Observational Signatures and Non-Gaussianities of General Single Field Inflation

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Inflationary Models and Observations

- WMAP measurement on CMBR

  Spectral index: $n_S = 0.951^{+0.015}_{-0.019}$

  Running of index: $dn_S/d\ln k = -0.102^{+0.050}_{-0.043}$

  Tensor to Scalar ratio: $r_{0.002} < 0.55$ (95%CL)

  Non-Gaussianity: $-54 < f_{NL} < 114$

- Currently available models

  Slow-roll inflation — using flat potential;

  DBI inflation — using speed-limit in warped space;

  K-inflation — inflation driven by kinetic energy.

Previous Studies on Non-Gaussianities in Single Field Inflation

• Non-Gaussianity in slow-roll inflation
(Maldacena 02; Acquaviva,Bartolo,Matarrese&Riotto 02; Seery&Lidsey 05)

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \sim f_{NL}\ F(k_1,k_2,k_3)$$

$$f_{NL} \sim \mathcal{O}(\epsilon) \lesssim \text{a few percent}$$

Adding higher derivative corrections (Creminelli 03)

$$f_{NL} < \mathcal{O}(1)$$

• Non-Gaussianity in DBI inflation
(Alishahiha,Silverstein&Tong 04; X.C. 05)

$$1 < f_{NL} \lesssim 100$$

• Non-Gaussianity in K-inflation

Previously not calculated.

Most General Non-Gaussianities in Single Field Theory

- Single field inflation:
  - — Inflaton is responsible for density perturbations;
  - — Lagrangian is arbitrary function of $g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$ and ϕ;
  - — Arbitrary sound speed and λ (to be defined).

- Motivations
  - — Null hypothesis on specific models;
    Fit or constrain parameters model-independently;
  - — Several string inspired models has distinctive predictions on non-Gaussianity;
  - — Straightforward evaluation of non-Gaussianities for future models in this general class;
  - — Non-Gaussianity signatures for some hypothesized trans-Planckian physics.

Review of Several Classes of Models

1. Slow-roll inflation (Linde 82; Albrecht & Steinhardt 82)

- Lagrangian

$$P(X,\phi) = X - V(\phi), \qquad X = -\tfrac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

Inflaton rolling on a flat potential $V(\phi)$

- Slow-roll parameters

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta_V = M_{\rm Pl}^2 \frac{V''}{V} \ll 1$$

- Scalar and tensor power spectrum

$$P_k^\zeta = \frac{1}{12\pi^2 M_{pl}^6} \frac{V^3}{V'^2} \,, \qquad P_k^h = \frac{2V}{3\pi^2 M_{pl}^4}$$

- Spectral index

$$n_s - 1 = \frac{d \ln P_k^\zeta}{d \ln k} = M_{pl}^2 \left(-3 \frac{V'^2}{V^2} + 2 \frac{V''}{V} \right)$$

- String models:

Branes; Tachyon; Axions; Radions(Kahler moduli).

2. DBI inflation

(Silverstein, Tong & Alishahiha 03,04; X.C. 04,05)

- Warped space

$$ds^2 = f(\phi)^{-1/2} ds_4^2 + f(\phi)^{1/2} ds_{\text{internal}}^2$$

- UV model (Silverstein, Tong & Alishahiha 03,04)

$$V(\phi) \approx \frac{1}{2} m^2 \phi^2$$
$$m \gg M_{\text{Pl}}/\sqrt{\lambda}$$

- IR model (X.C. 04,05)

$$V(\phi) \approx V_0 - \frac{1}{2} m^2 \phi^2$$
$$m \sim H$$

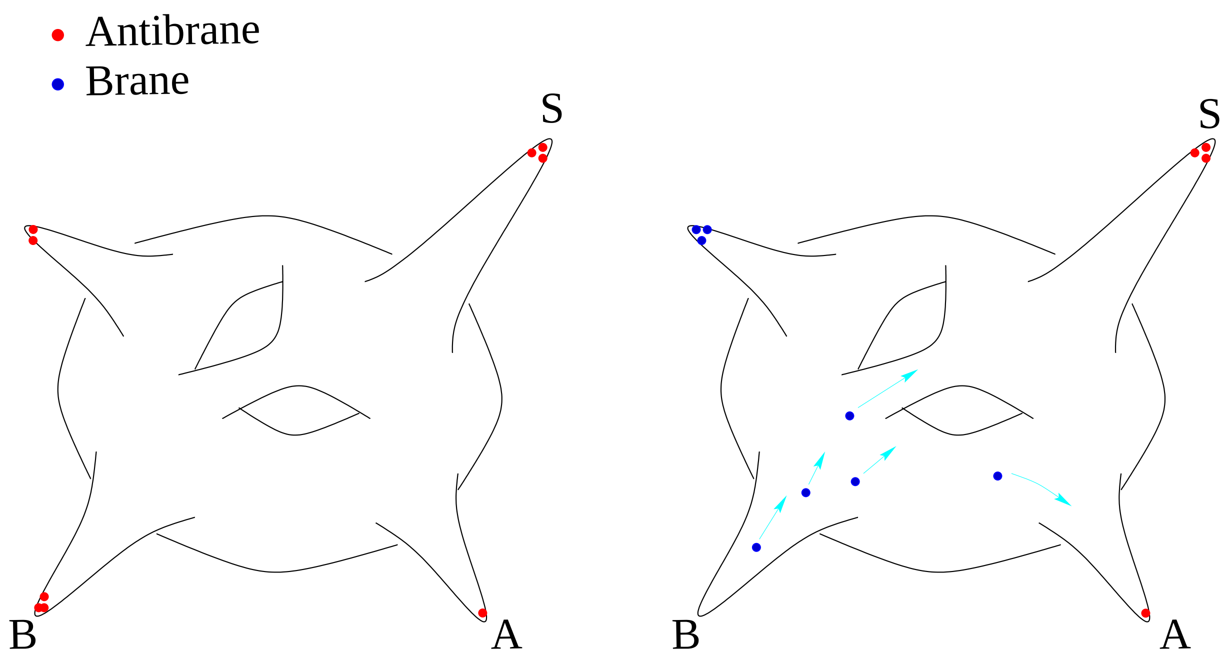
- Speed-limit constraint

$$|\dot{\phi}| \leq \frac{\phi^2}{\sqrt{\lambda}}$$

- Lagrangian

$$P = -f(\phi)^{-1}\sqrt{1 - 2Xf(\phi)} + f(\phi)^{-1} - V(\phi)$$

Multi-Throat Brane Inflation (X.C. 04)



→ Antibrane-flux annihilation; (Kachru,Pearson&Verlinde 01)

→ Generate branes as candidate inflatons;

→ Exit B-throat;
Roll through bulk;
Settle down in another throat (A or S);

→ DBI inflation, if enough warping;
Slow-roll inflation, if flat potential.

3. K-inflation (Armendariz-Picon, Damour & Mukhanov 99)

- Lagrangian

$$P(X,\phi) = \frac{1}{\phi^2} f(X) = \frac{4}{9}\frac{(4-3\gamma)}{\gamma^2}\frac{1}{\phi^2}(-X + X^2)$$

- Attractor solution

$$X \; = \; X_0 \; = \frac{2-\gamma}{4-3\gamma}$$
$$a(t) \sim t^{\frac{2}{3\gamma}}$$

- Inflation driven by kinetic energy if $0 < \gamma \ll 2/3$

- Using hybrid model to end inflation

- Not realized in string theory so far

A General Formalism (Garriga & Mukhanov 99)

- Lagrangian

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_{pl}^2 R - 2P(X, \phi) \right]$$

$$\text{where} \quad X = -\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

- Metric

$$ds^2 = -dt^2 + a^2(t) dx_3^2$$

- Equation of motion

$$3M_{pl}^2 H^2 = E = 2XP_{,X} - P$$

$$\dot{E} = -3H(E + P)$$

- Define parameters

$$c_s^2 = \frac{dP}{dE} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \quad \text{(sound speed)}$$

$$\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}$$

$$\Sigma = XP_{,X} + 2X^2 P_{,XX} = \frac{H^2 \epsilon}{c_s^2}$$

- Slow variation parameters

$$\epsilon = -\frac{\dot{H}}{H^2}\,, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}\,, \quad s = \frac{\dot{c}_s}{c_s H}\,, \quad l = \frac{\dot{\lambda}}{\lambda H}\,.$$

- More general than the usual slow-roll parameters

  — flat potential v.s.
  steep potential (DBI) or no potential (k-inflation)

  — non-relativistic slow-roll v.s.
  ultra-relativistic fast-roll

- Power spectrum

$$P_k^\zeta = \frac{1}{36\pi^2 M_{pl}^4}\frac{E^2}{c_s(P+E)} = \frac{1}{8\pi^2 M_{pl}^2}\frac{H^2}{c_s\epsilon}$$

$$P_k^h = \frac{2}{3\pi^2}\frac{E}{M_{pl}^4}$$

- Spectral index

$$n_s - 1 = \frac{d\ln P_k^\zeta}{d\ln k} = -2\epsilon - \eta - s$$

$$n_T = \frac{d\ln P_k^h}{d\ln k} = -2\epsilon$$

ADM Formalism (Maldacena 02)

- Metric

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Comoving gauge: $\delta\phi = 0 \quad , \quad h_{ij} = a^2 e^{2\zeta}\delta_{ij} \quad .$

- Action

$$S = \frac{1}{2}\int dt d^3x \sqrt{h} N (R^{(3)} + 2P) + \frac{1}{2}\int dt d^3x \sqrt{h} N^{-1}(E_{ij}E^{ij} - E^2)$$

where $E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$

- N and N^i are Lagrangian multipliers

In order to get the cubic expansion of $\mathcal{L}$, it is enough to solve N and N^i to the first order, because

$$S \supset \int d^4x \text{ [zeroth order constraint eqn]} \cdot N^{(n)}$$
$$+ \int d^4x \text{[first order constraint eqn]} \cdot N^{(n-1)} + \cdots$$

- Quadratic and cubic expansion

(Seery & Lidsey 05; Maldacena 02)

$$S_2 = \int dt d^3x \, [a^3 \frac{\epsilon}{c_s^2}\dot{\zeta}^2 - a\epsilon(\partial\zeta)^2] \, ,$$

$$S_3 = \int dt d^3x [-\epsilon a \zeta (\partial\zeta)^2 - a^3(\Sigma + 2\lambda)\frac{\dot{\zeta}^3}{H^3} + \frac{3a^3\epsilon}{c_s^2}\zeta\dot{\zeta}^2$$
$$+\frac{1}{2a}(3\zeta - \frac{\dot{\zeta}}{H})(\partial_i\partial_j\psi\partial_i\partial_j\psi - \partial^2\psi\partial^2\psi) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi] \, .$$

Quadratic Part

- Fourier transform $\quad u_k = \int d^3x \; \zeta(t, \mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}}$

  Define $\quad v_k \equiv z u_k \;,\; z \equiv \frac{a\sqrt{2\epsilon}}{c_s}$

  E.O.M. $\quad v_k'' + c_s^2 k^2 v_k - \frac{z''}{z} v_k = 0$

- For short wavelength, $\frac{k}{a} > \frac{H}{c_s}$, v_k is oscillating;

  For long wavelength, $\frac{k}{a} < \frac{H}{c_s}$, u_k is frozen.

- The solution

$$u_k = u(\tau, \mathbf{k}) = \frac{iH}{\sqrt{4\epsilon c_s k^3}} (1 + ikc_s\tau) e^{-ikc_s\tau}$$

$$\text{Condition} : -k\tau\Delta c_s \ll c_s k \Delta\tau$$

$$\Rightarrow s = \frac{\dot{c}_s}{Hc_s} \ll 1$$

- Only requires the variation of c_s be slow;

  the sound speed c_s can be arbitrary.

The Cubic Part

- Integrate by parts, bring S_3 to $\mathcal{O}(\epsilon^2)$ except for terms proportional to $1-c_s^2$ or λ.

$$S_3 = \int dt d^3x \{ -a^3 (\Sigma(1-\frac{1}{c_s^2}) + 2\lambda)\frac{\dot{\zeta}^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot{\zeta}^2$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - 2s + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot{\zeta}(\partial\zeta)(\partial\chi)$$
$$+ \frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}(\frac{\eta}{c_s^2})\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\frac{\delta L}{\delta\zeta}|_1 \} ,$$

where

$$\partial^2\chi = a^2 \frac{\epsilon}{c_s^2}\dot{\zeta} ,$$

$$\frac{\delta L}{\delta \zeta} |_1 \; = a \left(\frac{d\partial^2\chi}{dt} + H\partial^2\chi - \epsilon\partial^2\zeta \right) ,$$

$$f(\zeta) = \frac{\eta}{4c_s^2}\zeta^2 + \text{ (terms with at least one derivative on } \zeta) .$$

- The last term can be absorbed by a redefinition:

$$\zeta \to \zeta_n + f(\zeta_n) ,$$

which introduces an extra term in $\langle\zeta^3\rangle$:

$$\langle \zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\zeta(\mathbf{x}_3)\rangle = \langle \zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_2)\zeta_n(\mathbf{x}_3)\rangle$$
$$+ \frac{\eta}{2c_s^2}(\langle\zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_2)\rangle\langle\zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_3)\rangle + \text{sym}) + \mathcal{O}(\eta^2) .$$

• The 3-point function

$$\langle\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle = -i\int_{t_0}^{t} dt'\langle[\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3), H_{int}(t')]\rangle\ ,$$

where H_{int} is the interaction Hamiltonian given by S_3.

• The leading contributions from each term:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(P_k^\zeta)^2\frac{1}{\Pi_i k_i^3}\ \mathcal{A}\ ,$$

where,

$$\begin{aligned}\mathcal{A} \supset & \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2K^3} \\ & + \left(\frac{1}{c_s^2}-1\right)\left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3+\frac{1}{8}\sum_i k_i^3\right) \\ & + \frac{\epsilon}{c_s^2}\left(-\frac{1}{8}\sum_i k_i^3+\frac{1}{8}\sum_{i\neq j}k_ik_j^2+\frac{1}{K}\sum_{i>j}k_i^2k_j^2\right) \\ & + \frac{\eta}{c_s^2}\left(\frac{1}{8}\sum_i k_i^3\right) \\ & + \frac{s}{c_s^2}\left(-\frac{1}{4}\sum_i k_i^3-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right)\ .\end{aligned}$$

Correction Terms

- The solution for the quadratic part

$$
\begin{aligned}
u_k(y) \approx & \frac{iH}{\sqrt{4\epsilon c_s k^3}}(1 + ikc_s\tau)e^{-ikc_s\tau} \\
& \to -\frac{\sqrt{\pi}}{2\sqrt{2}} \frac{H}{\sqrt{\epsilon c_s}} \frac{1}{k^{3/2}}(1 + \frac{\epsilon}{2} + \frac{s}{2})\ e^{i\frac{\pi}{2}(\epsilon + \frac{\eta}{2})} \\
& \times\ y^{3/2} H^{(1)}_{\frac{3}{2}+\epsilon+\frac{\eta}{2}+\frac{s}{2}}((1+\epsilon+s)y)
\end{aligned}
$$

where $\qquad y \equiv \frac{c_s k}{aH}$

- Slowly-varying parameters H, c_s, λ and ϵ

$$
\begin{aligned}
f(\tau) \approx & f(\tau_K) \\
& \to f(\tau_K) - \frac{\partial f}{\partial t}\frac{1}{H_K} \ln \frac{\tau}{\tau_K} + \mathcal{O}(\epsilon^2 f)
\end{aligned}
$$

- The scale factor

$$
\begin{aligned}
a \approx & -\frac{1}{H_K \tau} \\
& \to -\frac{1}{H_K\tau} - \frac{\epsilon}{H_K\tau} + \frac{\epsilon}{H_K\tau}\ln(\tau/\tau_K) + \mathcal{O}(\epsilon^2)
\end{aligned}
$$

Final results

• The 3-point correlation function for a general single field inflation to $\mathcal{O}(\epsilon)$:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\tilde{P}_K^\zeta)^2\frac{1}{\Pi_i k_i^3} \times (\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s) \ ,$$

where we have decomposed the shape into six parts ($K \equiv k_1+k_2+k_3$)

$$\begin{aligned}
\mathcal{A}_\lambda &= \left(\frac{1}{c_s^2}-1-\frac{\lambda}{\Sigma}[2-(3-2\mathbf{c}_1)l]\right)_K \frac{3k_1^2k_2^2k_3^2}{2K^3} \ , \\
\mathcal{A}_c &= \left(\frac{1}{c_s^2}-1\right)_K \left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3+\frac{1}{8}\sum_i k_i^3\right) \ , \\
\mathcal{A}_o &= \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)_K (\epsilon F_{\lambda\epsilon}+\eta F_{\lambda\eta}+sF_{\lambda s}) \\
&\quad + \left(\frac{1}{c_s^2}-1\right)_K (\epsilon F_{c\epsilon}+\eta F_{c\eta}+sF_{cs}) \ , \\
\mathcal{A}_\epsilon &= \epsilon\left(-\frac{1}{8}\sum_i k_i^3+\frac{1}{8}\sum_{i\neq j}k_ik_j^2+\frac{1}{K}\sum_{i>j}k_i^2k_j^2\right) \ , \\
\mathcal{A}_\eta &= \eta\left(\frac{1}{8}\sum_i k_i^3\right) \ , \\
\mathcal{A}_s &= sF_s \ .
\end{aligned}$$

• Completely determined by 5 parameters:

$$c_s \ , \quad \frac{\lambda}{\Sigma} \ , \quad \epsilon \ , \quad \eta \ , \quad s.$$

- Size of non-Gaussianities

  WMAP ansatz:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(-\frac{3}{10}f_{NL}(P_k^\zeta)^2)\frac{\Sigma_i k_i^3}{\Pi_i k_i^3}$$

  Taking equilateral limit in our results:

$$f_{NL}^\lambda = -\frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)+(3-2\mathbf{c}_1)\frac{l\lambda}{\Sigma}\ ,$$
$$f_{NL}^c = \frac{35}{108}\left(\frac{1}{c_s^2}-1\right)\ ,$$
$$f_{NL}^o = \mathcal{O}\left(\frac{\epsilon}{c_s^2}\ ,\ \frac{\epsilon\lambda}{\Sigma}\right)\ ,$$
$$f_{NL}^{\epsilon,\eta,s} = \mathcal{O}(\epsilon)\ .$$

  Large non-Gaussianity $\rightarrow$ small c_s or large λ/Σ

- Current bound: $|f_{NL}| < 300$ for the first two,

  and $|f_{NL}| < 100$ for the rest.

  (WMAP team 06; Creminelli, Nicolis, Senatore, Tegmark & Zaldarriaga 05)

- WMAP will eventually reach $|f_{NL}| \lesssim 20$;
  Planck satellite $|f_{NL}| \lesssim 5$.

- Shape of non-Gaussianities

  Plot $\mathcal{A}(1,x_2,x_3)/x_2x_3$, as in Babich, Creminelli & Zaldarriaga, 04.

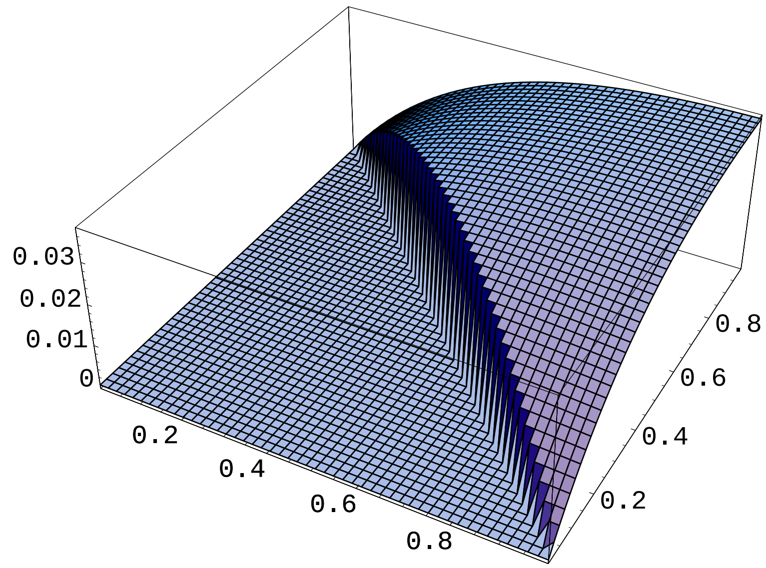


FIG. 1. The shape of $-\mathcal{A}_\lambda/k_1k_2k_3$

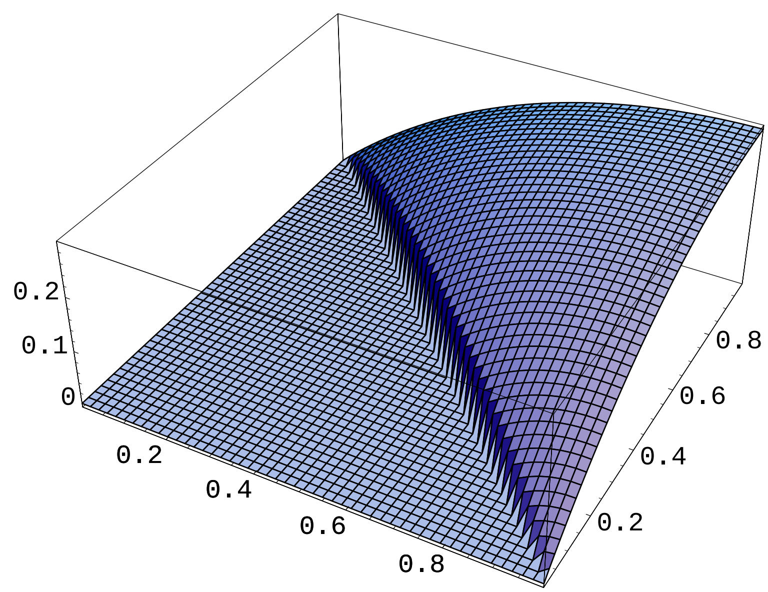


FIG. 2. The shape of $\mathcal{A}_c/k_1k_2k_3$

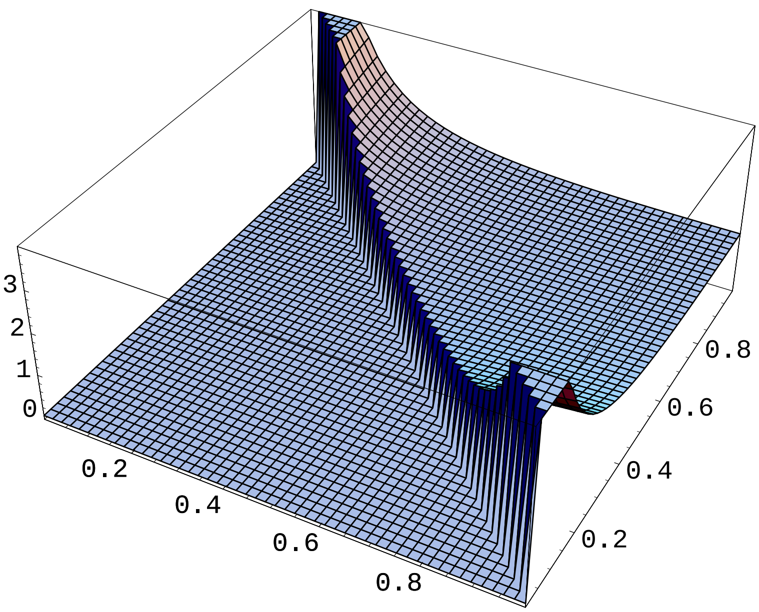


FIG. 3. The shape of $-\mathcal{A}_\epsilon/k_1k_2k_3$

Slow-Roll Inflation

- $c_s \approx 1$ & $\lambda = 0$

  In this limit, our formulae recover the slow-roll result:
  (Maldacena, 02; Seery & Lidsey, 05)

$$
\begin{aligned}
\mathcal{A} = &-\left(\frac{\epsilon}{3\epsilon_X}s + u\right)\frac{k_1^2 k_2^2 k_3^2}{K^3} \\
&- u\left(-\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8}\sum_i k_i^3\right) \\
&+ \epsilon\left(-\frac{1}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{K}\sum_{i>j} k_i^2 k_j^2\right) \\
&+ \eta\left(\frac{1}{8}\sum_i k_i^3\right)
\end{aligned}
$$

where, $\epsilon_X \equiv -\frac{\dot{X}}{H^2}\frac{\partial H}{\partial X}$, $u \equiv 1 - \frac{1}{c_s^2} \ll 1$

- In slow-roll inflation, the non-Gaussianity is unobservable, $f_{NL} = \mathcal{O}(\epsilon)$.

DBI Inflation

- $\frac{2\lambda}{\Sigma} = \frac{1}{c_s^2} - 1$, so the leading order of $\mathcal{A}_\lambda$ vanishes.

  In this limit, our formulae recovers the DBI result:
  (Alishahiha, Silverstein & Tong, 04)

$$\mathcal{A}_c = \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8}\sum_i k_i^3\right)$$

- Non-Gaussianity compatible with current bound, and potentially observable in future experiments:

$$f_{NL} \approx 0.32 c_s^{-2} \ .$$

  In the UV model, $c_s^{-1} \approx \sqrt{\frac{2\lambda}{3}\frac{m M_{\rm Pl}}{\phi^2}}$;

  In the IR model, $c_s^{-1} \approx \beta N_e/3 \ \ (\beta \sim 1)$.

  Possible to have $c_s^2 \ll \mathcal{O}(\epsilon)$
  $\Rightarrow$ even $\mathcal{A}_o$ becomes observable.

- Running of non-Gaussianity
  $\Rightarrow$ Shape of geometry in extra dimensions. (X.C. 05)

  Define: $$n_{NG} - 1 \equiv \frac{d \ln |f_{NL}|}{d \ln k} \approx -2s \ .$$

$$\text{AdS geometry}: \quad n_{NG} - 1 < 0 \ ;$$
$$\text{constant warp factor}: \quad n_{NG} - 1 > 0 \ .$$

K-inflation

- Another leading shape: (Gruzinov, 04)

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2K^3} \ .$$

- Potentially observable in k-inflation:

$$\mathcal{A}_\lambda = \frac{12}{\gamma}\left(\frac{k_1^2k_2^2k_3^2}{K^3}\right) \ ,$$
$$\mathcal{A}_c = \frac{8}{\gamma}\left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{8}\sum_i k_i^3\right) \ .$$

$$f_{NL} \approx \frac{2.1}{\gamma}$$

- Sound speed is constant, non-Gaussianity does not run, $n_{NG} - 1 = 0$.

A Probe of Inflationary Vacuum

- Deviation from Bunch-Davis vacuum:

$$u_k = \frac{iH}{\sqrt{4\epsilon c_s k^3}}(C_+(1+ikc_s\tau)e^{-ikc_s\tau} + C_-(1-ikc_s\tau)e^{ikc_s\tau})$$

$$C_+ = 1 \quad ; \quad C_- \neq 0, \quad \text{but small} .$$

- Replace one of k_i with $-k_i$:

$$\tilde{A}_\lambda = Re(C_-)(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma})\frac{3k_1^2k_2^2k_3^2}{2} \times (\frac{1}{(k_1+k_2-k_3)^3} + \frac{1}{(k_1-k_2+k_3)^3} + \frac{1}{(-k_1+k_2+k_3)^3}),$$

$$\tilde{A}_c = Re(C_-)(\frac{1}{c_s^2} - 1)\sum_{p=1}^{3}(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{8}\sum_i k_i^3)|_{k_p\to -k_p} \ .$$

- Shape of this non-Gaussianities are distinctive in folded triangle limit, for example, when $k_1 + k_2 = k_3$.

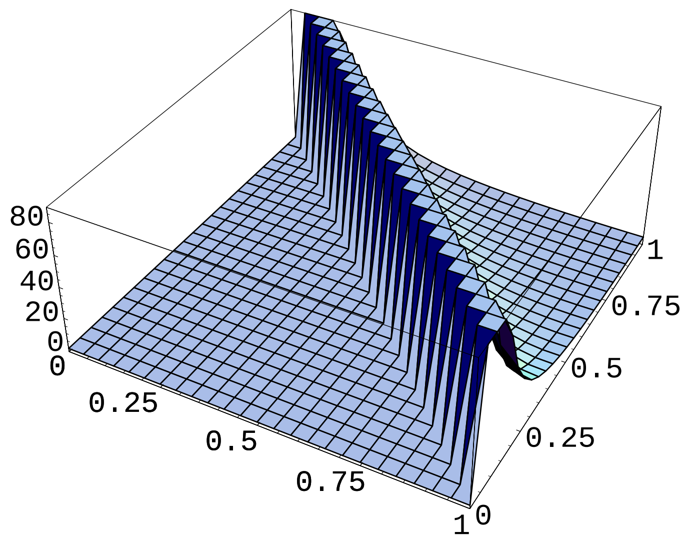


FIG. 4. The shape of $|\tilde{\mathcal{A}}_\lambda|/k_1k_2k_3$

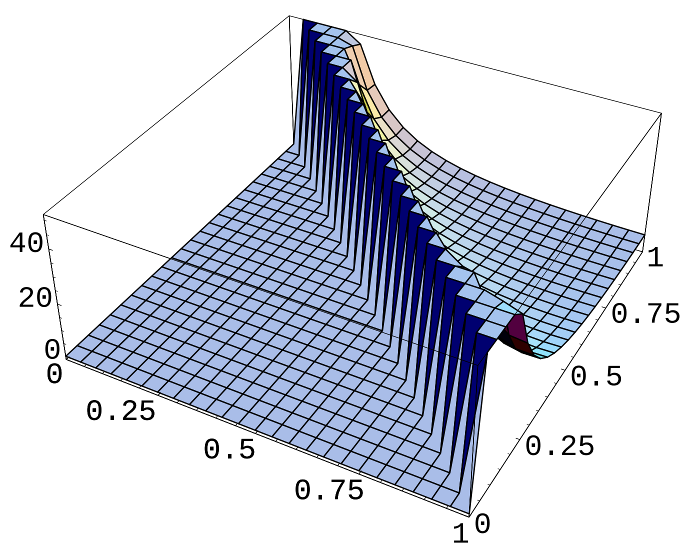


FIG. 5. The shape of $|\tilde{\mathcal{A}}_c|/k_1k_2k_3$

Conclusion and Future Directions

- Results:
  — A full non-Gaussianity specified by 5 parameters;
  — Explicit form of momentum dependence,
  including a few potentially observable;
  — Recovered all previous known results,
  and explored unknown regions.

- Future directions:

- Other models with large non-Gaussianities with sensible UV origin;

- Non-Gaussianities in general multi-field inflation;

- Derive non-Gaussianities from symmetry principle, such as those in dS/CFT correspondence;

- Novel features in dual non-gravitational theory for $c_s \ll 1$.