Standard-like Heterotic Vacua via Unitary Gauge Fluxes

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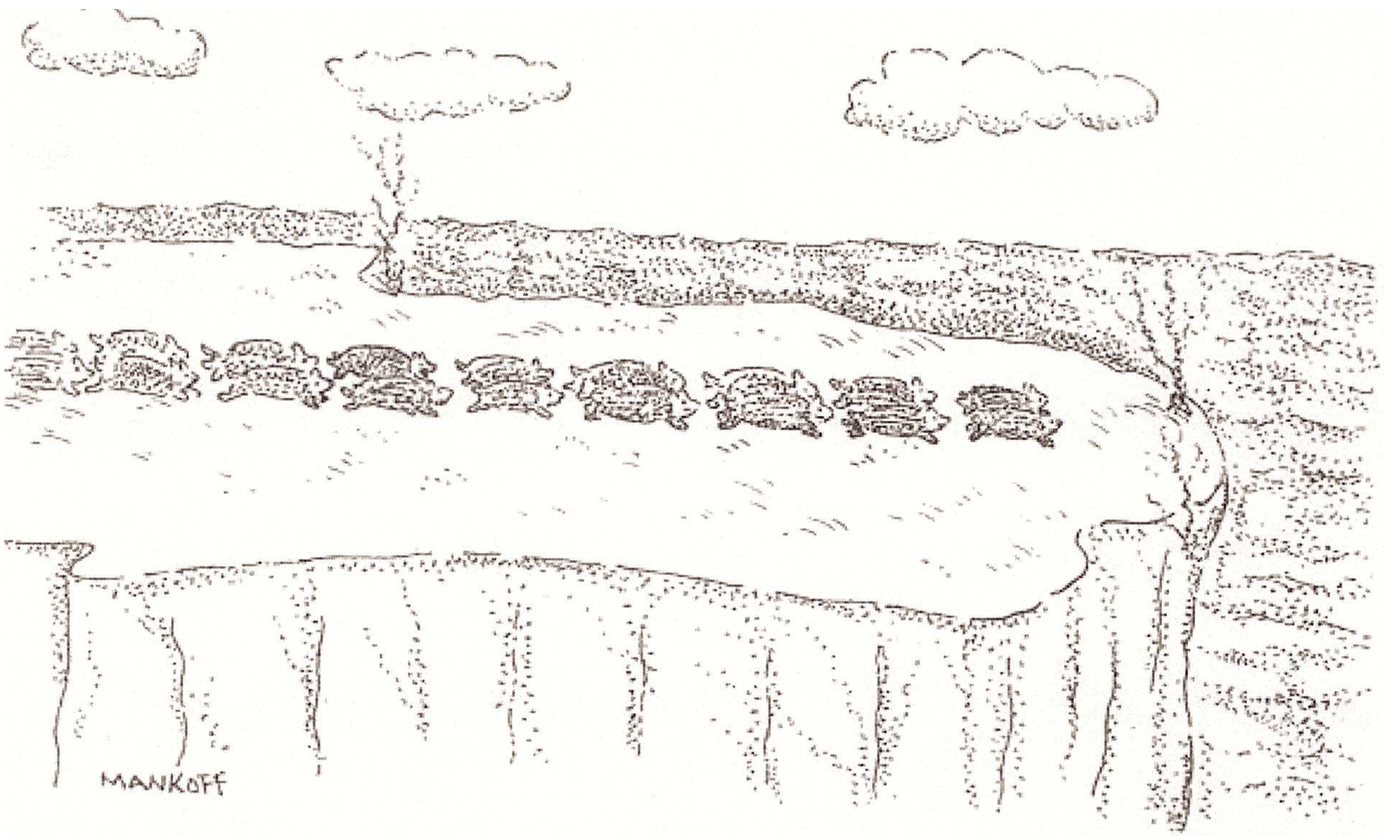
Briefing on string model building

How physical is String Theory?
How stringy is Physics?
Among the plethora of (semi-)stable string vacua,
how dense do the MSSM-like vacua lie?
Do they exhibit generic features beyond the Standard Model?
One step back: Can we give examples of string vacua in agreement with experiments up to 1 TeV?
Minimal wishlist includes:

- right gauge sector
- dynamical SUSY breaking
- all moduli stabilized
- cosmological constant fine

Need to be unprejudiced and explore all regions of the landscape we can!



"Look, I have my misgivings, too, but what choice do we have except to stay the course?"

Plan

- Intro: String Model building briefing
- Group theoretic embeddings of U(N) bundles into $E_8 \times E_8$:
  $\rightsquigarrow$ flipped SU(5)
  $\rightsquigarrow$ MSSM gauge group
- Physical implications:
  $\rightsquigarrow$ Anomalies
  $\rightsquigarrow$ Massless U(1)s
  $\rightsquigarrow$ SUSY and λ-stability
- Model building and phenomenology
- Conclusions

Briefing on string model building

Standard approaches to engineering of gauge sector:

M-theory corners:
Open Strings $\leftrightarrow$ Heterotic Strings
(or duals thereof: M-theory at singularities, F-Theory, ...)

Methods:
Exactly solvable CFTs $\leftrightarrow$ Geometric/Effective Field Theory limit
(depending on details of background manifold)

In this talk focus on geometric methods, mainly heterotic

Briefing on string model building

Open Strings

- Type II A orientifolds: Intersecting Braneworlds (IBW)
- Type I: magnetized D9/D5-branes
- Type II B orientifolds: D7/D3-branes, branes at singularities [Blumenhagen,Bianchi,Cremades,Cvetic,Douglas,Font, Honecker,Ibanez,Kiritsis,Koers,Kokorelis,Kumar,Luest,Maillard,Marchesano, Rabadan,Sagnotti,Shiu,Stieberger,Taylor,Uranga,Verlinde,Wijnholt]

Main ingredients of IBW in IIA orientifolds:

- stacks of N_a coincident D6-branes
  $\longrightarrow U(N_a)$ gauge field on worldvolume
- intersection of stacks of N_a and N_b branes
  $\longrightarrow$ chiral fermion in $(N_a, \overline{N}_b)$

Idea: Combine various $U(N_a)$ gauge modules

Briefing on string model building

Problem: SUSY constraint:
A-cycles must be special Lagrangian = real constraint
$\rightsquigarrow$ beyond the power of complex geometry!
Ways out:

- Very special $CY_3 \rightsquigarrow$ toroidal orientifolds
  $CY_3 = T^6/\mathbb{Z}_N \times \mathbb{Z}_M$
  particular class of factorizable branes wrap one-cycle on each T^2
- RCFT methods: Gepner model orientifolds
  abstract CFT describes string propagation at special small radius point in moduli space of certain CY_3
  branes $\longrightarrow$ boundary states
  plethora of appealing solutions
  [Dijkstra,Huiszoon,Schellekens'04], [Anastasopoulos, Dijkstra, Kiritsis,Schellekens'06]

Briefing on string model building

Question: Extension to more general background manifolds possible?
(Homological) Mirror symmetry (slightly simplified):
real objects $\leftrightarrow$ complex objects

SUSY condition more approachable in mirror dual Type I theory?
stack of N_a D6-branes $\rightarrow$ N_a D9-branes
non-trivial intersection $\rightarrow$ gauge flux (holomorphic bundles!)
Toroidal example:
Dual of factorizable cycles Π_a: VEV to diagonal $U(1)_a \subset U(N_a)$
extendable to more general CY_3 and non-abelian subgroups $U(n_a) \subset U(N_a)$

In fact: Standard procedure in the S-dual Heterotic picture!

Classic Heterotic Compactifications

Appealing GUT groups as natural subgroups of E_8 [Witten '85]

Ansatz: $\mathcal{M}^{(10)} = \mathbb{R}^{(1,3)} \times \mathcal{M}$

VEV to internal gauge field in subgroup $G \subset E_8 \to$

gauge group in 4D $=$ commutant $H \subset E_8$

$\rightsquigarrow$ concept of background gauge bundle V over $\mathcal{M}$

usually: structure group G of V semi-simple:

$$\begin{array}{llll} SU(3) \subset E_8 & \to \quad E_6 & SU(4) \subset E_8 & \to \quad Spin(10) \\ SU(5) \subset E_8 & \to \quad SU(5) & SU(6) \subset E_8 & \to \quad SU(3) \times SU(2) \end{array}$$

chiral matter from induced decomposition of 10D gaugino

$$\mathbf{248} \to \bigoplus_i (R_i, r_i)$$

r_i associated with bundle $U_i \to R_i$ counted by $H^*(\mathcal{M}, U_i)$

Classic Heterotic Compactifications

Ex.: $SU(5) \subset E_8 \to SU(5)$

$\mathbf{248} \to (\mathbf{24}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{24}) \oplus (\mathbf{5}, \mathbf{10}) \oplus c.c \oplus (\mathbf{10}, \overline{\mathbf{5}}) \oplus c.c$

$$\begin{array}{ccc} \mathbf{24} & \leftrightarrow & H^*(\mathcal{M}, \mathcal{O}) \\ \overline{\mathbf{5}} & \leftrightarrow & H^*(\mathcal{M}, \Lambda^2 V) \end{array} \qquad \begin{array}{ccc} \mathbf{10} & \leftrightarrow & H^*(\mathcal{M}, V) \\ \mathbf{1} & \leftrightarrow & H^*(\mathcal{M}, \mathrm{adj} V) \end{array}$$

GUT breaking?

No GUT Higgs in $\mathbf{24}$ since $H^*(\mathcal{M}, \mathcal{O}) = (1, 0, 0, 1)$ (for CY_3)

$\to$ Need non-trivial Wilson lines for GUT breaking

$\to$ approach restricted to $\mathcal{M}$ with $\pi_1(\mathcal{M}) \neq 0$

[Donagi,Ovrut,Pantev,Waldram '00]

[Braun,Ovrut,Pantev,Reinbacher '04]

[Braun,He,Ovrut,Pantev '05]

[Cvetic,Donagi,Bouchard '06]

Classic Heterotic Compactifications

Gauge sector hinges on very specific properties of $\mathcal{M}$
What if internal manifold even harder to describe than Calabi-Yau manifolds (non-Kähler, fluxes)?
Why restrict ourselves to gauge bundles with semi-simple structure group?
Can we replace Wilson line GUT breaking by suitable embedding of line bundles or more general unitary gauge bundles?
$\rightsquigarrow$ "decoupling" of gauge sector from topology of $\mathcal{M}$
$\rightsquigarrow$ restore symmetry between dual heterotic / Type I setup

$\rightsquigarrow$ new model building possibilities

Unitary embeddings

• Embedding of single $U(N)$ bundles not possible due to $SU(N) \subset E_8$, but
• $SU(N) \times U(1) \subset SU(N+1) \subset E_8 \to H_{N+1} \times U(1)$

$$\to W = V_N \oplus L^{-1}, \qquad c_1(V_N) = c_1(L) \quad V_N : U(N) - \text{bundle},$$
$$L : \text{line bundle}$$

spectrum:
$\mathbf{248} \to \bigoplus_i (R_i, r_i)_{q_i}$, $\qquad r_i, q_i$ from elementary decomp.

$$\begin{aligned}
\mathbf{Adj(N+1)} &\to \mathbf{Adj(N)}_0 + (1)_0 + (\mathbf{N})_{N+1} + (\overline{\mathbf{N}})_{-(N+1)}, \\
(\mathbf{N+1}) &\to (\mathbf{N})_1 + 1_{-N}, \\
\Lambda^{\mathbf{2}}(\mathbf{N+1}) &\to \Lambda^{\mathbf{2}}(\mathbf{N})_2 + (\mathbf{N})_{-(N-1)}, \\
\Lambda^{\mathbf{3}}(\mathbf{N+1}) &\to \Lambda^{\mathbf{3}}(\mathbf{N})_3 + \Lambda^{\mathbf{2}}(\mathbf{N})_{-(N-2)}
\end{aligned}$$

Example: Flipped $SU(5) \times U(1)_X$

$SU(5) \subset E_8 \to SU(5)$

$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) + (\mathbf{1},\mathbf{24}) + (\mathbf{5},\mathbf{10}) + (\overline{\mathbf{5}},\overline{\mathbf{10}}) + (\mathbf{10},\overline{\mathbf{5}}) + (\overline{\mathbf{10}},\mathbf{5})$$

$SU(4) \times U(1) \subset SU(5)$

$$\begin{aligned} \mathbf{24} &\longrightarrow \mathbf{15}_0 + \mathbf{1}_0 + \mathbf{4}_5 + \overline{\mathbf{4}}_{-5}, \qquad \mathbf{10} \longrightarrow \mathbf{6}_2 + \mathbf{4}_{-3} \\ \mathbf{5} &\longrightarrow \mathbf{4}_1 + \mathbf{1}_{-4}, \end{aligned}$$

$$\mathbf{248} \xrightarrow{SU(4)\times SU(5)\times U(1)_X} \left\{ \begin{array}{c} (\mathbf{15},\mathbf{1})_0 \\ (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{10})_{-4} + (\mathbf{1},\overline{\mathbf{10}})_4 + (\mathbf{1},\mathbf{24})_0 \\ (\mathbf{4},\mathbf{1})_5 + (\mathbf{4},\overline{\mathbf{5}})_{-3} + (\mathbf{4},\mathbf{10})_1 \\ (\overline{\mathbf{4}},\mathbf{1})_{-5} + (\overline{\mathbf{4}},\mathbf{5})_3 + (\overline{\mathbf{4}},\overline{\mathbf{10}})_{-1} \\ (\mathbf{6},\mathbf{5})_{-2} + (\mathbf{6},\overline{\mathbf{5}})_2 \end{array} \right.$$

Flipped $SU(5) \times U(1)_X$

associate $U(1)_X$-charge to V, L: $Q_X(V) = 1, Q_X(L^{-1}) = -4$

$SU(5) \times U(1)_X$	cohomology	SM part.
$\mathbf{10}_1$	$H^*(V)$	$(q_L, d_R^c, \nu_R^c) + [H_{10} + \overline{H}_{10}]$
$\mathbf{10}_{-4}$	$H^*(L^{-1})$	$-$
$\overline{\mathbf{5}}_{-3}$	$H^*(V \otimes L^{-1})$	(u_R^c, l_L)
$\overline{\mathbf{5}}_2$	$H^*(\bigwedge^2 V)$	$[(h_3, h_2) + (\overline{h}_3, \overline{h}_2)]$
$\mathbf{1}_5$	$H^*(V \otimes L)$	e_R^c

GUT Higgs as $(\mathbf{10}) + (\overline{\mathbf{10}})$ $\rightarrow$ possible on arbitrary CY

Note the flip $d_R^c \leftrightarrow u_R^c, \qquad e_R^c \leftrightarrow \nu_R^c$

Example II: $SU(3) \times SU(2) \times U(1)_Y$

$SU(5) \times U(1) \subset SU(6) \subset E_8 \longrightarrow SU(3) \times SU(2) \times U(1)$

$\rightarrow W = V_5 \oplus L^{-1}$

$SU(3) \times SU(2) \times U(1)_Y$	cohom.	SM part.
$(\mathbf{3}, \mathbf{2})_{\frac{1}{3}}$	$H^*(V)$	q_L
$(\mathbf{3}, \mathbf{2})_{-\frac{5}{3}}$	$H^*(L^{-1})$	$-$
$(\overline{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}}$	$H^*(\bigwedge^2 V)$	d_R^c
$(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}}$	$H^*(V \otimes L^{-1})$	u_R^c
$(\mathbf{1}, \mathbf{2})_{-1}$	$H^*(\bigwedge^2 V \otimes L^{-1})$	l_L
$(\mathbf{1}, \mathbf{1})_2$	$H^*(V \otimes L)$	e_R^c

nomalies

Caution: $U(1)$ are a priori anomalous!
Field theoretic anomaly six-forms (for flipped SU(5)):

$$A_{U(1)-G^2_{\mu\nu}} \simeq \int_{\mathcal{M}} c_1(L)\left[12\Big(-c_2(V)+c_1^2(L)\Big)+5\,c_2(T)\right],$$

$$A_{U(1)-SU(5)^2} \simeq \int_{\mathcal{M}} c_1(L)\left[2\Big(-c_2(V)+c_1^2(L)\Big)+c_2(T)\right],$$

$$A_{U(1)^3} \simeq \int_{\mathcal{M}} c_1(L)\left[-12c_2(V)+17c_1^2(L)+6\,c_2(T)\right],$$

→ Anomaly must be cancelled by Green-Schwarz mechanism
⇝ massive U(1)!
→ Careful analysis of effective $\mathcal{N}=1$ SUGRA required to find way out

General setup

$$W = W_1 \oplus W_2, \qquad W_i = V_{N_i} \oplus \bigoplus_{m_i=1}^{M_i} L_{m_i}, \qquad c_1(W_i) = 0$$

$$\leftrightarrow SU(N_i) \times U(1)^{M_i} \subset SU(N_i + M_i) \subset E_8^{(i)}$$

$$\rightarrow H = H_1 \times H_2, \qquad H_i = E_{(9-N_i-M_i)} \times U(1)^{M_i}$$

plus optional five-branes along holomorphic two-cycle γ_a $\rightarrow$
Anomaly cancellation condition [Donagi,Lukas,Ovrut,Waldram '99]

$$\sum_{i=1}^{2}\left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2}\sum_{m_i=1}^{M_i} c_1^2(L_{m_i})\right) - \sum_a N_a \overline{\gamma}_a = -c_2(T)$$

$\sum_a N_a \overline{\gamma}_a$: effective class!

General setup

Tree-level Killing spinor equation for gaugino:

$F_{ab} = F_{\overline{a}\overline{b}} = 0 \to V_{N_i}, L_{m_i}$ holomorphic bundles

$g^{a\overline{b}} F_{a\overline{b}} = 0 \to J \wedge J \wedge \overline{F}_{N_i/m_i} = 0$ (HYM)

zero-slope limit of

$$J \wedge J \wedge \overline{F}_{N_i/m_i} = 2\pi\, \mu(V_{N_i/m_i})\, \mathrm{vol}_{\mathcal{M}}\, \mathrm{id},$$

$$\text{where} \quad \mu(V_{N_i/m_i}) = \frac{1}{\mathrm{rk}(V_{N_i/m_i})} \int_{\mathcal{M}} J \wedge J \wedge c_1(V_{N_i/m_i}).$$

$\leftrightarrow$ V_{N_i} μ-stable (i.e. every proper subsheaf is of lower μ-slope)
(trivially satisfied for line bundles L_{m_i})

$\to$ in addition: $\int_{\mathcal{M}} J \wedge J \wedge c_1(V_{N_i/m_i}) = 0$ (DUY)
will find one-loop corrections later

nomalies

In general: all U(1) factors are anomalous

$$A_{U(1)_{m_i}-E^2_{9-\mathcal{N}_i}} \sim f_{m_i} \wedge \mathrm{tr} F_i^2 \left[\int_{\mathcal{M}} \overline{f}_{m_i} \wedge \left(\mathrm{tr} \overline{F}_i^2 - \frac{1}{2} \mathrm{tr} \overline{R}^2 \right) \right],$$

$$A_{U(1)_{m_i}-G^2_{\mu\nu}} \sim f_{m_i} \wedge \mathrm{tr} R^2 \left[\int_{\mathcal{M}} \overline{f}_{m_i} \wedge \left(12\, \mathrm{tr} \overline{F}_i^2 - 5\, \mathrm{tr} \overline{R}^2 \right) \right]$$

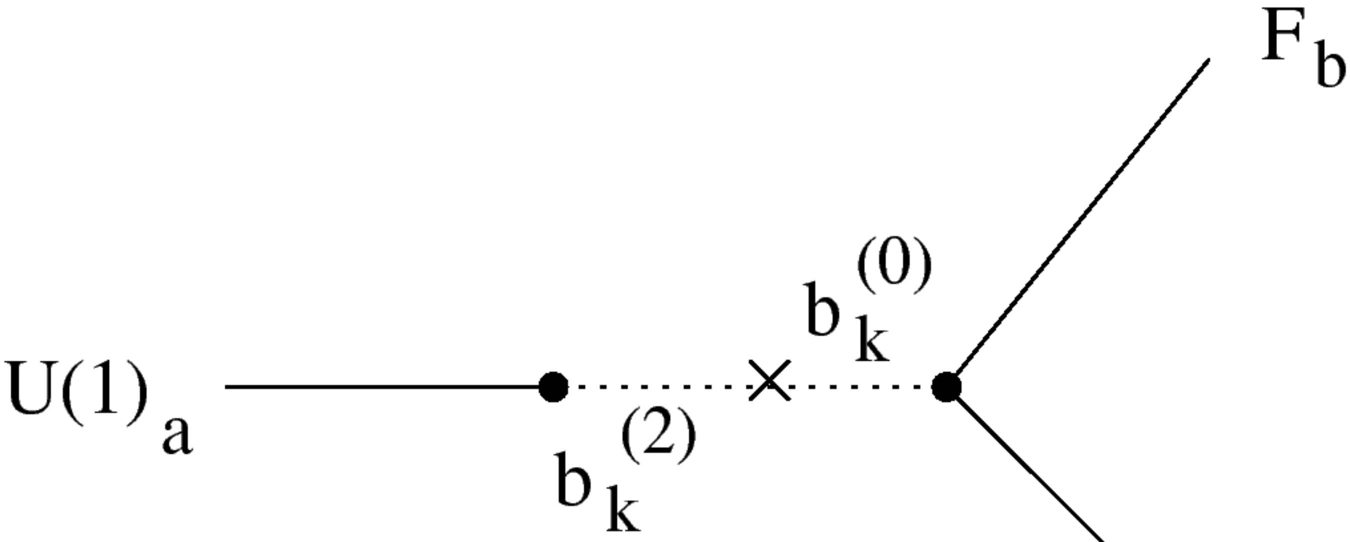
plus cubic abelian
$\rightarrow$ cancelled by 4D generalised GS-mechanism

in Type I: [Bianchi, Sagnotti; Ibanez, Rabadan, Uranga; Anastasopoulos, Bianchi, Dudas, Kiritsis]

important far beyond issue of anomalies
$\rightsquigarrow$ essential for determining effective gauge group (massive U(1)s!)
$\rightsquigarrow$ new physics (one-loop modifications of stability condition)

$\rightsquigarrow$ worthwhile to study in detail

4D GS-mechanism



4D GS-mechanism

$$S^j_{kin} = \alpha_j \int_{\mathbb{R}_{1,3}} db^{(2)}_j \wedge \star_4 db^{(2)}_j \qquad db^{(0)}_j = \beta_j \star_4 db^{(2)}_j$$

need as dynamical input:

$$\begin{aligned} S_{vertex} &= \sum_j \mathcal{A}_j \int_{\mathbb{R}_{1,3}} b^{(0)}_j \wedge \mathrm{tr}\mathcal{F}^2, \\ S_{mass} &= \sum_j \int_{\mathbb{R}_{1,3}} b^{(2)}_j \wedge \sum_m \mathcal{M}_{jm} f_m \end{aligned}$$

Anomalous coupling induces associated anomaly six-form:

$$A^{GS}_{U(1)_n - \mathcal{F}^2} \sim \sum_j \frac{\beta_j}{\alpha_j} \mathcal{A}_j \mathcal{M}_{jn} \left(f_n \wedge \mathrm{tr}\mathcal{F}^2 \right).$$

GS-mechanism

axions/2-forms via dimensional reduction from:

$$B^{(2)} \; = \; b_0^{(2)} + \ell_s^2 \sum_{k=1}^{h_{11}} b_k^{(0)} \, \omega_k , \quad B^{(6)} = \ell_s^6 \, b_0^{(0)} \, \mathrm{vol}_6 + \ell_s^4 \sum_{k=1}^{h_{11}} b_k^{(2)} \, \widehat{\omega}_k$$

Dynamical input:

$$S_{GS} \; = \; \frac{1}{24\,(2\pi)^5\,\alpha'} \int_{\mathcal{M}^{(10)}} B^{(2)} \wedge X_8(F_1, F_2, R)$$

$$S_{kin} \; = \; -\frac{1}{4\kappa_{10}^2} \int_{\mathcal{M}^{(10)}} e^{-2\phi_{10}} \, H \wedge \star_{10} H$$

collect vertex and mass terms and compute anomaly six-form

GS-mechanism

Upon using tadpole cancellation condition find:

$$A \sim A_{\text{pert}} - \frac{1}{24(2\pi)^4\alpha'} \sum_a N_a \int_{\gamma_a} \text{tr}(F_1 \overline{F}_1) \times$$

$$\left[\frac{1}{4}\left(\text{tr}F_1^2 + \text{tr}F_2^2 - \text{tr}R^2\right) + \frac{3}{4}\left(\text{tr}F_1^2 - \text{tr}F_2^2\right)\right] + (1 \leftrightarrow 2)$$

A_{pert} cancels field theoretic (mixed/cubic) abelian anomalies

But: Five-branes induce additional anomaly in presence of $U(1)$s!

Additional five-brane dependent counter terms required

GS-mechanism

First part cancelled by:

$$S_{GS}^{(1)} = \frac{1}{96(2\pi)^3\alpha'} \sum_a N_a \int_{\Gamma_a} B^{(2)} \wedge \left(\mathrm{tr}F_1^2 + \mathrm{tr}F_2^2 - \mathrm{tr}R^2\right)$$

On worldvolume Γ_a of five-brane a: self-dual 2-form $\widetilde{B}_a$

$$\begin{aligned} d\widetilde{B}_a &= \star_a d\widetilde{B}_a \\ \widetilde{B}_a &= \widetilde{b}_a^{(2)} + \ell_s^2\, \widetilde{b}_a^{(0)}\, \widehat{\gamma}_a \qquad \text{with} \quad d\widetilde{b}_a^{(0)} = \star_4 d\widetilde{b}_a^{(2)} \end{aligned}$$

($\widehat{\gamma}_a$: Hodge dual of $\overline{\gamma}_a$)

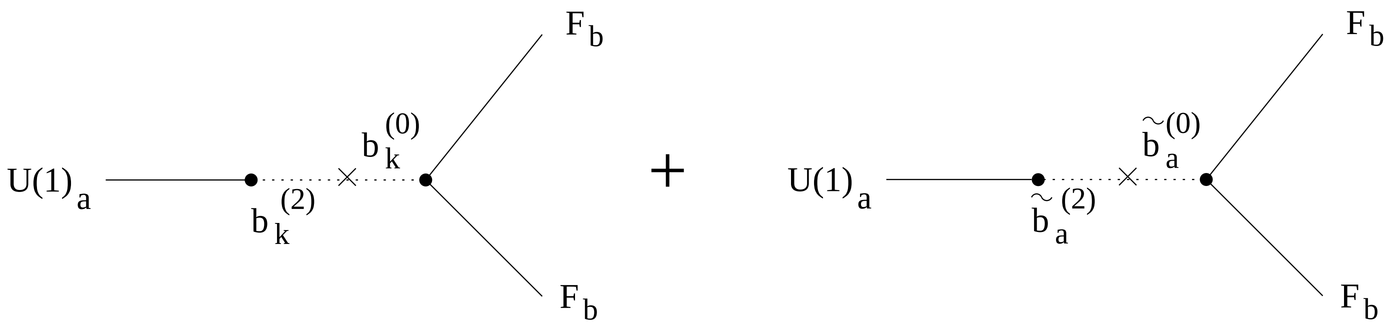
$$S_{GS}^{(2)} = \frac{1}{8(2\pi)^3\alpha'} \sum_a N_a \int_{\Gamma_a} \widetilde{B}_a \wedge \left(\mathrm{tr}F_1^2 - \mathrm{tr}F_2^2\right)$$

cancels second anomaly term

GS-mechanism

Both five-brane dependent GS-terms also follow from detailed reduction of heterotic M-theory

Summary:
One-loop counter terms of form:



$\rightsquigarrow$ Cancel all field-theoretic anomalies

Additional information:
$\rightsquigarrow$ encode threshold corrections of gauge kinetic functions
$\rightsquigarrow$ give rise to mass terms of $U(1)$ potentials
$\rightarrow$ decide on low-energy gauge group

Massive U(1)

From $\quad S_{mass} = \sum_j \int_{\mathbb{R}_{1,3}} b_j^{(2)} \wedge \sum_m \mathcal{M}_{jm} f_m \quad$ find

$$\begin{aligned} S_{Stuckelberg} &= -\sum_{m,n} (\mathrm{M})^2_{m,n} \, (A_m \wedge \star_4 A_n) \quad \text{where} \\ (\mathrm{M})^2_{m,n} &= \sum_j \frac{1}{\alpha_j} \mathcal{M}_{jm} \mathcal{M}_{jn} \end{aligned}$$

$$U(1)_f = \sum_m a_m U(1)_m \quad \text{is massless} \quad \Longleftrightarrow \quad \sum_m \mathcal{M}_{jm} a_m = 0$$

$\rightarrow$ need to identify mass terms in low-energy field theory!

Massive U(1)

Recall: $S_{mass} = \sum_j \int_{\mathbb{R}_{1,3}} b_j^{(2)} \wedge \sum_m \mathcal{M}_{jm} f_m$

$$\mathcal{M}_{0,n_i} \simeq \int_{\mathcal{M}} c_1(L_{n_i}) \wedge \left(\frac{1}{4\pi^2} [\mathrm{tr}(\overline{F}_i^2) - \frac{1}{2}\mathrm{tr}(\overline{R}^2)] - \frac{1}{4} \sum_a N_a \overline{\gamma}_a \right)$$

universal axio – dilaton

$$\mathcal{M}_{k,n_i} \simeq \int_{\mathcal{M}} c_1(L_{n_i}) \wedge \widehat{\omega}_k$$

Kahler axions

$$\mathcal{M}_{a,n_i} \simeq \pm \int_{\mathcal{M}} c_1(L_{n_i}) \wedge \overline{\gamma}_a$$

five – brane axions

For SUSY: bosonic partners in $\mathcal{N} = 1$ supermultiplet must get massive!

BUT: SUSY condition $\int_{\mathcal{M}} J \wedge J \wedge \overline{F}_{N_i/m_i} = 0$ only involves

Kähler moduli

SUSY and λ-stability

Aim: find missing terms in SUSY condition
Idea: Massive/anomalous U(1)s $\leftrightarrow$ Fayet-Iliopoulos D-terms

$\mathcal{N} = 1$ SUGRA:

$$\frac{\xi_m}{g_m^2} = \left.\frac{\partial \mathcal{K}}{\partial V_m}\right|_{V_m=0}$$

V_m: superfield containing A_m
$\rightsquigarrow$ How do gauge fields appear in Kähler potential?

SUSY and λ-stability

axionic U(1) coupling requires gauging of $\mathcal{N} = 1$ SUGRA:

$$S_{axion} = \int_{\mathbb{R}_{1,3}} db_j^{(0)} \wedge *db_j^{(0)} + \mathcal{M}_{jm}\, b_j^{(0)} \wedge d * A_m,$$

then unbroken $U(1)_m$ gauge symmetry requires that

$$A_m \to A_m + d\chi_m \quad \longleftrightarrow \quad b_j^{(0)} \to b_j^{(0)} + \frac{1}{2}\mathcal{M}_{jm}\,\chi_m.$$

for superfields:

$$\begin{array}{c} A_m^\mu \to A_m^\mu + \partial^\mu \chi_\mu \\ b_j^{(0)} \to b_j^{(0)} + \frac{1}{2}\mathcal{M}_{jm}\,\chi_m \end{array} \longleftrightarrow \begin{array}{c} V_m \to V_m + \Phi_m + \Phi_m^* \\ \widetilde{B}_j \to \widetilde{B}_j + \mathcal{M}_{jm}\,\Phi_m \end{array}$$

SUSY and λ-stability

$\rightsquigarrow$ guideline to construct gauge invariant Kähler potential

$$\mathcal{K} = -\ln[B_j + B_j - \sum \mathcal{M}_{jm} V_m]$$

B_j: superfield containing $b_j^{(0)}$ for axio-dilaton, Kähler axions, five-brane axions (more complicated: [Derendinger,Sauser'00] [Moore,Peradze,Saulina'00])

$$\begin{aligned} S &= e^{-2\phi_{10}} \frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + \sum_a N_a \frac{\lambda_a^2}{2\ell_s^2} \int_{\Gamma_a} J + i b_0^{(0)} \\ T_k &= -\frac{1}{\ell_s^2} \int_{\mathcal{M}} J \wedge \widehat{\omega}_k + i b_k^{(0)}, \\ \Lambda_a &= -\lambda_a \frac{\mathrm{Vol}(\gamma_a)}{\ell_s^2} + i \widetilde{b}_a^{(0)} \end{aligned}$$

SUSY and λ-stability

Result: FI-term vanishes iff

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(L_{m_i})$$

$$-\ell_s^4\, g_s^2 \int_{\mathcal{M}} c_1(L_{m_i}) \wedge \left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2} \sum_{n_i}^{M_i} c_1(L_{n_i}^2) + \frac{1}{2}\, c_2(T) \right)$$

$$+\ell_s^4\, g_s^2 \sum_a N_a \left(\frac{1}{2} \mp \lambda_a \right)^2 \int_{\gamma_a} c_1(L_{m_i}) = 0$$

involves all moduli fields, as required from mass terms
includes tree-level part of DUY equation
$\rightsquigarrow$ one-loop modification of DUY
$\rightsquigarrow$ there must exist a perturbative correction also of (local) Hermitian Yang-Mills equation

SUSY and λ-stability

Conjecture 1: perturbatively exact SUSY condition (HYM):

$$J \wedge J \wedge F_{k_i} - (2\pi\alpha')^2 \frac{g_s^2}{4} F_{k_i} \wedge d\left(\omega_{YMi} - \frac{1}{2}\omega_L\right) = 2\pi\,\lambda_{k_i}\,\mathrm{vol}_{\mathcal{M}}\,\mathrm{id}$$

λ_{k_i}: λ-slope

$$\lambda(V_{k_i}, g_s) \equiv \frac{1}{\mathrm{rk}(V_{k_i})} \int_{\mathcal{M}} J \wedge J \wedge c_1(V_{k_i}) - (2\pi\alpha')^2 \frac{g_s^2}{4} c_1(V_{k_i}) \wedge d\left(\omega_{YMi} - \frac{1}{2}\omega_L\right),$$

together with: $\lambda_{k_i} = 0$ (DUY) (in absence of charged matter VEV!)

Conceptually: SUSY comes in two parts: HYM + DUY

4D SUGRA D-term only knows about DUY

SUSY and λ-stability

Stability at one-loop?
Conjecture 2: if $g_s < g_s(V_{k_i})$: HYM $\leftrightarrow$ V_{k_i} stable w.r.t. $\lambda(g_s)$
(inspired by Gieseker stability [Leung'97;Enger,Lutken'03])

λ-stability as perturbative stability concept

for arbitrary λ-slope: μ-stablity $\rightarrow$ λ-stability
requires that tree-level part dominates over one-loop part
not true if one-loop term not vanishing by itself!

What is the correct stability concept in most general case?
Which are the additional non-pert. corrections?
$\rightsquigarrow$ Heterotic version of Π-stability of B-type branes?
Loop corrections directly from 10 D analysis?

Model building

Recall: natural appearance of

- flipped $SU(5) \times U(1)_X$
  $W_1 = V_4 \oplus L^{-1}$ in $E_8^{(1)} \leftrightarrow$
  $SU(4) \times U(1) \subset E_8^{(1)} \to SU(5) \times U(1)_1$
- $SU(3) \times SU(2) \times U(1)_Y$
  $W_1 = V_5 \oplus L^{-1}$ in $E_8^{(1)} \leftrightarrow$
  $SU(5) \times U(1) \subset E_8^{(1)} \to SU(3) \times \times SU(2) \times U(1)_1$

Lesson:
Massless $U(1)$ as linear combination of at least two U(1) factors
second U(1) from same $E_8 \rightsquigarrow$ chiral exotics, hard to control
simpler: second $U(1)$ from second E_8

Model building

Easiest: $W_2 = L \oplus L^{-1}$ in $E_8^{(2)}$ $\leftrightarrow$
$U(1) \times U(1) \subset E_8^{(2)} \to E_7 \times U(1)_2$

$$\mathbf{248} \xrightarrow{E_7 \times U(1)} \left\{ \; (\mathbf{133})_0 + (\mathbf{1})_0 + (\mathbf{56})_1 + (\mathbf{56})_{-1} + (\mathbf{1})_2 + (\mathbf{1})_{-2} \; \right\}$$

$$\text{TAD}: \quad \mathrm{ch}_2(V) + 3\,\mathrm{ch}_2(L) - \sum_a N_a \overline{\gamma}_a \stackrel{!}{=} -c_2(T).$$

Detailed analysis: $U(1)_X = \frac{1}{2}\left(U(1)_{X'} - \frac{5}{2}\,U(1)_2\right)$
anomaly-free and massless iff:

$$\int_{\mathcal{M}} c_1(L) \wedge c_2(V) \stackrel{!}{=} 0, \qquad \int_{\gamma_a} c_1(L) \stackrel{!}{=} 0 \quad \text{for all M5 branes}$$

Model building

repr	chirality	SM part.
$(\mathbf{10},\mathbf{1})_{\frac{1}{2}}$	$\chi(V)=g$	$(q_L, d_R^c, \nu_R^c)+[H_{10}+\overline{H}_{10}]$
$(\mathbf{10},\mathbf{1})_{-2}$	$\chi(L^{-1})=0$	$-$
$(\overline{\mathbf{5}},\mathbf{1})_{-\frac{3}{2}}$	$\chi(V\otimes L^{-1})=g$	(u_R^c, l_L)
$(\overline{\mathbf{5}},\mathbf{1})_{1}$	$\chi(\bigwedge^2 V)=0$	$[(h_3,h_2)+(\overline{h}_3,\overline{h}_2)]$
$(\mathbf{1},\mathbf{1})_{\frac{5}{2}}$	$\chi(V\otimes L)+\chi(L^{-2})=g$	e_R^c
$(\mathbf{1},\mathbf{56})_{\frac{5}{4}}$	$\chi(L^{-1})=0$	$-$

absence of e_R^c from $E_8^{(2)} \leftrightarrow \int_{\mathcal{M}} c_1(L)^3 = 0$

$\rightsquigarrow$one-loop part in DUY vanishes: $\int_{\mathcal{M}} J\wedge J\wedge c_1(L) \overset{!}{=} 0$

μ-stability $\to$ λ-stability $\to$ HYM

Model building

Task: Find, on arbitrary CY_3 $\mathcal{M}$, μ-stable, holomorphic rank 4/5 bundle V and complex non-trivial line bundle L, $c_1(V) = c_1(L)$, such that:

$$\sum_a N_a \overline{\gamma}_a = \mathrm{ch}_2(V) + 3\,\mathrm{ch}_2(L) + c_2(T) \quad \text{is} \quad \text{effective}$$

$$\int_{\mathcal{M}} c_1(L) \wedge c_2(V) = 0 = \int_{\mathcal{M}} c_1(L)^3 = \int_{\gamma_a} c_1(L)$$

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(L) = 0$$

$$\chi(V) = \frac{1}{2} \int_{\mathcal{M}} c_3(V) = 3$$

$\rightsquigarrow$ 3 gen. of flipped $SU(5) \times U(1)_X / \; SU(3) \times SU(2) \times U(1)_Y$

Model building

Any construction of μ-stable bundles does the job
Example: Spectral cover construction (SCC) on elliptically fibered CY_3:

[Friedman,Morgan,Witten'97],[Donagi´97],[Andreas,Ruiperez'04]
construct $SU(N)$ bundles on elliptic fiber and extend to $U(N)$ bundles over CY_3

Took as CY_3 fibration over del Pezzo surfaces
searched up to dP_4 in very restricted parameter space
$\rightsquigarrow$ found some dozens of chiral 3-gen. models in both setups

Model building

Next steps:

- More general constructions:

non-split extensions of lower rank SCC bundles

$$0 \to V_2 \to V_{4/5} \to V_{2/3} \to 0$$

μ-stable if

$\mu(V_2) < 0$ and $\mathrm{Ext}_{\mathcal{M}}(V_{2/3}, V_2) \simeq (H^1(\mathcal{M}, V_2 \otimes V^*_{2/3}) \neq 0$

[Donagi,Ovrut,Pantev,Waldram '00]

$\rightsquigarrow$ much more flexibility

- Compute cohomology classes, in part. $H^*(\mathcal{M}, V_a \otimes V_b)$ for SCC bundles

new techniques, F-theory inspired

Determine vector-like spectrum (in part. GUT Higgses for flipped SU(5))

Model building

Result: $\mathcal{O}(100)$ models with precisely 3 families -except weak Higgs sector- (and no vector-like exotics) of MSSM matter in both frameworks
for flipped $SU(5)$: precisely one GUT-Higgs pair
weak Higgs: under computation (need $H^*(\mathcal{M}, \Lambda^2 V)$)

Phenomenology-flipped SU(5)

- Good proton stability:

weak Higgs: $\overline{\mathbf{5}}_1 \longleftrightarrow (u_R^c, l)$: $\overline{\mathbf{5}}_{-\frac{3}{2}}$: different quantum numbers

gauge invariant Yukawas present:

$$\mathbf{10}^i_{\frac{1}{2}}\,\mathbf{10}^j_{\frac{1}{2}}\,\mathbf{5}_{-1}, \quad \mathbf{10}^i_{\frac{1}{2}}\,\overline{\mathbf{5}}^j_{-\frac{3}{2}}\,\overline{\mathbf{5}}_1, \quad \overline{\mathbf{5}}^i_{-\frac{3}{2}}\,\mathbf{1}^j_{\frac{5}{2}}\,\mathbf{5}_{-1},$$

dim 4 proton decay operators absent: (not gauge invariant)

$$\mathbf{l\,l\,e} \quad \in \quad \overline{\mathbf{5}}^i_{-\frac{3}{2}}\,\mathbf{1}^j_{\frac{5}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}}, \qquad \mathbf{q\,d\,l}, \quad \mathbf{u\,d\,d} \quad \in \quad \mathbf{10}^i_{\frac{1}{2}}\,\mathbf{10}^j_{\frac{1}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}}$$

see also [Tatar,Watari'06]

also dim.5 (doublet-triplet splitting) and dim-6 ops. [Dorsner,Perez'04] absent

- MSSM-like gauge coupling relations only for threshold corrected gauge couplings (to be checked in each case)

Conclusions

Unitary gauge fluxes provide alternative gauge breaking mechanism to Wilson line approach
Heterotic/ Type II model building on same footing
Possible also for $SO(32)$ heterotic theory
Works on general Calabi-Yau manifolds

Extendable to non-Kähler compactifications where geometry even harder to control?
Need even more general construction of stable holomorphic bundles
Understand λ-stability and non-pert. corrections
Explicit computation of Yukawas? μ-terms? Higgs-sector?
Statistical analysis? Heterotic landscape?