Making and Checking Inflation

Top-down approach to inflation:
seeks to embed it in fundamental theory

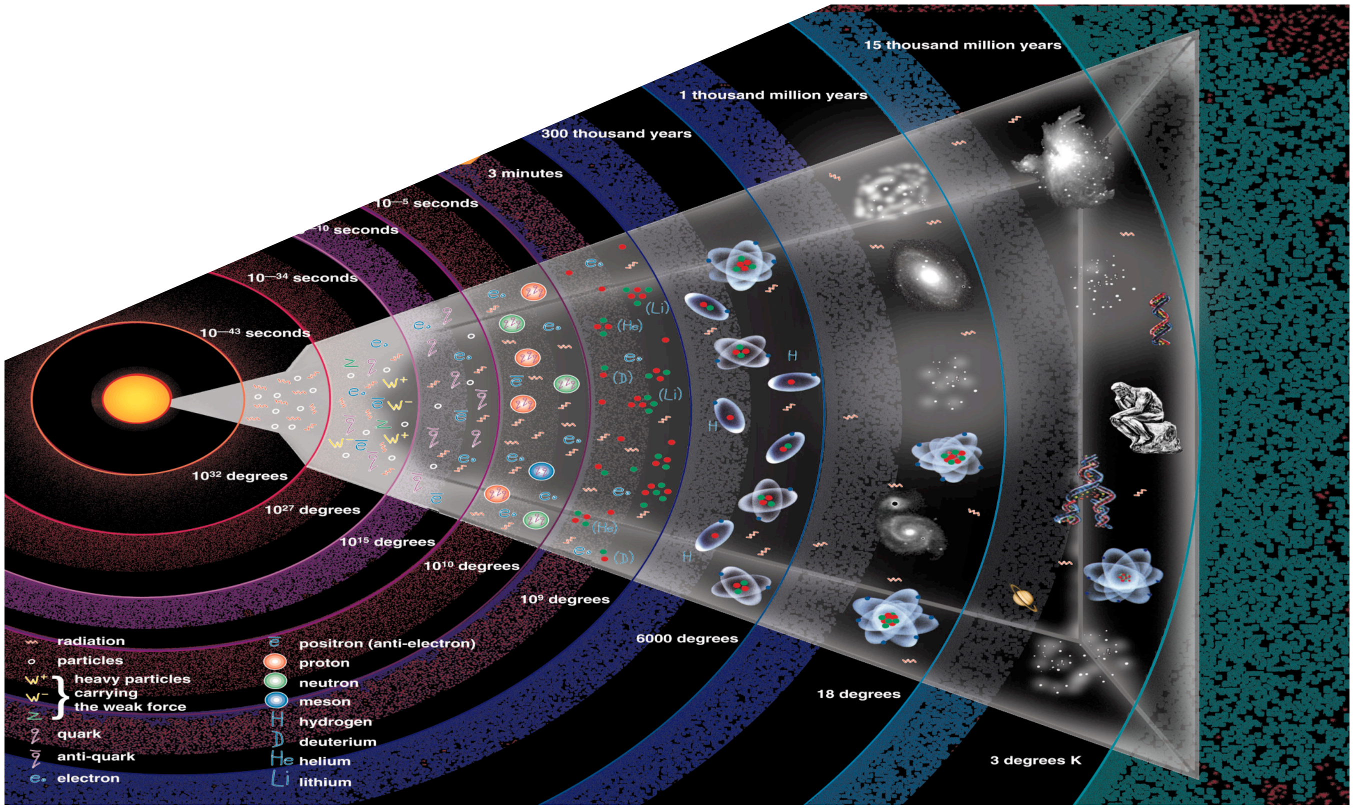
Bottom-up approach to inflation:
reconstruction of acceleration trajectories

Lev Kofman

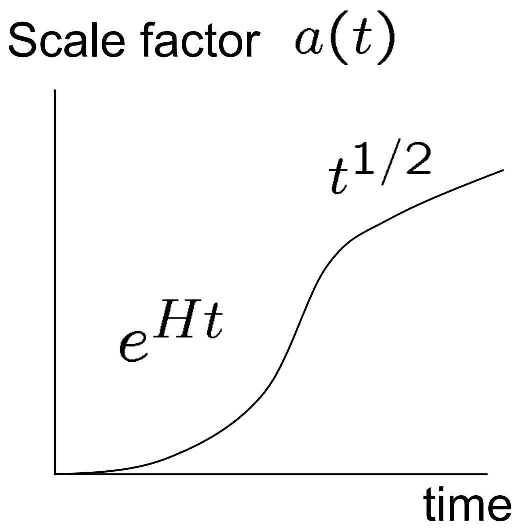
CITA ICAT
Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique

Workshop on String Vacua and the Landscape
ICTP May 31, 2006

Inflation with QF
Preheating with QFT
Reheating with String Theory
Reconstruction of (P)reheating with GW
Reconstruction of Inflationary Trajectory



Early Universe Inflation



Equation of State $t \leq 10^{-35}$ sec

$$p \approx -\epsilon$$

Inflation $a(t) \approx e^{Ht}$

Realization of Inflation

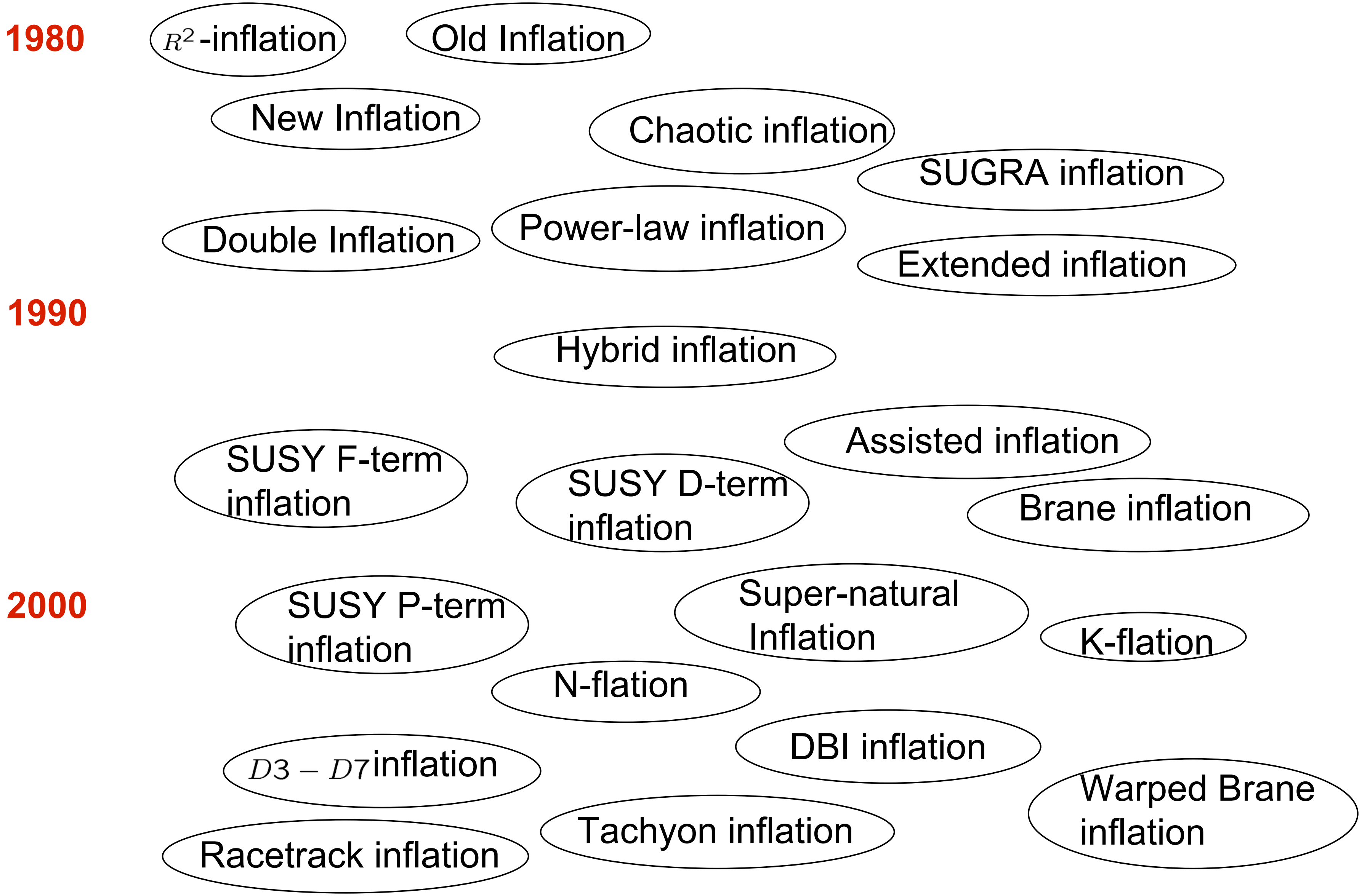
Scalar field

$$p = \frac{1}{2}\dot{\phi}^2 - V$$
$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

slow roll $\dot{\phi}^2 \ll V$

choise of $V(\phi)$

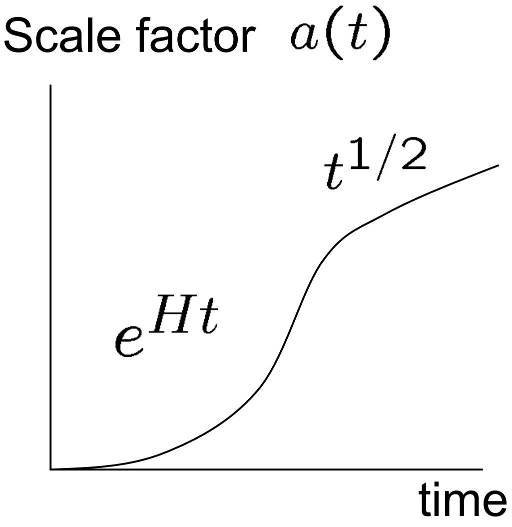
Inflation in the context of ever changing fundamental theory



4 dimensional Inflation predicts

- $\Omega_{tot} = 1$
- No classical inhomogeneities from the past $C_{\mu\nu\lambda\rho} = 0$
- Scale free gaussian fluctuations of all light scalars $\delta\phi_k(t)e^{ikx}$
- No vector perturbations $A_\mu = 0$
- Scalar (almost scale free gaussian) metric perturbations $\Phi \rightarrow \Phi_k(t)e^{ikx}$
- Tensor metric perturbations $h_{ik} \rightarrow h_k(t)e^{ikx}e_{ij}$
- Creation of all SM particles in preheating/thermalization T_{reh}

Early Universe Inflation



Equation of State $t \leq 10^{-35}$ sec

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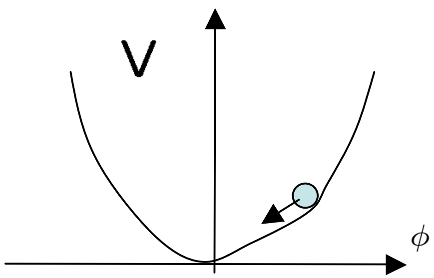
Realization of Inflation

Scalar field

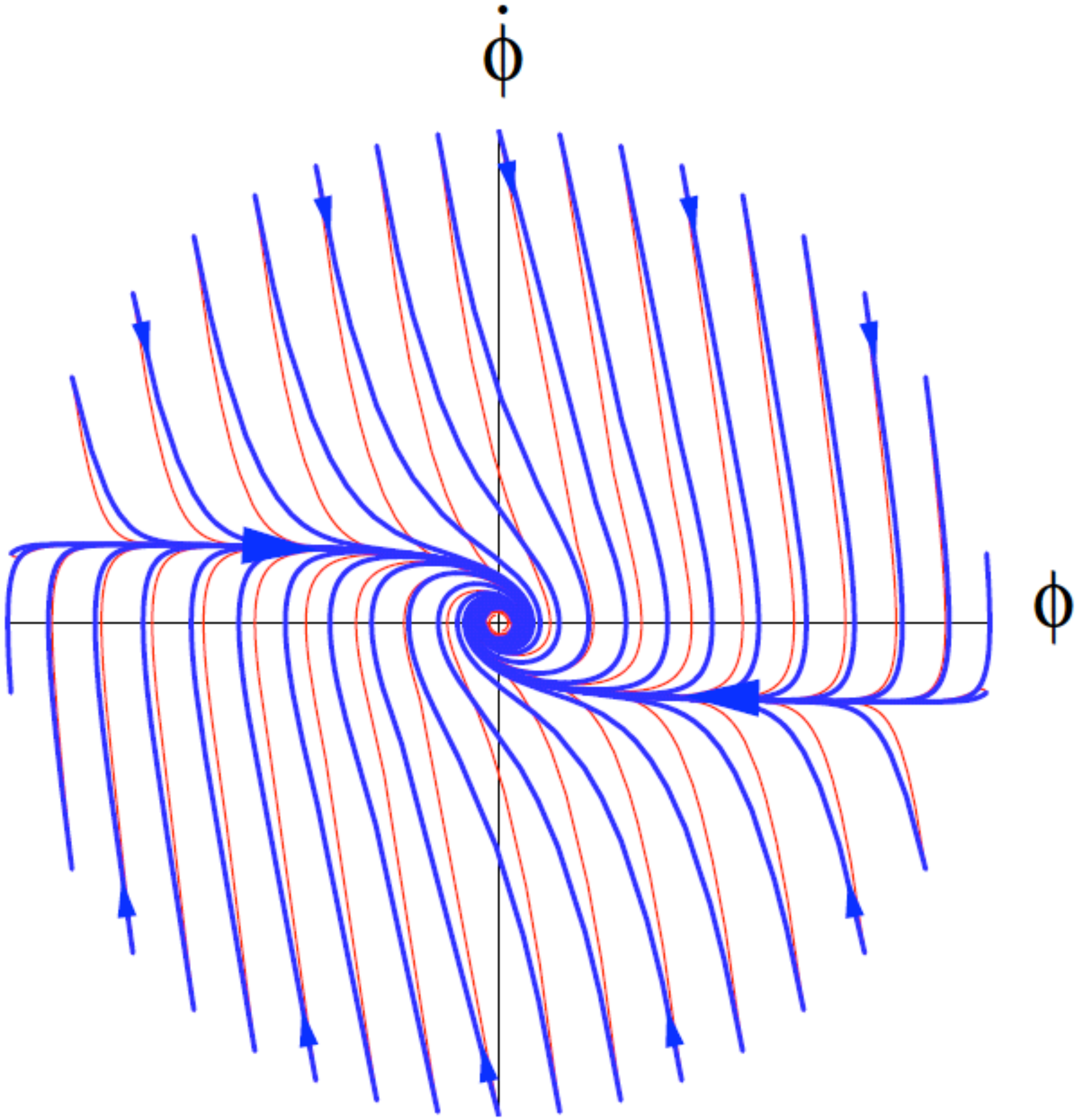
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

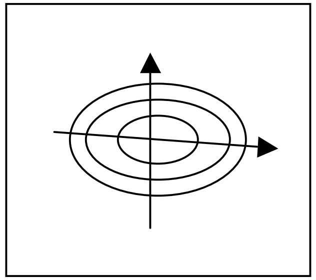


$$p = \frac{1}{2}\dot{\phi}^2 - V$$
$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

Phase portrait of inflation

$$\ddot{\phi} + 3H\phi + V_{,\phi} = 0$$
$$3H^2 = \frac{8\pi}{M_p^2}\left(\dot{\phi}^2/2 + V\right)$$

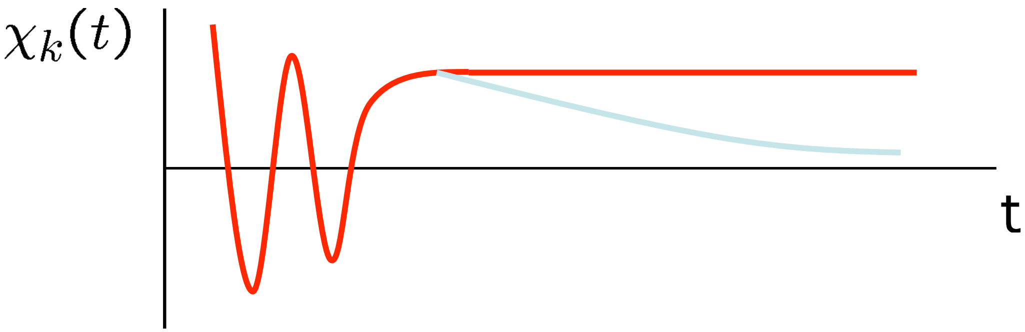




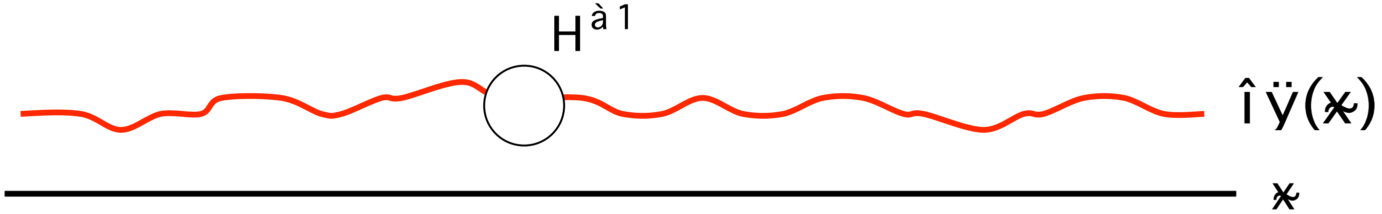
Light field at inflation

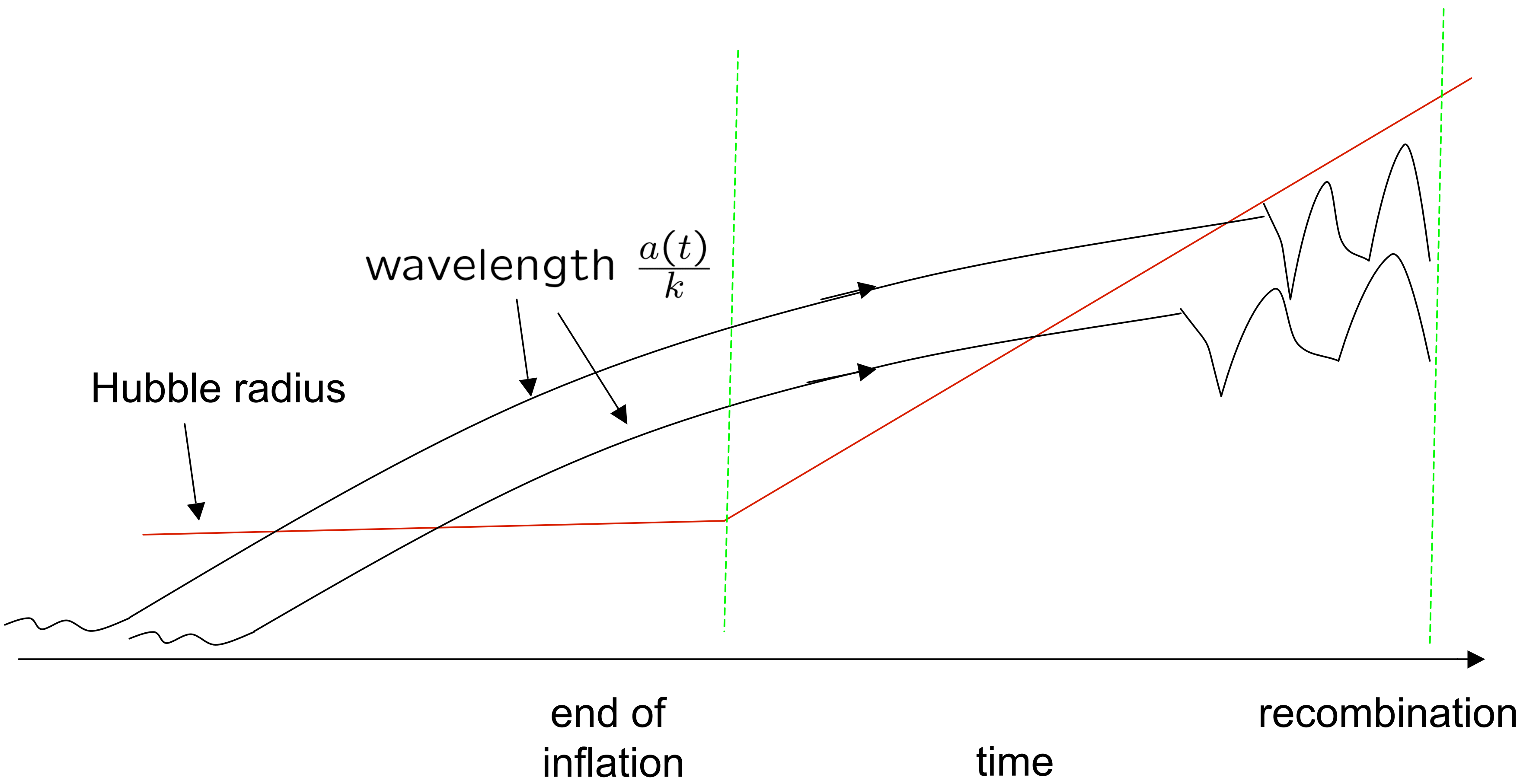
$$\delta\chi = \int d^3k\,(a_k\chi_k(t)e^{i\vec{k}\cdot\vec{x}} + h.c.)$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0$$



$$\chi_k \simeq \frac{H}{\sqrt{2}k^{3/2}}$$





wavelength $\frac{a(t)}{k}$
Hubble radius
end of
inflation
time
recombination

Scalar metric Fluctuations from Inflation

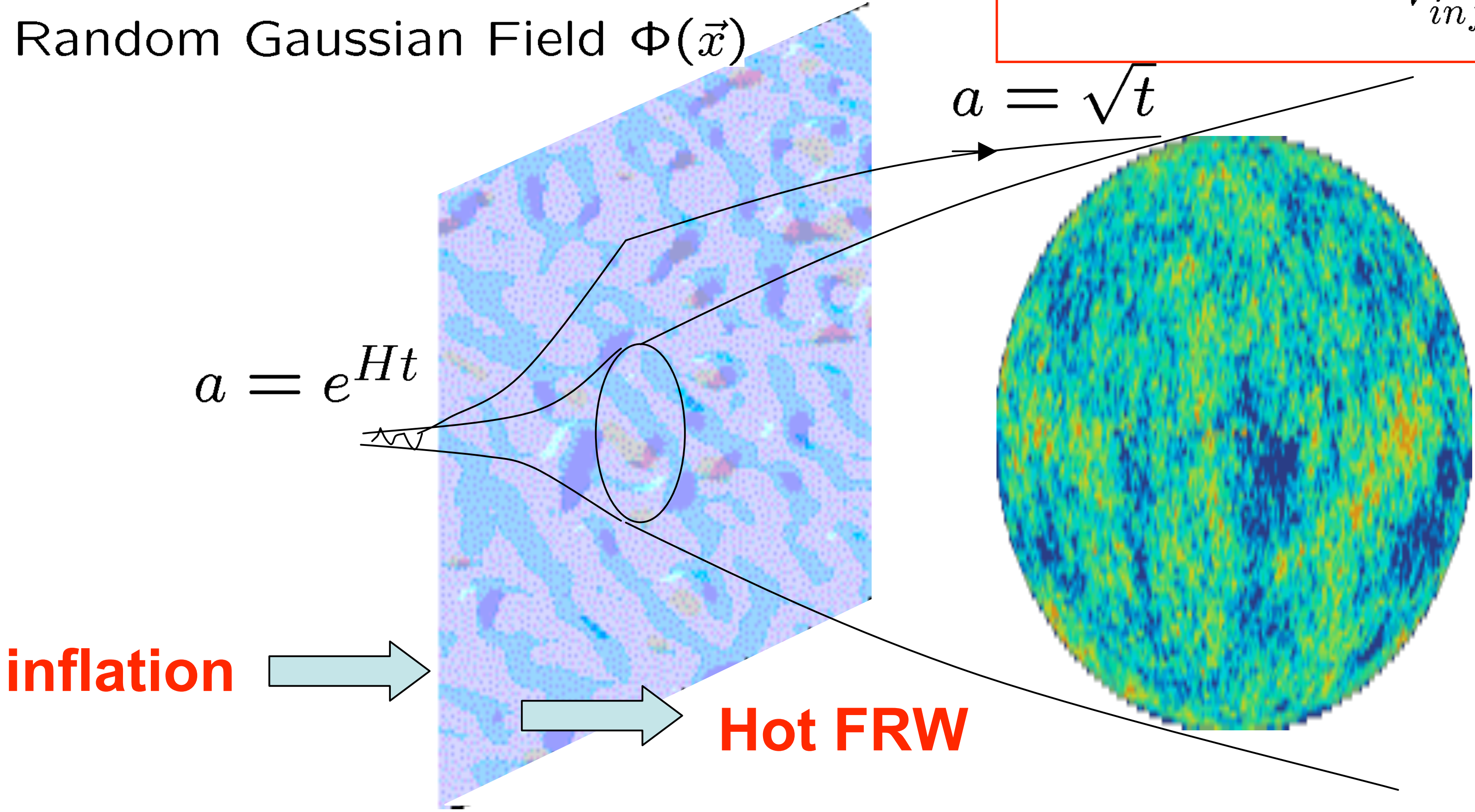
$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$

Initial conditions from Inflation ⇒

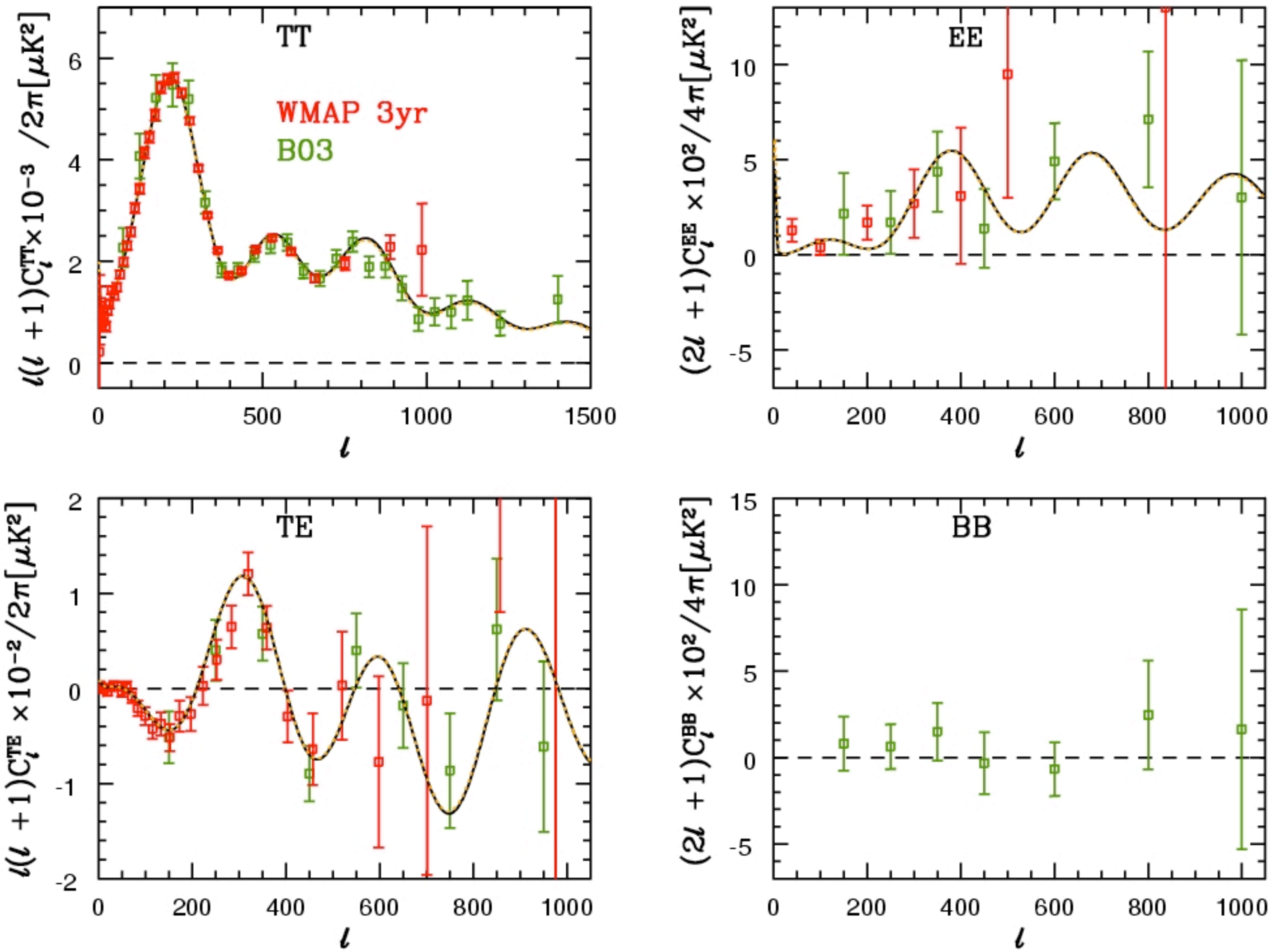
$$\Omega_{tot} = 1$$
$$k^3 \Phi_k^2 \rightarrow P_s = A_s k^{n_s - 1}$$
$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

⇓

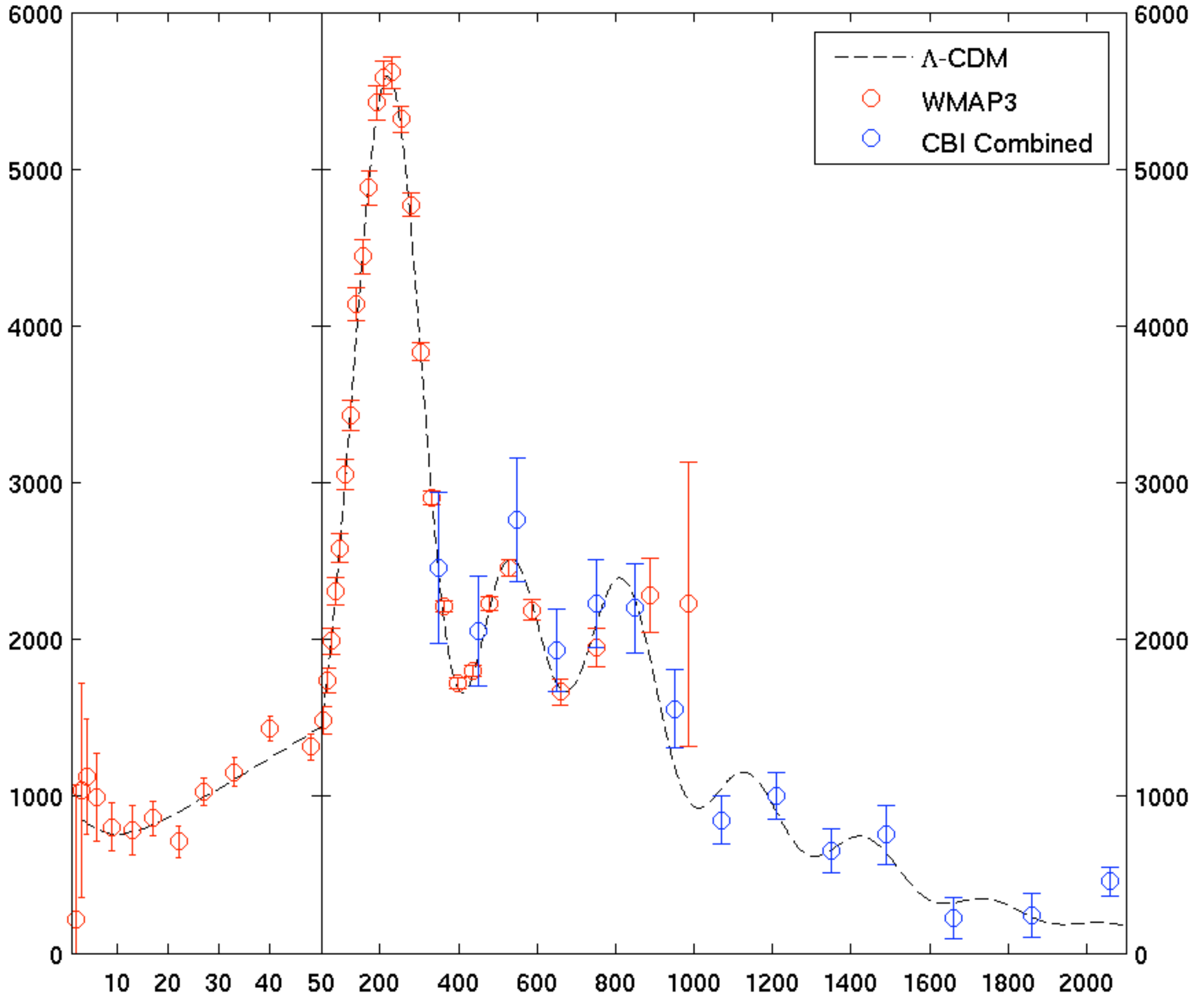
Random Gaussian Field $\Phi(\vec{x})$



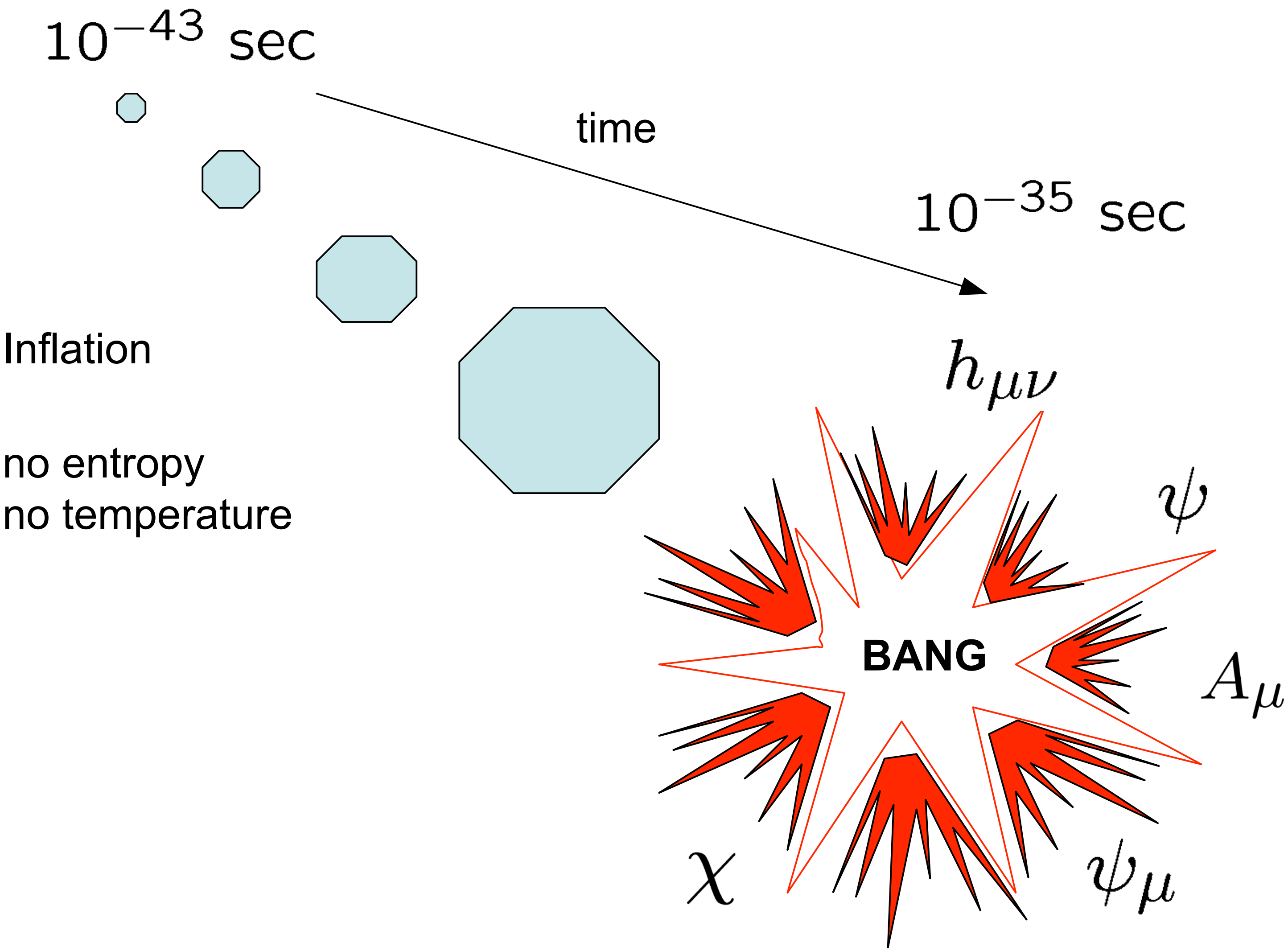
WMAP3 sees 3rd pk, B03 sees 4th



CBI combined TT sees 5th pk



Particlegenesis



$$\mathcal{L}(\phi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu})$$

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\;& -\tfrac{1}{2}M_P^2\left[R+\bar{\psi}_\mu R^\mu+\mathcal{L}_{SG,torsion}\right]-g_i{}^j\left[M_P^2(\hat{\partial}_\mu z^i)(\hat{\partial}^\mu z_j)+\bar{\chi}_j\,\not{\mathcal{D}}\chi^i+\bar{\chi}^i\,\not{\mathcal{D}}\chi_j\right]\\
&+(\mathrm{Re}\,f_{\alpha\beta})\left[-\tfrac{1}{4}F_{\mu\nu}^\alpha F^{\mu\nu\,\beta}-\tfrac{1}{2}\bar{\lambda}^\alpha\,\hat{\not{\mathcal{D}}}\lambda^\beta\right]+\tfrac{1}{4}\mathrm{i}(\mathrm{Im}\,f_{\alpha\beta})\left[F_{\mu\nu}^\alpha\tilde{F}^{\mu\nu\,\beta}-\hat{\partial}_\mu\left(\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta\right)\right]\\
&-M_P^{-2}\mathrm{e}^{\mathcal{K}}\left[-3WW^*+(\mathcal{D}^iW)g^{-1}{}_i{}^j(\mathcal{D}_jW^*)\right]-\tfrac{1}{2}(\mathrm{Re}\,f)^{-1\,\alpha\beta}\mathcal{P}_\alpha\mathcal{P}_\beta\\
&+\tfrac{1}{8}(\mathrm{Re}\,f_{\alpha\beta})\bar{\psi}_\mu\gamma^{\nu\rho}\left(F_{\nu\rho}^\alpha+\hat{F}_{\nu\rho}^\alpha\right)\gamma^\mu\lambda^\beta\\
&+\Big\{M_Pg_j{}^i\bar{\psi}_{\mu L}(\hat{\not{\partial}}z^j)\gamma^\mu\chi_i+\bar{\psi}_R\cdot\gamma\left[\tfrac{1}{2}\mathrm{i}\lambda_L^\alpha\mathcal{P}_\alpha+\chi_iY^3M_P^{-4}\mathcal{D}^iW\right]\\
&\quad+\tfrac{1}{2}Y^3M_P^{-3}W\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}-\tfrac{1}{4}M_P^{-1}f_{\alpha\beta}^i\bar{\chi}_i\gamma^{\mu\nu}\hat{F}_{\mu\nu}^{-\alpha}\lambda_L^\beta\\
&\quad-Y^3M_P^{-5}(\mathcal{D}^i\mathcal{D}^jW)\bar{\chi}_i\chi_j+\tfrac{1}{2}\mathrm{i}(\mathrm{Re}\,f)^{-1\,\alpha\beta}\mathcal{P}_\alpha M_P^{-1}f_{\beta\gamma}^i\bar{\chi}_i\lambda^\gamma-2M_P\xi_\alpha{}^ig_i{}^j\bar{\lambda}^\alpha\chi_j\\
&\quad+\tfrac{1}{4}M_P^{-5}Y^3(\mathcal{D}^jW)g^{-1}{}_j{}^if_{\alpha\beta i}\bar{\lambda}_R^\alpha\lambda_R^\beta\\
&\quad-\tfrac{1}{4}M_P^{-1}f_{\alpha\beta}^i\bar{\psi}_R\cdot\gamma\chi_i\bar{\lambda}_L^\alpha\lambda_L^\beta+\tfrac{1}{4}M_P^{-2}(\mathcal{D}^i\partial^jf_{\alpha\beta})\bar{\chi}_i\chi_j\bar{\lambda}_L^\alpha\lambda_L^\beta+\mathrm{h.c.}\Big\}\\
&+g_j{}^i\left(\tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\chi}^j\gamma_\sigma\chi_i-\bar{\psi}_\mu\chi^j\,\bar{\psi}^\mu\chi_i\right)\\
&+M_P^{-2}\left(R_{ij}^{k\ell}-\tfrac{1}{2}g_i{}^kg_j{}^\ell\right)\bar{\chi}^i\chi^j\bar{\chi}_k\chi_\ell\\
&+\tfrac{3}{64}M_P^{-2}\left((\mathrm{Re}\,f_{\alpha\beta})\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\beta\right)^2-\tfrac{1}{16}M_P^{-2}f_{\alpha\beta}^i\bar{\lambda}_L^\alpha\lambda_L^\beta g^{-1}{}_i{}^jf_{\gamma\delta j}\bar{\lambda}_R^\gamma\lambda_R^\delta\\
&+\tfrac{1}{8}(\mathrm{Re}\,f)^{-1\,\alpha\beta}M_P^{-2}\left(f_{\alpha\gamma}^i\bar{\chi}_i\lambda^\gamma-f_{\alpha\gamma i}\bar{\chi}^i\lambda^\gamma\right)\left(f_{\beta\delta}^j\bar{\chi}_j\lambda^\delta-f_{\beta\delta j}\bar{\chi}^j\lambda^\delta\right).
\end{aligned}
$$

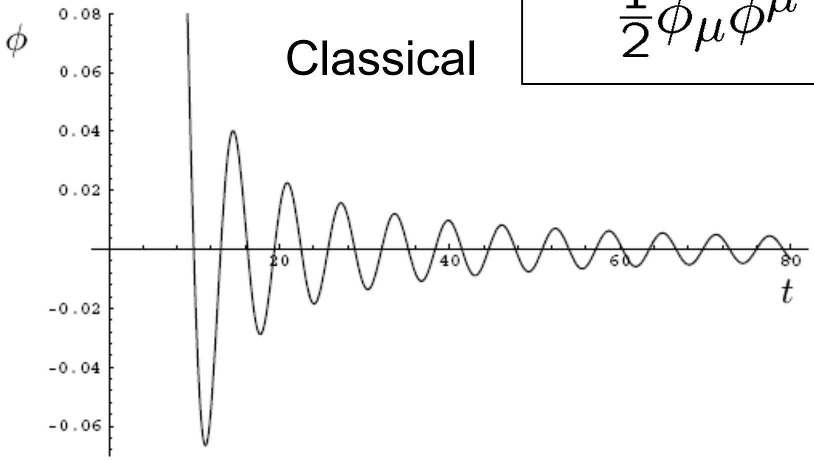
Output of Preheating

- Reheat temperature T_R
- Out-of-equilibrium state
- Evolution of EoS
- Number of efolds

$$N = 62 - \ln \frac{10^{16} Gev}{V_h^{1/4}} + \frac{1}{4} \ln \frac{V_h}{V_{end}} - \frac{1}{12} \ln \frac{V_{end}}{\rho_{rad}}$$

- Potential observables

Resonant Preheating in Chaotic Inflation



Classical

$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$

Quantum

$$\hat{\chi}(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\ \left(\hat{a}_k\chi_k(t)\,e^{-i\mathbf{kx}} + \hat{a}_k^+\chi\right.$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$

$$\tau = mt$$

$$\ddot{X}_k + \left(\frac{k^2}{m^2} + q\sin^2\tau\right)X_k = 0$$

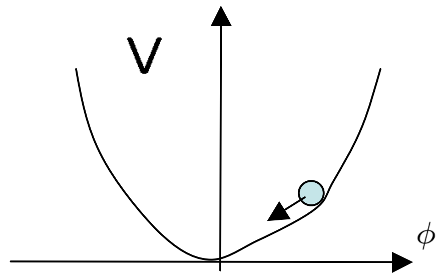
$$\text{parameter } q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10}$$

Occupation number

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}$$

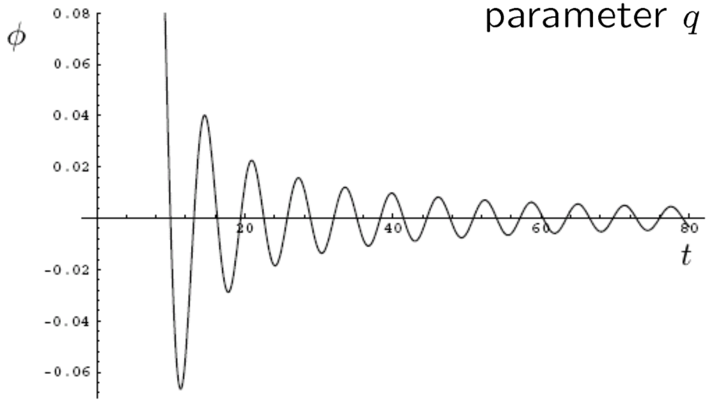
$$n_k \sim e^{\mu t}$$

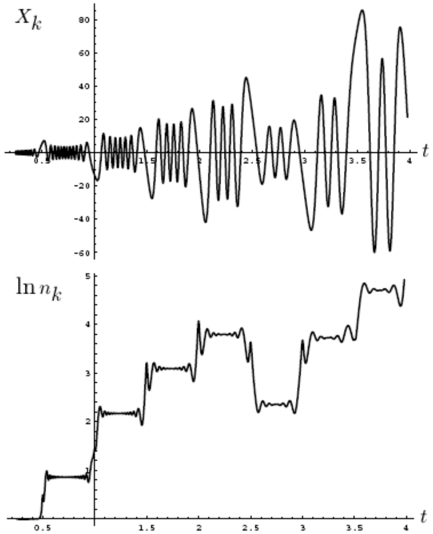
Resonant Preheating in Chaotic Inflation



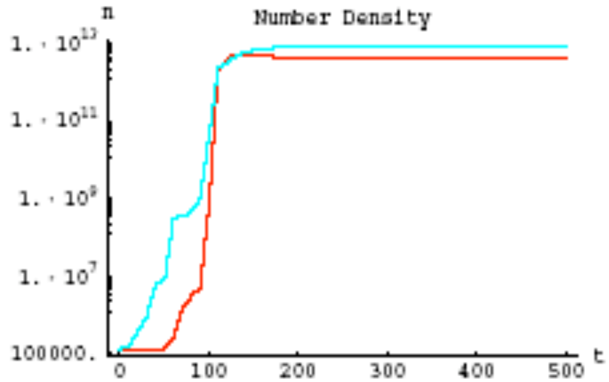
$$g^2\phi^2\chi^2$$

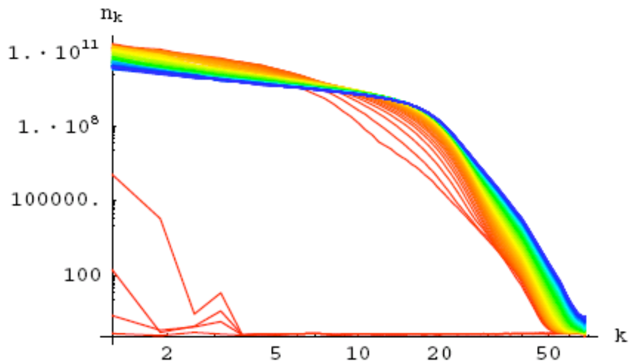
parameter $q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10} \gg 1$





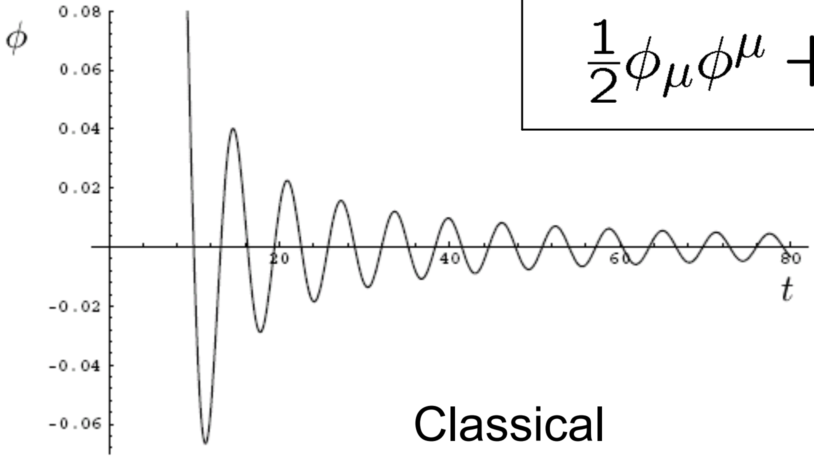
$\delta\chi$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$

Decay of inflaton and preheating after inflation



Classical

$$\phi_0 + \phi$$

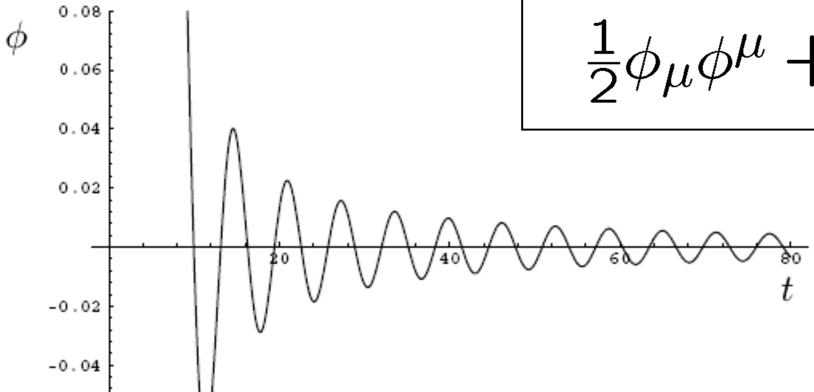
Quantum

$$\chi$$

Felder, LK, Peloso,05

movie

$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi 2$$



Decay of inflaton and preheating after inflation

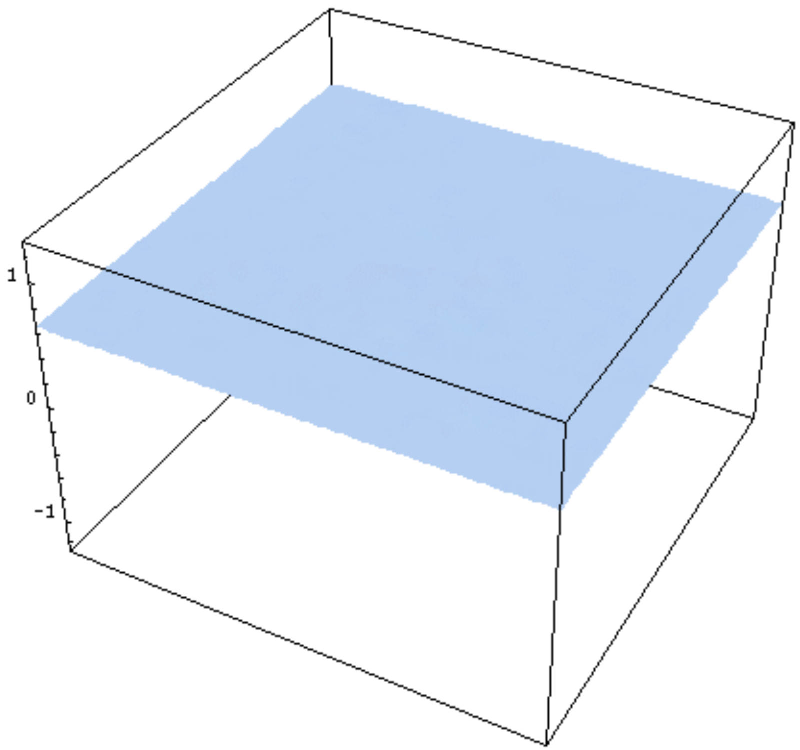
Classical

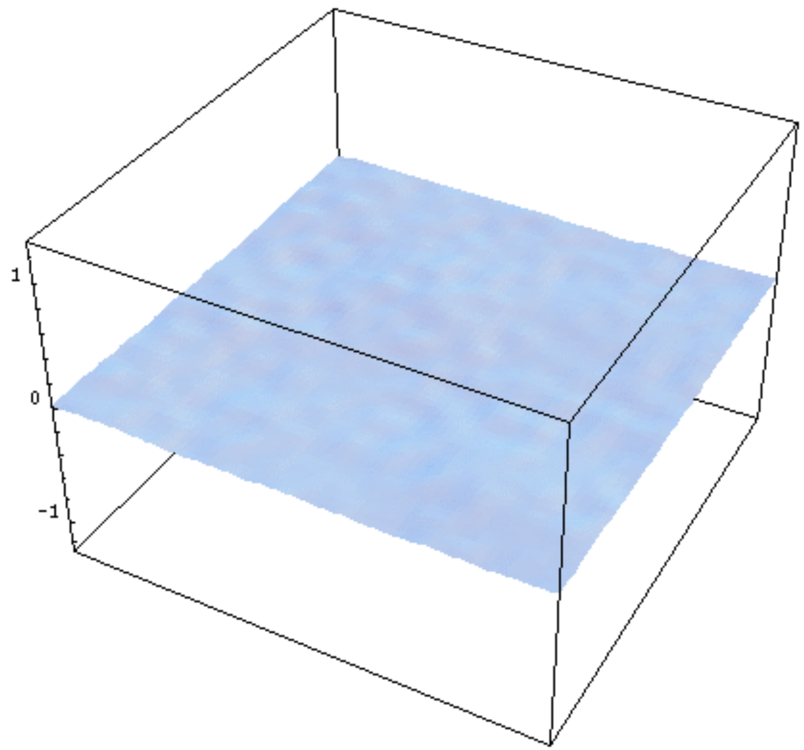
Quantum

$\phi_0 + \phi$

χ

Slices for t=95.2019

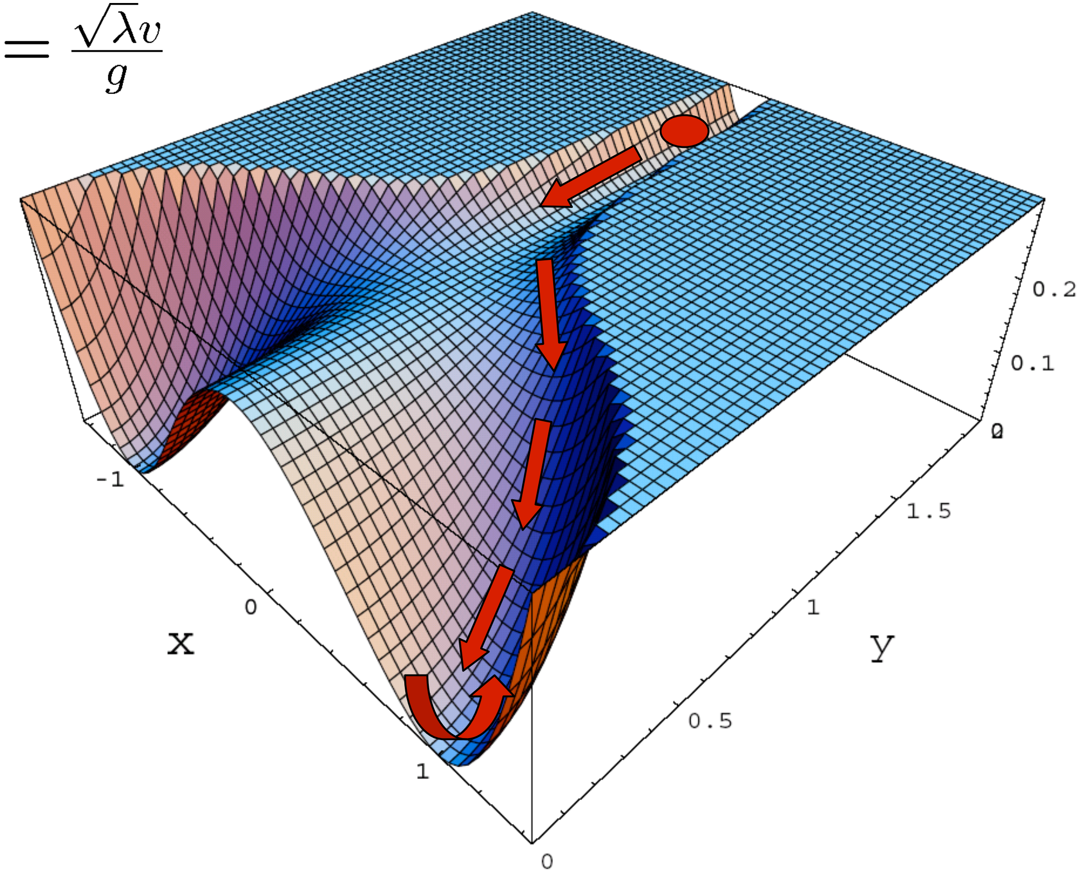




Tachyonic Preheating in Hybrid Inflation

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$$

bifurcation point $\phi_c = \frac{\sqrt{\lambda} v}{g}$



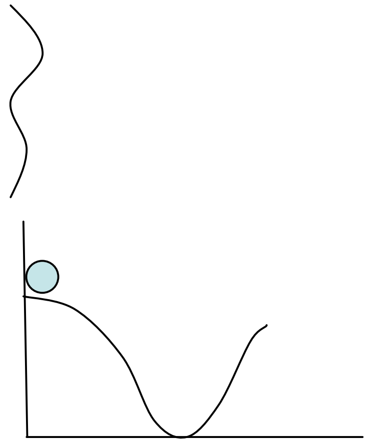
l

Felder, LK, Linde,01

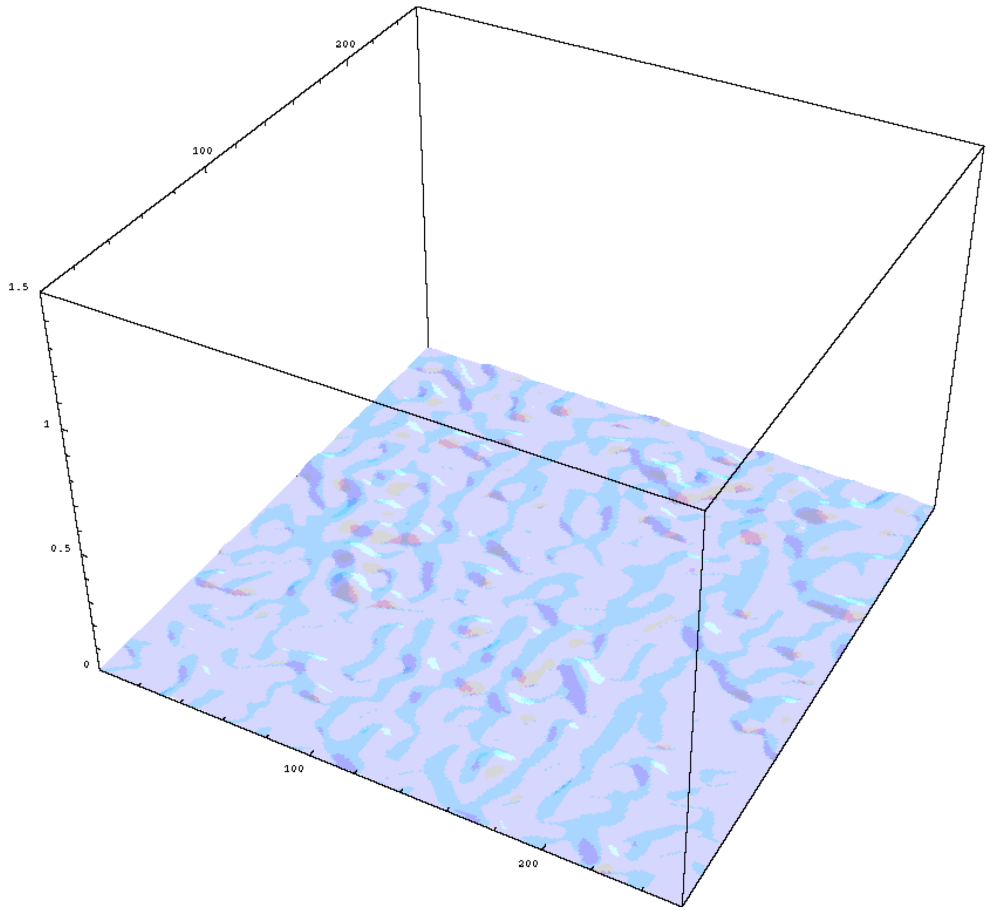
movie

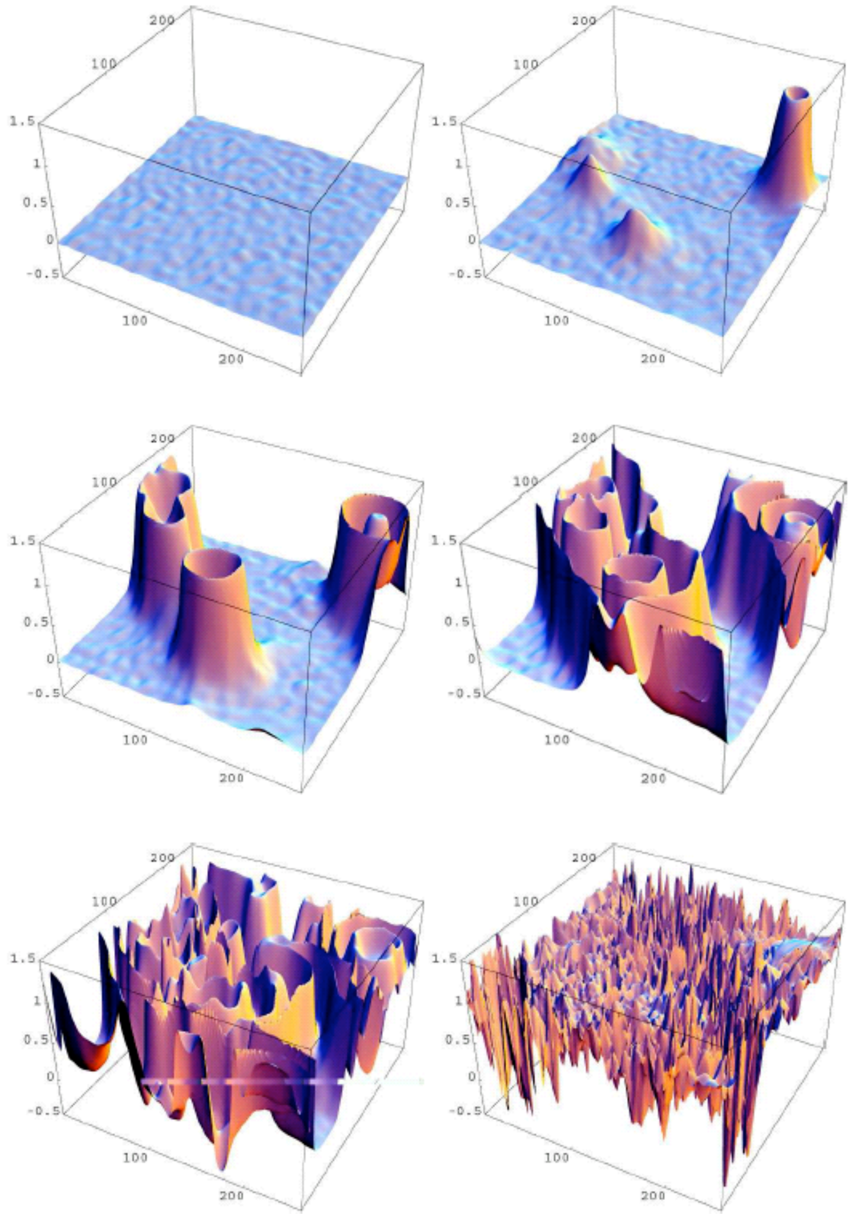
Tachyonic Preheating

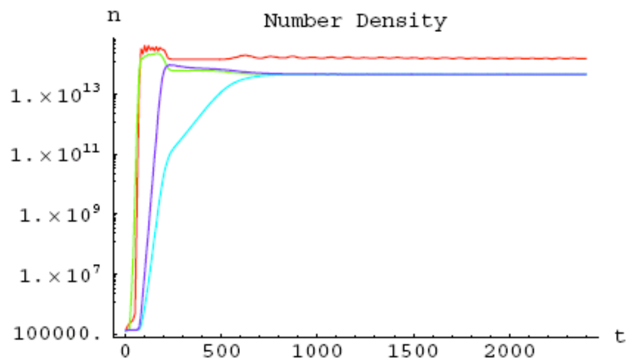
$$V_F = V_0 + \frac{\lambda}{4}\sigma^4 - \frac{\lambda^3}{4}\sigma^3 + \lambda\sigma^2$$



$\sigma(t, \vec{x})$







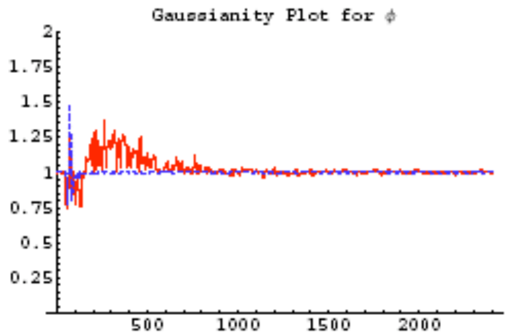


FIG. 10. Deviations from Gaussianity for the field ϕ as a function of time. The solid, red line shows $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$ and the dashed, blue line shows $3\langle\delta\dot{\phi}^2\rangle^2/\langle\delta\dot{\phi}^4\rangle$.

Generation of gravitational waves

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2/a^2 h_{ij} = 16\pi G \Pi_{ij}$$

Probing preheating with stochastic background gravitational radiation

$$\Omega_{gw} h^2 = \Omega_r h^2 \frac{d\rho_{gw}(a_e)}{d\ln\omega} \left(\frac{g_0}{g_*}\right)^{1/3}$$

estimation $$\frac{\rho_{gw}}{\rho_r} \sim (RH)^2$$

size of structures R vs Hubble radius $1/H$

$$f \sim \frac{M}{10^{15} Gev} 10^8 \text{ Hz}$$

numerics

$$\frac{dE}{d\Omega} = 2G\Lambda_{ij,lm}\omega^2 T^{ij*}(\vec{\mathbf{k}},\omega)T^{lm}(\vec{\mathbf{k}},\omega)d\omega$$

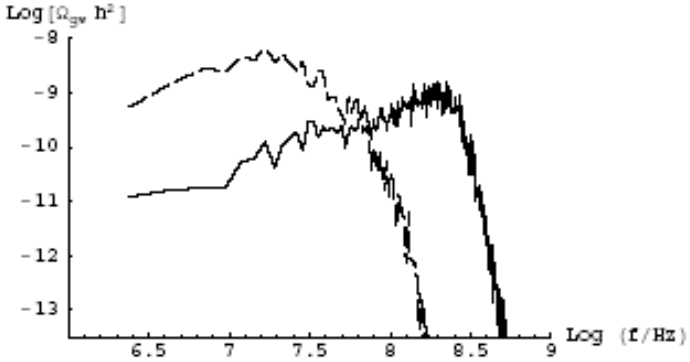
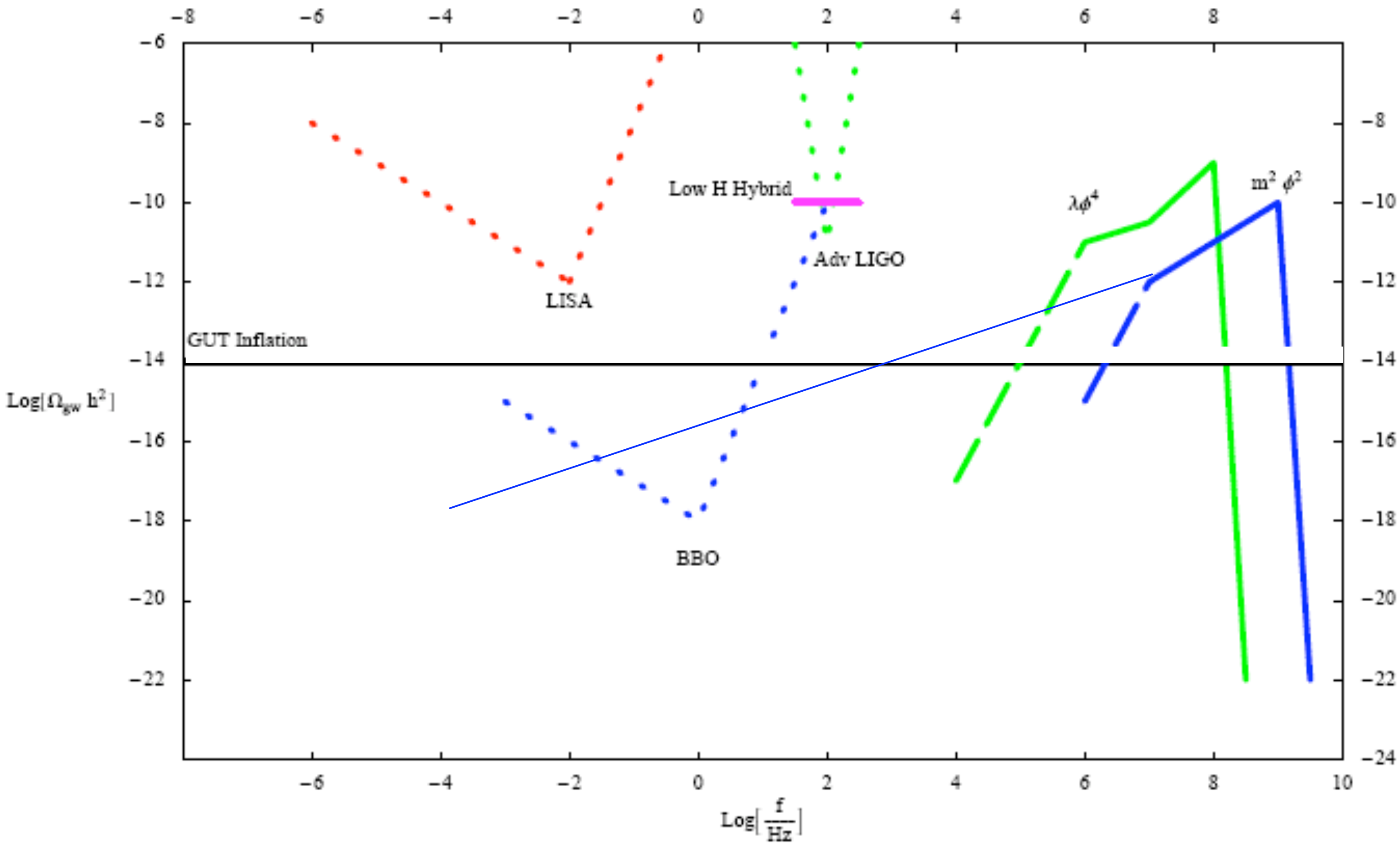


FIG. 1: The gravitational spectrum for the $\lambda\phi^4$ model with $\lambda = 10^{-14}$ and $g^2/\lambda = 1.2$ (dash line) and 120 (full line) respectively . As expected, it is peaked around $10^7 \sim 10^8$ Hz and spans about 2 decades. The horizon size at the time of preheating imposes the low frequency cut-off, while the high frequency cut-off is due to the fact that high momentum χ particles are energetically too expensive to be created. Notice that the power is roughly inversely proportional to g^2.



Low H Hybrid
Adv LIGO
LISA
GUT Inflation
BBO
$\lambda\phi^4$
$m^2\phi^2$
$\text{Log}[\Omega_{gw}h^2]$
$\text{Log}[\frac{f}{Hz}]$

Reheating after String Theory Inflation

Barnaby, Burgess, Cline, hep-th/0412095

LK, Yi, hep-th/0507257

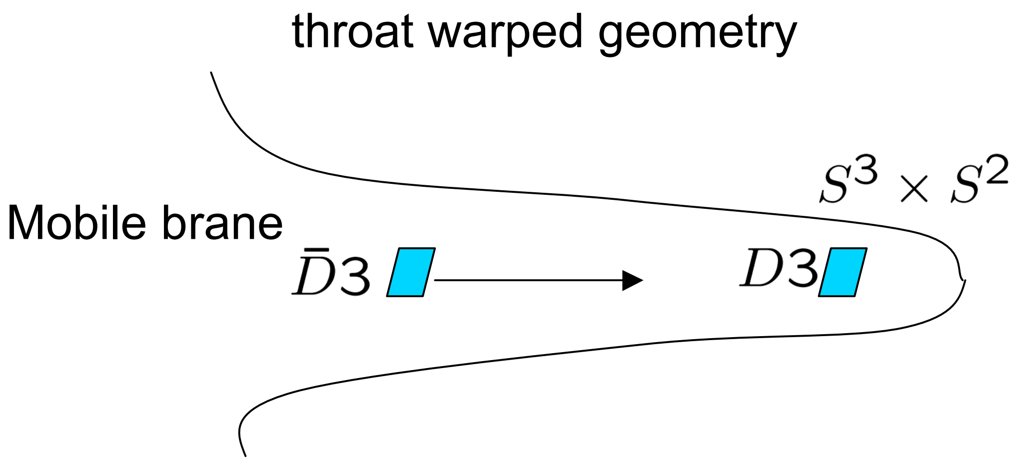
Frey, Mazumdar, Myers, hep-th/0508139

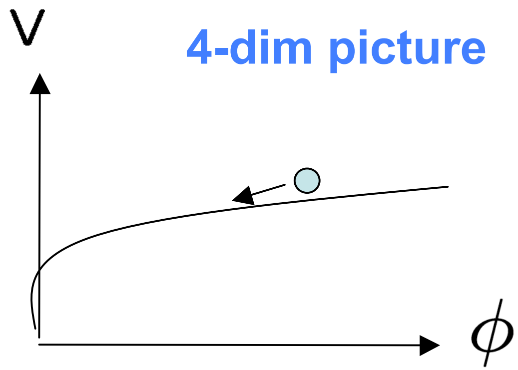
Chialva, Shiu, Underwood, hep-th/0508229

Chen, Tye, hep-th/05120000

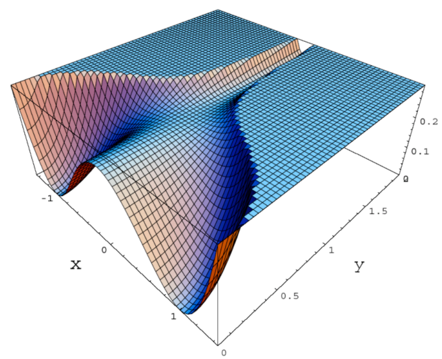
Realization of String Theory Hybrid Inflation

Warped brane inflation



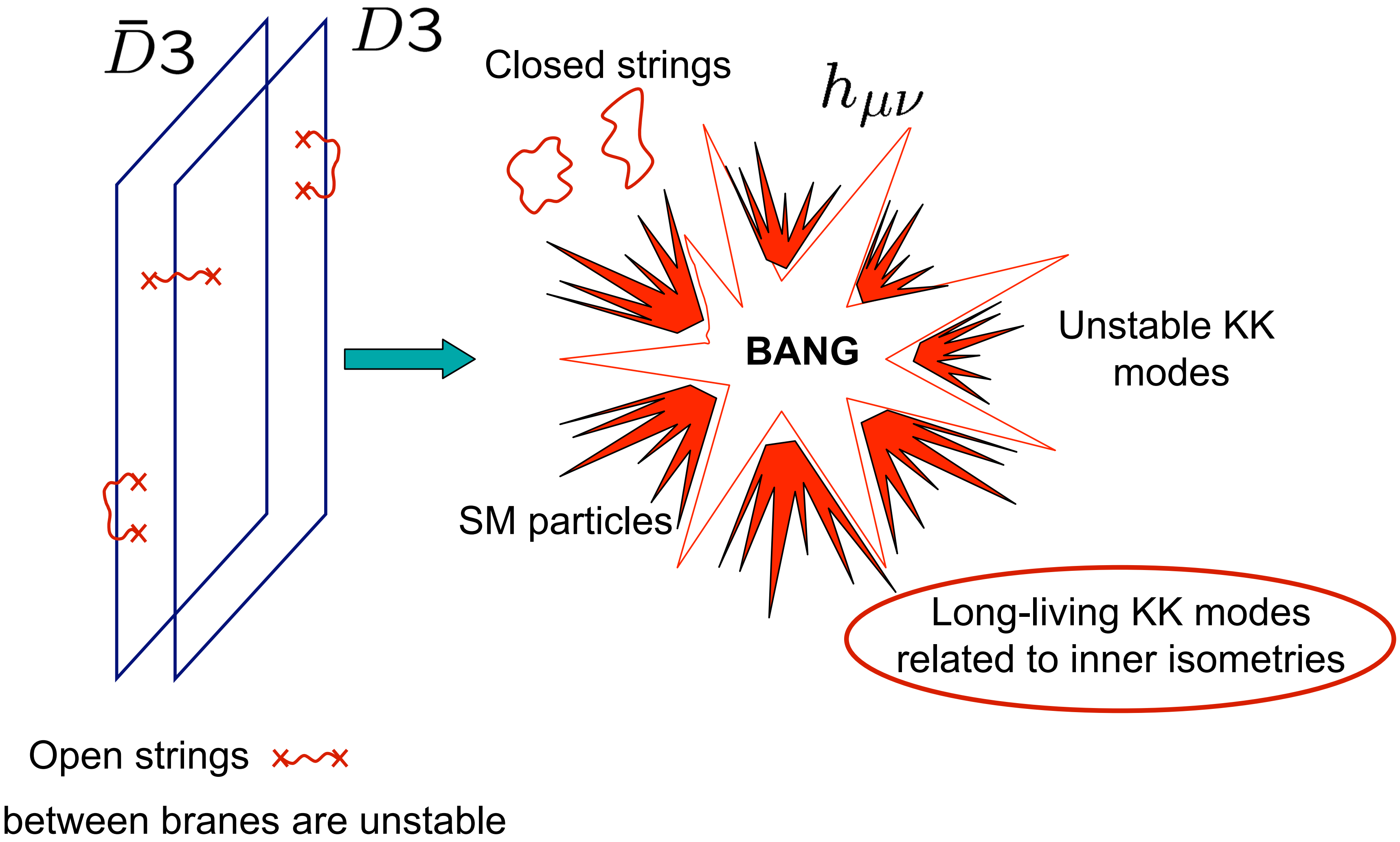


Prototype of
hybrid inflation



End point of inflation

LK, Yi 05



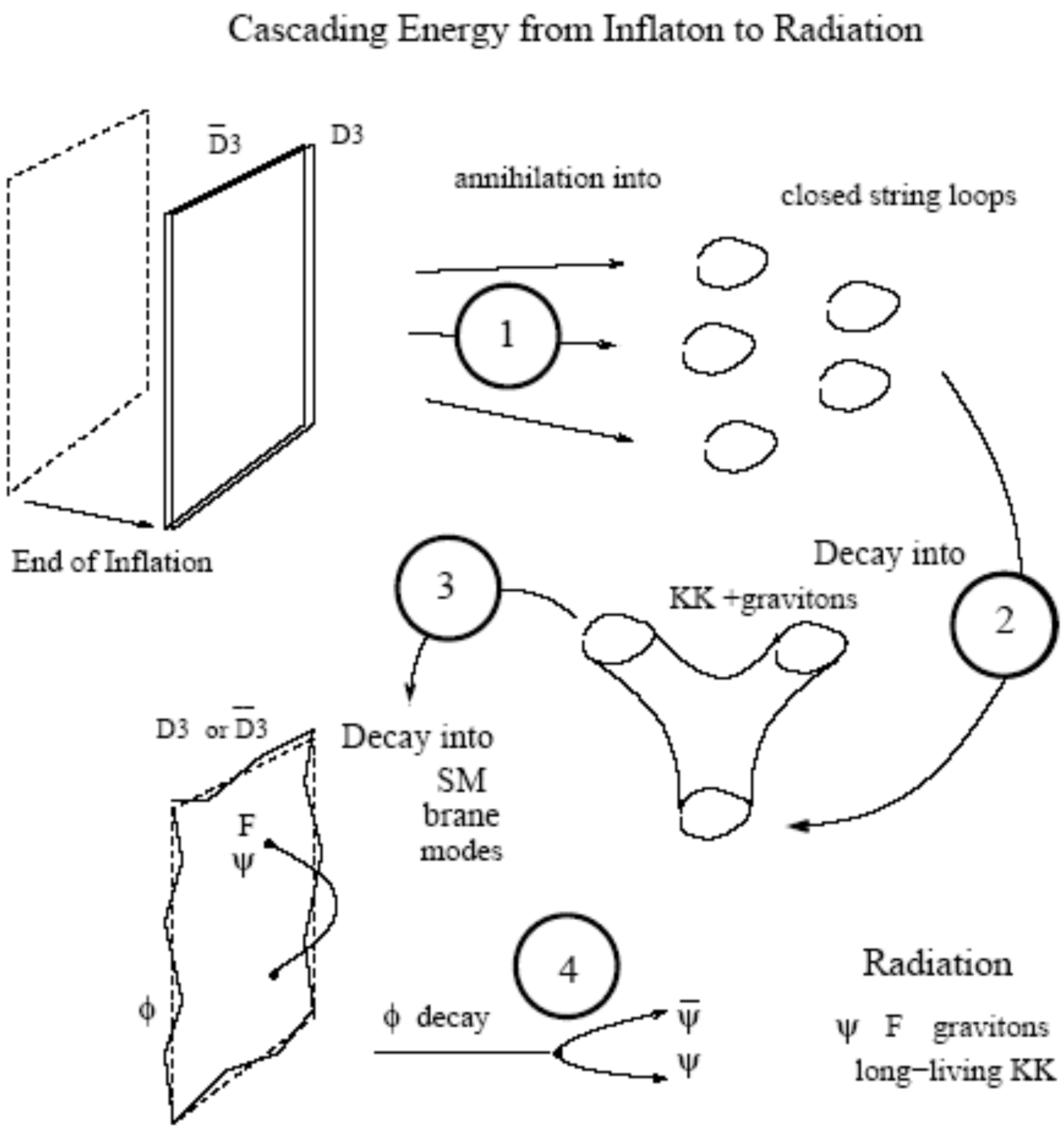
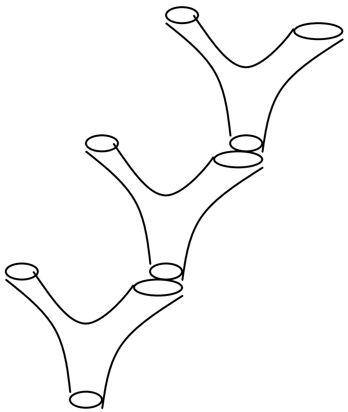


Figure 2: Identifying the channels of D-brane decay

KK story



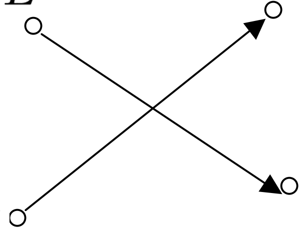
$$R^4 \times \mathcal{M}$$

$$h_{AB}(x,y) = \sum_m h^{(m)}(x) f_m(y)$$

$m = 0$: usual 4 dim gravitons $\Omega_{GW} \simeq e^{-2A}$

other m: modes $m_{KK} \simeq e^{-A}/R$

KK particles are thermalized first
SM particles are thermalized much later



KK from M with isometries
are stable

No complete decya

KK particles freeze out $\Omega_{KK} \gg 1$

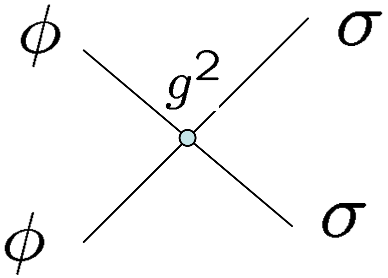
resolution?

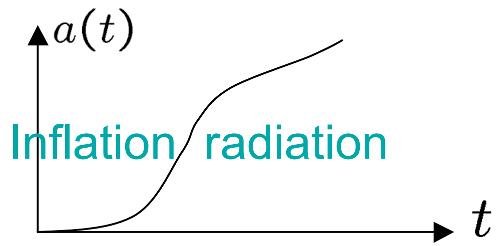
- Attachment of KS throat to a compact CY
Induces symmetry breaking perturbations.

- Tip of KS throat is a particular case of Sasaki-Einstein manifolds.
There are asymmetric SE manifolds, but no examples of asymmetric throats

Modulated Fluctuations

$g^2\phi^2\sigma^2$

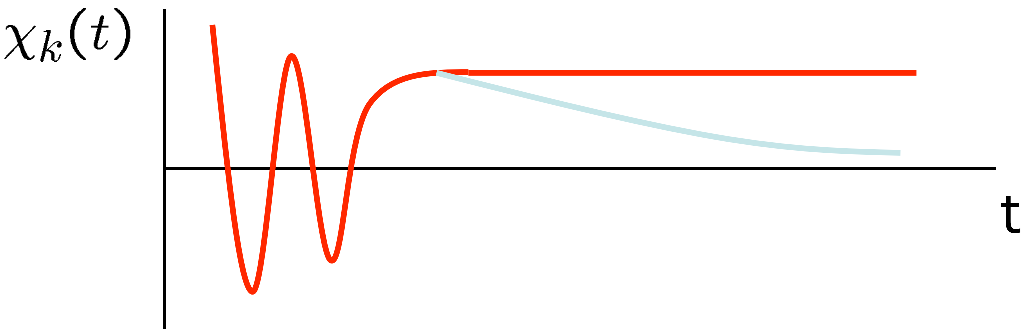




coupling depends on moduli $g^2 = g^2(\chi)$

Light field at inflation

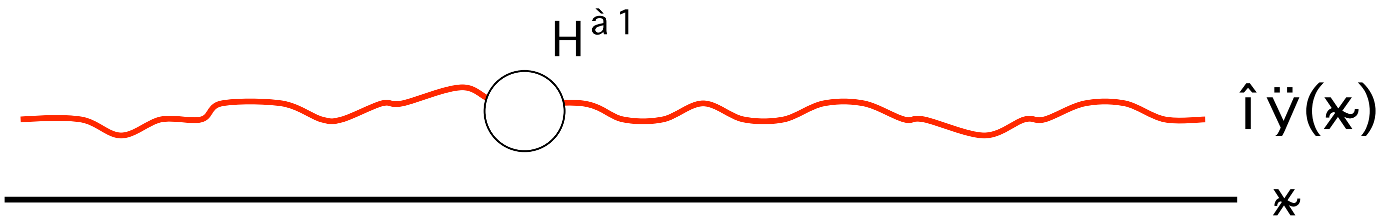
$$\delta\chi = \int d^3k\,(a_k \chi_k(t) e^{ikx} + h.c.)$$



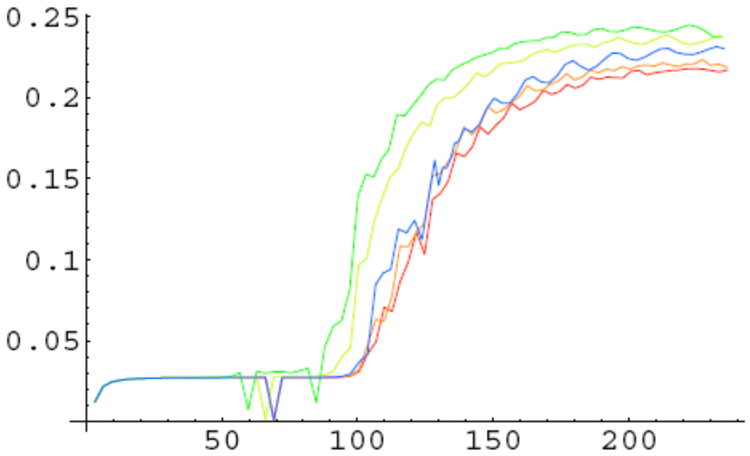
$$\chi_k \simeq \frac{H}{\sqrt{2}k^{3/2}}$$

LK03;Dvali et al,03

spacial variations $\delta g^2 = \frac{\partial g^2}{\partial \chi}\delta\chi$



Modulated fluctuations in Chaotic Inflation



equation of state $\frac{p}{\epsilon}$

varying $g^2 = 10^{-7}$ by 5%

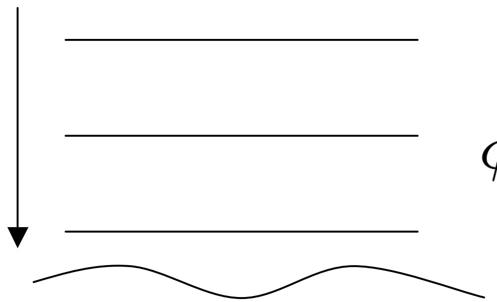
Generation of metric fluctuations

$\delta\chi_k \rightarrow \delta g^2 \rightarrow \Phi_k$

Modulated fluctuations in Hybrid Inflation

$$V(\phi,\sigma)=\frac{\lambda}{4}(\sigma^2-v^2)^2+\frac{g^2}{2}\phi^2\sigma^2$$

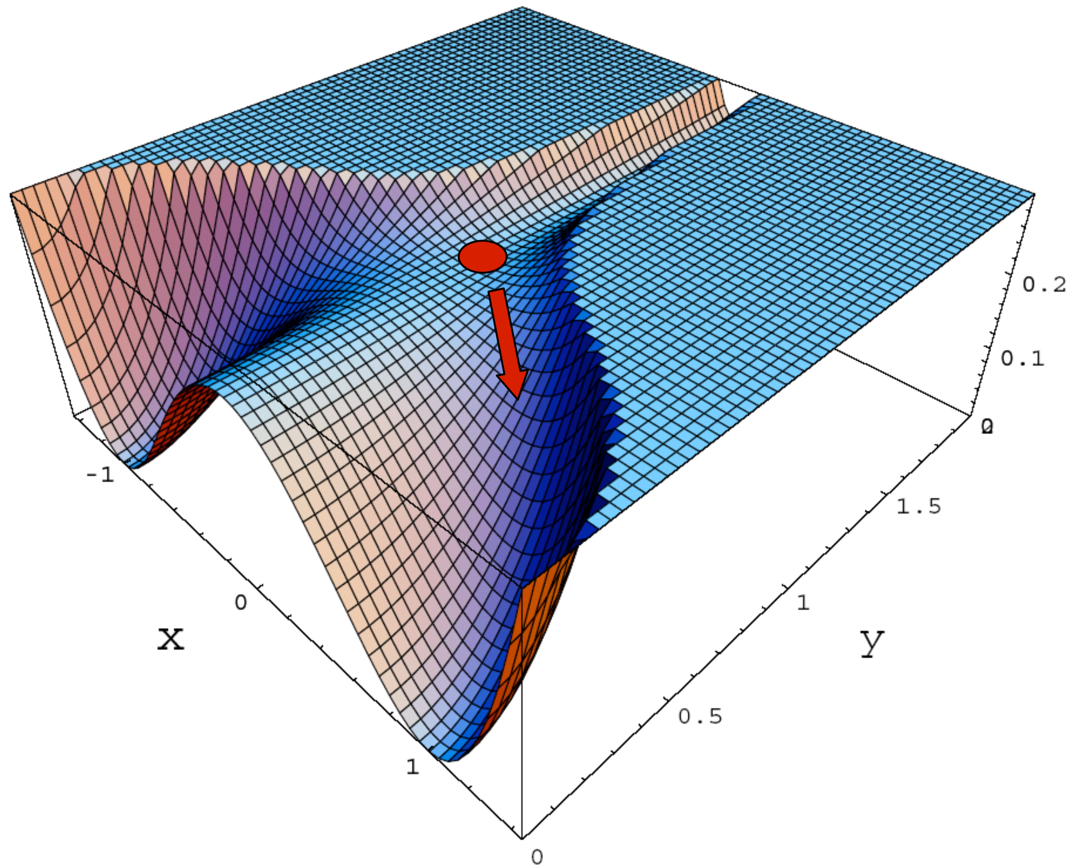
bifurcation point $\phi_c=\frac{\sqrt{\lambda}v}{g}$



$\phi(t)$

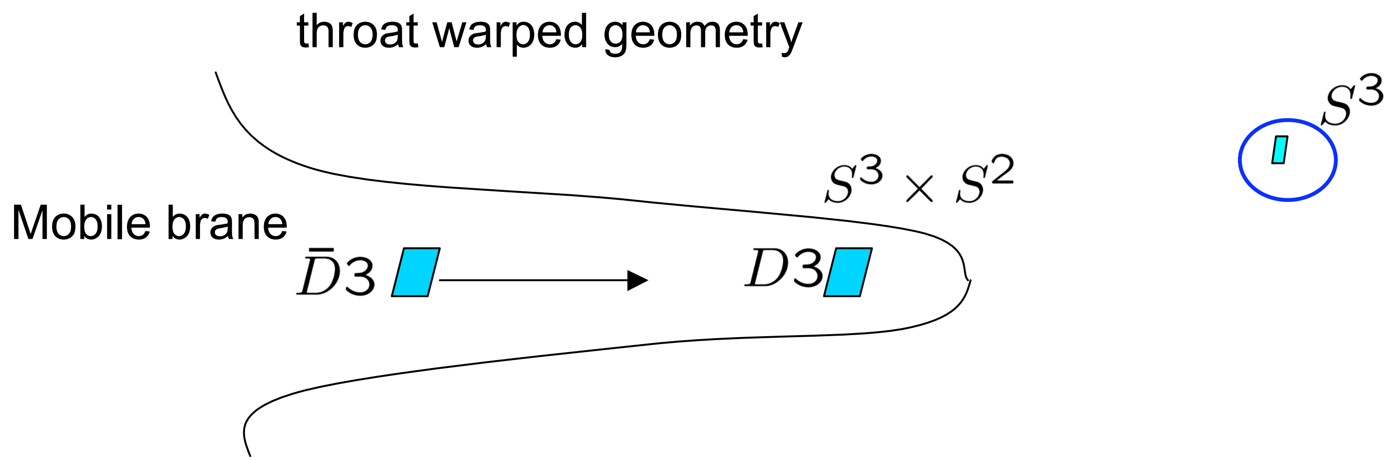
$\phi_c(\chi_0+\delta\chi)$

inhomogeneous waterfall



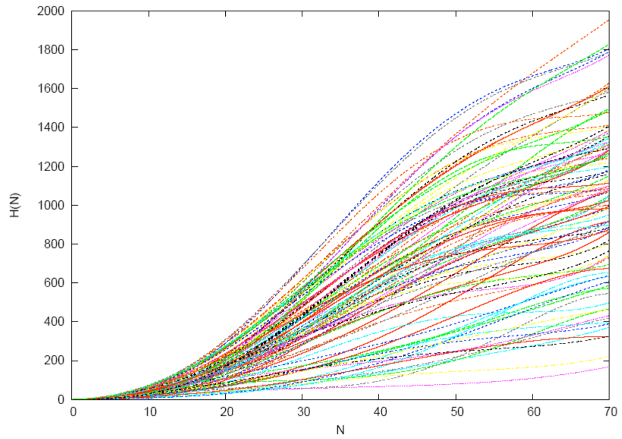
modulated fluctuations

scalar field associated with angular position at S^3



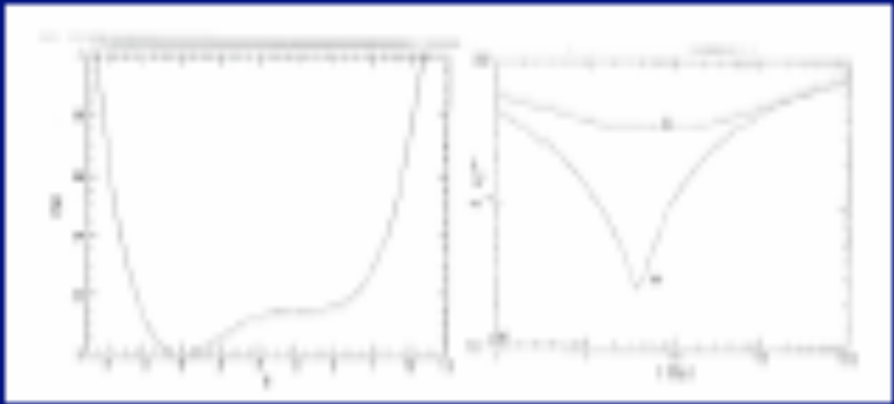
Ensemble of Inflationary trajectories

String theory lanscape



$$V(\phi_1, ..., \phi_{1000}, ...) \quad 10^{500} \text{ vacua}$$

$$V = m^2\phi^2 + \lambda_1\phi^3 + \lambda_2\phi^4$$

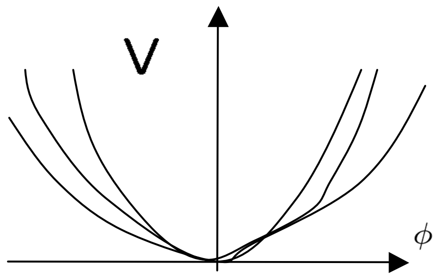


- renormalizable
- different choices of parameters give quite d powerspectra
- but also different shape potential:
  - exponential
  - SUGRA
  - ...

Chaotic Inflation with many scalars

VEV of scalar fields do not exceed Mp

Many scalar fields



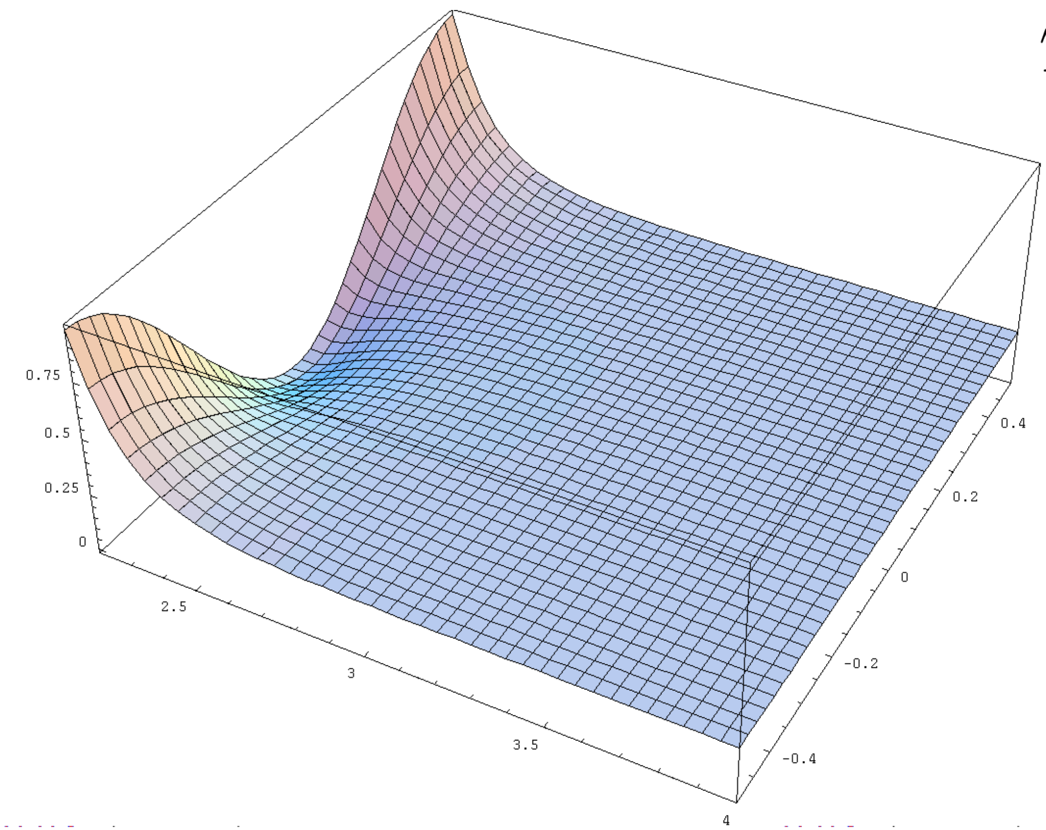
Assisted Inflation

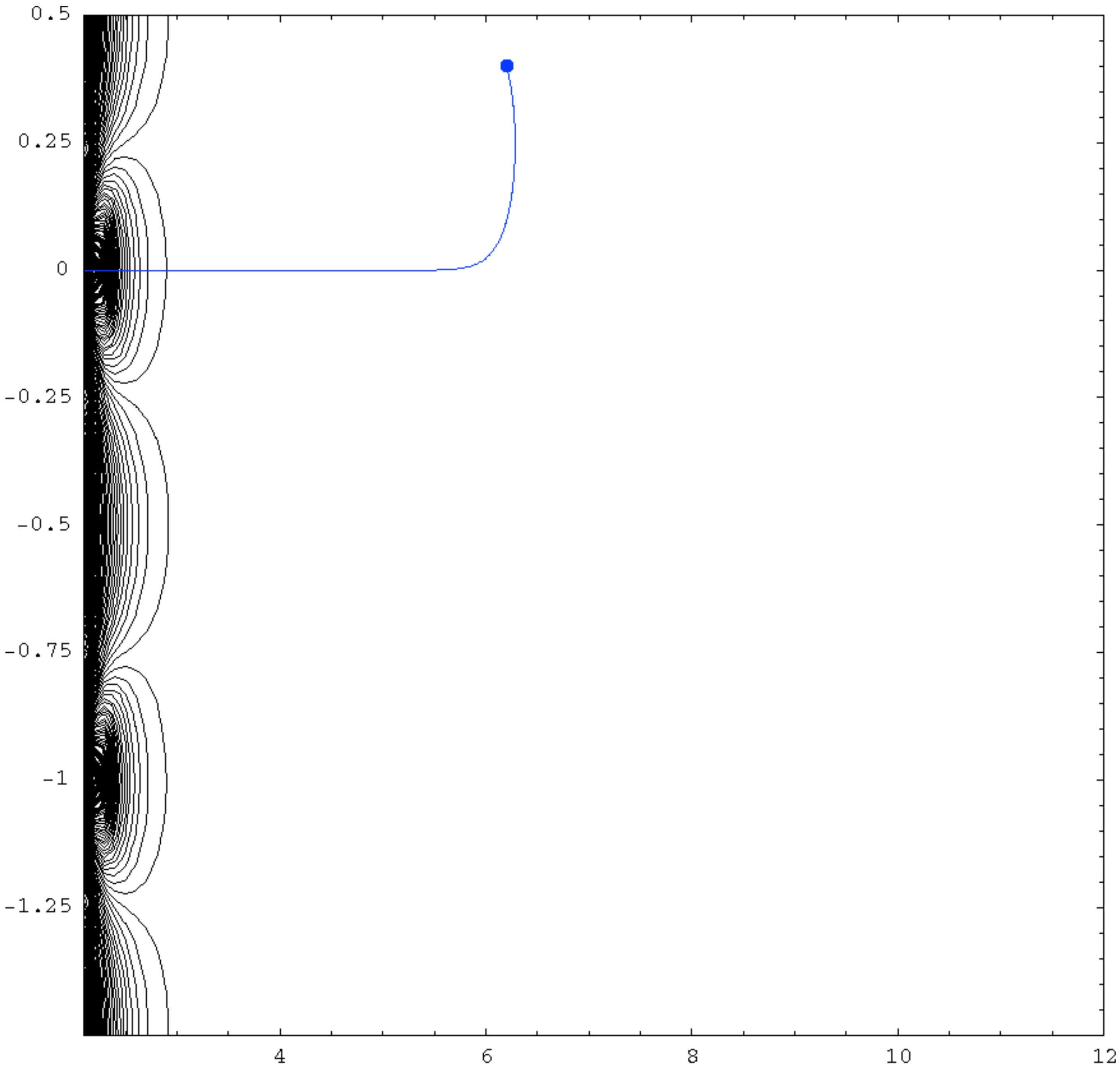
$$H^2 = \frac{8\pi}{M_p^2}\left(\frac{1}{2}\sum \dot{\phi}_j^2 + \frac{1}{2}\sum m_j^2 \phi_j^2\right)$$
$$\ddot{\phi}_j + 3H\dot{\phi} + m_j \phi_j = 0$$

Corresponds to a single inflaton field (collective coordinate)
With complicated effectice potential

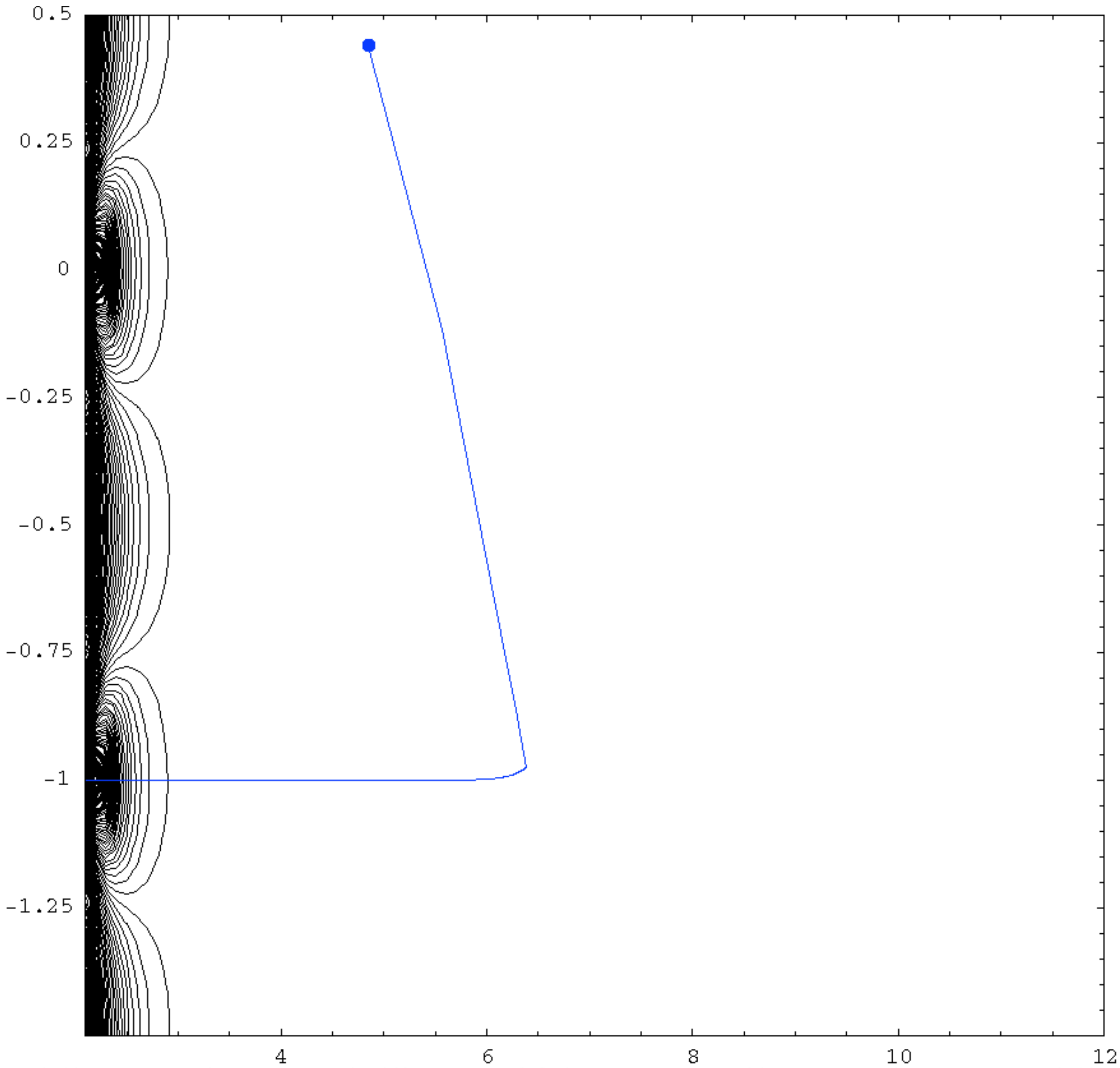
Kahler/axion moduli Inflation
Conton & Quevedo hep-th/0509012

$$W = W_0 + Ae^{-aT}$$
$$K = -2\log(V + \xi/2)$$
$$T = \tau + ib$$





0.5
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
4
6
8
10
12



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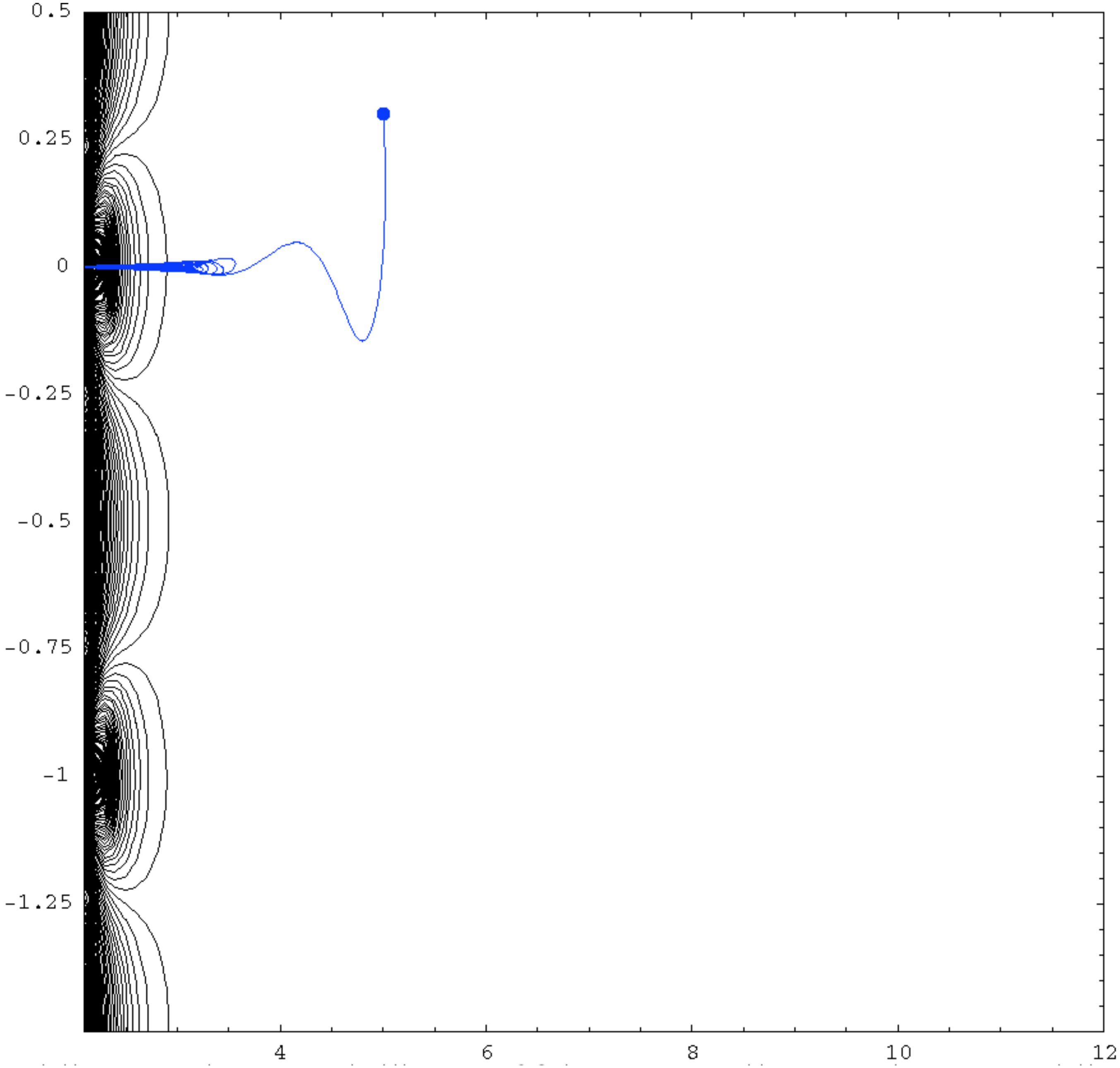

0.5
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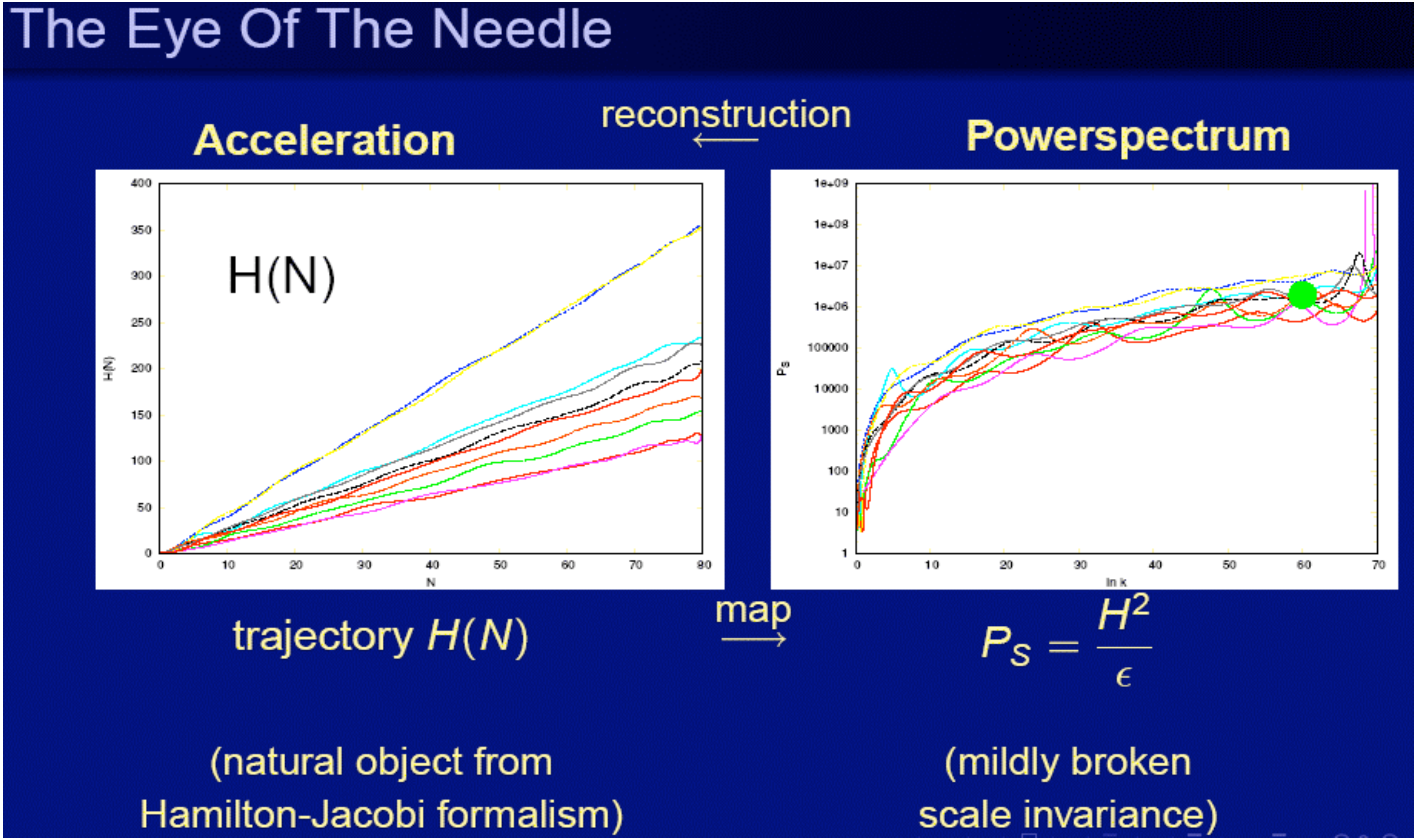
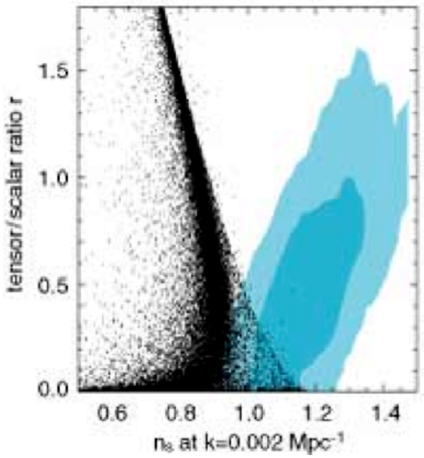


Scanning inflation Bond, LK, Contaldi, Vandrevange 06

Bottom-up

Scanning Inflation



$P_s(k) = A_s k^{n_s - 1}$, $r = P_{GW}/P_s$

RG flow method

Small slow roll parameters

$\epsilon = \frac{M_p^2}{4\pi}\frac{H'^2}{H^2}$, $\eta = \frac{M_p^2}{4\pi}\frac{H''}{H^2}$, $\xi \sim H'''$ etc.

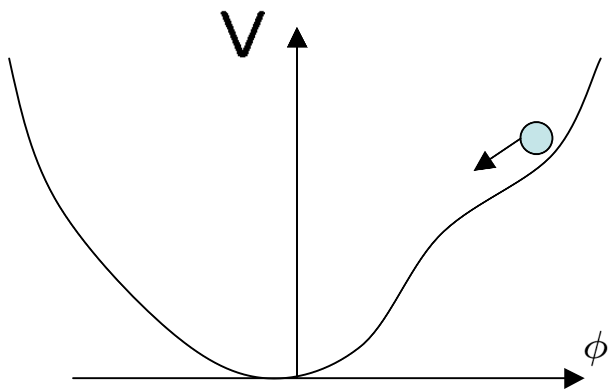
Flow eqs.

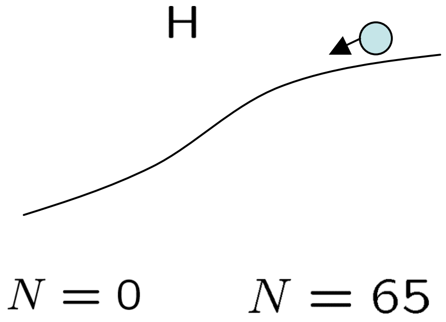
$\frac{d\epsilon}{dN} = \epsilon(\sigma + 2\epsilon)$

$\frac{d\sigma}{dN} = -5\sigma\epsilon - 12\epsilon^2 + 2\xi$

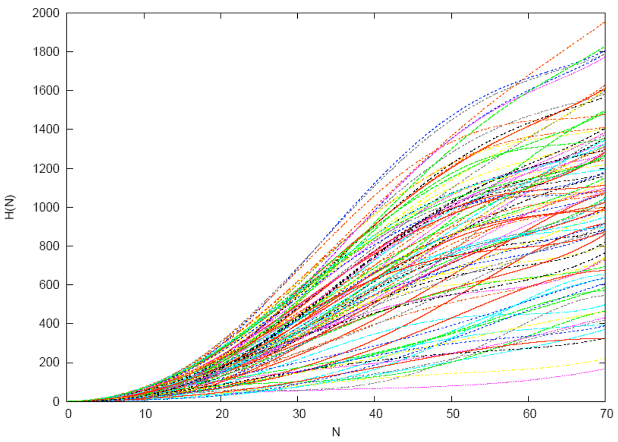
$\frac{d\xi}{dN} \sim$ cubic in $\epsilon, \sigma, \xi + \lambda$

etc





Ensemble of Inflationary trajectories



Chebyshev decomposition

$$H(x) = \sum c_n T_n(x) \ , \ x = \frac{2N - N_{\max}}{N_{\max}}$$

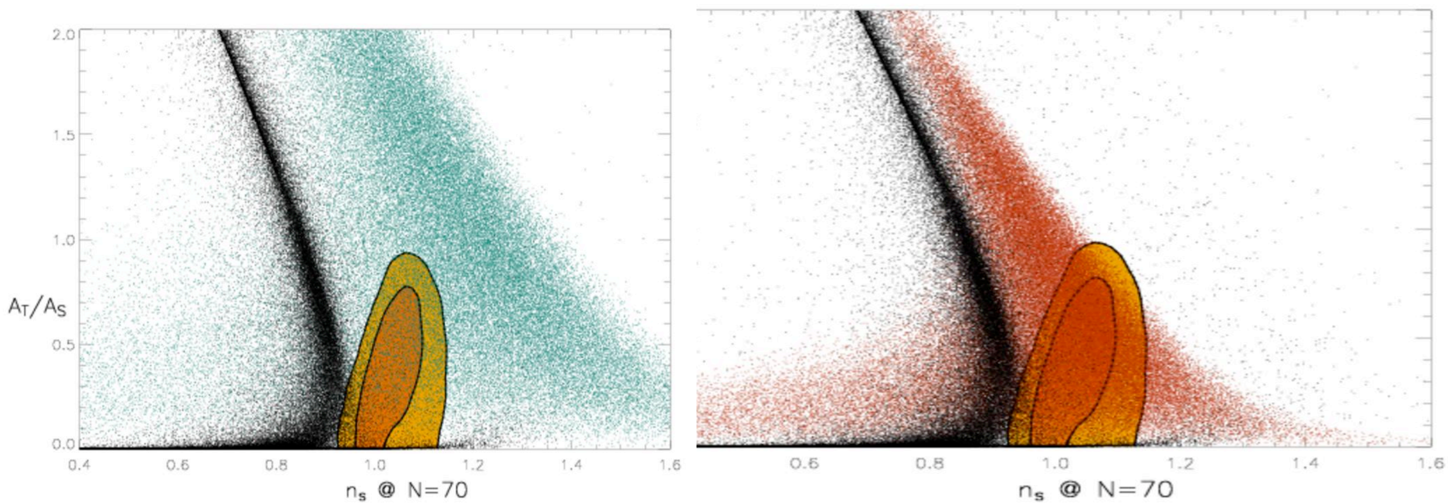
$$||H(x) - \sum^{M} c_n T_n(x)|| = min \ .$$

$0 < \frac{dH}{dN} < H$ $\frac{dH}{dN} = H$ at $N = 65$

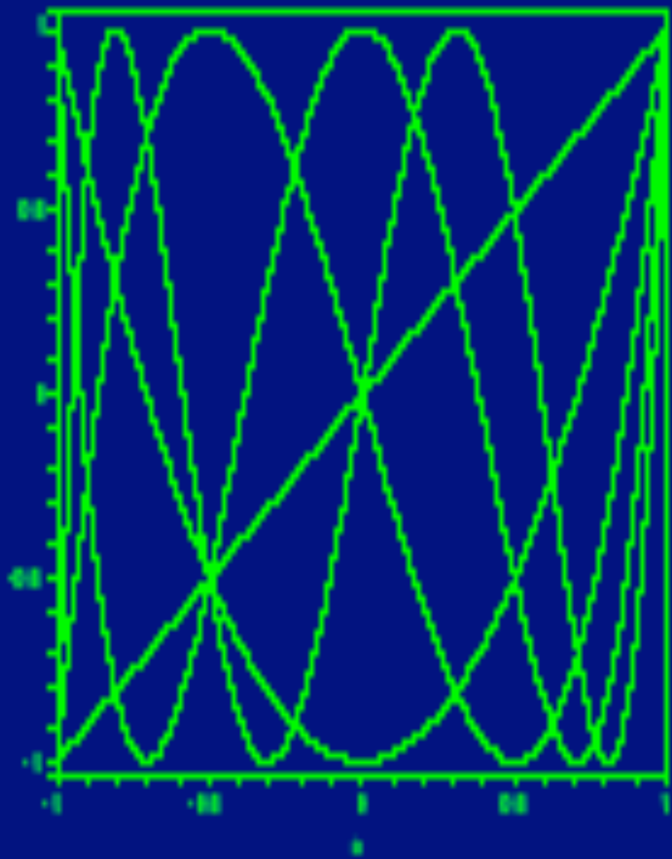
Related methods of trajectory generation

speed 10^5 up vs RGF

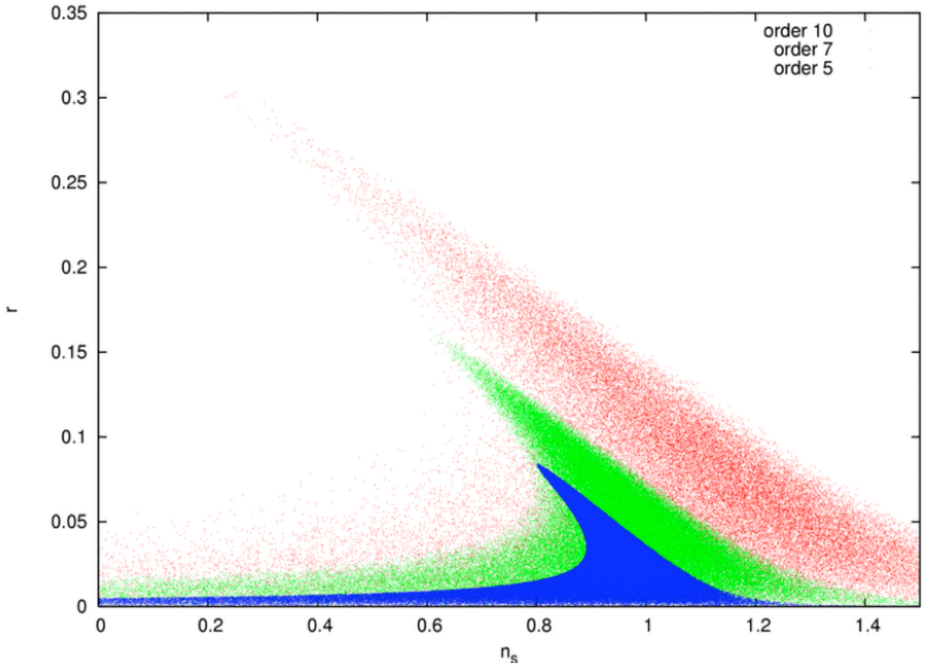
Space of models opens wide



- N: # of efolds $dN = -Hdt$
- Constraints during inflation
  - $0 \leq \epsilon \leq 1$
  - $H > 0$
- at the end of inflation $\epsilon = 1$
- Expansion to arbitrary order
  - $H(N) = \sum_l c_l T_l(x)$
  - with $x = \frac{2N - N_{max}}{N_{max}}$, Chebyshev polynomials $T_l(x)$ (uniformly best approximation to "true" function)

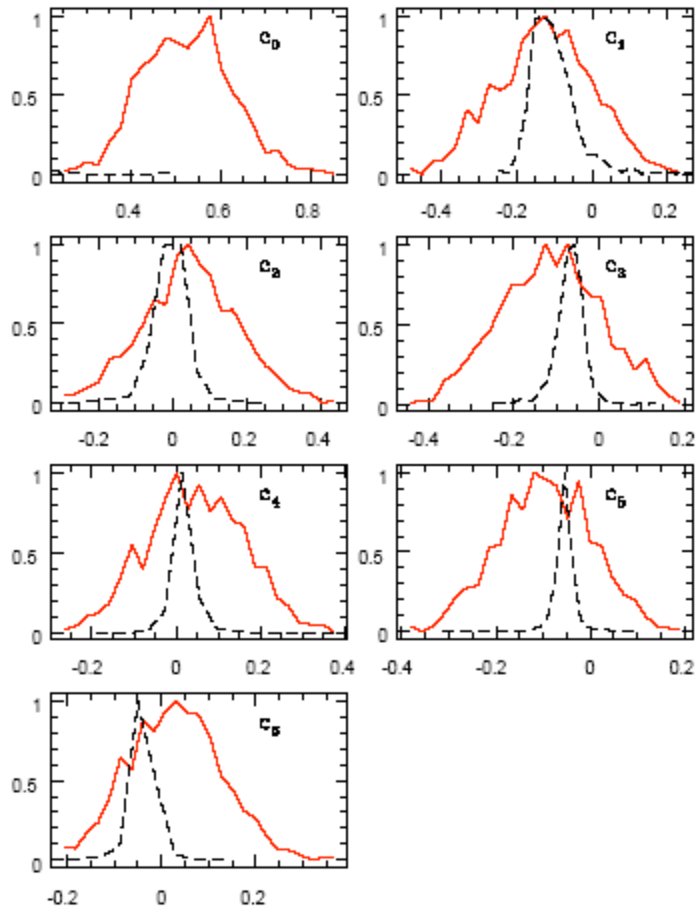


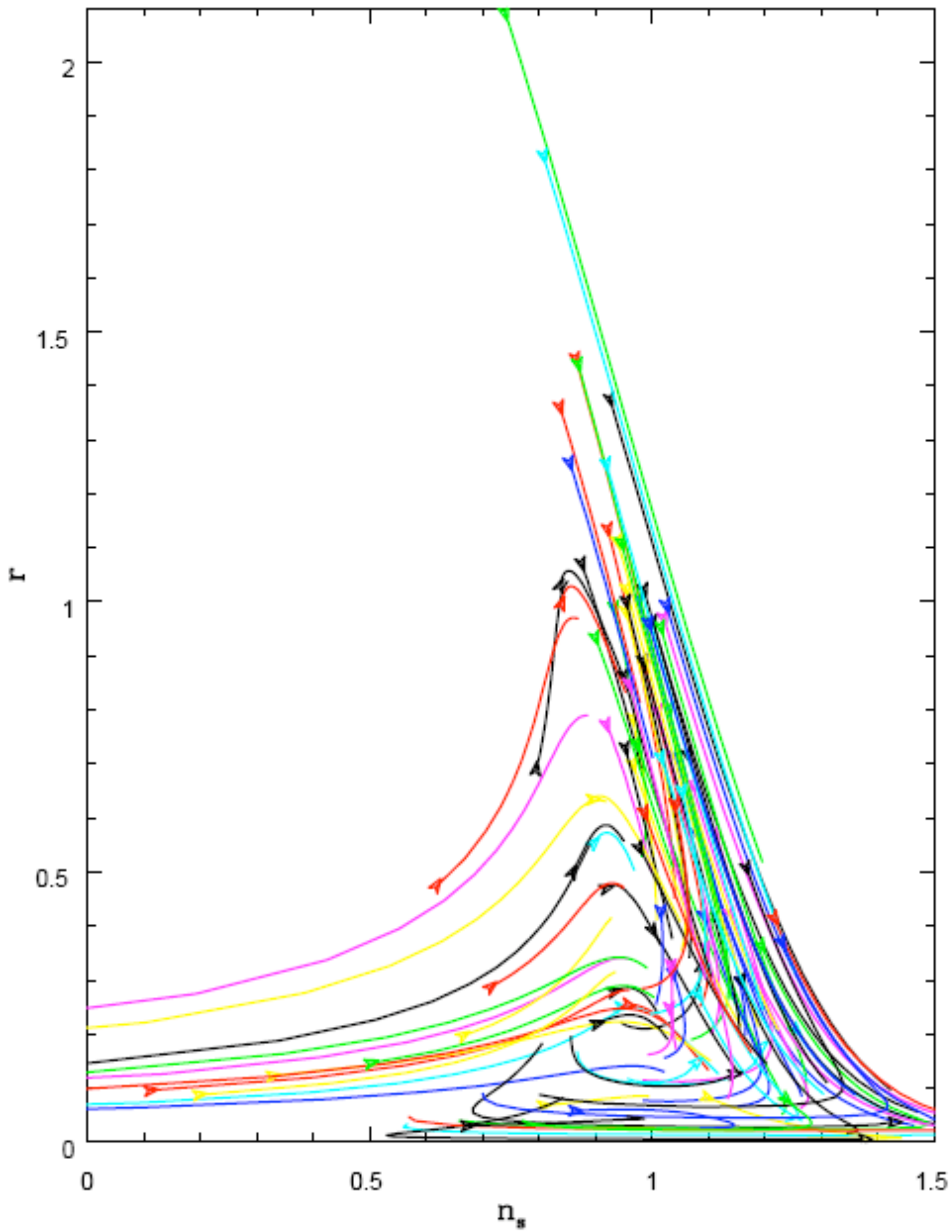
$T_n(\cos(x)) = \cos(nx)$



space opens more with higher order polynoms

comparison of c_n (red) in our method vs c_n (black) of Chebyshev transform of trajectories generated with RG flow



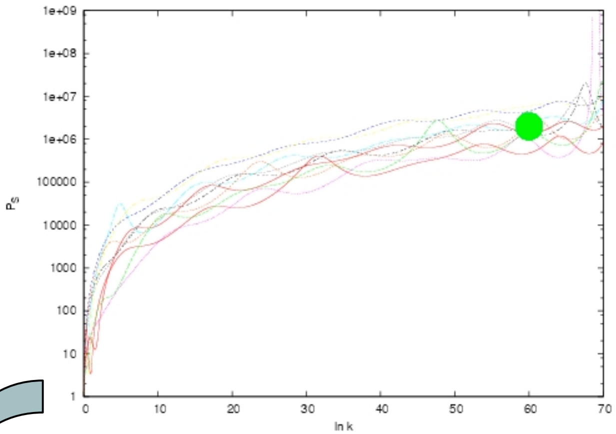


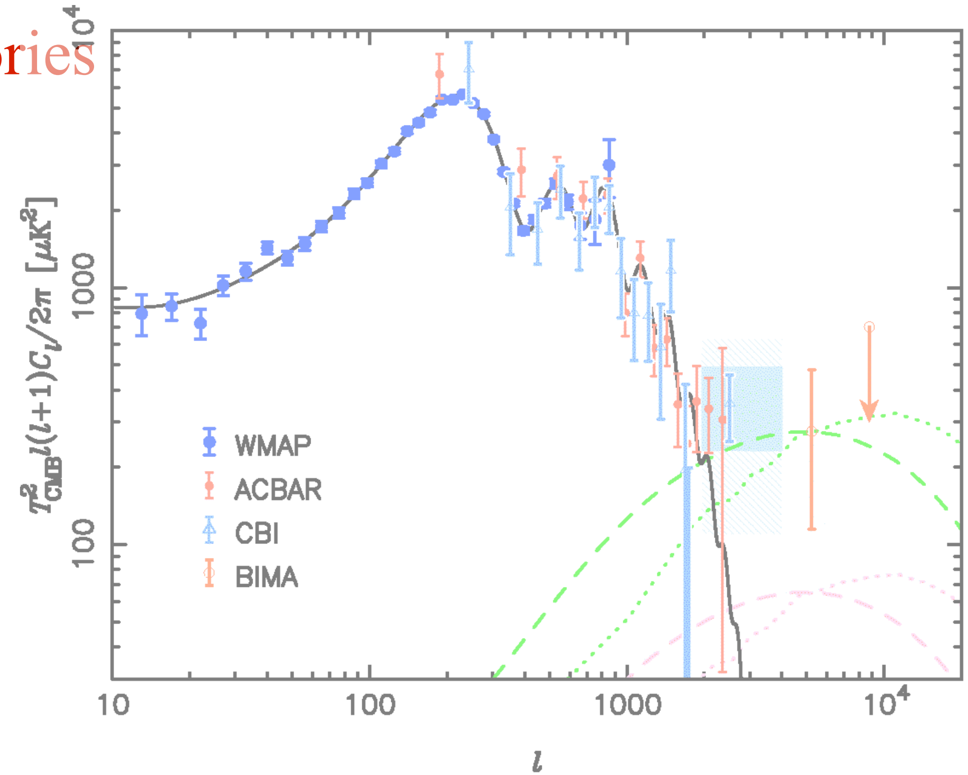
ï; ñ:::

$H(N) \longrightarrow P(k)$

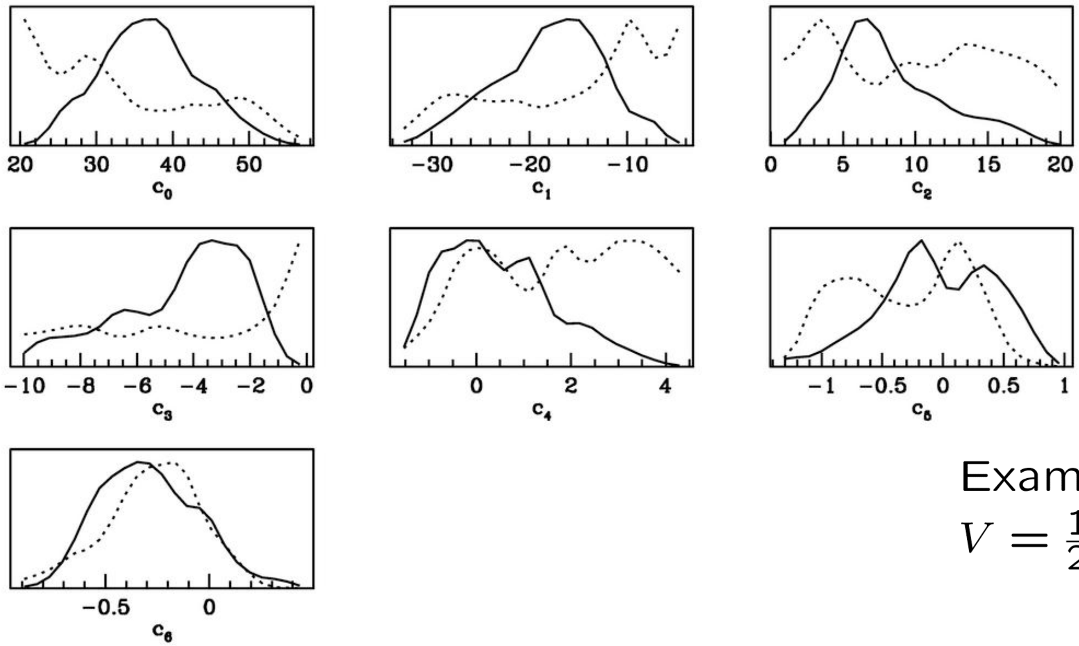
~~$n_s; n_t; r; dn/d\ln k; A_s; :::$~~

Observational constraints on trajectories

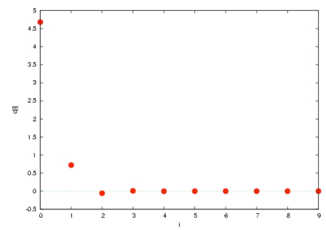




Markov Chain Monte Carlo



Example of c_n for $V = \frac{1}{2}m^2\phi^2$



$$f(x) = \sum^{j=n} c_j T_j(x)$$

Chebyshev Polynoms Nodal Points Method

$$f(x) = \sum f(x_j) \phi_j^{(n)}(x)$$

set of nodal points $x_j^{(n)}$

Variations

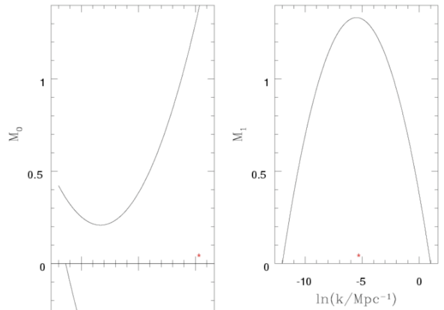
$$f(x) = H, \log H, \epsilon, \log \epsilon, P_s, P_t$$

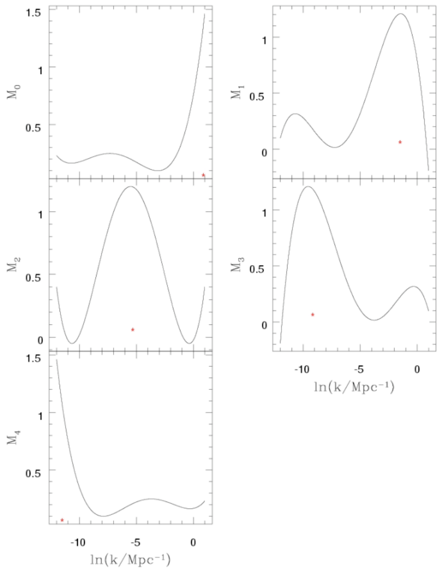
$$x = N, \log k$$

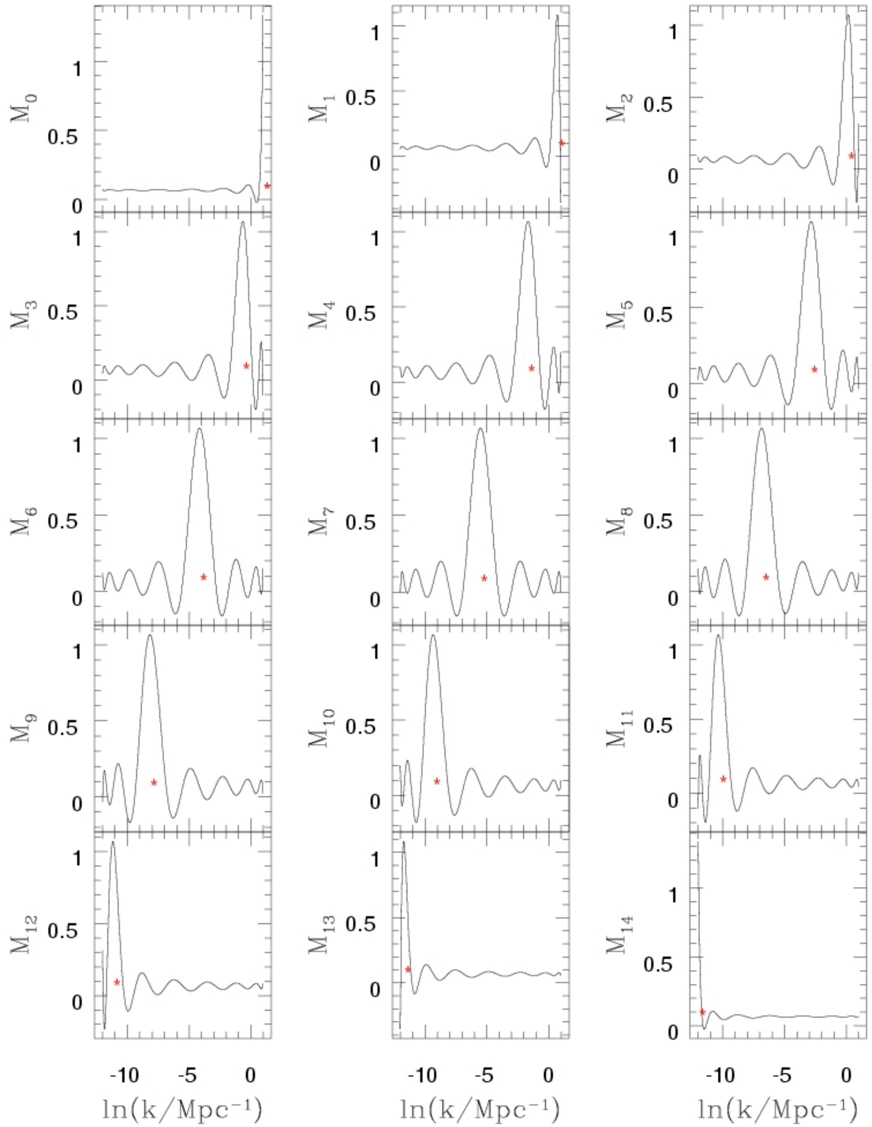
Trajectories cf.
WMAP1+BO3+CBI+DASI+VSA+Acbar+Maxima+SDSS+2dF

Chebyshev nodal modes: order 3, 5, 15 (Fourier at high order)
Chebyshev modes are liner conbinatin

How far does choice of values at nodal points “feel” out?







Displaying Trajectory constraints:

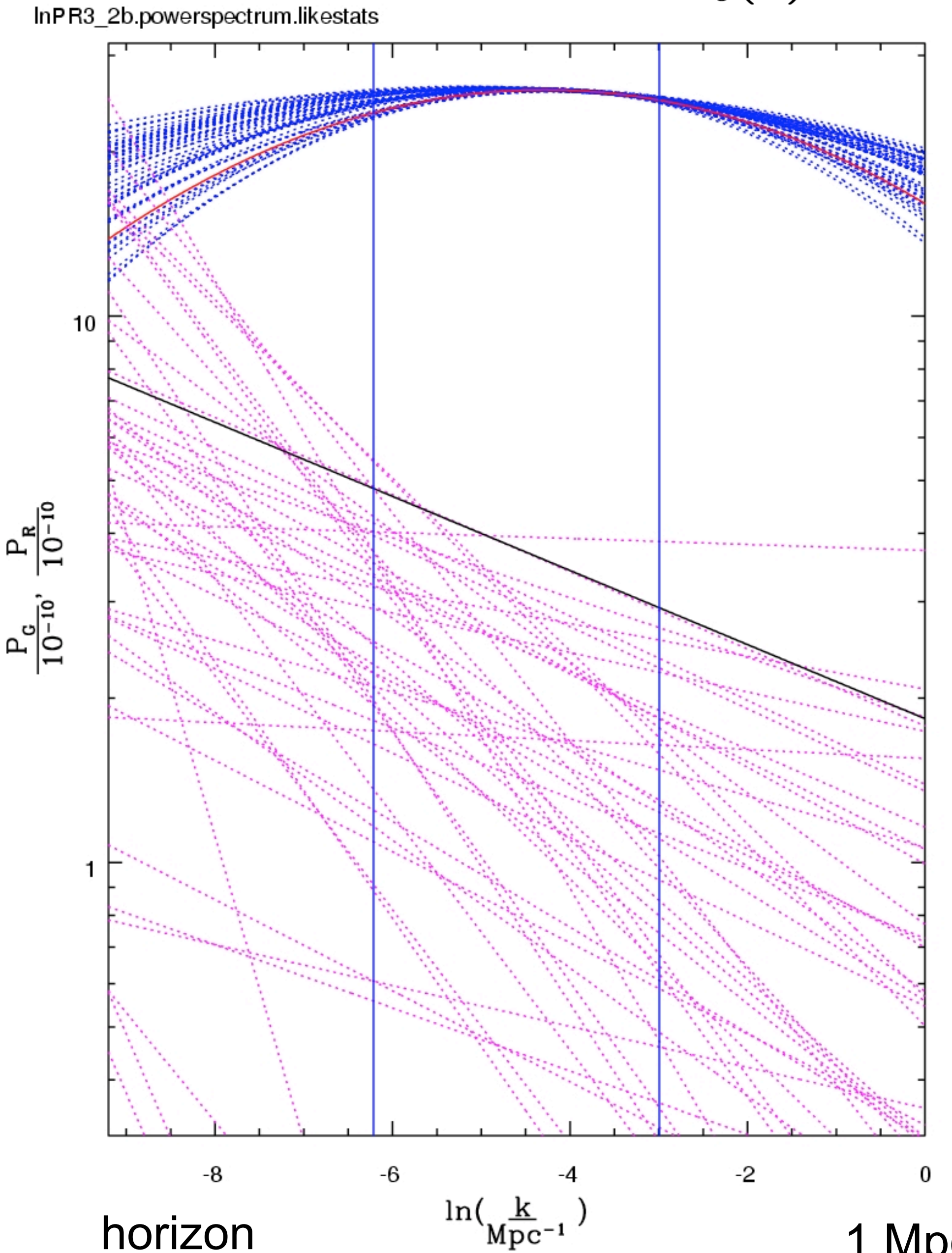
If Gaussian likelihood, compute χ^2 where 68% probability, and follow the ordered trajectories to

$$\ln L/L_m = -\chi^2/2,$$

displaying a uniformly sampled subset.

Errors at nodal points in trajectory coefficients can also be displayed.

$$P_s(k) = A \exp\left[(n_s - 1)\ln k/k_* + \frac{1}{2}\frac{dn}{d\ lnk}\ln^2 k/k_*\right]$$



horizon

1 Mpc

Independent expansion
(not assuming consistency)

Chebyshev of ln P_s and ln P_t

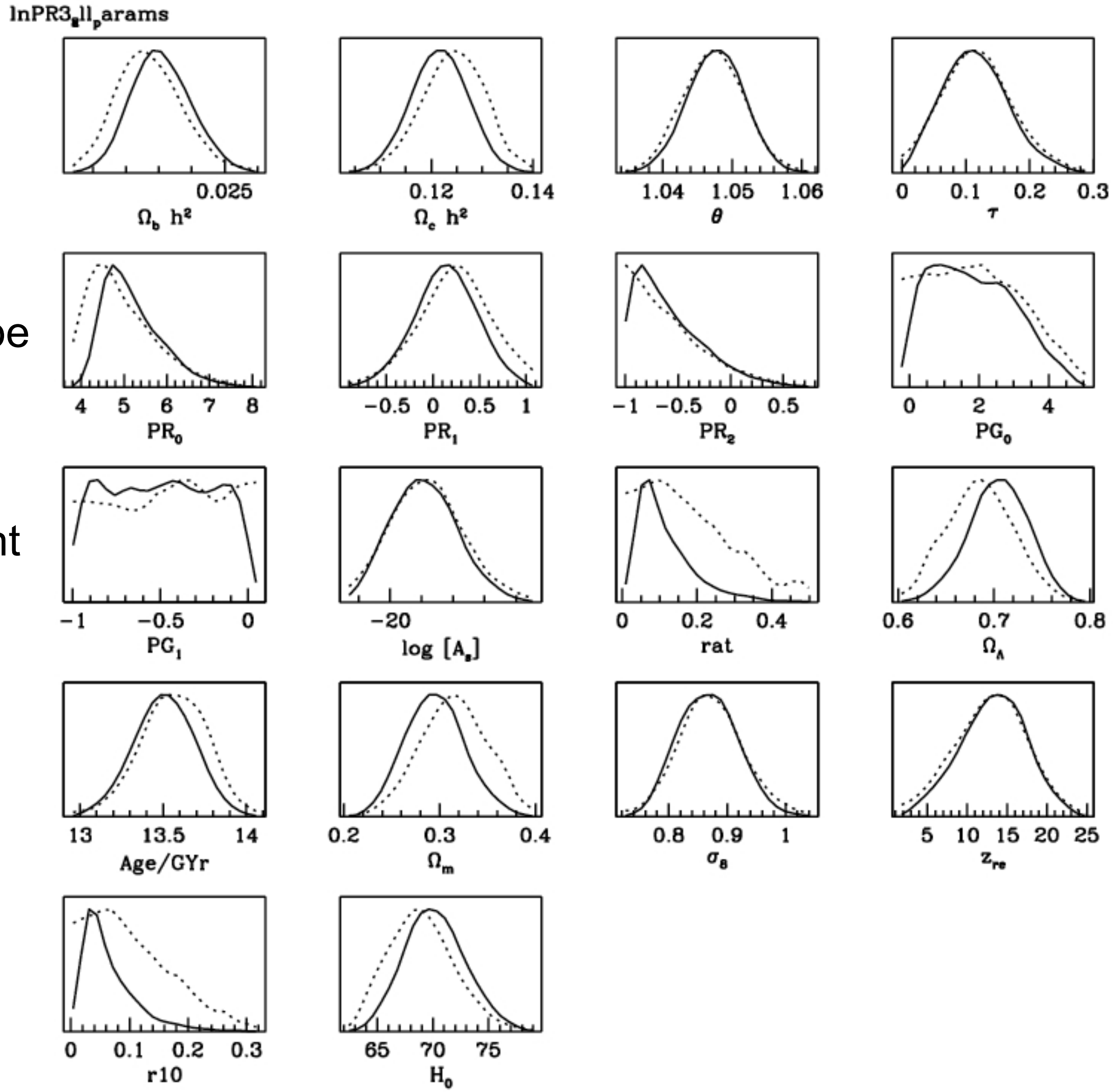
P_r up to the third order
P_g to order 2
ln k between -12 and 1
Chebyshev argument x=11/13 +2/13 ln k
Pivot point k=0.05 /Mpc
Best fit
n_s -1 = -0.06138 at k=0.002 Mpc^-1
Other cosmological parameters fixed

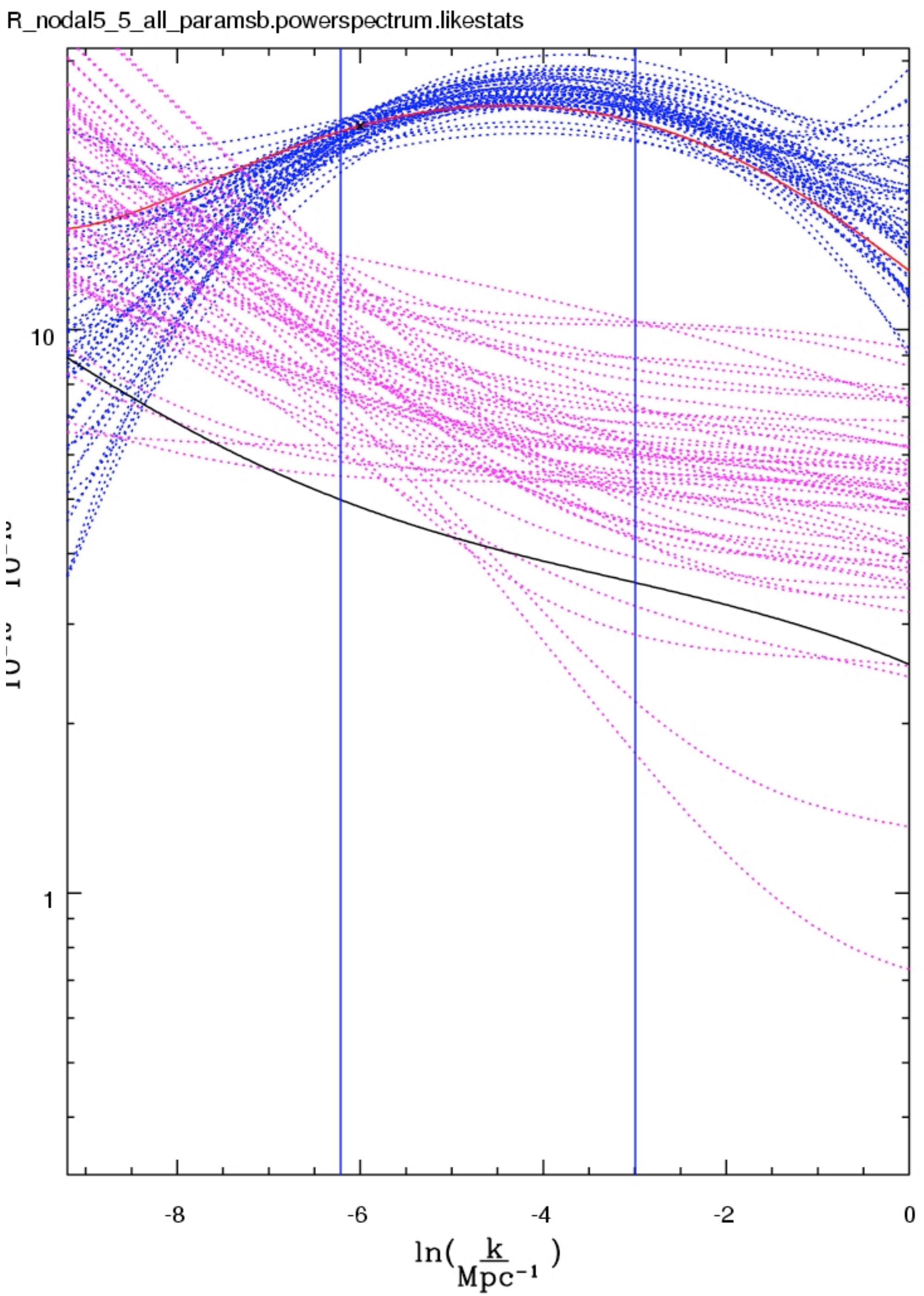
$P_s, P_t(lnaH)/10^{-5}M_p$ Inflation trajctory reconstructed from CMB+LSS data
Using Chebyshev mode expansion & MCMC

Expand scalar power in
Slope and running index
Tensor power in slope
(not usual to do both, and
Not usual to allow tensor slope
to be unslaved)

Parameters come out OK
Cf. more restrictive treatment

Nodl points unifrm prior
Implies different measures
On n_s, n_t and running
Than usual uniform

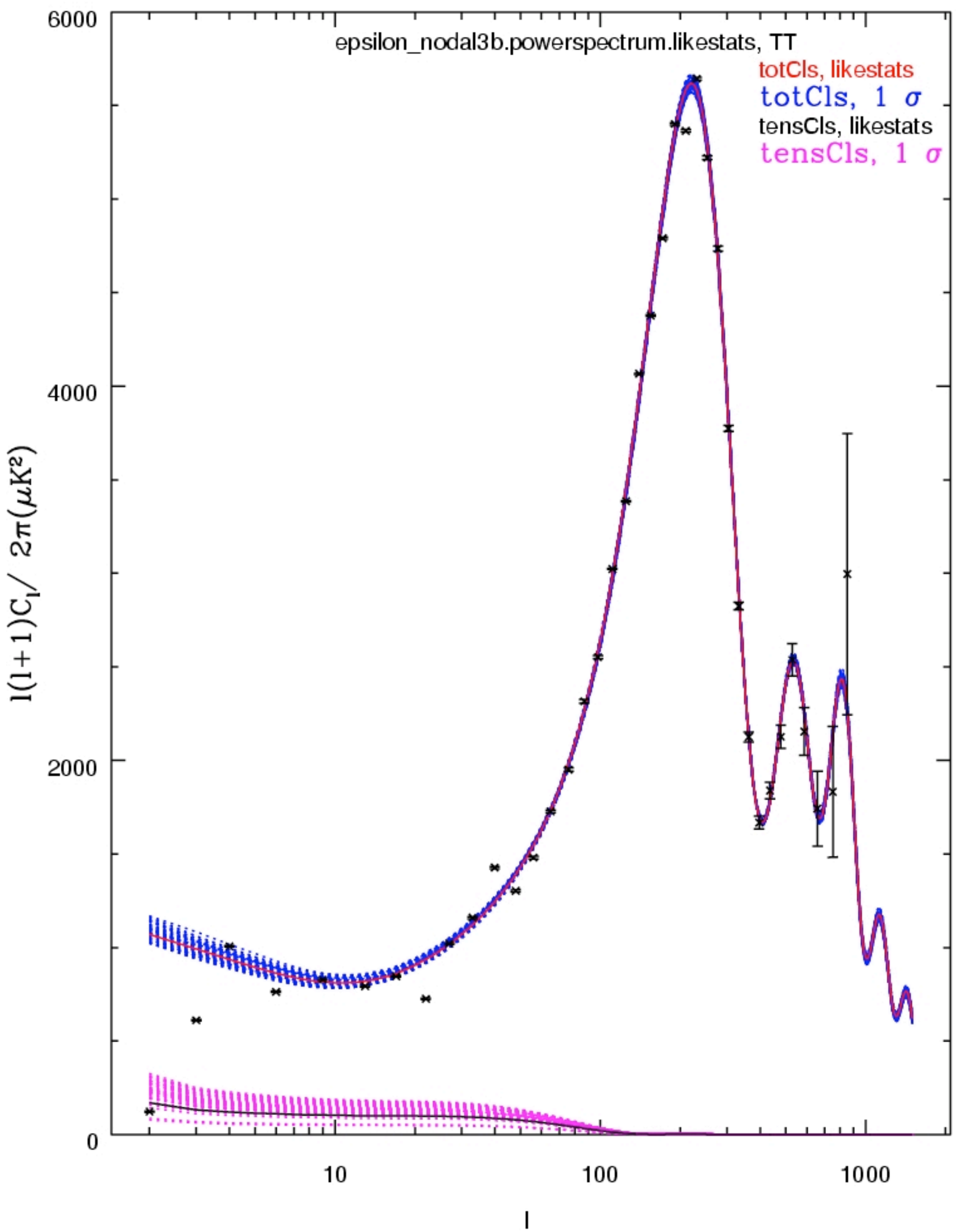




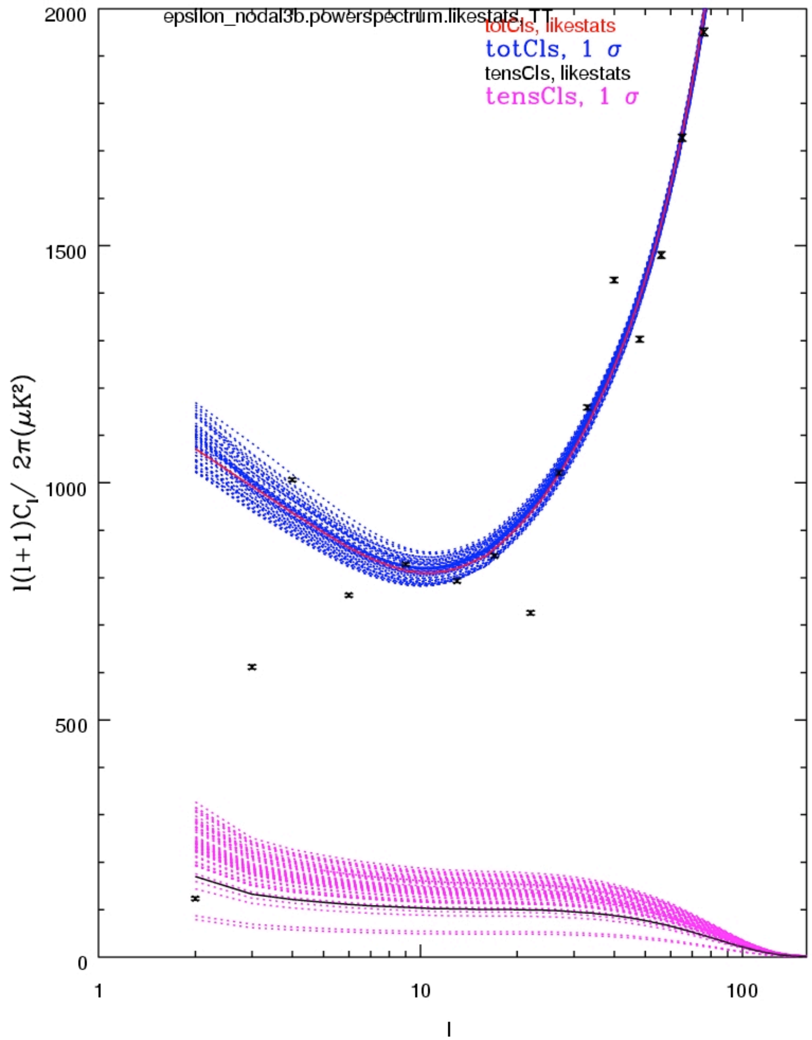
Independent expansion
(not assuming consistency)

P_s and P_t at nodal points

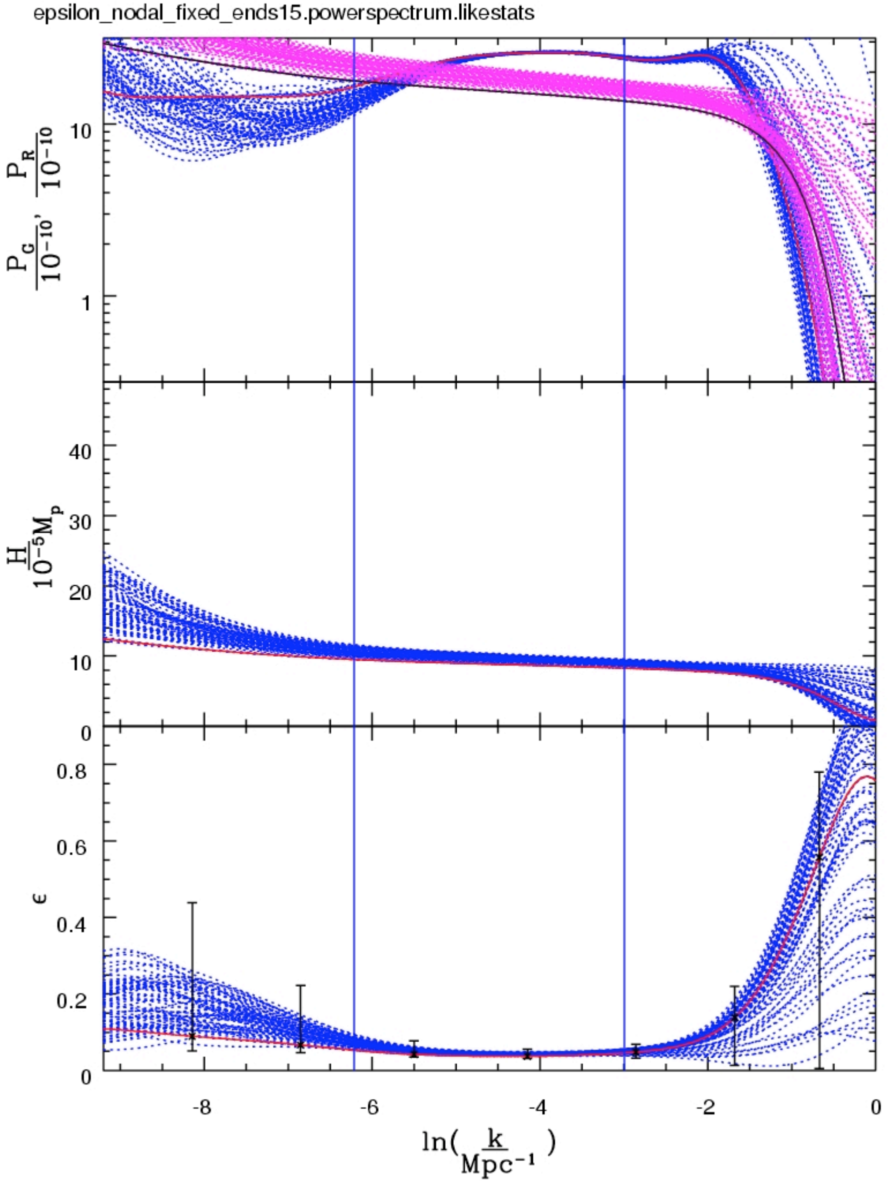
P_s up to order 5
P_t to order 5
ln k between -17 and 5
Other cosmological parameters varying



C_l for the model vs WMAPI



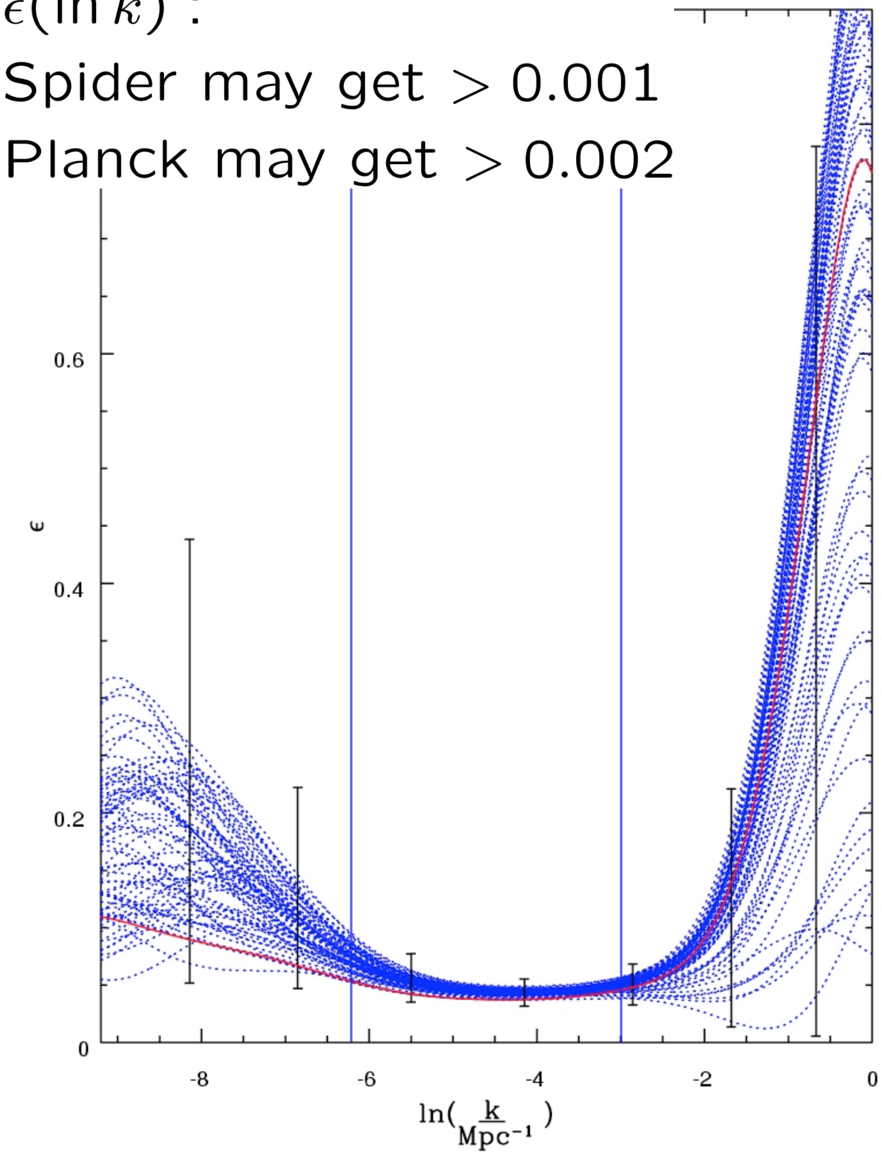
$\epsilon(\ln k)$ reconstructed from CMB+LSS data using Chebyshev expansion (order 15 nodal points) and Markov Chain Monte Carlo Method. T/S consistency function is imposed
Probe of CMB+LSS window only 1- folds

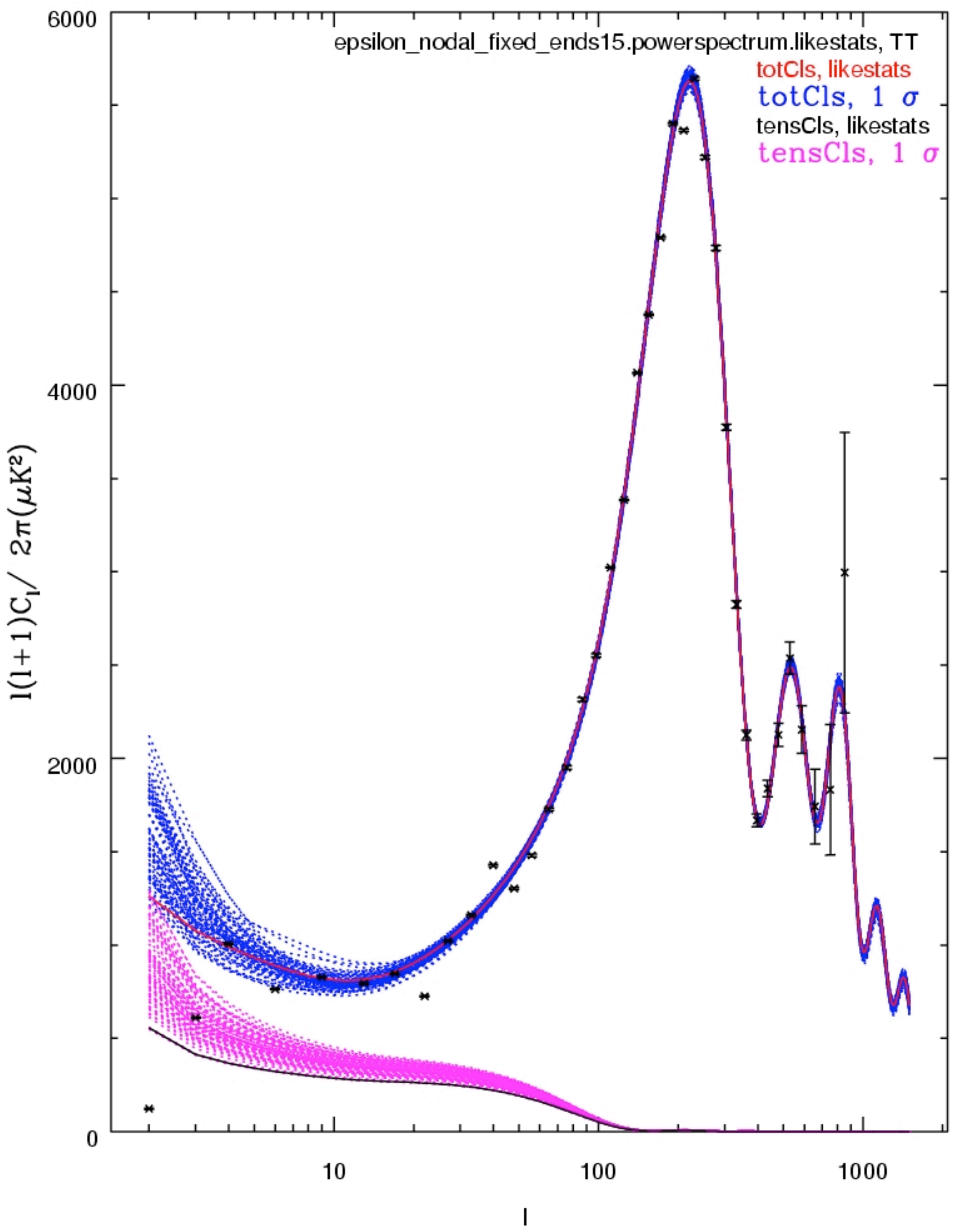


$\epsilon(\ln k)$:

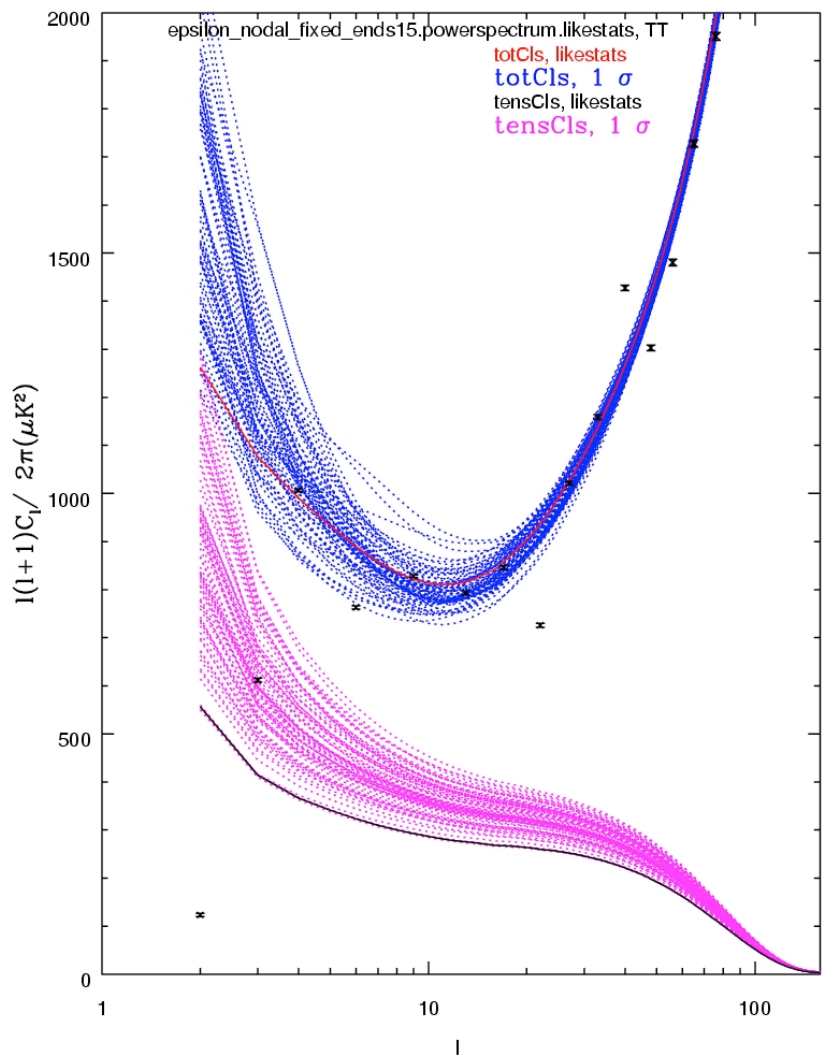
Spider may get > 0.001

Planck may get > 0.002

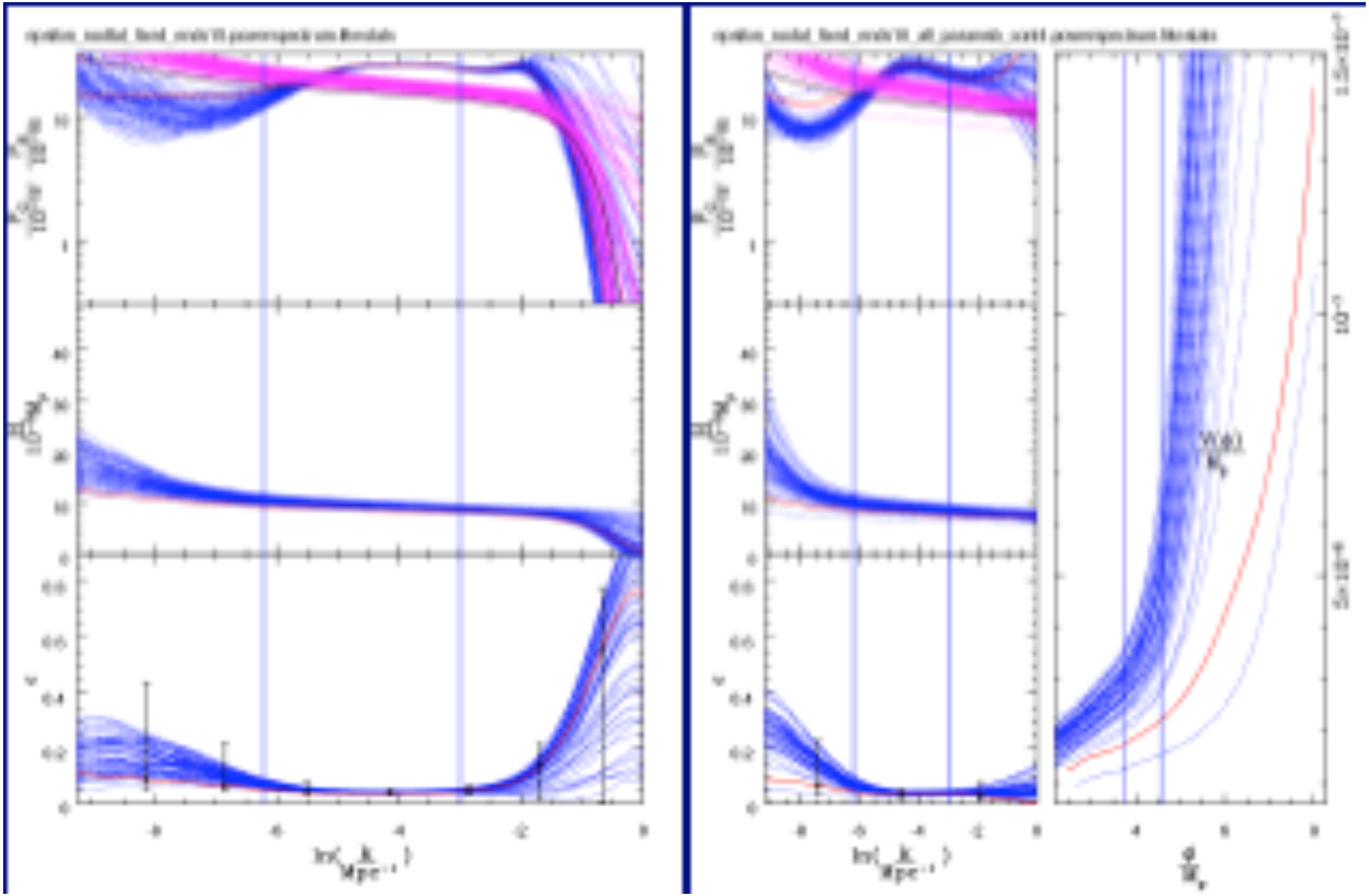




C_l for the model
vs WMAPI



WMAP1 (left)   WMAP3 (right)



Top-down approach:
no priors

"Best fit" model is not usual:

features in the potential
suppression of scalar mode at large scales
large tensor mode

mutually (almost) compensated features in

$$\left(\frac{\Delta T}{T}\right)^2_{tot} = \left(\frac{\Delta T}{T}\right)^2_s + \left(\frac{\Delta T}{T}\right)^2_t$$

Bottom-up:
personal priors

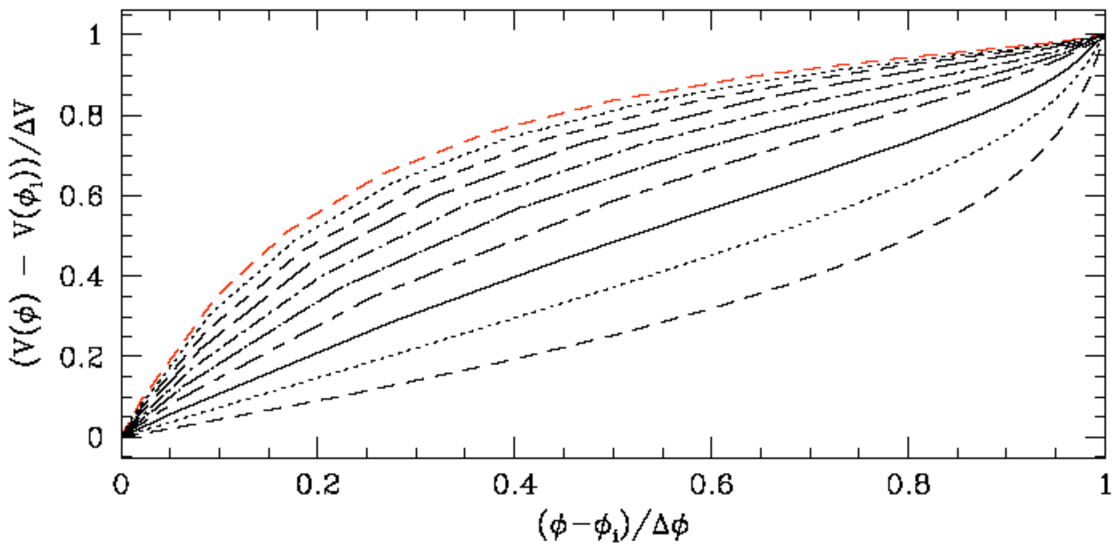
Non-vanishing probability

Degeneracy of the Potential Reconstruction

known $P_s(k) \rightarrow$ reconstruct $V(\phi)$

$$P_s(k) = \frac{8\pi H^4}{M_p^4 H'^2}$$

$$V(\phi) = \frac{M_p^4}{32\pi^2}\left(\frac{12\pi}{M_p^2}H^2 - H'^2\right)$$

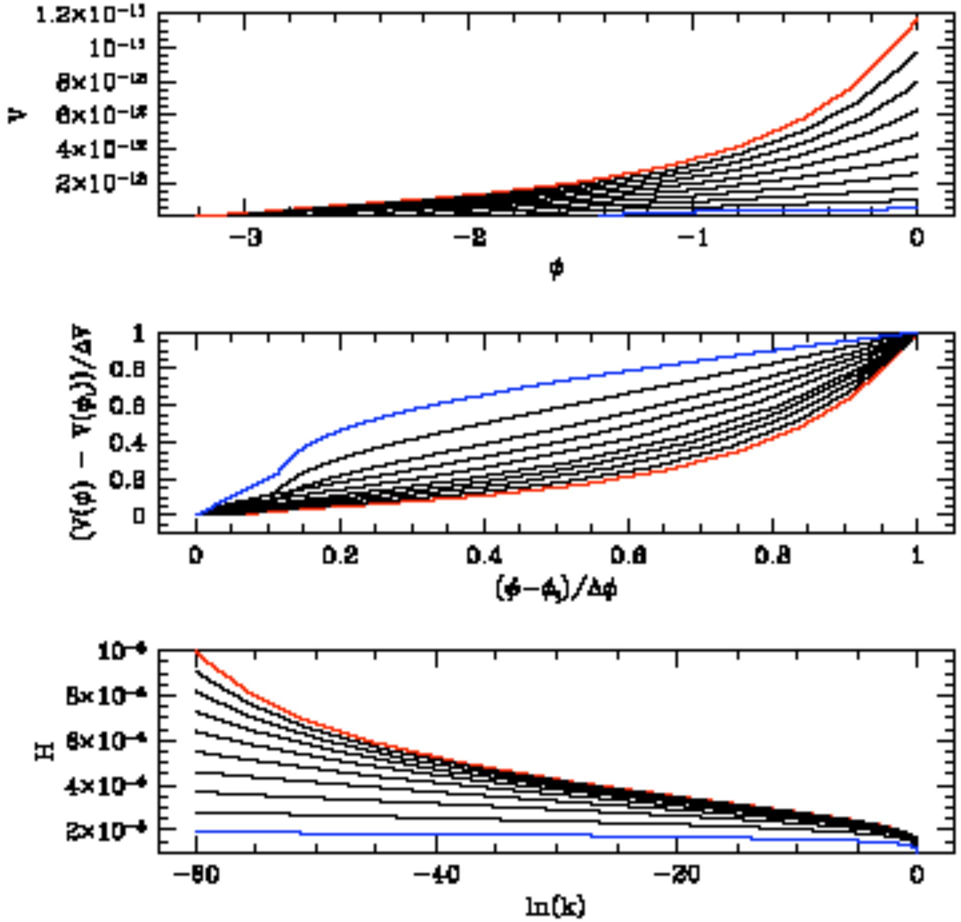


$$\phi - \phi_0 = \frac{M_p^2}{2}\int_{\ln k_0}^{\ln k} d\ln k' \, \frac{\sqrt{P_s(k')}}{H^2(k')}\frac{dH}{d\ln k'}$$

$$\frac{dH}{d\log k} = \frac{H^3}{H^2 - \pi M_p^2 P_s(k)}$$

Degeneracy is lifted by fixing $P_{GW} = \frac{H_*^2}{M_p^2}$

Example $P_s(k) = k^{n_s - 1}$
$n_s = 0.98$



4 dim Inflation in 10dim String Theory predicts

- All what 4 dim inflation predicts
- Creation of non-SM particles (KK modes) in reheating/thermalization T_{KK}
- Short-wavelength gravitational radiation $\Omega_{GW} \sim 10^{-8}$
- Scale free gaussian fluctuations of many light scalars $\hat{\imath}\,\ddot{y}_k(t)e^{ikx}$
- Modulated cosmological fluctuations
- String theory Cosmic strings $\mu G < 10^{-7}$