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WORKSHOP ON DRIVEN STATES IN SOFT AND BIOLOGICAL MATTER
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Spontaneous Flows and Instabilities in Active Gels: Applications to Cytoskeleton Dynamics

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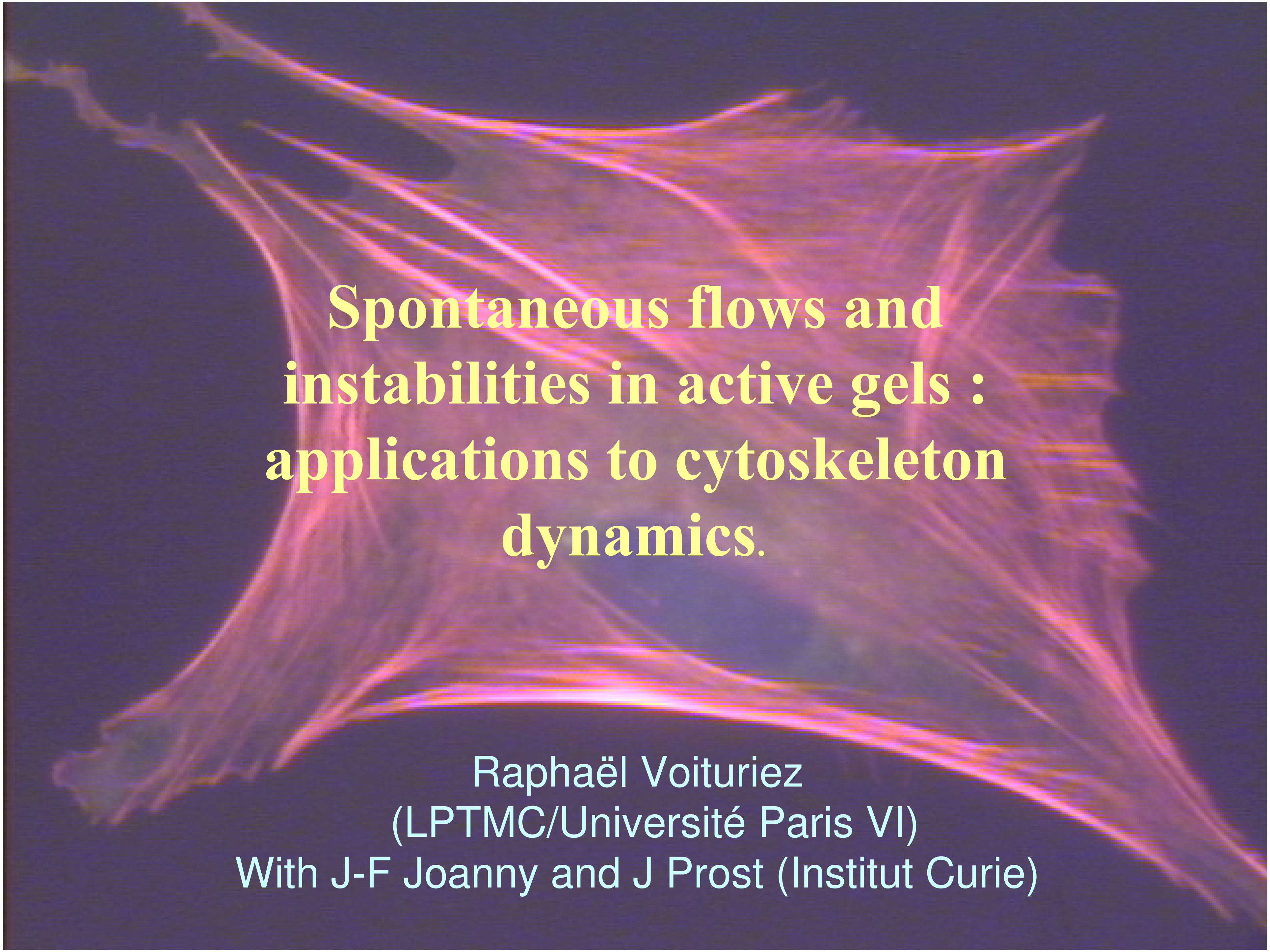


Spontaneous flows and instabilities in active gels : applications to cytoskeleton dynamics.

Raphaël Voituriez
(LPTMC/Université Paris VI)
With J-F Joanny and J Prost (Institut Curie)

Headlines

-Phases and instabilities of the (actin) cytoskeleton

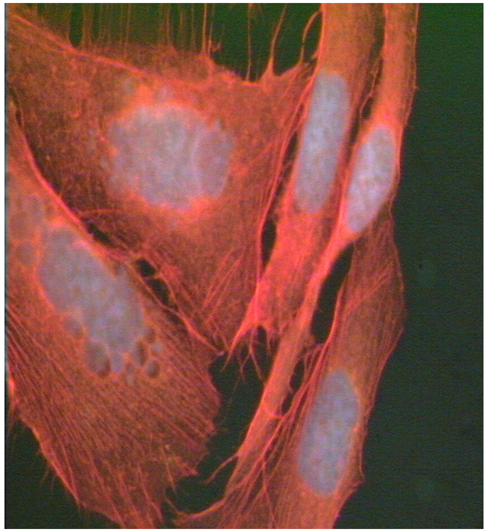
-Generalized hydrodynamics of active gels

-Stationnary states : spontaneous flows

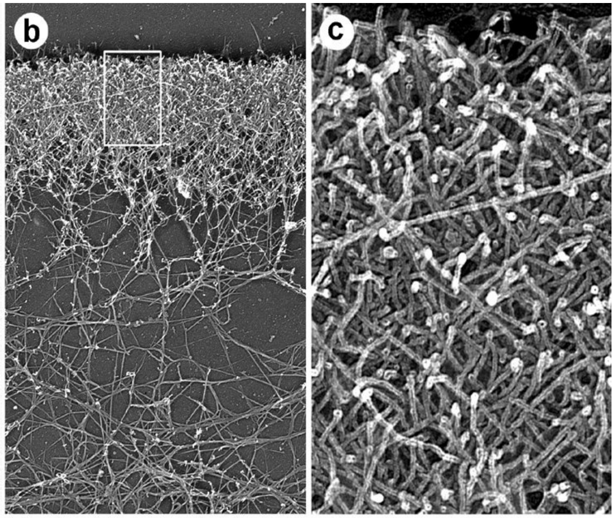
-Instabilities and waves : dynamic phase diagram

-Qualitative applications: cytoskeleton, bacterial colonies…

The (actin) Cytoskeleton



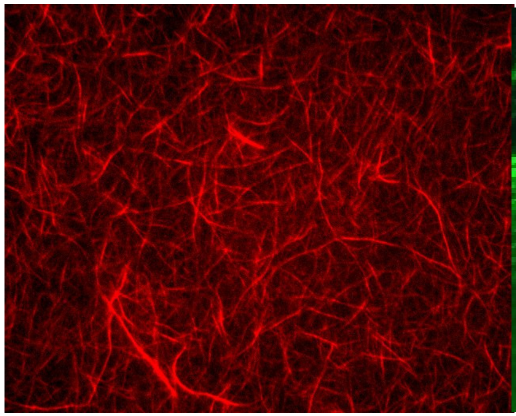
[Kline]



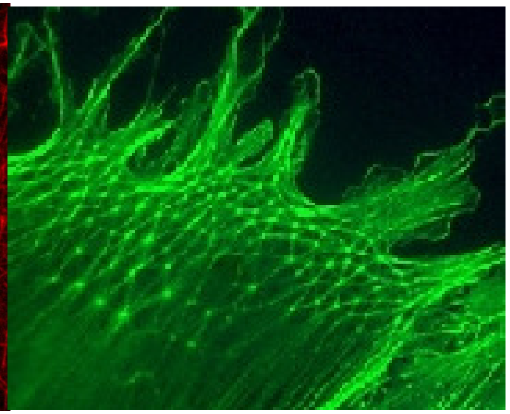
[Svitkina]

Skeleton of a cell:

-cell shape
-mechanical properties
-dynamical properties (migration, cell division)



[Langford]



[Ingber]

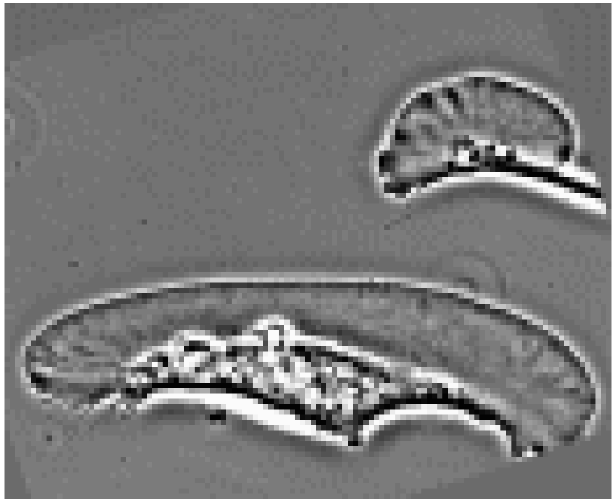
Different phases:

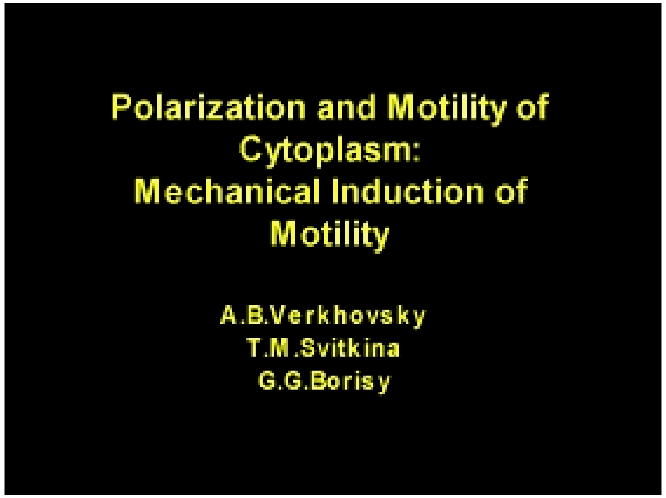
-ordered (polar), isotropic
-homogeneous, nonhomogeneous
-topological defects
-patterns

Can we provide an effective continuous description ?

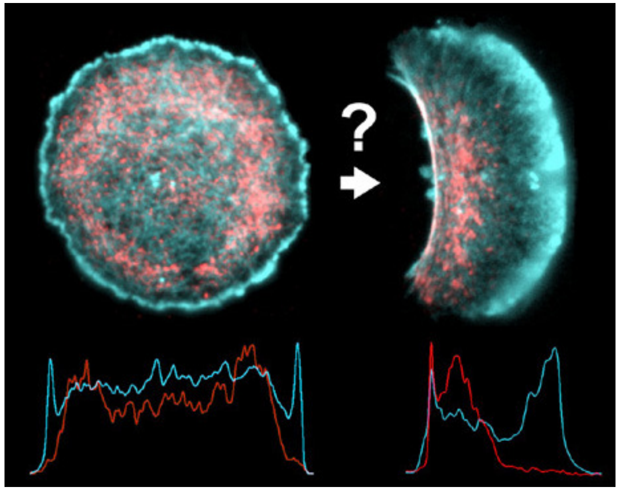
Cell motility : keratocytes

[Verkhovsky]





Active material generating forces

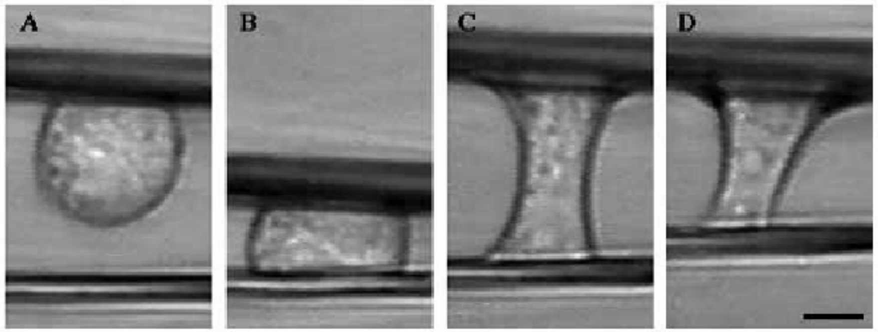


Motility as a phase transition:

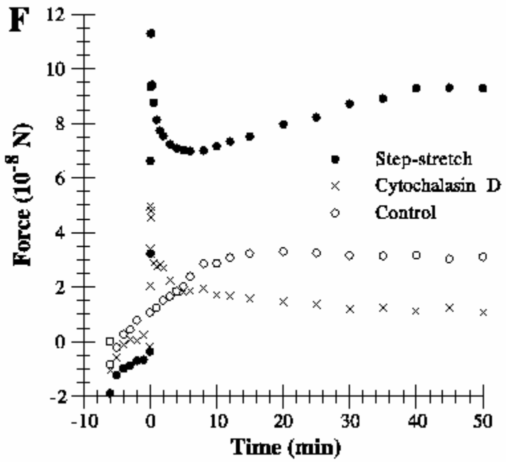
-symmetry breaking

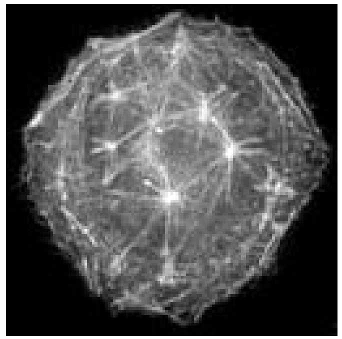
-transition homogeneous/nonhomogeneous

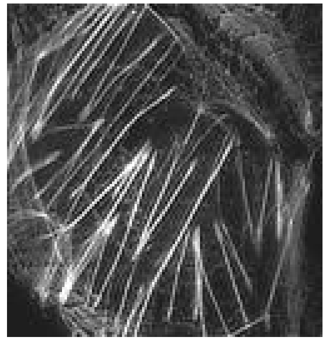
“Active” elasticity of cells

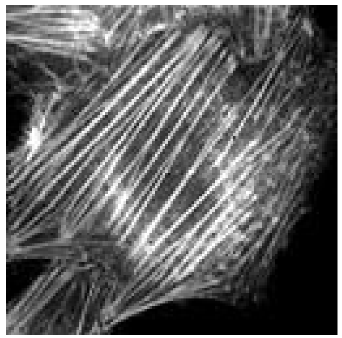


[Ott, Thoumine]





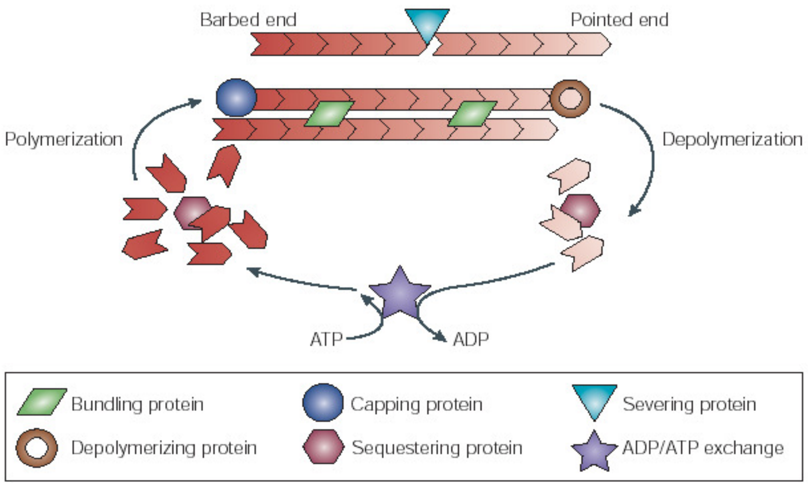


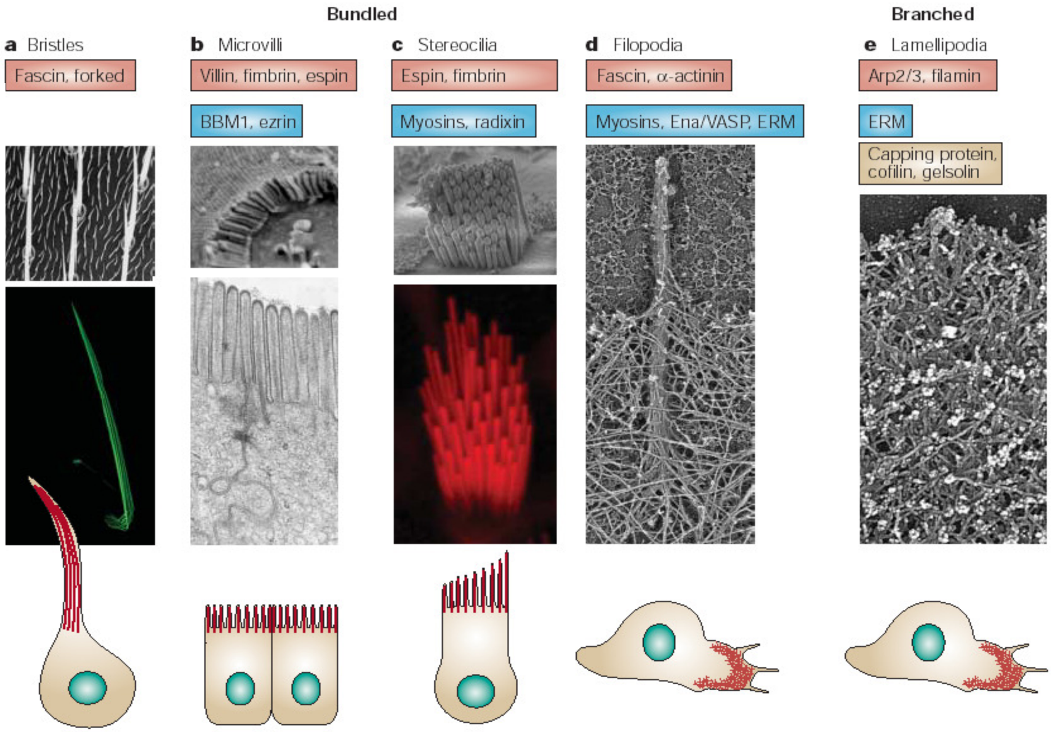


[X.Wang]

Force generation as a phase transition (stress fibres)

Ingredients (I) : actin filaments

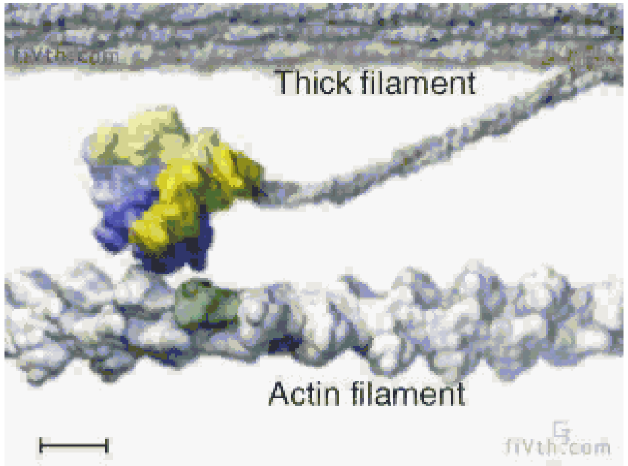




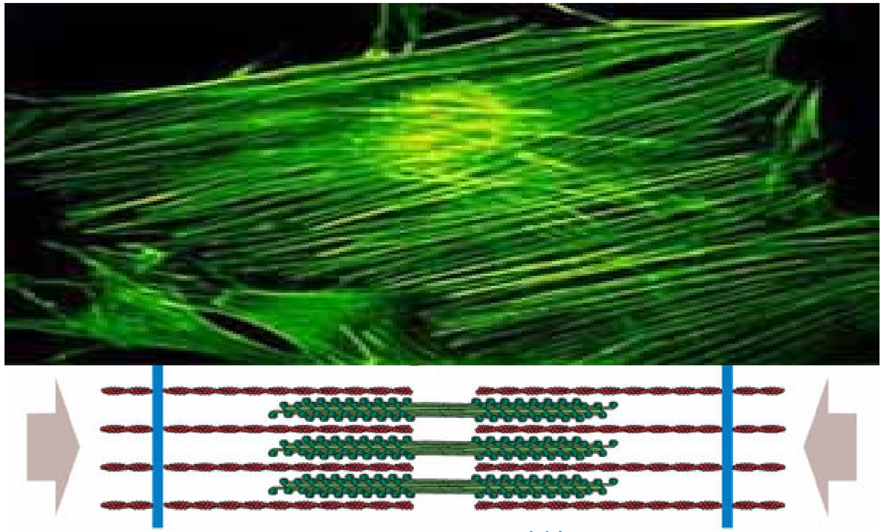
[C.Revenu]

- Polymerization/depolymerization (treadmilling): actin circulation
- Filaments are polar (local orientation) : polarization vector **p**
- Cross-linkers: gel structure

Ingredients (II) : myosin II motor+ATP

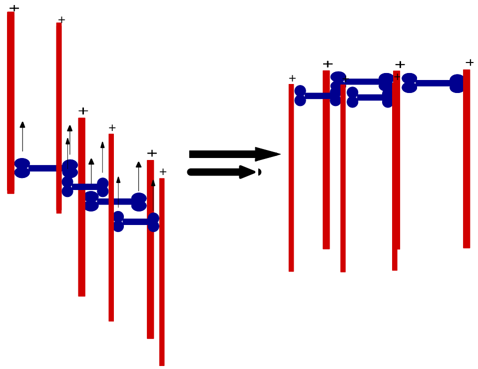


[R.Vale]

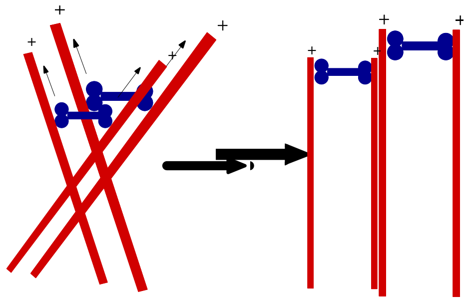


Contractility

-Chemical energy (ATP)
-Oriented displacement
 along actin filaments
-Aggregation : mini-filaments



Contraction
[Kruse]



Polarization
[Liverpool]

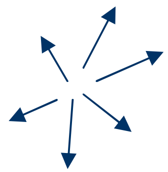
➡ Stress in the gel : polarization, contraction (stress fibers)

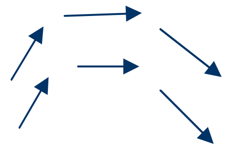
Model (I) : actin polar order

Polarization vector:
p=<**n**>, **n** : filament director

→ Nematic order parameter
[Simha,Hatwalne,Ahmadi,Zumdieck]
Ferroelectric order parameter

Frank elastic free energy: $$F = \int dr \frac{K}{2}\left[(\nabla \cdot p)^2 + (\nabla \times p)^2 \right]$$

splay 

 bend

Force → Conjugated field (molecular field) $$h = -\frac{\delta F}{\delta p}$$

Torque → $$\Gamma = p \times h$$

Model (II) : visco-elastic gel

Maxwell approximation:

Elastic at short times, viscous at long times :

1 relaxation time τ

$$2\eta u_{ij} = \left(1 + \tau \frac{D}{Dt}\right)\sigma_{ij}$$

reactive elastic term   dissipative viscous term

…. but cytoskeleton rheology is much more subtle:

-viscous at shorter times, with several characteristic times [Sackmann]

-power law response [Balland et al., Desprat et al.]

Model (III) : Myosin Activity ?

Phenomenological description:

-conservation laws (mass+momentum)

-linear response theory: fluxes are proportional to forces

Force	h	u_{ij}	$\Delta\mu$
Flux	$P = \frac{Dp}{Dt}$	σ_{ij}	r

ATP → ADP+P (arrows pointing to $\Delta\mu$ and r)

Constraints on the response matrix (symmetries)

- time reversal → symmetry of the reactive *and* dissipative parts of the response matrix

- translation/rotation → tensor invariants

General hydrodynamic theory of active polar gels

General equations of active gels

(incompressible case: [Kruse])

Maxwell

active terms:

stress fibers

isotropic contractility

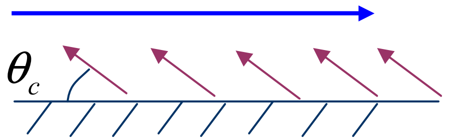
$\Rightarrow \zeta, \bar{\zeta} < 0$

$$2\eta u_{ij} = \left(1 + \tau \frac{D}{Dt}\right)\left(\sigma_{ij} + \zeta \Delta\mu\, p_i p_j + \bar{\zeta}\Delta\mu\,\delta_{ij}\right.$$

antisymetric stress

$$\left. - \frac{\nu}{2}\left(p_i h_j + p_j h_i\right) - \bar{\nu}\, p_k h_k \delta_{ij} + \frac{1}{2}\left(p_i h_j - p_j h_i\right)\right)$$

Nematic under shear



$\cos(2\theta_c) = 1/\nu$

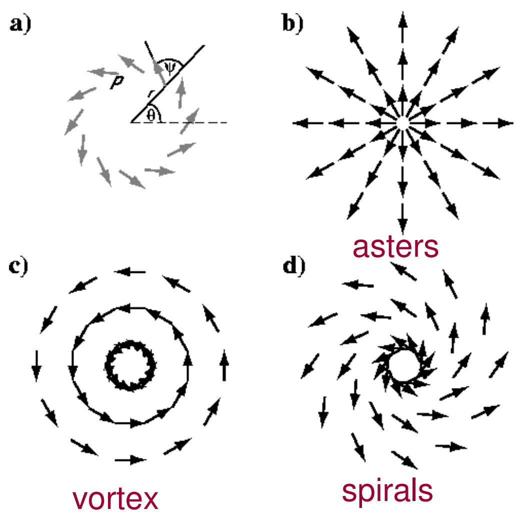
Polarizing active term

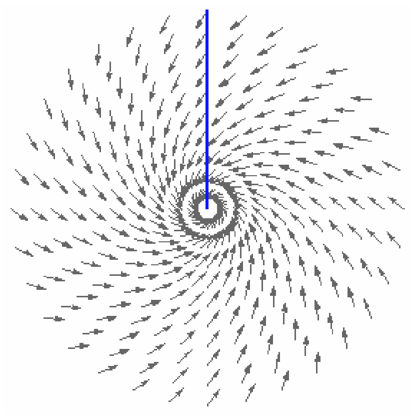
$$\frac{Dp_i}{Dt} = \frac{1}{\gamma} h_i + \lambda p_i \Delta\mu - \nu u_{ij} p_j + \bar{\nu} u_{kk} p_i$$

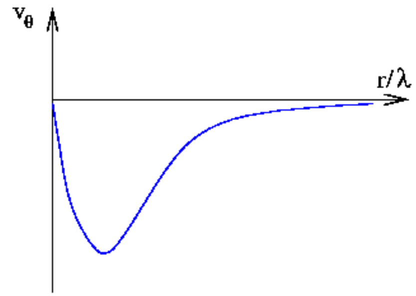
+ mass conservation and force balance

Polarization defects (active films)

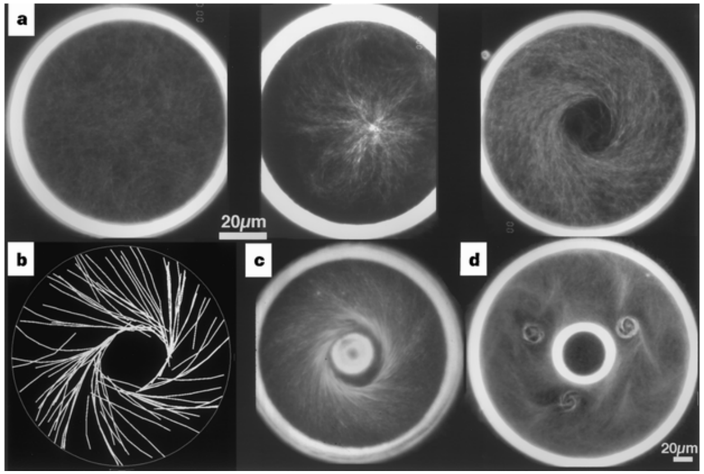
topological defects of charge 1 [Kruse]







Spontaneous flow



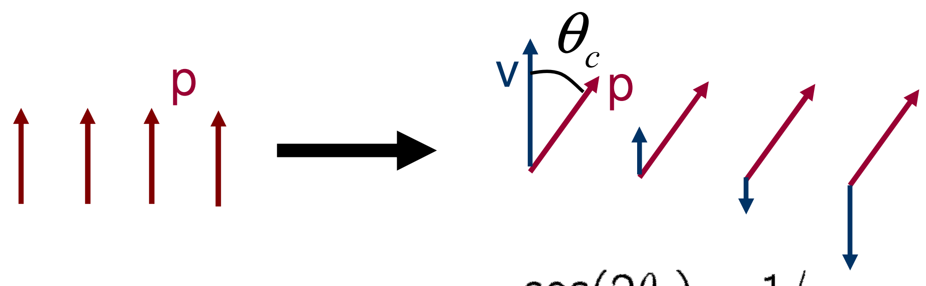
[Nédélec,Surrey]

Microtubules+kinesin in vitro

Spontaneous flow transition

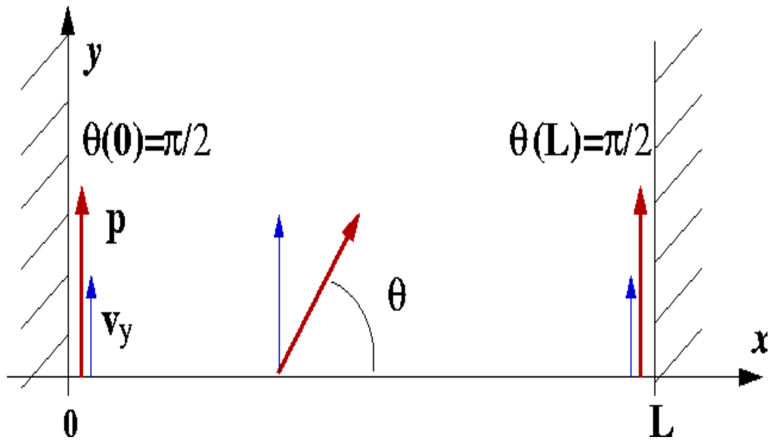
R.V., J-F Joanny, J. Prost

The uniform steady state
flows with constant shear
For $\zeta\Delta\mu < 0$



$$\cos(2\theta_c) = 1/\nu$$

Effects of confinement (quasi 1d geometry) with strong polarization anchoring [Free standing films, microchannels…?]



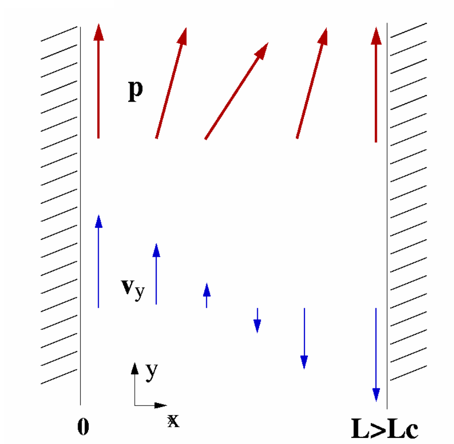
Frederiks transition with $B \approx \zeta\Delta\mu$

Boundary conditions

Free hydrodynamic boundary conditions [$\sigma_{xy} = 0$]:

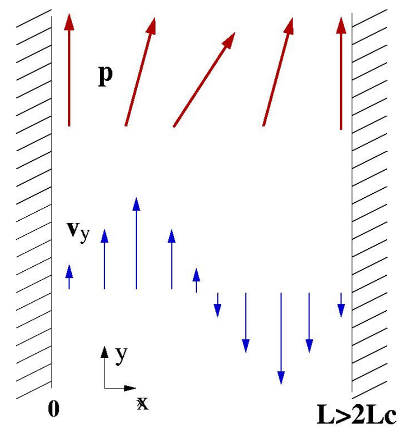
$$\partial_x^2\theta = \frac{\tilde{\zeta}\Delta\mu \sin 2\theta(\nu\cos 2\theta - 1)}{K[4\frac{\eta}{\gamma} + \nu^2 - 2\nu\cos 2\theta + 1]} = \Phi_f(\theta)$$ [Frederiks for ν=0]

Non uniform p and spontaneous flow for the critical activity:



$$\Delta\mu_c = \frac{\pi^2 K(4\eta/\gamma + (\nu+1)^2)}{-2L^2\tilde{\zeta}(\nu+1)}$$

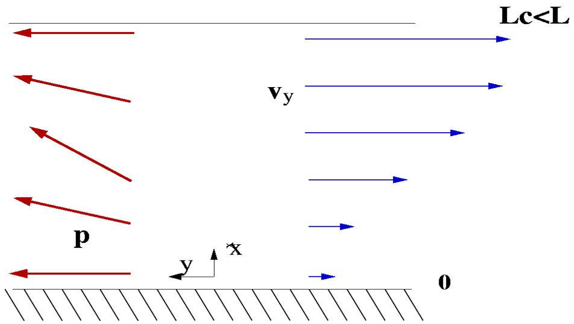
No slip hydrodynamic boundary conditions : transition for

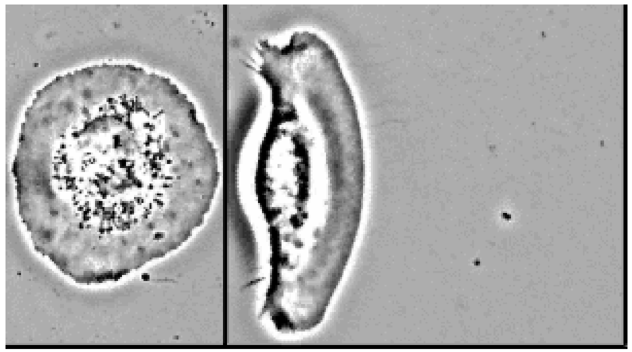


$$\Delta\mu > 4\Delta\mu_c$$

Application: self-moving material

Non symmetric boundaries : net flow





$$Q = \int_0^L v_y \, \mathrm{d}x = -\frac{4L\tilde{\zeta}\Delta\mu \quad \epsilon_m}{\pi[4\eta + \gamma(\nu+1)^2]} \qquad \text{with} \qquad \epsilon_m \propto \sqrt{\Delta\mu - \Delta\mu_c}$$

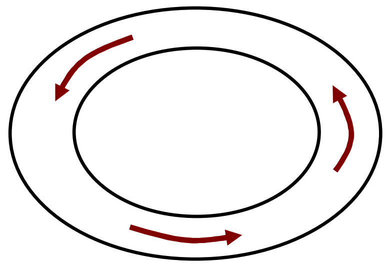
Schematic scenario of cell motility

Experimental perspectives :

-microfluidics / liquid crystal techniques

-artificial systems: “self-moving gels”

-mixed or active boundary conditions



Compressible active films

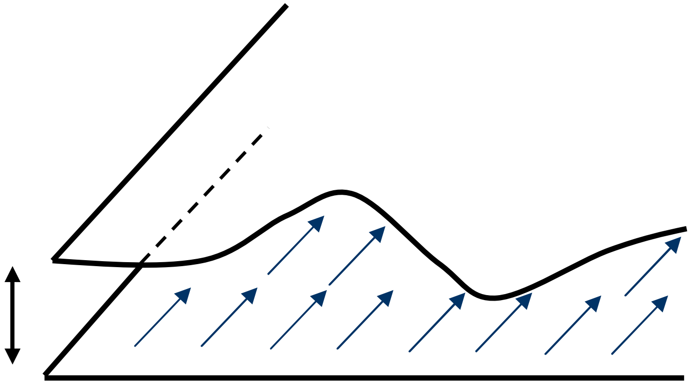
R. V., J-F Joanny et J. Prost

Compressible gel model:

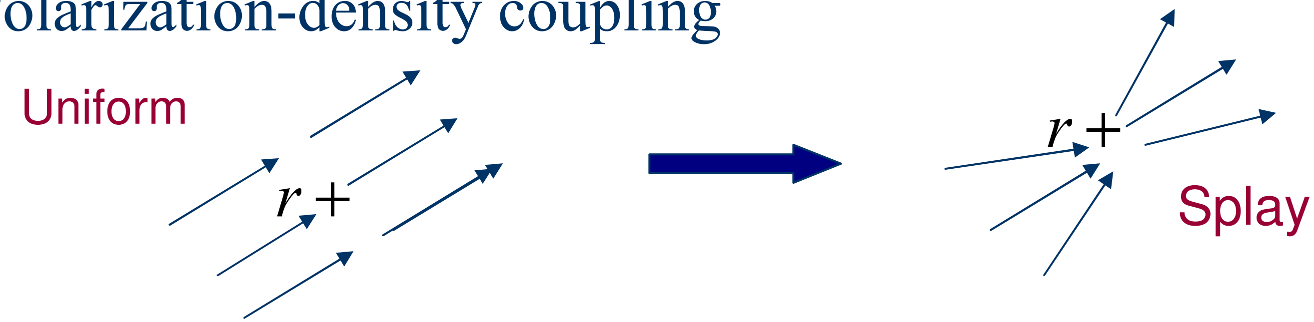
-free standing thin film

-two fluids gel

$\rho(\mathbf{r}) \propto h(\mathbf{r})$



Polarization-density coupling



$\delta\rho(\mathbf{r}) = 0$

$\delta\rho(\mathbf{r}) \neq 0$

What's changed?

Elastic free energy [Hinshaw,Blankschtein]:

$$F = \int d\mathbf{r} \left[\frac{K}{2} \left((\nabla \cdot \mathbf{p})^2 + (\nabla \times \mathbf{p})^2 \right) + w\rho \nabla \cdot \mathbf{p} + \frac{\beta}{2} (\nabla \rho)^2 + \frac{\alpha}{2} \rho^2 \right]$$

Conjugated fields

$$\Pi = \frac{\delta F}{\delta \rho} = w \nabla \cdot \mathbf{p} + \alpha \rho - \beta \Delta \rho \quad \text{and} \quad h = -\frac{\delta F}{\delta p}$$

Mass conservation and force balance

$$\partial_t \rho + \nabla [(\rho_0 + \rho) \mathbf{v}] = 0$$

$$\partial_\alpha (\sigma_{\alpha\beta} - \Pi \delta_{\alpha\beta}) = 0$$

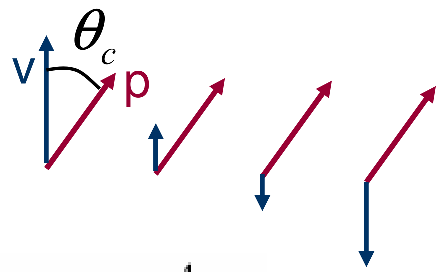
Dynamical equations:

$$2\eta u_{\alpha\beta} = \sigma_{\alpha\beta} + [\Delta\mu, p] + [p, h]$$

$$\frac{Dp}{Dt} = h/\gamma + [p, \Delta\mu] + [p, u]$$

Stability : patterns, phase separation…

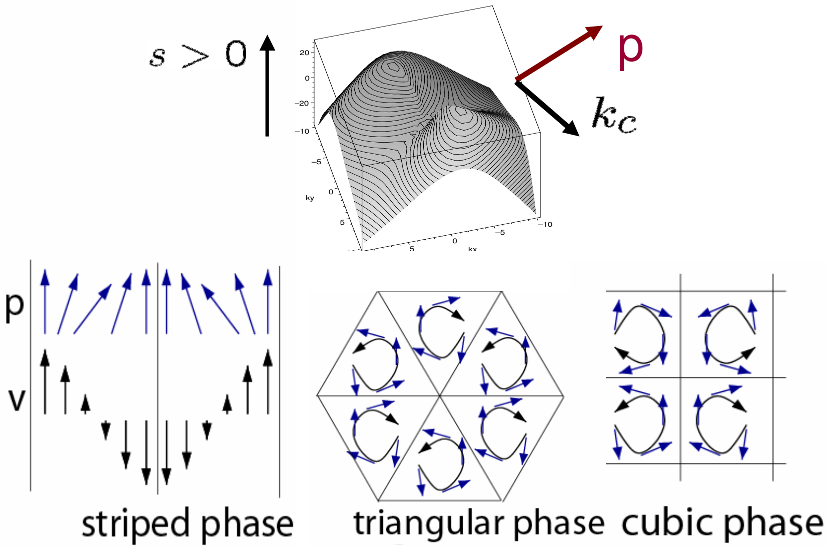
Is the uniform state stable ($\zeta\Delta\mu < 0$)?



→ Solutions of type $\exp\left(st + i\vec{k}\cdot\vec{r}\right)$

→ $$2\eta\gamma a s^2 + s[(2\eta\beta + Kab\gamma)k^2 + 2\eta\alpha'] + bK\beta k^4 + b(K\alpha - w^2)k^2 + ikdw\zeta\Delta\mu - \zeta\Delta\mu\alpha(\nu+1) = 0$$

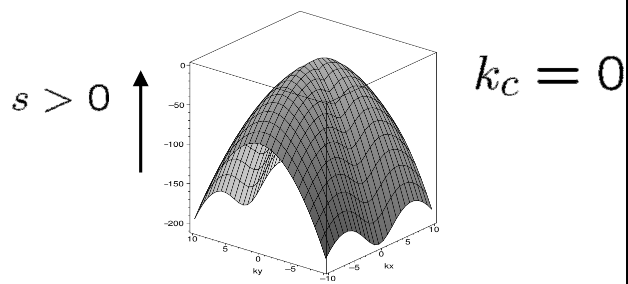
Organized phase



[Zumdieck]

Increasing → activity

Phase separation



Effective compressibility

$\alpha' < 0$

…and waves

$\mathrm{Im}(s) \neq 0$ ⟶ oscillatory instability for $\zeta \Delta\mu < 0$

[Simah]
[Ahmadi]
[Zumdieck]

⟶ Stationary or traveling waves (boundary conditions)

Low activity expansion:

$$k_c \approx \frac{1}{w}\sqrt{\frac{w^2 - K}{c_1 + c_2(w^2 - K)}} + g(w,K)\zeta\Delta\mu$$

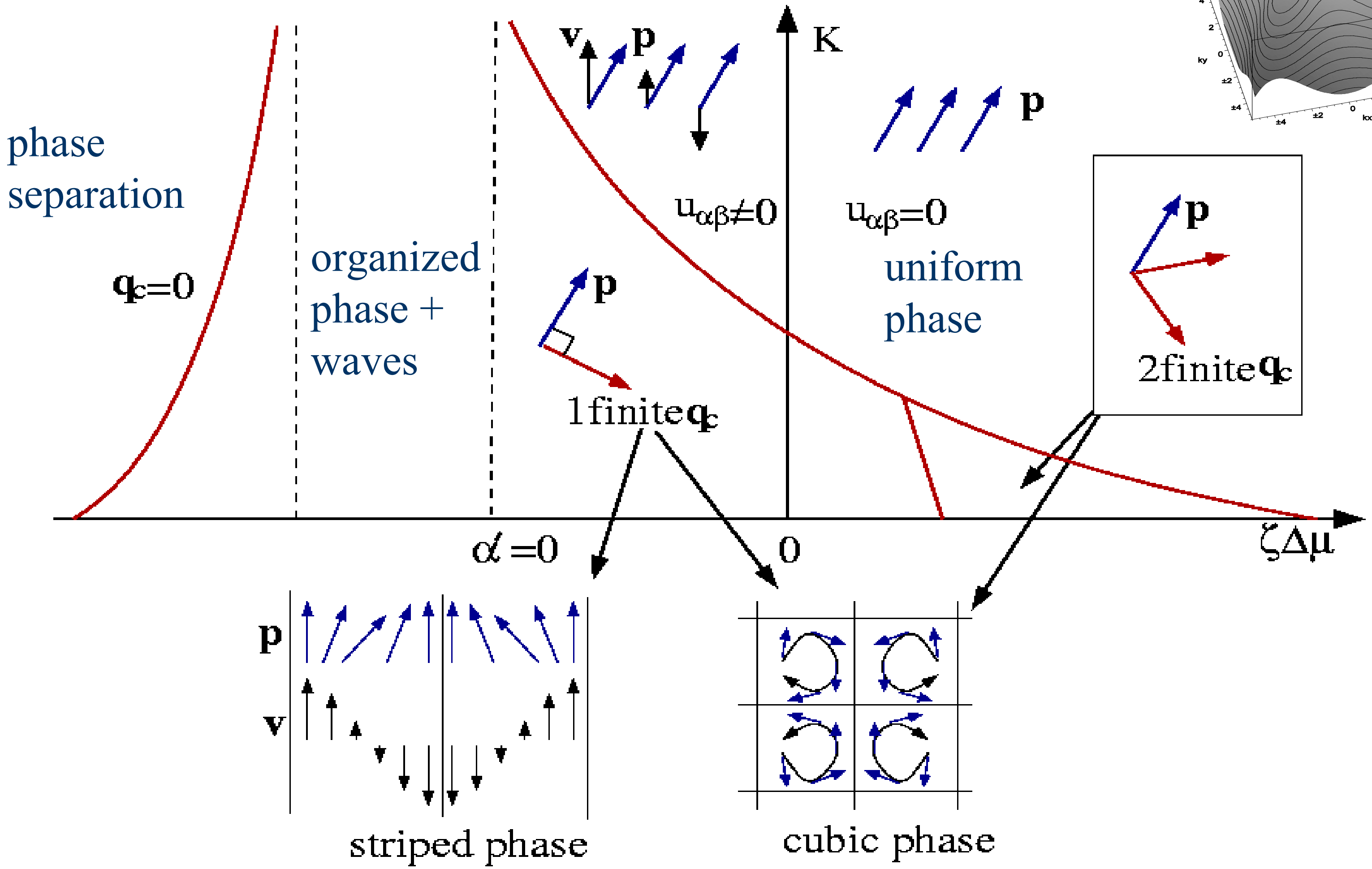
[Hinshaw,Blankschtein]

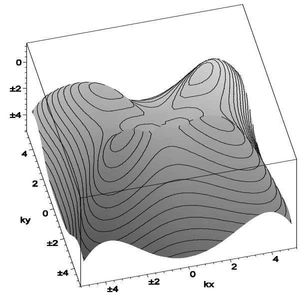
$$\omega_c = \mathrm{Im}(s) \propto w\zeta\Delta\mu k_c$$

Phase diagram

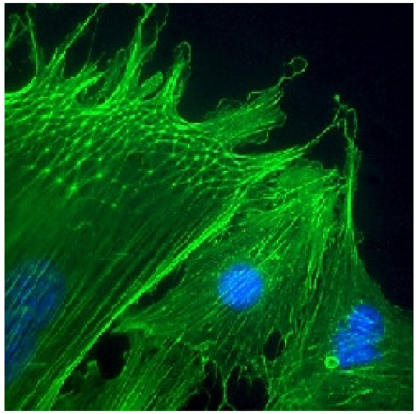
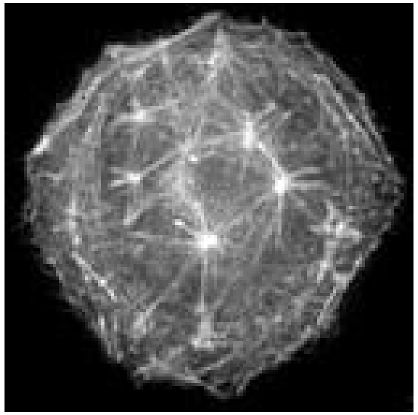
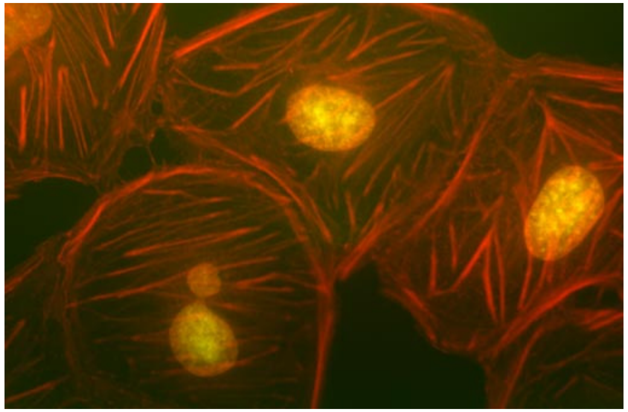
out of equilibrium

(2D infinite film)



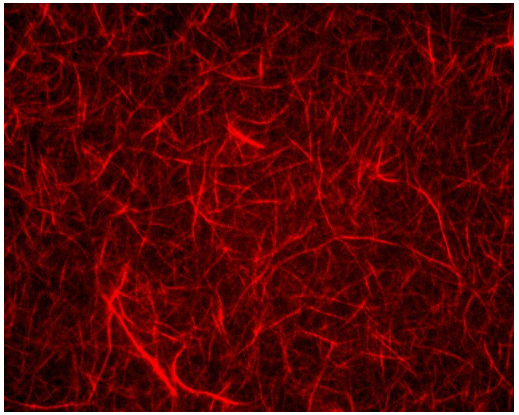


Applications : Actin

patterns phase separation …and waves

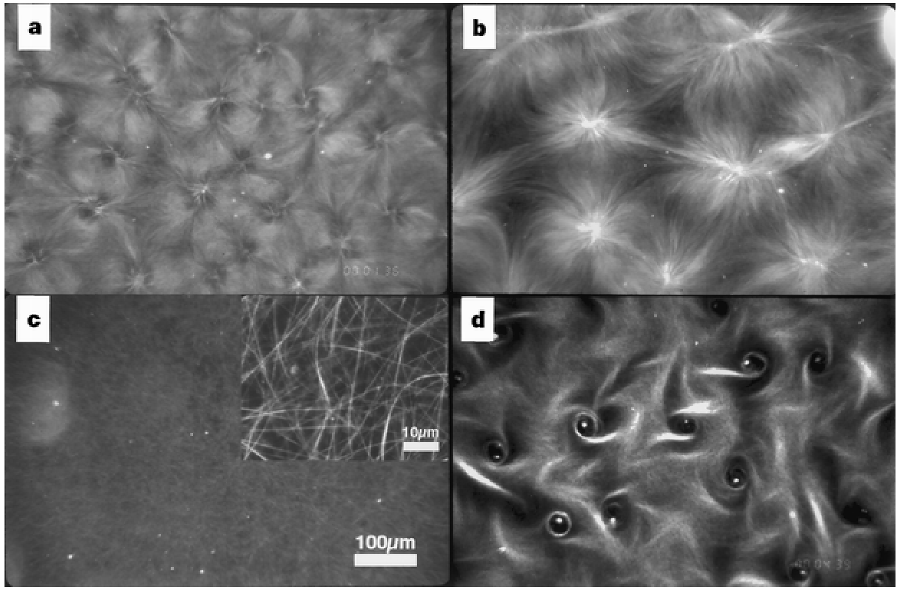
In-vitro ?



Superprecipitation [Sekine], [Käs]

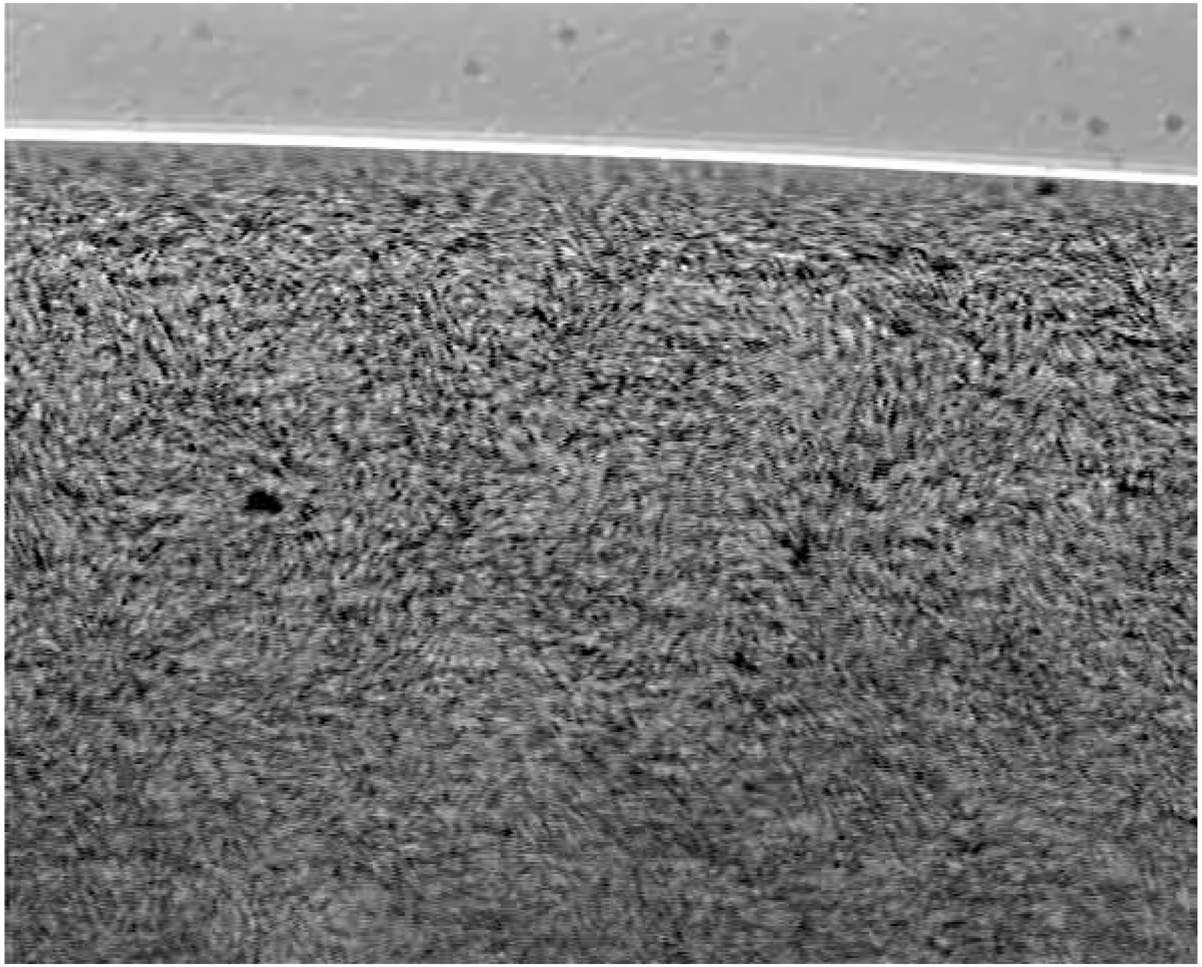
Experimental perspectives:
Freely suspended films:
(polarization and flow measurements)

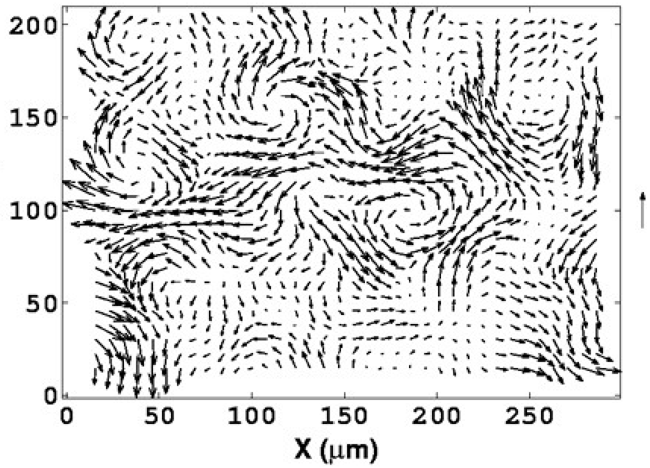
Equivalent in-vitro system: Microtubules+kinesin



For increasing activity:
-patterns
-phase separation
[Nédélec,Surrey]

Bacterial Turbulence





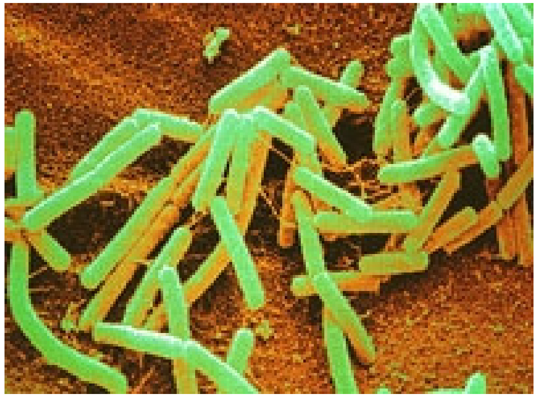
[Goldstein]

Low Reynolds number turbulence: $R_e = \frac{vL}{\mu} = 10^{-4}$

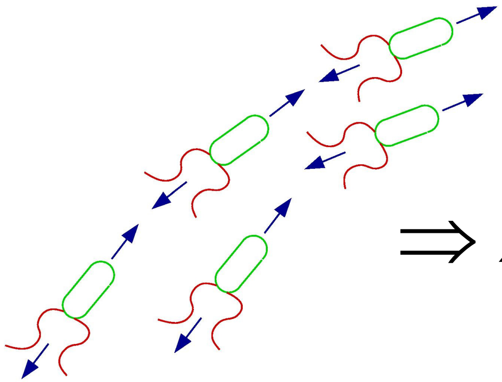
Bacterial Turbulence

as a vortex liquid

Bacterial colonies as active gels:

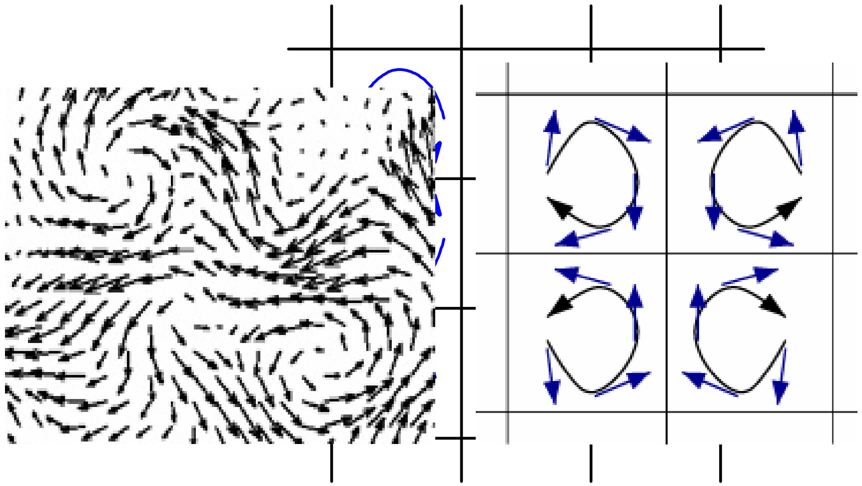


- -Polar
- -Visco-elastic
- -Active (oxygen)



$\Rightarrow \lambda, \zeta$?

[Hatwalne]



Organized vortex phase

Conclusions and perspectives

-phenomenological model of active polar gels
-spontaneous flow transition (observable)
-phase diagram : instabilities
-patterns and phase separation

→ -experiments :
  -in vitro (simplified systems, self-motion)
  -in vivo (controlled geometry)
-theory :
  -non-linear dynamics
  -noise (thermal and stochastic)

Thanks

Jean-François Joanny

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F.Amblard
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Y. Kafri
D. Nelson
D. Riveline
P. Dewimille