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Hard Exclusive Reactions and Hadron Structure:
Some New Results

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These are preliminary lecture notes, intended only for distribution to participants
Hard exclusive reactions and hadron structure: some new results

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Factorization of Hard Exclusive processes

- **DIS : INCLUSIVE / Large vs Short distance →**

- **DVCS : EXCLUSIVE γ* N → γ N’**
  → Amplitude = Pert. Coef. Funct. × GPD
  *Generalized Parton Distributions*

- Deep EXCLUSIVE meson production
  Amplitude = Pert. Coef. Funct. × GPD × DA

- **CROSSING → γ*γ → M_1 M_2 near threshold**
  Amplitude = Pert. Coef. Funct. × GDA
  *Generalized Distribution Amplitude*
Successes of Factorized framework

- Consistent picture in QCD
  Evolution Equations interpolate between
  DGLAP (e.g. for structure functions) and ERBL (e.g. for form-factors) equations

- SCALING, e.g.
  handbag dominance $\equiv$ (generalized) Bjorken scaling
Successes of Factorized framework

- Right order of magnitudes with experimental results, for DVCS (Guzey + Polyakov 2005)
Successes of Factorized framework

- Angular dependence of asymmetries coming from Bethe Heitler / DVCS interference

JLab data at $Q^2 = 2.3 \text{ GeV}^2$, $t = -0.28$ and $-0.23 \text{ GeV}^2$
Generalized Parton Distributions

- **Non forward** Matrix elements of non-local light-cone operators, e.g. for a nucleon

\[
\langle N(p, \lambda) | \bar{\psi}(-z/2) \Gamma[-z/2; z/2] \psi(z/2) | N'(p', \lambda') \rangle
\]

\[
\Gamma = \gamma_\mu, \quad \gamma_\mu \gamma^5, \quad \sigma_{\mu \nu}
\]

- Fourier Transform + Decomposition → 8 GPDs:
  - chiral even:
    \( H(x, \xi, t), E(x, \xi, t), \tilde{H}(x, \xi, t), \tilde{E}(x, \xi, t) \)
  - chiral odd:
    \( H_{Ti}(x, \xi, t), i = 1, \ldots, 4 \quad \text{(transversity)} \)
Kinematics:

- Notation: \( \Delta^+ = -2\xi P^+ \)
  
  \( 2P = p + p', \Delta = p' - p \) \( \xi = \text{skewness} \)

- \( \Delta^2 = t \ll Q^2 \) \( t \)-dependence parametrized as in Form Factors
Properties of Generalized Parton Distributions

- Two quite distinct regions: \( x > \xi \): DGLAP
- \( x < \xi \): ERBL

- Limits at zero skewedness \( \rightarrow \) Usual parton dist.
Properties of Generalized Parton Distributions

- First $x-$moment $\rightarrow$ Form Factors ($\xi$ independent), e.g.

$$F^q_1(t) = \int_{-1}^{1} dx \ H^q(x, \xi, t)$$

- Second $x-$moment $\rightarrow$ Spin Sum Rule (through energy-momentum tensor), e.g.

$$2\langle J^3_q \rangle = \int_{-1}^{1} dx \ x \ [H^q(x, \xi, t = 0) + E^q(x, \xi, t = 0)]$$
Properties of Generalized Parton Distributions

- Lorentz invariance $\rightarrow$ Polynomiaality ($\rightarrow$ Double distributions), e.g.

$$
\int_{-1}^{1} dx \, x^n \, H^q(x, \xi, t) = \sum_{i=0}^{n} (2\xi)^i A^q_{n+1,i}(t) + \text{'D-term'}
$$

- Positivity constraints in DGLAP region, e.g.

$$
|H^q_{\pi}(x, \xi, t)| \leq \sqrt{q_{\pi}\left(\frac{x + \xi}{1 + \xi}\right) \cdot q_{\pi}\left(\frac{x - \xi}{1 - \xi}\right)}
$$
When do we access the factorization regime?

- dVCS → wait for experimental talks today ...
- crossed process → LEP2 data: EARLY SCALING

$Q^2$ dependence of $\gamma^*\gamma \rightarrow \rho^+\rho^-$ and $\gamma^*\gamma \rightarrow \rho^0\rho^0$

- blue
- red
- $t$ dependence of GPDs maps transverse position of quarks in proton.

Fourier transform GPD at zero skewedness

$$ q(x, b_T) = (2\pi)^{-2} \int d^2 \Delta e^{i\Delta \cdot b} H(x, \xi = 0, t) $$

Generalize at $\xi \neq 0 \rightarrow \text{Quantum femtophotography.}$

- $W^2$ dependence of $\gamma^*\gamma \rightarrow M_1 M_2$ maps impact representation of hadronization.
Some new results

- Transversity GPDs

- Searching for EXOTIC HADRONS

- Describing other processes through TDAs
  \[ \bar{N}N \rightarrow \gamma^*\gamma \text{ and } \bar{N}N \rightarrow \gamma^*\pi \]
Transversity GPDs

Transversity dependent quark distribution $h_1(x) \rightarrow 4$ transversity GPDs

- How to access them?

Chiral odd functions come in pairs ->
  try electroproduction of $\rho_T$

- **BUT** zero amplitude for $\gamma^* N \rightarrow \rho_T N'$:
  use Pomeron analog

$$\mathcal{P} N \rightarrow \rho_T N' \ i.e. \ \gamma^* N \rightarrow \rho_L \rho_T N'$$
Transversity GPDs

\[ P = 2 \text{ gluons, at Born order 6 diagrams, e.g.} \]
Models for transversity TDA, $H_T$:

(i) axial meson $A = b_1(1235)$ exchange dominance

$$H_T^a(x, \xi) = \frac{g_{ANN} f_{A_1}^a}{2M_N m_A^2} \frac{(\Delta \cdot S_T)^2 \phi_\perp \left(\frac{x+\xi}{2\xi}\right)}{2\xi},$$

with $b_1$ distribution amplitude $\phi_\perp^A(u)$ (only ERBL)

(ii) the bag model of transversity (Scopetta 2005)
Diff. cross sec. for
\( \gamma^\ast(Q) p \rightarrow \rho_L^0 \rho_{L,T}^+ n \)

Our model: \( \xi = 0.1, 0.3, 0.5 \)
Scopetta: \( \xi = 0.1, 0.3, 0.5 \)
Exotic meson exclusive production

Exotic Hybrid Meson $\pi_1$ with $J^{PC} = 1^{-+}$

Define $\pi_1$ Distribution Amplitude as usual:

$$\langle \pi_1(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle =$$

$$i f_{\pi_1} M_{\pi_1} \left[ p_\mu \frac{e^{(\lambda)} \cdot z}{p \cdot z} \int_0^1 dy e^{i(\bar{y} - y) p \cdot z/2} \phi_H^H(y) \right]$$

- same twist as $\rho$ Distribution Amplitude
- QCD sum rules $\rightarrow f_{\pi_1} \sim 50$ MeV
- Similar electroproduction cross sections in $e p$ collisions.
- Also possible in $e \gamma$ collisions
Exotic meson exclusive production

\[ \gamma^* (q) \rightarrow H(p) \]

\[ \gamma^* (q) \rightarrow \pi(p_{\pi}) \]

\[ \gamma^* (q) \rightarrow \eta(p_\eta) \]

\[ N(p1) \rightarrow N(p2) \]

\[ N(p1) \rightarrow N(p2) \]

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Comparison of $\rho^0$ and $H \equiv \pi_1(1400)$ electroproduction cross sections

Figure 1:

seems visible → COMPASS, e-RHIC
Extension

- What can pQCD say about other exclusive reactions at large $Q^2$ such as those of

$$\bar{p}N \rightarrow \gamma^*\gamma \text{ and } \bar{p}N \rightarrow \gamma^*\pi$$

*PANDA-PAX programs at GSI-FAIR*

New factorization $P \rightarrow \gamma, \ P \rightarrow \pi$ TDA

**Transition Distribution Amplitudes**

$$\langle \pi(p')|\epsilon^{ijk}u^i_\alpha(z_1 n)u^j_\beta(z_2 n)d^k_\gamma(z_3 n)|p(p, s)\rangle \bigg|_{z^+ = 0, z_T = 0}$$

$$\langle \gamma(p', \epsilon')|\epsilon^{ijk}u^i_\alpha(z_1 n)u^j_\beta(z_2 n)d^k_\gamma(z_3 n)|p(p, s)\rangle \bigg|_{z^+ = 0, z_T = 0}$$
Arguments for Factorization

PROOFS EXIST for

- Factorization of deep exclusive $\pi$ electroproduction on meson target.  
  Collins Frankfurt Strikman
- Time inversion: Factorization of $\pi M \rightarrow \gamma^* M'$ on meson target.  
  Berger Diehl BP

\[ \gamma^*(q) \rightarrow \pi(q') \rightarrow \phi_\pi \rightarrow x+\eta \rightarrow \eta-x \]

\[ \tilde{H}, \tilde{E} \rightarrow N(p) \rightarrow \pi \rightarrow \phi_\pi \rightarrow x+\eta \rightarrow \eta-x \]

\[ \gamma^*(q') \rightarrow \pi(q) \rightarrow \phi_\pi \rightarrow x+\eta \rightarrow \eta-x \]

\[ \tilde{H}, \tilde{E} \rightarrow N(p) \rightarrow \pi \rightarrow \phi_\pi \rightarrow x+\eta \rightarrow \eta-x \]
Arguments for Factorization (continued)

- Choose $N = \pi$ and $N' = \rho$

\[ \rightarrow \text{Factorization of } \pi \pi \rightarrow \gamma^* \rho \]

- Change $\rho \rightarrow \gamma$

Remember: Photon structure function factorizes in the same way as meson structure function!

\[ \rightarrow \text{Factorization of TDA in } \pi \pi \rightarrow \gamma^* \gamma \]

in the forward direction (where cross section is bigger.)
The factorization of $\pi\pi \rightarrow \gamma^*\gamma$
Arguments for Factorization - continued

- Change Meson $\rightarrow$ Baryon

*More problematic since 3 quark exchange!*

**BUT Remember:** Baryon Form Factor factorizes in the same way as Meson Form Factor!

$\rightarrow$ Factorization of the $p \rightarrow \gamma$ TDA

in $\bar{p}p \rightarrow \gamma^*\gamma$

*This is NOT a proof ... Hope for a technical derivation*
The factorization of $\bar{N} N \rightarrow \gamma^* \gamma$
From DAs to TDAs

- Recall definition of Distribution Amplitudes

\[ 4 \langle 0 | e^{ijk} u_\alpha^i (z_1 n) u_\beta^j (z_2 n) d_\gamma^k (z_3 n) | B(p, s) \rangle = f_N \]

\[ V(\hat{p} C)^{\alpha \beta} (\gamma^5 B)_\gamma + A(\hat{p} \gamma^5 C)^{\alpha \beta} B_\gamma + T(\gamma^\mu \gamma^5 B)^{\alpha \beta} \]

\[ i, j, k = \text{color indices} \quad n = \text{light cone + direction} \]

- Define Transition Distribution Amplitudes

\[ 4 \langle \pi^0 (p') | e^{ijk} u_\alpha^i (z_1 n) u_\beta^j (z_2 n) d_\gamma^k (z_3 n) | p(p, s) \rangle \Bigg|_{z^+ = 0, z_T = 0} = \]

\[-\frac{f_N}{2f_\pi} \left[ V_1^0 (\hat{P} C)^{\alpha \beta} (B)_\gamma + A_1^0 (\hat{P} \gamma^5 C)^{\alpha \beta} (\gamma^5 B)_\gamma - 3T_1^0 (P^\nu i \sigma_{\mu \nu} C)^{\alpha \beta} (\gamma^\mu B)_\gamma \right] + V_2^0 (\hat{P} C)^{\alpha \beta} (\Delta_T B)_\gamma + A_2^0 (\hat{P} \gamma^5 C)^{\alpha \beta} (\Delta_T \gamma^5 B)_\gamma + T_2^0 (\Delta_T P^\nu \sigma_{\mu \nu} C)^{\alpha \beta} (B)_\gamma + T_3^0 (P^\nu \sigma_{\mu \nu} C)^{\alpha \beta} (\sigma^\mu \rho \Delta_T^\rho B)_\gamma + \frac{T_4^0}{M} (\Delta_T^\mu P^\nu \sigma_{\mu \nu} C)^{\alpha \beta} (\Delta_T B)_\gamma \]

\[ B = \text{nucleon spinor}. \]
• **Fourier transform** each TDA, → momentum fractions representation

\[ F(z_i P \cdot n) = \int \frac{1+\xi}{1+\xi} d^3 x \delta(\sum x_i - 2\xi) e^{-i P n \sum x_i z_i} F(x_i, \xi, t, Q^2) \]

• **Factorize process amplitude** :

\[ \mathcal{M}(Q^2, \xi, t) = \int dx dy \phi(y_i) T_H(x_i, y_i, Q^2) F(x_i, \xi, t) \]
Evolution equations

- QCD radiative corrections → logarithmic scaling violations.
- The scale dependence of $N \rightarrow \pi$ or $N \rightarrow \gamma$ TDAs is governed by evolution equations = an extension of DGLAP/ERBL equations for DAs and GPDs
- Start with quark fields having definite chirality or helicity $q^{(\uparrow(\downarrow))} = \frac{1}{2} \left(1 \pm \gamma^5\right)q$
- Separate “minus” components → dominant twist-2 with $\hat{n} = n^\mu \gamma_\mu$
Evolution equations (2)

- Two relevant operators in our problem:
  \[ B^{1/2}_{\alpha\beta\gamma}(z_i) = \epsilon^{ijk}(\hat{n}q_i^\dagger)\alpha(z_1n)(\hat{n}q_j^\dagger)\beta(z_2n)(\hat{n}q_k^\dagger)\gamma(z_3n) \]
  \[ B^{3/2}_{\alpha\beta\gamma}(z_i) = \epsilon^{ijk}(\hat{n}q_i^\dagger)\alpha(z_1n)(\hat{n}q_j^\dagger)\beta(z_2n)(\hat{n}q_k^\dagger)\gamma(z_3n) \]

- They obey renormalisation group equation
  \[ \mu \frac{d}{d\mu} B = H \cdot B \text{ with} \]
  \[ H = -\frac{\alpha_s}{2\pi} [(1 + 1/N_c) H_h + 3C_F/2] \]

- \[ H_{3/2} = H_{12}^v + H_{23}^v + H_{13}^v \text{ with } H_{12}^v B(z_i) = \]
  \[ = \frac{1}{\alpha} \int_0^1 \frac{d\alpha}{\alpha} \{ \bar{\alpha} [B(z_{12}^\alpha, z_2, z_3) - B(z_1, z_2, z_3)] \]
  \[ + \bar{\alpha} [B(z_1, z_{21}^\alpha, z_3) - B(z_1, z_2, z_3)] \} \]
Evolution equations (3)

- $H_{1/2} = H_{3/2} - \mathcal{H}_{12}^e - \mathcal{H}_{23}^e$ where $\mathcal{H}_{12}^e B(z_i) = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) B(z_{1\alpha_1}^{\alpha_1}, z_{2\alpha_2}^{\alpha_2}, z_3)$

- Derive the corresponding equation for the matrix element of operators $B$ from the RGE
$$Q \frac{d}{dQ} F_{\uparrow\downarrow\uparrow}(x_i) = -\frac{\alpha_s}{2\pi} \left[ \frac{3}{2} C_F F_{\uparrow\downarrow\uparrow}(x_i) - (1 + \frac{1}{N_c}) A \right]$$

$$A = \left[ \left( \int_{-1+\xi}^{1+\xi} dx'_1 \left[ \frac{x_1 \rho(x'_1, x_1)}{x'_1(x'_1 - x_1)} \right] + \int_{-1+\xi}^{1+\xi} dx'_2 \left[ \frac{x_2 \rho(x'_2, x_2)}{x'_2(x'_2 - x_2)} \right] \right) F_{\uparrow\downarrow\uparrow}(x'_1, x'_2, x_3) \right]$$

$$+ \left( \int_{-1+\xi}^{1+\xi} dx'_1 \left[ \frac{x_1 \rho(x'_1, x_1)}{x'_1(x'_1 - x_1)} \right] + \int_{-1+\xi}^{1+\xi} dx'_3 \left[ \frac{x_3 \rho(x'_3, x_3)}{x'_3(x'_3 - x_3)} \right] \right) F_{\uparrow\downarrow\uparrow}(x'_1, x_2, x'_3)$$

$$+ \left( \int_{-1+\xi}^{1+\xi} dx'_2 \left[ \frac{x_2 \rho(x'_2, x_2)}{x'_2(x'_2 - x_2)} \right] + \int_{-1+\xi}^{1+\xi} dx'_3 \left[ \frac{x_3 \rho(x'_3, x_3)}{x'_3(x'_3 - x_3)} \right] \right) F_{\uparrow\downarrow\uparrow}(x_1, x'_2, x'_3)$$

$$+ \frac{1}{2\xi - x_3} \left( \int_{-1+\xi}^{1+\xi} dx'_1 \left[ \frac{x_1 \rho(x'_1, x_1)}{x'_1} \right] + \int_{-1+\xi}^{1+\xi} dx'_2 \left[ \frac{x_2 \rho(x'_2, x_2)}{x'_2} \right] \right) F_{\uparrow\downarrow\uparrow}(x'_1, x'_2, x_3)$$

$$+ \frac{1}{2\xi - x_1} \left( \int_{-1+\xi}^{1+\xi} dx'_2 \left[ \frac{x_2 \rho(x'_2, x_2)}{x'_2} \right] + \int_{-1+\xi}^{1+\xi} dx'_3 \left[ \frac{x_3 \rho(x'_3, x_3)}{x'_3} \right] \right) F_{\uparrow\downarrow\uparrow}(x_1, x'_2, x'_3) \right]$$
• with integration region restricted by:
\[ \rho(x, y) = \theta(x \geq y \geq 0) - \theta(x \leq y \leq 0), \]
and \( x'_i \in [-1 + \xi, 1 + \xi] \)

• Different evolution in the various \( x_i \) sectors.

When \( x_i > 0 \rightarrow \) usual ERBL (\( x_i \rightarrow x_i/2\xi \) rescaling).

• Other regions need further study!
CHIRAL LIMIT of $p \rightarrow \pi$ TDA

- Soft pion theorems →

\[
\langle \pi^a(k)|O|P(p, s)\rangle = \frac{-i}{f_\pi} \langle 0|[Q^a_5, O]|P(p, s)\rangle \\
+ \frac{ig_A}{4f_\pi p \cdot k} \sum_{s'} \bar{u}(p, s) \hat{k}\gamma_5\tau^a u(p, s') \langle 0|O|P(p, s')\rangle
\]

1st term → TDA at threshold; 2nd term → nucleon pole.

- Since $[Q^b_5, \psi] = \frac{i}{2} \tau^b \gamma^5 \psi$
CHIRAL LIMIT ($\xi \to 1$)

\[ V_1^0(x_1, x_2, x_3) \to V(x_1, x_2, x_3) \]
\[ = (\phi_N(x_i) + \phi_N(x_2, x_1, x_3)) / 2 \]

\[ A_1^0(x_1, x_2, x_3) \to A(x_1, x_2, x_3) \]
\[ = \frac{1}{2} (\phi_N(x_i) - \phi_N(x_2, x_1, x_3)) \]

\[ T_1^0(x_i) \to T(x_i) = \frac{1}{2} (\phi_N(x_i) + \phi_N(x_2, x_3, x_1)) \]

where $\phi_N(x_1, x_2, x_3) = \text{standard leading twist DA}$
Interpretation

- The proton DA selects the valence contribution and analyses it from large angle scattering (and Form Factors).

- The proton $\rightarrow \pi$ TDA allows a pion (cloud) around the valence contribution.

- The proton $\rightarrow \gamma$ TDA allows a photon (cloud) around the valence contribution.

- The proton $\rightarrow \rho$ TDA...
Impact parameter interpretation

- As for GPDs and GDAs, Fourier transform $t \rightarrow b_T$
- Transverse picture of *pion cloud* in the proton
To test these ideas:
Model-independent predictions

- scaling law for the amplitude: \( \mathcal{M}(Q^2, \xi) \sim \frac{\alpha_s(Q^2)^2}{Q^4} \), (up to logarithmic corrections).

- Ratio: \( \frac{d\sigma(\overline{p}p \rightarrow l^+ l^- \pi^0)/dQ^2}{d\sigma(\overline{p}p \rightarrow l^+ l^-)/dQ^2} \) almost \( Q^2 \) independent.

- \( \gamma^*_T \) dominates \( \rightarrow \frac{d\sigma(p\overline{p} \rightarrow l^+ l^- \pi)}{\sigma d\theta} \sim 1 + \cos^2 \theta \) 
  \((\theta = \text{lepton angle in } \gamma^*_T \text{ CMS})\)

- Choose \( V, A \) and \( T \) \( \rightarrow \) Estimate threshold cross section in terms of e-m form factor.
This description also applies to crossed reactions

- Backward VCS $\gamma^* P \rightarrow P' \gamma$
  
  Data exist (JLab) for $Q^2$ up to 1 GeV$^2$.

Data from HERMES?

- and backward meson electroproduction
  $\gamma^* P \rightarrow P' \pi$; $\gamma^* P \rightarrow P' \rho$ ...

- Data exist (JLab) Analysis to be done
\( \gamma^* \gamma \) collisions

- One may describe along the same lines the crossed reactions

\[
\gamma^* \gamma \rightarrow \pi^+ \pi^- \quad (1)
\]

\[
\gamma^* \gamma \rightarrow \pi^\pm \rho^\mp \quad (2)
\]

and

\[
\gamma^* \gamma \rightarrow \rho^+ \rho^- \quad (3)
\]

in the near forward region and for large virtual photon invariant mass \( Q \), which may be studied in detail at intense electron colliders such as BABAR and BELLE.

- wait for talk by Jean Philippe Lansberg ...
CONCLUSIONS on TDAs

- FAIR will help to understand the deep structure of the proton
- Transition Distribution Amplitudes will reveal the dynamics of the next to lowest Fock state
- $\bar{p}p \rightarrow \gamma^*\pi$ explores the pion cloud.
- $\bar{p}p \rightarrow \gamma^*\rho$ explores the $\rho$ cloud.
- $\bar{p}p \rightarrow \gamma^*\gamma$ explores the photon cloud.
- Detectors should be ready to measure these reactions!
- If Polarized beam and target $\rightarrow$ spin structure too!
- NOT SO SMALL CROSS-SECTIONS AND BIG REWARDS.
CONCLUSIONS

- Exclusive Hard Reactions are revealing much about Hadron structure
- Theoretical progress ongoing ...
- Extremely Good Experiments are being done and prepared
- Nature seems to help us with early scaling!