On D-branes in Flux Vacua

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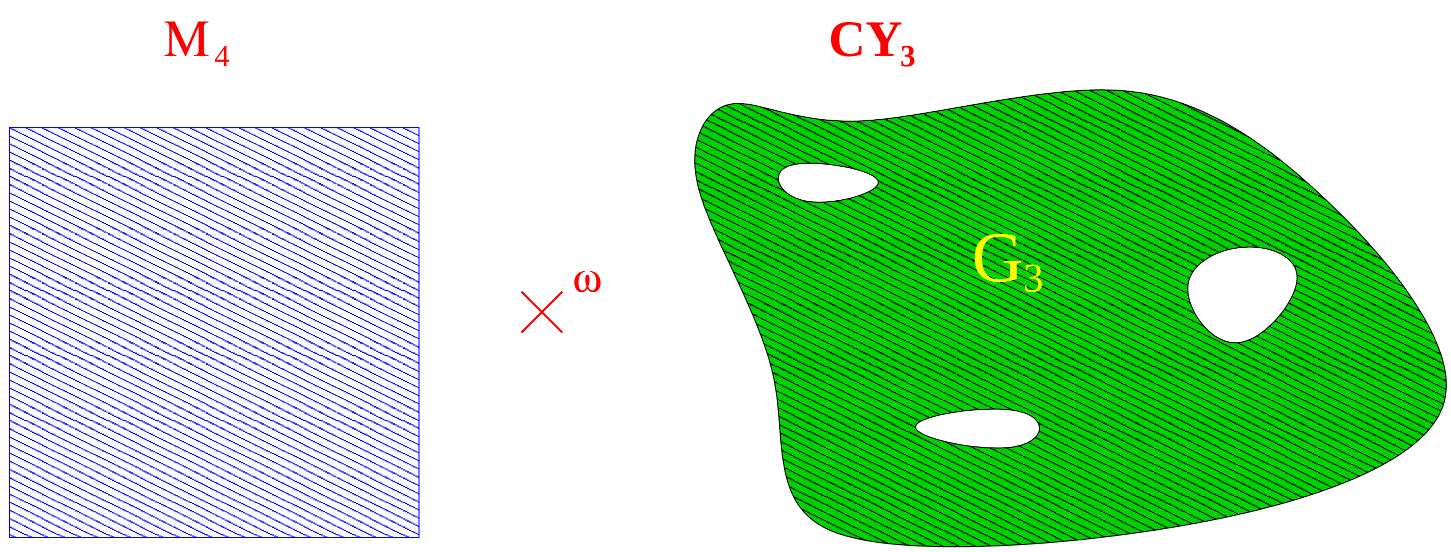
Motivation

Why fluxes?

- Recently, it has been realized that many new interesting effects can be achieved by going beyond conventional Calabi-Yau compactifications.

- Classical example: type IIB on a conformal Calabi-Yau, threaded by ISD background fluxes

$$G_3 = F_3 - \tau H_3 \qquad \begin{cases} F_3 & \text{RR flux} \\ H_3 & \text{NSNS flux} \\ \tau & \text{complex dilaton} \end{cases}$$



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- Classical example: type IIB on a conformal Calabi-Yau, threaded by ISD background fluxes

  ⋆ Moduli stabilization

  ⋆ Hierarchies via warped metrics

  ⋆ Supersymmetry breaking

  ⋆ Landscape of vacua

  ⋆ Inflationary models

  ...

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Fluxes and Model Building

- All these achievements point towards a greater flexibility in finding $D = 4$ semi-realistic vacua

- However, if interested in semi-realistic physics, one should eventually worry about embedding the SM in these class of constructions

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- However, if interested in semi-realistic physics, one should eventually worry about embedding the SM in these class of constructions

- This problem has been explored in the $\mathbf{CY_3}$ case but . . .

Question

How do fluxes affect (chiral) Model Building?

Motivation

Model Building and D-branes

- In type II vacua, the SM should arise from space-filling D-branes

- Some basic properties of D-branes which are quite relevant for model-building are

  - Spectrum of consistent/stable/BPS D-branes

  - D-brane moduli space

  - Chirality from D-branes

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. . . so how do fluxes affect these features?

D-branes in type IIB flux vacua

We already know that the spectrum of D-branes will change by adding ISD G_3 fluxes, because

- The D3-brane charge is no longer a $\mathbb{Z}$-valued quantity but takes values in $\mathbb{Z}_N$, where N depends on the H_3 quanta

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- D9-branes are removed from the spectrum, because of a Freed-Witten anomaly: the Bianchi identity $d\mathcal{F} = H_3$ does not have a global solution.

What about D7-branes?

Fluxes and D7-branes

- For closed strings, fluxes lift moduli by means of a GVW superpotential. This produces a discretum of closed string backgrounds.

- In some simple cases the D-brane moduli can also be lifted by G_3

- Examples:

  – D7-branes

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Let us now understand the general picture of D-brane moduli stabilization, and see whether there is a D-brane discretum as well.

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- We consider an $\mathcal{N} = 1$ closed string background

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- We consider an $\mathcal{N} = 1$ closed string background

- We compute the supersymmetry conditions for D-branes in such background

- We look at the D-brane deformations that don't spoil such conditions

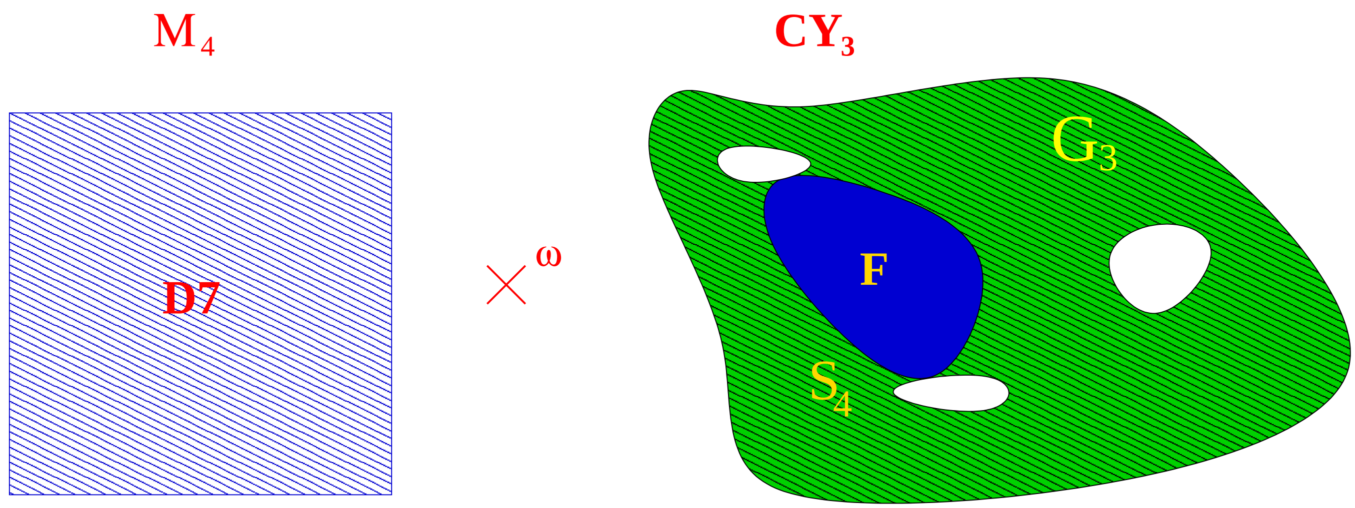
$\longrightarrow$ local moduli space

- We look at supersymmetric solutions which are disconnected

$\longrightarrow$ open string landscape

D7-branes

• Let us consider a type IIB flux compactification with a D7-brane wrapping a 4-cycle $\mathcal{S}_4$ on the internal dimensions



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$\mathcal{S}_4$ holomorphic

$\mathcal{F}$ is a (1,1)-form and $\mathcal{F} \wedge J = 0$

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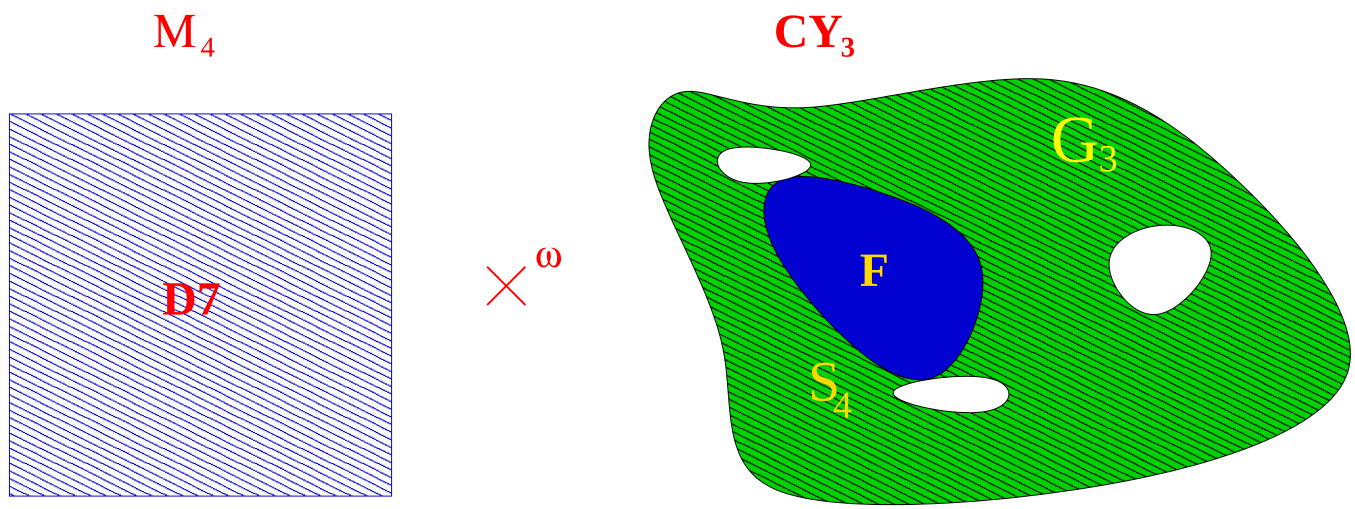

• Via a κ-symmetry analysis, we obtain that the BPS conditions are

$$\mathcal{S}_4 \text{ holomorphic}$$
$$*_4\mathcal{F} = -\mathcal{F} \;\Rightarrow\; \mathcal{F} \text{ induces D3-brane charge}$$

D7-brane moduli

- In absence of fluxes the moduli space of a D7-brane is given by

$$\text{Geometric moduli}: \quad \zeta^a \quad a = 1, \ldots, h^{(0,2)}(\mathcal{S}_4)$$

$$\text{Wilson line moduli}: \quad \xi^b \quad b = 1, \ldots, h^{(0,1)}(\mathcal{S}_4)$$

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- Recall that

$$\mathcal{F} = 2\pi\alpha' F + B_2|_{\mathcal{S}_4} \quad \left\{ \begin{array}{l} F = dA \in H^2(\mathcal{S}_4, \mathbb{Z}) \\ dB_2 = H_3 \end{array} \right\} \Rightarrow \quad d\mathcal{F} = H_3|_{\mathcal{S}_4}$$

- The Wilson lines ξ^b are inequivalent choices of A for the same $F = dA$

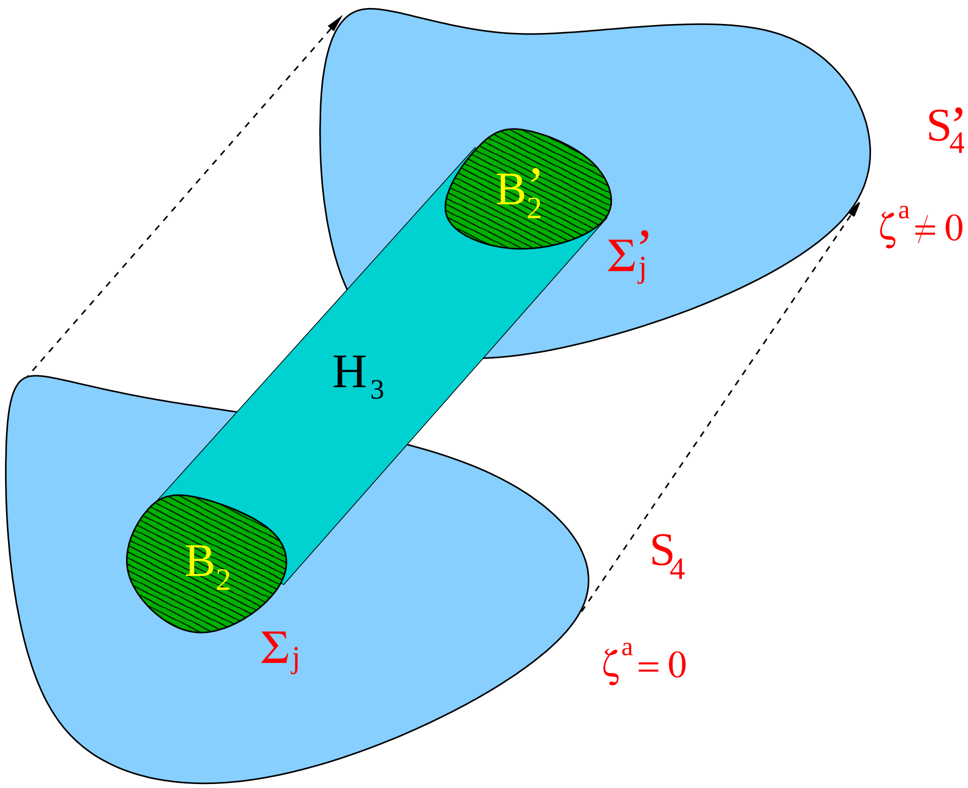
$\Rightarrow$ They don't change $\mathcal{F}$ nor the $\mathcal{N} = 1$ D-brane equations

$$\Downarrow$$

Non-lifted moduli

D7-brane moduli II

• On the other hand, since B_2 is non-closed, its pull-back depends on the 4-cycle $\mathcal{S}_4$, i.e., on the geometric moduli ζ^a



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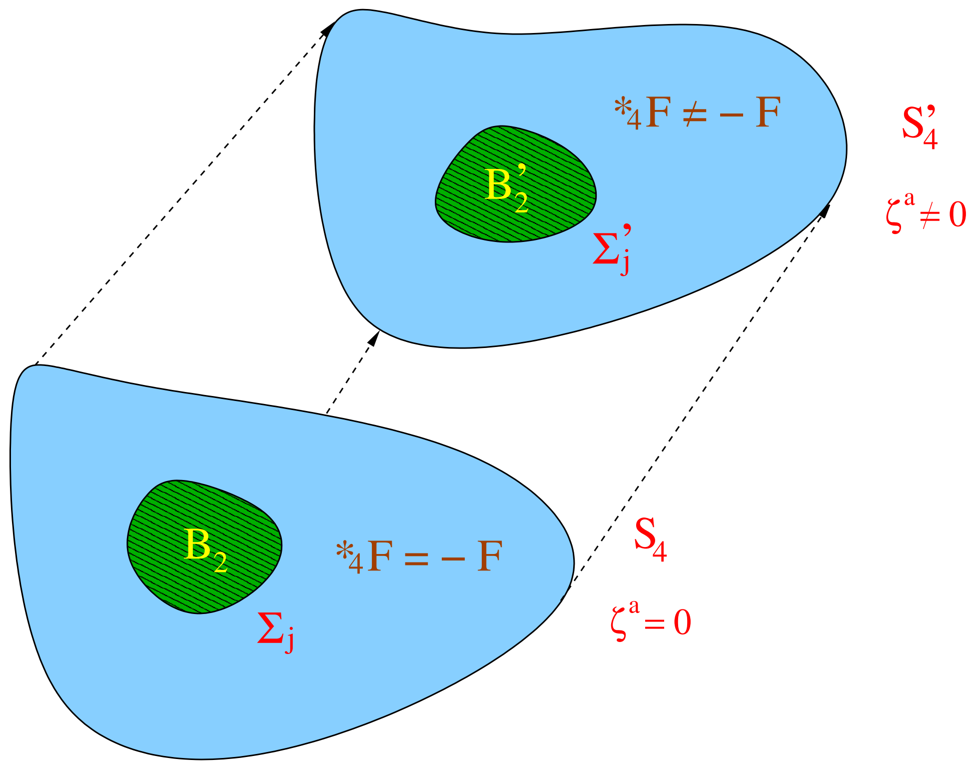

- A deformation $\mathcal{S}_4 \to \mathcal{S}'_4$ may not preserve the supersymmetry condition $*_4\mathcal{F} = -\mathcal{F} \quad \Rightarrow \quad$ Lifted modulus

Moduli lifting

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$$\mathcal{F}^{har} = \sum_j b_j(\{\zeta^a\})\,\beta^j + \sum_i a_i(\{\zeta^a\})\,\alpha^i$$

$$+\,c_J(\{\zeta^a\})\,J|_{\mathcal{S}_4} + \text{c.c.}$$

$$\bar{\alpha}^i \in \mathcal{H}^{2,0} \quad \beta^j \in \mathcal{H}^{1,1}_p \quad \alpha^i \in \mathcal{H}^{0,2}$$
$$J|_{\mathcal{S}_4} \in \mathcal{H}^{1,1}_{np}$$

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- The supersymmetry conditions translate into the system

$$a_1(\zeta^1,\ldots,\zeta^{h^{0,2}}) = 0,$$
$$\vdots$$
$$a_{h^{0,2}}(\zeta^1,\ldots,\zeta^{h^{0,2}}) = 0,$$
$$c_J(\zeta^1,\ldots,\zeta^{h^{0,2}}) = 0,$$

Moduli lifting II

- Supersymmetry of the closed string background implies that G_3 is a primitive (2,1)-form

$\longrightarrow G_3 \wedge J = 0 \quad \Rightarrow \quad c_J = \text{const.}$

$\longrightarrow G_3$ is (2,1) $\quad \Rightarrow \quad a_i =$ holomorphic on $\{\zeta^a\}$

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- We are left with

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$$\Downarrow$$

$h^{(2,0)}$ equations for $h^{(2,0)}$ unknowns

All geometrical moduli generically lifted

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⇒ Generically all the D7-brane moduli are lifted

$D = 4$ Perspective:

• Adjoint fields Φ^i get a mass from a superpotential W^{open}

• Interpretation: $a_i(\xi^a) = 0 \leftrightarrow \partial_{\Phi^i} W^{open} = 0$

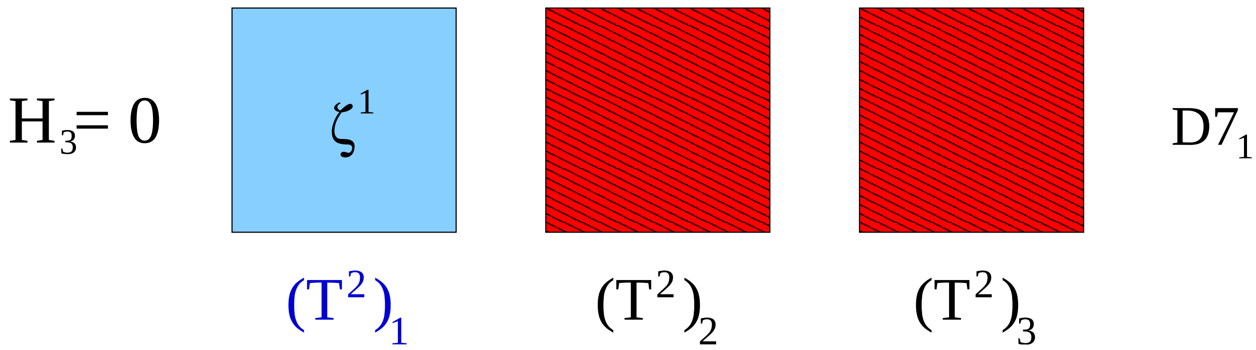
Checked by [Martucci]
Same W as in the **CY** case [Jockers, Louis]

The D-brane discretuum

- So far we have made a local analysis, and found that the D7-brane moduli space looks like a point

- Globally, however, it may look like a lattice! $\Rightarrow$ D7-brane discretum

Simple Example: A D7-brane on T^6



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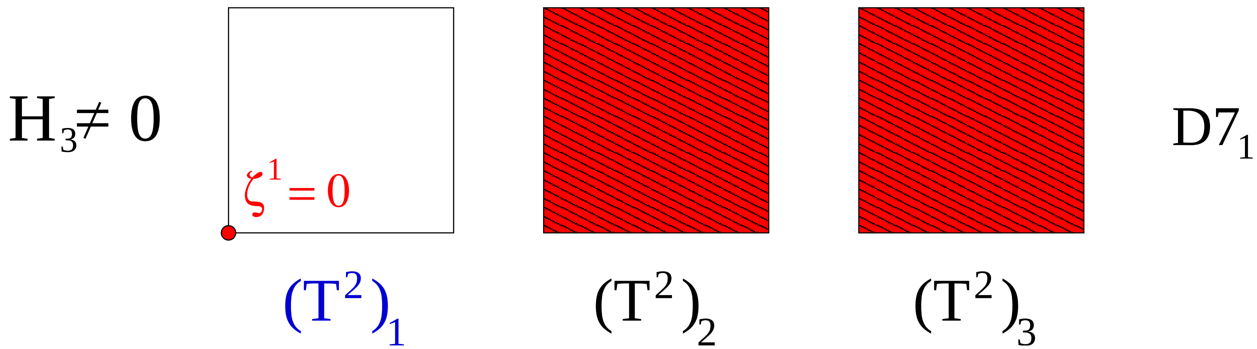

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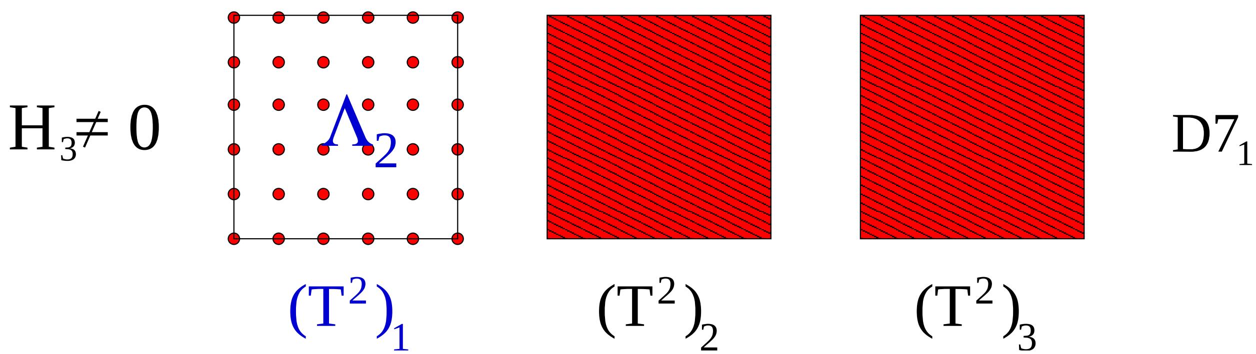

- $B_2|_{D7}$ is a continuous function of ζ^1, and supersymmetric iff $\zeta^1 = 0$

- But $\mathcal{F} = F + B_2|_{D7}$, and we can make discrete choices on F to find new (isolated) $\mathcal{N} = 1$ solutions

Summary I

• The supersymmetry conditions for D7-branes in flux compactifications allows us to understand moduli lifting from a general and geometrical point of view

• The fact that all (geometrical) moduli are lifted does not mean that the D7-brane can be located at only one point

$\Rightarrow$ There is a discretum of D7-brane positions

Summary I

• The supersymmetry conditions for D7-branes in flux compactifications allows us to understand moduli lifting from a general and geometrical point of view

• The fact that all (geometrical) moduli are lifted does not mean that the D7-brane can be located at only one point

$\Rightarrow$ There is a discretum of D7-brane positions

• This defines an Open String Landscape directly connected to the physics of $D = 4$ non-Abelian gauge theories and chiral matter. Any statistical prediction derived from it should have an interpretation in terms of particle physics

D-branes in type IIB revisited

- D3-branes are BPS, and their moduli space remains untouched. Their charge takes values in $\mathbb{Z}_N$, where N depends on the H_3 quanta.

- D7-branes have their geometrical moduli lifted, and their spectrum is multiplied by some integer M, which again depends on the H_3 quanta.

- D9-branes are removed from the spectrum, because of a Freed-Witten anomaly: the Bianchi identity $d\mathcal{F} = H_3$ does not have a global solution.

Notice that all of these effects happen because of the NSNS flux H_3, rather than F_3

What about type IIA?

- Upon mirror symmetry, all these D-branes get mapped to D6-branes in type IIA flux vacua

$$\Downarrow$$

We should be able to describe all these effects in terms of D6-branes

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Mirror Symmetry:

- Our type IIB CY_3 background is mapped to a half-flat, symplectic manifold $\mathcal{M}_6$ in type IIA, with $H_3 = 0$

[Gourrieri, Louis, Micu, Waldram]
+[Graña, Minasian, Petrini, Tomasiello]

- The topological information of H_3 is mapped to the torsion factors in $H^n(\mathcal{M}_6, \mathbb{Z}) \Rightarrow$ invisible to de Rham cohomology

[Tomasiello]

$$H^n(\mathcal{M}_6, \mathbb{Z}) \simeq H^n(\mathcal{M}_6, \mathbb{R}) \oplus \text{Tor}\, H^n(\mathcal{M}_6, \mathbb{Z})$$

A twisted torus example

- The simplest examples of such flux vacua are given by type IIA on twisted six-tori $\tilde{\mathbf{T}}^6$

[Kachru, Schulz, Tripathy, Trivedy]

- Mirror map:

$$\text{type IIB on } (\mathbf{T}^6, H_3 \neq 0) \longrightarrow \text{type IIA on } (\tilde{\mathbf{T}}^6, H_3 = 0)$$

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 + (dx^5)^2 + (dx^6)^2$$

$$H_3 = N\, dx^1 \wedge dx^5 \wedge dx^6 - N\, dx^4 \wedge dx^2 \wedge dx^6$$

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Twisted torus cohomology

- Let us compute the cohomology groups $H^n(\mathcal{M}_6, \mathbb{Z}) \simeq \mathbb{Z}^{b^n} \oplus \text{Tor } H^n(\mathcal{M}_6, \mathbb{Z})$ before and after the twist

	$b^n(\mathbf{T}^6)$	Tor $H^n(\mathbf{T}^6, \mathbb{Z})$	$b^n(\tilde{\mathbf{T}}^6)$	Tor $H^n(\tilde{\mathbf{T}}^6, \mathbb{Z})$
$n = 1$	6	-	4	-
$n = 2$	15	-	9	$\mathbb{Z}_N^2$
$n = 3$	20	-	12	$\mathbb{Z}_N^4$
$n = 4$	15	-	9	$\mathbb{Z}_N^4$
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. . . so geometical fluxes reduce the de Rham cohomology and produce torsional cohomology

Freed-Witten anomaly

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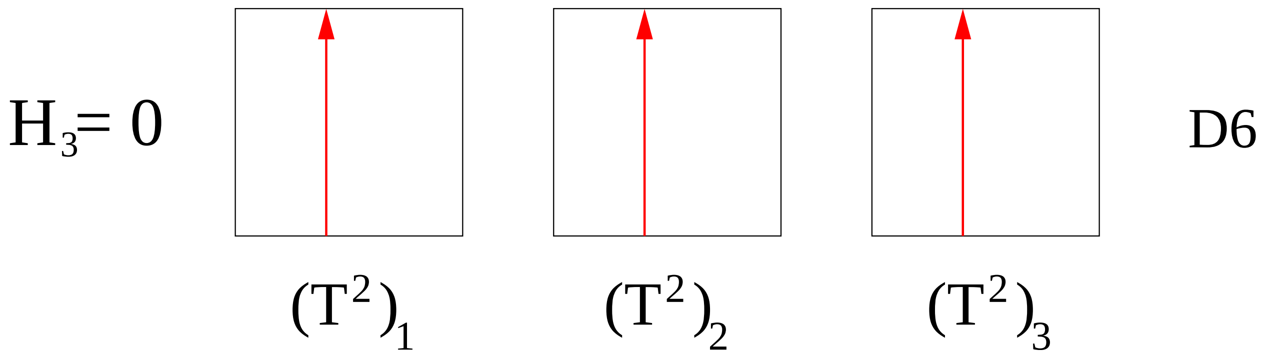
- For instance, the 3-chain

$$\Pi_3 = (0,1)_1 \times (0,1)_2 \times (0,1)_3$$

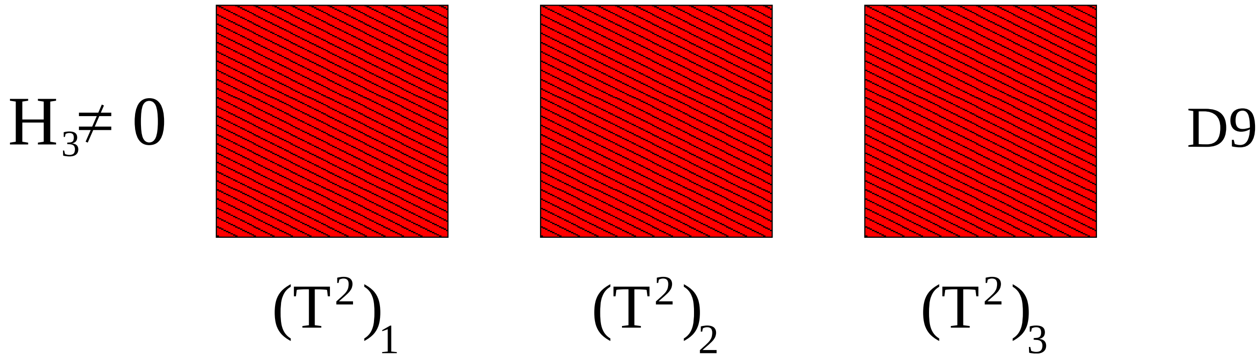


is closed on $\mathbf{T}^6$ but not on $\tilde{\mathbf{T}}^6$ $\Rightarrow$ inconsistent D6-brane.

Freed-Witten anomaly

• Because $b_3(\tilde{\mathbf{T}}^6) < b_3(\mathbf{T}^6)$ some 3-cycles 'disappear' from the de Rham cohomology

• In the mirror type IIB picture such D6-brane would correspond to a D9-brane in the presence of $H_3 \Rightarrow$ Freed-Witten anomaly



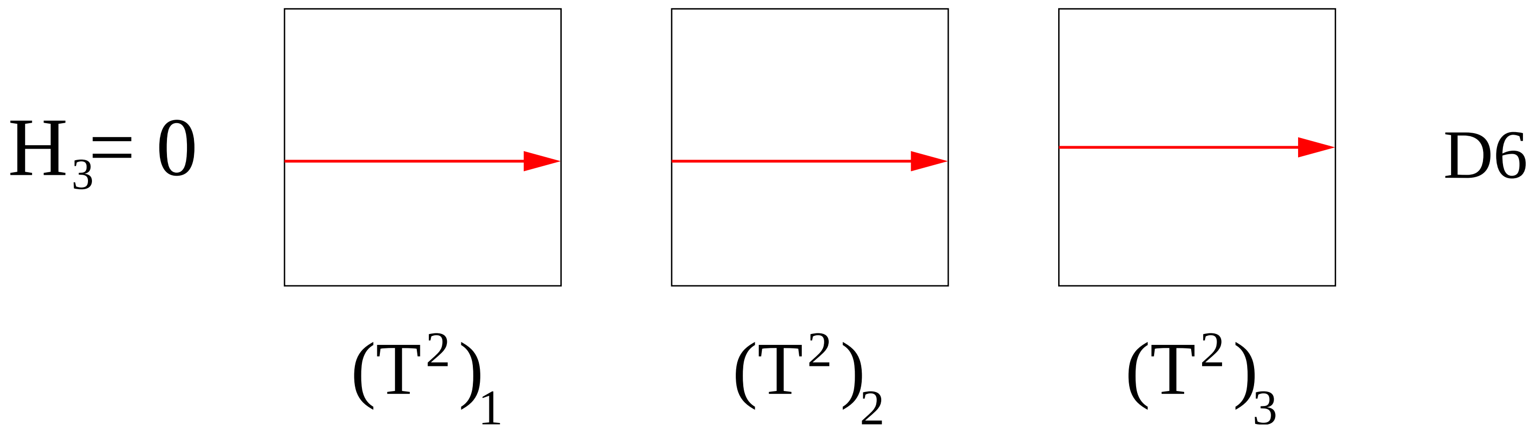
In general, the Freed-Witten anomaly is mirror to $\partial\Pi_3 \neq 0$

See also [Cámara, Font, Ibáñez] [Villadoro, Zwirner]

Torsional charges

• Some other 3-cycles Π_3 do not contribute to b_3 because they are torsional 3-cycles, like

$$\Pi_3 = (1,0)_1 \times (1,0)_2 \times (1,0)_3$$



which is the mirror of the type IIB D3-brane. Hence N D6-branes wrapped on Π_3 can be deformed to no D6-branes at all $\Rightarrow$ $\mathbb{Z}_N$-valued charge

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- In addition, we know that Π_3 is a supersymmetric 3-cycle, which implies

$$\int_{\Pi_3} \Omega = \text{Vol}(\Pi_3) \neq 0$$

- However, because $[\Pi_3] \in \text{Tor}\, H_3(\mathcal{M}_6, \mathbb{Z})$, this is only possible if $d\Omega \neq 0$

Torsion $\longleftrightarrow$ Intrinsic torsion

D6-brane moduli

- The SUSY conditions for a D6-brane wrapping Π_3 can be written as

$$\begin{aligned} \text{Im } \Omega|_{\Pi_3} &= 0 \\ J_c|_{\Pi_3} + F &= 0 \end{aligned}$$

- In Calabi-Yau manifolds $d\Omega = dJ = 0$. By McLean's theorem, the moduli space of a D6-brane on Π_3 has dimension $b_1(\Pi_3)$.

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- For half-flat, symplectic manifolds Ω is no longer a calibration. However, the weaker condition $d\,\text{Im } \Omega = dJ = 0$, is satisfied, and the proof of McLean's theorem still applies

$\Rightarrow$ A D6-brane has $b_1(\Pi_3)$ complex moduli

Moduli lifting

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- Let us consider a D6-brane wrapped on the 3-cycle

$$\Pi_3 = (1,0)_1 \times (0,1)_2 \times (0,-1)_3$$

- The topology of Π_3 changes when we introduce geometric fluxes. More precisely:

On $\mathbf{T}^6$: $\quad H_1(\Pi_3, \mathbb{Z}) = \mathbb{Z}^3 \ \Rightarrow \ b_1(\Pi_3) = 3$

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• The topology of Π_3 changes when we introduce geometric fluxes. More precisely

On $\tilde{\mathbf{T}}^6$: $\quad H_1(\Pi_3, \mathbb{Z}) = \mathbb{Z}^2 \times \mathbb{Z}_N \quad \Rightarrow \quad b_1(\Pi_3) = 2$

$\Rightarrow$ Geometric fluxes lift moduli because they reduce $b_1(\Pi_3)$

Torsion and light fields

- The lifted modulus Φ is a D6-brane deformation which spoils the SUSY conditions. It also corresponds to the generator of Tor $H_1(\Pi_3, \mathbb{Z}) = \mathbb{Z}_N$.

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- We can compute its mass by a direct DBI analysis or by means of a D6-brane superpotential

[Martucci]

$$W = \int_{\Sigma_4} (F + J_c)^2, \qquad \partial\Sigma_4 = \Pi_3' - \Pi_3$$

We obtain

$$W = \frac{1}{2} N \Phi^2 \quad \Rightarrow \quad m_\Phi^2 = N^2 \left(\frac{R_1}{R_5 R_6}\right)^2$$

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• In the large complex structure limit $m_\Phi << m_{KK}$, and this field is much lighter than other massive D6-brane states.

$\Rightarrow$ $\text{Tor}\, H_1(\Pi_3, \mathbb{Z})$ corresponds to light D6-brane modes

The D6-brane discretum

- The presence of torsion also explains the D-brane discretum

- In our last example, $\text{Tor}\, H_1(\Pi_3, \mathbb{Z}) = \mathbb{Z}_N$

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- All these 3-cycles have the same intersection numbers, $[\Pi_3^r] \cdot [\Sigma_3]$ because

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Same intersection numbers $\Longleftrightarrow$ Same chiral spectrum

$\Rightarrow$ Landscape of D6-branes

Torsion versus chirality

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- Besides stabilizing moduli, we know that fluxes make D-brane charges torsional $\Rightarrow$ tension between fluxes and chirality

Summary II

- We have analyzed the moduli space of D-branes in type IIB flux vacua via simple geometrical methods

- The key point was that, because we have $H_3 \neq 0$, B_2 varies along the internal space and it does matter where the D-branes sit

  $\Rightarrow$ All geometrical moduli are lifted

- Perhaps more surprisingly, we have also shown the presence of a D-brane discretum or open string landscape

Summary II

- A similar analysis can be performed for D6-branes in type IIA, but now the key objects are the groups Tor H_n invisible to de Rham cohomology

- Tor H_n give us information about the D6-brane spectrum, their moduli and their light fields. On the other hand, they obstruct $D = 4$ chirality

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- Tor H_n give us information about the D6-brane spectrum, their moduli and their light fields. On the other hand, they obstruct $D = 4$ chirality

- It has been recently realized that we can introduce plenty of fluxes in type II compactifications, which stabilize more moduli

  However, it seems to be some tension between fluxes and chirality...

  It could happen that, by enlarging the spectrum of background fluxes, we reduce our chances of obtaining realistic string vacua!