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Axial Segregation of a Settling Suspension in a Rotating Cylinder

(Dynamic and pattern formation in a rotating suspension)

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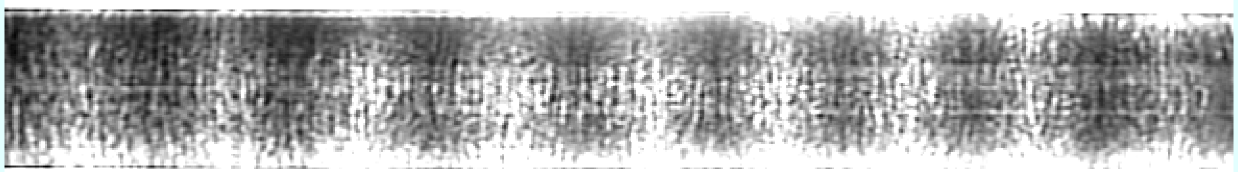
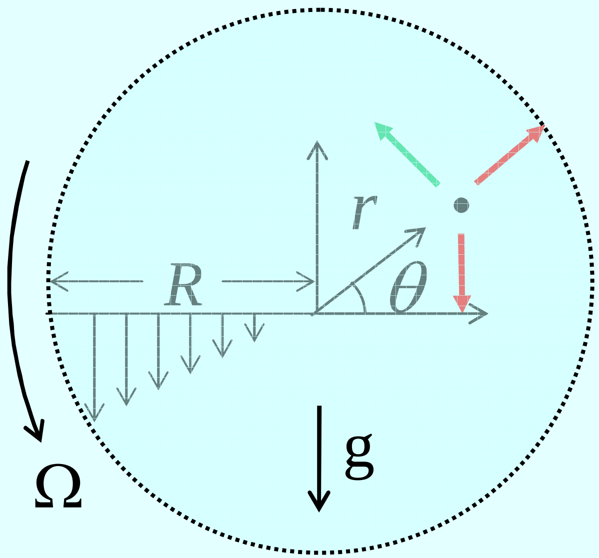
Rotating suspensions and granular solids show a variety of non-equilibrium patterns

1) Granular segregation. Mixtures of particles of *different* size or mass in a partially filled tube segregate into bands of each component.
2) Suspensions of neutrally buoyant particles in a *partially filled* cylinder segregate into bands of densely packed particles and pure fluid. Interfacial segregation.
3) Suspensions of non-neutrally buoyant particles in a *filled* cylinder form weak axial bands of high and low concentration, as shown below. At higher frequencies stronger bands develop, possibly due to inertial effects.



Dynamics of a single non-buoyant particle

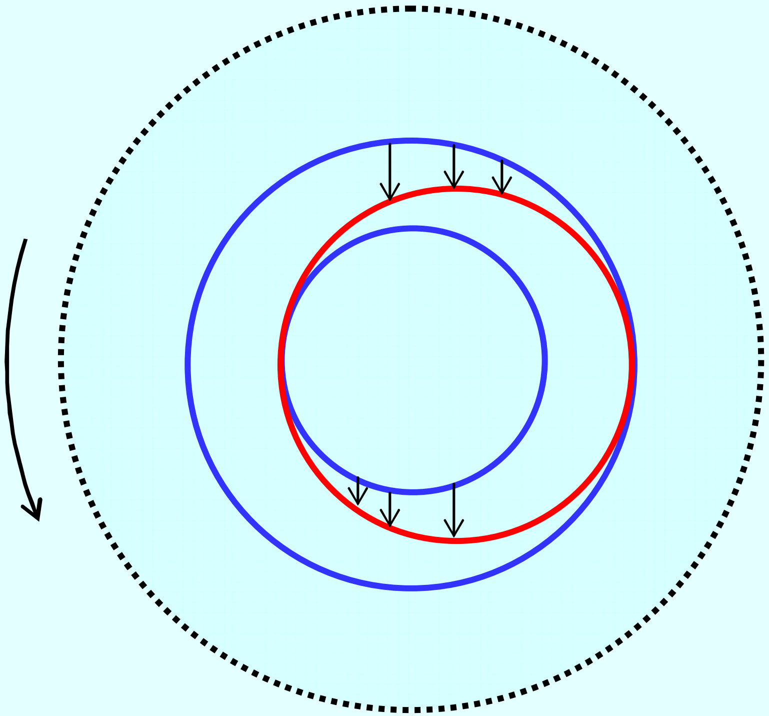


$$u_r = u_0\left[\frac{r}{D_1 R} - \sin\theta\right], \quad D_1 = \frac{g}{\Omega^2 R}$$

$$u_\theta = u_0\left[\frac{r}{D_2 R} - \cos\theta\right], \quad D_2 = \frac{u_0}{\Omega R}$$

Forces: Viscous drag, gravity, centrifugal
Fixed point where 3 forces balances
Heavy particles spiral outwards from the fixed point
Light particles spiral towards the fixed point

Centrifugal force is small under conditions of most experiments (low Ω).



At low Ω, particles rotate in nearly circular off-center orbits (red)

Particles are continually falling from the fluid streamlines (blue)

Dynamics characterized by $D_2 = u_0 / \Omega R$

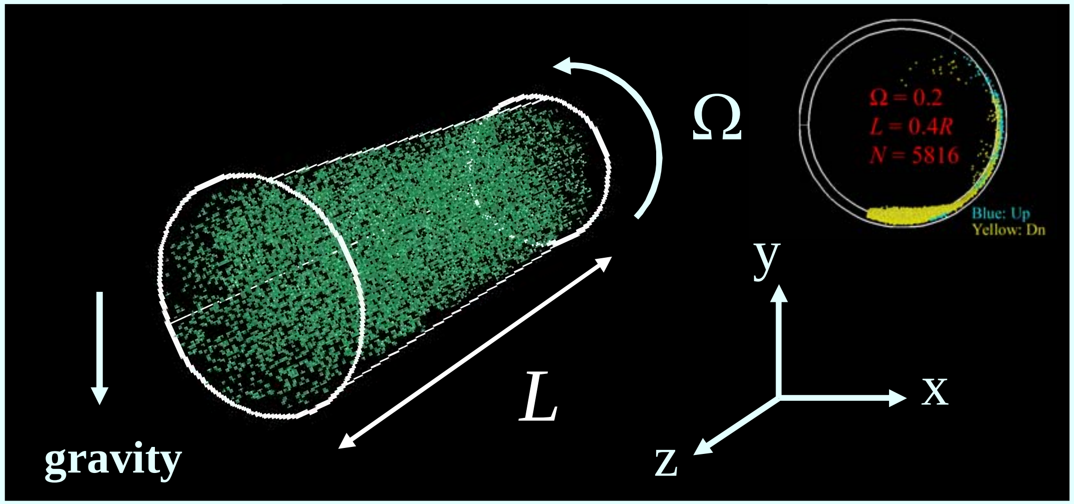
Dynamics of a dilute rotating suspension

Experiments (Matsen et al): Filled cylinder, radius $R \sim$ 1 cm

$\phi \sim 2$ %	***Dilute***
$a = 100$ μm	***Non-Brownian***
$\rho = 2.40$	***Heavy***
$\mu = 5$ -75 cp	***Viscous***
$\Omega \sim 1$ Hz;	***Stokes flow:*** $Re_a << 1$, $Re_R \sim$ 1-10
$\Omega^2 R \sim 10^{-3} g$	

Questions:

1. Dimensionless groups?
2. Steady state?
3. Axial bands?

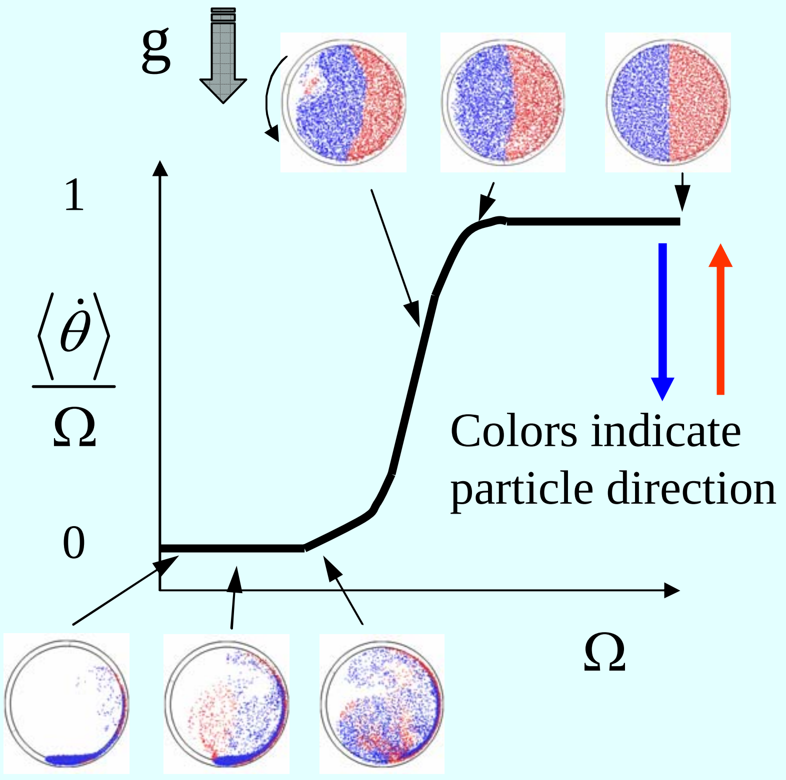


Pattern formation in a rotating suspension Order parameter separated dense and dilute phases

Low rotational velocities:
Particles lifted by tube rotation
Particles slip down
Backflow lifts particles off bottom

High rotational velocities:
Particles are dispersed
Near solid-body rotation

Mean rotational velocity about the cylinder axis, $Q = \langle \dot{\theta} \rangle / \Omega$ is an order parameter

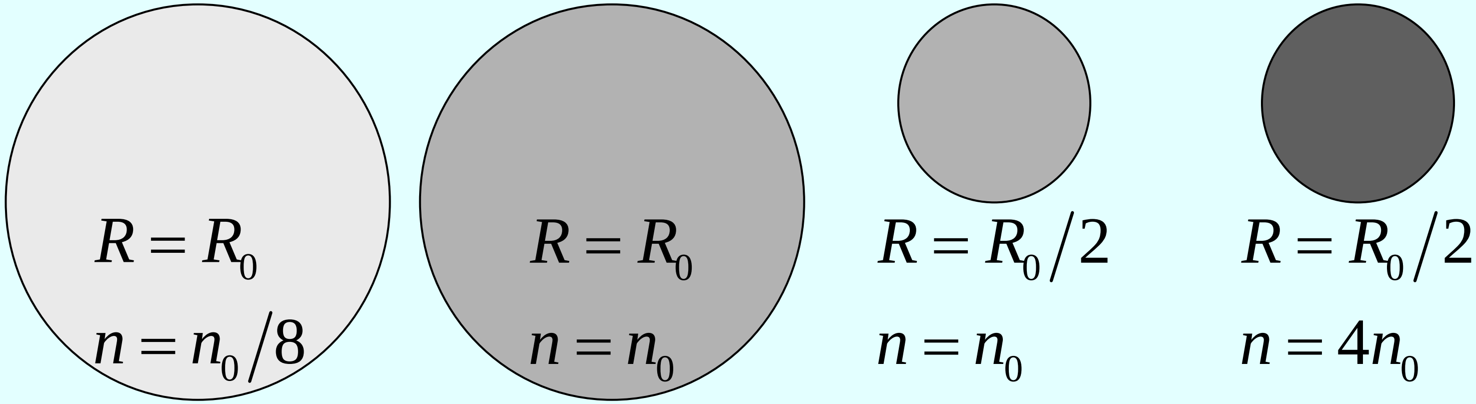


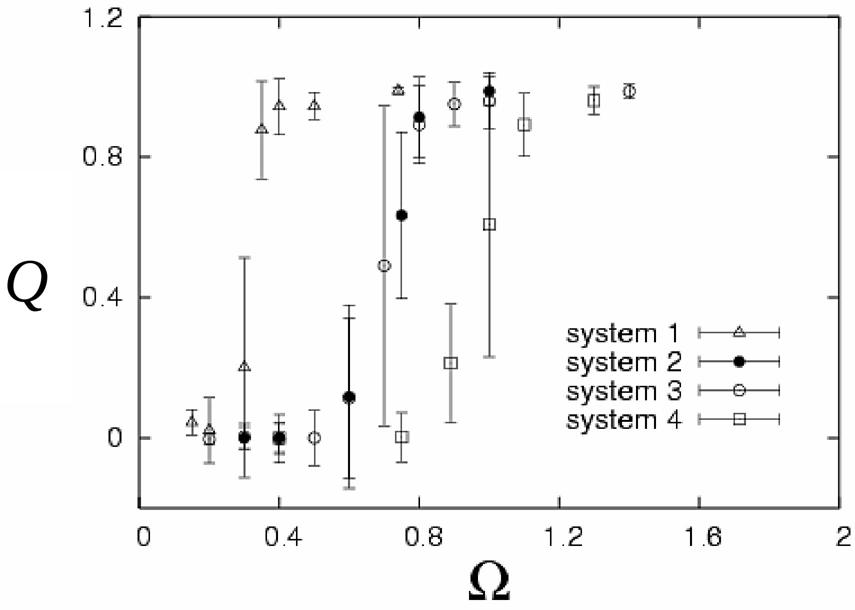
$L = 0.4R$

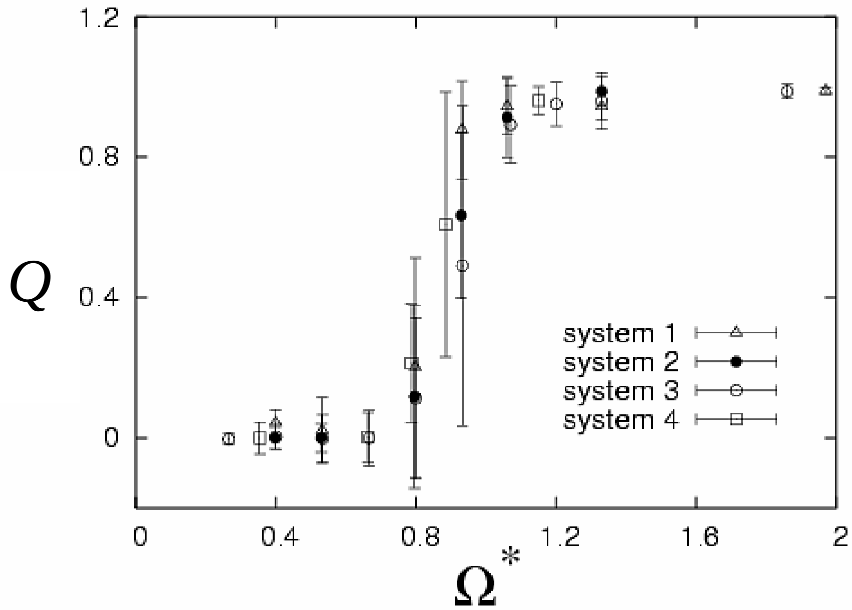
(suppress axial fluctuations)

Characteristic length:
MIS = $n^{-1/3}$

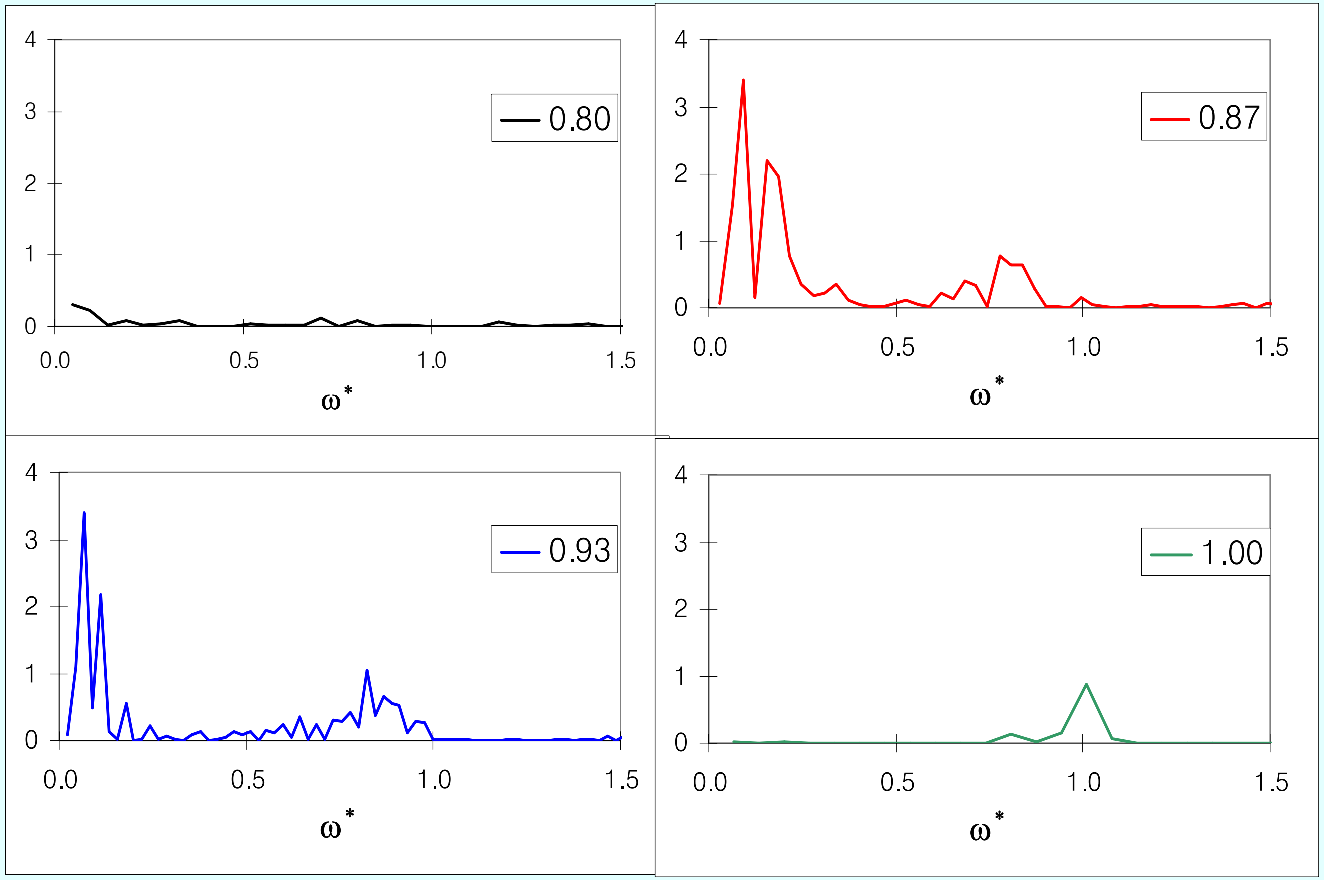
$$\Omega^* = \frac{\Omega d}{u_0}, \text{where } d = n^{-1/3}$$

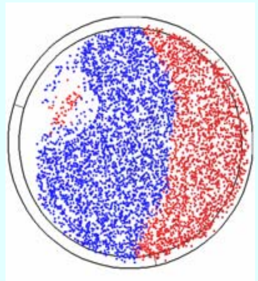




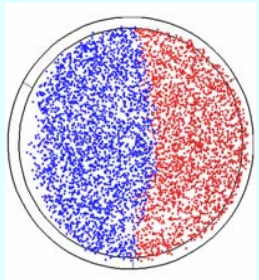


Possible dynamical phase transition between segregated (Q=0) and dispersed phase (Q=1)





$\Omega* = 0.8$



$\Omega* = 1.0$

Hydrodynamic interactions (HI). Long-range fluid mediated forces, driven by fluid motion

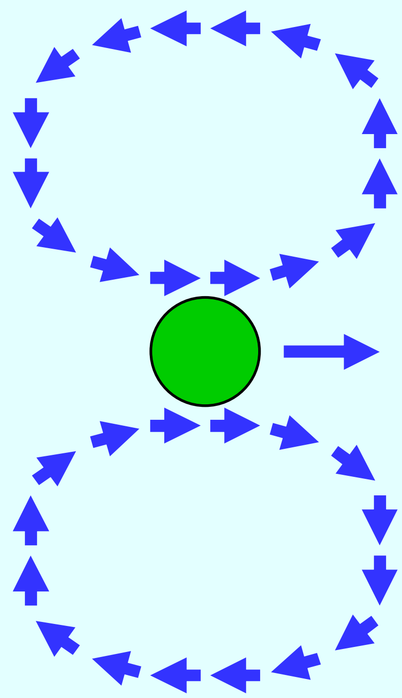
For dilute suspensions in Stokes flow (time independent):

$$\dot{\mathbf{r}}_i = \mathbf{v}_i = \frac{\mathbf{f}}{6\pi\mu a} + \sum_{j \neq i} \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j).\mathbf{f}; \quad \text{(force balance)}$$

The tensor **G** is the Green's function for the geometry.

$$\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j) = \mathbf{T}(\mathbf{r}_i - \mathbf{r}_j) + \text{wall corrections}$$

$$\mathbf{T}(\mathbf{r}) = \frac{1}{8\pi\mu r}\left(\mathbf{I} + \frac{\mathbf{rr}}{r^2}\right) \quad \text{(Oseen tensor)}$$



Calculated **G** from a Fourier-Bessel sum (Liron and Shahar) and also a sum of residues.

Order N algorithm for source field: $v_i = \sum_{j \neq i} G_{ij} F_j$

$$G_{ij} \sim \sum_{k,\lambda} \cos(k\theta_i - k\theta_j)\cos(\lambda z_i - \lambda z_j) f(k,\lambda,r_i,r_j)$$

Absence of translational invariance in radial direction

$$f(k,\lambda,r_i,r_j) \sim I_k(\lambda r_i) K_k(\lambda r_j)$$

Expand G_{ij}:

$$G_{ij} \sim \sum_{k,\lambda} g(k,\lambda,\mathbf{r}_i) g'(k,\lambda,\mathbf{r}_j)$$

$$g(k,\lambda,\mathbf{r}_i) \sim I_k(\lambda r_i)\cos(k\theta_i)\cos(\lambda z_i)$$

$$g'(k,\lambda,\mathbf{r}_j) \sim K_k(\lambda r_j)\cos(k\theta_j)\cos(\lambda z_j)$$

Plus many more terms …

Complications

1) FB expansion requires $r_i < r_j$

   Sort particles by radial position

   Two series sums: ascending and descending

2) Large r divergence: Define new “Bessel” functions with powers of r taken out

   Keep explicit track of powers and cancel products

3) Large k divergence:

   Use hyperbolic Bessel functions. Cancel exponential terms

4) Use ascending and descending recursion series for Bessel functions to maintain stability

5) Explicitly subtract logarithmic divergences from K_0

Order N algorithm for cancelling field

Cancelling field: Solution of Stokes flow, $w(R,\mathbf{r}^N) = -v(R,\mathbf{r}^N)$

$$\sum_{k,\lambda} \cos(k\theta_i - k\theta_j)\cos(\lambda z_i - \lambda z_j) I_k(\lambda r_i)\psi(\lambda)$$

Solve linear system for 3 coefficients $\psi(\lambda), \pi(\lambda), \omega(\lambda)$

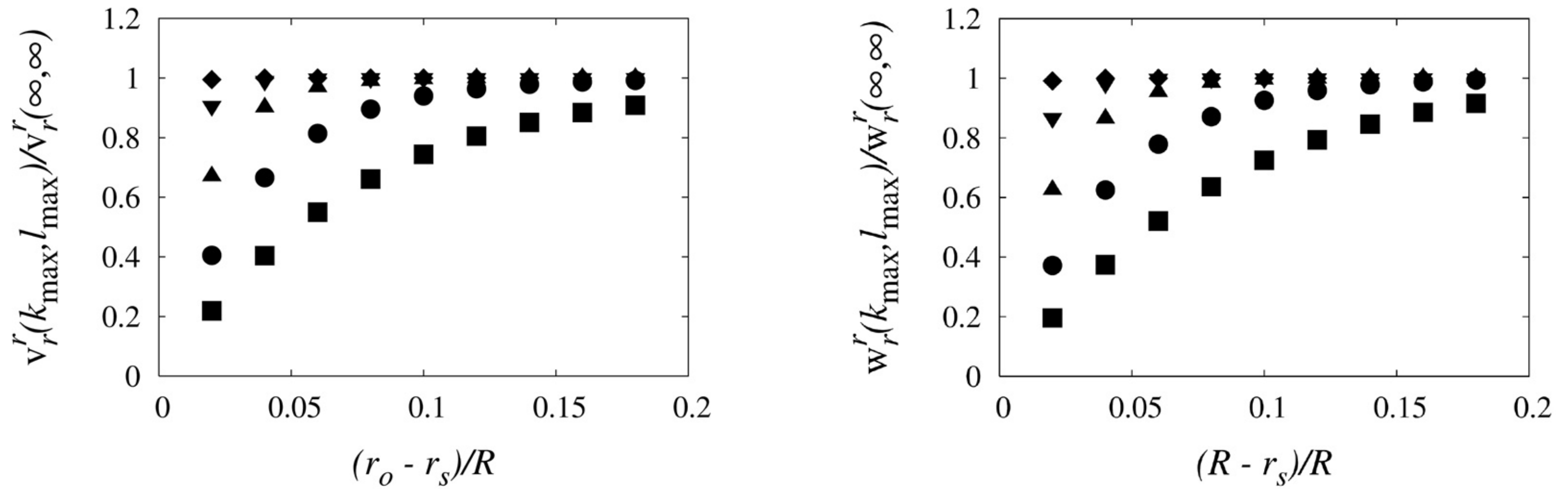
Cancel source field from all particles at once, mode by mode for order *N*

But only order *N* for fixed number of Fourier modes.

$N^{5/3}$ for fixed resolution

Spatial grid more complicated and error prone in cylindrical coordinates

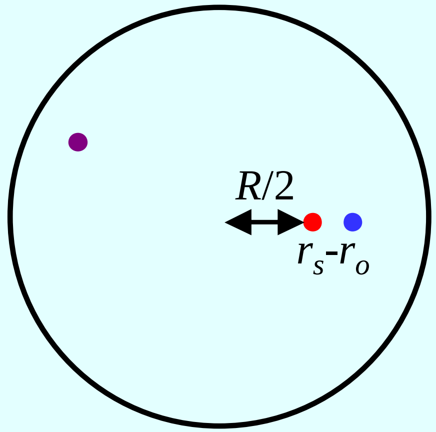
Convergence slow with FB series



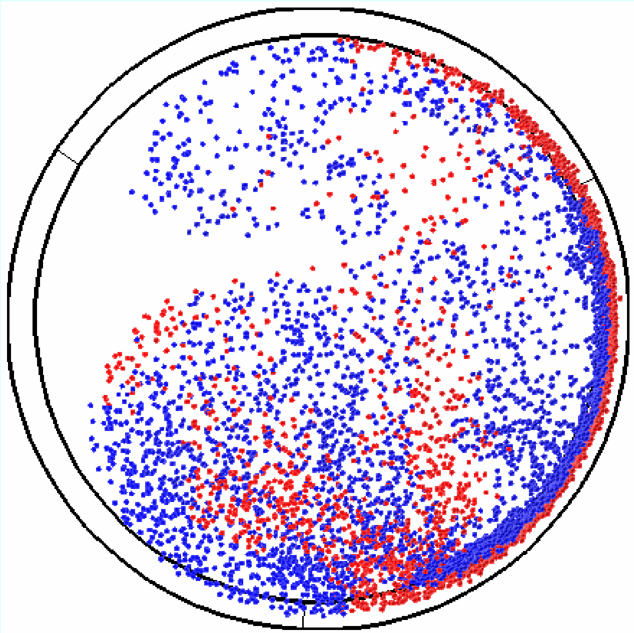
k_{max}, l_{max}: 8 squares 16 circles 32 triangles 64 inverted triangles 128 diamonds

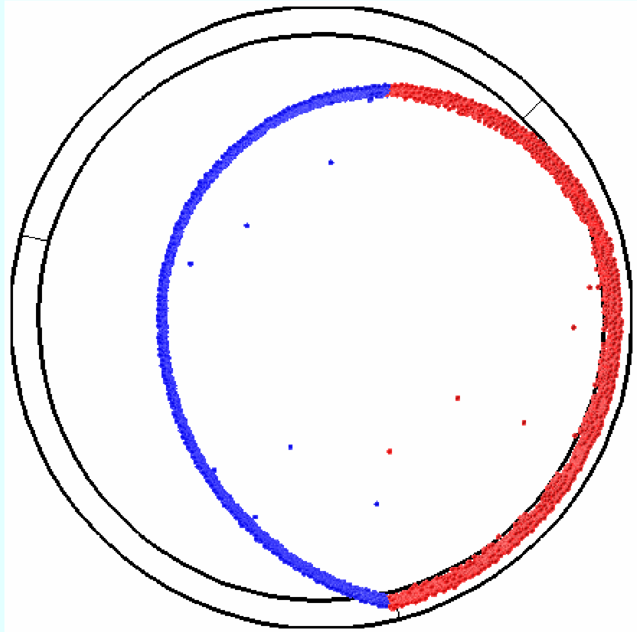
Simulations used k_{max}, l_{max} = 32

Code checked by comparison with residue sums



Hydrodynamic interactions play a key role in pattern formation





With HI
Particles remain dispersed indefinitely

Without HI:
Particles spiral out to a single limiting trajectory constrained by cylinder wall

Mass balance apparently sets the observed length scale d = n$^{-1/3}$

In segregated phase HI dominates

Sedimentation velocity determined by MIS ~ *d*

$$u_{sed} \sim u_0 a d^{-1}$$

Up-current of particles carried by wall as a monolayer:

$$\dot{M}_{up} \sim \Omega R a L n m$$

Settling flux is distributed over a length proportional to *R*.

$$\dot{M}_{dn} \sim u_{sed} R L n m$$

Mass balance controlled by single dimensionless parameter

$$\Omega^* = \frac{\Omega a}{u_{sed}} = \frac{\Omega d}{u_0}$$

Characteristic length is *d* in agreement with simulation.

Different mass balance in dispersed phase Change in scale of flow

In dispersed phase:

$$u_{sed} \sim u_0$$

Up-current carried by wall as a thick layer proportional to *R*:

$$\dot{M}_{up} \sim \Omega R^2 Lnm$$

Settling flux is distributed over a length proportional to *R:*

$$\dot{M}_{dn} \sim u_0 RLnm$$

Characteristic frequency:

$$\Omega^* = \frac{\Omega R}{u_0}$$

Change in scale from *d* to *R* (in agreement with simulation)

At higher rotation frequencies particles are very uniformly dispersed-rotating bioreactors

Three different motions:

1) Primary circles:

$$l = R, \ \tau = 2\pi / \Omega$$

2) Secondary circles (counter):

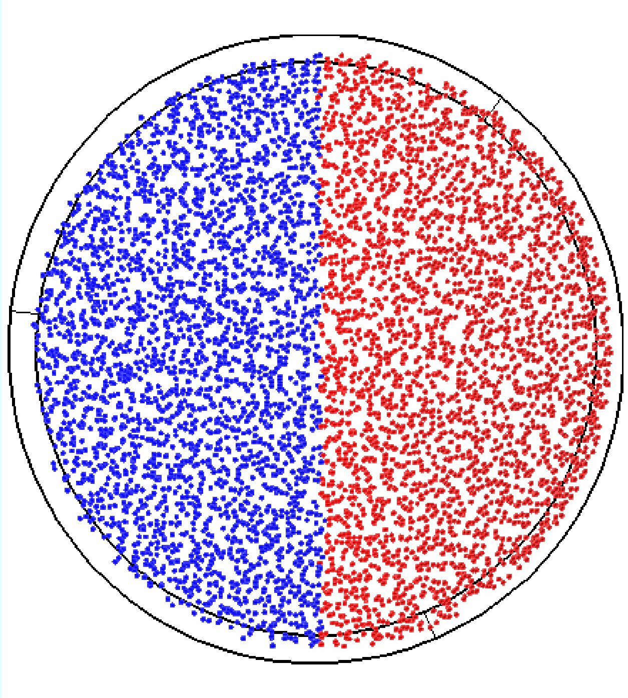
$$d = u_0 \tau, \ \tau = \pi / \Omega$$

3) Diffusive:

$$\frac{D}{u_0 R} \sim 0.1(\Omega^*)^{-2}$$

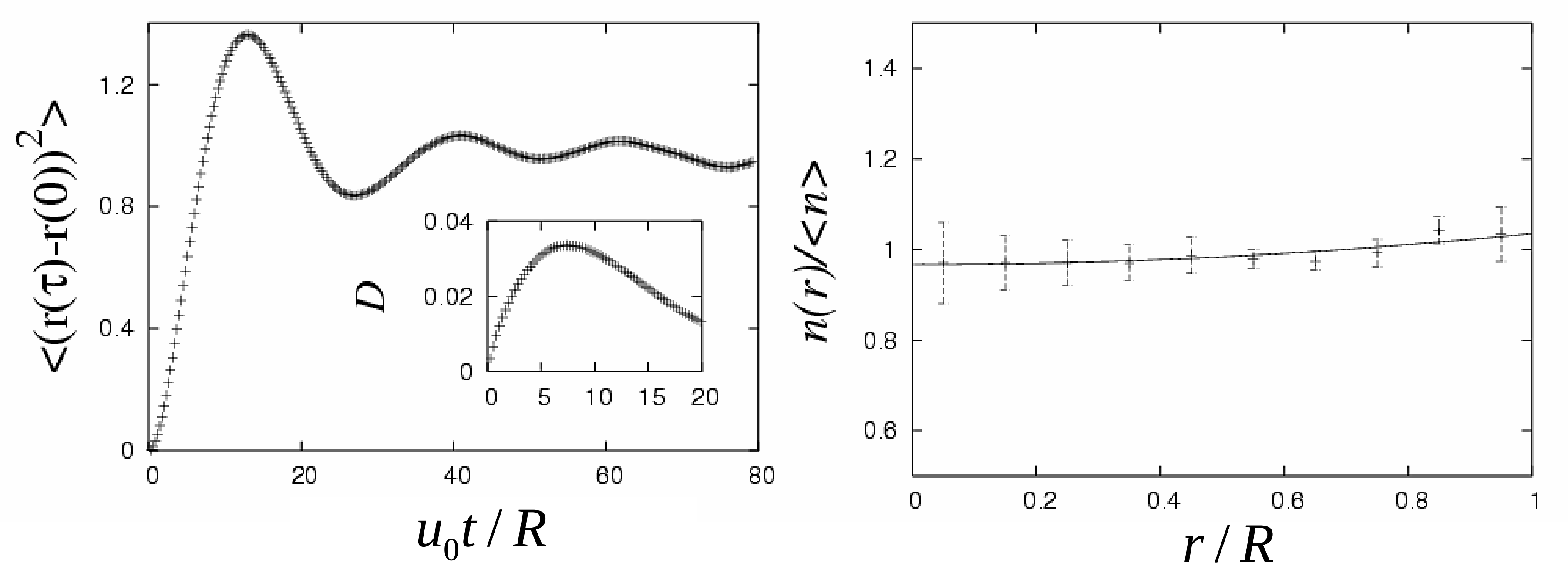
But only in a narrow frequency range:

$$1 < \Omega^* < 4$$



$$\Omega^* \sim 3$$

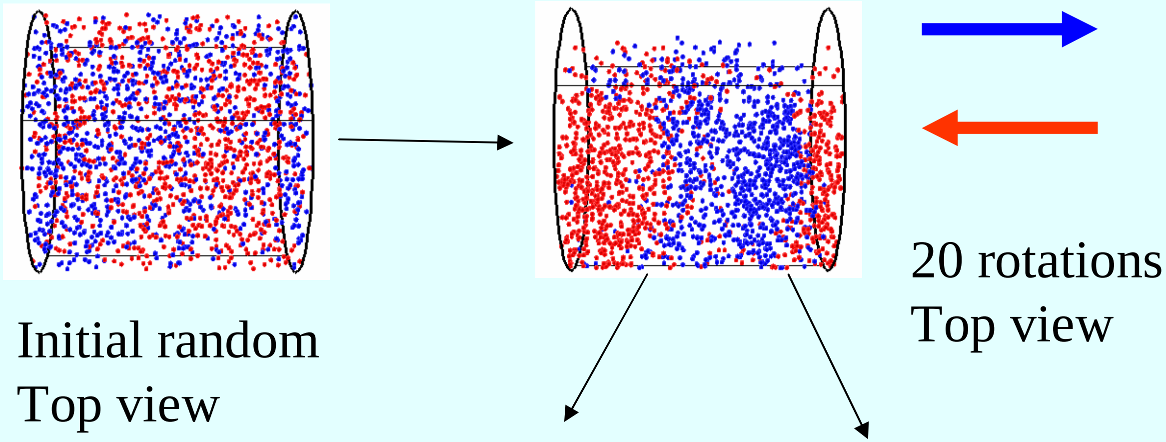
Dispersed state maintained by hydrodynamic diffusion against centrifuging



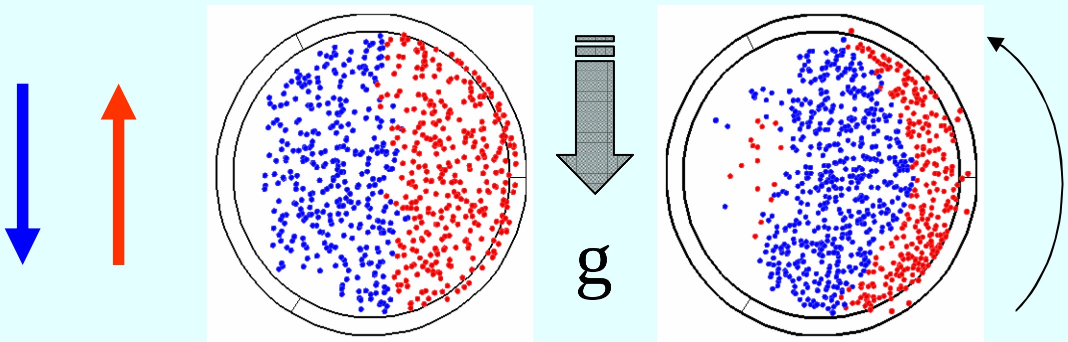
Mean particle density is quantitatively predicted by flux balance

$$\frac{1}{r}\partial_r (rJ_r) = 0; \quad J_r = -\frac{m_b \Omega^2 rn}{6\pi\mu a} + D_{\max}\partial_r n$$

Radial particle distribution varies with axial posiition in band phase: $\Omega^* = 1,\ L = 2R$



At onset of banding, transient aggregates are falling much faster than Stokes velocity.



Initiates secondary flows in axial direction.

Dilute region:
- More dispersed
- Slower settling

Denser region:
- Less dispersed
- Faster settling

In longer cylinders bands of fixed wavelength (~ 2R) develop

$\Omega^* = 1$

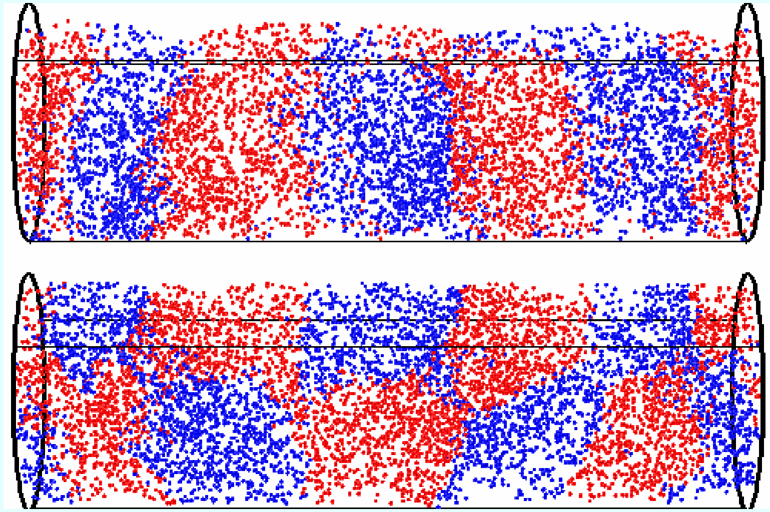
$L \sim 6R$

Suspension organizes into bands of high (blue/red) and low (red/blue) concentration

High density regions (Top) generate secondary flows as the particles fall (Front)

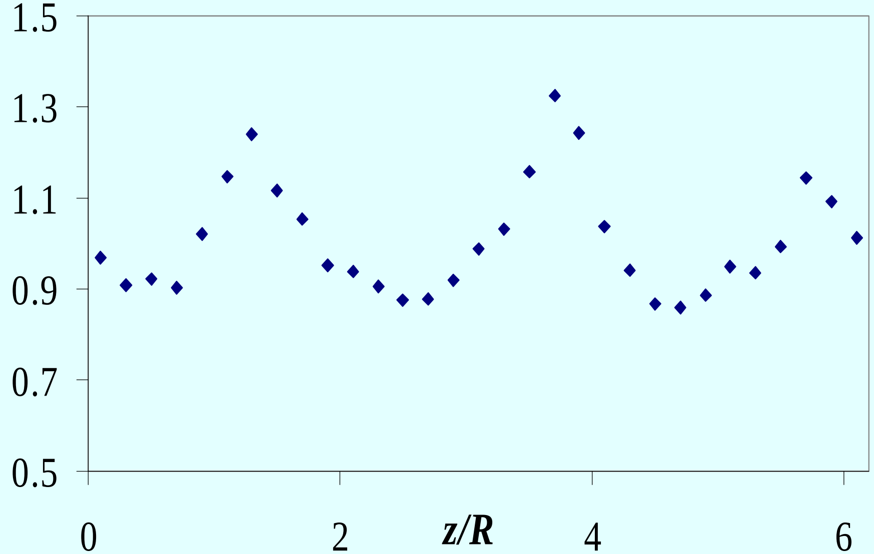
From the front the bands appear diagonal because of the axial flow.

30% variations in $\langle n \rangle$



Top

Front



A suspension in an oscillating gravitational field also segregates

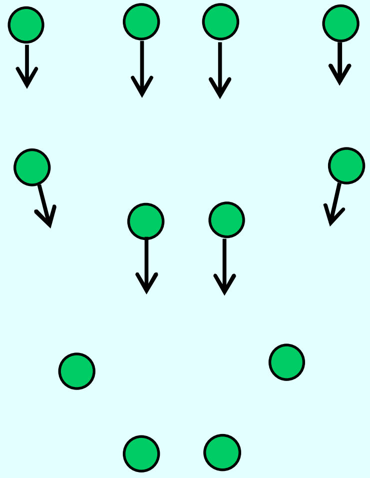
Free-settling suspension is unstable (Crowley)

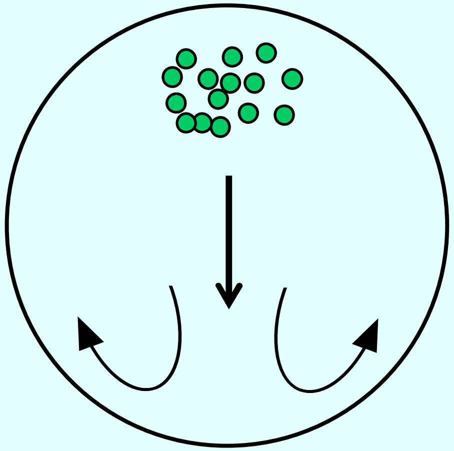
Similar to Rayleigh-Taylor instability

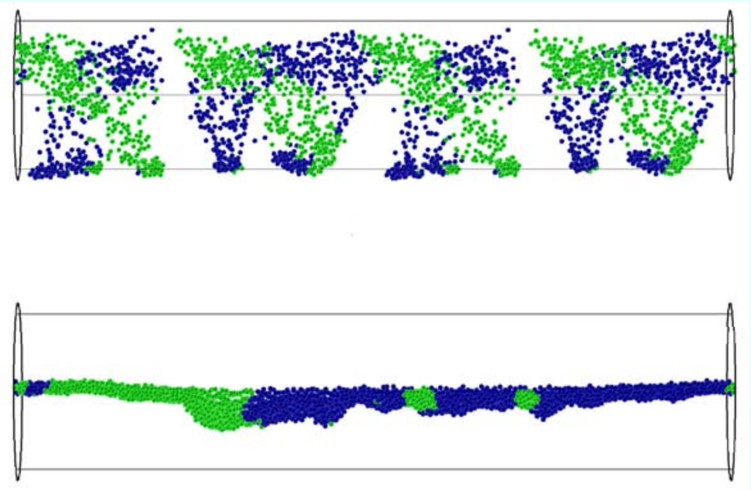
Fluid flows around cluster and denser regions fall faster.

A dispersion is stabilized by backflow-denser regions fall more slowly

Cylinder radius screens HI and sets bandwidth







Summary

Hydrodynamic interactions qualitatively important, even in dilute suspensions, because of large density fluctuations

Characteristic length scale underlying pattern formation $d = n^{-1/3}$

Dynamics characterized by single dimensionless frequency $\Omega d/u_0$

Transition from segregated to dispersed phases has an order parameter $\langle\dot{\theta}\rangle$

Stable dispersed phase around $\Omega^* \sim 1$ due to HI

Band formation generated by concentration instabilities with hydrodynamic screening

Secondary flows generated by instability to axial density variations