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Magnetospheric optics for Cosmic Rays

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1. Classification of particle trajectories in magnetosphere.

Large variety, depending on particle energy and on magnetic and electric fields.

Energies: from < 1eV (ionospheric plasma) to > 10^{17} eV (cosmic rays)

Classification (of particles) according to characteristic dimensions of their trajectories (d) with respect to dimension of magnetosphere (d_m):

$d \ ^{3} \ d_m$ $\qquad$ $d \ll d_m$

Equation of motion (particle with rest mass m, charge q, velocity **v** in static magnetic field **B**):

(1) $d(\gamma.m.\mathbf{v})/dt = d(\mathbf{p})/dt = q\ .\ \mathbf{v} \times \mathbf{B}$ $/\ .\ \mathbf{p}$ $(\ \mathbf{p} \,||\, \mathbf{v}\ )$

$\partial\mathbf{B}/\partial t = 0$ (-rot ($\mathbf{E}$)) $\Rightarrow$ $d(\mathbf{p})^2/dt = 0$

$p = |\mathbf{p}|$ is constant of motion

$\mathbf{v} = d\mathbf{x}/dt = c^2\ \mathbf{p}\ /\ U$

Total energy:

$U = (c^2.p^2 + m^2.c^4)^{1/2}$ $U = \gamma\ m\ c^2$ $\gamma = (1-v^2/c^2)^{-1/2}$

U, $|\mathbf{v}|$ and γ are constants of motion (in static **B**)

Dipolar magnetic field

$\mathbf{B}$ = - grad (μ_o . $\mathbf{M}$. $\mathbf{r}$ / (4 π r^3)) (2)

$\mathbf{r}$ – radiusvector (from center of dipole), $\mu_o = 4\pi . 10^{-7}$ (SI)

Since $\mathbf{B}$ is static, the operator d/dt can be replaced by ($\mathbf{v}$. ∇) and (1) has form

m . ($\mathbf{v}$. ∇) $\mathbf{v}$ = q. $\mathbf{v} \times \mathbf{B}$ (3)

Eq. (3) with $\mathbf{B}$ (2) can be rewritten in non-dimensional form by using the Stormer length as a unit of length

Stormer length

$l_s = (\mu_o . M . |q| / (4 \pi m v))^{1/2}$

For Earth : $M_E = 8.1 . 10^{22}$ A.m^2

Physical meaning of l_s:

Curvature radius **R** of any smooth curve is

$$\mathbf{R}/R^2 = (\mathbf{t} \cdot \nabla)\, \mathbf{t} \tag{4}$$

t – unit tangent vector to the curve at given point

$\mathbf{t} = \mathbf{v}/v$

RHS of (4) is equivalent to $[(\mathbf{v}/v) \cdot \nabla] (\mathbf{v}/v)$

Using (2 and 3)

$$\mathbf{R}/R^2 = - (\mu_o\, q / (4 \pi\, m\, v^2))\, \mathbf{v} \times \nabla (\mathbf{M} \cdot \mathbf{r}/r^3) \tag{5}$$

Estimate of R from (5) is

$R = r\, (r/l_s)^2\, F$, where F is geometric coefficient of order of unity (depending on angle between **B** and **v**) (6)

Meaning of (6) : $R = r\,(r/l_s)^2\,F$

If $l_s > r$, curvature radius (R) is significantly smaller than r

If $l_s < r$, $R > r$

Characteristic distance (r) is size of magnetosphere

Example. Compute the Stormer length of

a. electron with kinetic energy 1 eV, 1 MeV, 10 GeV

b. proton with these energies

in Earth magnetosphere and compare the values with distance to the sub-solar point of the magnetosphere (~10 Re).

How the lengths are changed for Jupiter (M_J = 20.000 M_E, 1 R_J = 71.400 km, sub-solar point at 50 – 100 R_J) ?

Two categories of particles (their trajectories) in the magnetosphere of Earth:

a. Particles for which the Stormer length is comparable or smaller than dimensions of magnetosphere. For protons of energy above ~10 MeV (kinetic), for electrons above ~100 MeV.

   Cosmic rays (galactic, anomalous, solar and accelerated in the interplanetary space to those energies) – Stormer trajectories (part 3).

b. Particles with Stormer length substantially larger than that of magnetosphere.

   Trapped particles in radiation belts, auroral particles, particles of magnetospheric plasma populations - guiding center approach (part 2).

2. Low energy particles: short summary

If $l_s >> d_m$, theory of perturbation is used [1,2,4]. Magnetic and electric fields are "given", not influenced by particle motion.

2.1. Perturbation theory.

2.1.1. Homogeneous (unperturbed) magnetic field

Rotation with gyroradius ($r_c = m.v_\perp /(|q|.B)$), gyrofrequency ($\omega_c = |q| .B / m$). Magnetic flux in Larmor ring (Φ) is proportional to the diamagnetic moment of particle motion (μ)

($\mu = \pi . r_c^2 . J = m.v_\perp{}^2/2B = W_\perp /B$; $J = |q| . \omega_c / 2\pi$; W – kinetic energy)

$$\Phi = \pi . r_c^2 . B = 2 \pi \mu m / q^2 \qquad (7)$$

Collisions. Change of velocity is $\delta v = 1/m .\int F_i \, dt$, F_i is interaction force, integration over time of interaction. Gyration center is shifted.

If the force has a predominant direction (average F - 0), drift appears $\mathbf{v}_F = (\mathbf{F} \times \mathbf{B})/(q.B^2)$ (8)

Presence of electric field **E** -0. Reference system S' with **E'** = 0 is found. Its velocity with respect to S ($\mathbf{v}_{E\perp} = (\mathbf{E} \times \mathbf{B}) / (B^2)$) is superimposed on rotation motion.

Slow variations of magnetic field (relative variation of **B** is small over one gyroperiod). Change of energy over gyroperiod is

$$\Delta W_\perp / W_\perp = \Delta B / B \Rightarrow \qquad \mu = W_\perp / B = \text{const} ; \qquad (9)$$

μ is saved if temporal and/or spatial variations of magnetic and electric field met by the gyrating particle over one gyroperiod are small (*not valid for high energy particles – cosmic rays*).

2.1.2. Drift motion [4].

Guiding center system GCS (*if at any instant of time we can define a (moving) frame of reference in which an observer sees the particle in a periodic orbit perpendicular to magnetic field (with single periodicity and during at least one full cycle).*

Order of drifts: by value of ($r_c \, .|\nabla B|/B$).

Zero order drifts: *uniform static **B** plus external non-magnetic force **F** which is constant in time and space.* To find GCS: perpendicular velocity ($\mathbf{v}_F$) of the reference system is found in which the external force $\mathbf{F}^*$ (* are quantities in GCS) is balanced by the induced electric field ($q \,.\, \mathbf{E}^* = q \,.\, \mathbf{v}_F \times \mathbf{B}$)

Similar to (8) $\mathbf{v}_F = (\mathbf{F} \times \mathbf{B})/(q.B^2)$

First order drifts: *non-uniform **B**.* Gradients of B cause 1-st order drifts which are *energy dependent.*

Combined gradient-curvature drift

$\mathbf{v}_{CG} = (m/2.q.B^2).(v_\perp{}^2 + 2.v_{||}{}^2)\, \mathbf{e} \times \nabla_\perp B =$

$= m.v^2/(2.q.B^2).(1+\cos^2\alpha)\, \mathbf{e} \times \nabla_\perp B$ (**e** is unit vector along **B,** α PA) (10)

If $r_c \,.|\nabla B|/B << 1 \quad \Rightarrow \quad v_{CG} << v$

Second order drifts: *time-dependent zero or first order drifts* (e.g. time-dependent **E**, or direction-changing drift along a curved equipotential). If frame of reference moves with **v** (*changing in time*), it is *accelerated* and *inertial force appears* to an observer in that frame $-m \, . \, d\mathbf{v}/dt$ $\Rightarrow$ "true" GCS is moving with velocity **v** plus additional drift ($\mathbf{v}_s$) :

$$\mathbf{v}_s = -(m/qB) \, . \, (d\mathbf{v}/dt \times \mathbf{e}) \qquad (11)$$

which is necessary to induce electric field that cancels the effect of inertial force in GCS. For GCS approximation to remain valid, the change of **v** during one gyroperiod must be very small (if $T_c.(dv/dt)/v \ll 1$, (11) implies $v_s \ll v$).

Combining (11, 10, 8) more general expression for drift is:

$$\mathbf{v}_D = \mathbf{e}/(q.B) \times [\,-\mathbf{F} + (m/2B).(v_\perp^2 + 2v_{||}^2)\, \nabla_\perp B + m.d\mathbf{v}_D/dt] \qquad (12)$$

F represents all external forces, including all electric field forces (electrostatic and induced). For computations of $\mathbf{v}_D$ the iterations are needed. Complete expression for $\mathbf{v}_D$ including higher orders is in [7].

2.2. Adiabatic invariants.

If particle motion is *possible to assume* as superposition of more (3 in case of geomagnetic trapped population) *cyclic motions* (with different periodicities), the useful approach to trajectory checking is by *adiabatic invariants* connected with these motions. Theory is e.g. in [9].

First invariant. Gyroradius is defined in GCS: $r_c = p_\perp^*/q.B^*$; the associated gyroperiod $T_c = (2\pi.r_c)/v_\perp^* = (2\pi.m^*)/q.B^*$ and the gyrofrequency $\omega_c = 2\pi/T_c = q.B^*/m^*$. These expressions are valid relativistically assuming $m = m_o.\gamma$, $\gamma = (1-\beta^2)^{-1/2}$; $\beta = v^*/c$.

When either a *zero order or first order drift acts alone*, the *kinetic energy of particle is conserved* in original frame of reference (OFR). When *both drifts act together*, *kinetic energy does not remain constant*.

There is, however, *quantity (related to gyration) is conserved when GCS approximation holds*, that is, as long as a spatial variations are very small over r_c and time variations are very small over T_c.

$\mu = p_\perp^{*2}/(2m_oB)$ = const ($p_\perp^* = p^*$ is particle momentum in GCS) (13)
Relativistic magnetic moment or 1st adiabatic invariant.

Second invariant.

Related to the *bounce motion* (between the mirror points, for trapped particles in the magnetic mirror). **p** = m.**v** + e.**A, A** is magnetic field vector potential

The quantity

$$J = \oint p_{||}\, ds \qquad (14)$$

is conserved (the path of integration is *along a field-line*: from the equator to the upper mirror point, back along the field-line to the lower mirror point, and then back to the equator).

Third invariant.

is associated with the *cyclic longitudinal drift of particles* around the earth. Similarly to guiding center associated with a particle's gyromotion around field-lines, the *bounce center lies on the equatorial plane, when particles are azimuthally drifting around the earth*.

$$K = e \cdot \oint A_\phi \; ds = e \cdot \Phi \qquad (15)$$

Where A_ϕ is magnetic vector potential (its ϕ component). *Integration is along the guiding drift shell*. Φ is the total magnetic flux enclosed by the drift trajectory (by the orbit of the bounce centre around the earth).

Adiabatic approximation holds for $T_c \ll T_{bounce} \ll T_{drift}$.

Periodic forces on time scales T_c, T_{bounce}, T_{drift} may yield in violation of corresponding invariants.

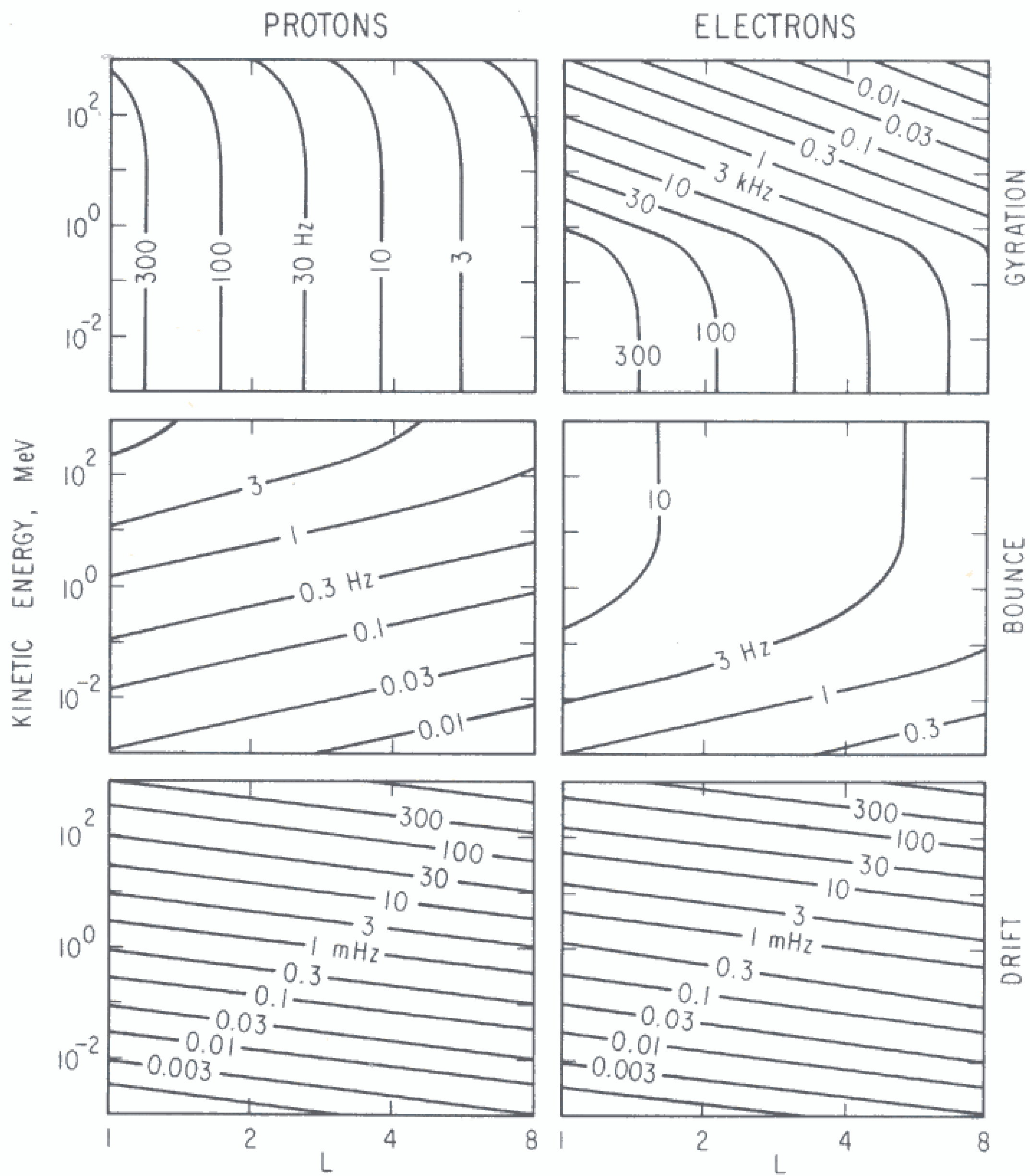


Fig. 6. Contours of constant adiabatic gyration, bounce, and drift frequency for equatorially mirroring particles in a dipole field. Adiabatic approximation fails in upper right-hand corners ($E \sim 1\,\mathrm{GeV}$, $L \sim 8$), since $\Omega_1 \sim \Omega_2 \sim \Omega_3$ implies $|\varepsilon| \sim 1$.

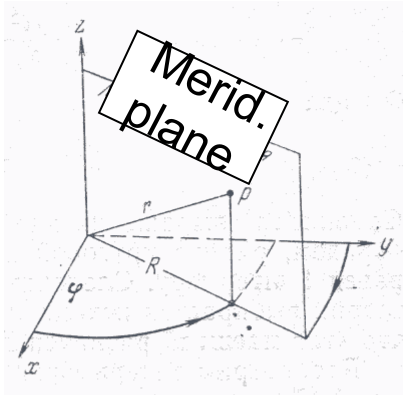
Frequencies of cyclic motions for equatorially trapped charged particles.

Copied from [10].

3. High energy particles

3.1. Stormer theory for dipolar field.

$$B(\mathbf{r},\lambda) = \frac{\mu_0}{4\pi}\frac{\mathrm{M}}{r^3}\sqrt{1+3\sin^2\lambda}$$



λ– magnetic latitude
ψ-angle (z,r, in Rz plane)

Equation of motion (m.d**v**/dt=q.**v** x **B**) is solved as (a) *motion of meridian plane* and (b) *motion within the meridian plane*.

For (a) using *equivalence of moment of force* and *time derivative of the moment of momentum* with respect to z axis ($\mathcal{M}$=RF_ϕ=d/dt(Rm.v_ϕ)) [11]

(RF_ϕ = R^2.m.dϕ/dt) ↓

R^2.dϕ/dt = l_s^2 . (R^2/r^3) + b, where l_s is Stormer length (A)

Adjusting the scale as l_s = 1, a universal equation is obtained independent on mass and velocity of particle. Since v is constant, time is substituted by geometrical variable s = v.t .

Meaning of l_s in equatorial plane of dipolar field:

Stable trajectory: Centrifugal force = Lorentz force

$m.v^2/R_o = q.v.\mu_o.M/(4.\pi.R_o^3) \;\; \Rightarrow \;\; R_o = (\mu_o.M.|q|/(4.\pi.m.v))^{1/2} = l_s$

Circular periodic orbit of particle in dipolar field is that at l_s.

Limit of (A) for $r \rightarrow \infty$ is moment of of particle momentum with respect to z outside the influence of dipolar field. By putting $\mathcal{M} \,|_{r \rightarrow \infty} = - 2.\gamma$, the form of motion of meridian plane (with the particle) is

$R^2.d\phi/ds = \pm (R^2/r^3) - 2.\gamma$ (± for sign of charge) (B)

<u>For (b)</u> (motion in meridian plane, (Rz)). Kinetic energy ($T=1/2.m.v^2$) is conserved. Using coordinate system with m = v = 1,

$T=(1/2).[\underline{R^2.(d\phi/ds)^2} + \underline{(dr/ds)^2 + r^2.(d\psi/ds)^2}] = 1/2.$ (C)

Motion is divided into (1) parallel and (2) perpendicular to meridian. Kinetic energy in Rz plane is Q'.

According to (C,B) $\Rightarrow$ $Q' = (1/2).[1-(\pm(R/r^3) - 2.\gamma/R)^2]$ (D)

Form (D): $Q' = (1/2).[1-(\pm(R/r^3) - 2.\gamma/R)^2]$ (D)

1. Motion in meridian plane is independent on rotation of that plane (Q' is indepentent on ϕ).

2. For given γ the kinetic energy Q' is only function of coordinates. Dipolar field is a potential one and forces are repulsive (Q' is smaller when approaching to the dipole). For Q = 2Q' there exist 2 characteristic values:

   maximum (Q=2, T=1). Particle is moving in meridian plane.

   minimum (Q=0). Particle is moving perpendicular to meridian plane and velocity projection to Rz plane changes its sign (particle after reaching the point Q=0 is moving in reversed direction). The line Q=0 at plane Rz is enclosing the region forbidden for particle.

(D) Provides system of equipotential lines. *Q=0 is most important*. *By rotating Q=0 around z axis, the rotational surface is obtained: segment of space near the dipole to which the particles cannot enter.* Shape and extent of this segment of space depends on γ.

3.2. Allowed and forbidden trajectories.

When using the system m = v = 1 then $Q = v_{mer}^2$, $v_n = \pm(1\text{-}Q)^{½}$ (meridian and normal components of v).

Adjusting ω as angle between instantaneous velocity vector **v** of the particle and normal to meridian plane which follows the particle in its orbit (measured consistently with ϕ, i.e. from west to east)

$v_n^2 = v^2 \,.\, \cos^2(\omega) = \cos^2(\omega) = 1\text{-}Q$

According to (D) for positively charged particles

$$\cos(\omega) = (R/r^3) - (2.\gamma/R) \qquad \text{(E)}$$

$$\cos(\lambda) = R/r \quad \Rightarrow \quad \cos(\omega) = \cos(\lambda)/r^2 - 2.\gamma/(r.\cos(\lambda)) \qquad \text{(F)}$$

λ is latitude.

Solving (F): $$r = (\cos^2(\lambda)/(\gamma \pm (\gamma^2 + \cos(\omega).\cos^3(\lambda))^{1/2}) \qquad \text{(G)}$$

For limiting surface Q=0, $\cos(\omega)= \pm\ 1$ and

$$r = \cos^2(\lambda)/(\gamma \pm (\gamma^2 \pm \cos^3(\lambda))^{1/2} \qquad \text{(H)}$$

The solutions:

$r_1 = \cos^2(\lambda)/(\gamma + (\gamma^2 + \cos^3(\lambda))^{½})$

$r_2 = \cos^2(\lambda)/(\gamma + (\gamma^2 - \cos^3(\lambda))^{½})$

$r_3 = \cos^2(\lambda)/(\gamma - (\gamma^2 - \cos^3(\lambda))^{½})$

3 solutions for $\gamma^2 > \cos^3(\lambda)$ (a)

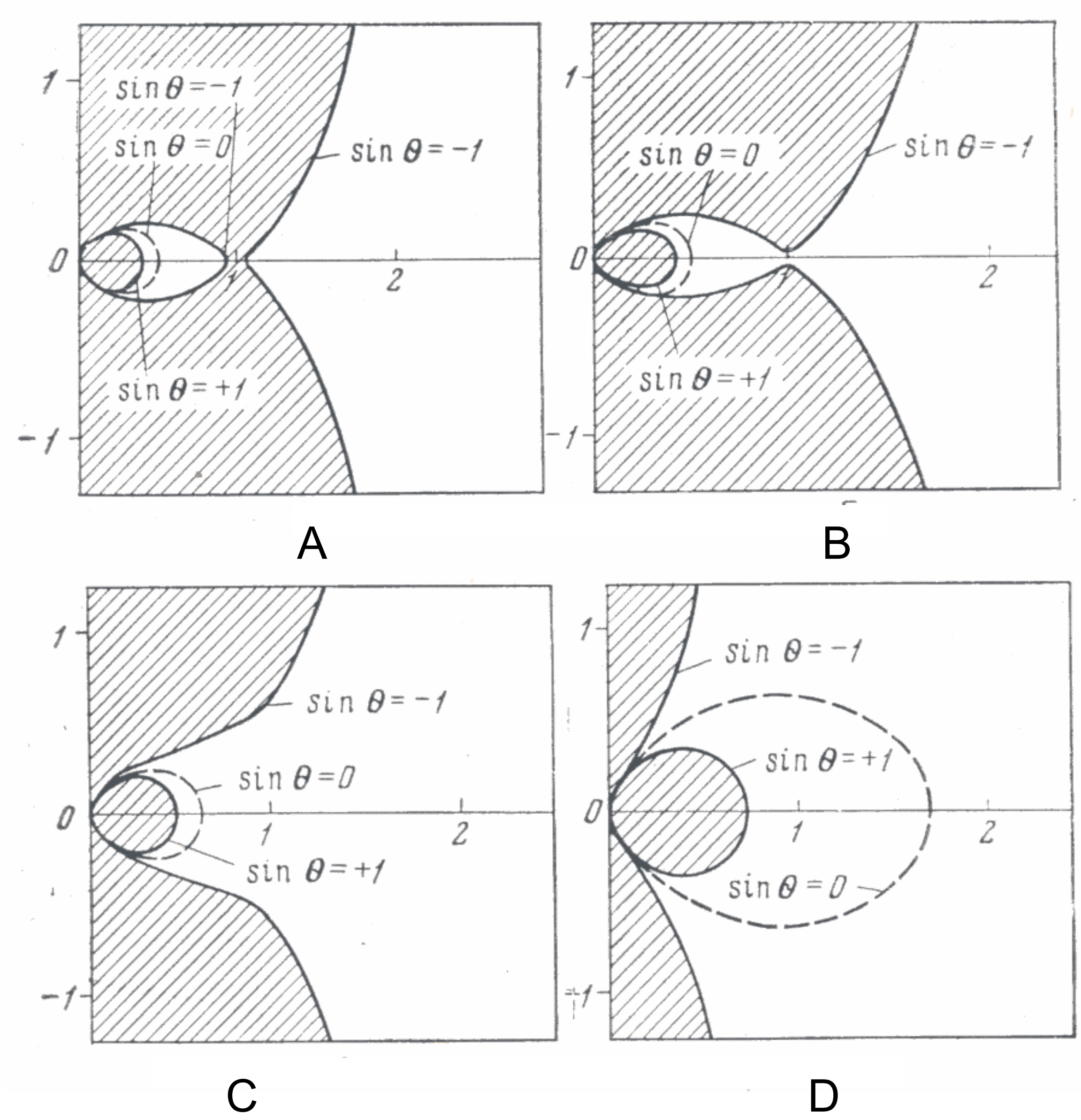
1 solution for $\gamma^2 < \cos^3(\lambda)$ (b)

In case (a) the radius-vector crosses surface Q=0 three times (region of allowed trajectories Q>0 is composed of two isolated regions);

In case (b) there is only one region of allowed trajectories.

Max. of $\cos^3(\lambda) = 1$, for $\gamma > 1$ ⇨ two regions of allowed trajectories

Details and complete calculations regarding the problem “given a particle approaching a magnetic dipole from infinity, what is its trajectory?” is e.g. in book [13]. *No general solution to the problem, but it is possible to define permitted and forbidden zones for any particle approaching from infinity.*



Regions (in polar coordinates r,λ) accessible to high energy particles entering Earth's magnetic field. Shaded regions have $|\sin(\theta)|>1$ – forbidden.

The distance units are Stormer units, i.e. distances in units $l_s=(\mu_o.M.q/(4\pi p))^{1/2}$

$\theta = \pi/2 - \omega$

A: $\gamma = 1.001$

B: $\gamma = 0.999$

C: $\gamma = 0.8$

D: $\gamma = 0.3$

Particle momentum is "hidden" in r and γ.

Specifying particle momentum – the Earth can be drawn in these diagrams. For γ=1 there is a critical momentum at which Stormer length is equal to radius of Earth (p_1=59.6GeV/c). For this momentum the Earth is a circle with r_1=1. For p=4p_1 the r=2.

The access of positively charged particles is “more difficult” from east.

For given position and given momentum there is a cone with axis in east direction for which the access of particles is forbidden.

Its extent is decreasing with increase of momentum and with latitude.

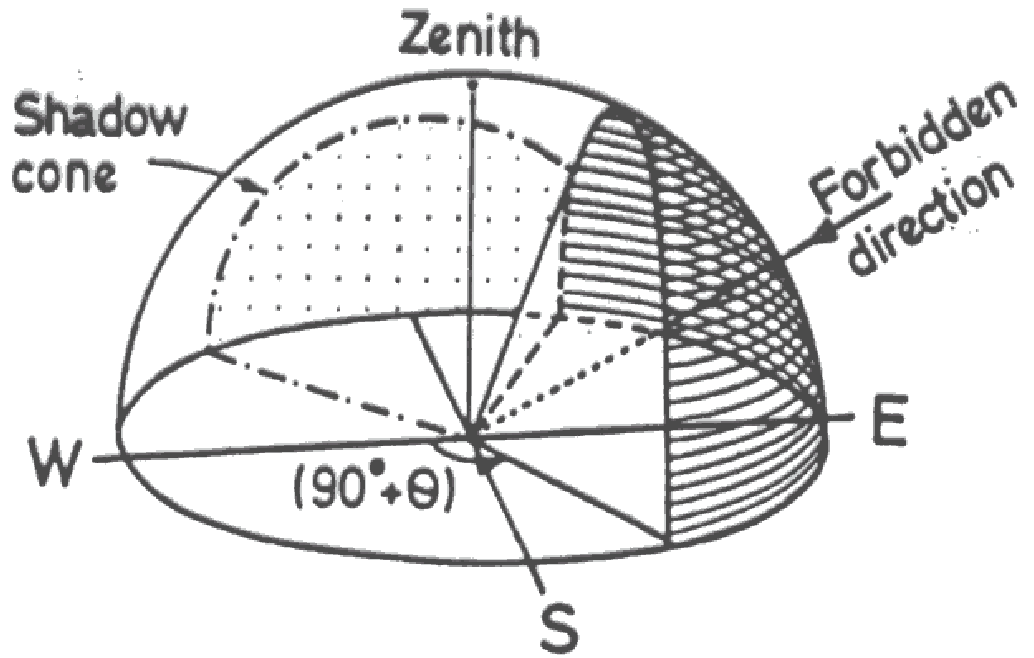


FIG. 5. Størmer's forbidden cone, and the shadow cone.

Shadow cone is due to the presence of earth body.
(from A.M. Hillas, Cosmic Rays, 1972)

3.3. Concept of cut-off rigidity and asymptotic directions.

Usually the effects of geomagnetic field on cosmic rays are expressed in terms of rigidity.

total energy U

$$\boxed{U = \underbrace{m}_{\text{relat.}} \cdot c^2}$$

rest mass

mass

$$\boxed{\begin{aligned} m &= m_0 \cdot \gamma \\ \gamma &= \frac{1}{\sqrt{1-\beta^2}} \\ \beta &= v/c \end{aligned}} \qquad \boxed{U = \gamma m_0 c^2}$$

velocity

momentum

$$\boxed{p = m_0 \cdot v \cdot \gamma}$$

$$p^2c^2 = \frac{m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}} = \frac{m_0^2 \frac{v^2}{c^2} \cdot c^4}{1 - \frac{v^2}{c^2}} \quad \Big| \pm$$

$$p^2c^2 = \frac{m_0^2 c^4 \left[\frac{v^2}{c^2} - 1\right]}{1 - \frac{v^2}{c^2}} + \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} = -m_0^2 c^4 + m_0^2 c^4 / \left(1 - \frac{v^2}{c^2}\right)$$

$$p^2c^2 = -m_0^2 c^4 + U^2 \longrightarrow \boxed{U^2 = p^2c^2 + m_0^2 c^4}$$

$$p^2c^2 = -m_0^2c^4 + U^2 \rightarrow \boxed{U^2 = p^2c^2 + m_0^2c^4}$$

$$U = E_0 + E_{kin}$$

$$E_0 = m_0c^2$$

$$\gamma m_0 c^2 = m_0c^2 + E_{kin} \rightarrow E_{kin} = (\gamma - 1)\cdot E_0$$

rigidit

(16) $\boxed{R = \frac{pc}{Z}} \Rightarrow$

$$\frac{p^2c^2}{Z^2} = (\gamma^2 - 1)\cdot\frac{m_0^2c^4}{Z^2} \rightarrow$$

Z – charge number (ratio of charge to elementary charge)

Definition by [12] (in V)

$$\rightarrow \frac{p^2c^2}{Z^2} = \left(\left(1+\frac{E_K}{E_0}\right)^2 - 1\right)\frac{m_0^2c^4}{Z^2} = \left(1 + 2\frac{E_K}{E_0} + \frac{E_k^2}{E_0^2} - 1\right)\frac{m_0^2c^4}{Z^2} =$$

$$= \left(\frac{2E_kE_0 + E_k^2}{\cancel{E_0^2}}\right)\frac{\cancel{m_0^2c^4}}{Z^2} = \frac{1}{Z^2}\left(E_k\cdot(E_k + 2E_0)\right) =$$

$$= \frac{1}{Z^2}\left(T_n\cdot A\cdot(T_n\cdot A + 2\cdot A\cdot E_{0p})\right) = \frac{A^2}{Z^2}\left(T_n\cdot(T_n + 2E_{0p})\right)$$

A-atomic mass number (nucleon number)

kinet. en./nucl.

proton rest energy

$$\boxed{R = \frac{A}{Z}\sqrt{T_n\cdot(T_n + 2E_{0p})}} \quad (17)$$

Why "rigidity"?

$$R = \frac{A}{Z}\sqrt{T_M \cdot (T_M + 2E_{0p})} \qquad (\quad GV)$$

centrifugal f.

Lorentz f.

$$m \cdot r \cdot \omega^2 = Z \cdot e \cdot v \cdot B \qquad (\perp \vec{B})$$

$$\omega = \frac{Z \cdot e \cdot B}{m} \qquad v = r \cdot \omega$$

gyroradius

$$r = \frac{v}{\omega} = \frac{m\,v}{ZeB} = \frac{p}{ZeB} = \frac{pc}{Z} \cdot \frac{1}{e \cdot c \cdot B} = R\,\frac{1}{e \cdot c \cdot B}$$

$$R \nearrow \infty \rightarrow r \nearrow \infty$$

For given B the gyroradius is proportional to rigidity rest energy (mass) of selected particles

e: 0.511 MeV ($9.109 \cdot 10^{-31}$ kg)

p: 938.27 MeV ($1.673 \cdot 10^{-27}$ kg)

n: 939.57 MeV ($1.675 \cdot 10^{-27}$ kg)

μ: 105.66 MeV

d: 1875.6 MeV

α: 3727.4 MeV

Directional definitions for cosmic ray access (from *Cooke, D.J. et al., Il Nuovo Cimento, 14 C, N 3, 213-233, 1991*):

Cones (each definition is for charged particles of a *single specified rigidity* value arriving at *particular point* in geomagnetic field):

<u>Allowed cone</u> – solid angle with all directions of arrival of all trajectories which *do not intersect the earth and which cannot possess sections asymptotic to bound periodic orbits* (because rigidity is too high to permit such sections to exist in the directions of arrival concerned).

<u>Main cone</u> – *The boundary of allowed cone*. It is composed of trajectories which are asymptotic to the simplest bound periodic orbits and trajectories which graze the surface of earth (For this purpose the surface of earth is generally taken to be at the top of effective atmosphere).

<u>Forbidden cone</u> – The solid angle region within which all directions of arrival correspond to *trajectories which, in absence of solid earth, would be permanently bound in the geomagnetic field*. Access in these directions from outside the field is therefore impossible.

Stormer cone – *the boundary of forbidden cone*. In axially symmetric field the surface forms a right circular cone.

Shadow cone – the solid angle containing all *directions of particle arrival which are excluded due to short range earth intersections of the approaching trajectories, while the particle loops around the local field lines*

Penumbra – *the solid angle region contained between the main cone and the Stormer cone*. It contains a complex structure of allowed and forbidden bands of arrival directions.

Rigidity picture definitions (each definition refers to particles *arriving at a particular site* within geomagnetic field *from a specified direction*.

Cut-off rigidity – location of *transition, in rigidity space, from allowed to forbidden trajectories as rigidity is decreased*

Stormer cut-off rigidity, R_s – the rigidity value for which the Stormer cone lies in the given direction. In a dipole field *the direct access for particles of all rigidity values lower than Stormer cut-off rigidity is forbidden from outside the field*. In a dipole approximation (in GV)

$$R_s = (M.\cos^4(\lambda))/\{r^2.[1 + (1 - \cos^3(\lambda).\cos(\varepsilon).\sin(\xi))^{1/2}]^2\} \quad (18)$$

M is the dipole moment and has a normalized value of 59.6 when r is expressed in units of earth radii

λ is magnetic latitude

r is distance from the dipole in earth radii

ε is azimuth angle measured clockwise from geomagnetic east direction (for positive particles)

ξ is the angle from the local magnetic zenith direction

Upper cut-off rigidity (R_U) – rigidity value of the *highest* detected *allowed/forbidden transition* among a set of computed trajectories.

Lower cut-off rigidity (R_L) – the lowest detected cut-off value (i.e. the rigidity value of the *lowest allowed/forbidden transitions* observed in a set of computed trajectories). If no penumbra exist $R_L = R_U$.

Effective cut-off rigidity (R_E) – the total effect of penumbral structure in a given direction may be represented usefully by this single value. Effective cut-offs may be either linear averages of the allowed rigidity intervals in penumbra, or functions weighted for cosmic ray spectrum and/or detector response. For linear weighting

$$R_E = R_U - \sum_{R_L}^{R_U} \Delta R_i \text{ (allowed)} \qquad (19)$$

Where the trajectory calculations are performed at rigidity interval (step) ΔR_i.

Example :

What is the Stormer cut-off for cosmic ray access to Trieste position

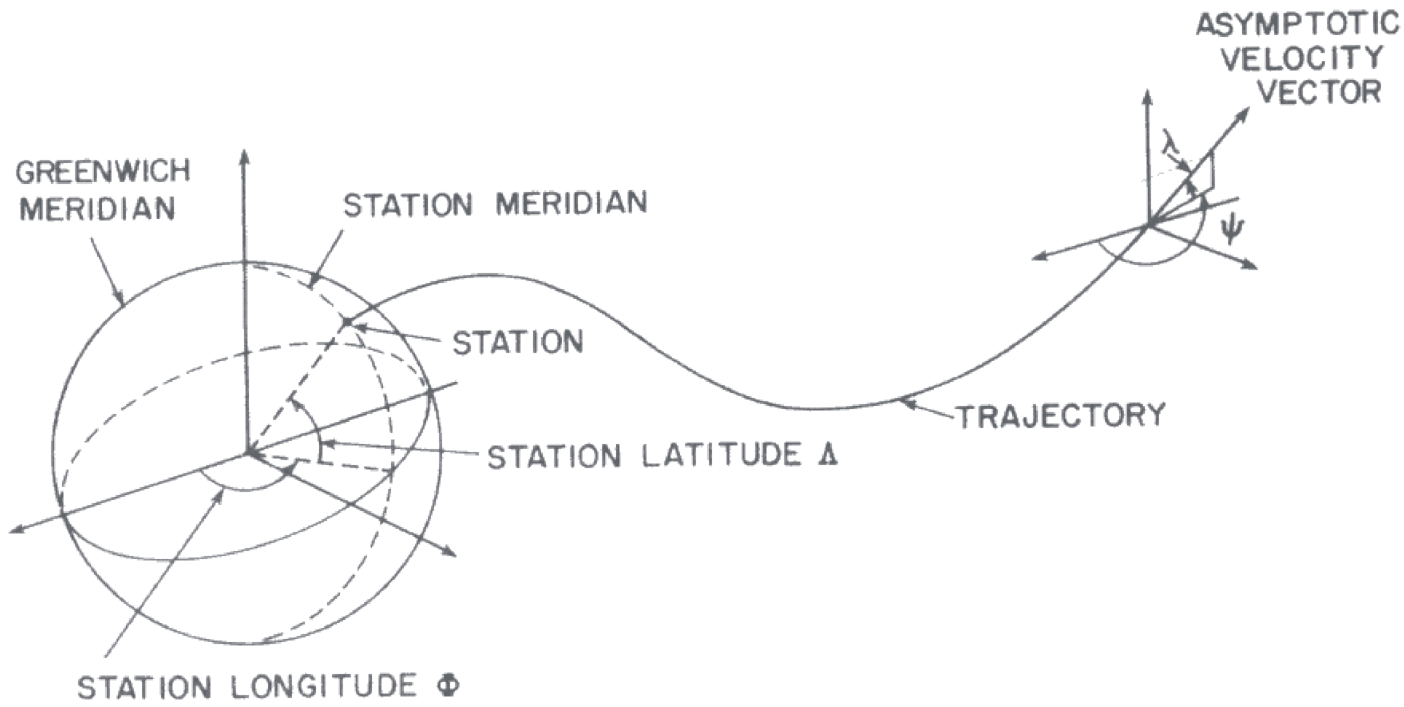
a. from the vertical direction, b. from direction 30^o declined from zenith from east (magnetic) and c. from west (magnetic). What kinetic energy of protons and of anomalous cosmic ray oxygen (A=16, Z=1) these values correspond

Lat = 45.64861, Long = 13.78000

http://swdcwww.kugi.kyoto-u.ac.jp/igrf/gggm/index.html

Conversion from Geographic to Geomagnetic coordinates
Geomagnetic 45.50 N   96.07 E

<u>Asymptotic directions.</u>



Allowed trajectory. Charged particle of cosmic rays arriving above the site of particular station from a given direction (local) arrived formerly to magnetospheric boundary at another direction (asymptotic direction). Figure (*from Shea,M.A. and Smart, D.F., ERP No 524, AFCRL-TR-75-0381, 1975*) illustrates the definition of asymptotic latitude and longitude.

The "asymptotic latitude", Λ, is given by

$$\tan \Lambda = \frac{-v_\theta \sin \theta + v_r \cos \theta}{[v_\theta^2 + (v_\phi \cos \theta + v_r \sin \theta^2]^{1/2}}$$

and the "asymptotic longitude", ψ, by

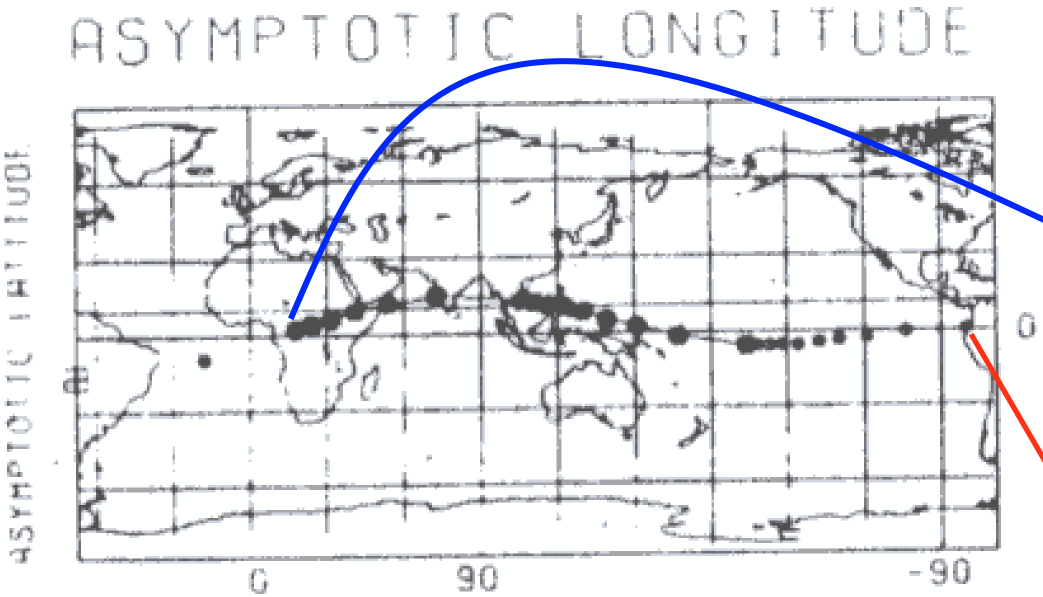
$$\psi = \phi + \arc \tan \left(\frac{v_\phi}{v_\theta \cos \theta + v_r \sin \theta} \right) \tag{20}$$

where

θ = co-latitude

ϕ = longitude measured eastward from the Greenwich meridian, and

v_r, v_θ, v_ϕ = velocity components in r, θ, and ϕ directions.



CHACALTAYA, BOLIVIA

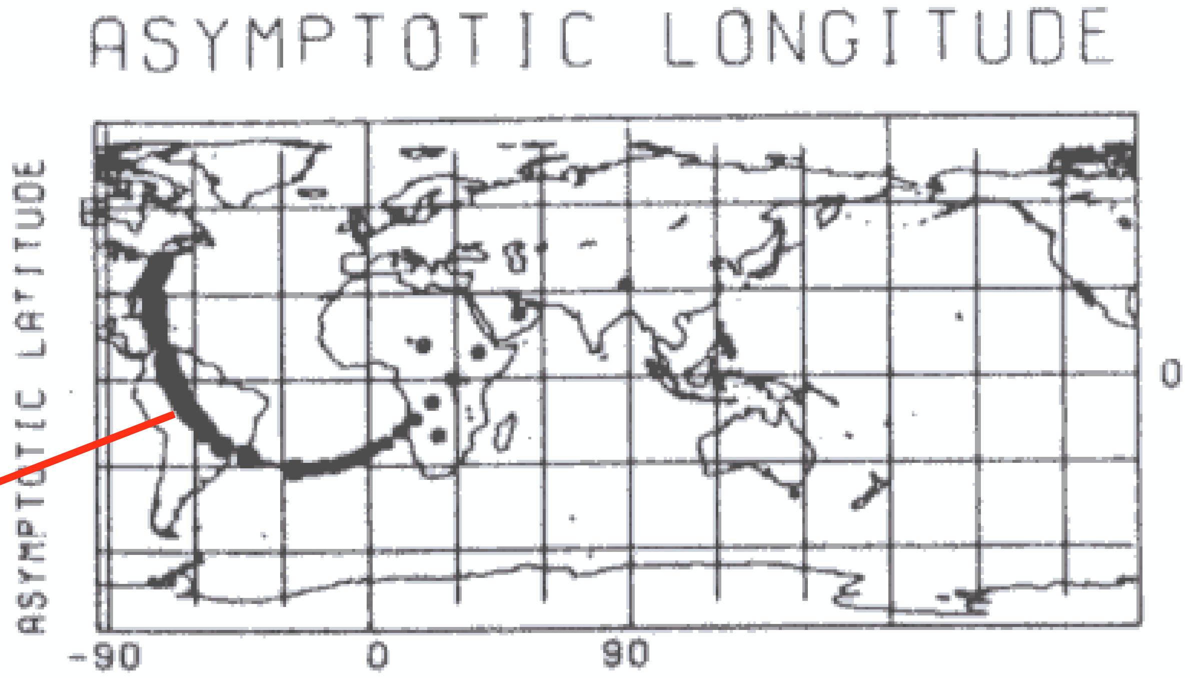
GEOGRAPHIC LATITUDE = -16.31
GEOGRAPHIC LONGITUDE = 291.85

EFFECTIVE VERTICAL CUTOFF
RIGIDITY = 13.10 GV

RIGIDITY (GV)	ASYMPTOTIC LAT.	ASYMPTOTIC LONG.
20.00	2	19
19.00	4	25
18.00	7	32
17.00	10	42
16.00	13	55
15.00	15	73
14.00	14	106
13.90	13	111
13.80	12	117
13.70	10	123
13.60	8	131
13.50	6	140
13.40	2	151
13.30	-2	167
13.20	-6	195
13.19	-7	199
13.18	-7	204
13.17	-7	209
13.16	-6	215
13.15	-6	222
13.14	-5	231
13.13	-4	242
13.12	-2	256
13.11	-1	280
13.10	-11	343
13.09	R	R

Figure A11. Station Orientated World Map Illustrating the Asymptotic Directions for Chacaltaya, Bolivia

Asymptotic directions are usually plotted as (long, lat) projection on the map. The computed asymptotic longitudes and latitudes for a high cut-off rigidity station and vertical access. Wide range of asymptotic longitudes for 13-20 GV. (*from Shea, M.A. and D.F. Smart, 1975*)



Atmospheric cut-off

Figure A12. Station Orientated World Map Illustrating the Asymptotic Directions for Churchill, Canada

CHURCHILL, CANADA

GEOGRAPHIC LATITUDE = 58.75
GEOGRAPHIC LONGITUDE = 265.91

EFFECTIVE VERTICAL CUTOFF
RIGIDITY = .21 GV

RIGIDITY (GV)	ASYMPTOTIC LAT.	LONG.
20.00	42	-72
19.00	41	-72
18.00	40	-72
17.00	39	-73
16.00	38	-73
15.00	37	-74
14.00	36	-74
13.00	35	-75
12.00	34	-75
11.00	34	-76
10.00	33	-76
9.00	33	-76
8.00	32	-75
7.00	30	-74
6.00	26	-74
5.00	23	-75
4.00	21	-73
3.00	16	-73
2.00	7	-71
1.90	6	-71
1.80	5	-71
1.70	4	-70
1.60	2	-69
1.50	1	-69
1.40	-1	-68
1.30	-2	-68
1.20	-4	-66
1.10	-6	-65
1.00	-9	-64
.90	-12	-62
.80	-15	-60
.70	-19	-56
.60	-23	-51
.50	-28	-43
.40	-31	-26
.39	-31	-24
.38	-31	-21
.37	-31	-19
.36	-31	-16
.35	-30	-13
.34	-29	-9
.33	-28	-6
.32	-27	-2
.31	-25	2
.30	-22	6
.29	-19	11
.28	-14	18
.27	-9	22
.26	-1	29
.25	9	38
.24	23	53
.23	32	90
.22	-28	384
.21	13	379
.20	F	F

Asymptotic directions computed for a low cut-off station. The interval of asymptotic longitudes is narrower.

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Adiabatic invariants. Copied from [8].

for the motion of magnetized particles – points having the same guiding centre at a certain time will continue to have approximately the same guiding centre at a later time. An approximate Poincaré invariant may thus be obtained by choosing the curve C to be a circle of points corresponding to a gyrophase period. Thus I

$$\mathcal{I} \simeq I = \oint \mathbf{p} \cdot \frac{\partial \mathbf{q}}{\partial \gamma} d\gamma .$$

is adiabatic invariant. p,q are coordinate and momentum in phase space.

p = m**v** + e**A** , **A** is vector potential