The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR. 1745-5

SPRING SCHOOL ON SUPERSTRING THEORY
AND RELATED TOPICS

27 March - 4 April 2006

Worldsheet instanton effects in heterotic Calabi-Yau compactifications

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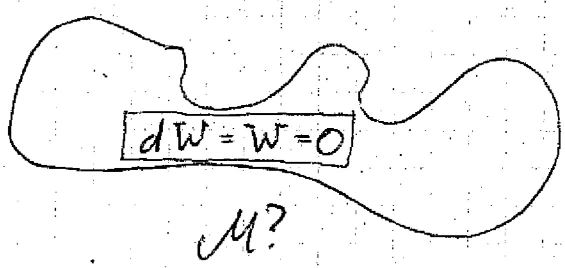
Worldsheet Instanton Effects in Het CY Compactifications, C. Beasley, Trieste 2006

$d=4$, $\mathcal{N}=1$ SUSY; $\mathbb{R}^{3,1}\times X$

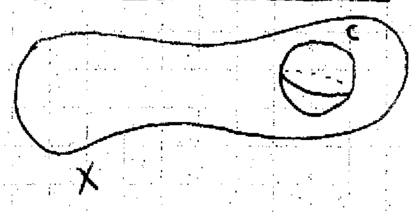


Superpotential: $W(\Phi^i)$ ← massless χsf

Weakly-coupled Het on (pert. in g_s) $\left\{\begin{array}{l} E \quad \text{stable, holomorphic bundle} \\ \downarrow \\ X \quad \text{CY 3fold}\end{array}\right.$

[DSWW 86]

Worldsheet Instantons



C = smooth, genus 0, holomorphic curve [$\mathbb{CP}^1$]

$$W_C = \exp\Big[\underbrace{-\frac{\mathrm{Area}(C)}{2\pi\alpha'} + i\int_C B}_{\chi\mathrm{sf}\ R\ =\ \text{radial modulus}}\Big]\times(\text{1-loop Dets})$$

$\neq 0$ in general

$$W = \sum_{C\subset X} W_C \overset{?}{\neq} 0$$

$= 0!$ for LINEAR (X, E) [SW 95, BS, BW 03]

I. Pert Het CY Compactification

[CHSW 85]

Start **classically**: $d=10$, $\mathcal{N}=1$ SUGRA+YM ($\alpha'=g_s=0$)

Gauge gp: $G = E_8\times E_8$ or $Spin(32)/\mathbb{Z}_2$

Massless spectrum:

gravity: $g \quad B \quad \phi \quad \lambda \quad \psi$ (dilatino, gravitino)

YM: $A \quad \chi$ (gaugino)

Ansatz: $\left\{\begin{array}{l}\mathbb{R}^{3,1}\times X \\ \nabla_X\phi = H = 0\end{array}\right.$

SUSY $\Rightarrow \delta_\varepsilon \lambda = \delta_\varepsilon\psi = \delta_\varepsilon \chi = 0$

$\delta\psi = \nabla_X \varepsilon = 0 \iff X$ is Calabi-Yau

$$\delta\chi = 0 \iff \underbrace{\underset{\underset{dA+A\wedge A}{\parallel}}{F^{(0,2)}} = F^{(2,0)}}_{\bar\partial_A^2 = F_A^{(0,2)} = 0,\ E\ \text{holomorphic}} = \underbrace{g^{i\bar j}F_{i\bar j} = 0}_{\text{stable}}$$

(*)

Standard Embedding

$E = TX$ solves $(*)$ [see page ①]

TX: holomorphic tangent bundle, $SU(3)$

$$\text{cf. } \overbrace{R_{\bar{i}\bar{j}k}{}^{l} = R_{ijk}{}^{l}}^{\text{Kähler}} = \overbrace{g^{i\bar{j}} R_{i\bar{j}k}{}^{l} = 0}^{\text{Ricci-flat}}$$

$i, j, k, l = 1 \dots 3$

$*$ (0,2) worldsheet SUSY

$\overset{E=TX}{\Longrightarrow}$ (2,2) worldsheet SUSY!

1-loop in α'

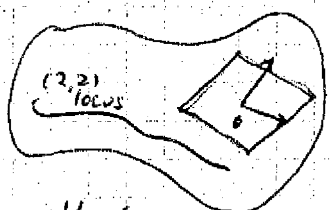
$$dH = \frac{\alpha'}{4\pi}\left(\mathrm{tr}(R\wedge R) - \mathrm{tr}(F\wedge F)\right)$$

~~Anomalies~~: $\left[\frac{1}{8\pi^2}\mathrm{tr}(R\wedge R)\right] = \left[\frac{1}{8\pi^2}\mathrm{tr}(F\wedge F)\right]$ in $H^2(X, \mathbb{Z})$

$$\Longrightarrow \begin{cases} dH = 0 & (2,2) \text{ locus} \\ dH = O(\alpha') & (0,2) \text{ locus} \end{cases}$$

Do classical solns remain valid in pert th?

Classical Moduli



$\mathcal{M}_{cl}(X, E)$

$$\text{Kähler}_{\mathbb{C}} = \underset{(dB=0)}{B} + i\omega \leftarrow \text{Kahler form}$$

$$[\delta T_{\bar{x}j}] \in H^1_{\bar\partial}(X, \Omega^1_X) = H^{1,1}_{\mathbb{C}}(X)$$

$\uparrow$

Radial modulus: $(B + i\omega)|_0 = (b + ir)\omega_0$, $\quad r, b \in \mathbb{R}$

$$\omega_0 \in H^{1,1}(X) \cap H^2(X, \mathbb{Z})$$

$$\chi_{sf} \quad R = (r - ib) + \cdots$$

Dilaton

$$\chi_{sf} \quad S = (e^{-2\phi} + ia) + \cdots$$

a: axion, dual to $B_{\mu\nu}$ in $\mathbb{R}^{3,1}$

Cplx Str X:

$$[\delta U_{\bar{z}}{}^{j}] \in H^1_{\bar\partial}(X, TX) \simeq H^{2,1}(X)$$

Cplx Str E:

$$[\delta A_{\bar{x}}{}^{b}{}_{a}] \in H^1_{\bar\partial}(X, \mathrm{End}\, E), \quad \mathrm{End}\, E = E^* \otimes E$$

a: gauge index

<u>Nonrenormalization of W_{cl} [DS; W 86]</u>

① $(r, \phi) \longleftrightarrow (\alpha', g_s)$

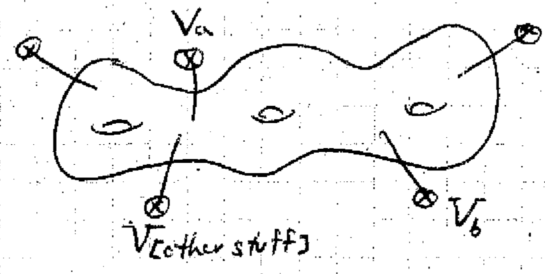
② Holomorphy $\Longrightarrow W = W(R, S)$

③ $\underbrace{(b, a)}_{B}$ decouple in pert th ....

$\Longrightarrow W = W_{cl}$ to all orders in (α', g_s)

W_{pert}

$[k=0]$



$(\text{n}\ell\sigma\text{m}) \; \Phi : \Sigma \longrightarrow \mathbb{R}^{3,1} \times X$

$$V_b = -i \int_\Sigma \Phi^*(\omega_0) = 0 \quad \underline{\text{if}} \quad [\Phi(\Sigma)] = 0 \in H_2(X)$$

pert in α'

$$V_a = -i \int_\Sigma \Phi^*(\cancel{e_{\mu\nu}}\, dx^\mu \wedge dx^\nu) \equiv 0$$

0 in $H^2(\mathbb{R}^{3,1}, \mathbb{Z})$

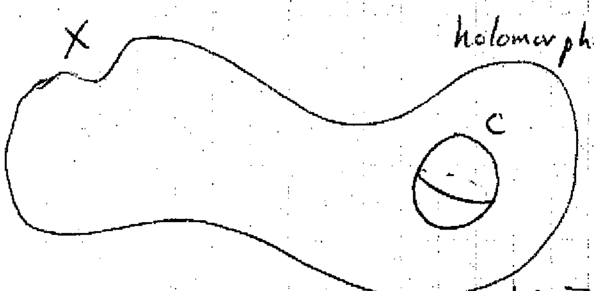
polarization tensor in $\mathbb{R}^{3,1}$

<u>II. Worldsheet Instantons</u>

$W_{cl} \longleftarrow$ non-pert. corrections in α' at string tree-level

<u>Req'd</u>: $[\Phi(\Sigma) = C] \neq 0$ in $H_2(X)$

holomorphy?



$$S_E = \int_\Sigma \Phi^*(\omega - iB) = R \overbrace{\int_\Sigma \Phi^*(\omega_0)}^{d \in \mathbb{Z}}$$

$$\Longrightarrow \boxed{W_a = \exp(-dR) \times (\cdots\cdots)}$$

$(\alpha' = 1)$

$$S_E = \frac{i}{2} \int_\Sigma d^2z \, g_{i\bar{j}} \left(\partial_z \phi^i \, \partial_{\bar{z}} \bar{\phi}^{\bar{j}} + \partial_z \bar{\phi}^{\bar{j}} \, \partial_{\bar{z}} \phi^i \right) + \cdots$$

$$= \| \partial_z \phi^i \|^2 + \| \partial_{\bar{z}} \phi^i \|^2$$

$$\geq \| \;\; " \;\; \|^2 - \| \;\; " \;\; \|^2$$

$$\left[\omega = \frac{i}{2} g_{i\bar{j}} \, d\phi^i \wedge d\bar{\phi}^{\bar{j}} \right]$$

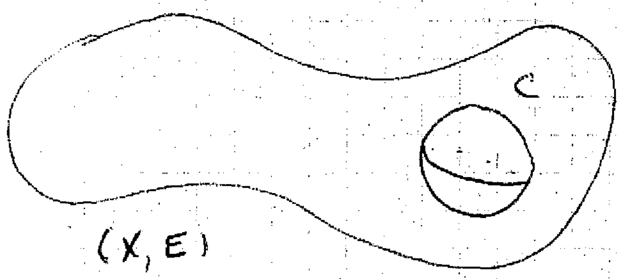
$$= \int_\Sigma \Phi^*(\omega) = \text{Area}(C)$$

for

$$\text{BPS} \Longleftrightarrow \boxed{\partial_{\bar{z}} \phi^i = 0} \text{ or c.c.} \Longleftrightarrow C \text{ holomorphic}$$

II. Worldsheet Instantons (cont'd)

C = genus 0, holomorphic



When is $W_C \neq 0$?

RNS, Berkovits, Green-Schwarz, ...

[BBS, HM, W 99]

Physical Gauge: Type I D1-brane

$C \longrightarrow \mathbb{C}^2 \times X$ smooth embedding

Bosons: (x^μ, y^m) $\quad \mathcal{O}^{\oplus 2} \oplus \mathcal{N}$

$\mu, m = 1, 2$

Fermions:

$(L, \bar\partial)$ $\quad \lambda \quad S_-(TC) \otimes E|_C$

(R, ∂) $\quad \Psi \quad \underbrace{S_+(TC) \otimes S_+(\mathcal{O}^2 \oplus \mathcal{N})}$

$[S_+(\mathcal{O}^2) \otimes S_+(\mathcal{N})] \oplus$
$\oplus [S_-(\mathcal{O}^2) \otimes S_-(\mathcal{N})]$

Twist: $\overset{SU(4):\ 4 \ \oplus\ \bar{4}}{S_+(TX) \otimes S_-(TX)} \simeq \bigoplus_{p=0}^{3} \bar{\Omega}^p_X$

Clifford: $\{\Gamma^i, \Gamma^j\} = \{\Gamma^{\bar\imath}, \Gamma^{\bar\jmath}\} = 0$

$\{\Gamma^i, \Gamma^{\bar\jmath}\} = 2 g^{i\bar\jmath}$

$$\overset{4 \oplus \bar 4}{\Psi} = \Big(\overset{(0,0)}{a} + \overset{(0,1)}{a_{\bar\imath}} \Gamma^{\bar\imath} + \overset{(0,2)}{a_{\bar\imath\bar\jmath}} \Gamma^{\bar\imath\bar\jmath} + \overset{(0,3)}{a_{\bar\imath\bar\jmath\bar k}} \Gamma^{\bar\imath\bar\jmath\bar k}\Big)\underbrace{|\ \rangle}_{K_X^{1/2} \simeq \mathcal{O}_X}$$

Near C, $0 \to TC \to TX|_C \to \mathcal{N} \to 0$

$\Longrightarrow S_+(TC) \otimes S_-(\mathcal{N}) \simeq \mathcal{O} \oplus \Lambda^2 \mathcal{N}^* \simeq \bar{\mathcal{O}} \oplus \bar{\Omega}^1_C$

$S_+(TC) \otimes S_+(\mathcal{N}) \simeq \mathcal{N}^* \simeq \bar{\mathcal{N}}$

Twisted Fermions (R,∂) — 0-modes

$$\theta^\alpha \quad S_-(\underset{\mathbb{C}^2}{\overset{\shortparallel}{\theta^2}}) \otimes \bar{\mathcal{O}} \qquad d^2\theta$$

$$\theta^\alpha_{\bar z} \quad S_-(\theta^2)\otimes \bar\Omega^1_C \qquad \overset{\text{genus}}{2g}$$

$$\bar\chi^{\bar m}_{\dot\alpha} \quad S'_+(\theta^2)\otimes \bar N \qquad 2p,$$

$$p=\dim_{\mathbb{C}} H^0_{\bar\partial}(C,N)$$

Computing $W_C(g=p=0)$

$$\delta S'_{\rm eff} = \overbrace{\int d^4x\, d^2\theta}^{\text{Goldstone}}\ W_C$$

$$W_C = \exp\left[-\frac{\mathrm{Area}(C)}{2\pi\alpha'} + i\int_C B\right]\times$$

$$\times\ \frac{\mathrm{Pfaff}(\bar\partial_{E(-1)})}{(\det{}'\bar\partial_{\mathcal{O}})^2\,(\det\bar\partial_{\mathcal{O}(-1)})^2}$$

$$\boxed{E(-1) \equiv S_-(TC)\otimes E|_C \simeq \mathcal{O}(-1)\otimes E|_C}$$

$$W_C \neq 0 \iff E|_C \text{ trivial}$$

Vanishing Result for W_C

- $W_C = 0 \iff \mathrm{Pfaff}(\bar\partial_{E(-1)}) = 0$

$$\iff \dim \mathrm{Ker}\,\bar\partial_{E(-1)} > 0$$

$$\iff \underbrace{\exists\ \lambda\ \text{0-modes}}_{\text{count!}}$$

Fact: $V|_{\mathbb{CP}^1} \cong \bigoplus_{k=1}^{rk(V)} \mathcal{O}(a_k)$, $a_k \in \mathbb{Z}$

V: hol. vector bundle

[See Witten, hep-th/9907041]

Result: Always λ 0-modes unless

$$E|_C \simeq \mathcal{O}^{\oplus 32} \quad \text{(trivial)}$$

Application: $E = TX \longleftarrow$ (2,2) locus

λ 0-modes from L w.s. SUSY

$$\Longrightarrow E|_C = \underset{\substack{\shortparallel \\ \mathcal{O}(+2)\ \text{nontrivial}}}{TC} \oplus N$$

$$\Longrightarrow W_C = 0 \ \text{for}\ (2,2)$$

See also [Distler, Greene 88]

[BCOV, AGNT]

Type IIA Worldsheet Instantons

Ⓛ Ⓡ

$$\left\{ \begin{array}{ll} \theta^\alpha_L \sim S_-(\theta^?) \otimes \Theta & \theta^\alpha_R \sim S_-(\theta^?) \otimes \overline{\Theta} \\ \theta^\alpha_z \sim S_-(\theta^?) \otimes \Omega^1_C & \theta^\alpha_{\bar z} \sim S_-(\theta^?) \otimes \overline{\Omega}^1_C \\ \overline{\chi}^m_{\dot\alpha} \sim S_+(\theta^?) \otimes N & \overline{\chi}^{\bar m}_{\dot\alpha} \sim S_+(\theta^?) \otimes \overline{N} \end{array} \right.$$

Set $\rho^\alpha = \theta^\alpha_z\, dz + \theta^\alpha_{\bar z}\, d\bar z$.

Assume C isolated $(p=0)$, genus g.

Interaction: $I_C^{int} = \int_C \underbrace{\mathcal{W}_{\alpha\beta}}_{\mathcal{N}=2 \text{ Weyl multiplet}} \rho^\alpha \wedge \rho^\beta$, $\quad \mathcal{W}_{\alpha\beta} = \mathcal{W}_{\mu\nu} \Gamma^{\mu\nu}_{\alpha\beta}$

$$\delta S_{eff} = \int d^4x\, d^4\theta\, d^{4g}\rho\, (I_C^{int})^{2g} \times \exp\left[-\frac{Area(C)}{2\pi\alpha'} + i\int_C B\right]$$

Use $J(\eta, \xi) = \int_C \eta \wedge \xi$ for $\eta, \xi \in H^2(C, \mathbb{C})$

$$\Rightarrow \delta S_{eff} = \int d^4x\, d^4\theta\, (\mathcal{W}_{\alpha\beta}\mathcal{W}^{\alpha\beta})^g \times \exp\left[-\frac{Area(C)}{2\pi\alpha'} + i\int_C B\right] \checkmark$$

Heterotic Generalizations

$C = \mathbb{CP}^1$ moves in a 1-parameter family $(g=0, p=1)$

Eg.
$$\left.\begin{array}{ccc} (\text{flat}) & \otimes & E \\ \downarrow & & \downarrow \\ X = \underbrace{T^2}_{\text{dilaton } \phi \in \text{ vector}} & \times & \underbrace{K3}_{\text{hyper}} \end{array}\right\} \mathcal{N}=2 \text{ SUSY}$$

SUSY $\Rightarrow$ $C \hookrightarrow K3$ genus 0 and holomorphic in <u>some</u> cplx str on K3

$\Rightarrow$ 1-param family parametrized by T^2

$$\delta S_{eff} = \int d^6x\, d^4\widetilde{\theta}\ \Psi_C \qquad (d=6 \text{ Weyl})$$

$$\Psi_C = \exp\left[-\frac{Area(C)}{2\pi\alpha'} + i\int_C B\right] \times \frac{\mathrm{Pfaff}(\bar\partial_{E(-1)})}{(\det' \bar\partial_\Theta)^3 (\det' \bar\partial_{\Theta(-2)})}$$

Correction to <u>metric</u> on hyper moduli space!

III. Vanishing Results for W

A Sketch of [SW, 95]:

$$\begin{array}{c} (0,2)\ \text{GLSM} \\ \downarrow \scriptstyle IR \\ (0,2)\ \text{NLSM} \end{array}$$

X = quintic in $\mathbb{CP}^4$

$\longrightarrow$ complete int in cpct toric Y

E = deformation of TX

$\longrightarrow$ monad bundle

$$A \xrightarrow{\alpha} B \xrightarrow{\beta} C\,, \quad E = \frac{\mathrm{Ker}\,\beta}{\mathrm{Im}\,\alpha}$$
$$\beta \circ \alpha = 0$$

Holomorphy + Compactness

- Global SUSY: W is a function on $\mathcal{M}$

- Local SUSY: W is a section of
  [Bagger, Witten 82]

$$\begin{array}{c} \mathcal{L} \\ \downarrow \\ \mathcal{M} \end{array} \quad \text{negative curvature}$$

-

$$\mathcal{K}(\Phi^i, \bar{\Phi}^{\bar{i}}) \longmapsto \mathcal{K} + f(\Phi^i) + \bar{f}(\bar{\Phi}^{\bar{i}})$$
$$W \longmapsto \exp\left(-\tfrac{1}{M_{pl}^2} f(\Phi^i)\right) W$$

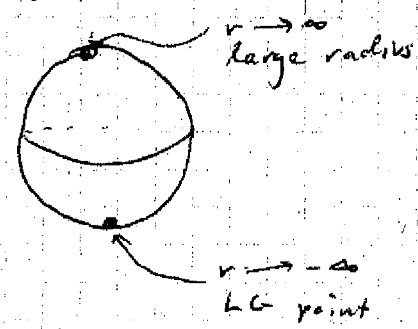
(Kodaira) If $\mathcal{M}$ cpct (non-singular), $\mathcal{L}$ has no non-zero holomorphic sections.

$\overset{\text{holomorphy}}{\Longrightarrow}$ $W = 0$ or W has a pole on $\mathcal{M}$

Fact: GLSM compactifies $\mathcal{M}_{cl}(X, E)$

Example: Kähler modulus of X

$$q = \exp\left(-2\pi(r - ib)\right) \in \mathbb{C}^*$$



$\Longrightarrow \mathbb{C}^*$ compactified to $\mathbb{CP}^1$

[SW] $\Longrightarrow$ W has no poles on $\mathcal{M}$.

$\Longrightarrow W = 0.$ ∎

Corollary of [SW]:

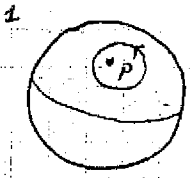
$$W_C = \exp\Big[-\underbrace{\frac{\overbrace{\mathrm{Area}(C)}^{\int_C \omega}}{2\pi\alpha'} + i\int_C B}_{-dR}\Big] \times \left.\frac{\mathrm{Pfaff}(\bar\partial_{E(-1)})}{(\det{}'\,\bar\partial_{\mathcal{O}})^2(\det\bar\partial_{\mathcal{O}(-1)})^2}\right\}\begin{matrix}\text{cplx}\\ \text{str}\end{matrix}$$

X has 2875 lines, $\{C_I\}$

$$\xRightarrow{W=0}\quad \sum_{I=1}^{2875} \exp\Big(i\int_{C_I} B\Big)\left.\frac{\mathrm{Pfaff}(\bar\partial_{E(-1)})}{(\det\bar\partial_{\mathcal{O}(-1)})^2}\right|_{C_I} = 0$$

Residue Thm:

$\mathbb{CP}^1$



z

ϖ = meromorphic 1-form on $\mathbb{CP}^1$, simple poles p

$$= \frac{g(z)\,dz}{f(z)}$$

$$\mathrm{Res}_p(\varpi) = \frac{1}{2\pi i}\oint_\gamma \varpi = \frac{g}{\partial f/\partial z}(p).$$

$$\text{Thm: } 0 = \sum_p \mathrm{Res}_p(\varpi) = \sum_p \frac{\overset{\mathrm{Pfaff}}{g}}{\underset{\det\bar\partial}{\partial f/\partial z}}(p).$$

Twisted Het String [AGNT 96]

Bosons $\Phi: \Sigma \to X$

$(\phi^i, \bar\phi^{\bar\imath})$ local coords on $\mathcal{M}^\infty = \{\Phi: \Sigma \to X\}$

Fermions $\psi^i_{\bar z}$ $\quad \bar\Omega^1_\Sigma \otimes \Phi^*(TX)$

coords on $\left\{\begin{matrix} \mathcal{V}^\infty \\ \downarrow \\ \mathcal{M}^\infty\end{matrix}\right\}$

$\psi^{\bar\imath}$ $\quad \Phi^*(\overline{TX})$

coords on $\overline{T\mathcal{M}^\infty}$

$+\ \lambda$'s

BRST-operator Q

$$\boxed{\begin{matrix}\delta\phi^i = 0 & \delta\bar\phi^{\bar\imath} = \psi^{\bar\imath} \\ \delta\psi^i_{\bar z} = \bar\partial_{\bar z}\phi^i & \delta\psi^{\bar\imath} = 0\end{matrix}}$$

$Q^2 = 0$

$$S_E = \int_\Sigma d^2z\, \delta\big(g_{j\bar\jmath}\,\partial_z\bar\phi^{\bar\jmath}\,\psi^j_{\bar z}\big) + \cdots$$

Toy Model

NB: Not the moduli space $\mathcal{M}$!

Data: M cpct, cplx, $\dim_{\mathbb{C}} = n$

V hol. vector bundle, rk $= r$

s hol. section of V

g hol. section of $\Omega^n_M \otimes \Lambda^r V$

Local coords $(z^i, \bar{z}^{\bar{\imath}})$ on M

Grassmann coords $\theta^{\bar{\imath}}$ on $\overline{TM}$

Grassmann coords χ^α on V

SUSY: $\boxed{\begin{matrix} \delta z^i = 0 & \delta \bar{z}^{\bar{\imath}} = \theta^{\bar{\imath}} \\ \delta \chi^\alpha = s^\alpha & \delta\theta^{\bar{\imath}} = 0 \end{matrix}}$

$\delta^2 = 0$, as s holomorphic

"operator"

$$\mathcal{O}(z, \bar{z}, \theta^{\bar{\imath}}, \chi^\alpha) = \sum_{p,q} \underbrace{\mathcal{O}_{\alpha_1 \cdots \alpha_p \bar{\imath}_1 \cdots \bar{\imath}_q}}_{\overline{\Omega}^q_M \otimes \Lambda^p V^*} \chi^{\alpha_1} \cdots \chi^{\alpha_p} \theta^{\bar{\imath}_1} \cdots \theta^{\bar{\imath}_q}$$

Complex: $\bigoplus_{(p,q)} \overline{\Omega}^q_M \otimes \Lambda^p V^*$, grading $= q - p$

$$\delta = \bar{\partial} + \iota_s$$

[Carrell, Lieberman, K. Liu, Andersson]
"Holomorphic Equivariant Cohomology"

A SUSY Integral

$$Z = \int g\, d\mu\, \exp(-t\, S_E)$$

$t > 0$ real parameter

$$g\, d\mu \equiv \overbrace{g(z)\, \underbrace{d^n z\, d^n \bar{z}\, d^n \theta}_{\text{NOT canonical}}\, d^r \chi}^{\text{canonical}}$$

$$S_E = \delta(h_{\bar{\alpha}\alpha} \bar{s}^{\bar{\alpha}} \chi^\alpha)$$
$$= h_{\bar{\alpha}\alpha} \bar{s}^{\bar{\alpha}} s^\alpha + h_{\bar{\alpha}\alpha} D_{\bar{\jmath}} \bar{s}^{\bar{\alpha}} \theta^{\bar{\jmath}} \chi^\alpha$$

positive hermitian metric on V

If $n - r \neq 0$, $Z = 0$ trivially.

Z independent of (t, h):

Pf: $\dfrac{dZ}{dt} = -\int_M g\, d\mu\, S_E \exp(-t S_E) = -\langle S_E \rangle$

But for any $\mathcal{O}$,

$$\langle \delta \mathcal{O} \rangle = \int_M g\, d\mu\, \delta\mathcal{O} \exp(-t S_E)$$
$$\overset{\delta = \bar{\partial} + \iota_s}{=} \int_M d\mu\, \delta\big(g\, \mathcal{O} \exp(-t S_E)\big)$$

$[M \text{ cpct}] = 0$ Stokes Thm.

As $S_E = \delta(\cdots)$, $\dfrac{dZ}{dt} = \langle \delta(\cdots) \rangle = 0$. ∎

Sim'ly for metric h.

Residue Thm: [as $\int d\theta \cdot 1 = 0$ for Grassmann integration]

① Set $t = 0 \Rightarrow Z = \int_{\mathcal{M}} g\, d\mu \cdot 1 = 0$

② $t \longrightarrow \infty \Rightarrow Z$ localizes on $\{s = 0\} \subset \mathcal{M}$

$\mathcal{M}$

dim $\mathcal{M}$ = rk V

Sps s vanishes at isolated points $p \in \mathcal{M}$, with $\det(\partial_j s^\alpha)|_p \neq 0$.

Thm: $0 \overset{\text{from ①}}{=} Z \overset{\text{from ②}}{=} \sum_p Z(p)$, where $Z(p)$ is local contribution to Z from p.

Compute $Z(p)$ — expand S_E around point p in local coords $(z, \bar{z})$

$$Z(p) = \int_{\mathcal{M}} g\, d\mu \exp\Big[-t\big(h_{\bar{\alpha}\alpha} \partial_{\bar{j}} \bar{s}^{\bar{\alpha}} \partial_j s^\alpha|_p\, \bar{z}^{\bar{j}} z^j + h_{\bar{\alpha}\alpha} \partial_{\bar{j}} \bar{s}^{\bar{\alpha}}|_p\, \theta^{\bar{j}} \chi^\alpha + \ldots\big)\Big]$$

- Evaluate Z as a Gaussian as $t \longrightarrow \infty$

$$\Rightarrow Z(p) = \frac{g \overbrace{\det(h_{\bar{\alpha}\alpha} \partial_{\bar{j}} \bar{s}^{\bar{\alpha}})}^{\text{[from fermions]}}}{\underbrace{\det(h_{\bar{\alpha}\alpha} \partial_{\bar{j}} \bar{s}^{\bar{\alpha}} \partial_j s^\alpha)}_{\text{[from bosons]}}}\Bigg|_p$$

$$\Rightarrow Z(p) = \frac{g}{\det(ds)}\Big|_p \Longleftarrow \boxed{\text{residue}}$$

cf $[\mathrm{Pfaff}(\partial_{E(-1)})]$

cf $[s = \partial_{\bar{z}} \phi^i,\ \det(\partial_{\bar{z}})]$

Main Ideas

1) (0,2) worldsheet SUSY

2) Cpctness of $\mathcal{M}$

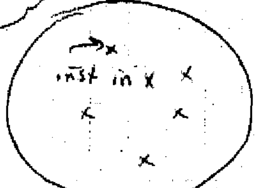
In nlsm, $\mathcal{M} \overset{\infty}{=} \cancel{\{\Phi : \Sigma \to X\}}$ NOT cpct!

Use quintic $X \longrightarrow Y = \mathbb{CP}^4$

LIFT

$\mathcal{M}_d^{inst}(X) = \left\{ \begin{matrix} \text{deg } d \text{ hol.} \\ \text{maps } \Phi : \mathbb{CP}^1 \to X \end{matrix} \right\} \subseteq \mathcal{M}_d^{inst}(Y) = \left\{ \begin{matrix} \text{deg } d \text{ hol.} \\ \text{maps } \Phi : \mathbb{CP}^1 \to Y \end{matrix} \right\}$

instantons in Y



$$\sum_{\mathcal{M}_d^{inst}(X)} (\ldots) = \int_{\mathcal{M}_d^{inst}(Y)} (\ldots) = 0 \text{ ! (Fermion 0-modes)}$$

Half-Linear Heterotic String

- X = quintic in $Y = \mathbb{CP}^4$ $[\Phi^0 : \Phi^1 : \cdots : \Phi^4]$

$\downarrow$

$J(\Phi)$ hol. sect. of $\mathcal{O}(5)$ on Y

- E on X pulls back from Y

[NB: in GLSM, E is a monad on X]

$\boxed{\text{Bosons}}$ $\Phi : \Sigma \longrightarrow Y$

nlsm $\underbrace{(\phi^i, \bar{\phi}^{\bar{i}})}$ $i = 1 \cdots 4$

local, NOT homog., coords on $\mathbb{CP}^4$

Fermions

$$R\begin{cases}\psi^{i}_{\bar z} & \bar\Omega^{1}_{\Sigma}\otimes\Phi^{*}(TY)\\ \psi^{\bar\imath} & \Phi^{*}(\overline{TY})\end{cases}$$

$$L\begin{cases}\chi_{z} & \Omega^{1}_{\Sigma}\otimes\Phi^{*}(\mathcal{O}(-5))\\ \bar\chi & \Phi^{*}(\mathcal{O}(5))\\ \lambda^{a},\ a=1\ldots32 \leftarrow \text{irrelevant for following}\end{cases}$$

conjugate $(\chi_z,\bar\chi)$

Anomaly Cancellation

$TY|_{X}\cong TX\oplus\mathcal{O}(5)$ smoothly

normal bundle $\begin{cases}\psi_{\bar z} & \bar\Omega^{1}_{\Sigma}\otimes\Phi^{*}(\mathcal{O}(5))\\ \bar\psi & \Phi^{*}(\mathcal{O}(-5))\end{cases}$

SUSY
$$\boxed{\begin{array}{ll}\delta\phi^{i}=0 & \delta\phi^{\bar\imath}=\psi^{\bar\imath}\\ \delta\psi^{i}_{\bar z}=\partial_{\bar z}\phi^{i} & \delta\psi^{\bar\imath}=0\\ \delta\bar\chi=J(\phi^{i}) & \delta\chi_{z}=0\end{array}}$$

Action: $S=S_0+S_X+\int_{\Sigma}\Phi^{*}(\omega_{\mathbb{C}})+\underset{\text{usual kinetic terms for }\lambda\text{'s}}{S_\lambda}$

$$S_0'=t\int_{\Sigma}d^2z\,\delta\left(g_{\bar\imath i}\,\partial_z\bar\phi^{\bar\imath}\psi^{i}_{\bar z}+\bar J\bar\chi\right)$$

$$\Rightarrow S_0=t\int_{\Sigma}d^2z\left(g_{\bar\imath i}\,\partial_z\bar\phi^{\bar\imath}\,\partial_{\bar z}\phi^{i}+g_{\bar\imath i}D_z\psi^{\bar\imath}\psi^{i}_{\bar z}+\bar J J+\psi^{\bar\imath}\bar D_{\bar\imath}\bar J\bar\chi\right)$$

[Annihilated by δ!]

curvature of $\mathcal{O}(5)$

$$S_X'=\int_{\Sigma}d^2z\left(D_{\bar z}\bar\chi\,\chi_z-\psi^{i}_{\bar z}D_iJ\,\chi_z+F_{\bar\imath i}\psi^{\bar\imath}\psi^{i}_{\bar z}\bar\chi\chi_z\right)$$

$t\to\infty$, localizes on $\boxed{\partial_{\bar z}\phi^{i}=J(\phi)=0}$

Half-Linear Residue Thm - 3 Steps

1) $\bar J\longrightarrow\infty$

X nonsingular $\Longleftrightarrow$ $dJ\neq0$ on X

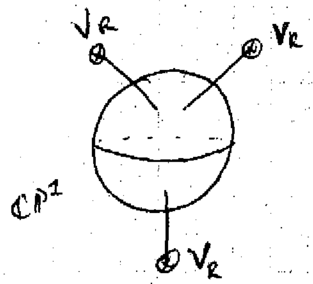
normal

masses $\int_{\Sigma}d^2z\left(\bar\phi\,\overline{DJ}\,DJ\phi+\bar\psi\,\overline{DJ}\,\bar\chi-\psi_{\bar z}DJ\chi_z\right)$

$\int$-out massive modes at 1-loop $\Rightarrow$ nlσm on X

2) Probe for W

$W = \exp(-dR) \times (\cdots)$



pulls back from Y

$V_R = \omega_{\bar{\imath} i}\, \partial_z \varphi^i \psi^{\bar{\imath}}$

$\langle V_R V_R V_R \rangle = 0 \Leftrightarrow W = 0$

$\overset{\shortparallel}{\dfrac{\partial^3 W}{\partial R^3}}$

3) Set $\bar{J} = 0 \Rightarrow \langle V_R V_R V_R \rangle = 0$

$\bar{J} \gg g_{\bar{\imath} i} \Rightarrow$ (twisted) nlσm on X

$\bar{J} \ll g_{\bar{\imath} i} \Rightarrow \int$ over $\mathcal{M}_d^{inst}(Y)$

Recall $\mathcal{M}_d^{inst}(Y) = \{\text{hol } \Phi : \mathbb{CP}^1 \xrightarrow{d} Y = \mathbb{CP}^4$

dense, open $\subset \mathbb{CP}^{5d+4}$

To describe $\mathcal{M}_d^{inst}$ explicitly, consider

$\Phi : \mathbb{CP}^1 \xrightarrow{d} Y = \mathbb{CP}^4$

$[U:V] \qquad [\Phi^0 : \cdots : \Phi^4]$

$\Phi^0 = P^0(U,V) \overset{(= U\tilde{P}^0)}{=} \underbrace{a_0^0 U^d + a_1^0 U^{d-1} V + \cdots a_d^0 V^d}_{d+1}$

$\Phi^4 = P^4(U,V) \overset{(= U\tilde{P}^4)}{=} a_0^4 U^d + a_1^4 U^{d-1} V + \cdots + a_d^4 V^d$

$\underline{a} \in (\mathbb{C}^{5(d+1)} \setminus \{0\}) / \mathbb{C}^* \simeq \mathbb{CP}^{5d+4}$

$[0:0:0:0:0] \notin \mathbb{CP}^4 \qquad$ "quasimaps"

$\mathcal{M}_d^{inst}(Y) = \{(P^0, \ldots, P^4)$ have no common zeroes on $\mathbb{CP}^1\}$

compactified by maps of lower degree

0-modes on $\mathcal{M}_d^{inst}(Y)$

$(\varphi^i, \varphi^{\bar{\imath}})$	$5d+4 - 3 = 5d+1$	fix $SL(2,\mathbb{C})$
$\psi^{\bar{\imath}}$	$5d+4 - 3 = 5d+1$	
$\bar{\chi}$	$5d+1$ from $\Phi^*(\mathcal{O}(5)) = \mathcal{O}(5d)$	
$\psi^i_{\bar{z}}$	0	
χ_z	0	

$\bar{J} = 0 \Rightarrow$ 0-modes of $(\psi^{\bar{\imath}}, \bar{\chi})$ kill $\langle V_R V_R V_R \rangle = 0$

$\Rightarrow W = 0 \quad \blacksquare$

Open Questions

1) Nonvanishing Results for W?

2) Generalization to Brane Instantons in $\mathcal{N}=1$ Backgds?

3) Non-perturbative Existence of $\mathcal{N}=1$ Minkowski Vacua? ($\Lambda_{cc}=0$)