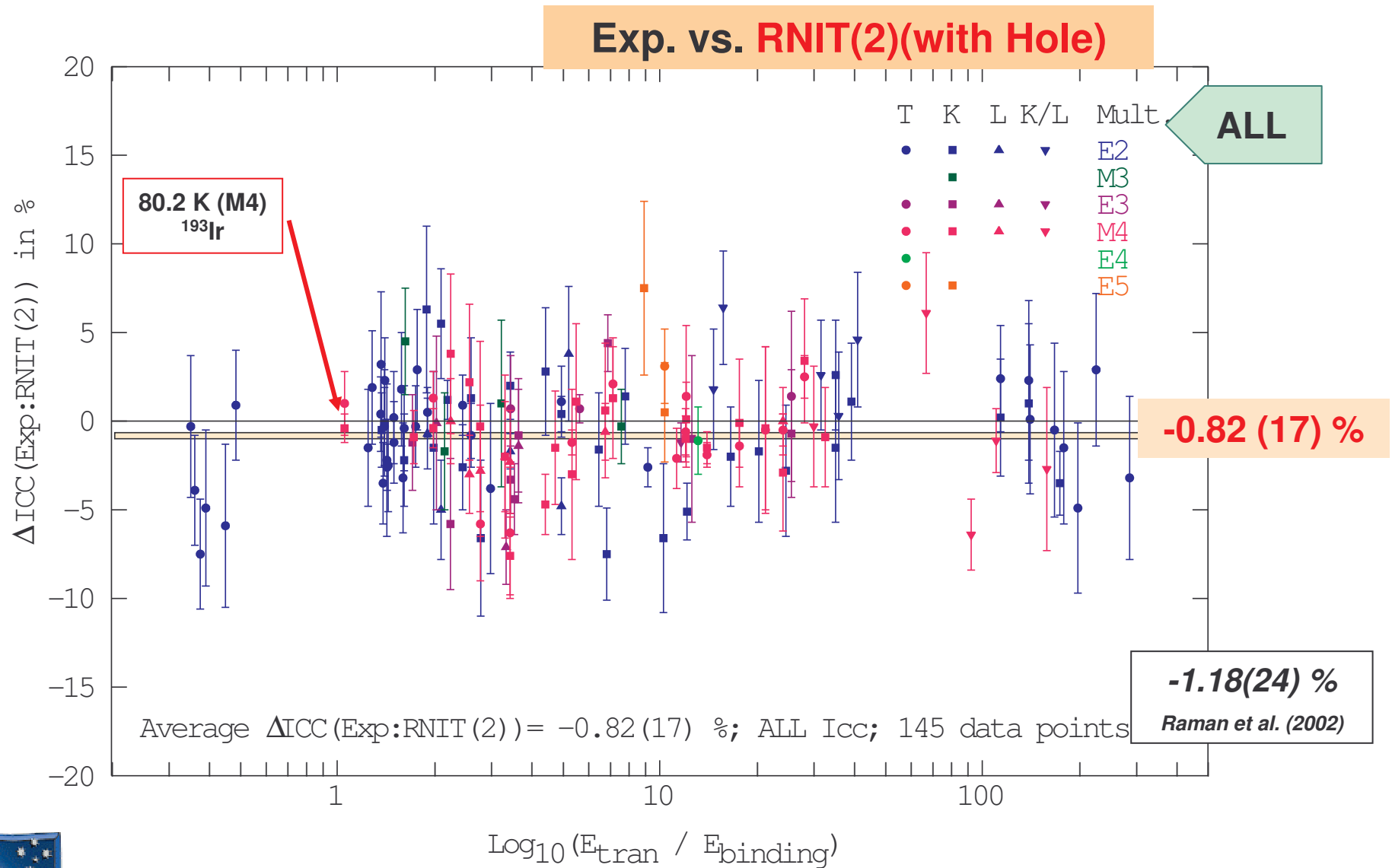
Can we ignore the atomic vacancies?



How good are the internal conversion coefficients now?



How good are the internal conversion coefficients now?



Acknowledgements

G.D. Dracoulis , G.J. Lane, P. Nieminen, H. Maier (ANU)
F.G. Kondev (ANL)
T.W. Burrows (NNDC)
P.E. Garrett (*University of Guelph and TRIUMF*)
S.W. Yates (*Univ. of Kentucky*)
P. Greenlees (University of Jyväskylä)
P.M. Walker (*Surrey*)

