The Abdus Salam
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SCHOOL ON ION BEAM ANALYSIS AND ACCELERATOR APPLICATIONS

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Interaction of particles with matter

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Interaction of particles with matter

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Scope of talk:

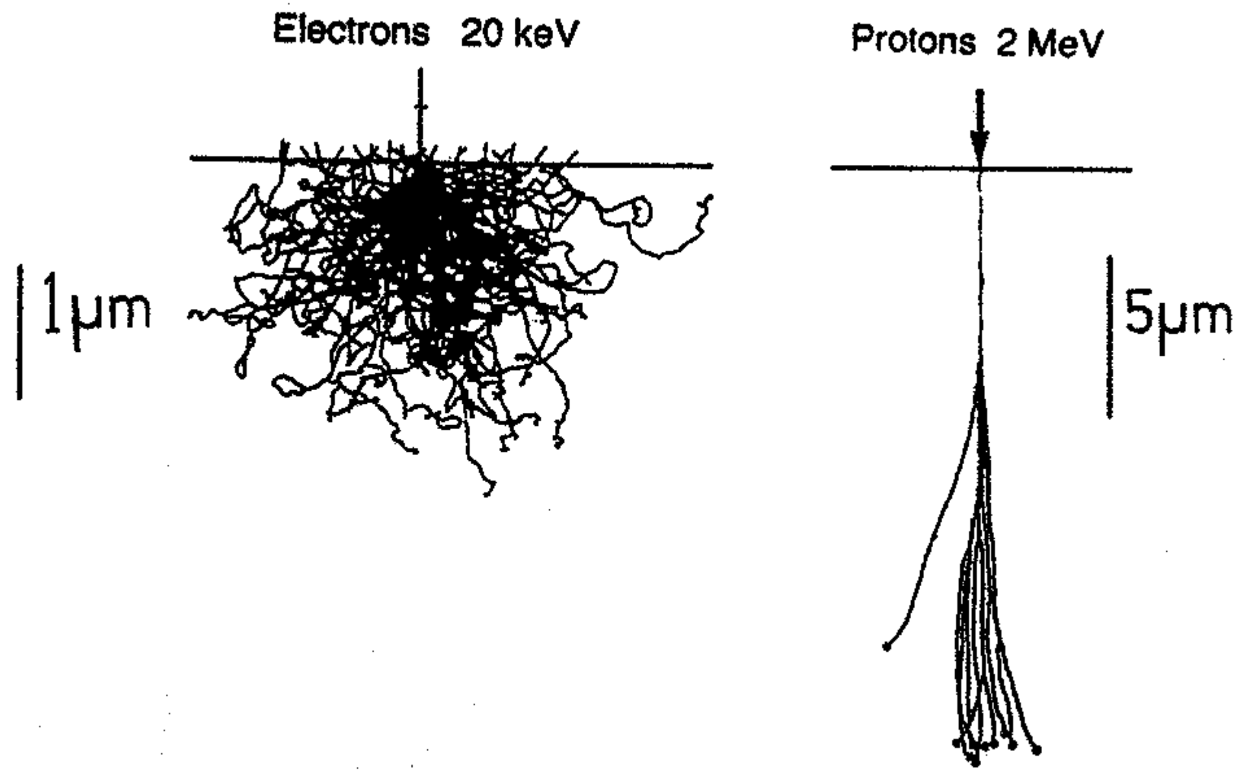
1) Particle penetration through matter (light ions, electrons, basic phenomena: scattering, ionization)

2) Scattering:
   - Coulomb collision
   - Rutherford cross section
   - Multiple scattering

3) Ionization and production of X-rays
   - X-ray lines
   - Ionization models and computer codes
   - Experimental data

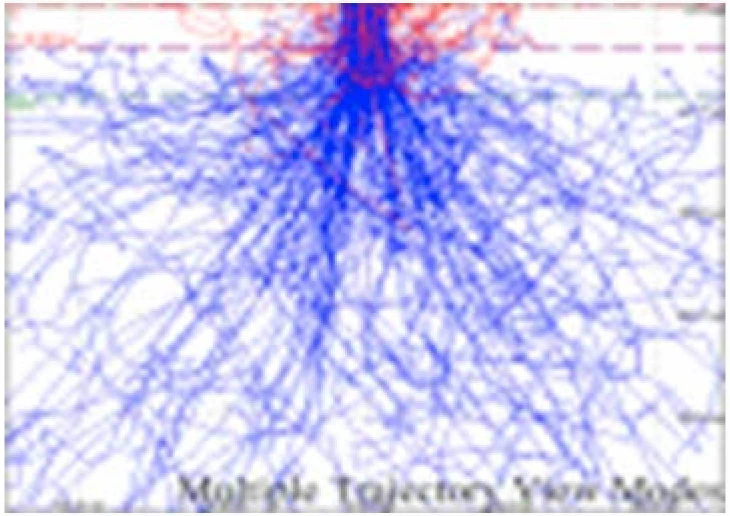
4) Nuclear reactions
5) Biologic aspects, dosimetry
6) Photon-induced charged particles

Particles in matter – trajectories of electrons and protons



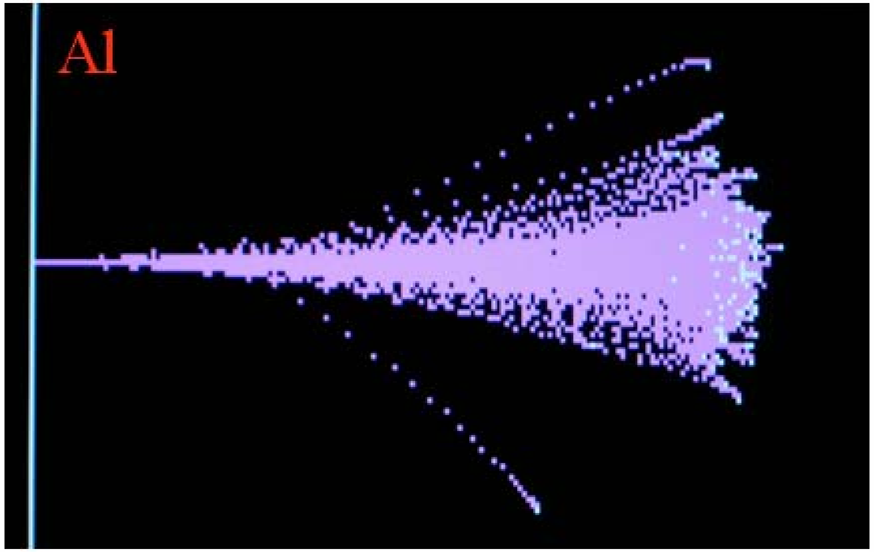
Cu target (Demortier)

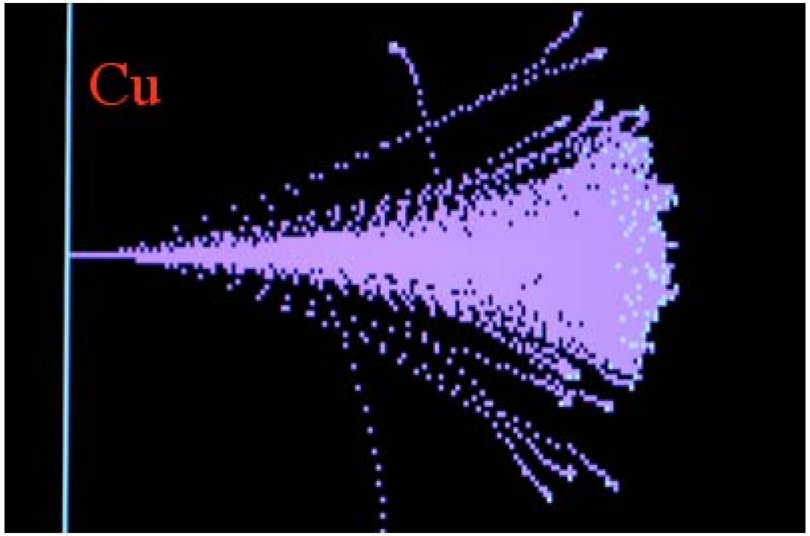
Computer simulations
of particle trajectories



(CASINO)

1 MeV protons (TRIM89)





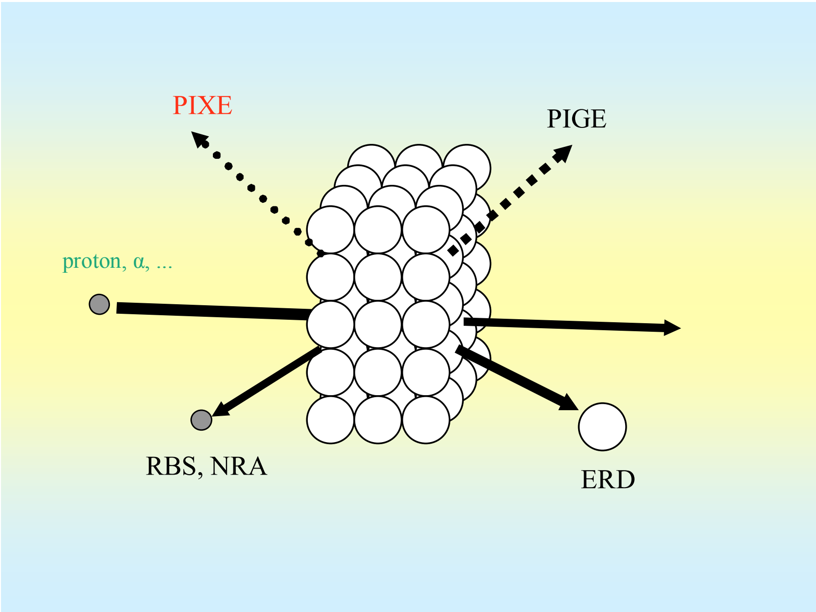
Principal effects in solid targets

1) Scattering

- with nuclei: zig-zag trajectory → lateral spread of the beam,
  backscattering (RBS)
- with electrons: stopping → finite range for ions

2) Ionization → emission of X-rays, Auger electrons (PIXE)

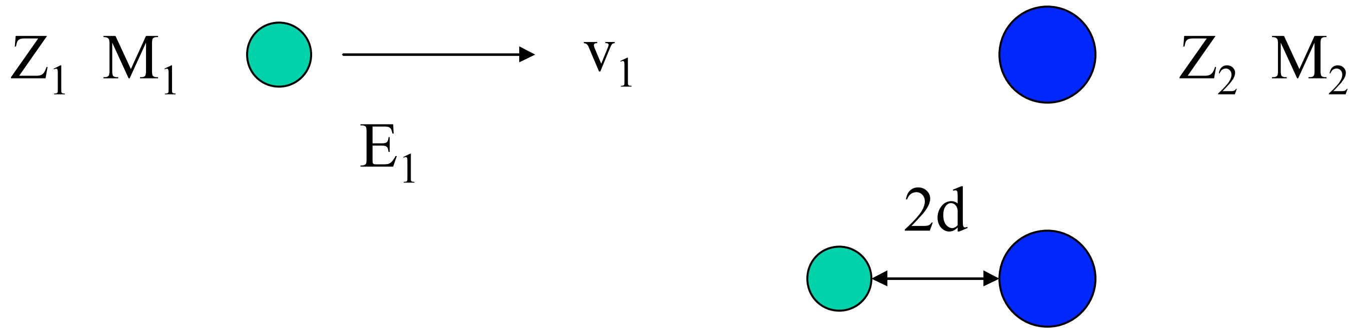
3) Displacement of atoms: lattice defects,
   emission of light atoms (ERD)



Coulomb collision – target charge infinitely heavy

1) Head-on collision

half-distance of closest approach:



$$\tfrac{1}{2} M_1 v_1^2 = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 (2d)} \longrightarrow \boxed{d = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 M_1 v_1^2}}$$

Atomic units

Length

$$\frac{\lambda_c}{2\pi} = \frac{\hbar}{mc}$$

Energy

$$mc^2$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$$

fine-structure constant

$$a_B = \frac{\hbar}{\alpha mc}$$

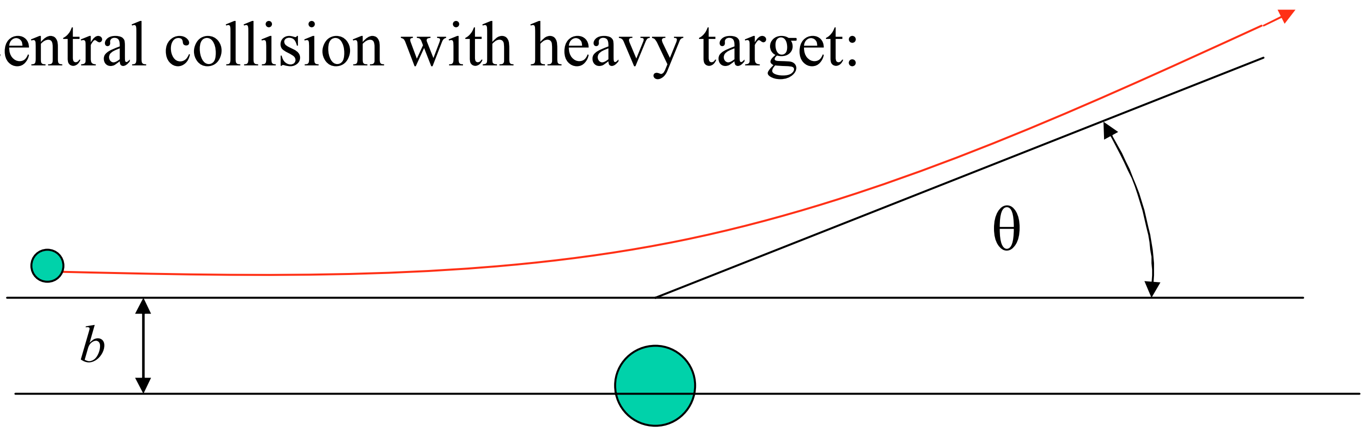
Bohr radius (0.053 nm)

$$Ry = \tfrac{1}{2}\alpha^2 mc^2$$

Rydberg (13.6 eV)

$$\frac{d}{a_B} = Z_1 Z_2 \frac{Ry}{\frac{1}{2} M_1 v_1^2}$$

Non-central collision with heavy target:



scattering angle:

$$\boxed{\tan\frac{\theta}{2} = \frac{d}{b}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Scattering (Rutherford) cross section

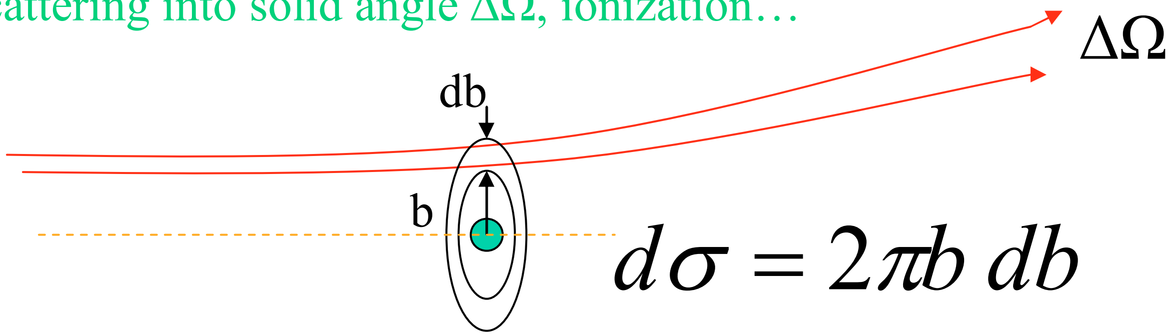
cross section: $$\frac{dN}{dt} = \sigma \; j$$

no. of events per unit time [s^{-1}]

density of projectile flux [$m^{-2}\, s^{-1}$]

events: scattering into solid angle $\Delta\Omega$, ionization…

$$b = d\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$



$$d\sigma = 2\pi b\, db$$

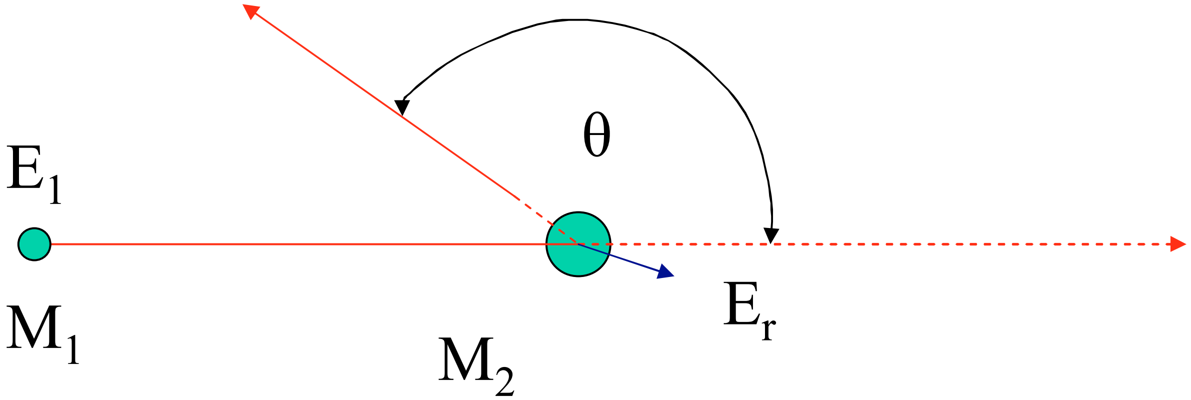
differential scattering cross section:

$$\boxed{\frac{d\sigma}{d\Omega}} = \frac{2\pi b\, db}{2\pi\, d(\cos\theta)} = d^2\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\,\frac{d}{d(\cos\theta)}\left(\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right) = \boxed{\frac{d^2}{(1-\cos\theta)^2}}$$

$$1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

Target atom heavy, but not infinitely → mass analysis

$\boxed{E = K\,E_1}$



$$K = \left[\frac{(M_2^2 - M_1^2 \sin^2\theta)^{\frac{1}{2}} + M_1 \cos\theta}{M_1 + M_2} \right]^2$$

kinematic factor

$$\theta = \pi : \qquad K = \left(\frac{M_2 - M_1}{M_2 + M_1} \right)^2$$

$$v_2 = \frac{2m_1}{m_1 + m_2} v_1^0 + \frac{m_2 - m_1}{m_1 + m_2} v_2^0$$

Concept of mass resolution $(\theta = \pi)$:

1) Energy of scattered particles is measured with a resolution ΔE
2) What is the smallest difference ΔM_2 that can be detected?

$$E = KE_1$$

$$\Delta E = E_1 \frac{dK}{dM_2} \Delta M_2$$

$$\Delta M_2 = \frac{\Delta E}{E_1} \frac{(M_1 + M_2)^3}{4M_1(M_2 - M_1)}$$

$\longrightarrow$ minimal at $M_2 = 2M_1$ $\qquad \Delta M_2 = \frac{\Delta E}{E_1} \frac{27}{8} M_2$

Example: 1 MeV alpha (ΔE=15 keV): $M_2 \sim 16 \rightarrow \Delta M_2 = 0.6$ a.m.u. (0.8 for $M_1 = 8$)
$M_2 \sim 100 \rightarrow \Delta M_2 = 11$ a.m.u. (5 for $M_1 = 50$)

Thick targets: projectiles gradually loose energy

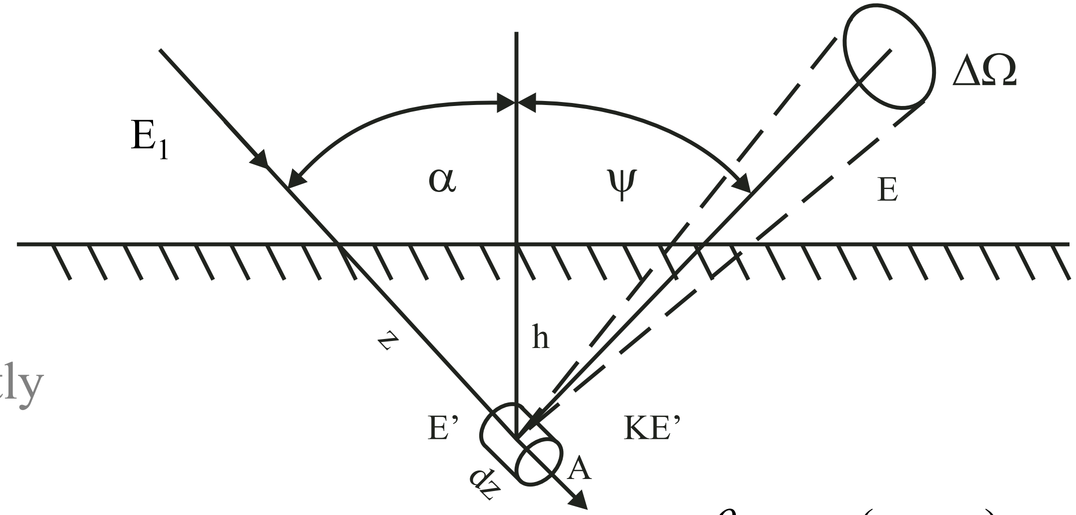
$$\frac{dE}{dx} = \rho\, S(E)$$

stopping power

energy of surface-scattered ions: KE_1

energy oh ions scattered at depth h: E

$$\Delta E = KE_1 - E$$



$$\theta = \pi - (\alpha + \psi)$$

Variation of S with energy only partly taken into account: $S(E_1)$, $S(E)$

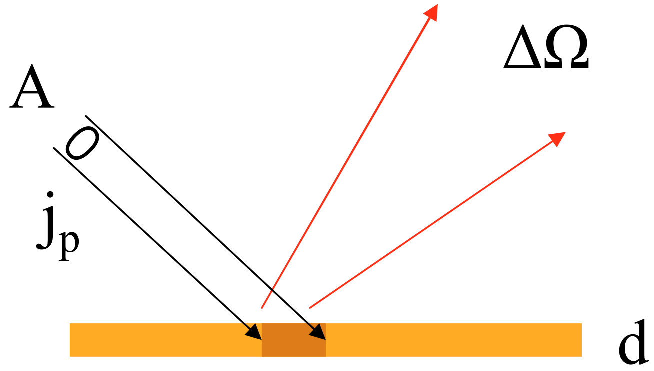
$$E' = E_1 - \rho\, S(E_1) \frac{h}{\cos\alpha} \qquad E = KE' - \rho\, S(E) \frac{h}{\cos\psi}$$

$$\boxed{\Delta E = \rho \left(K \frac{S(E_1)}{\cos\alpha} + \frac{S(E)}{\cos\psi} \right) h = [S]\, h}$$

Rutherford spectrum from a thin slice of material

Number of scattered ions:

$$N = \sigma \, j_p N_{at} t$$



Detected ions – yield Y:

$$Y = \frac{d\sigma}{d\Omega} \Delta\Omega \, j_p N_{at} t$$

$$N_{at} = \frac{m_i}{M} N_A = \frac{\rho \, Ad}{M_i \cos\alpha} N_A x_i$$

x_i – weight fraction

N_p – number of projectiles

areal density

$$Y_i = \frac{d\sigma_i}{d\Omega} \Delta\Omega \, j_p \frac{\rho \, A \, d}{M_i \cos\alpha} N_A x_i t = \frac{d\sigma_i}{d\Omega} \Delta\Omega \, N_p N_A \frac{x_i \rho d}{M_i \cos\alpha}$$

$$n_i = \frac{\rho N_A}{M_i} x_i \longrightarrow \boxed{Y_i = \frac{d\sigma_i}{d\Omega} \Delta\Omega \, N_p \frac{n_i \, d}{\cos\alpha}}$$

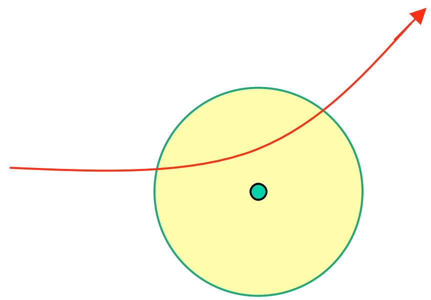
density of atoms

Rutherford scattering from a thick target

I. Scattering cross section – finite mass, electron screening

$$\frac{d\sigma}{d\Omega} = d^2 \frac{\left[\left(M_2^2 - M_1^2 \sin^2\theta\right)^{\frac{1}{2}} + M_2 \cos\theta\right]^2}{M_2 \sin^4\theta \left(M_2^2 - M_1^2 \sin^2\theta\right)^{\frac{1}{2}}}$$

$$d \sim \frac{Z_1 Z_2}{E_1}$$



Correction factor for screening of atomic electrons:

$$f = \frac{1}{1 + \frac{V}{E_1}} \qquad V = 1.586 \frac{Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 .0.8853 a_B} \sqrt{Z_1^{\frac{2}{3}} + Z_2^{\frac{2}{3}}}$$

H.H. Andersen et al., Phys. Rev. A21 (1980) 2070.

II. Infinitely thick target

smearing due to stopping – energy spectrum dY/dE

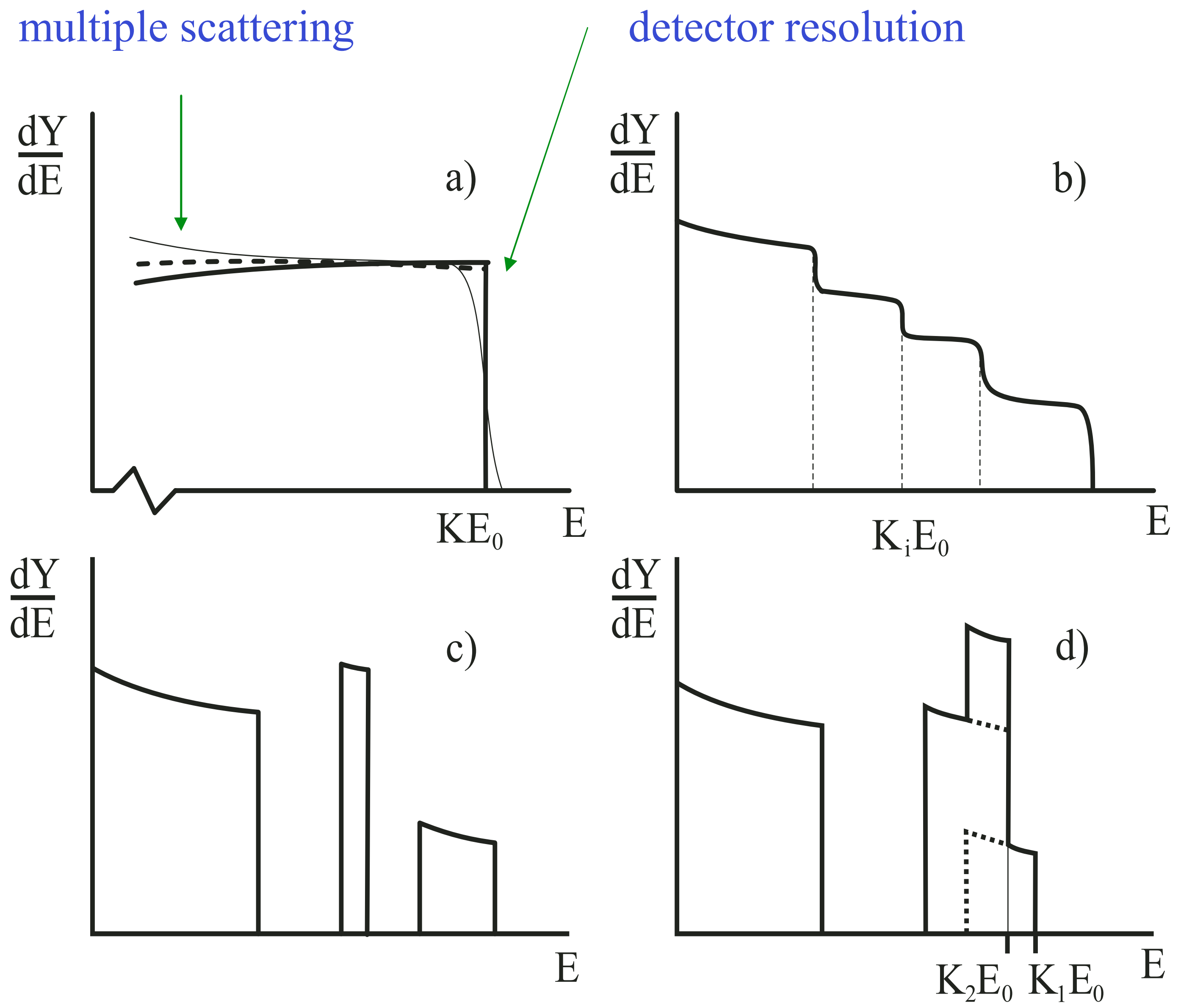
distribution of the target into slices: $dz = \dfrac{d}{\cos\alpha}$

$$\frac{dY_i}{dE} = \frac{d\sigma_i}{d\Omega}(E')\ \Delta\Omega\, N_p\, \frac{\rho N_A}{M_i}\, x_i\, \frac{dz}{dE}$$

analytical approximations: $\dfrac{d\sigma_i}{d\Omega}(E') = \dfrac{d\sigma_i}{d\Omega}(E_1).(\frac{E_1}{E'})^2$

$$z = \frac{h}{\cos\alpha} \longrightarrow \frac{\rho\, dz}{dE} = \left[K_i S(E_1) + \frac{\cos\alpha}{\cos\psi} S(E)\right]^{-1}$$

$$\boxed{\frac{dY_i}{dE} = \frac{d\sigma_i}{d\Omega}(E')\ \Delta\Omega\, N_p\, \frac{N_A}{M_i}\, \frac{x_i}{K_i S(E_1) + \frac{\cos\alpha}{\cos\psi} S(E)}}$$



impact energy: E_0

1

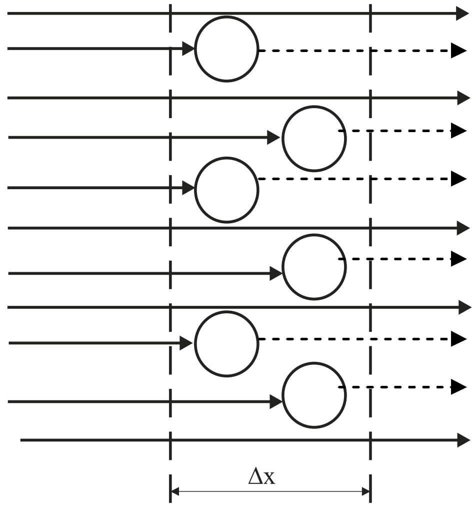
1

Target chemical compound $A_{m1}B_{m2}\ldots Z_{mn}$

$\longrightarrow$ stopping cross section $\varepsilon = \Delta E_1 \sigma$

ΔE_1 – energy loss in a single atomic collision

in Δx: $\Delta E_{tot} = \Delta E_1 N = \Delta E_1 \sigma\, j\, t\, N_{at}$



$$N_{at} = \frac{\rho\, A\, dx}{M} N_A$$

$$\Delta E_{tot} = \varepsilon \underbrace{\boxed{j\, t\, A}}_{N_p} dx \frac{\rho N_A}{M}$$

energy loss of a single ion: $\Delta E = \dfrac{\Delta E_{tot}}{N_p}$

$$\frac{\Delta E}{\Delta x} = \varepsilon \frac{\rho N_A}{M} \longrightarrow \boxed{\varepsilon = S \frac{M}{N_A}}$$

Stopping cross section of a molecule $A_{m1}B_{m2}\ldots Z_{mn}$:

$$\varepsilon = \frac{S\,M}{N_A} = \sum_i m_i \varepsilon_i$$

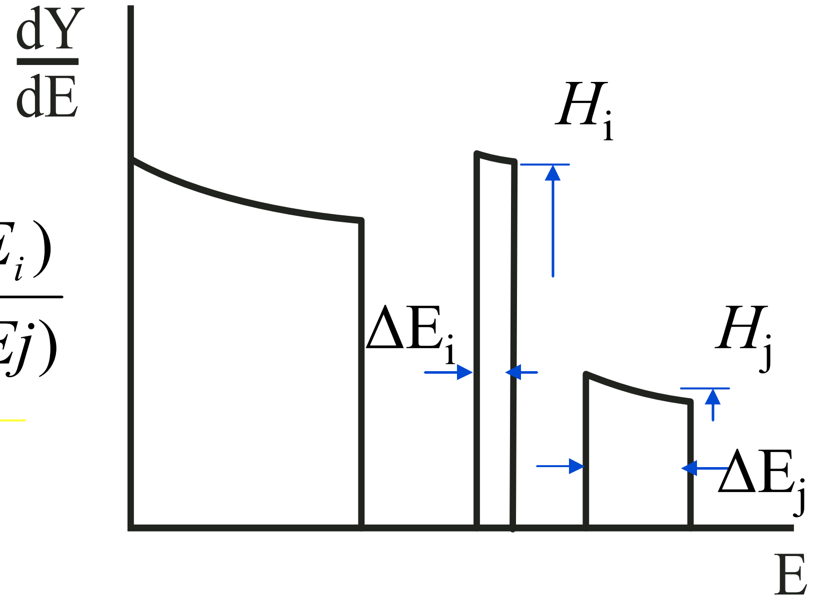
Scattering spectrum of a thick target:

$$\boxed{\frac{dY_i}{dE} = \frac{d\sigma_i}{d\Omega}(E')\Delta\Omega\, N_p \frac{m_i}{K_i\varepsilon(E_1) + \frac{\cos\alpha}{\cos\psi}\varepsilon(E)}}$$

Measured quantity: $$H_i = \frac{dY_i}{dE}\underbrace{\Delta E_{ch}}_{\text{energy width of the channel}}$$

Example: a layer of compound $A_{m_i}B_{m_j} \longrightarrow$ $m_i, m_j = ?$

$$H_i = \frac{dY_i}{dE} \Delta E_{ch}$$

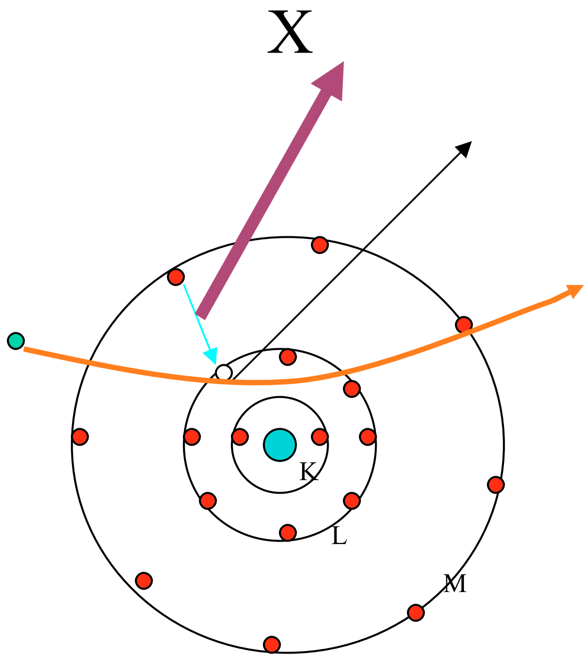


$$\frac{m_i}{m_j} = \frac{H_i}{H_j} \frac{\frac{d\sigma_j}{d\Omega}(E_1)}{\frac{d\sigma_i}{d\Omega}(E_1)} \frac{K_i \varepsilon(E_1) + \frac{\cos\alpha}{\cos\psi}\varepsilon(E_i)}{K_j \varepsilon(E_1) + \frac{\cos\alpha}{\cos\psi}\varepsilon(Ej)}$$

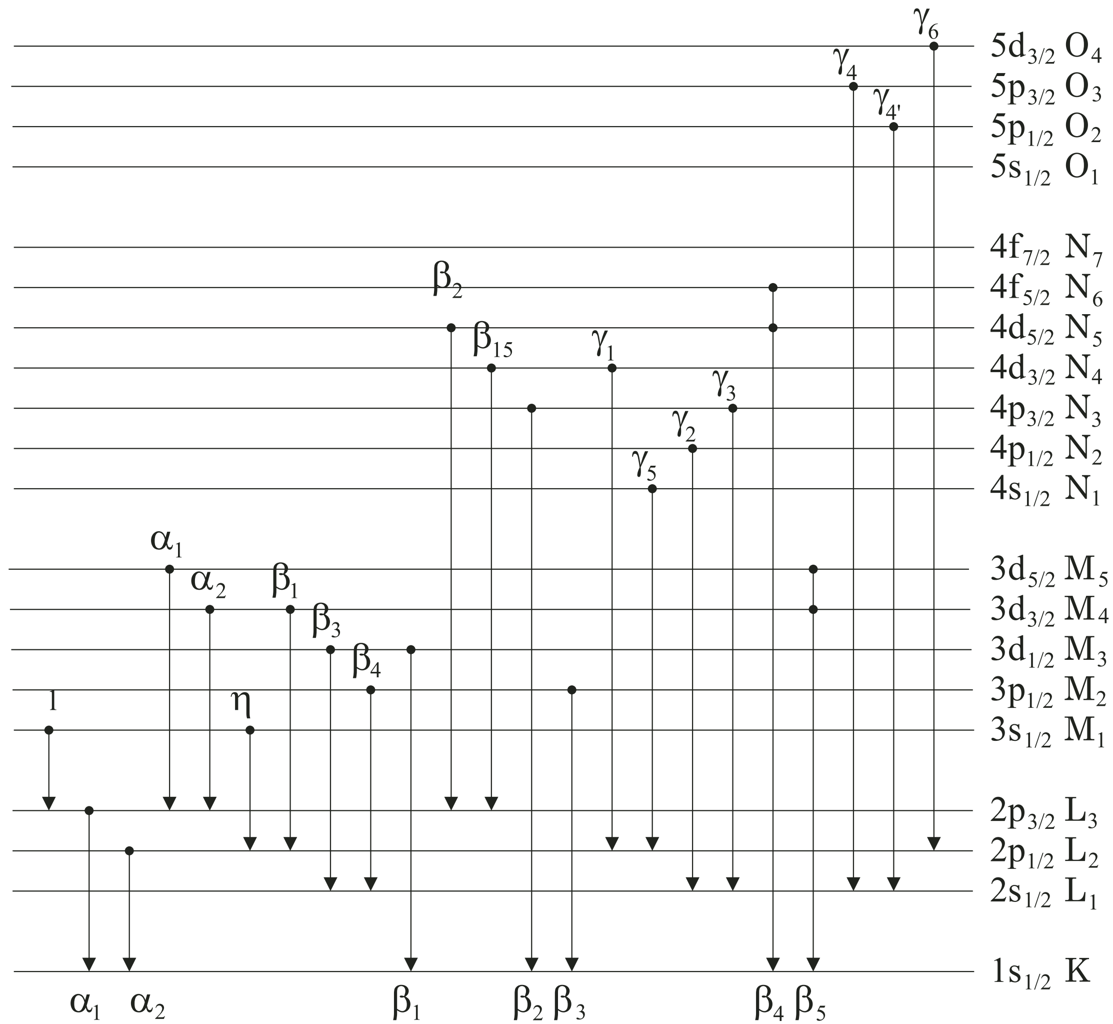
$$\frac{\Delta E_i}{\Delta E_j} = \frac{K_i \varepsilon(E_1) + \frac{\cos\alpha}{\cos\psi}\varepsilon(E_i)}{K_j \varepsilon(E_1) + \frac{\cos\alpha}{\cos\psi}\varepsilon(Ej)}$$

$$\boxed{\frac{m_i}{m_j} = \frac{H_i}{H_j} \frac{\frac{d\sigma_j}{d\Omega}(E_1)}{\frac{d\sigma_i}{d\Omega}(E_1)} \frac{\Delta E_i}{\Delta E_j} \approx \frac{\frac{d\sigma_j}{d\Omega}(E_1)}{\frac{d\sigma_i}{d\Omega}(E_1)} \frac{Y_i}{Y_j} \approx \frac{Z_j^2}{Z_i^2} \frac{Y_i}{Y_j}}$$

Ionization

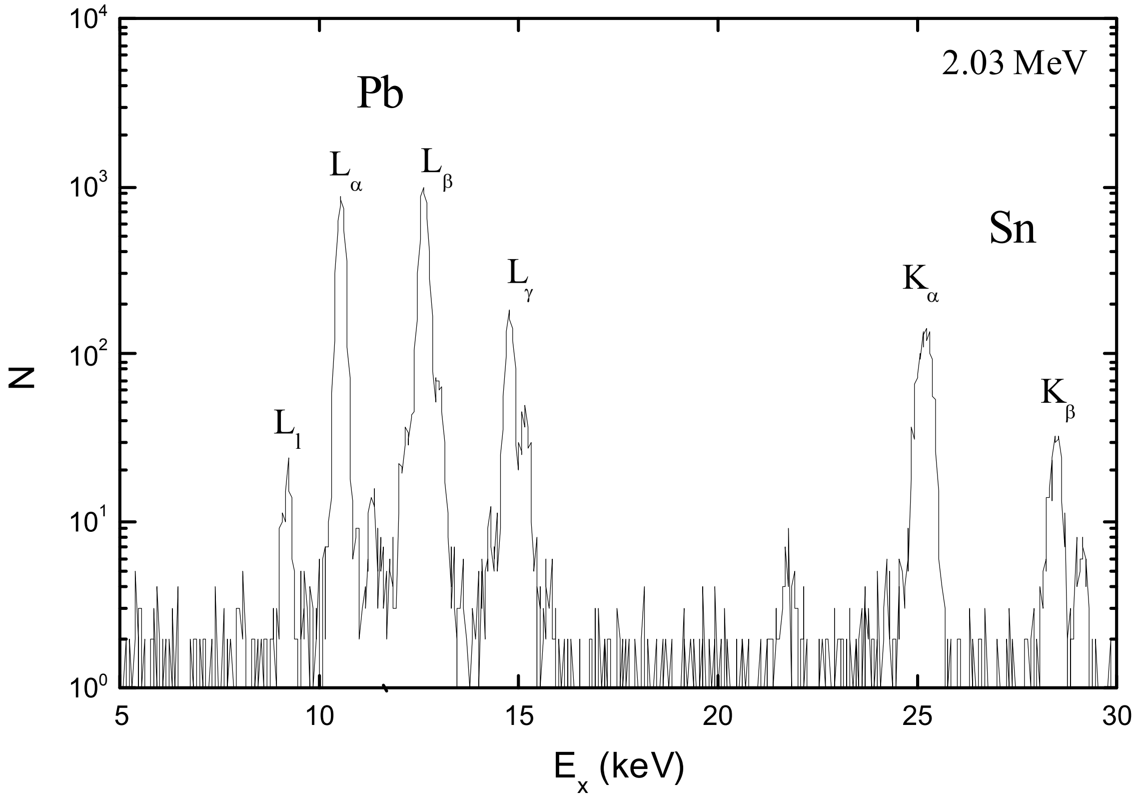


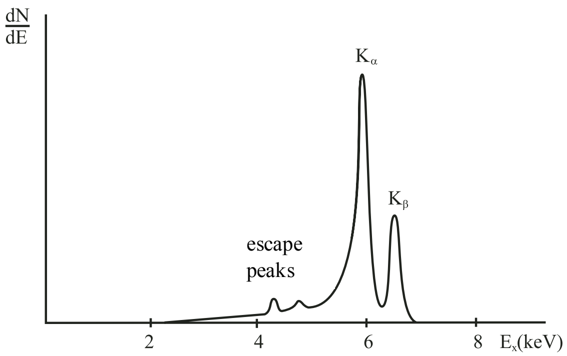
X
K
L
M

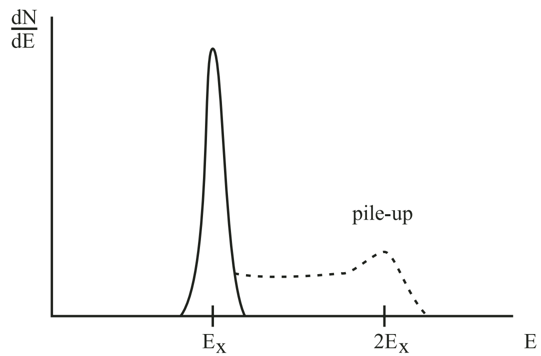


$5d_{3/2}$ O_4
$5p_{3/2}$ O_3
$5p_{1/2}$ O_2
$5s_{1/2}$ O_1
$4f_{7/2}$ N_7
$4f_{5/2}$ N_6
$4d_{5/2}$ N_5
$4d_{3/2}$ N_4
$4p_{3/2}$ N_3
$4p_{1/2}$ N_2
$4s_{1/2}$ N_1
$3d_{5/2}$ M_5
$3d_{3/2}$ M_4
$3d_{1/2}$ M_3
$3p_{1/2}$ M_2
$3s_{1/2}$ M_1
$2p_{3/2}$ L_3
$2p_{1/2}$ L_2
$2s_{1/2}$ L_1
$1s_{1/2}$ K
γ_6 γ_4 $\gamma_{4'}$
β_2 β_{15} γ_1 γ_3 γ_2 γ_5
α_1 α_2 β_1 β_3 β_4
l η
α_1 α_2 β_1 β_2 β_3 β_4 β_5

X-ray spectra

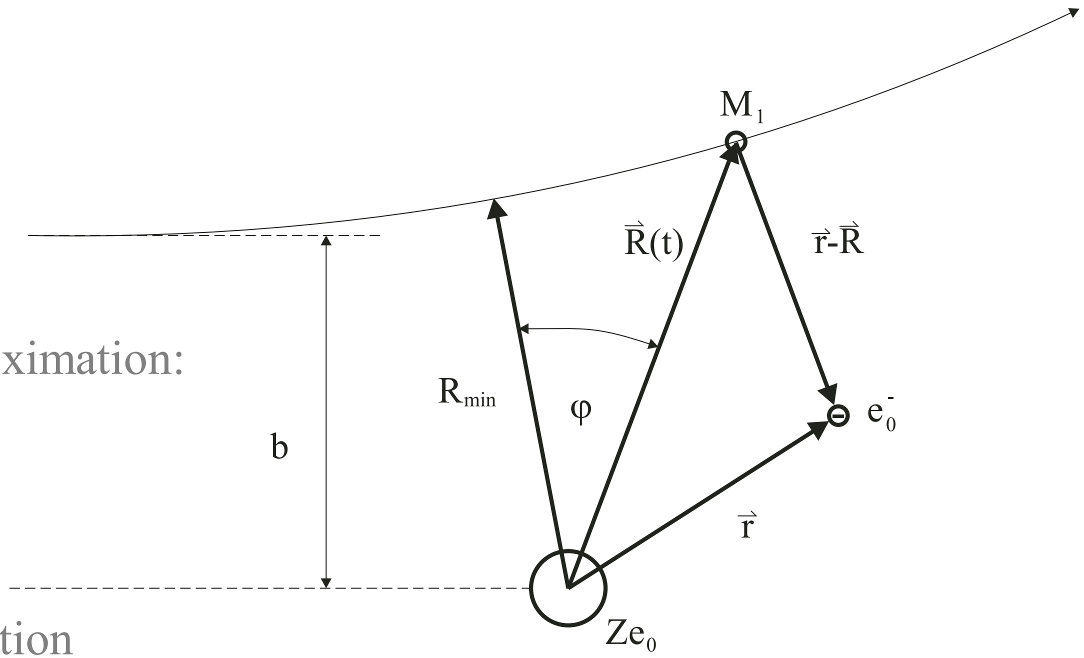






Ionization models

I. Semiclassical picture



electrons: independent particle approximation:

$$\Psi = \sum_n a_n(t)\,|n\rangle\, e^{i\omega_n t}$$

time-dependent electronic wave function

$$t=0: \quad a_0=1 \quad a_{i>0}=0 \qquad t=\infty: \qquad P=\sum_{f>0}|a_f|^2$$

$$V = -\frac{Z_1 e_0}{4\pi\varepsilon_0 \left|\vec{r}-\vec{R}(t)\right|}$$

Hamiltonian: $H = H_0 + V$

$$\frac{da_f}{ds} = -i\, Z_1\, \alpha \frac{c}{v_1} \sum_n \langle f | \frac{-1}{\left|\vec{R}(t)-\vec{r}\right|} | n \rangle\; e^{i\frac{\omega_f-\omega_n}{v_1}s}$$

$$s = v_1 t$$

Separation of electronic and projectile coordinates:

1. Momentum approach (P. Amundsen)

$$\frac{1}{\left|\vec{r}-\vec{R}\right|} = \frac{1}{2\pi^2}\int \frac{d^3 s}{s^2} e^{i\vec{s}(\vec{r}-\vec{R})} = 8\int ds \sum_{LM} j_L(sr)\, j_L(sR) Y_{LM}(\vec{r}) Y^*_{LM}(\vec{R})$$

Bethe's integral

$$\boxed{a_{lm} = iZ_1 \alpha \frac{c}{v_1} \int ds\, F_l(s) B_{lm}(s,q)}$$

$$\boxed{F_l(s) = \int_0^\infty R_{kl}\, j_l(sr) R_0\, r^2 dr}$$

electronic form factor

$$\boxed{B_{lm}(s,q) = \frac{4}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{iqT} j_l(sR)\, Y^*_{lm}(\vec{R})\, dT}$$

path factor

$$q = \frac{\Delta\omega}{v_1}$$ momentum transfer

1. Coordinate approach (Trautmann et al.)

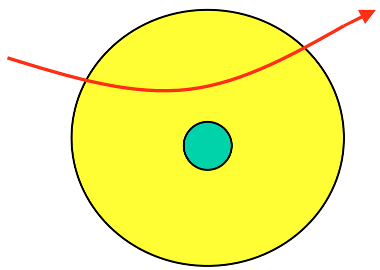
$$\frac{1}{\left|\vec{r}-\vec{R}\right|}=\sum_{LM}\frac{4\pi}{2L+1}\frac{r_<^L}{r_>^{L+1}}Y_{LM}^*(\vec{R})Y_{LM}(\vec{r})$$

multipole expansion

$$\longrightarrow \quad \boxed{G_l=\frac{1}{R^{l+1}}\int_0^R R_{kl}R_0 r^{l+2}dr+R^l\int_R^\infty R_{kl}R_0 r^{1-l}dr}$$

$$\boxed{a_{lm}=iZ_1\alpha\frac{c}{v_1}\int e^{iqT}G_l(R)\frac{4\pi}{2l+1}Y_{lm}^*(\vec{R})\,dT}$$

$\longrightarrow$ any $\vec{R}(T)$



atomic potential

Time-variation of atomic wave function: coupled channels

$$\frac{da_f}{ds} = -i\, Z_1\, \alpha \frac{c}{v_1} \sum_n \langle f | \frac{-1}{|\vec{R}(t) - \vec{r}|} | n \rangle\; e^{i \frac{\omega_f - \omega_n}{v_1} s}$$

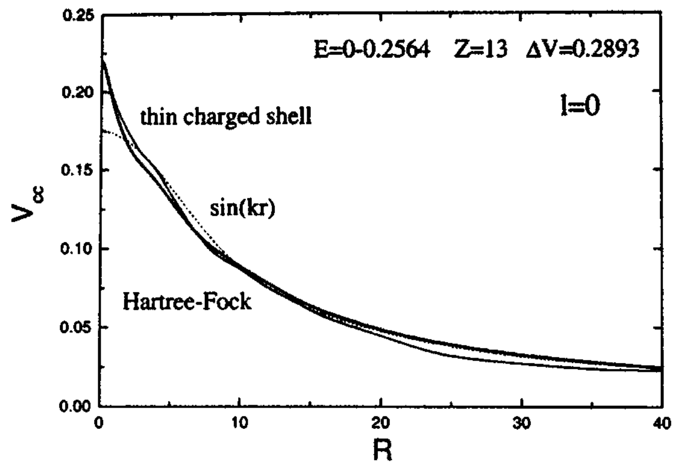
discrete states in continuum:

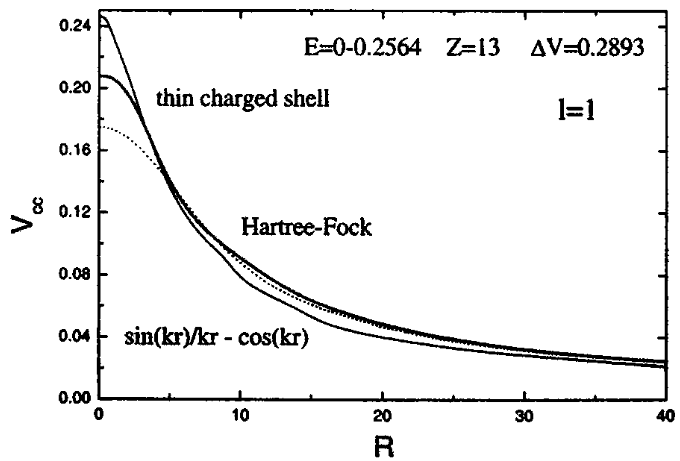
wave packets (Mehler, Greiner, Soff, 1987)

$$w_{k-\frac{\Delta k}{2}, k+\frac{\Delta k}{2}}(r) = \frac{1}{\sqrt{\Delta k}} \int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} u_k(r)\, dk$$

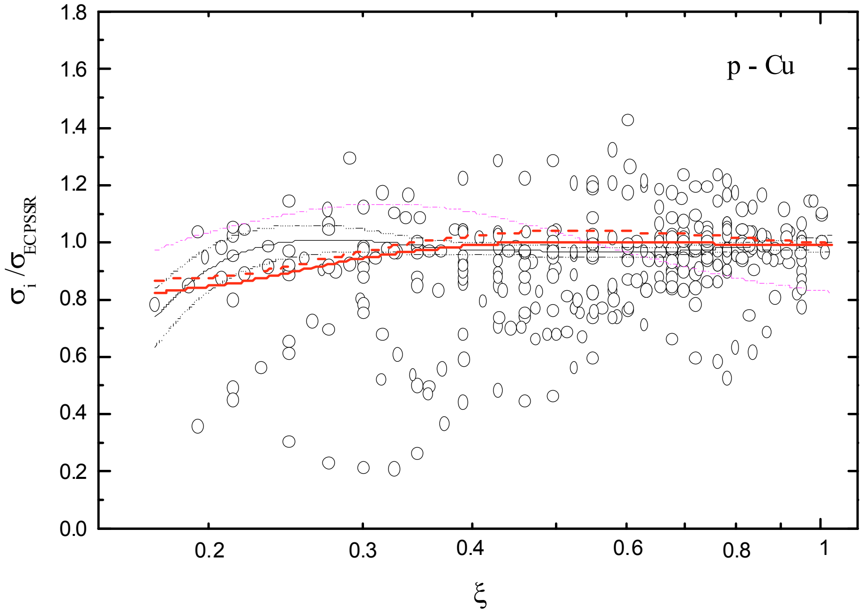
$$w_{E-\frac{\Delta E}{2}, E+\frac{\Delta E}{2}}(r) = \frac{1}{\sqrt{\Delta E}} \int_{-\frac{\Delta E}{2}}^{\frac{\Delta E}{2}} u_E(r)\, dE$$

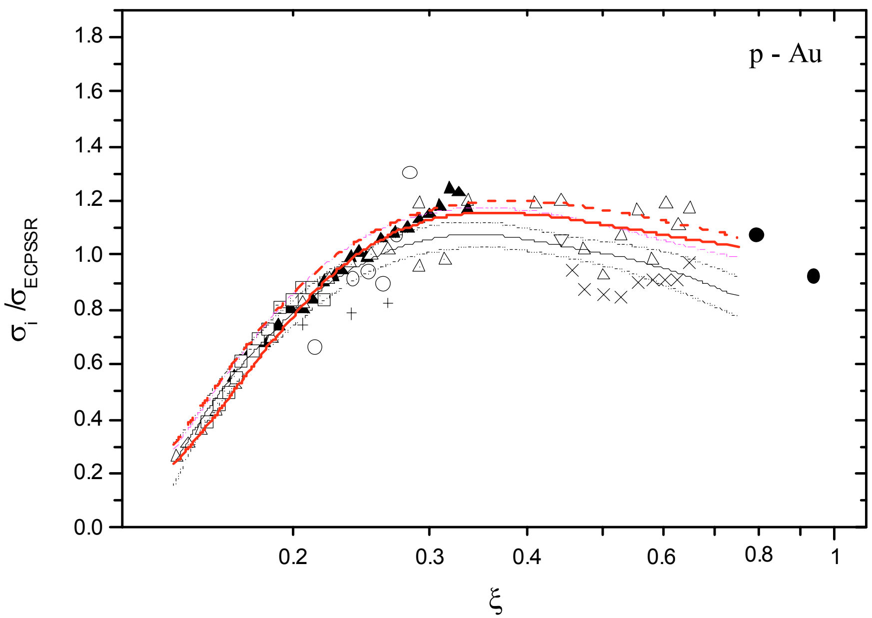
$$\int_0^\infty w^2(r)\, dr = 1$$





Comparison of experimental and calculated cross section K-shell ionization





——— coupled channel calculation, transition to a single final state (wave packet), Hartree-Fock wave functions

- - - - hydrogenic bound state

- - - - united atom approximation

——— reference cross section of Paul (with error limits - - - - - -)

$$\boxed{\xi = \sqrt{\frac{2}{\Theta}}\frac{v_1}{(Z_2 - 0.3)\alpha c}}$$

$$E_B = \Theta(Z_2 - 0.3)^2 Ry$$

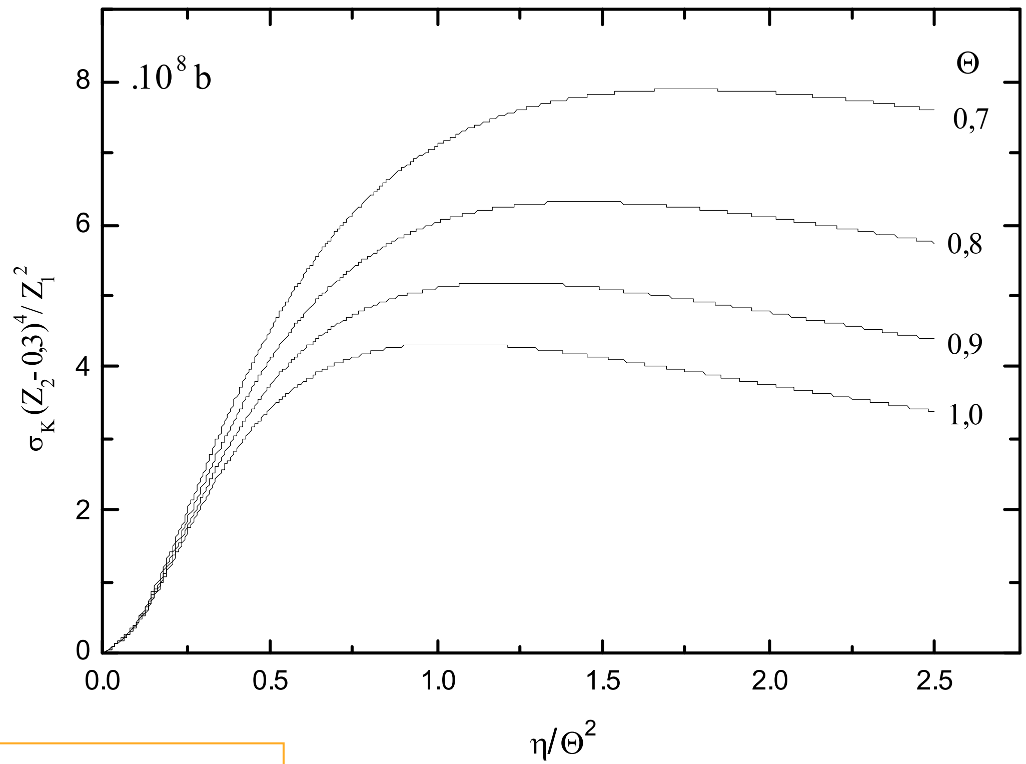
Ž. Šmit, Nucl. Instr. and Meth. B 189 (2002) 1.

II. Plane-wave Born approximation

projectile-atomic electron wave functions:

$$\chi_i = e^{\frac{i}{\hbar}\vec{p}_i\vec{R}}\Psi_i(\vec{r})$$

$$\chi_f = e^{\frac{i}{\hbar}\vec{p}_f\vec{R}}\Psi_f(\vec{r})$$



adiabatic limit (K shell):

$$\sigma_K = 8\pi a_B^2 \frac{Z_1^2}{(Z_2-0.3)^4}\frac{2^{17}}{45}\frac{1}{\Theta}\left(\frac{\eta}{\Theta^2}\right)^4$$

$$\eta = \left(\frac{v_1}{(Z_2-0.3)\alpha c}\right)^2$$

Improvements of the PWBA approach: ECPSSR theory

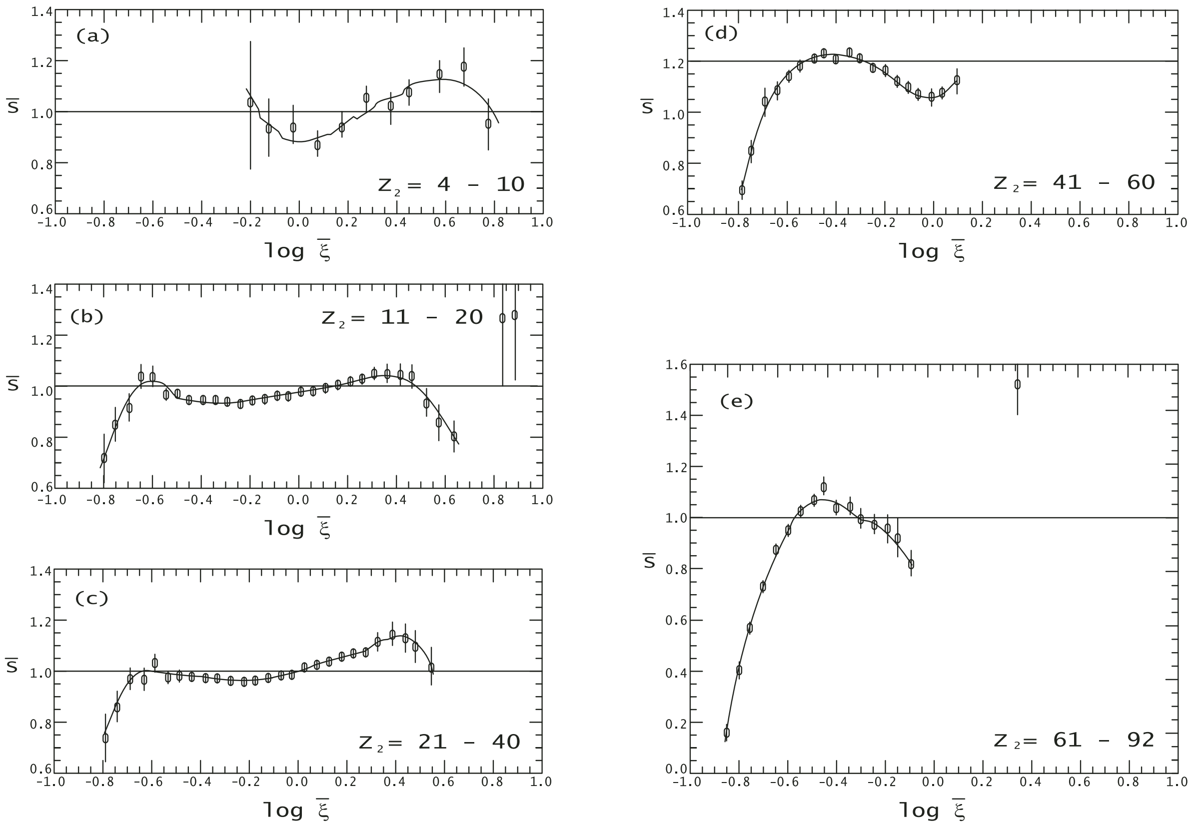
(Basbas, Brandt, Lapicki, Laubert, 1978-1981)

	ECPSSR	SCA
projectile kinematics	corr. for hyperbolic paths	exact hyperbolic paths exact kinematics in screened potential
atomic wave functions	PSS (averaged binding energies)	united atom coupled channels
relativistic effects	electron mass	hydrogenic Dirac w.f.
recoil	Ø	+
inelasticity	exact momentum transfer	symmetrized v
CODES	Paul, Cipolla, Šmit	IONHYD, HIPGL

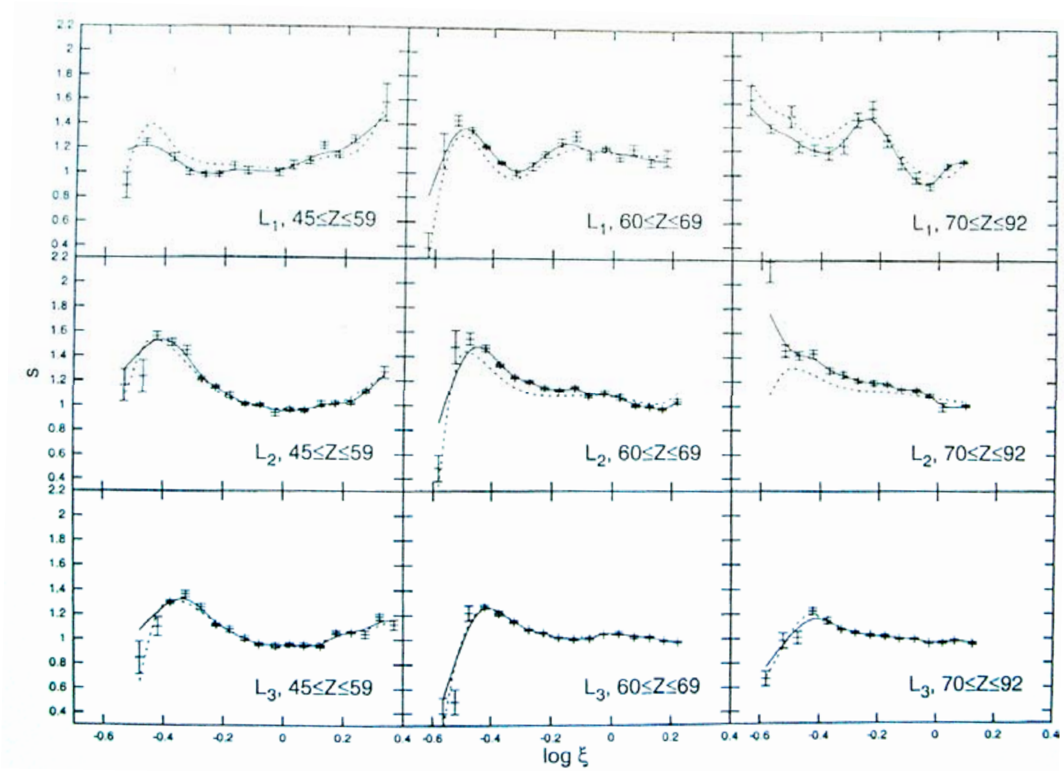
K-shell cross sections – experimental values (H. Paul et al.)

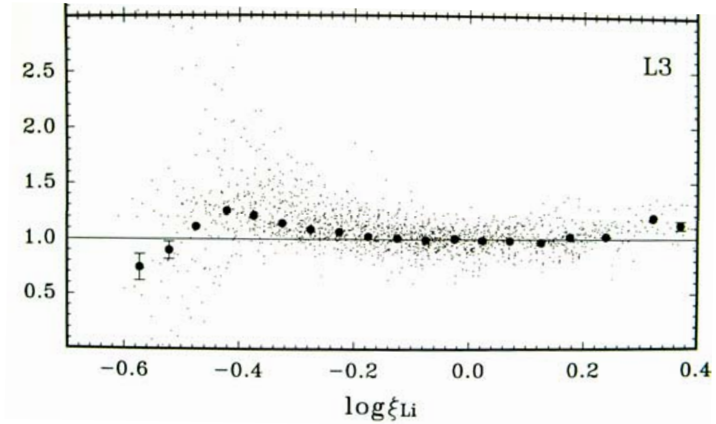
$$\sigma_{K\alpha} = \sigma_{ion}\omega_K \frac{1}{1+K_\alpha / K_\beta}$$

$$\sigma_{ion} = \sigma_{ECPSSR}\ s_c$$



L- shell reference cross sections



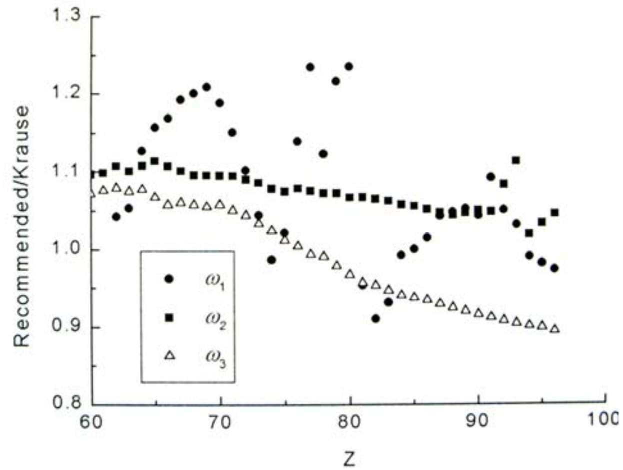


Sow, Orlić, 1993

Campbell, Hopman, Maxwell, Nejedly, 2000

New set of L-subshell fluorescence and Coster-Kronig yields for proton induced ionization

J.L. Campbell, K. Dinelle, PIXE 2004.
J.L. Campbell, ADNDT 85 (2003) 291.



Ratios between new and 1979 (Krause) yields

Cross section data:

- K-shell

reference cross section of Paul:

H. Paul, J. Muhr, Phys. Rep. 135 (1986) 47
H. Paul, J. Sacher, At. Data Nucl. Data Tables 42 (1989) 105

- L-shell

averaging procedures:

I. Orlić, C.H. Sow, S.M. Tang, At. Data Nucl. Data Tables 56 (1994) 159. →
S.H. Sow, M.S. Thesis, National University of Singapore (1993)
J.L. Campbell, T.L. Hopman, J.A. Maxwell, Z. Nejedly, Nucl. Instr. and Meth. B170 (2000) 193. (appl. in GUPIX)

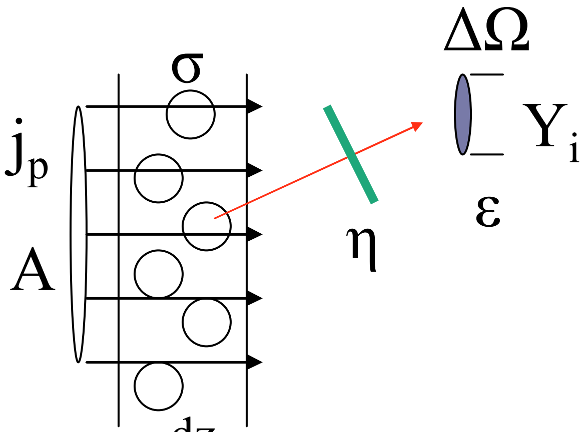
M.A. Reis, A.P. Jesus, At. Data Nucl. Data Tables 63 (1996) 1.

X-ray production in thick targets

$$N_i = \sigma\, j_p \frac{m}{M} N_A\, t = \sigma\, j_p \frac{\rho A\, dz}{M} N_A\, t$$

no. of atoms

no. of projectiles N_p



thin target

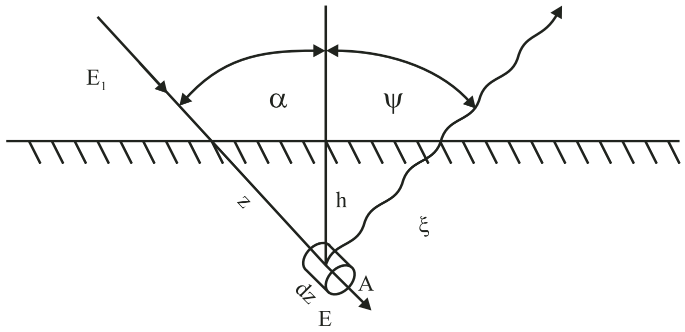
$$Y_i = \frac{\Delta\Omega}{4\pi} N_p N_A \frac{\varepsilon\, \eta\, \sigma}{M} \rho\, dz$$

$$\boxed{\rho\, dz = \rho \frac{dz}{dE} dE = \frac{dE}{S(E)}}$$

$$Y_i = \frac{\Delta\Omega}{4\pi} N_p N_A \frac{\varepsilon_i \eta_i T_i}{M_i} x_i$$

mass fraction

thick target X-ray yield



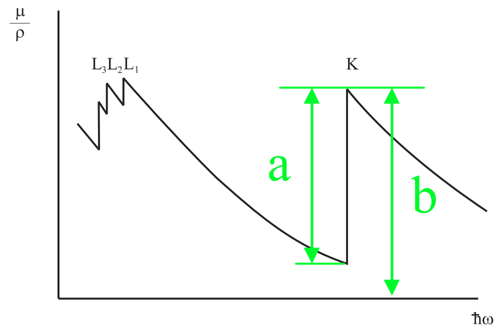
thick target

$$T_i = \int_0^{E_1} \frac{\sigma_i^x(E)}{S(E)} \mathrm{e}^{-\mu_i \xi(E)} dE$$

thick target factor

$$z = \int_E^{E_1} \frac{dE}{\rho S(E)} \qquad \xi = z \frac{\cos\alpha}{\cos\psi}$$

Secondary fluorescence effects $\boxed{i \rightarrow j}$

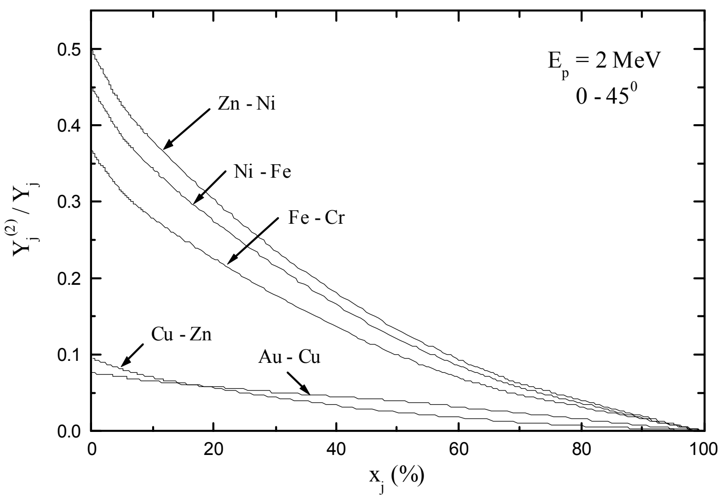


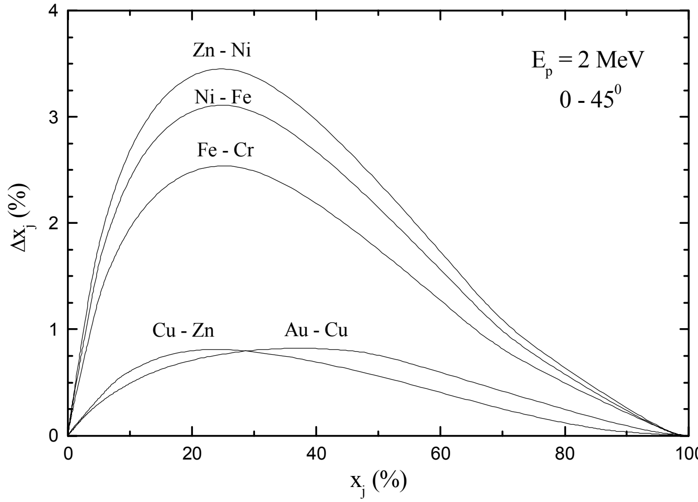
$$\frac{Y_j^{(2)}}{Y_j} = \frac{1}{2}\omega_j r_j \frac{\mu_{ij}}{\mu_j}\cos\psi \frac{M_j}{M_i} x_i \frac{T_i^{(2)}}{T_j}$$

$$r = \frac{a}{b}$$

$$T_j = \int_0^{E_1} \frac{\sigma_j^x(E)}{S(E)} \exp(-\mu_j \xi(E))dE \qquad T_j^{(2)} = \int_0^{E_1} \frac{\sigma_j^x(E)}{S(E)} \Xi(E)\, dE$$

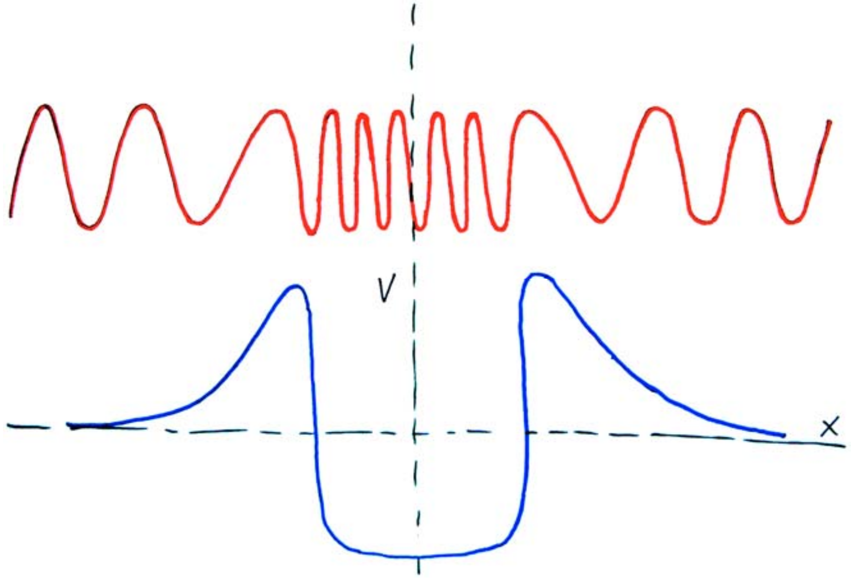
$$\Xi = \exp\left(-\mu_j z \frac{\cos\alpha}{\cos\psi}\right)\left[\ln\frac{\mu_i\cos\psi + \mu_j}{\left|\mu_i\cos\psi - \mu_j\right|} - \mathrm{E}_1\left(\left|\mu_i z\cos\alpha - \mu_j z\frac{\cos\alpha}{\cos\psi}\right|\right)\right] + \mathrm{E}_1\left(\mu_i z\cos\alpha\right)$$





$$\sum_i x_i = 1$$

Nuclear reactions



elastic scattering:

$$\frac{d\sigma}{d\Omega} = \left| f_{Coul} + f_{nucl} \right|^2$$

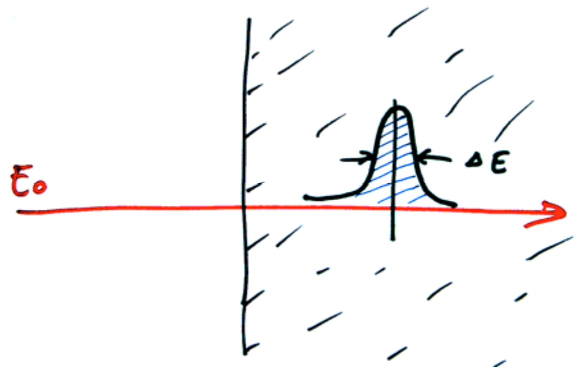
excitation of compound nucleus:

$$a + A \rightarrow X^* \rightarrow B + b$$

$$\sigma(E) = \frac{\pi}{k^2} \frac{g\Gamma_i\Gamma_f}{(E-E_R)^2 + \frac{\Gamma^2}{4}}$$

Breit-Wigner formula

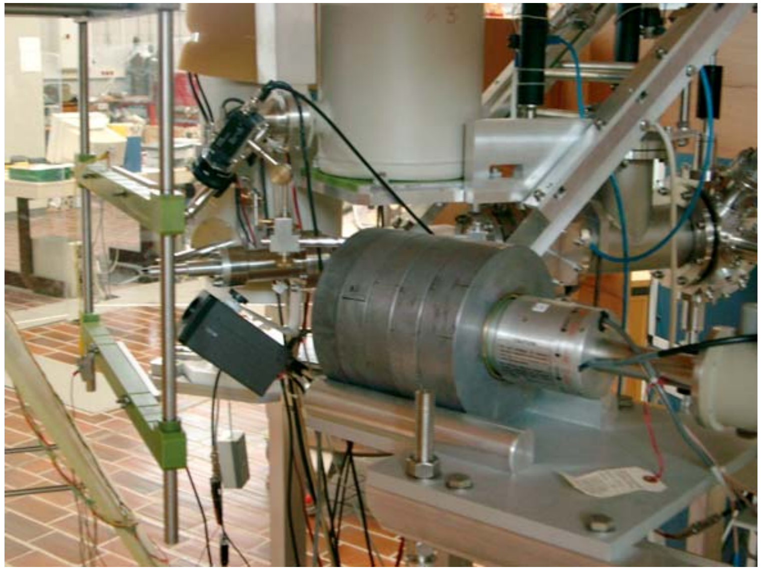
Reactions:
-particle-particle
-particle-photon (γ)

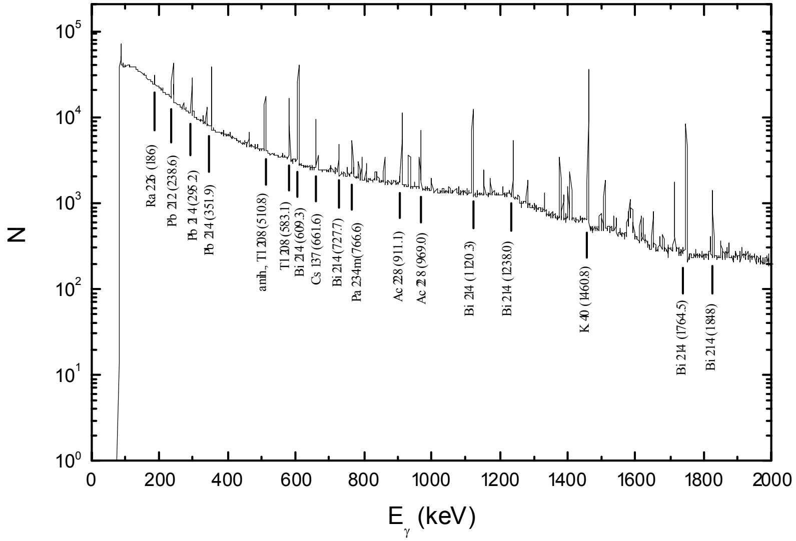


Concentration profiles:

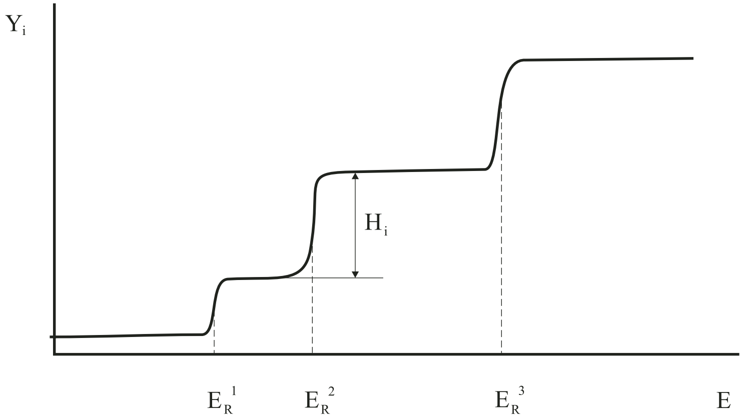
$$h = \frac{E-E_R}{\rho\, S(E)} \qquad \Delta h = \frac{\Gamma}{\rho\, S(E)}$$

PIGE

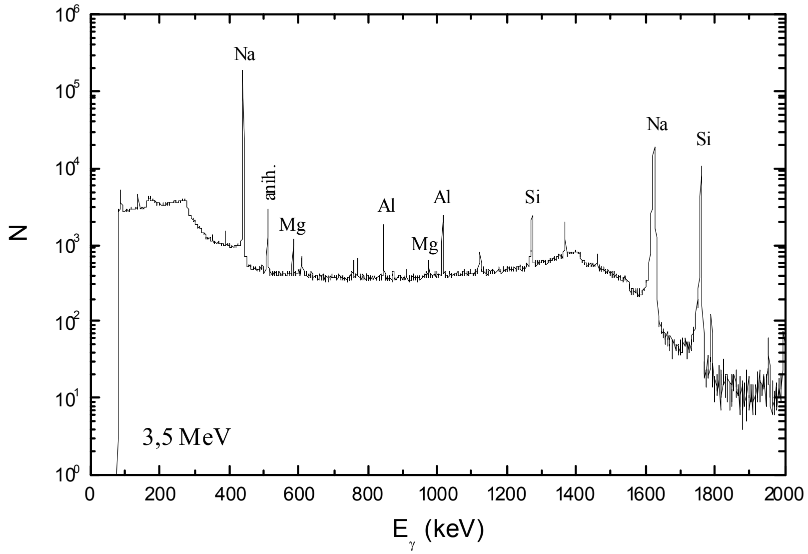




Natural background



Gamma yields from a thick target



PIGE spectrum on glass

Particle-particle reactions for protons and alphas

$$E_1 = E_{cm} + Q$$

Reaction	E_R(MeV)	Γ(keV)	$\frac{d\sigma}{d\Omega}$(mb)
$^{15}N(p,\alpha)^{12}C$ Q=4,966 MeV	~1 1,21		
$^{18}O(p,\alpha)^{15}N$ Q=3,9804 MeV	0,152	0,05	
	0,629	2,1	60
	1,165	0,05	
	1,766	4,5	135
$^{19}F(p,\alpha)^{16}O$ Q=8,1137 MeV	1,347	36	3,2
	1,713	72	2,9
	1,842	122	3,2
$^{19}F(\alpha,p)^{22}Ne$ Q=1,6746 MeV	2,443		
$^{31}P(\alpha,p)^{34}S$ Q=6,316 MeV	2,79 3,048 3,64 3,97		

Energy of gamma rays, induced by 3-4 MeV protons

Thick targets:

$$Y_i = \frac{\Delta\Omega}{4\pi} N_p \frac{\varepsilon_i \eta_i}{M_i} N_A \, x_i \int \frac{\sigma_\gamma}{S(E)} dE$$

$$\Lambda \equiv \int \sigma_\gamma dE = \frac{2\pi^2 g}{k^2} \frac{\Gamma_i \Gamma_f}{\Gamma}$$

Projectile energy exceeding energies of several resonances:

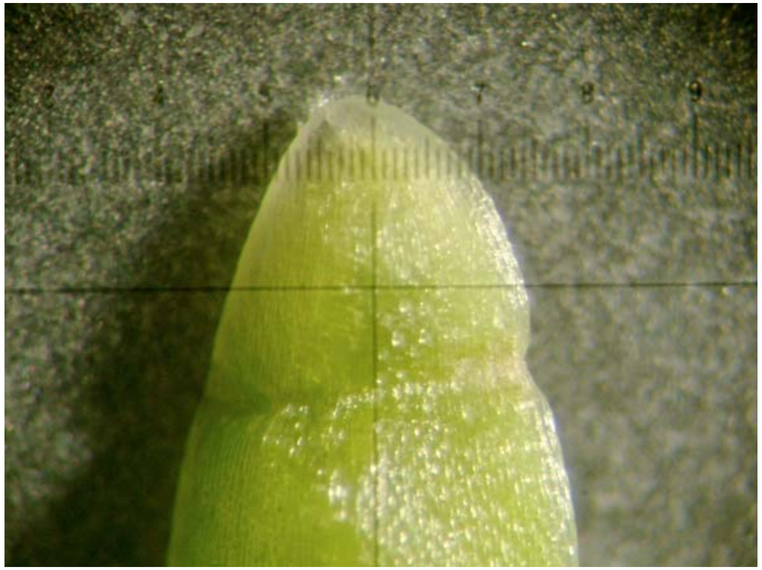
$$\boxed{Y_i = \frac{\Delta\Omega}{4\pi} N_p \frac{\varepsilon_i \eta_i}{M_i} N_A \frac{x_i}{\langle S \rangle} \sum_j \Lambda_{ij}}$$

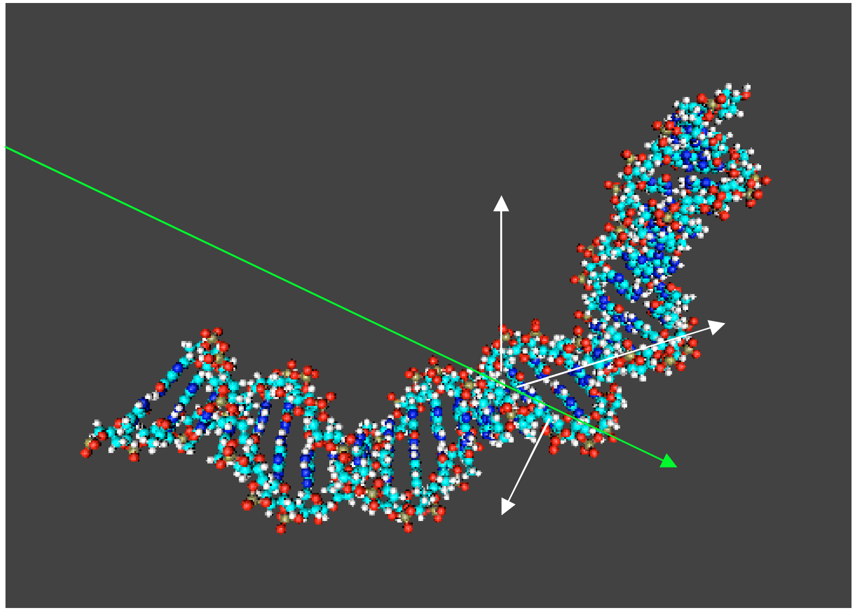
Element	E_γ(keV)	Reaction
O	459	$^{16}O(p,\gamma_1)^{17}F$
	871	$^{17}O(p,p_1\gamma)^{17}O$
	1982	$^{18}O(p,p_1\gamma)^{18}O$
F	197	$^{19}F(p,p_2\gamma)^{19}F$
	1236	$^{19}F(p,p\gamma_{3-1})^{19}F$
	1349	$^{19}F(p,p\gamma_{4-1})^{19}F$
	1357	$^{19}F(p,p\gamma_{5-2})^{19}F$
	1459	$^{19}F(p,p_4\gamma)^{19}F$
Na	440	$^{23}Na(p,p_1\gamma)^{23}Na$
	1634,1636	$^{23}Na(p,p\gamma_{2-1})^{23}Na$
Mg	585	$^{25}Mg(p,p_1\gamma)^{25}Mg$
	975	$^{25}Mg(p,p_2\gamma)^{25}Mg$
Al	844	$^{27}Al(p,p_1\gamma)^{27}Al$
	1014	$^{27}Al(p,p_2\gamma)^{27}Al$
Si	1266	$^{30}Si(p,\gamma_1)^{31}P$
	1273	$^{29}Si(p,p_1\gamma)^{29}Si$
	1779	$^{28}Si(p,p_1\gamma)^{28}Si$
	2028	$^{29}Si(p,p_2\gamma)^{29}Si$
P	1266	$^{31}P(p,p_1\gamma)^{31}P$
S	1219	$^{34}S(p,\gamma_1)^{35}Cl$
Cl	1219	$^{35}Cl(p,p_1\gamma)^{35}Cl$
K	980	$^{41}K(p,p_1\gamma)^{41}K$
	1294	$^{41}Si(p,p_2\gamma)^{41}K$
Ca	371	$^{48}Ca(p,n\gamma_{2-1})^{48}Sc$

Element	Reaction	E_R (keV)	σ(mb) or Σ(eV)	Γ (keV)	Energies of strongest γ (MeV)
Li	$^{7}Li(p,\gamma)^{8}Be$	441,4	6 mb	10	17,65 (63 %), 14,75 (37 %)
C	$^{12}C(p,\gamma)^{13}N$	457	0,127 mb	35	2,36
	$^{13}C(p,\gamma)^{14}N$	551	1,44 mb	25	8,06 (80 %)
N	$^{15}N(p,\alpha\gamma)^{12}C$	429	300 mb	0,12	4,43
		897	800 mb	1,7	4,43
		1028	15 mb	140	4,43
O	$^{18}O(p,\gamma)^{19}F$	630		2,0	8,39 (42 %)
F	$^{19}F(p,\alpha\gamma)^{16}O$	340,5	102 mb	2,4	6,13 (96,5 %)
		483,6	32 mb	0,9	6,13 (79 %), 7,12 (20 %)
		668	57 mb	6	6,13 (81 %), 7,12 (19 %)
		872,1	661 mb	4,5	6,13 (68 %), 6,71 (24 %)
		935	180 mb	8,6	6,13 (76 %), 7,12 (21 %)
		1371	300 mb	11	6,13 (87 %)
Na	$^{23}Na(p,\alpha\gamma)^{20}Ne$	1011	55 eV	<0,1	1,634
		1164	160 eV	1,2	1,634
Mg	$^{26}Mg(p,\gamma)^{27}Al$	1548	3 eV	0,020	7,552–9,761
Al	$^{27}Al(p,\gamma)^{28}Si$	632	5,3 eV	0,016	10,42 (74 %)
		992	31 eV	0,10	10,76 (76 %)
Si	$^{30}Si(p,\gamma)^{31}P$	620	3,9 eV	0,068	7,897 (93 %)

Important proton-induced resonant reactions in light atoms.

Biological aspects, dosimetry





dose: absorbed energy per unit mass $D = \frac{E}{m}$ $[J / kg = gy]$

equivalent dose: multiplication by RBE $[J / kg = sv]$

Relative Biological Effectiveness:

(density of ionization)

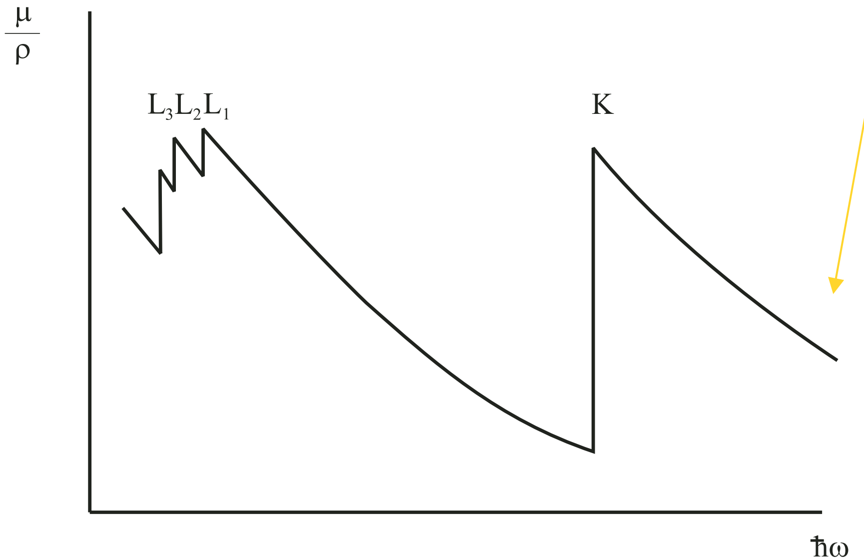
photons, fast electrons:	1	slow neutrons:	3
protons 1-10 MeV:	2	fast neutrons:	10
alpha particles:	10- 20	fission products:	20

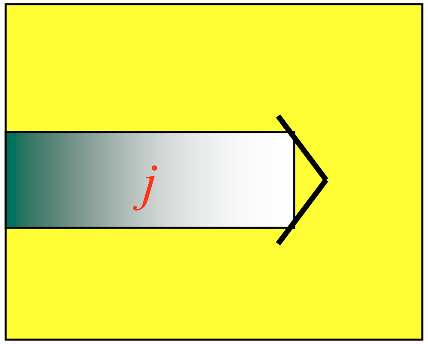
Photon-induced charged particles

1) Photoelectrons
2) Compton electrons
3) Pairs electron-positron

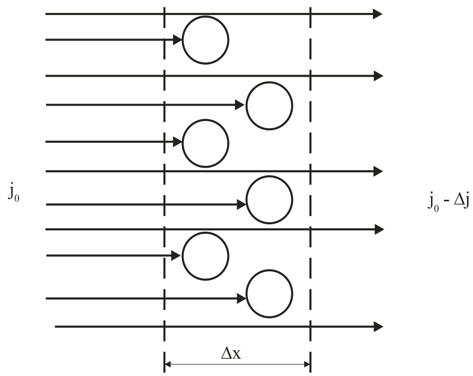
1. Photo effect

K-shell, hydrogenic atom:





$$j = j_0 e^{-\mu x}$$



$$dj\,\cancel{S} = -\underline{\sigma}\; j\; \underline{n}\,\cancel{S}\,dx$$

μ

$$\frac{\mu}{\rho} = \frac{\sigma}{M} N_A$$

$$\sigma_K = \frac{2^9\pi^2}{3}\alpha\, a_K^2 \left(\frac{\hbar\omega}{(Z-0.3)^2 Ry}\right)^{-4} \frac{\exp\left(-\frac{4}{a_K k}\arctan a_K k\right)}{1-\exp\left(-\frac{2\pi}{a_K k}\right)}$$

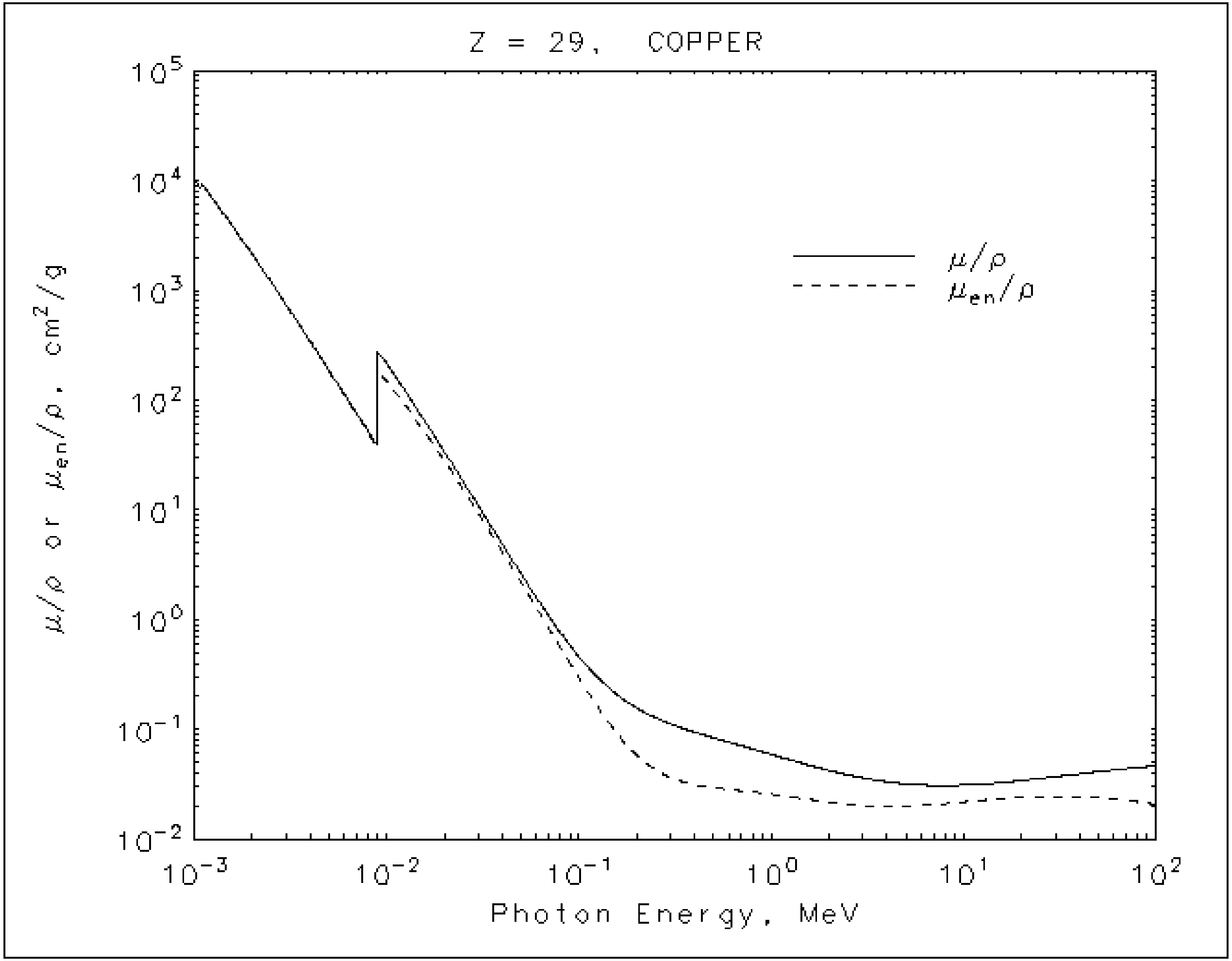
$$a_K = \frac{a_B}{(Z-0.3)^2}$$ K-shell radius

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega - E_B$$ Photoelectron energy

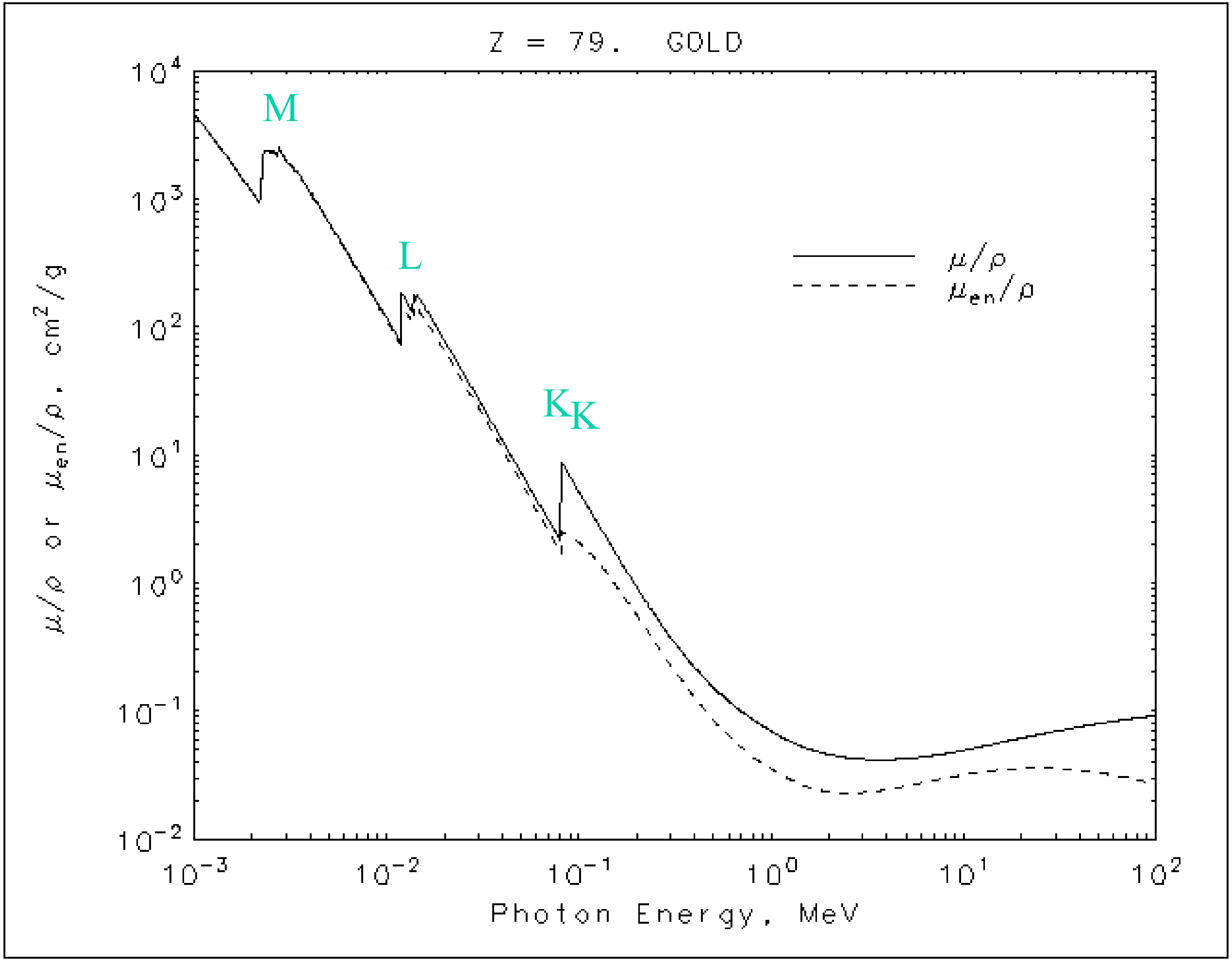
Scaled photon energy: $\varepsilon_{ph} = \frac{\hbar\omega}{(Z-0.3)^2 Ry}$

High-energy limit:

$$\boxed{\sigma_K = \frac{2^8\pi}{3}\alpha\, a_K^2\, \varepsilon_{ph}^{-\frac{7}{2}}}$$

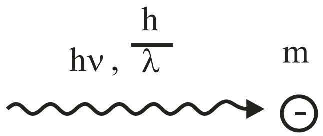


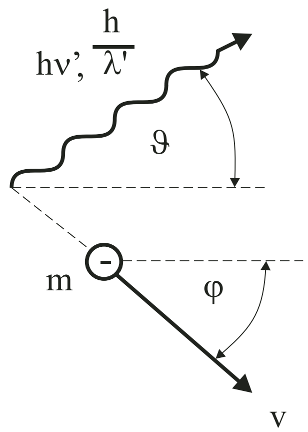
Hubbell, Seltzer, NIST



Hubbell, Seltzer, NIST

2. Compton scattering





$$\boxed{h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\vartheta)}}$$

Limits: 1) $h\nu << mc^2$: $h\nu' \approx h\nu$

2) $h\nu >> mc^2$: $h\nu' = \dfrac{mc^2}{1 - \cos\vartheta}$

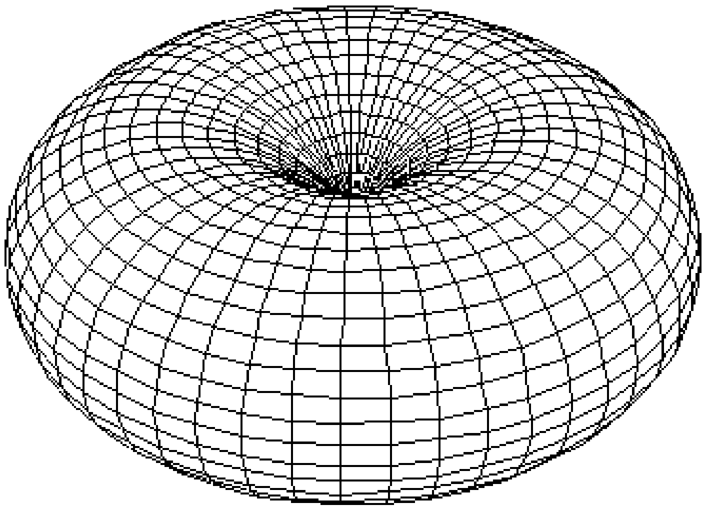
backscattering $(\vartheta = \pi)$: $\boxed{h\nu' = \dfrac{mc^2}{2}}$

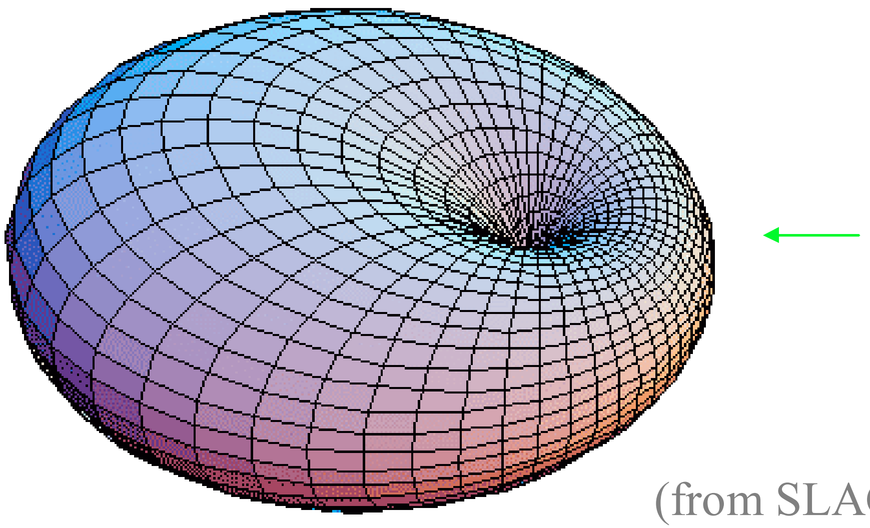
Differential cross section:

$$\frac{d\sigma_c}{d\Omega} = r_e^2 \underbrace{\frac{1+\cos^2\vartheta}{2}}_{} \cdot \frac{1+\dfrac{\varepsilon^2(1-\cos\vartheta)^2}{(1+\cos^2\vartheta)(1+\varepsilon(1-\cos\vartheta))}}{(1+\varepsilon(1-\cos\vartheta))^2}$$

Thompson cross section

$$\varepsilon = \frac{\hbar\omega}{mc^2}$$

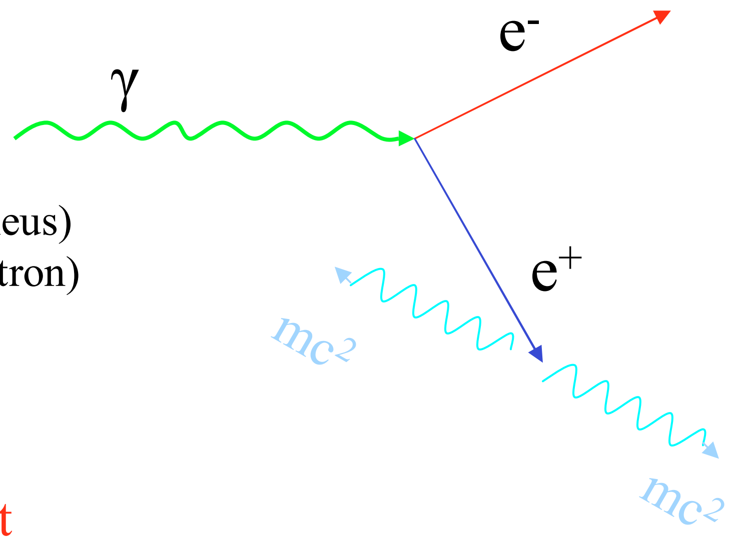




(from SLAC)

Total cross section: $\varepsilon \to \infty :$ $\sigma_C \approx \pi r_e^2 \dfrac{\ln 2\varepsilon}{\varepsilon} Z$

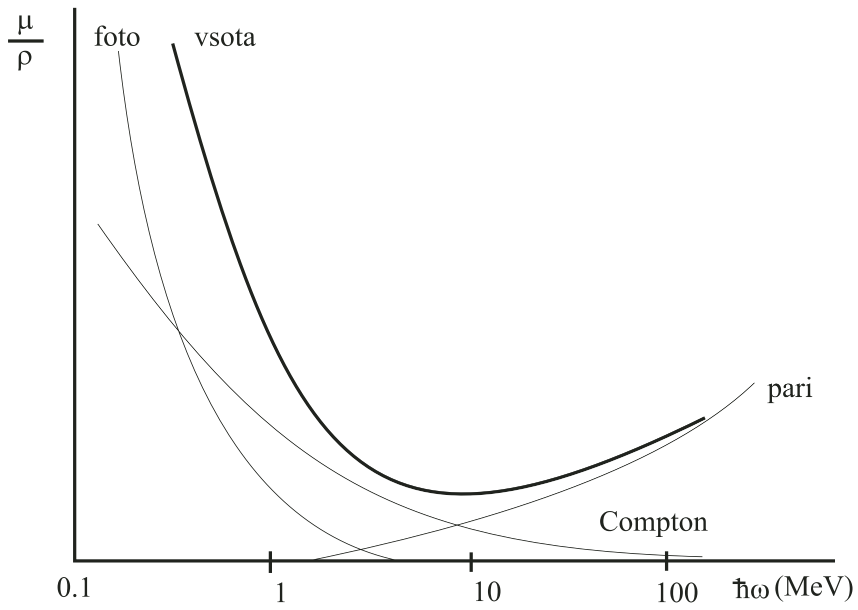
3) Pair production



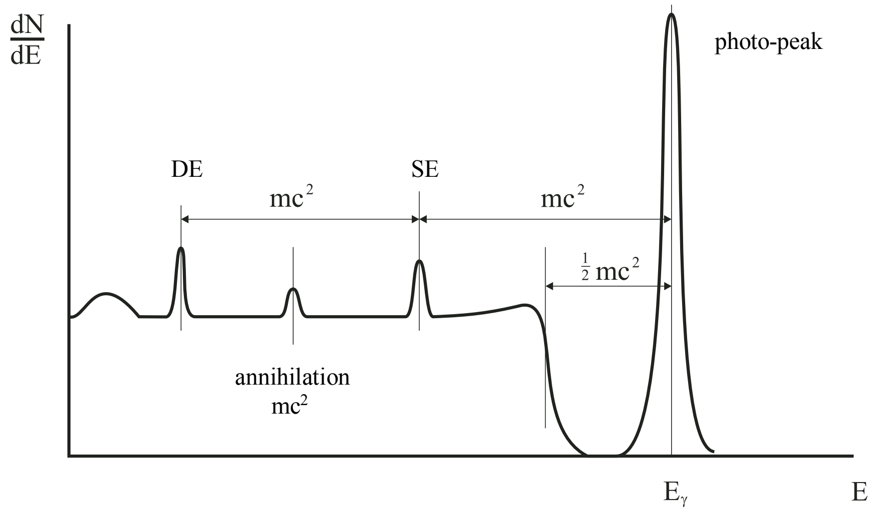
Threshold energy: $2mc^2$ (field of nucleus)
$4mc^2$ (field of electron)

$$\sigma_p \sim Z^2 \ln 2\varepsilon$$

Total attenuation coefficient



Measured gamma spectrum



X-ray attenuation coefficients

Tabulations

1) W.H. Mc Master et al. UCRL-50174 (1969) → CRC Handbook of Chemistry and Physics (…1975…)
2) **E.B. Saloman, J.H. Hubbell, J.H. Scofield, ADNDT 38 (1988) 1.**

Semiempirical fits

1) Wm. J. Veigele, Atomic Data 5 (1973) 51.
2) T.P. Thinh, J. Leroux, X-ray Spectrom. 8 (1979) 85.
3) I. Orlić et al., Nucl. Instr. and Meth. B74 (1993) 352.

Programs

XCOM (M.J. Berger, J.H. Hubbel, NBSIR 87-3595, 1987)

	E_x(keV)	μ_{LT}(cm^2/g)	μ_{XCOM}(cm^2/g)	diff.
Fe	1	1.11e4	9.08e3	22%
	3	555	558	-0.5%
	10	174	171	1.8%
N	1	3180	3310	-3.9%
	3	141	146	-3.4%
	10	3.52	3.88	-9.3%