GEOMETRY, DUALITY + NON-GEOMETRIC STRING BACKGROUNDS

CMH hepth 0406102

0604178 + ...

0605149

CMH + DABHOLKAR

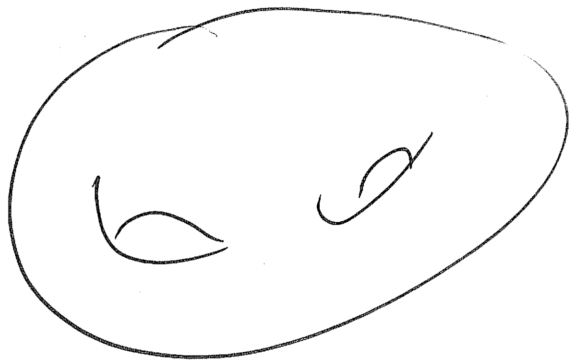
0210209

0512005

CMH + REID-EDWARDS

0503114

0603094



GEOMETRIC
BACKGROUND
$M(g_{ij}, H_{ijk}, \Phi, \ldots)$

$\Downarrow$ DUALITY
(T, U, MIRROR...)

??

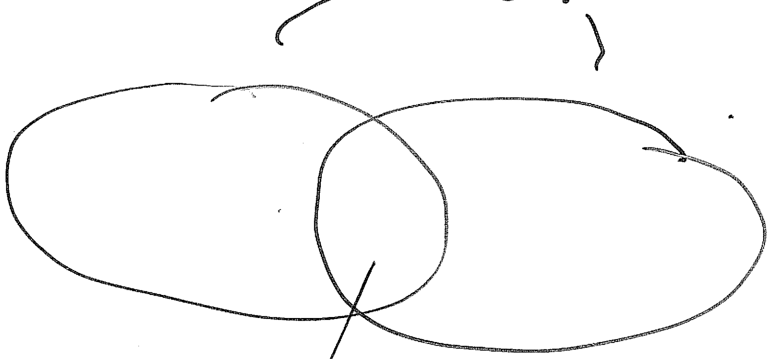
<u>USUALLY</u> TO ANOTHER
GEOMETRIC BACKGROUND

BUT SOMETIMES NOT

- OBSTRUCTION TO DUALITY?
- NON-GEOMETRIC BACKGROUND?

GEOMETRIC BACKGROUND

PATCHES

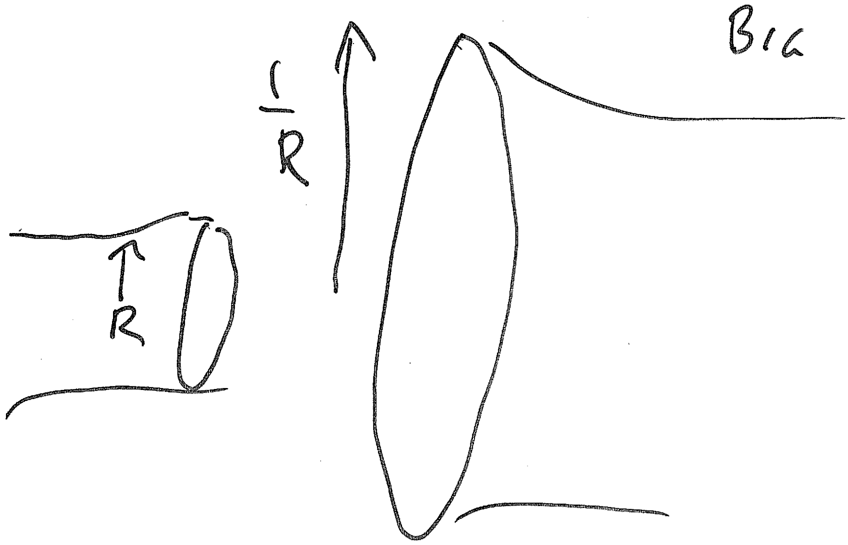


"GLUE" USING DIFFEOS/
GAUGE TRANSFORMATIONS —
ALL GAUGE SYMMS

T-FOLD

GLUE USING T-DUALITIES — DISCRETE GAUGE SYMS

GLUE MOMENTUM MODES TO
WINDING MODES,
BIG S^1 TO SMALL S^1



NON GEOMETRIC BACKGROUNDS

- STRINGY, NOT SUPERGRAVITY

- NEW "COMPACTIFICATIONS"

- FIX MODULI e.g. SCALES

- T-DUALS / MIRRORS OF
  FLUX COMPACTIFICATIONS

- SWAMP CLEARANCE

GENERIC $D=4$ SUGRA + MATTER
CAN'T ARISE FROM COMPACTIFICATION
OF $D=10$ OR 11 SUGRA

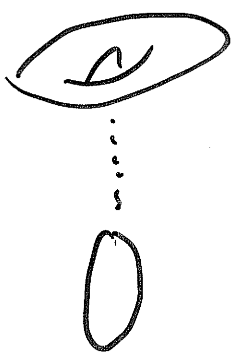
MANY LIFT TO NGB'S
NGB $\rightarrow$ CONVENTIONAL $D=4$ SUGRA

- GENERIC SOLUTIONS OF STRING
  THEORY ARE NGBs

T^3 H-FLUX $\quad H = N \times \mathrm{Vol}$

$\Downarrow$ T-DUAL ON S^1

T^2
$\downarrow$
S^1



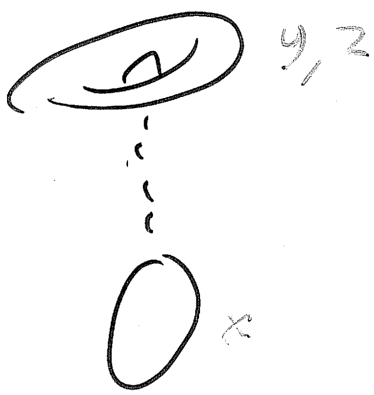
MONODROMY $\begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in SL(2,\mathbb{Z})$

$H = 0$

$\Downarrow$ T-DUAL ON S^1 FIBRE

T^2

S^1



$$ds^2 = \frac{1}{1+N^2x^2}\left(dy^2 + dz^2\right) + dx^2$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$

BUT x PERIODIC?

$$E(x+2\pi) = \frac{aE+b}{cE+d}$$

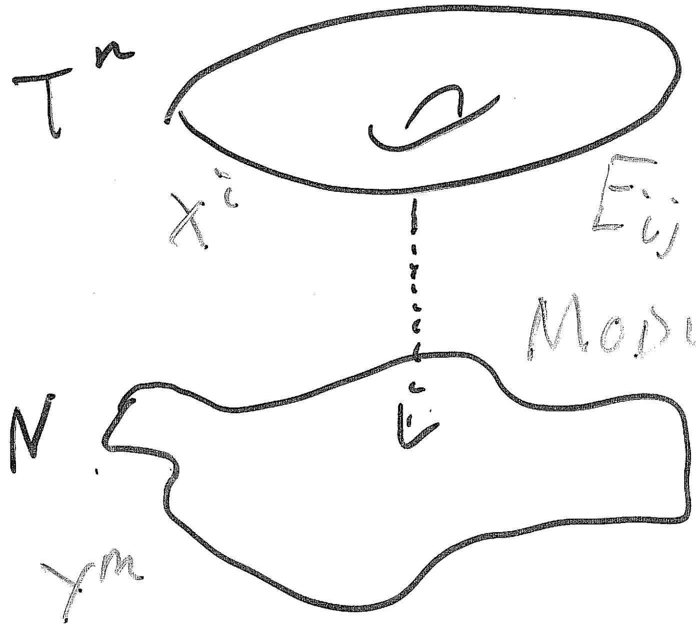
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;\mathbb{Z})$

$\times$ T-DUALITY?

MONODROMY MIXES MOMENTUM + WINDING

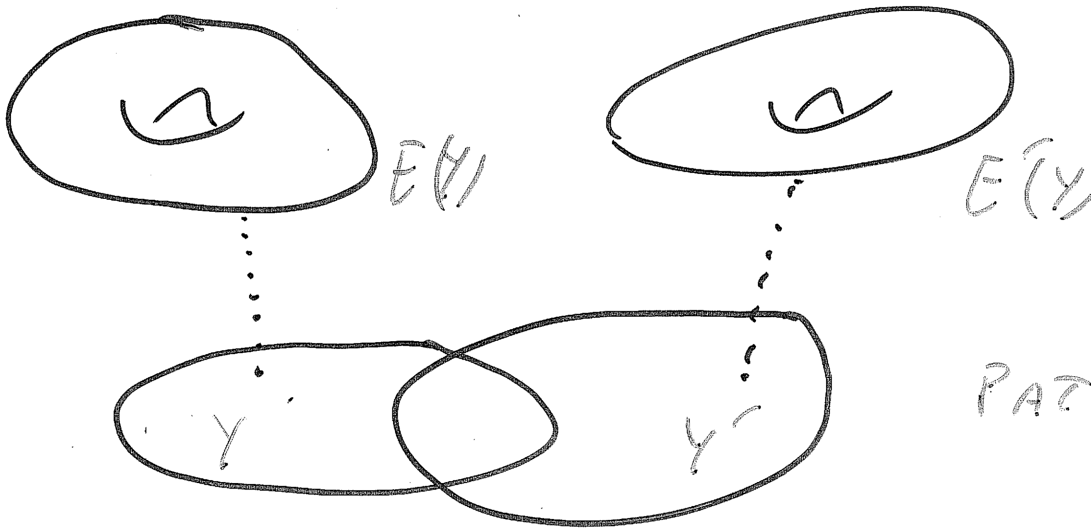
Torus Bundles

$$\mathcal{M} = \begin{array}{c} T^n \\ \downarrow \\ N \end{array}$$



$E_{ij} = G_{ij} + B_{ij}$

Moduli $E(y)$



Patches in N

Glue T^n fibres with $h \in GL(n, \mathbb{Z})$

(and $U(1)^n$)

$$E' = h E h^t$$

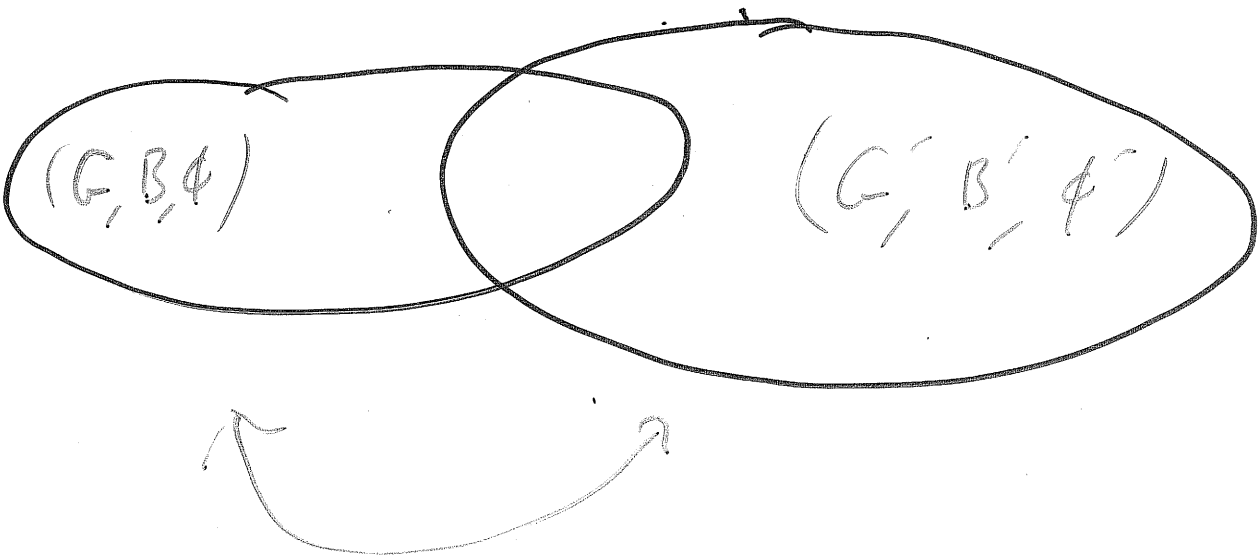
$\Downarrow$ T-Duality $g \in O(n,n;\mathbb{Z})$

$$h \to k = g h g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(n,n;\mathbb{Z})$$

$$E' = \frac{aE + b}{cE + d}$$

Not Tensor!

MANIFOLD



TRANSITION FNS:

DIFFEO + GAUGE TRANS

$$\Downarrow$$

"T-FOLD"

TRANS FUNCTIONS ARE
T-DUALITIES

$$E' = \frac{aE + b}{cE + d} \qquad E_{ij} = G_{ij} + B_{ij}$$

MANY CONSISTENT NON-GEOMETRIC STRING BACKGROUNDS

NOT MANIFOLDS, BUT

T-FOLDS, U-FOLDS, ...

WITH TRANSITION FUNCTIONS IN

T-DUALITY, U-DUALITY, MIRROR MAPS...

CMH + DABOLKHAR

HELLERMAN MCGREEVY WILLIAMS

KACHRU SCHULZ TRIPATHY TRIVEDI

LOWE NASTASE RAMGOOLAM

FLOURNOY WECHT WILLIAMS

VAFA

SHELTON TAYLOR WECHT

HELLERMAN WALCHER

STRINGS ON T^n

$\Box \underline{X} = 0$

$$\underline{X} = X_L(\sigma+\tau) + X_R(\sigma-\tau)$$

$$X = X_L + X_R \qquad \tilde{X} = X_L - X_R$$

CONJ TO WINDING NO., T-DUAL OF X

$$\partial_\alpha X = \epsilon_\alpha{}^\beta \partial_\beta \tilde{X} \qquad \boxed{dX = * d\tilde{X}}$$

NEED "AUXILIARY" $\tilde{X}$:

1) VERTEX OPS $\quad e^{ik_L \cdot X_L}, \; e^{ik_R \cdot X_R}$

2) STRING FIELD THEORY

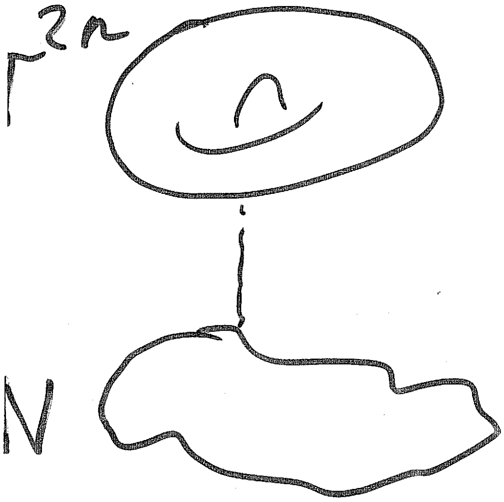
$$\Psi[X, \tilde{X}, a_n, \tilde{a}_n]$$

T-DUALITY: $O(n,n)$ ACTS ON $\begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix}$

T-FOLDS $\begin{pmatrix} X' \\ \tilde{X}' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} X \\ \tilde{X} \end{pmatrix}$

MIXES $X, \tilde{X}$; NO GLOBAL WAY OF SEPARATING OUT "REAL SPACE" X

DOUBLED FORMALISM



DOUBLED FIBRE

$$\mathbb{X}^I = \begin{pmatrix} X \\ \tilde{X} \end{pmatrix}$$

y^m

$\bar{M}$

$$O(n,n;\mathbb{Z}) \subset GL(2n;\mathbb{Z})$$

"NON-GEOMETRIC" TRANSITION FNS

$\longrightarrow$ T^{2n} BUNDLE OVER N

– STRING THEORY ON $\bar{M}$

$O(n,n)$ DUALITY COVARIANT

– CONSTRAINT : $\frac{1}{2}$'s $\mathbb{X}$ d.o.f.

$$d\mathbb{X} = S * d\mathbb{X} + \cdots$$

$$S^2 = \mathbb{1}, \quad S(Y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(Y) \ldots$$

$$\longrightarrow \quad dX \sim * d\tilde{X}$$

DOUBLED METRIC

$L=1: \quad g = R^2 dX^2 + R^{-2} d\tilde{X}^2$

T-DUALITY

ACTIVE

$R \to R^{-1}$

PHYSICAL X HAS RADIUS $R \to \tilde{R} = R^{-1}$

$\mathcal{H} \to \Lambda^t \mathcal{H} \Lambda$

PASSIVE

R FIXED, CHANGE "POLARIZATION" SO THAT $\tilde{X}$ IS REGARDED AS 'PHYSICAL'

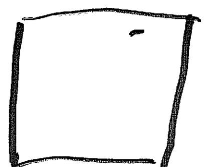
POLARISATION

CHOOSE $\frac{1}{2}$ OF $\mathbb{X}$ AS PHYSICAL x

$\frac{1}{2}$ AS AUXILIARY $\tilde{x}$

$$\mathbb{X}^I \longrightarrow \begin{cases} x^i \\ \tilde{x}_i \end{cases}$$

$\tilde{x}$



x

$$ds^2 = 2\, dX^i\, d\tilde{X}_i$$

$\{x^i\}$ MAXIMAL NULL

T-DUALITY CHANGES POLARISATION

CHANGES PHYSICAL SPACE

DOUBLED TORUS

FOR EACH S^1, COORD X^i

INTRODUCE T-DUAL S^1, COORD $\tilde{X}_i$

$$\mathbb{X}^I = \begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix} \qquad \mathbb{X}^I \sim \mathbb{X}^I + 1$$

$$T^{2n}$$

DOUBLED METRIC

$$G_{ij},\ B_{ij} \text{ on } T^n \longrightarrow \mathcal{H}_{IJ} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

on T^{2n}

T-DUALITY ACTS LINEARLY

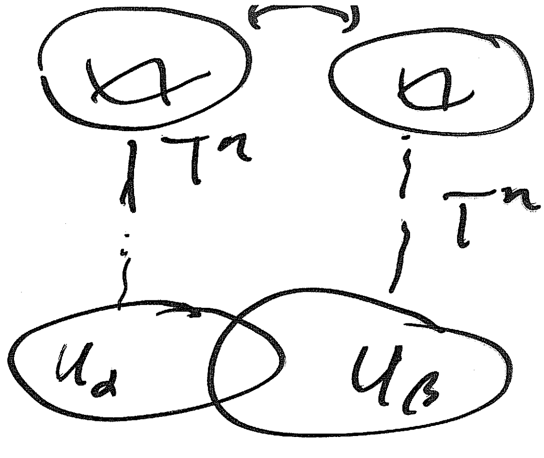
$$\mathbb{X}^I \longrightarrow \Lambda^I{}_J \mathbb{X}$$

$$\mathcal{H} \rightarrow \Lambda^t \mathcal{H} \Lambda$$

$$\eta \rightarrow \Lambda^t \eta \Lambda = \eta \qquad \eta = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$S^1 \longrightarrow T^2 \qquad \mathcal{H} = \begin{pmatrix} R^2 & 0 \\ 0 & R^{-2} \end{pmatrix} \quad \begin{pmatrix} X \\ \tilde{X} \end{pmatrix}$$

T- FOLD



LOCALLY $\quad U_\alpha \times T^n$

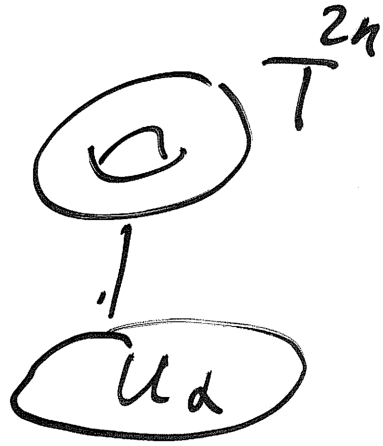
TRANSITION FUNCTIONS

INCLUDE $\quad O(n,n;\mathbb{Z})$

$\begin{cases} n \quad \text{CHERN CLASSES} & F^i_{\mu\nu}\, dY^\mu \wedge dY^\nu \\ n \quad \text{H - CLASSES} & H_{\mu\nu i}\, dY^\mu \wedge dY^\nu \end{cases}$

SAME DATA CONSTRUCTS

DOUBLED BUNDLE $\hat{M}$



TRANSITION FUNCTIONS

$O(n,n;\mathbb{Z}) \subset SL(2n;\mathbb{Z})$

T^{2n} BUNDLE OVER N

$2n$ CHERN CLASSES $\quad F^I_{\mu\nu} \sim \begin{cases} F^i_{\mu\nu} \\ H_{\mu\nu i} \end{cases}$

$\mathcal{H}_{IJ}$ TENSOR FIELD

σ - MODEL ON $\hat{M}$

CHOOSE <u>POLARIZATION</u>

$$\mathbb{X}^I \longrightarrow \begin{cases} x^i & \text{— 'FUNDAMENTAL' SPACETIME} \\ \tilde{x}_i & \text{— AUXILIARY} \end{cases}$$

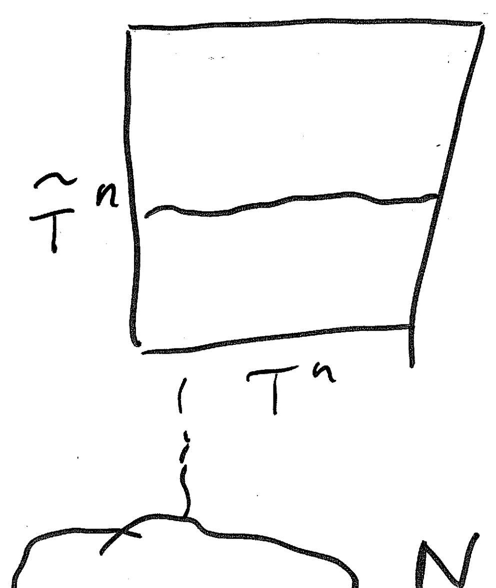
i.e. PICK $T^n \subset T^{2n}$ SPACETIME

$O(n,n)$ METRIC $\quad ds^2 = 2\, dx^i\, d\tilde{x}_i$

$\longrightarrow$ T^n MAXIMAL NULL (ISOTROPIC)

'BREAK' $O(n,n;\mathbb{Z}) \to GL(n,\mathbb{Z})$

T^{2n}



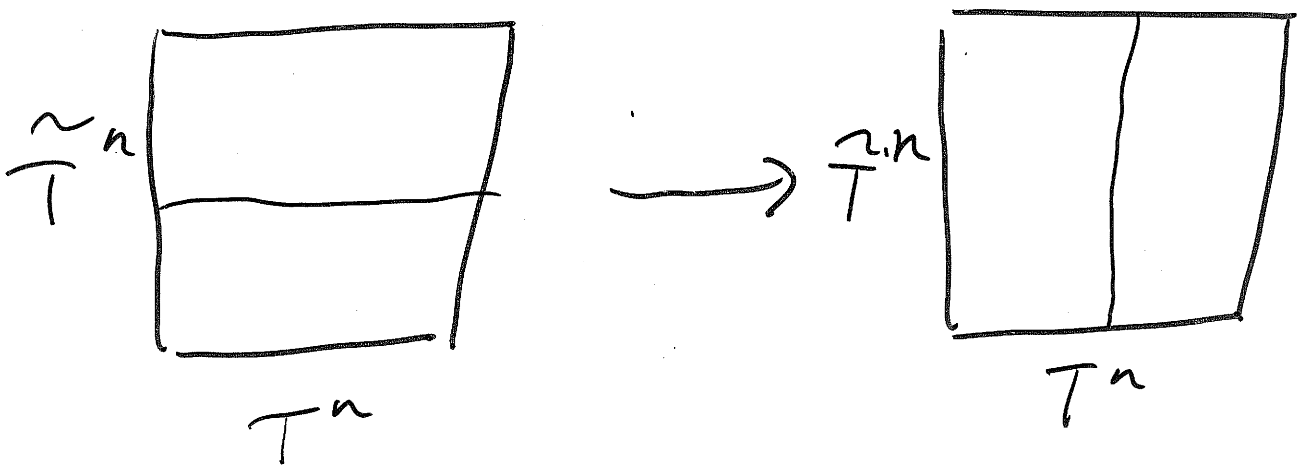
PICK "SECTION" TO GET SPACETIME

$$\begin{array}{c} T^n \\ \downarrow \\ N \end{array}$$

SOLVE $d\mathbb{X} = S \star d\mathbb{X} + \cdots$ IN TERMS OF X $\longrightarrow$ CONVENTIONAL FORMULATION

T-DUALITY CHANGES

POLARIZATION



T-DUALITY A SYMMETRY $\Rightarrow$

PHYSICS INDEPENDENT OF

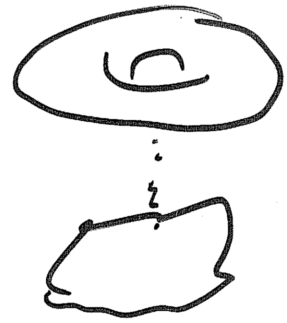
POLARIZATION CHOICE

SPACETIME A 'BRANE'

IN DOUBLED SPACE

T-FOLD

BUNDLE



$$\begin{array}{c} T^{2n} \\ \downarrow \\ N \end{array}$$

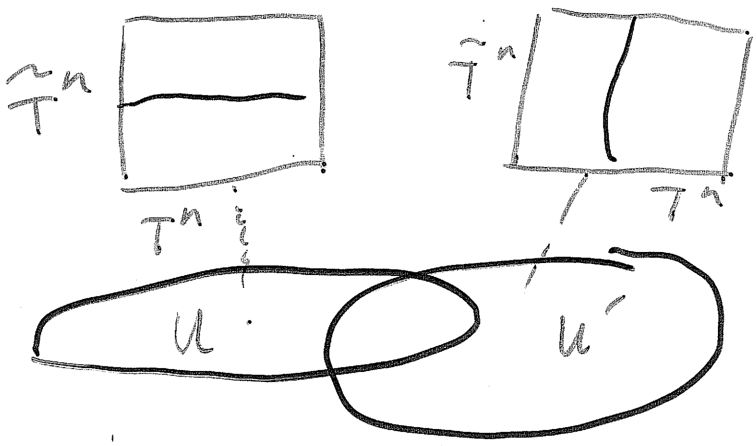
TRANSITION FUNCTIONS

IN $O(n,n;\mathbb{Z}) \subset GL(2n,\mathbb{Z})$

LOCAL SPACETIME

FOR EACH OPEN SET $U \subset N$: $\begin{array}{ccc} T^n & \subset & T^{2n} \\ \downarrow & & \downarrow \\ U & & U \end{array}$

PICK "SPACETIME" $T^n \times U$



T-DUALITY

TRANSITION

FNS

CHANGE POLARIZATION

FROM PATCH TO PATCH

GEOMETRIC BACKGROUND $GL(n,\mathbb{Z})$

LOCAL SECTIONS PATCH TOGETHER TO

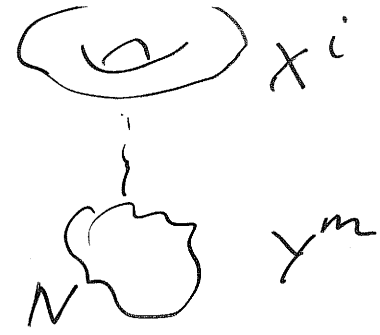
GIVE SPACETIME SUBMANIFOLD

NON-GEOMETRIC

LOCAL SPACETIME PATCHES,

BUT <u>DON'T</u> FIT TOGETHER TO MANIFOLD

TORUS BUNDLE



$$g = g_N + G_{ij}\, \xi^i \otimes \xi^j$$

$$\xi^i = dX^i + A^i \qquad A^i = A^i_m\, dy^m$$

$$\iota_i \iota_j H = -d B_{ij} \qquad G_{ij}(y) = g(k_i, k_j)$$

$$b = b_N + \xi^i \wedge \tilde{A}_i + \tfrac{1}{2} B_{ij}\, \xi^i \wedge \xi^j$$

$O(d,d)$

$$E_{ij} = G_{ij} + B_{ij} \qquad \mathcal{A} = \begin{pmatrix} A^i \\ \tilde{A}_i \end{pmatrix}$$ 1-FORMS ON N

$$h = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in O(d,d)$$

$$E \to (\alpha E + \beta)(\gamma E + \delta)^{-1}$$

$$\mathcal{A} \to h^{-1} \mathcal{A}$$

$$\mathcal{Y} = d\mathcal{A}$$ 2d CHERN CLASSES

Doubled Formalism

$$(y^m, x^i) \longrightarrow (y^m, \mathbb{X}^I) \qquad I = 1, \ldots 2d$$

L_{IJ} $O(d,d)$ metric, constant

$\mathcal{H}_{IJ}(y)$ +ve def metric

$$S^I{}_J \equiv L^{IK} \mathcal{H}_{KJ}$$

Constraint $\quad S^2 = \mathbb{1}$

$$\Rightarrow \quad \mathcal{H} \leftrightarrow (g, b)$$

Sigma-Model $\quad \mathcal{L}_k + \mathcal{L}_{WZ} + \mathcal{L}_{TOP}$

$$\mathcal{L}_k = \frac{1}{4} \mathcal{H}_{IJ} (d\mathbb{X}^I + A^I) \wedge * (d\mathbb{X}^J + A^J) + \mathcal{L}(y)$$

$$\mathcal{L}_{WZ} = -\frac{1}{2} L_{IJ}\, d\mathbb{X}^I \wedge A^J$$

$$\mathcal{L}_{TOP} = \frac{1}{2} \Omega_{IJ}\, d\mathbb{X}^I \wedge d\mathbb{X}^J$$

Const 2-form

Constraint $\quad \frac{1}{2}$ left-movers $\frac{1}{2}$ right

$$(d\mathbb{X} + A) = S * (d\mathbb{X} + A)$$

QUANTISATION

HOW TO IMPOSE CONSTRAINT?
OF CHIRAL BOSONS

1) CHOOSE POLARIZATION (LOCALLY)
$$\mathbb{X} \longrightarrow (X, \tilde{X})$$

2) CONSTRAINT GENERATES SHIFTS IN $\tilde{X}$. GAUGE THESE SHIFTS

$\longrightarrow$ CONVENTIONAL SIGMA MODEL

$$\mathcal{L} = \frac{1}{2} g_{ij}\, dX^i \wedge * dX^j + \frac{1}{2} b_{ij}\, dX^i \wedge dX^j + \cdots$$

3) $$e^{i\int \mathcal{L}_{TOP}} = e^{\pi i n \tilde{n}} = \pm 1$$

NEEDED TO PROVE EQUIVALENCE ON ARBITRARY RIEMANN SURFACE

[ASIN GIVEON + ROCEK]

LANDSCAPE OF GAUGED SUGRAS

$$\mathcal{L}_D = e^{-2\Phi}\left[R + H^2 + (\nabla\Phi)^2\right]$$

TOROIDAL REDUCTION ON T^d:

$U(1)^{2d}$ $\quad A_{mi} \sim g_{mi}$

$\quad\quad\quad \tilde{A}_{mi} \sim b_{mi}$

$O(d,d;\mathbb{Z})$ $\quad\quad g_{ij} + b_{ij} \in \dfrac{O(d,d)}{O(d)\times O(d)}$

$O(d,d)$ SYMMETRY OF SUGRA

SCHERK - SCHWARZ REDUCTION

$\rightarrow$ GAUGED SUGRA

$A, \tilde{A}$ BECOME GAUGE FIELDS

FOR $\boxed{G_{2d} \subset O(d,d)}$

2d - DIM GAUGE GROUP

KALOPER + MYERS

FORMAL SYMMETRY $O(d,d)$

CHANGES EMBEDDING OF GAUGE GROUP

CMH + REID-EDWARDS

GENERAL GAUGE GROUP

$A_i \to$ GENERATORS Z_i

$\hat{A}^i \to$ X^i

$$[Z_i, Z_j] = \gamma_{ij}{}^k Z_k + \beta_{ijk} X^k$$

$$[Z_i, X^j] = \hat{\gamma}_i{}^j{}_k X^k + \delta_i^{jk} Z_k$$

$$[X^i, X^j] = \tilde{\gamma}^{ij}{}_k X^k + \tilde{\beta}^{ijk} Z_k$$

WHICH $G_{2d} \subset O(d,d)$

LIFT TO STRING COMPACTIFICATIONS

GEOMETRIC SUGRA OR NON-GEOMETRIC?

CMH + DABHOLKAR

SHELTON, TAYLOR, WECHT

ALL TYPES OF STRUCTURE CONSTANT CAN ARISE!

CMH + AD

1) TWISTED TORUS + FLUX

$\gamma, \beta, \hat{\gamma} \neq 0$

COMPACTIFICATION ON G_d / Γ

G_d: d-dim GROUP MANIFOLD
STRUCTURE CONSTANTS $\gamma_{ij}{}^{k}$

Γ: DISCRETE SUB GROUP
G_d / Γ COMPACT

FLUX $H_{ijk} \sim \beta_{ijk}$

γ : "TWIST"
β : FLUX

CMH + REID-EDWARDS

$$[Z, Z] \sim \gamma Z + \beta X$$
$$[X, X] \sim 0$$
$$[Z, X] \sim \gamma X$$

$\hat{\gamma} = \gamma$

2) REDUCTION WITH DUALITY TWIST [CMH + AD]

) REDUCE ON T^{d-1}, $H = O(d-1, d-1, \mathbb{Z})$

DUALITY SYMMETRY

2) REDUCE ON FURTHER S^1,

WITH H - TWIST $\qquad X \in S^1$

$$\psi(y^m, X) = g(X)\, \psi(y)$$

$$g(X) \in O(d-1, d-1)$$

MONODROMY $\quad g(2\pi) \in O(d-1, d-1, \mathbb{Z})$

GEOMETRIC TWIST: $\quad GL(d, \mathbb{Z}) \ltimes \mathbb{Z}^{d(d-1)/2}$

$$\gamma, \beta, \hat{\gamma} = \gamma$$

T - DUALITY TWIST: $\quad$ T- FOLD

$$\gamma, \beta, \hat{\gamma}, \quad \delta$$

$$[Z, X] = \hat{\gamma} X + \delta Z$$

3) ASYMMETRIC ORBIFOLDS

CMH + AD

AT SPECIAL POINTS IN MODULI SPACE, DUALITY TWIST $\rightarrow$ ORBIFOLD

$\mathbb{Z}_n$: SYMMETRY OF CFT ON T^{d-1} (AT SPECIAL POINT)

SHIFT $X \rightarrow X + \frac{2\pi}{n}$

4) ORBIFOLDS WITH DUAL TWISTS + SHIFTS

$\mathbb{Z}_{\tilde{n}}$ SYMM OF CFT ON T^{d-1}

SHIFT $\tilde{X} \longrightarrow \tilde{X} + \frac{2\pi}{\tilde{n}}$

$$|p, w\rangle \longrightarrow \exp\left(\frac{2\pi i p}{n}\right) \exp\left(\frac{2\pi i w}{\tilde{n}}\right) |p, w\rangle$$

MOMENTUM WINDING

TURNS ON $\tilde{\gamma}, \tilde{\beta}$!

$$[X, X] \sim \tilde{\gamma} X + \tilde{\beta} Z$$

5) REDUCTIONS WITH DUAL TWISTS

CMH + AD

MOVING AWAY FROM ORBIFOLD PTS:

X-SHIFT $\longrightarrow$ X TWIST

$\psi(Y, X) = g(X)\, \psi(Y)$ SUGGESTS

$\tilde{X}$-SHIFT $\rightarrow$ $\tilde{X}$ TWIST

$\psi(Y, X, \tilde{X}) = \tilde{g}(\tilde{X})\, \psi(Y)$

DEPENDENCE ON DUAL COORDINATE

GENERAL CASE: TWISTS

$g(X), \quad \tilde{g}(\tilde{X}) \qquad [g, \tilde{g}] = 0$

MONODROMIES $\quad g(2\pi), \tilde{g}(2\pi)$

e.g. T^3, $H = N \times \text{VOL}$

3 T-DUALITIES $\longrightarrow$

T^2 BUNDLE OVER DUAL $\tilde{S}^1$

MONODROMY $O(2,2;\mathbb{Z})$

6) WZW MODEL

"NON-GEOMETRIC" STRUCTURE CONSTANTS CAN ARISE GEOMETRICALLY FROM DIFFERENT ANSATZ

SO(3) WZW

$$\gamma_{ij}{}^{k} = \tilde{\gamma}^{ij}{}_{k} = \hat{\gamma}_{ij}{}^{k} = k\epsilon_{ijk}$$

$$\underline{\beta = \tilde{\beta} = \delta = 0}$$

ALL STRUCTURE CONSTANTS

$$\gamma, \beta, \tilde{\gamma}, \tilde{\beta}, \hat{\gamma}, \delta$$

CAN ARISE FROM STRING THEORY, IF NON-GEOMETRIC BACKGROUNDS INCLUDED.

T-DUALITY REQUIRES ISOMETRY IN CIRCLE DIRECTION – STANDARD

GENERALISED T-DUALITY

CMH + AD

$X^i \sim X^i + 1$ TORUS FIBRATION

BUT ALLOWS X^i DEPENDENCE

– NO ISOMETRY NEEDED

X^i DEPENDENCE $\longrightarrow$

$\tilde{X}^i$ DEPENDENCE

OR DEPENDENCE ON BOTH

$X, \tilde{X}$

DOUBLED GEOMETRY

NEED NOT ADMIT POLARIZATION,

NO LOCAL SPACETIME ?

STRING FIELD THEORY

$$|Y^\mu, X^i, \tilde{X}_i, I\rangle \leftrightarrow |Y^\mu, m, n, I\rangle$$

I : ALL OTHER LABELS INCLUDING ALL OSCILLATORS

$$|\Psi\rangle^R = \int dY \sum \Psi^R_{m,n,I}(Y)\,|m,n,Y,I\rangle^R$$

$$|\Psi\rangle^{\tilde{R}} \longrightarrow \Psi^{\tilde{R}}_{\tilde{m},\tilde{n},I}(Y) \qquad \tilde{R} = \frac{1}{R}$$

T-DUALITY EXACT MAP BETWEEN $|\ \rangle^R$ AND $|\ \rangle^{1/R}$ HILBERT SPACES

MAP BETWEEN STRING FIELDS

$$\Psi^R_{m,n\,I}(Y) \longleftrightarrow \Psi^{\tilde{R}}_{m,n,I}$$

BUSCHER : $\Psi^R_{0,0,I} \longleftrightarrow \Psi^{\tilde{R}}_{0,0,I}$

DOUBLED FIELDS

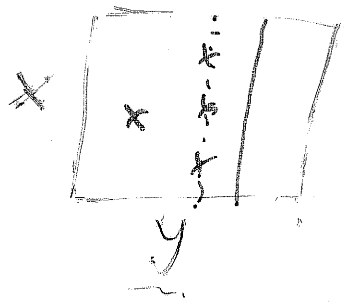
$$\Psi^R_I(Y,X,\tilde{X}) \longleftrightarrow \Psi^{\tilde{R}}_I(Y,X,\tilde{X})$$

NS5 - BRANES & KK MONOPOLES

NS5-BRANE: TRANSVERSE $\mathbb{R}^4$

HARMONIC FUNCTION $H(x, y_1, y_2, y_3)$

SMEARED $H = H(\underset{\sim}{y})$



TAKE x PERIODIC, T-DUALITY

$\rightarrow$ KK MONOPOLE $\quad x \rightarrow \tilde{x}$

PERIODIC ARRAY $H(x, \underset{\sim}{y}) = H(x + 2\pi R, \underset{\sim}{y})$

T-DUALITY $\rightarrow$ $\quad H(\tilde{x}, \underset{\sim}{y}) = H_0(\underset{\sim}{y}) + \cdots$

KK MONOPOLE

GIVES $\tilde{x}$ CORRECTIONS TO KK MONOPOLE, "THROAT" SEEN BY WINDING MODES

$\tilde{x}$ DEPENDENCE CAN BE UNDERSTOOD AS WORLD-SHEET INSTANTON CORRECTIONS

GREGORY, HARVEY MOORE; TONG; HARVEY - JENSEN

GENERALISED GEOMETRY AND ITS GENERALISATIONS

GENERALISED GEOMETRY: HITCHIN

DOUBLE TANGENT SPACE

$$T \to T \oplus T^*$$

EQUIVALENT TO ORDINARY GEOMETRY ON (M, g, b) WITH b-FIELD

$$\mathcal{H}_{IJ} \longleftrightarrow g, b$$

$$\mathcal{J}_1, \mathcal{J}_2 \longleftrightarrow J_1, J_2$$

WRITE IN $O(D,D)$ COVARIANT WAY

DOUBLED GEOMETRY

DOUBLE MANIFOLD

$$\begin{array}{ccc} T^d \longrightarrow M & & T^{d,d} \longrightarrow \bar{M} \\ \downarrow & \Longrightarrow & \downarrow \\ N & & T^*N \end{array}$$

NOT EQUIVALENT TO ORDINARY GEOMETRY, BUT TO T-FOLD

DISCRETE $O(d,d;\mathbb{Z})$

TRANSITION FUNCTIONS

GEOMETRY (M, g) DIFFEOS

$TM,\ T^*M,\ T\oplus T^*M$ $\quad GL(D,\mathbb{R})$

GENERALISED GEOMETRY (M, g, b)

DIFFEOS + b- GAUGE $\quad \delta b = d\lambda$

GENERALISED TANGENT BUNDLE

$GL(D,\mathbb{R}) \ltimes \mathbb{R}^{D(D-1)/2}$ $\quad$ B - SHIFTS

TORUS FIBRATION $\quad \begin{array}{ccc} T^n & \longrightarrow & M \\ & & \downarrow \\ & & N \end{array}$

$\mathrm{DIFF}(N) \times GL(n;\mathbb{Z}) \ltimes U(1)^n$

WITH B - FIELD

$\mathrm{DIFF}(N) \times (\delta b = d\lambda \text{ on } N)$
$\times \left[GL(n,\mathbb{Z}) \ltimes \mathbb{Z}^{n(n-1)/2}\right] \ltimes \left[U(1)^n \times U(1)^n\right]$

T- FOLD

$\mathrm{DIFF}(N) \times (\delta b = d\lambda \text{ on } N)$
$\times \left[O(n,n;\mathbb{Z}) \ltimes U(1)^{2n}\right]$

D-BRANES AND OPEN STRINGS

IF X^i NEUMANN B.C.'S (DIRICHLET)

THEN $\tilde{X}_i$: DIRICHLET (NEUMANN)

$$2n\ \mathbb{X}^I \longrightarrow \begin{cases} n \text{ DIRICHLET } X_D \\ n. \text{ NEUMANN } X_N \end{cases}$$

$\{X_D\}$ MAXIMAL NULL SUBSPACE

$\{X_D\} \cap \{X^i\} \rightarrow$ 'PHYSICAL' D-BRANE

CHANGING POLARIZATION $\{X^i\}$

CHANGES NUMBER p OF

DIRICHLET DIRECTIONS SEEN

p-BRANE $\longrightarrow$ p'-BRANE

BOUNDARY STATES IN DOUBLED FORMALISM: LAWRENCE, SCHULZ, WECHT

OBSTRUCTION TO T-DUALITY

NO H-FLUX ON T^n FIBRES

- IN GENERAL GIVES T-FOLDS, NGBs

LOCAL SPACETIME PATCHES,

NO GLOBAL SPACETIME MANIFOLD

- D-BRANES + SUSY CAN BE INCORPORATED

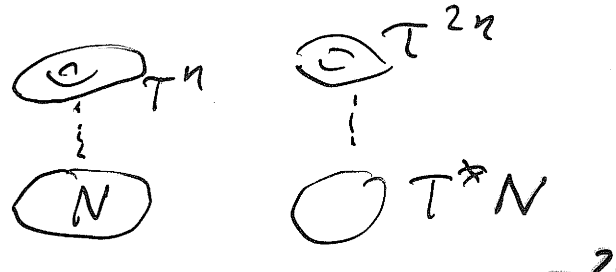
- T-FOLDS MOMENTUM + WINDING MIX
- U-FOLDS MOM^m + BRANE WRAPPING MODES MIX

- GENERALISED GEOMETRY HITCHIN

DOUBLE TANGENT SPACE $T \to T \oplus T^*$

- DOUBLED FORMALISM

DOUBLES SPACETIME



LOCAL SPACETIME FROM LOCAL CHOICE $T^n \subset T^{2n}$

- GENERALISATION TO GENERAL BACKGROUNDS, NOT TORUS FIBRATIONS?
- FIELDS $\phi(x, \tilde{x})$?