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Heavy Ion Collisions at Relativistic Energies:
Testing a Nuclear Matter at High Baryon and Isospin Density

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These are preliminary lecture notes, intended only for distribution to participants
Heavy Ion Collisions at Relativistic Energies: Testing a Nuclear Matter at High Baryon and Isospin Density

Differential Flows

Pion/Kaon Production

Deconfinement Precursors?

Mean Fields
Effective Masses
In-medium cross sections

Self-energies → Relativistic Transport Dynamics

HIC → probing:
- different densities
- high momenta
- covariant structure:
genuine rel. effects
- limits of the hadronic picture
Gas-Liquid Density $\rho/\rho_0$

Big Bang

Phases of Nuclear Matter

Plasma of Quarks and Gluons

Crab nebula

July 5, 1054

Collisions Heavy Ion 5?

Phases of Nuclear Matter

Isospin?

Neutron Stars

Philippe Chomaz artistic view
The Nuclear EOS Uncertainties

...iso-scalar sector

Binding energy/nucleon

\[ E/A = T_{00} \] (from 00-component of energy-momentum tensor)

**Hard EoS**

\[ \kappa \approx 380 \text{ MeV} \]

(less compression)

**Soft EoS**

\[ \kappa \approx 200 \text{ MeV} \]

(more compression)

...iso-vector sector

Symmetry energy

\[ E_{\text{sym}} \] from second derivative of \( E/A \) with respect to asymmetry \((N-Z)/(N+Z)\)

Nuclear matter at supra-normal densities not fixed

(critical differences between models)

High density symmetry energy → neutron star:
- structure (mass/radius, hybrid)
- cooling (proton fraction → direct URCA)
Physics at High Baryon (and Isospin?) Density

SIS 100/300 (FAIR)

SIS

AGS

CERN SPS

Optimum production of baryons with strange quarks

maximum compression in heavy-ion collisions

~4-5 \( \rho_0 \)

nuclear matter density (blue curve)

threshold for strange quarks

threshold for antiprotons

threshold for charm quarks

production of strange quarks (red curve)

ion lab-energy [AGeV]
Quantum Hadrodynamics (QHD)

NN scattering $\rightarrow$ nuclear interaction from meson exchange:
main channels (plus correlations)

- $\sigma(0^+,0)$
- $\omega(1^-,0)$
- $\delta(0^+,1)$
- $\rho(1^-,1)$

Isoscalar
Isovector
Scalar
Vector

Nuclear interaction by Effective Field Theory as a covariant Density Functional Approach

★ Attraction & Repulsion $\rightarrow$ Saturation

\[ L = \overline{\psi} \left[ i \partial^\mu - g_\sigma \hat{V}^\mu - (M - g_\sigma \hat{\Phi}) \right] + \frac{1}{2} \left( \partial^\mu \hat{\Phi} \partial_\mu \hat{\Phi} - m_\sigma^2 \hat{\Phi}^2 \right) - \frac{1}{4} \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{V}_\mu \hat{V}^\mu \]

\[ \sigma : \quad \left( \partial^\mu \partial_\mu + m_\sigma^2 \right) \hat{\Phi} = g_\sigma \overline{\psi} \psi = g_\sigma \hat{\rho}_S \]

\[ \omega : \quad \partial^\mu \hat{W}^{\mu\nu} + m_\omega^2 \hat{V}^\nu = g_\omega \overline{\psi} \gamma^\nu \psi = g_\omega \hat{J}^\nu \]

No linear
No perturbative
Self-interacting terms (NL models) 
Charged effective mesons (QHD-II) 

\[ \varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k)n_i(k) + \frac{g_s^2}{2m^2_\sigma} \rho^2_\sigma + \frac{g^2_\omega}{2m^2_\omega} \rho^2_\omega \] 
QHD-I

\[ \varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k)n_i(k) + \frac{m^2_\sigma}{2} \Phi^2 + U(\Phi) + \frac{g^2_\omega}{2m^2_\omega} \rho^2_\omega + \frac{g^2_\rho}{2m^2_\rho} \rho^2_\rho + \frac{g^2_\delta}{2m^2_\delta} \rho^2_\delta \] 
QHD-II

\( \rho \sim 0.15 \) 
\( a_4 = 30.7 \text{ MeV} \) 
\( m^*/M = 0.75 \) 
\( E_{\text{bind}} = -16 \text{ MeV} \) 
Soft EoS (K=220 MeV)
Isospin degrees of freedom in QHD

- σ − ω model  ⇒  Only kinetic contribution to \( E_{\text{sym}} \)

- Charged “mesons”:
  - \( \bar{\delta}[a_0(980)] \) (scalar isovector)
  - \( \bar{\rho} (770) \) (vector isovector)

\[
N : \left[ \gamma_{\mu} i \partial^\mu - g_V \gamma_0 V^0 - g_\rho \gamma_0 \tau_3 b^0 - (M - g_s \Phi - g_\delta \tau_3 \delta_3) \right] \Psi = 0
\]

\[
\bar{\rho} : \quad b_0 = \frac{g_\rho}{m_\rho^2} (\rho_p - \rho_n)
\]

\[
\bar{\delta} : \quad \delta_0 = \frac{g_\delta}{m_\delta^2} (\rho_{sp} - \rho_{sn})
\]

\[
\text{Nuclear Matter} \quad f_{\rho,\delta} \equiv \left( \frac{g_{\rho,\delta}}{m_{\rho,\delta}} \right)^2
\]

Splitting n & p  \( M^* \)

Relativistic structure also in isospin space!

\( E_{\text{sym}} = \text{cin.} + (\rho \text{–vector}) - (\delta \text{–scalar}) \)
RMF Symmetry Energy: \(\delta - \text{contrib.}\)

\[
E_{\text{sym}} = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{M^*}{E^*} \right)^2 \right] \rho_B
\]

\(a_4 = E_{\text{sym}}(\rho_0)\) fixes \((f_\rho, f_\delta)\)

\[f_\rho \approx 1.5 f_\rho^{\text{FREE}}\]
\[f_\delta = 2.5 \text{ fm}^2\]
\[f_\delta \approx 2.0 \div 2.5 \text{ fm}^2\]

RMF Symmetry Energy: \(\delta - \text{contrib.}\).
**Self-Energies: kinetic momenta and (Dirac) effective masses**

\[ k_i^{\mu*} = k_i^\mu - \Sigma_i^\mu \]
\[ m_i^* = M - \Sigma_{s,i} \]

\[ \Sigma_s(n, p) = f_\sigma \sigma(\rho_s) \mp f_\delta \rho_{s3} \]
\[ \Sigma^\mu(n, p) = f_\omega j^\mu \mp f_\rho j_3^\mu \]

Dirac dispersion relation: single particle energies

\[ (\rho, j)_3 = (\rho, j)_p - (\rho, j)_n \]
\[ \rho_{B3} = \rho_{Bp} - \rho_{Bn} < 0, n - rich \]

\[ \varepsilon_i + M = + \sum_i^0 + \sqrt{k^2 + m_i^{*2}} \]

Chemical Potentials (zero temp.)

\[ \mu_i = \sqrt{k_F^2 + m_i^{*2}} + f_\omega \rho_B \mp f_\rho \rho_{B3} \]

Symmetry Energy

\[ \mu_n - \mu_p = 4 E_{sym} I \]
\[ I \equiv \frac{\rho_n - \rho_p}{\rho} \quad \text{Asymmetry parameter} \]

A parametrization of the isovector dependence

\[ \rho \delta \]

\[ \text{NL} \rho \delta \]

\[ \text{NL} \rho \]

\[ \sigma, \omega, \rho, \delta \] couplings

Symmetry energies

\[ a_4 = E_{\text{sym}}(\rho_0) \approx 33 \text{ MeV} \]

Gaitanos, Di Toro, Typel, Baran, Fuchs, Greco, Wolter, NPA 732 (04) 24
Collection of EOS “Realistic” Covariant Models

compact stars & heavy ion data
T.Klaen et al. nucl-th/0602038

Pressure

Exp boundaries (flow, kaon multiplicities)

Slope at normal density:
Isospin transport at Fermi energies

Soft Symm Matter, Stiff (super-stiff) Symmetry Energy

Soft Symm Matter, Soft Symmetry Energy
Neutron Star (npeµ) properties

Direct URCA threshold

Mass/Radius relation

compact stars & heavy ion data
T.Klaen et al. nucl-th/0602038
RBUU transport equation

\[ \frac{\partial f}{\partial t} + \frac{\bar{p}}{m} \cdot \nabla_r f + \nabla_r U \cdot \nabla_p f = I_{\text{coll}} \]

Non-relativistic Boltzmann-Nordheim-Vlasov, Landau-Vlasov...

Collision term:

\[ I_c = \frac{g}{(2\pi)^3} \int \frac{dp_2^*}{p_{20}^*} \frac{dp_3^*}{p_{30}^*} \frac{dp_4^*}{p_{40}^*} \int d\Omega (p^* + p_2^*)^2 \frac{d\sigma}{d\Omega} \delta^4(p^* + p_2^* - p_3^* - p_4^*) \times \{ f_3 f_4 [1 - f][1 - f_2] - f f_2 [1 - f_3][1 - f_4] \} \]
Relativistic Landau Vlasov Propagation

Discretization of $f(x,p^*) \to$ Test particles represented by covariant Gaussians in xp-space

$$f(x, p^*) = \sum_{i=1}^{AN_{\text{test}}} \int_{-\infty}^{+\infty} \tau \, g(x - x_i(\tau))g(p^* - p_i^*(\tau))$$

→ Relativistic Equations of motion for $x^\mu$ and $p^{*\mu}$ for centroids of Gaussians

$$\frac{d}{d\tau} x_i^\mu = \frac{p_i^*(\tau)}{M_i^*(x_i)} ,$$

$$\frac{d}{d\tau} p_i^{*\mu} = \frac{p_i^{*\nu}(\tau)}{M_i^*(x_i)} F_i^{\mu\nu}(x_i(\tau)) + \partial^\mu M_i^*(x_i)$$

$u_\nu$ Test-particle 4-velocity → Relativity: - momentum dependence always included due to the Lorentz term $\left(u_\nu F^{\mu\nu}\right)$
- $E^*/M^*$ boosting of the vector contributions

Collision Term: local Montecarlo Algorithm imposing an average Mean Free Path plus Pauli Blocking → in medium reduced Cross Sections
Collective flows

**In-plane**

\[ V_1(y, p_t) = \frac{\langle p_x \rangle}{\langle p_t \rangle} \]

\( y = \text{rapidity} \)

\( p_t = \text{transverse momentum} \)

\[ V_1^{p-n}(y, p_t) = V_1^p(p_t) - V_1^n(p_t) \]

- Repulsion
- Attraction

**Out-of-plane**

\[ V_2(p_t) = \frac{\left( p_x^2 - p_y^2 \right)}{p_x^2 + p_y^2} \]

\[ V_2 \begin{cases} = 1 & \text{full out} \\ = 0 & \text{spherical} \\ = -1 & \text{full in} \end{cases} \]

\[ V_2^{p-n}(p_t) = V_2^p(p_t) - V_2^n(p_t) \]

Differential flows

\[ \text{Isospin} \]
Differential Transverse Flow

$^{132}\text{Sn} + ^{132}\text{Sn} \ @ \ 1.5 \ AGeV \ b=6\text{fm}$

- Sensitivity to the isovector part of the mean field
  $\rightarrow E_{sym}$ around $\rho \approx 3-4\rho_0$

0 NLH-$\rho$
- NLH-$(\rho+\delta)$
  Greater $E_{sym}$
  stiffer $< F^{p-n} >$
Elliptic flow

132Sn+132Sn, 1.5AGeV, b=6fm: Test with NL-ρ & NL-(ρ+δ)

Difference at high p_t ⇔ first stage

High p_t neutrons are emitted “earlier”

Equilibrium (ρ,δ) dynamically broken
Importance relativistic structure

Dynamical boosting of the vector contribution

\[ \langle V_2 \rangle \]

\[ \frac{dp^{*}_p}{d\tau} - \frac{dp^{*}_n}{d\tau} \approx 2 \left[ \gamma f_\rho - \frac{f_\delta}{\gamma} \right] \nabla \rho_3 = \frac{4}{\rho_B} E^{*}_{sym} \nabla \rho_3 \]

\[ 2 \left[ f_\rho - f_\delta \frac{M^{*}}{E^{*}_F} \right] = \frac{4}{\rho_B} E^{pot}_{sym} \]

PLB562(2003)
Not just a symmetry energy effect: DDH → density dep. f_ρ to reproduce the same symmetry term of NLρδ

Strong isospin dependence of isospin flow

→ Pt-dependence: Chronometer of collision (high pt’s reflect earlier high compression)

→ NLρδ: more I-Flow due to Lorentz decomposition of iso-vector channel: \( \frac{dρ^*_p}{dt} - \frac{dρ^*_n}{dt} \approx 2 \left[ γ_f - \frac{f_δ}{γ} \right] \nabla ρ_i \)

\( ρ \)-meson enhanced by \( γ \)  \( δ \)-meson suppressed by scalar density

One needs neutron (light isobars) detection from experiments!
PION PRODUCTION

**Main mechanism**

\[ NN \Rightarrow N \Delta \]

- \( n\Delta^- \rightarrow p\Delta^- \)
- \( n\Delta^0 \rightarrow n\Delta^0 \)
- \( n\Delta^+ \rightarrow p\Delta^+ \)
- \( p\Delta^- \rightarrow n\Delta^- \)
- \( n\pi^- \rightarrow p\pi^- \)
- \( p\pi^- \rightarrow n\pi^- \)
- \( n\pi^+ \rightarrow p\pi^+ \)
- \( p\pi^+ \rightarrow n\pi^+ \)

\[ \Rightarrow \begin{array}{c} \pi^- \\ \pi^+ \end{array} \]

**1. C.M. energy available: “threshold effect”**

\[ \epsilon_{n,p} = E_{n,p}^* + f_\omega \rho_B + f_\rho \rho_{B3} \]

\[ s_{nn}(NL) < s_{nn}(NL\rho) < s_{nn}(NL\rho\delta) \]

\[ s_{pp}(NL) > s_{pp}(NL\rho) > s_{pp}(NL\rho\delta) \]

\( \pi(\cdot) \) enhanced

\( \pi(\cdot) \) reduced

**2. Fast neutron emission: “mean field effect”**

\[ \frac{n}{p} \Rightarrow Y(\Delta^{0,-}) \downarrow \Rightarrow \pi^- \downarrow \Rightarrow decrease: NL \rightarrow NL\rho \rightarrow NL\rho\delta \]

**3. Pion absorption**

At low energies \( \pi(\cdot) \) more absorbed since more energy is available in their production

**Compensation in “open” systems:**

- HIC
Pion production at SIS energies: $^{96}$Ru+$^{96}$Ru at 1.53AGeV

Central selection

$N/Z=1.18$, still some Iso-EOS sensitivity

Coulomb effect: less $\pi^+$ present in the high density region

$N\rho\delta$ more efficient in “transforming” neutrons into protons at high density, producing $\pi^-$

Pion/Kaon production in “open” system: Au+Au 1AGeV, central

Pions: compensation

Kaons:
- early production: high density phase
- isovector channel effects
  but mostly coming from second step collisions...
→ reduced asymmetry of the source
Au+Au 1AGeV central: Phase Space Evolution in a CM cell

Testing EoS → CBM

K production
Kaon production in “open” system: Au+Au 1AGeV, central
Main Channels

\[ \text{NN} \rightarrow \text{BYK} \]
\[ \text{N}\Delta \rightarrow \text{BY} \]
\[ \Delta\Delta \rightarrow \text{BYK} \]
\[ \pi\text{N} \rightarrow \text{YK} \]
\[ \pi\Delta \rightarrow \text{YK} \]

opposite contribution of the \( \delta \)-coupling
Au+Au central: \( \pi \) and \( K \) yield ratios vs. beam energy

**Pions:**
- Less sensitivity ~10%, but larger yields

**Kaons:**
- ~15% difference between DDF and NL\( \rho \delta \)

No sensitive to the K-potential

Inclusive multiplicities

132Sn+124Sn
Au+Au 1AGeV: density and isospin of the Kaon source

Time interval of Kaon production

“central” density

n,p at High density

Drop: Competition of fast neutron emission and Inelastic channels: \( n \rightarrow p \) transformation
Au+Au 1AGeV: time evolution of the total number of nucleons

Large $n \rightarrow p$ transformation at early times:
Less asymmetry in the Kaon source

Check: $\pi^-/\pi^+$, free $n/p$, $K(0)/K(\pm)$ vs. emission time ($p_t$)

Free nucleons

→ Different behavior at lower energies, reduced inelastic competition
Density and temperature like in Au+Au 1AGeV at max. compression

Larger isospin effects:
- no neutron escape
- $\Delta$’s in chemical equilibrium $\rightarrow$ less $n$-$p$ “transformation”

vs. asymmetry

Nuclear Matter Box Results

NPA762(2005) 147
UrQMD: not fully covariant symmetry term

208Pb+208Pb at 0.4AGeV

Inelastic channels less important but still crossing at high p_t

Q.Li et al. PRC 72 (2005) 034613
IBUU: not fully covariant symmetry term

$^{132}\text{Sn} + ^{124}\text{Sn}$ at 0.4AGeV

$\pi^-/\pi^+$ always decreasing with the iso-stiffness?

Bao-An Li PRC 71 (2005) 014608
In a C.M. cell

\[ ^{238}U + ^{238}U, 1AGeV, \ b = 7 \text{ fm} \]

Exotic matter over 10 fm/c?
Transition to deconfined phase at high baryon density

H. Mueller NPA618 (1997)

Hadron EOS: QHD
Quark EOS: MIT-Bag Model

Symmetric

Mixed Phase

Asymmetric: I=0.4

Reduced transition density

1. Earlier transition at high isospin density
2. Worse model choice? Hadron: rho-meson only, Quark: B1/4=190 MeV
large Bag-Pressure
Lower Boundary of the Binodal Surface vs. NM Asymmetry

Hadron: \( NL\rho\delta \)

vs. Bag-constant choice

\( \rho_{cr}/\rho_0 \)

\( \delta - \alpha = 0 \)

Proton-fraction

\( \text{symmetric} \)

\( GM3 \iff NL\rho \)
Temperature variation of the crossing density

Reduction of the crossing density vs. T:
  delta-meson very efficient!
Isospin content of the Quark Clusters in the Mixed Phase

Signatures? Neutron migration to the quark clusters (instead of a fast emission)

nucl-th/0602052→NPA2006
1. $n - p$ collective flows $\rightarrow$ light isobar flows

2. Kaon Yields, $(\pi^-/\pi^+)$ ? , flows?

3. Deconfinement precursors

Violent Collisions of Relativistic Radioactive Beams?

Genuine relativistic effects: - boosting of vector potentials
- baryon and scalar densities
  (vector vs. scalar field competition)
- Dirac masses

+ A.Drago, A.Lavagno
Relativistic Transport Dynamics

Effective Lagrangian $\rightarrow$ Transport Equations $\rightarrow$ Event simulation

**Hadronic: High Baryon and Isospin Densities $\rightarrow$ New Physics?**
- Pion, proton multiplicities (saturation?)
- Meson, baryon spectra vs. transverse momentum
- Elliptic Flows (EOS softening?)
- Isospin structure of particles at high $p_t$

**Partonic: beyond “Cascade”**
- Hadronization (coalescence) dynamics
- Hydro limit
- Collective flows
- Spinodal mechanism for hadronization
Why Intermediate Energies?
Proton stopping at mid-rapidity: Au+Au central

Graph showing the distribution of dN/dy net-protons as a function of y_{CM} for different energy levels (AGS, SPS, RHIC).