



SMR 1760 - 17

**COLLEGE ON
PHYSICS OF NANO-DEVICES**

10 - 21 July 2006

***Introduction to Spintronics III:
Spin-orbit coupling***

Presented by:

Allan H. MacDonald

University of Texas at Austin, U.S.A

Commercial MRAM !!

By Chris Nuttall in Austin

Updated: 8:40 a.m. ET July 10, 2006

A new memory technology based on magnetism that could eventually supersede existing types of computer memory has made its commercial debut. Freescale, the chipmaker spun out of Motorola, is announcing the first Magnetoresistive Random Access Memory (MRAM) device, more than ten years after it first began research into the technology.

<http://msnbc.msn.com/id/13796288/>

Introduction to Spintronics

ICTP July 2006

University of Texas at Austin



Ramin Abolfath Tomas Dietl Rembert Duine Tomas Jungwirth Brian Gallagher
Paul Haney Juergen Koenig Hsiu-Hau Lin Alvaro Nunez Enrico Rossi
Nitin Samarth Peter Schiffer John Schliemann Jairo Sinova Maxim Tsoi

- I Metal Spintronics
- II Magnetic Semiconductors
- III Spin-Orbit Coupling

Reviews

Bir and Pikus - Wiley (1974)
Engel, Rashba, Halperin - cond-mat/0603306
Winkler - Springer-Verlag (2003)

Spin-orbit Interactions

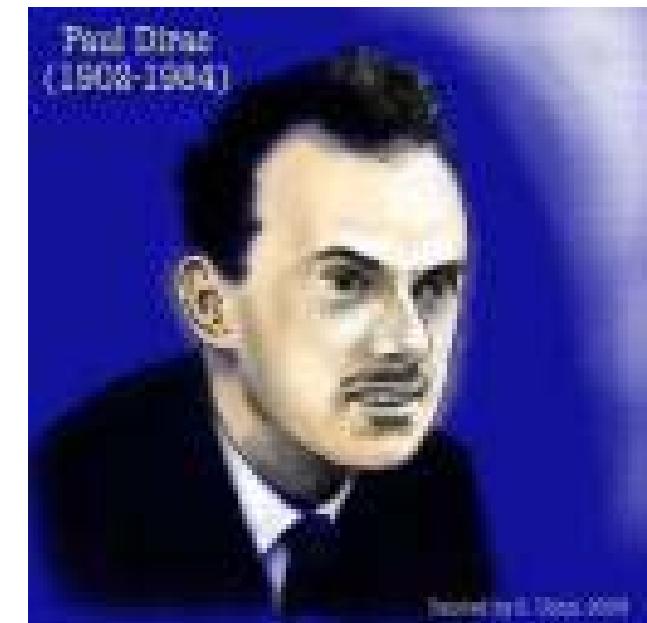
Optical Orientation

Anomalous Hall Effect

Spin Hall Effect

Spin-Orbit Interactions

Microscopic



$$\left(\alpha_0 mc^2 + \sum_{j=1}^3 \alpha_j p_j c \right) \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi}{\partial t}(\mathbf{x}, t)$$

Four Component Spinors

$$\alpha_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad \alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

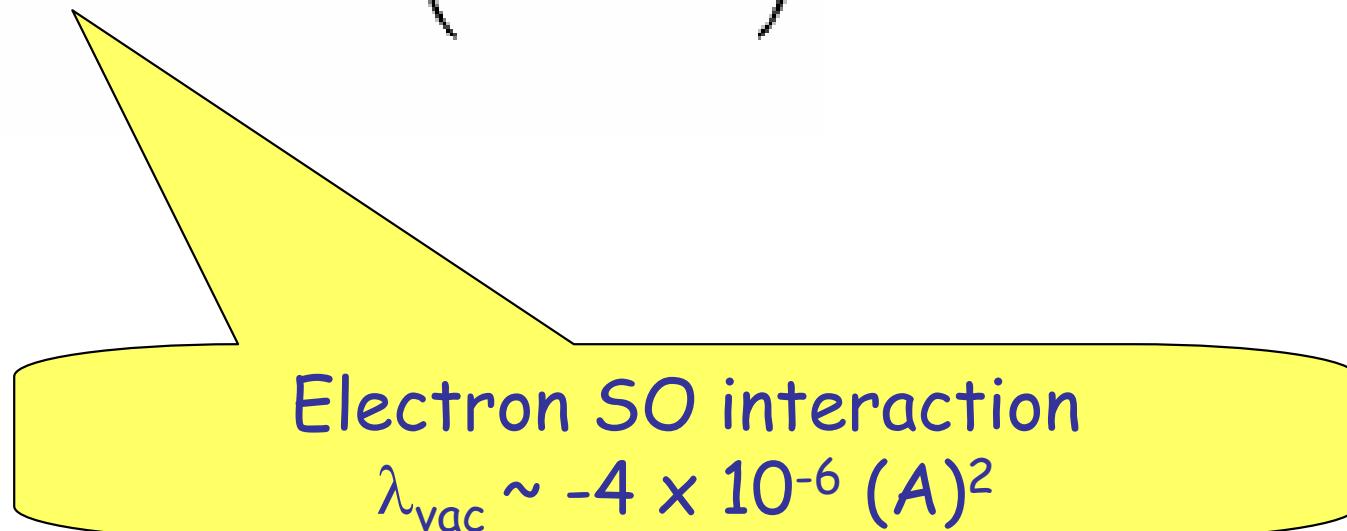
$$\alpha_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \quad \alpha_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

Pauli Matrices

Spin-Orbit Interactions

Microscopic

$$H_{SO, \text{vac}} = \lambda_{\text{vac}} \sigma \cdot (\mathbf{k} \times \nabla \tilde{V})$$



Foldy & Wouthuysen – PRB (1950)

k.p & Envelope Functions

$$\Psi_{\vec{k}}(\vec{r}) = \exp(i\vec{k} \cdot \vec{r}) u_{\vec{k}}(\vec{r})$$

Bloch's Theorem

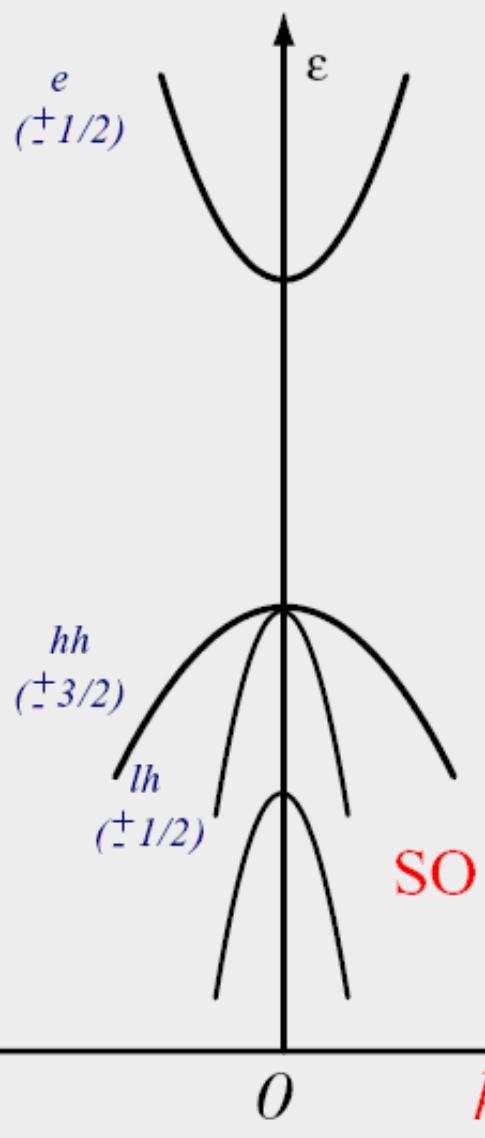
$$\left[\frac{1}{2m} (\vec{p} + \hbar\vec{k})^2 + V(\vec{r}) \right] u_{\nu, \vec{k}}(\vec{r}) = E_{\nu, \vec{k}} u_{\nu, \vec{k}}(\vec{r})$$

Primitive Cell
Hamiltonian

$$H_{\nu', \nu}(\vec{k}) = [E_{\nu, \vec{k}=0} + \frac{\hbar^2 k^2}{2m}] \delta_{\nu', \nu} + \frac{\hbar}{m} \vec{k} \cdot \vec{p}_{\nu', \nu}$$

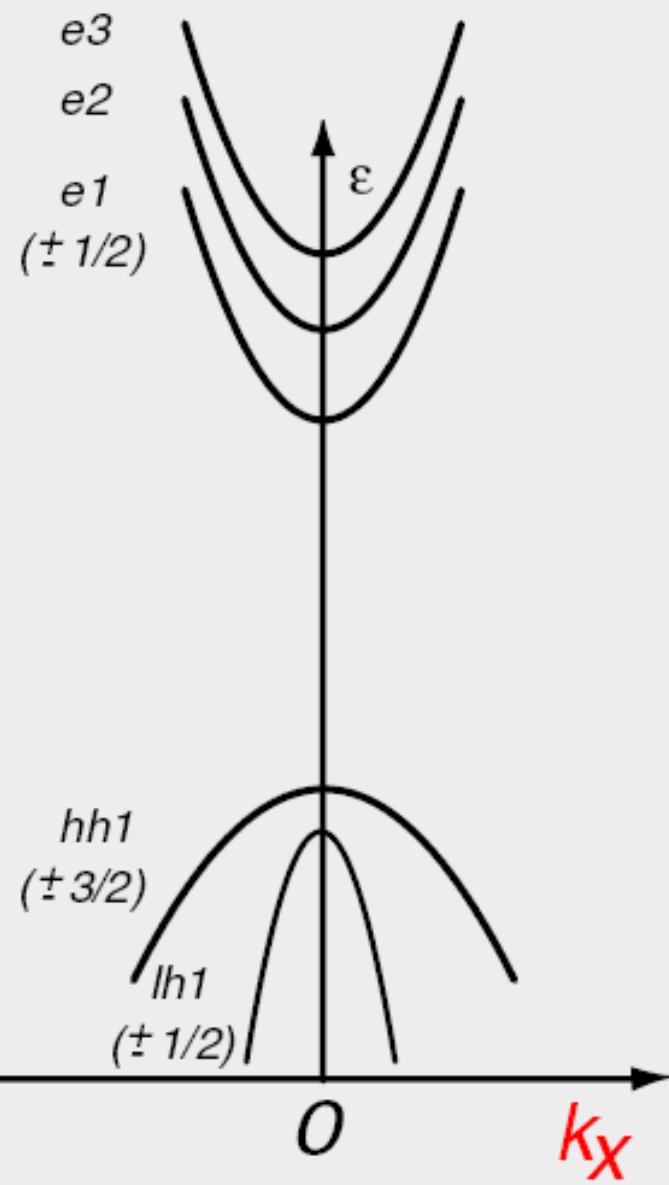
Envelope Function
Hamiltonian

Bulk



Bulk

<100> QW



<100> QW

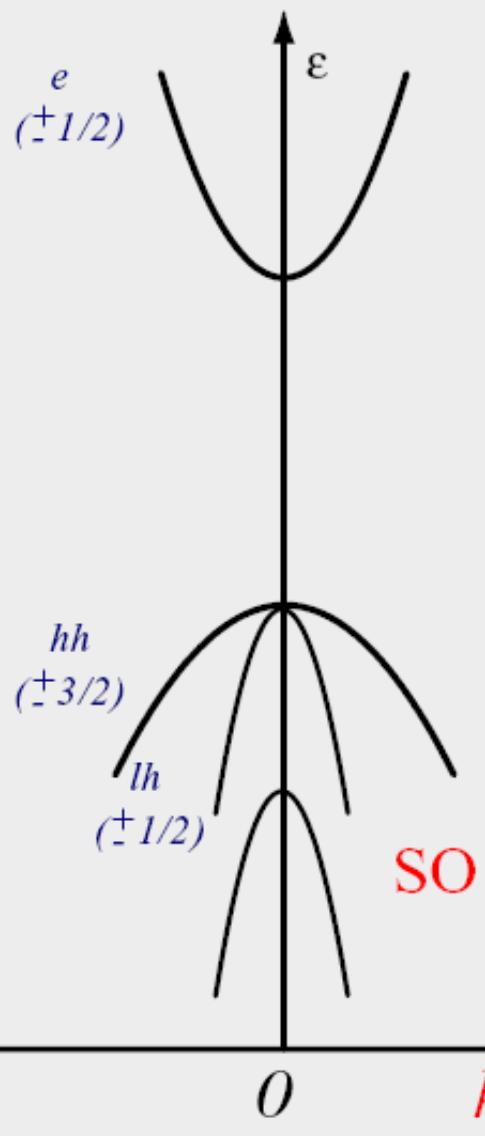
Luttinger Hamiltonian

$$H_L = \frac{\hbar^2}{m_0} \left[\left(\gamma_1 + \frac{5}{2}\gamma_2 \right) \frac{k^2}{2} - \gamma_3 (\mathbf{k} \cdot \mathbf{J})^2 + (\gamma_3 - \gamma_2) \sum_i k_i^2 J_i^2 \right]$$



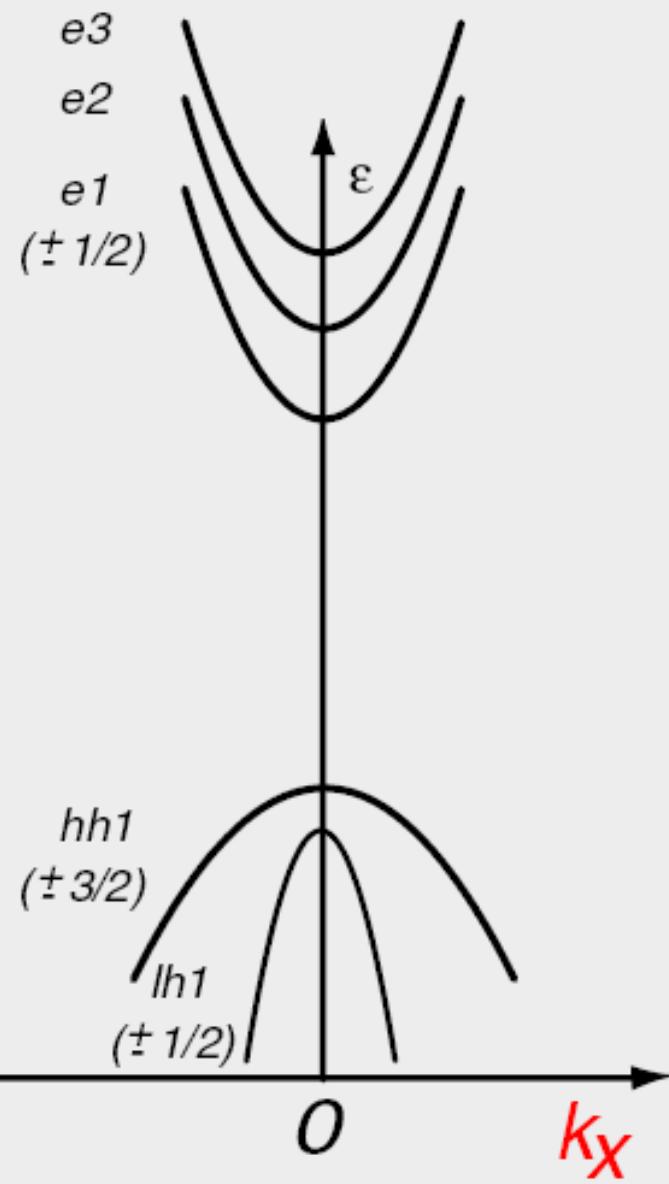
J=3/2
Spin Operators

Bulk



Bulk

<100> QW



<100> QW

Spin-Orbit Interactions

k.p - conduction

$$H_{D,3d} = B k_x (k_y^2 - k_z^2) \sigma_x + c.p.$$

BIA
Dresselhaus

$$H_{\text{ext}} = \lambda \boldsymbol{\sigma} \cdot (\mathbf{k} \times \nabla V)$$

$\lambda \sim .05 \text{ (nm)}^2$

$$H_\alpha = \alpha (k_y \sigma_x - k_x \sigma_y)$$

BIA
Rashba

Spin-Orbit Interactions

k.p - Valence

$$H_L = \frac{\hbar^2}{m_0} \left[\left(\gamma_1 + \frac{5}{2}\gamma_2 \right) \frac{k^2}{2} - \gamma_3 (\mathbf{k} \cdot \mathbf{J})^2 + (\gamma_3 - \gamma_2) \sum_i k_i^2 J_i^2 \right]$$

Luttinger
Hamiltonian

$$H_{\alpha,h} = i\alpha_h (k_-^3 \sigma_+ - k_+^3 \sigma_-)$$

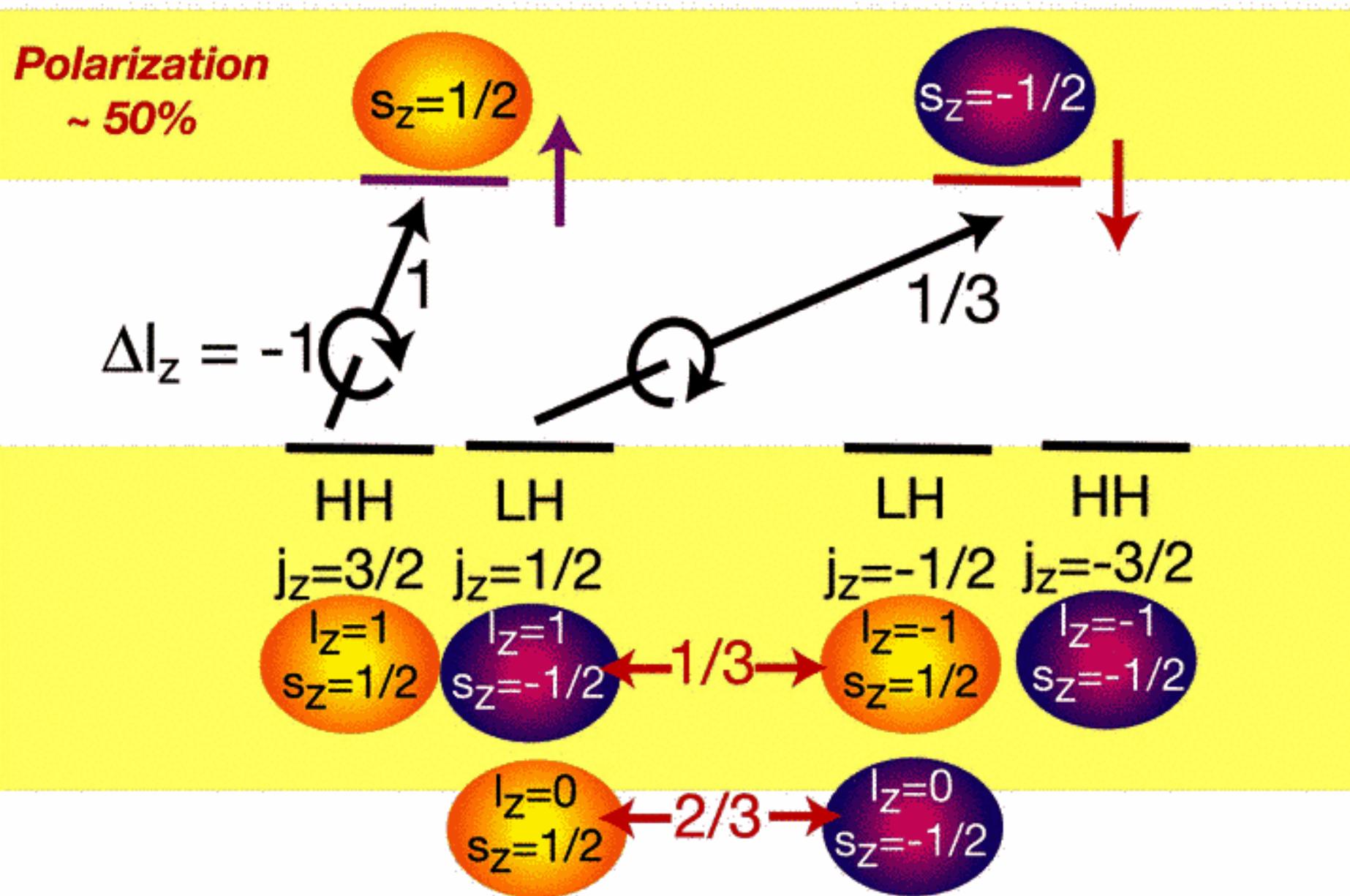
Valence Band BIA
k-cubic Rashba

Spin-orbit Interactions

Optical Orientation

Anomalous Hall Effect

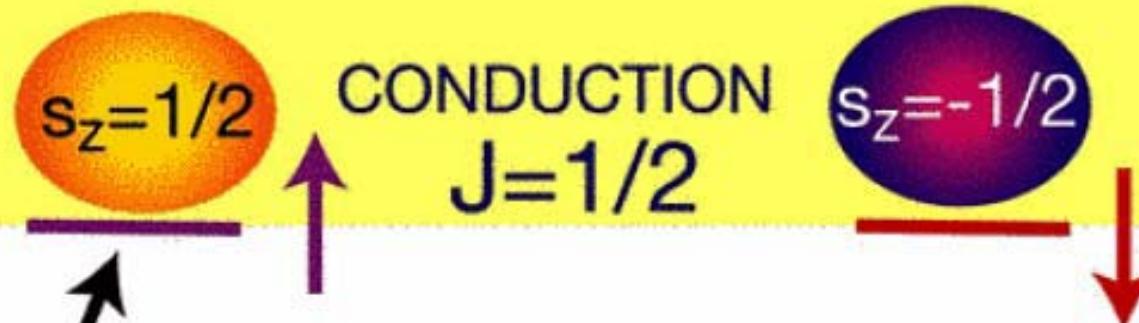
Spin Hall Effect





Optical excitation of spin-polarized distributions

Polarization
~ 100%



$$\Delta l_z = -1$$

Circularly polarized light
changes orbital quantum number

HH

$j_z = 3/2$

$l_z = 1$

$s_z = 1/2$

VALENCE
 $J=3/2$

Strain
confinement

LH

HH

$j_z = -3/2$

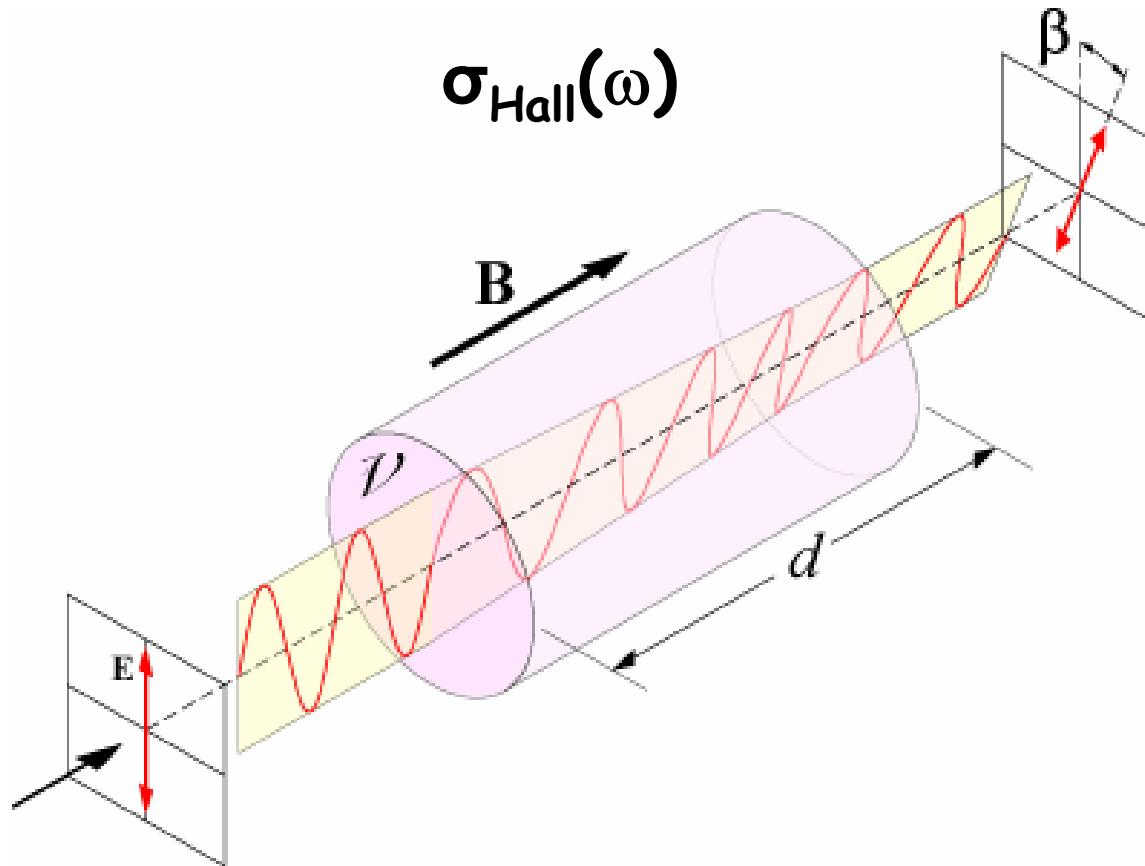
$l_z = -1$

$s_z = -1/2$

LH

Low-dimensional structures (e.g. quantum wells)

Faraday Effect



(Magneto-optical Kerr Effect - Reflection)

Spin-orbit Interactions

Optical Orientation

Anomalous Hall Effect

Spin Hall Effect

Anomalous Hall Effect

$$(\sigma_{xy} - \sigma_{yx})/2 = \sigma_H$$



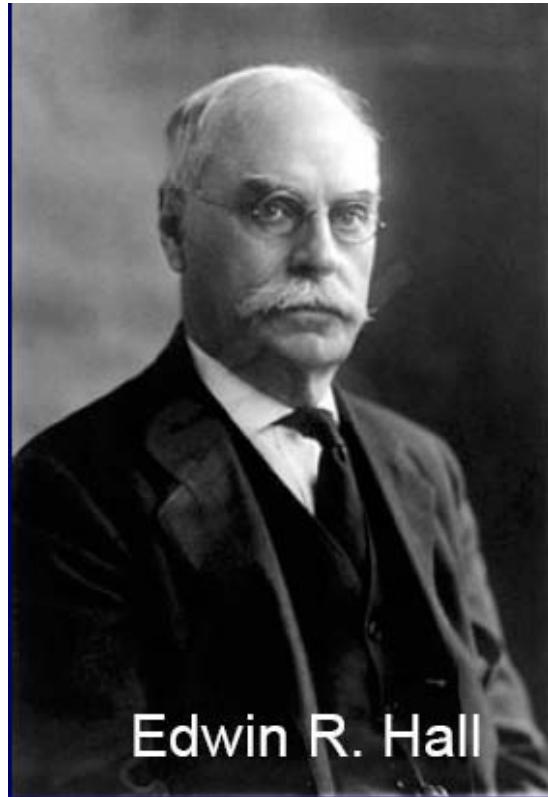
no time reversal invariance

+

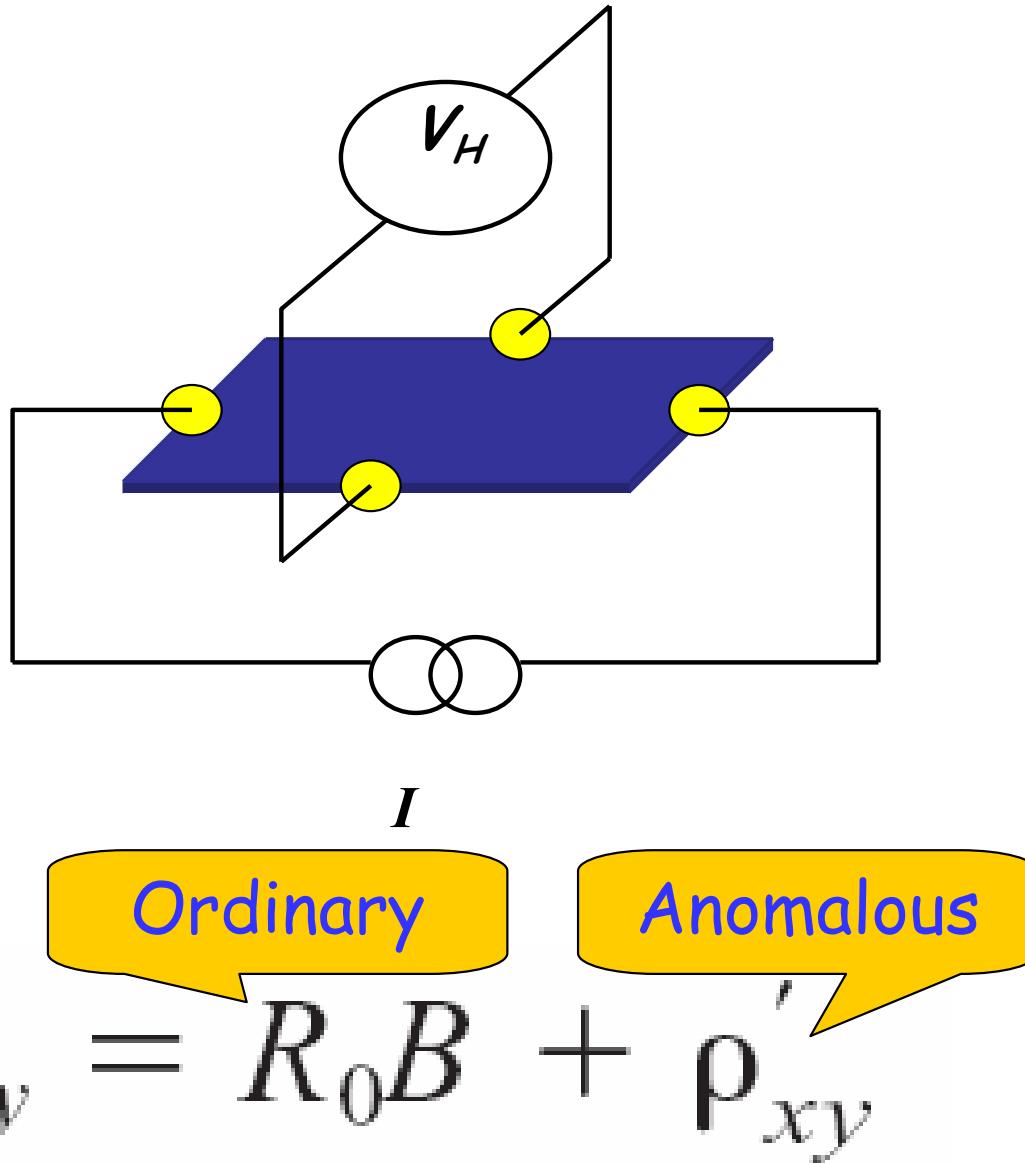
spin-orbit coupling

(not sufficient)

Anomalous Hall Effect



Edwin R. Hall

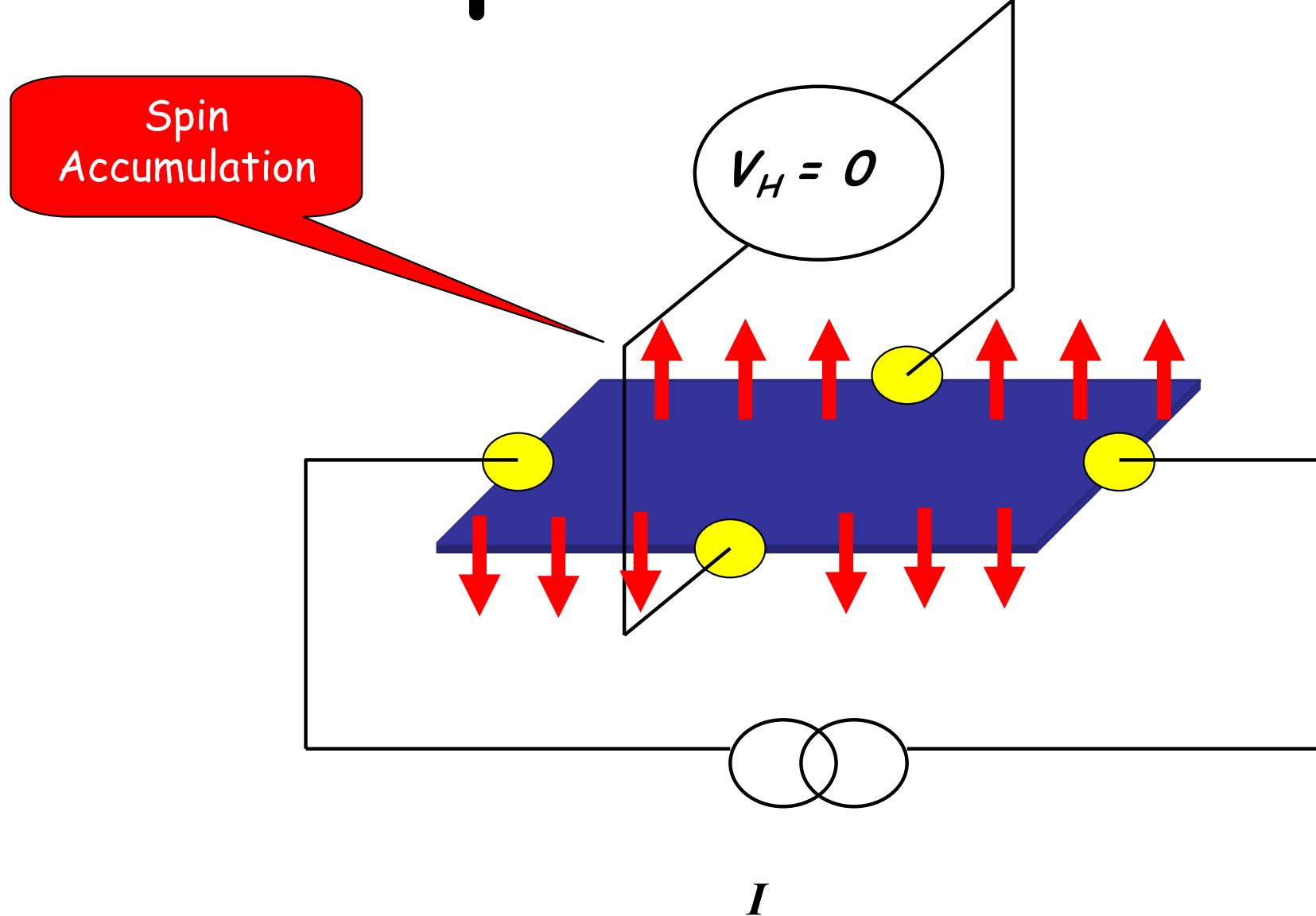


Ordinary

Anomalous

$$\rho_{xy} = R_0 B + \rho'_{xy}$$

Spin Hall Effect



I

Dyakanov & Perel (1971)
Kato et al., Wunderlich et al. (2005)

AHE is
complicated
because
it is small

Traditional Transport Theory: Boltzmann Equation

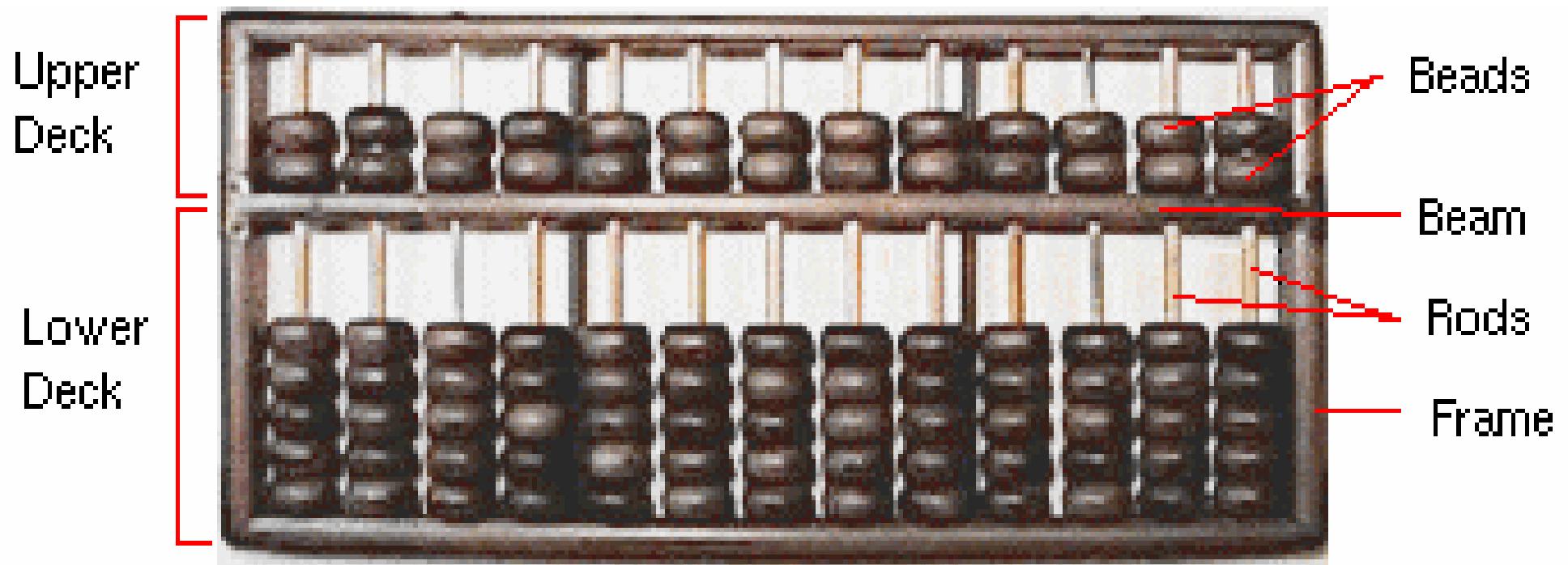
$$\frac{\partial f_l}{\partial t} + eE_x|v_l|\cos(\phi)\frac{df_0(\epsilon_l)}{d\epsilon_l} = - \sum_{l'} \omega_{ll'}(f_l - f_{l'})$$

$l = (n, k)$

f_l = occupation probability of Bloch state l

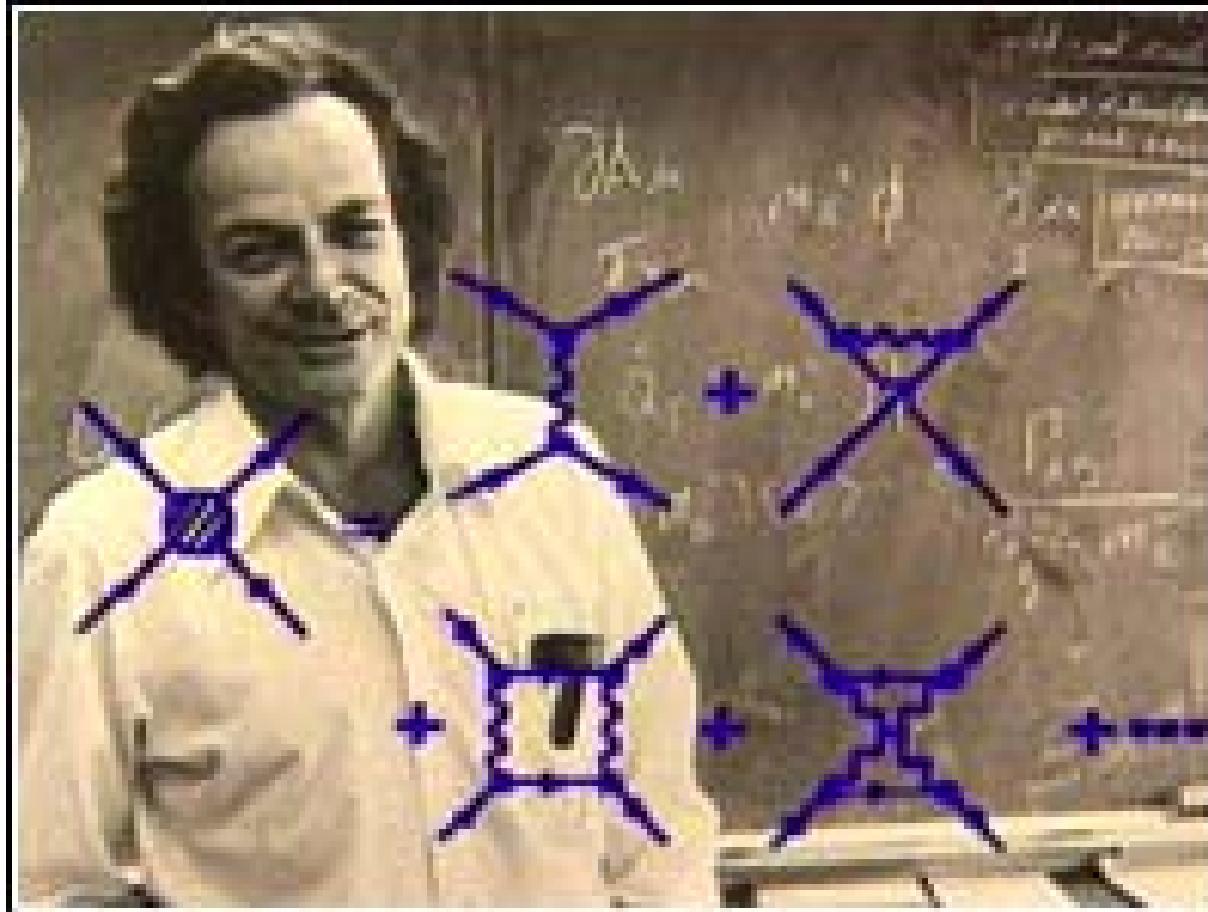
$$j = -e \sum_l v_l f_l$$

Pictorial Arithmetic



First Used in China circa 1200 A.D.

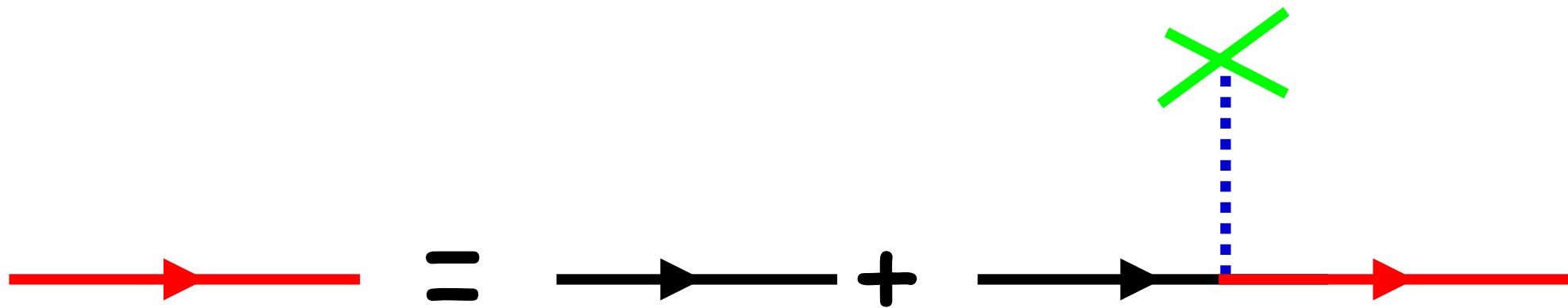
Pictorial Perturbation Theory



Feynman earned his Nobel for creating these diagrams

First used by Richard Feynman circa 1950 A.D.

Perturbation Theory

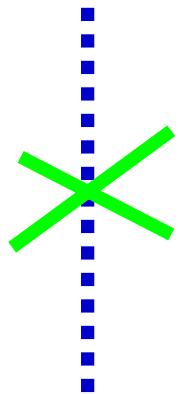


Bloch Electron

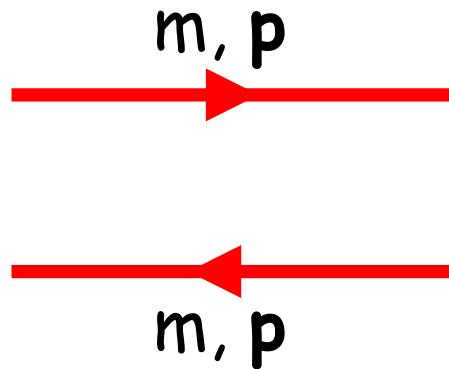


Real Eigenstates

Perturbation Theory

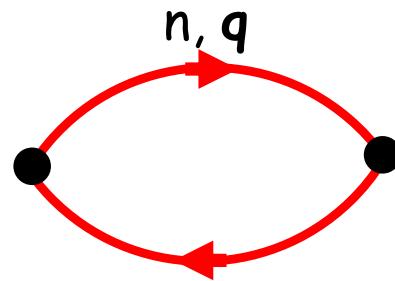


$$= \tau^{-1} / v_0$$

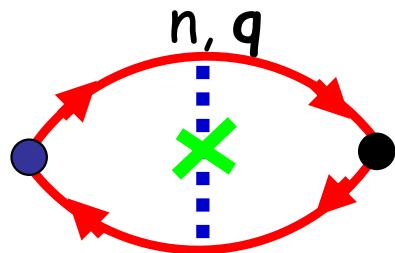


$$= v_0 / \tau^{-1}$$

Perturbation Theory

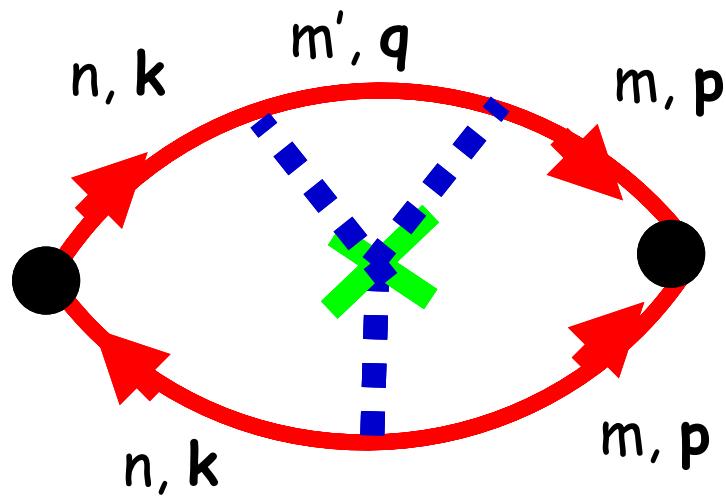


Drude Conductivity
 $\sigma = ne^2\tau/m^*$



Vertex Corrections
 $\sim 1 - \cos(\theta)$

Perturbation Theory - Skew



$$\sigma_H^{\text{Skew}} \sim (\tau_{\text{Skew}})^{-1} \tau^2$$

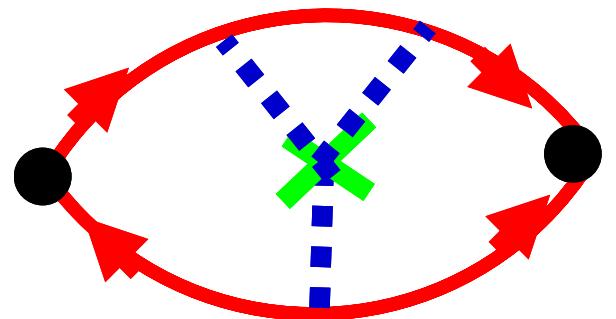
$$\bullet = j_v = -ev_v$$

Principle of Microscopic Reversibility

$$\left[\frac{\partial f_{\mathbf{k}}}{\partial t} \right]_{\text{scatt.}} = \int \{f_{\mathbf{k}'}(1-f_{\mathbf{k}}) - f_{\mathbf{k}}(1-f_{\mathbf{k}'})\} Q(\mathbf{k}, \mathbf{k}') d\mathbf{k}'. \quad (7.6)$$

The process of scattering depends on $f_{\mathbf{k}}$, the number of vacancies in the state \mathbf{k} , and on $(1-f_{\mathbf{k}'})$, the number of vacancies available in the final state. There is also the inverse process, from \mathbf{k}' into \mathbf{k} , which increases $f_{\mathbf{k}}$, and which is weighted with $f_{\mathbf{k}'}(1-f_{\mathbf{k}})$. We sum over all possible other states \mathbf{k}' . For each value of \mathbf{k} and \mathbf{k}' , however, there is a basic transition probability $Q(\mathbf{k}, \mathbf{k}')$, which would measure the rate of transition if, say, \mathbf{k} were known to be occupied and \mathbf{k}' known to be empty. The principle of *microscopic reversibility* tells us that the same function measures the transition rate from \mathbf{k}' to \mathbf{k} , so this is a common factor in the integrand.

Skewness

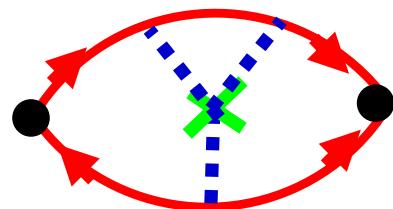


$$S = Q(k,p)/Q(p,k) - 1$$

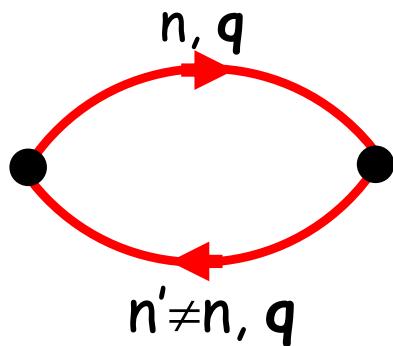
~

$$\nabla v_0 \operatorname{Im}[\langle k|q\rangle\langle q|p\rangle\langle p|k\rangle]$$

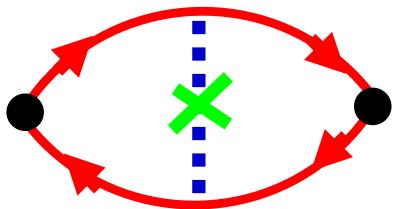
AHE - Perturbation Theory



Skew
 $\sim \sigma_0 S$

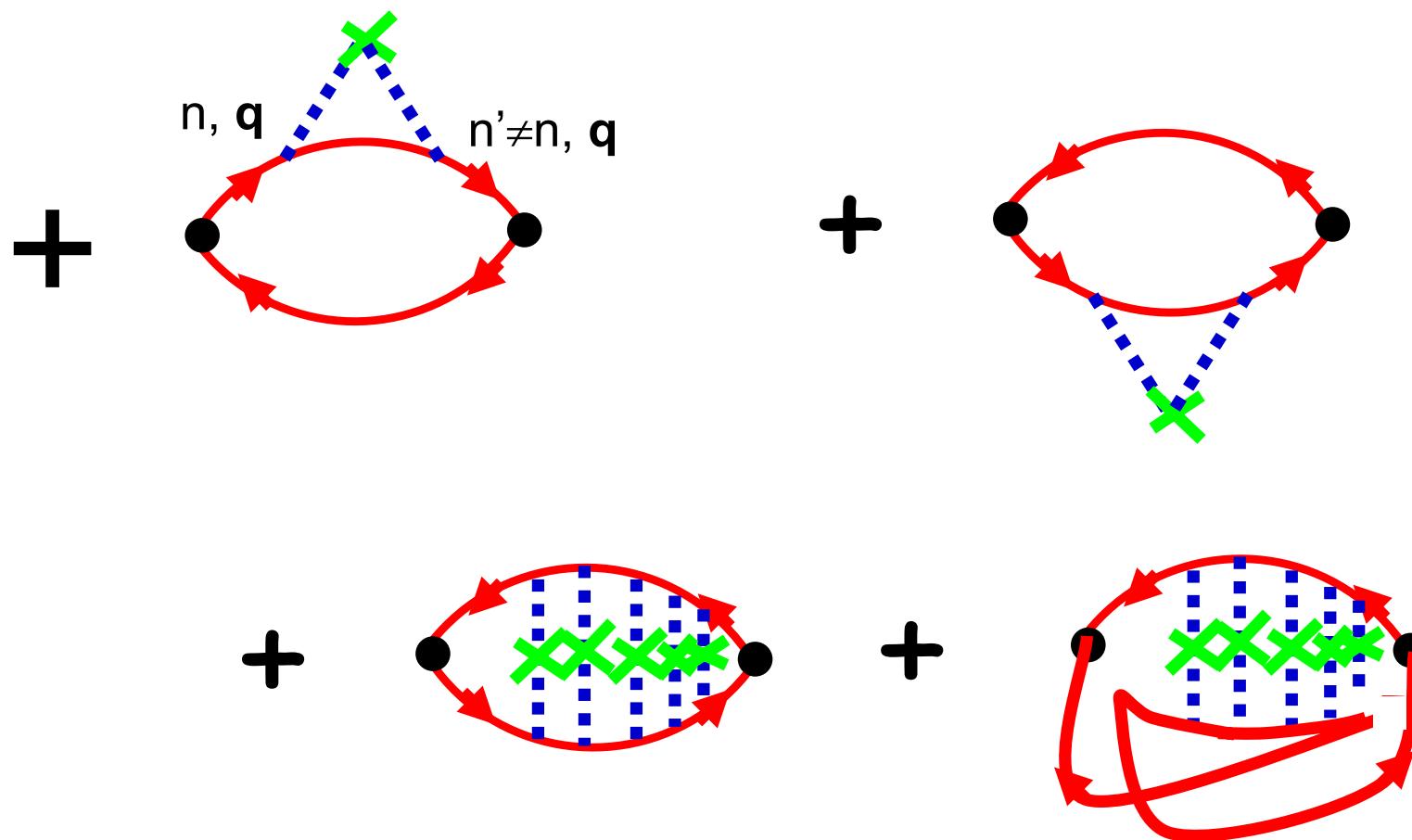


Intrinsic
 $\sim \sigma_0 / \epsilon_F \tau$



Vertex Corrections
 $\sim \sigma_{\text{Intrinsic}}$

AHE - Perturbation Theory



Intrinsic Anomalous Hall Effect

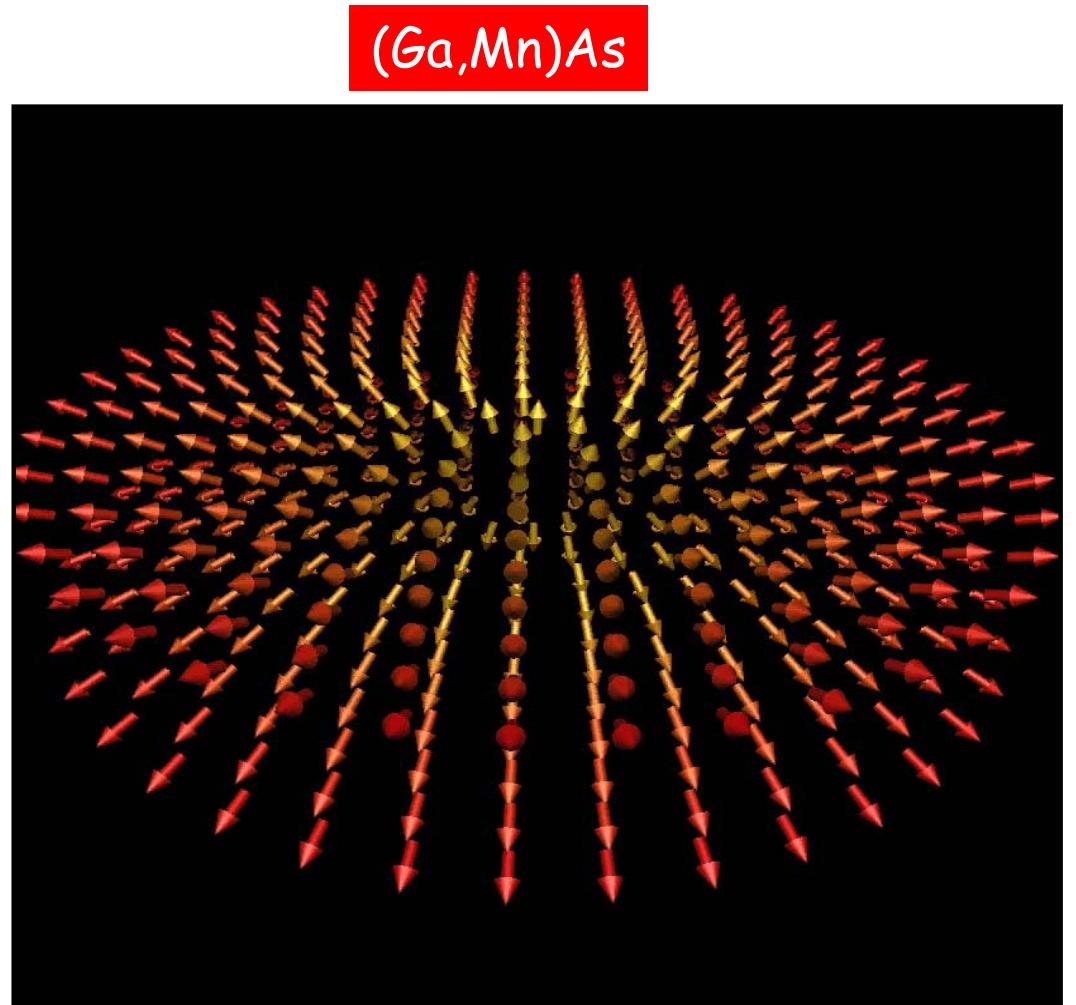
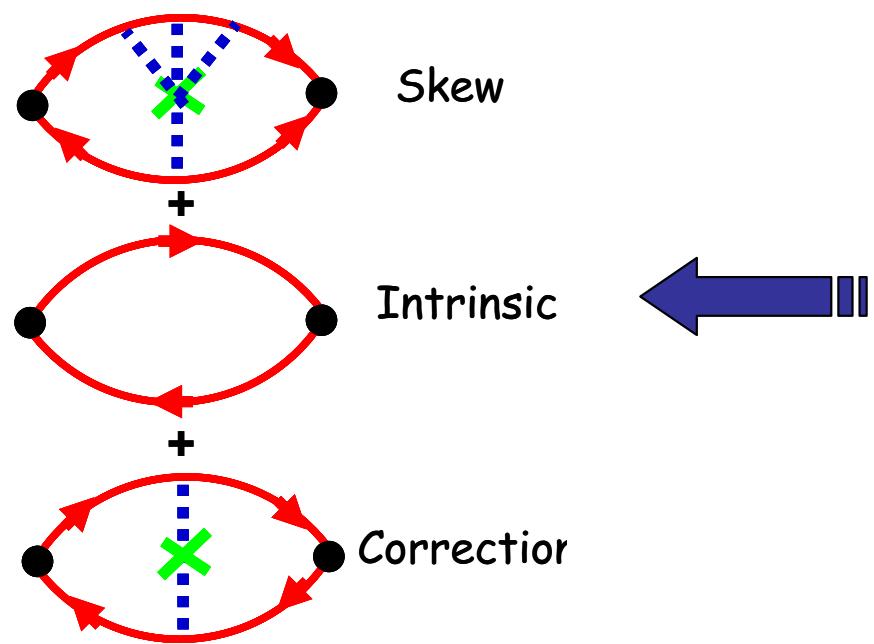
$$\frac{\partial \langle \Psi'_n | H | \Psi_n \rangle}{\partial \pi_x} = 0 = \langle \Psi'_n | \frac{\partial \Psi_n}{\partial \pi_x} \rangle (E_{n',\mathbf{k}} - E_{n,\mathbf{k}}) + \langle \Psi_{n'} | \frac{\partial H}{\partial \pi_x} | \Psi_n \rangle$$

TKNN Trick

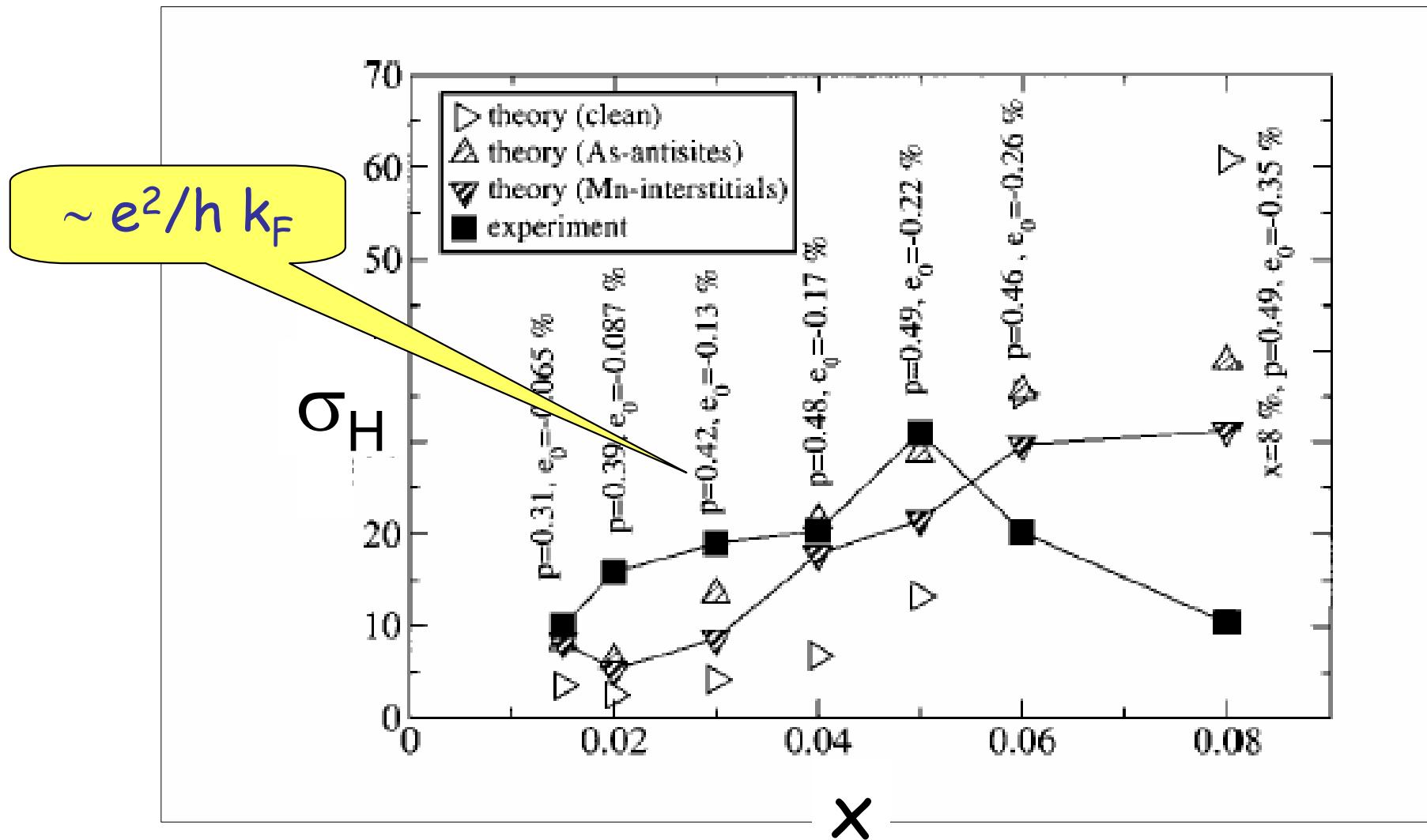
Anomalous Velocity

$$\sigma_{xy}^{INT} = \frac{2e^2}{\hbar A} \sum_{\vec{k},n} f_{\vec{k},n} \text{Im} \left[\langle \frac{\partial \vec{u}_{\vec{k},n}}{\partial k_x} | \frac{\partial \vec{u}_{\vec{k},n}}{\partial k_y} \rangle \right]$$

$(Ga,Mn)As$ k-space Berry phase

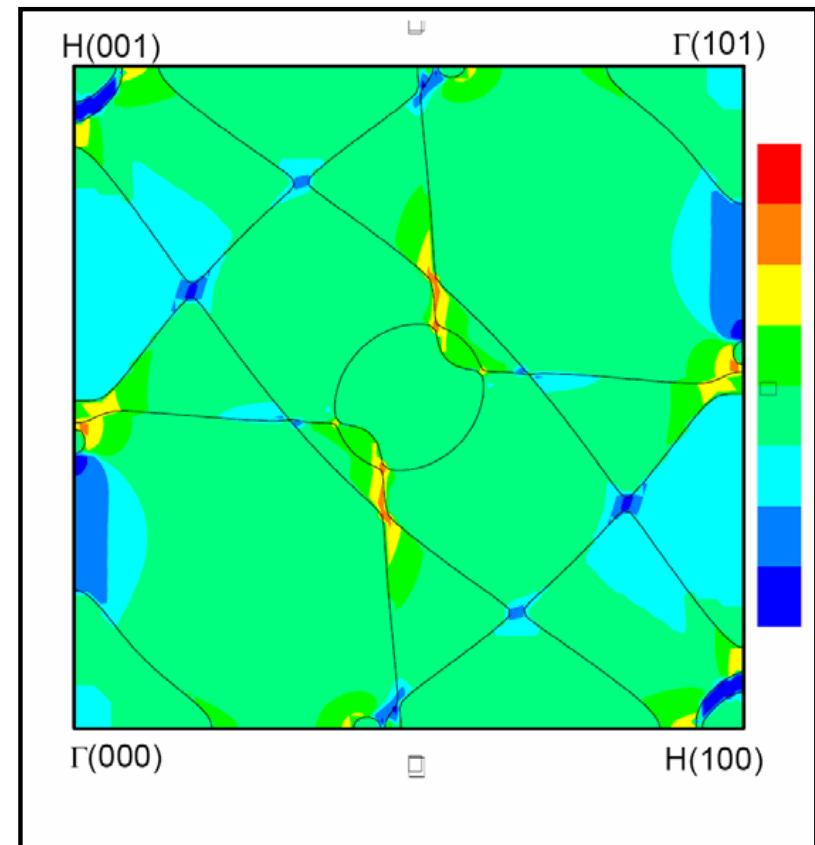
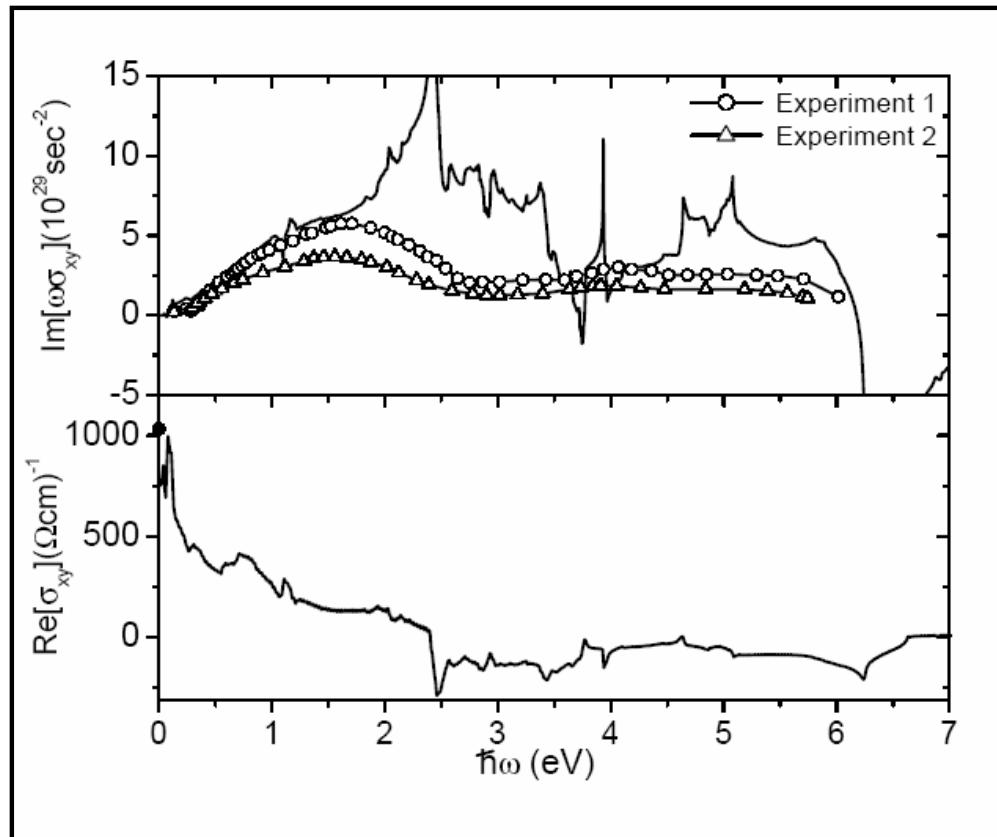


AHE in (III,Mn)V Ferromagnets



Jungwirth, Niu, et al. PHYSICAL REVIEW LETTERS 2002
Jungwirth, Gallagher et al. APPLIED PHYSICS LETTERS 2003

TRANSITION METAL (Fe) AHE



Yao et al. - PRL 2004 also Z Fang et al. Science 302

AHE is
complicated
because
it is small

AHE is often
not so complicated
because God is kind

Spin-orbit Interactions

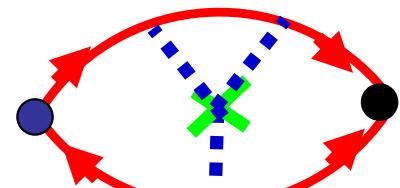
Optical Orientation

Anomalous Hall Effect

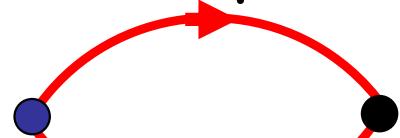
Spin Hall Effect

SHE - Perturbation Theory

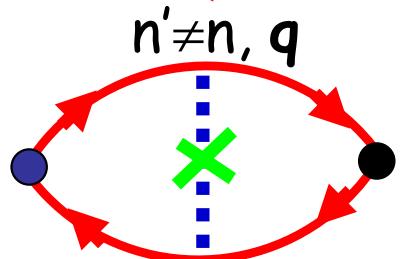
Dyakanov (1971)
Hirsch, PRL (1999)



Skew
 $\sim \sigma_0 S$



Intrinsic
 $\sim \sigma_0 / \epsilon_F \tau$



Vertex Corrections
 $\sim \sigma_{\text{Intrinsic}}$

$$\bullet = j_v = -e v_v$$

$$\bullet = j_v^z = \{v_v, s^z\}$$

What's special about k^1 Rashba?

Dimitrova, PRB 71 (2005)

Chalaev and Loss PRB 71 (2005)

$$\mathcal{H} = \frac{\hbar^2}{2m^*} \mathbf{k} \cdot \mathbf{k} - k_x \mathbf{b}^x \cdot \mathbf{s} - k_y \mathbf{b}^y \cdot \mathbf{s} + V_{dis}(\mathbf{r})$$

$$-\langle \hbar \dot{s}_x \rangle = b_x^y \langle J_x^z \rangle + b_y^y \langle J_y^z \rangle = 0$$

$$\langle \hbar \dot{s}_y \rangle = b_x^x \langle J_x^z \rangle + b_y^x \langle J_y^z \rangle = 0$$

Spin-Current ?

$$\partial_t \rho = -\partial_x J$$

Charge
Conserved

$$\partial_t S_z = -\partial_x J_{sz} + \text{precession}$$

Spin not
Conserved

Coupled Spin-Charge Response

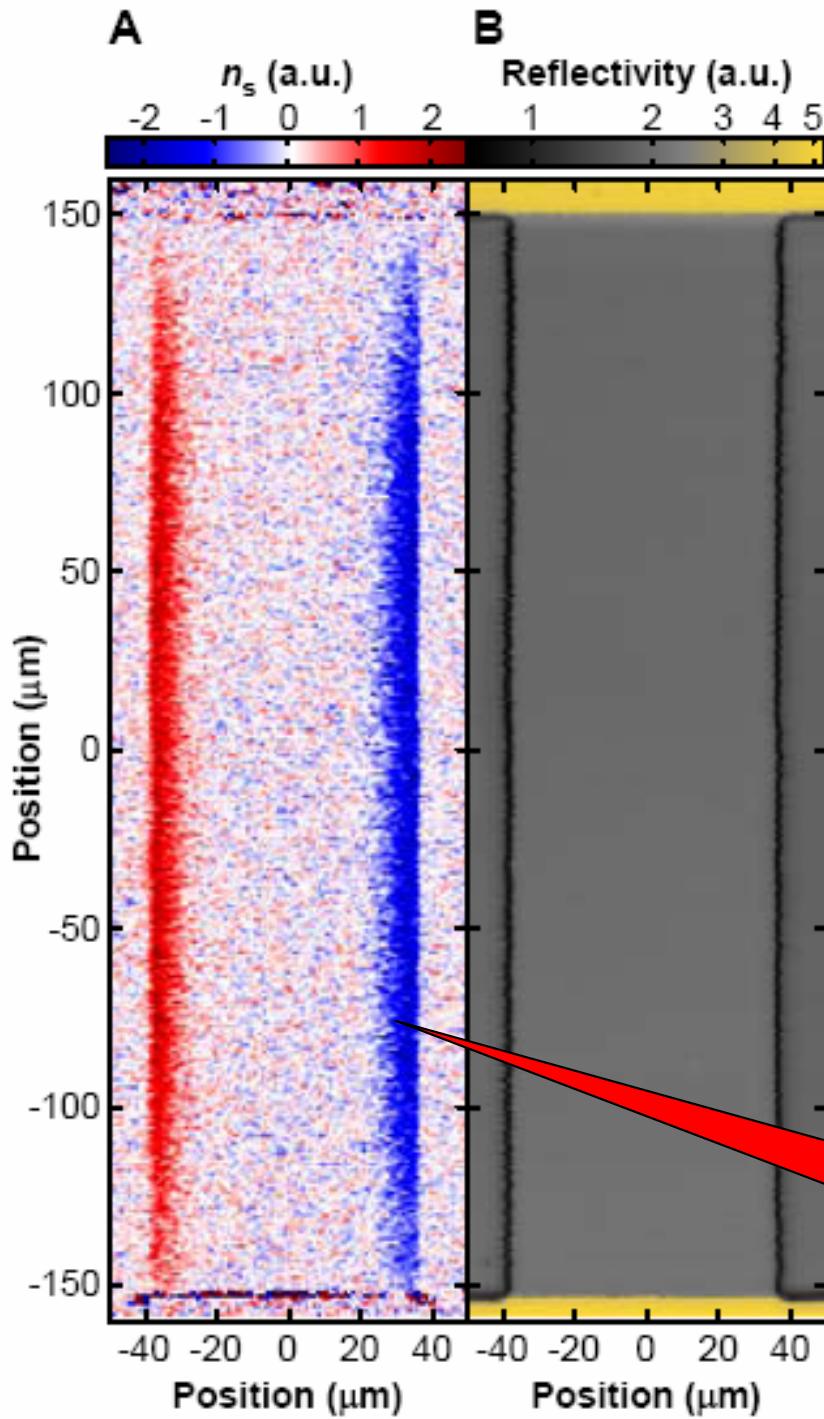
Burkov and AHM PRB 70 (2004)

$$\frac{\partial N}{\partial t} = D \nabla^2 (N + \varrho_0 V_c) + 2 \Gamma_{sc} (\hat{z} \times \nabla) \cdot (\mathbf{S} - \varrho_0 \mathbf{h}) + I^c,$$

$$\frac{\partial S^a}{\partial t} = \left(D \nabla^2 - \frac{1}{\tau_a} \right) (S^a - \varrho_0 h^a) + \Gamma_{ss} [(\hat{z} \times \nabla) \times (\mathbf{S} - \varrho_0 \mathbf{h})]_a$$

$$+ \frac{\Gamma_{sc}}{2} (\hat{z} \times \nabla)_a (N + \varrho_0 V_c) + I^{s,a}.$$

$$\Delta_{so} \tau \ll 1$$



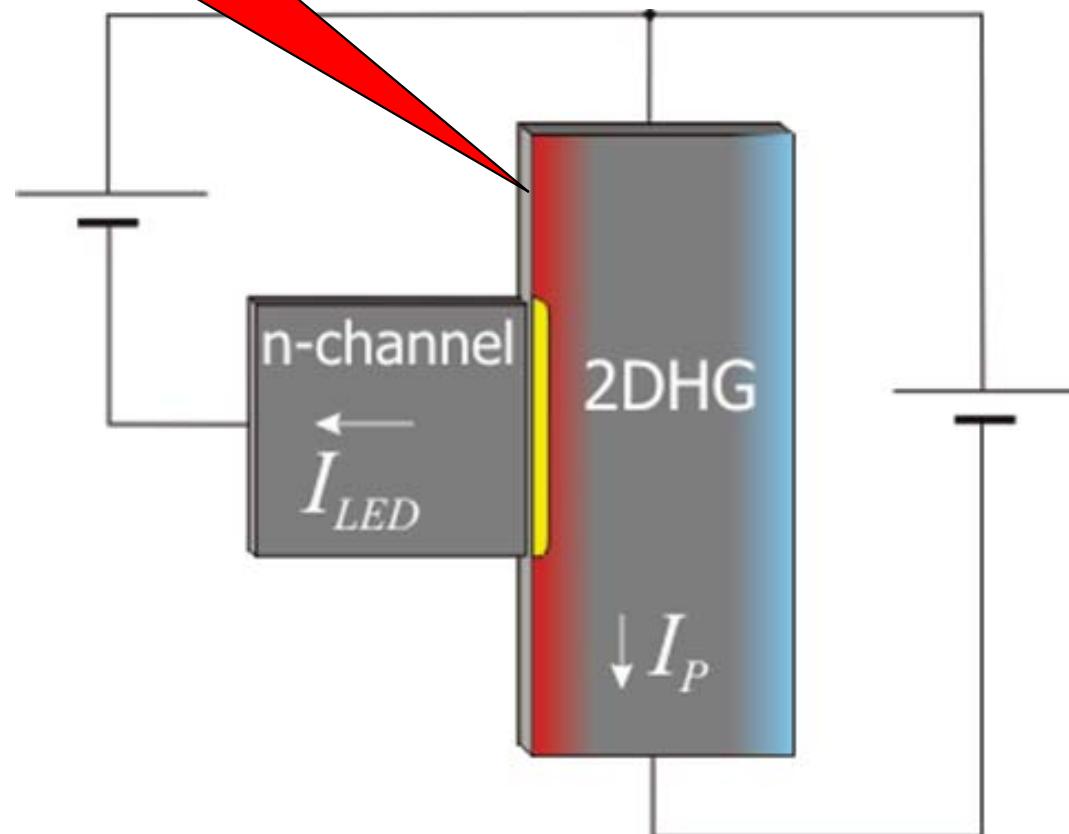
Spin Hall Effect

Kato et al.
Science (2004)

Spin
Accumulation

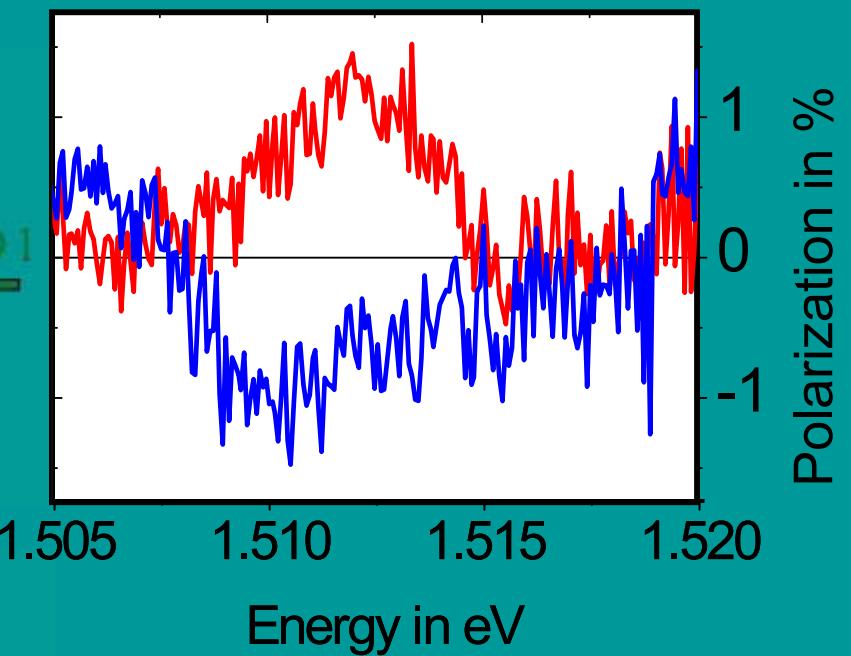
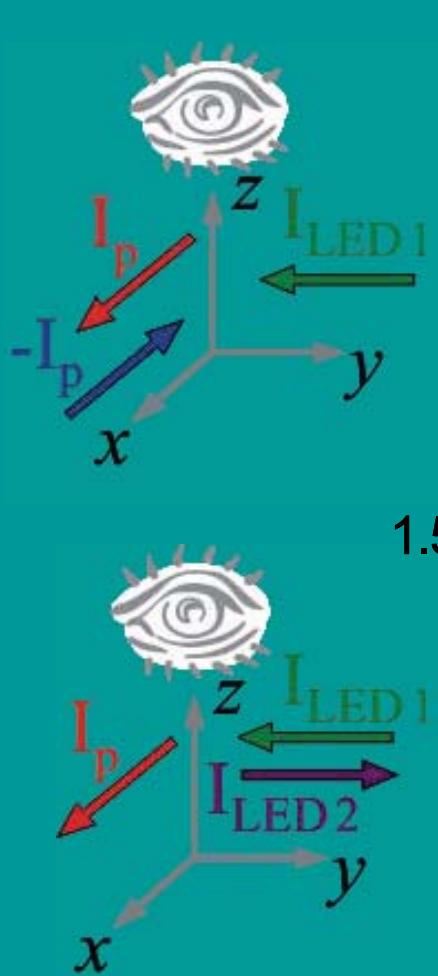
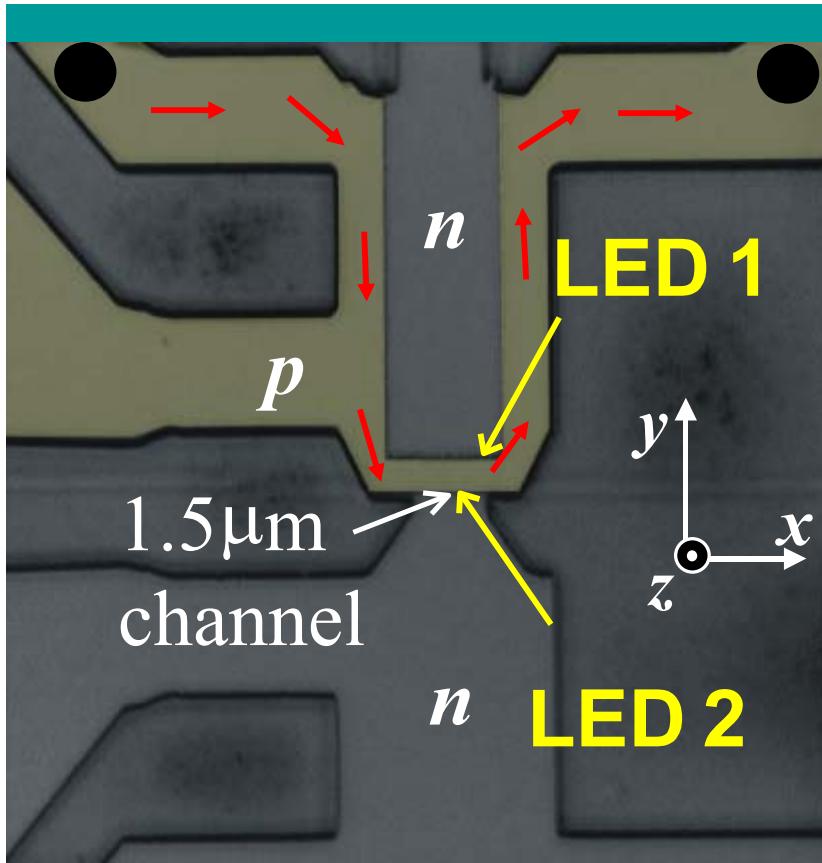
Spin Hall Effect

Spin
Accumulation



Wunderlich et al.
PRL (2005)

Co-planar spin LED in GaAs 2D hole gas: $\sim 1\%$ polarization



- I Metal Spintronics
- II Magnetic Semiconductors
- III Spin-Orbit Coupling

Thanks for Listening !!