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***Statistically-degenerate indirect excitons in  
coupled quantum wells***

Presented by:

**Alexei L. Ivanov**

Department of Physics and Astronomy  
Cardiff University, United Kingdom



# Statistically-degenerate indirect excitons in coupled quantum wells

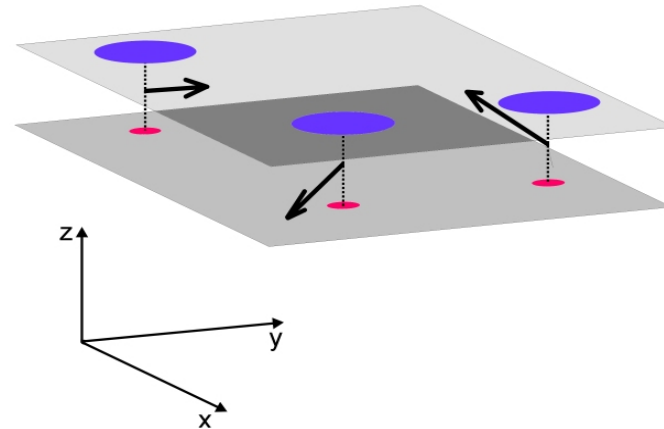
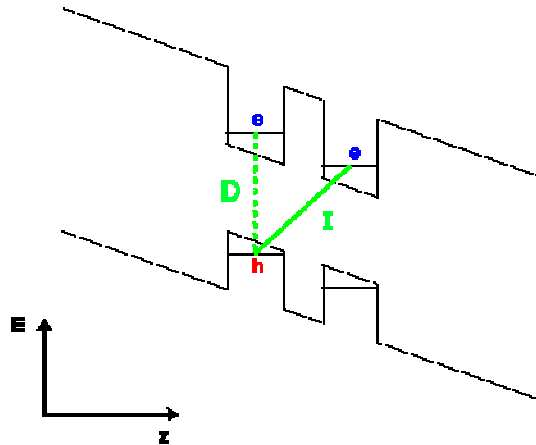
Alexei L. Ivanov

Department of Physics and Astronomy, Cardiff University, United Kingdom

## Outline

- Indirect excitons in coupled quantum wells
- Screening of QW disorder by dipole-dipole interacting indirect excitons
- Relaxation thermodynamics of indirect excitons
- Heating of indirect excitons by a laser pulse
- Recombination heating or cooling of indirect excitons
- Origin of the inner ring in PL patterns of indirect excitons
- Optical trapping of indirect excitons

## Indirect excitons in GaAs/AlGaAs coupled quantum wells



- A unique object for studying the transport and collective properties of interacting quasi-2D composite bosons :
  - a well-defined dipole-dipole repulsive interaction between indirect excitons (not sensitive to the spin structure) ,
  - the absence of quasi-2D excitonic molecules ,
  - a long radiative lifetime of excitons ( $\tau_{\text{opt}} \sim 10 - 100 \text{ ns}$ ) ,
  - effective screening of in-plane QW disorder .

## Thermodynamics of quasi-2D bosons

- The degeneracy temperature:  $k_{\text{B}}T_0 = \frac{2\pi}{g} \left( \frac{\hbar^2}{M_{\text{x}}} \right) \rho_{2\text{D}}$ ,

$\rightarrow g = 4$  and  $T_0 = 0.64 \text{ K}$  for  $\rho_{2\text{D}} = 10^{10} \text{ cm}^{-2}$ .

- Thermodynamic relationships for quasi-2D bosons:

$$\mu = k_{\text{B}}T \ln \left( 1 - e^{-T_0/T} \right), \quad N_E = \frac{1 - e^{-T_0/T}}{e^{E/k_{\text{B}}T} + e^{-T_0/T} - 1}.$$

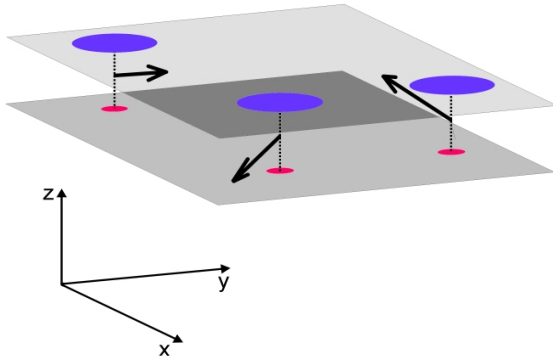
- Maxwell-Boltzmann statistics:

$$\mu = k_{\text{B}}T \ln (T_0/T) \quad \text{and} \quad N_{E=0} = T_0/T \ll 1.$$

- Bose-Einstein statistics:

$$\mu = -k_{\text{B}}T e^{-T_0/T} \quad \text{and} \quad N_{E=0} = e^{T_0/T} \gg 1.$$

## Mean-field energy and BEC of indirect excitons



- A well-defined dipole-dipole interaction :

$$u_0 = \pi \frac{\hbar^2}{\mu_x \chi(d)}, \quad \text{where} \quad \chi(d \geq a_{2d}^B) = \frac{a_{2d}^B}{2d}.$$

→ consistent with the plate capacitor formula,  $u_0 = 4\pi(e^2/\epsilon_b)d$ .

[S. Ben-Tabou and Boris Laikhtman (PRB, 2001)]

- **Bose-Einstein condensation of quasi-2D excitons**

Condensation temperature :

$$T_c = \frac{4\pi\hbar^2\rho_{2D}}{2M_x g k_B} \frac{1}{\ln(\rho_{2D}S/g)},$$

where  $S$  is the area of a meso-structure.

- Two key-questions:

→ Is it possible to build up  $N_E \gg 1$  ?

→ Is it possible to cool the indirect excitons down  $T_b \ll 1K$  ?

- The Einstein relation for degenerate quasi-2D excitons:

$$\mu^{(2D)} = \frac{D_x^{(2D)}}{k_B T_0} [e^{T_0/T} - 1] ,$$

where  $\mu^{(2d)}$  and  $D_x^{(2d)}$  are the mobility and diffusion coefficient,  $T_0 \propto \rho_{2D}/M_x$  is the degeneracy temperature.

- Quantum diffusion equation:

$$\begin{aligned} \frac{\partial \rho_{2D}}{\partial t} = & \nabla [D_x^{(2D)} \nabla \rho_{2D} + \frac{2}{\pi} \left( \frac{M_x}{\hbar^2} \right) D_x^{(2D)} (e^{T_0/T} - 1) \\ & \times \nabla (u_0 \rho_{2D} + U_{QW})] - \frac{\rho_{2D}}{\tau_{opt}} + \Lambda , \end{aligned}$$

where the diffusion coefficient is given by

$$D_x^{(2D)} = D_x^{(2D)}(T, T_0) = \frac{\hbar}{M_x} \left( \frac{a_{2d}^B}{2d} \right)^2 \left( \frac{\mu_x}{M_x} \right)^2 \left[ \frac{1}{2k_B} \frac{\partial E_{kin}}{\partial T_0} + 2 \frac{E_{kin}}{k_B T_0} \right] ,$$

→  $E_{kin}$  is the average thermal energy of QW excitons.

- Self-thermodiffusion  $\propto \nabla T$  is absent, i.e.,  $D_T^{(2D)} = 0$ .
- **Screening of long-range-correlated disorder**

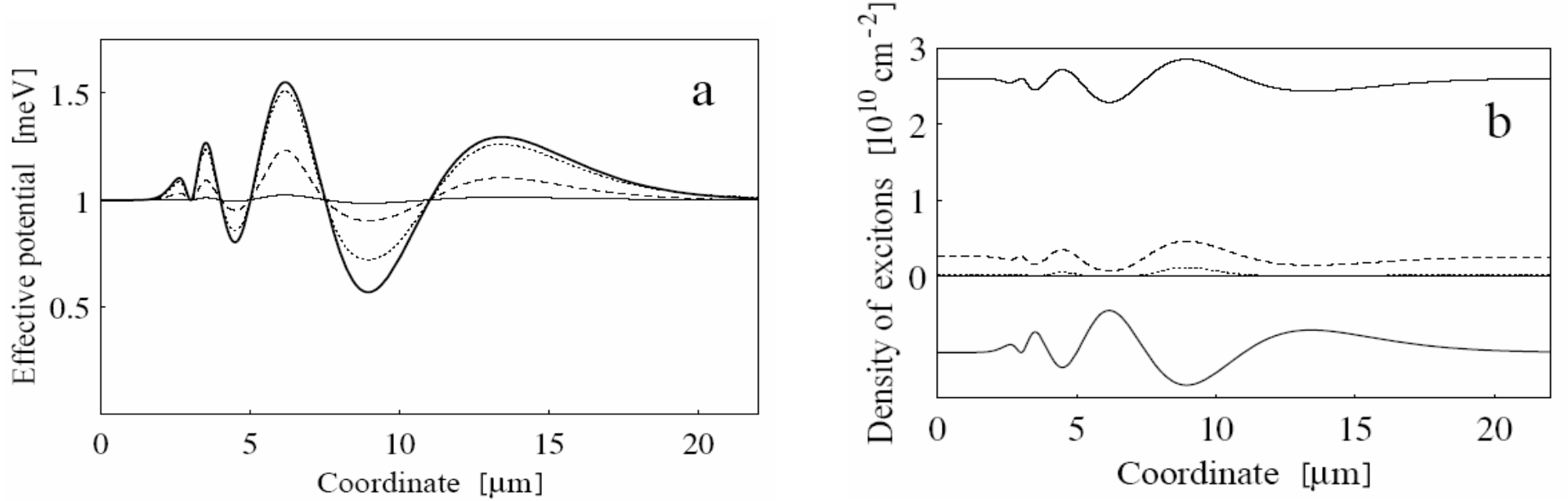
$$\delta\rho_{2D}(\mathbf{r}_{\parallel}) = - \frac{U_{\text{rand}}(\mathbf{r}_{\parallel})\rho_{2D}^{(0)}}{k_B T + u_0\rho_{2D}^{(0)}} ,$$

$$U_{\text{eff}}(\mathbf{r}_{\parallel}) = u_0\rho_{2D}^{(0)} + \frac{U_{\text{rand}}(\mathbf{r}_{\parallel})k_B T}{k_B T + u_0\rho_{2D}^{(0)}} .$$

- Thermionic model:  $\tilde{D}_x^{(2D,\text{cl})} = D_x^{(2D,\text{cl})} \exp\left[-\frac{U_{\text{rand}}(\mathbf{r}_{\parallel})}{k_B T + u_0\rho_{2D}^{(0)}}\right] .$

$\rightarrow |U_{\text{rand}}(\mathbf{r}_{\parallel})| \simeq 0.35 - 0.50 \text{ meV}$  in GaAs/AlGaAs coupled QWs ,

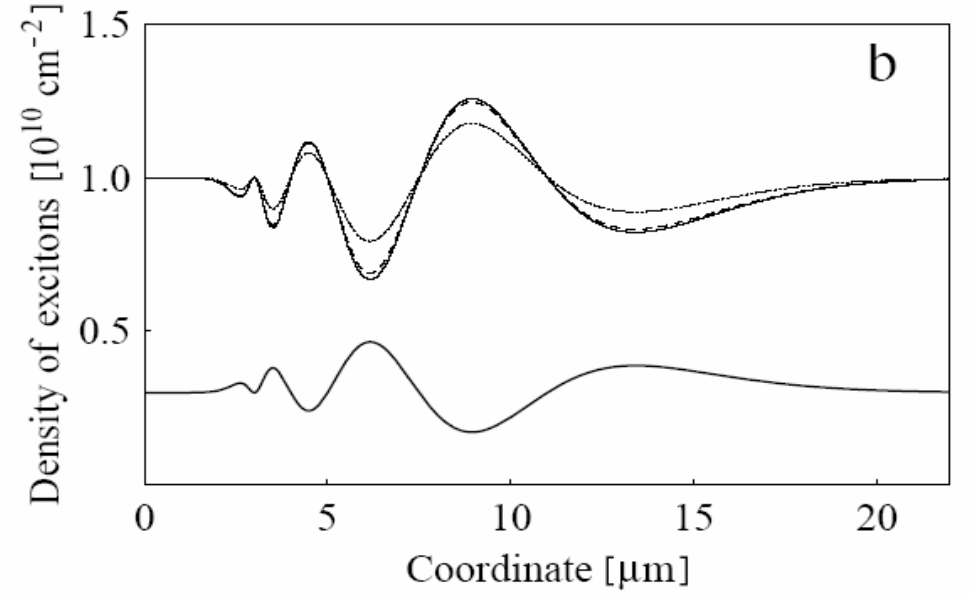
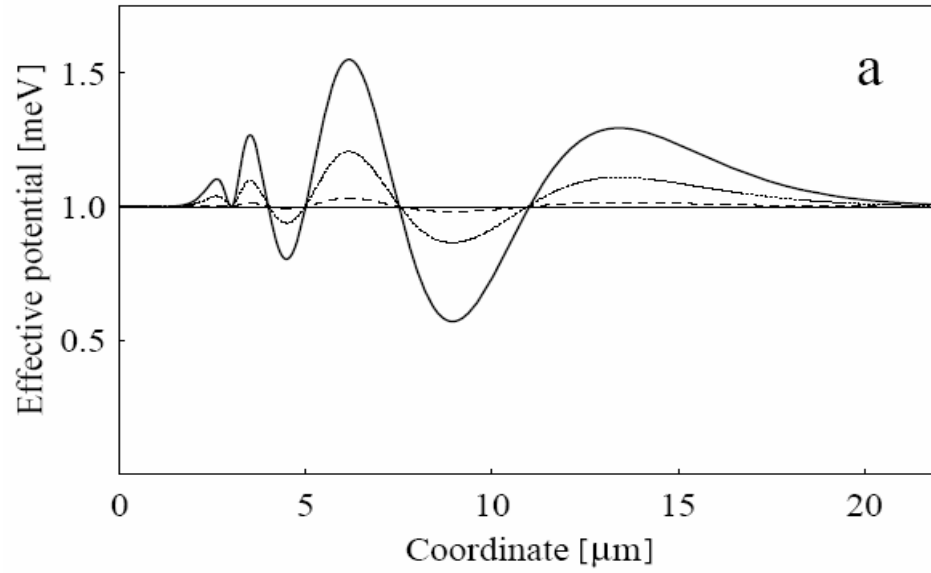
$\rightarrow$  For  $\rho_{2D}^{(0)} \geq 10^{10} \text{ cm}^{-2}$  the in-plane momentum,  $\mathbf{p}_{\parallel}$ , becomes a good quantum number even for low-energy particles .



### Relaxation of long-range-correlated disorder by the dipole-dipole interaction of indirect excitons in GaAs/AlGaAs coupled QWs.

(a) The effective, screened potential  $U_{\text{eff}}(x) - u_0 \rho_{2\text{D}}^{(0)}$  and (b) local concentrations of indirect excitons  $\rho_{2\text{D}}(x)$  versus in-plane coordinate  $x$ . The average concentrations are  $\rho_{2\text{D}}^{(0)} = 2.6 \times 10^{10} \text{ cm}^{-2}$  (thin solid line),  $2.6 \times 10^9 \text{ cm}^{-2}$  (dashed line), and  $2.6 \times 10^8 \text{ cm}^{-2}$  (dotted line). Temperature  $T=2 \text{ K}$ , diffusion coefficient  $D_{\text{x}}^{(2\text{D})} = 100 \text{ cm}^2/\text{s}$ , and radiative lifetime  $\tau_{\text{opt}} = 20 \text{ ns}$ . In both figures the input, unscreened potential  $U_{\text{rand}}(x)$  is shown by bold solid lines.





### Temperature dependence of the narrowing effect.

The bold solid line shows the unscreened potential  $U_{\text{rand}}(x)$ . (a) The effective potential  $U_{\text{eff}}(x)$  for  $T = 10$  K (dotted line),  $T = 1$  K (dashed line), and  $T = 0.1$  K (thin solid line). The average density of indirect excitons  $\rho_{2\text{D}}^{(0)} = 1.0 \times 10^{10} \text{ cm}^{-2}$ , diffusion coefficient  $D_{\text{x}}^{(2\text{D})} = 100 \text{ cm}^2/\text{s}$ , and radiative lifetime  $\tau_{\text{opt}} = 20 \text{ ns}$ . (b) The local concentrations of indirect excitons,  $\rho_{2\text{D}} = \rho_{2\text{D}}(x)$ .

## Quantum synergetics for indirect excitons

- *Nonlinear* quantum diffusion equation :

$$\frac{\partial \rho_{2D}}{\partial t} = \nabla [D_x^{(2D)} \nabla \rho_{2D} + \frac{2}{\pi} \left( \frac{M_x}{\hbar^2} \right) D_x^{(2D)} (e^{T_0/T} - 1) \\ \times \nabla (u_0 \rho_{2D} + U_{QW})] - \frac{\rho_{2D}}{\tau_{opt}} + \Lambda .$$

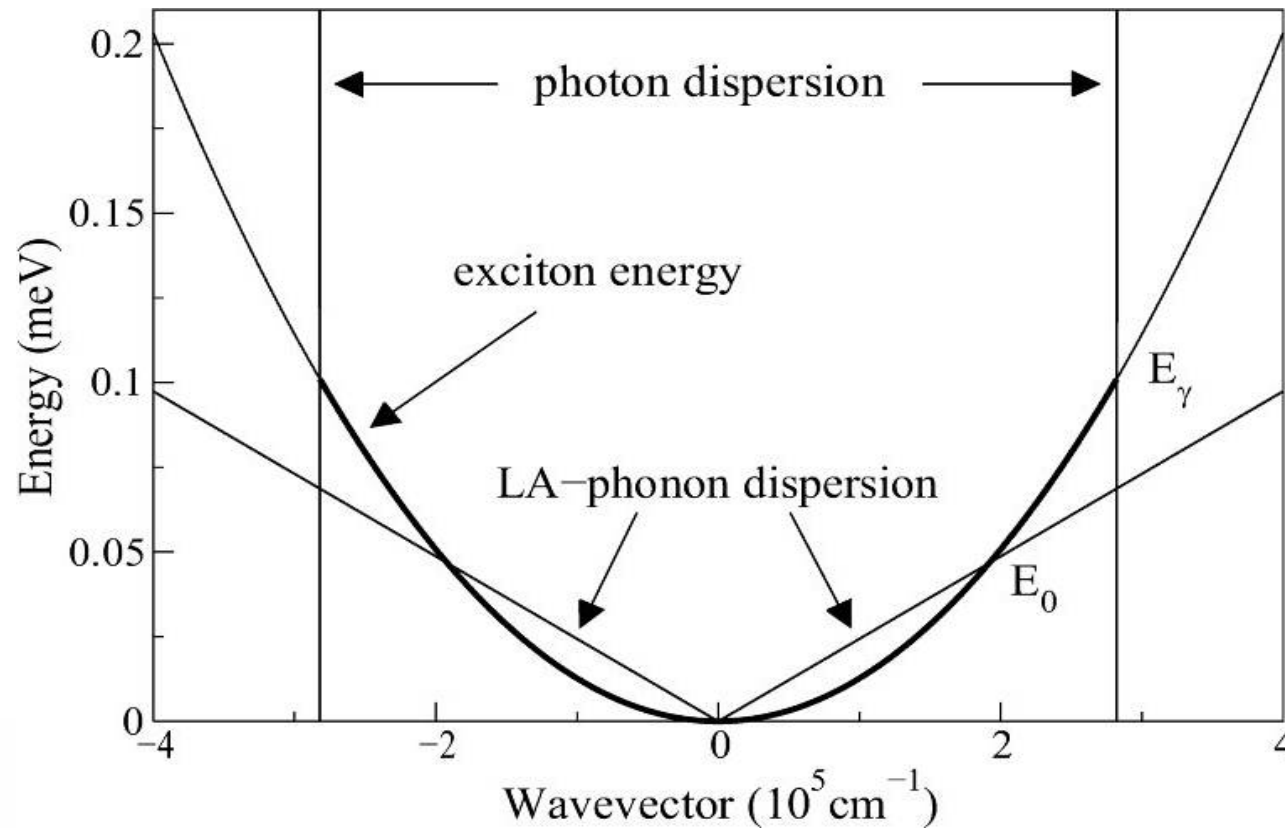
- *Nonlinear* LA-phonon-assisted thermalization :

$$\frac{\partial T}{\partial t} = - \frac{2\pi}{\tau_{sc}} \left( \frac{T^2}{T_0} \right) (1 - e^{-T_0/T}) \int_1^\infty d\varepsilon \, \varepsilon \sqrt{\frac{\varepsilon}{\varepsilon - 1}} |F_z(a\sqrt{\varepsilon(\varepsilon - 1)})|^2 \\ \times \frac{e^{\varepsilon E_0/k_B T_b} - e^{\varepsilon E_0/k_B T}}{(e^{\varepsilon E_0/k_B T} + e^{-T_0/T} - 1)} \frac{1}{(e^{\varepsilon E_0/k_B T_b} - 1)} + \Lambda_T .$$

- Intrinsically *nonlinear* optical decay :

$$I_{PL} = \hbar \omega_t \frac{\rho_{2D}}{\tau_{opt}(\rho_{2D}, T)} .$$

## Energy diagram for indirect QW excitons

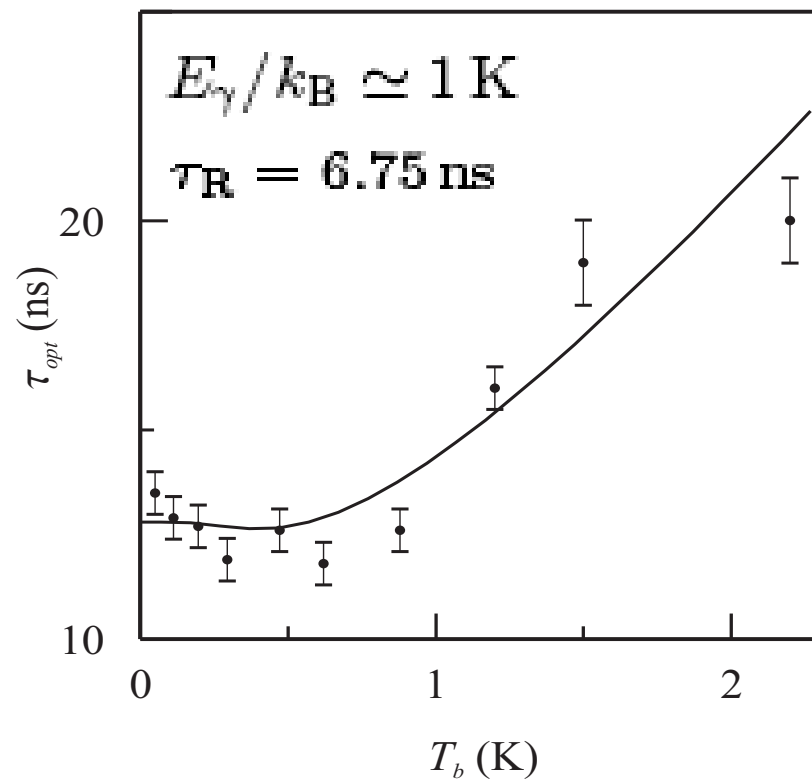


- the bath temperature  $T_b$ ;
- the effective temperature  $T$ ;
- the degeneracy temperature  $T_0 \propto \rho_{2D}$ ;
- the energy  $E_0 = 2M_x v_s^2$ ;
- the energy  $E_\gamma = (\hbar k_0)^2 / 2M_x$ , where  $k_0 = (\omega_t \sqrt{\epsilon_b}) / c$ .

# Optical decay of statistically-degenerate indirect excitons

$$\Gamma_{\text{opt}} \equiv \frac{1}{\tau_{\text{opt}}} = \frac{1}{2\tau_R} \left( \frac{E_\gamma}{k_B T_0} \right) \int_0^1 \frac{1+z^2}{A e^{-z^2 E_\gamma / k_B T} - 1} dz, \quad A = A(T, T_0) = \frac{e^{E_\gamma / k_B T}}{1 - e^{-T_0/T}}.$$

[A L Ivanov, P B Littlewood, and H Haug (PRB, 1999)]



- Well-developed classical statistics:

$$\tau_{\text{opt}}^{\text{cl}} = \left( \frac{3}{2} \frac{k_B T}{E_\gamma} \right) \tau_R + \left( \frac{9}{10} - \frac{3}{4} \frac{k_B T_0}{E_\gamma} \right) \tau_R.$$

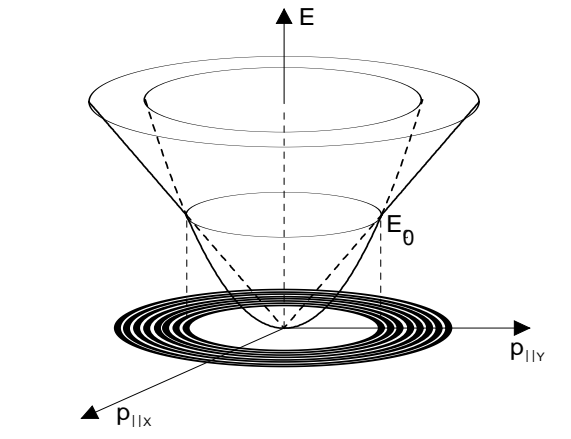
- Well-developed quantum statistics:

$$\Gamma_{\text{opt}}^q = \left[ 1 - \frac{T}{T_0} + \frac{T}{T_0} \ln \left( \frac{4E_\gamma}{k_B T} \right) \right] \frac{\Gamma_0}{2},$$

where  $\Gamma_0 = 1/\tau_R$ .

- Experiment (points, L V Butov et al., 2001) against theory (solid line).

# Relaxation thermodynamics of indirect excitons (high-density limit)

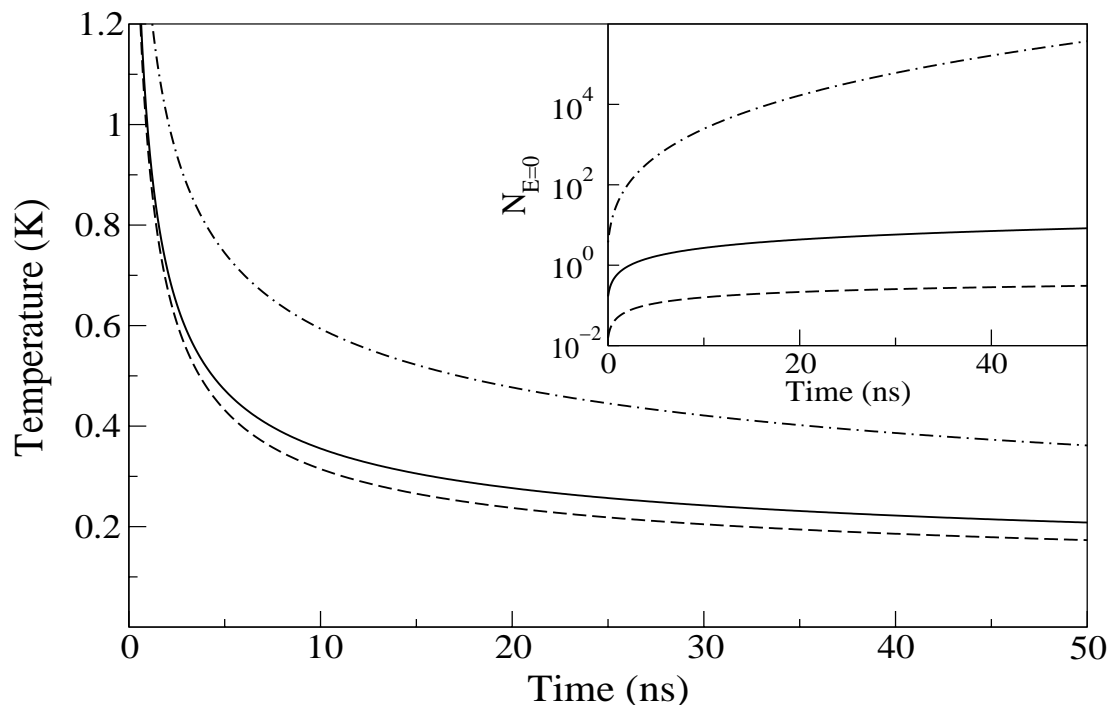


$$\frac{\partial}{\partial t} T = -\frac{2\pi}{\tau_{sc}} \left( \frac{T^2}{T_0} \right) (1 - e^{-T_0/T}) \int_1^\infty d\varepsilon \varepsilon \sqrt{\frac{\varepsilon}{\varepsilon - 1}} \left| F_z(a\sqrt{\varepsilon(\varepsilon - 1)}) \right|^2$$

$$\times \frac{e^{\varepsilon E_0/k_B T_b} - e^{\varepsilon E_0/k_B T}}{(e^{\varepsilon E_0/k_B T} + e^{-T_0/T} - 1)} \frac{1}{(e^{\varepsilon E_0/k_B T_b} - 1)},$$

where  $\tau_{sc} = (\pi^2 \hbar^4 \rho) / (D_{dp}^2 M_x^3 v_s)$ .

$$\rho_{2D} \geq 10^9 \text{ cm}^{-2} ;$$



- Classical slowing down:

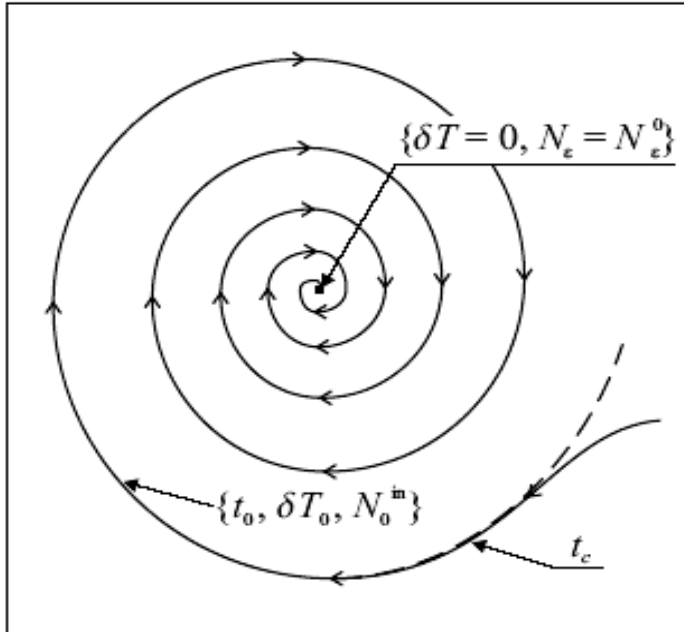
$$N_{E=0}(\tau) = \tau^{k_B T_0 / E_0},$$

$$\tilde{T}(\tau) = 1 / \ln(\tau) \text{ for } \tau \gg 1,$$

$$\tau = (2\pi^{3/2} E_0 t) / (k_B T_0 \tau_{sc}).$$

- Quantum slowing down.

# Generic solutions for phonon-assisted kinetics of indirect excitons (low-density limit)



- Kinetic equation for QW excitons:

$$\begin{aligned} \frac{\partial}{\partial t} N_{\mathbf{k}_{\parallel}} = & -\frac{2\pi}{\hbar} \sum_{\mathbf{q}} |M(\mathbf{q}, q_z)|^2 \{ [N_{\mathbf{k}_{\parallel}} (1 + n_{\mathbf{q}}^{\text{ph}}) (1 + N_{\mathbf{k}_{\parallel} - \mathbf{q}_{\parallel}}) \\ & - (1 + N_{\mathbf{k}_{\parallel}}) n_{\mathbf{q}}^{\text{ph}} N_{\mathbf{k}_{\parallel} - \mathbf{q}_{\parallel}}] \delta(E_{\mathbf{k}_{\parallel}} - E_{\mathbf{k}_{\parallel} - \mathbf{q}_{\parallel}} - \hbar q v_s) \\ & + [N_{\mathbf{k}_{\parallel}} n_{\mathbf{q}}^{\text{ph}} (1 + N_{\mathbf{k}_{\parallel} + \mathbf{q}_{\parallel}}) - (1 + N_{\mathbf{k}_{\parallel}}) (1 + n_{\mathbf{q}}^{\text{ph}}) N_{\mathbf{k}_{\parallel} + \mathbf{q}_{\parallel}}] \\ & \times \delta(E_{\mathbf{k}_{\parallel}} - E_{\mathbf{k}_{\parallel} + \mathbf{q}_{\parallel}} + \hbar q v_s) \} . \end{aligned}$$

→ low densities of QW excitons,  $\rho_{2\text{D}} \leq 10^9 \text{ cm}^{-2}$  ,

→ low temperatures,  $T \leq T_0$ , so that  $N_{\mathbf{k}_{\parallel}} \gg 1$  .

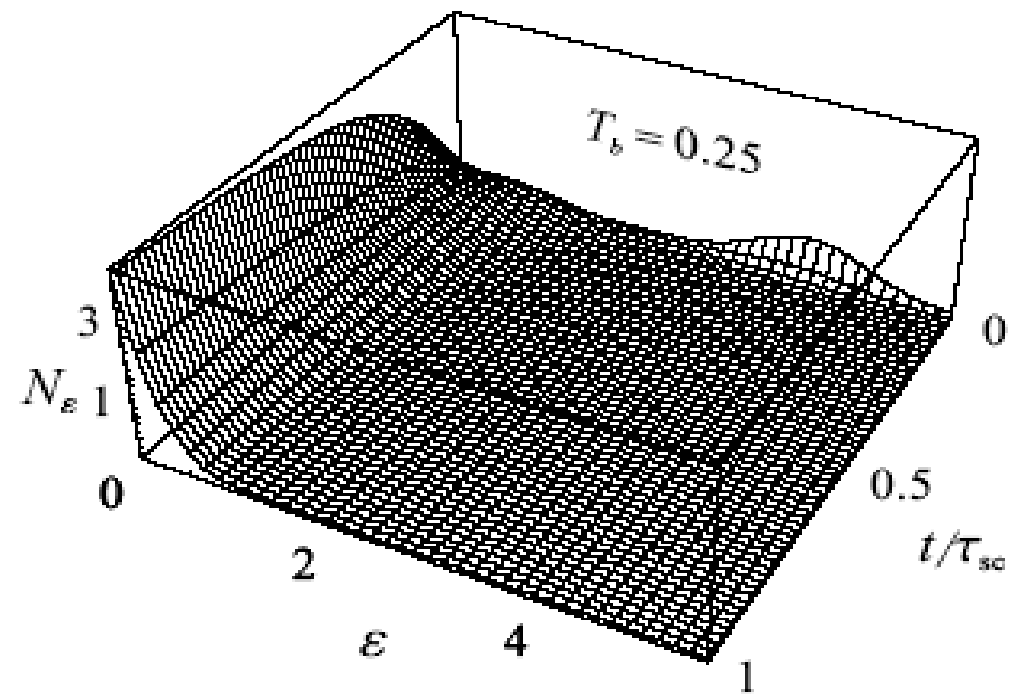
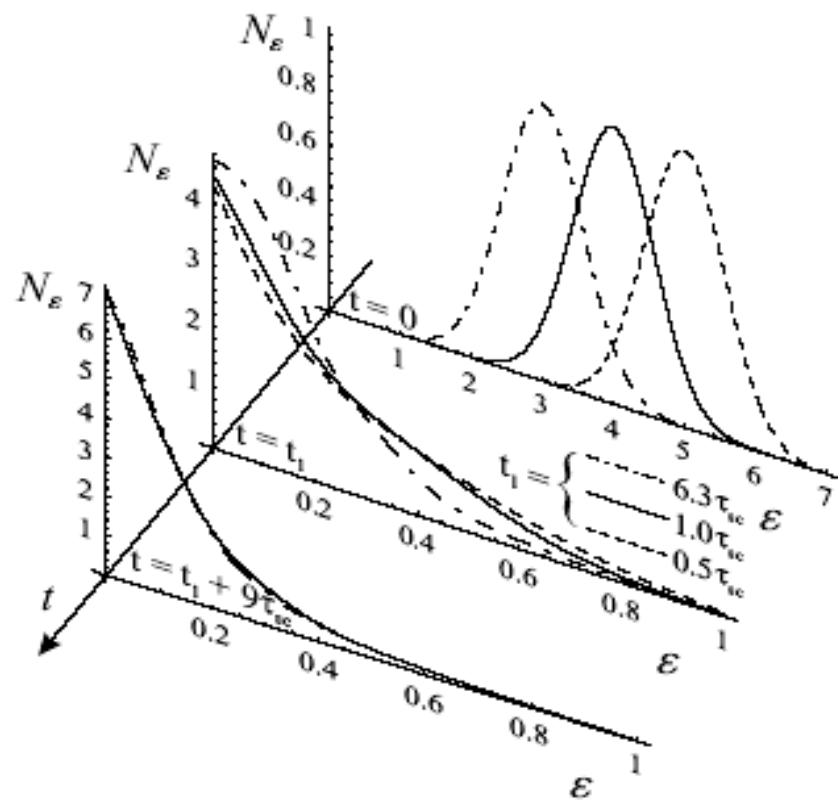
- **A generic solution:**

→ takes many  $\tau_{\text{sc}}$ ,

→ depends only upon two control parameters,  $T_b$  and  $T_0 \propto \rho_{2\text{D}}$ ,

→ a quasi-equilibrated distribution of high-energy QW excitons  
with  $T = T(t)$ .

## Transient relaxation towards the adiabatic stage of evolution



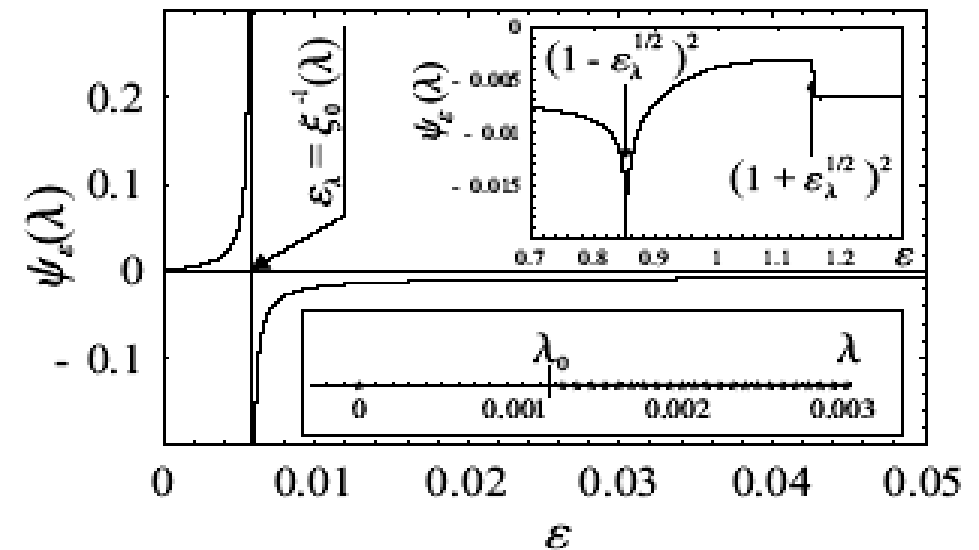
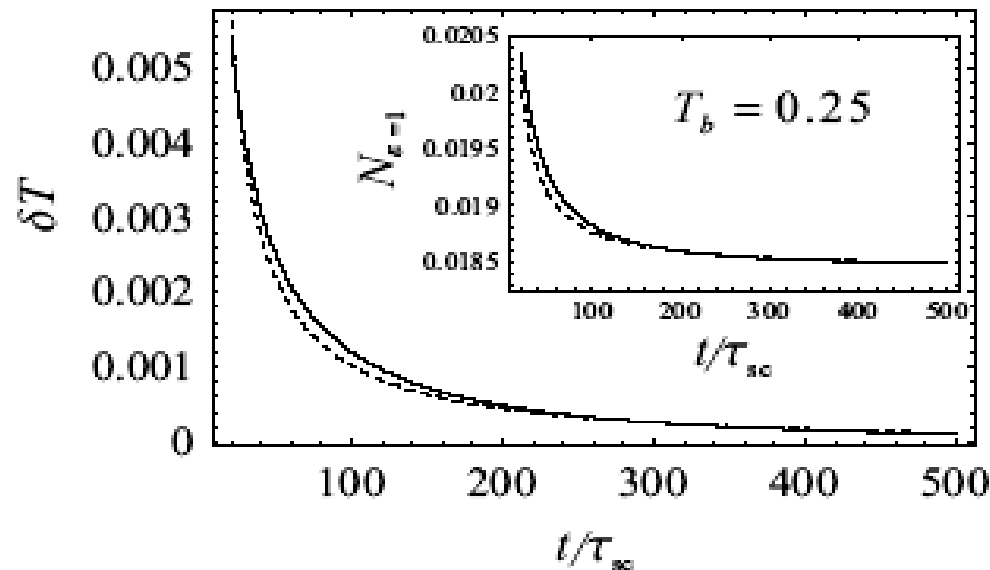
## Adiabatic stage of relaxation

- The thermalization law is given by

$$\delta T(t) = \left( \frac{\delta T_0}{\lambda_1 - \lambda_0} \right) \frac{e^{-\lambda_0(t-t_0)} - e^{-\lambda_1(t-t_0)}}{t - t_0},$$

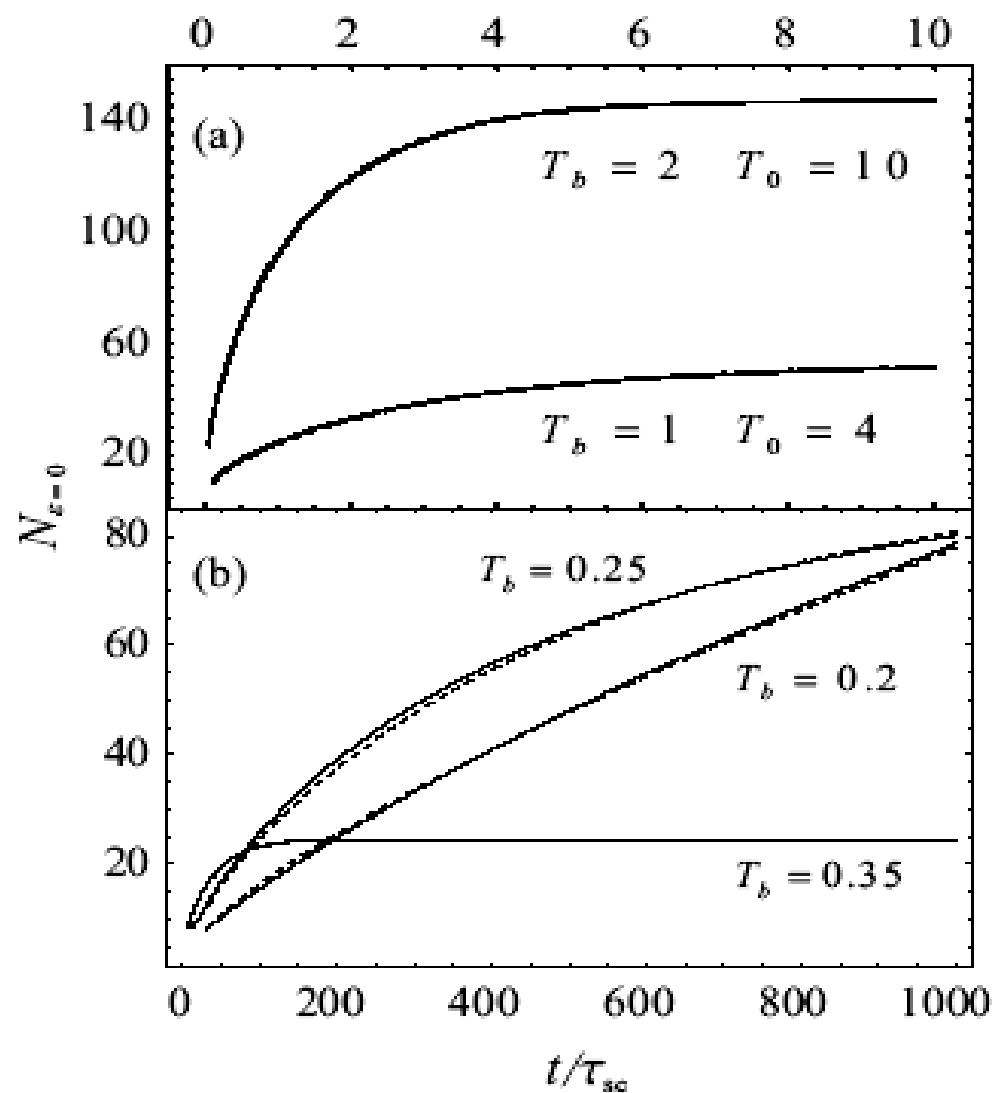
→  $\lambda_0$  is a marginal value of the continuous eigenvalue spectrum of the linearized kinetics ,

→  $t_0 \sim \tau_{sc}$  and  $\lambda_1 \gg \lambda_0$  .





## Occupation dynamics of the ground-state mode



• Nonexponential kinetics:

$$N_{\epsilon=0}(t) = N_0^{\text{in}} [1 + \chi(t - t_0)]^\nu.$$

## The relaxation and photoluminescence dynamics of indirect excitons

- The relaxation and PL dynamics :

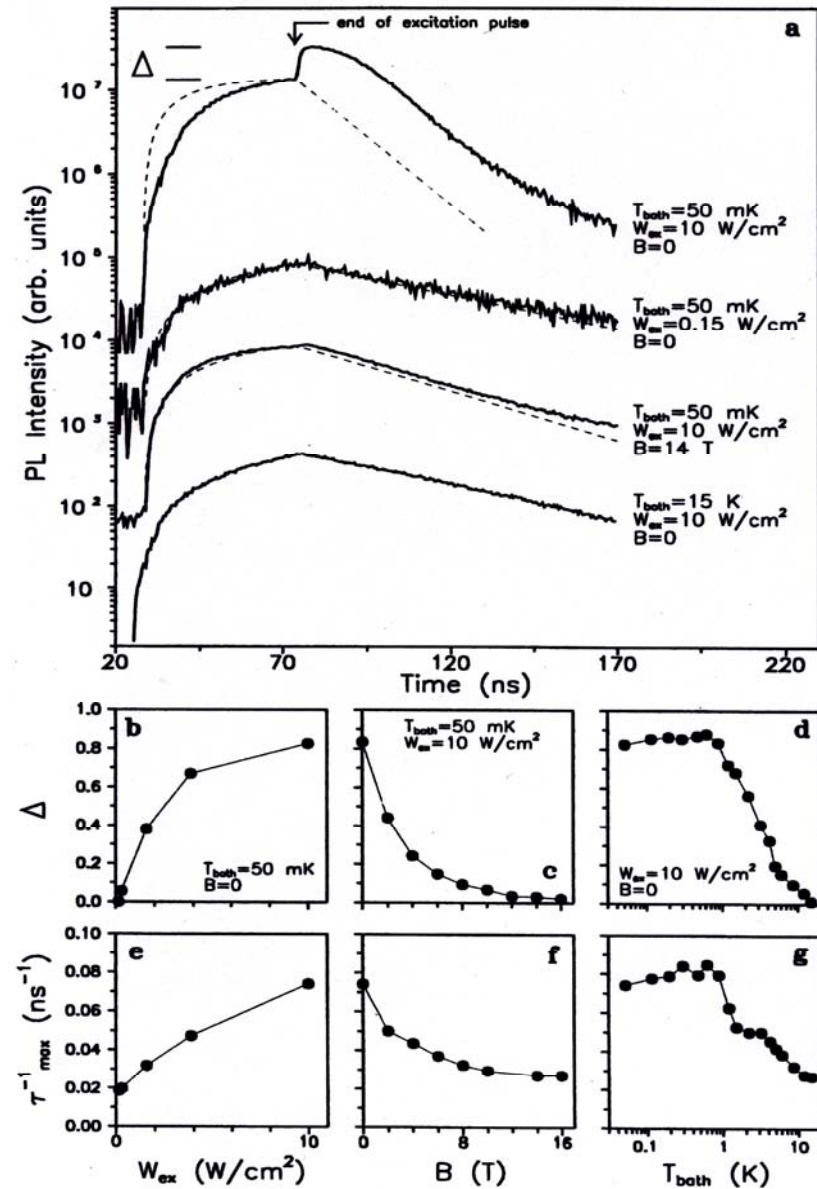
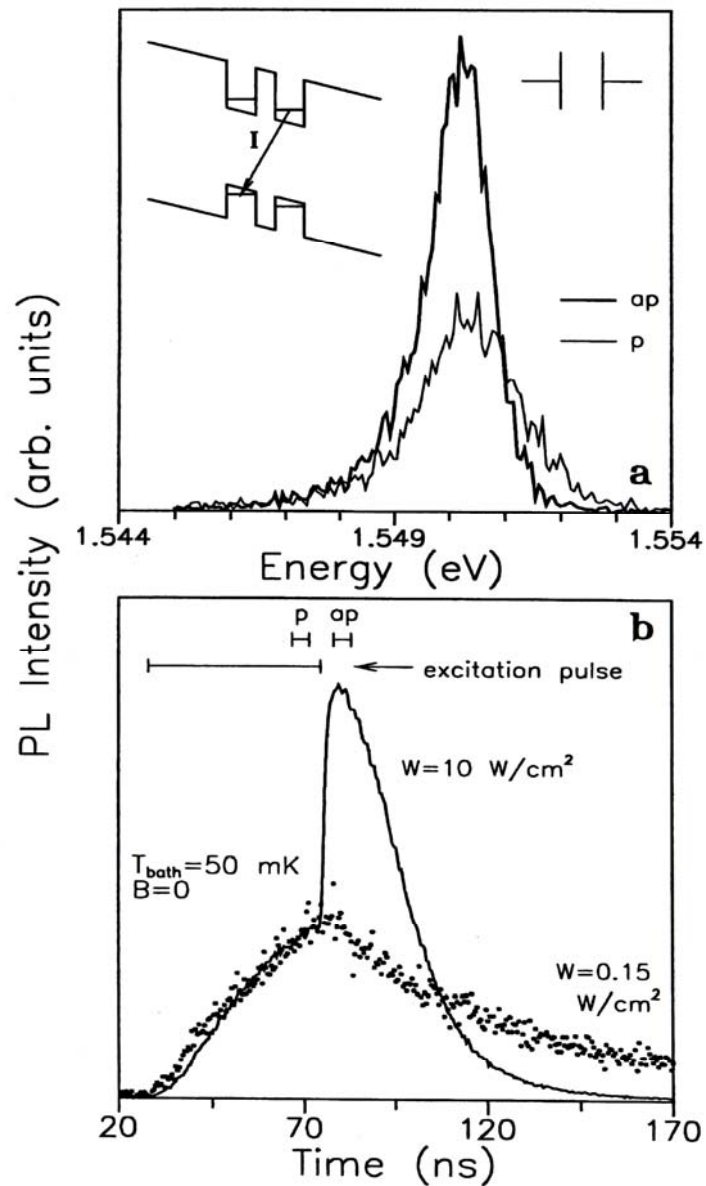
$$\begin{aligned}\frac{\partial}{\partial t}\rho_{2D} &= -\frac{\rho_{2D}}{\tau_{\text{opt}}} + \Lambda(t) , \\ \frac{\partial}{\partial t}T &= \left(\frac{\partial T}{\partial t}\right)_{\rho_{2D}} + S_T(t) .\end{aligned}$$

- Three contributions to the relaxation thermodynamics

$$S_T = S_{\text{pump}} + S_{\text{opt}} + S_{\text{diff}} ,$$

- $S_{\text{pump}}$  – heating of indirect excitons by the pump pulse ;
- $S_{\text{opt}}$  – recombination heating or cooling ;
- $S_{\text{diff}}$  – heating of indirect excitons by drift and diffusion .

# PL jump

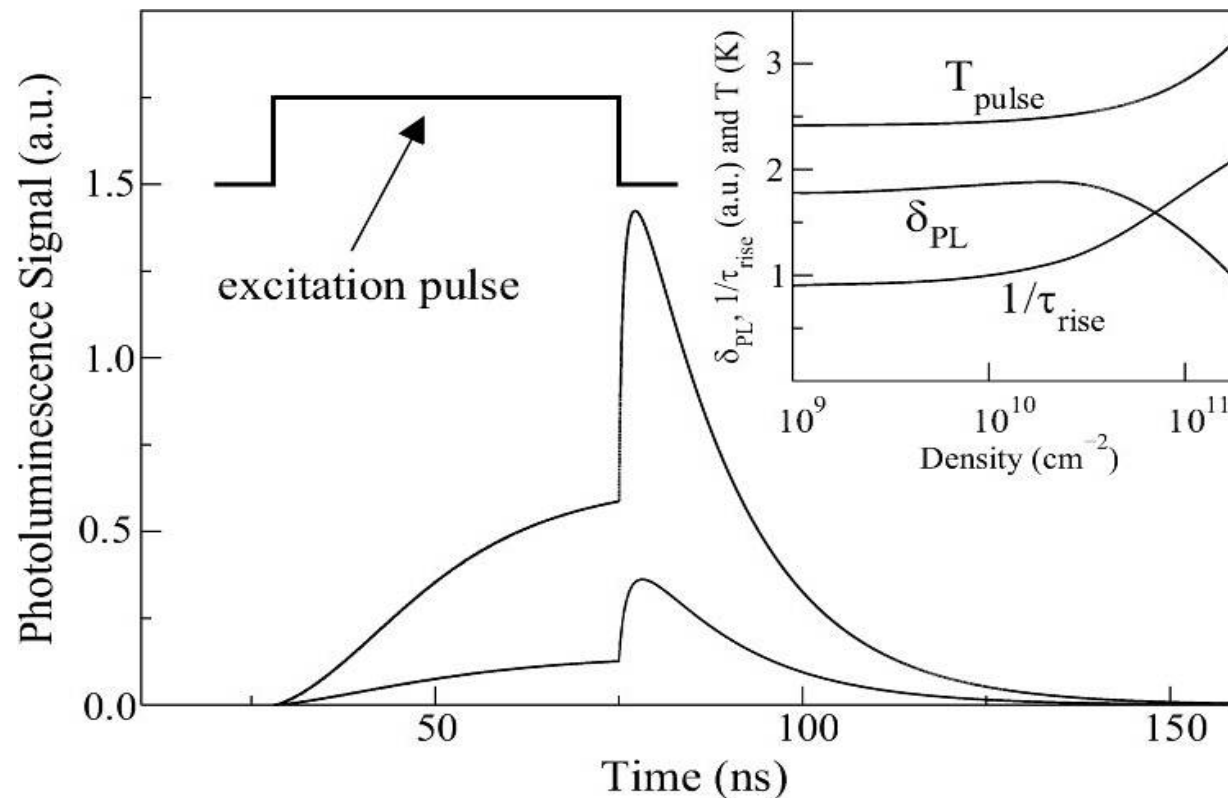


[L V Butov, A L Ivanov, A Imamoglu, P B Littlewood, A A Shashkin, V T Dolgoplov, and A Gossard (PRL, 2001)]

## The PL jump: origin and modelling

- The relaxation and PL dynamics:

$$\frac{\partial}{\partial t} \rho_{2D} = -\frac{\rho_{2D}}{\tau_{\text{opt}}} + \Lambda(t),$$
$$\frac{\partial}{\partial t} T = \left( \frac{\partial T}{\partial \rho_{2D}} \right) \frac{\partial \rho_{2D}}{\partial t} + S_T(t),$$

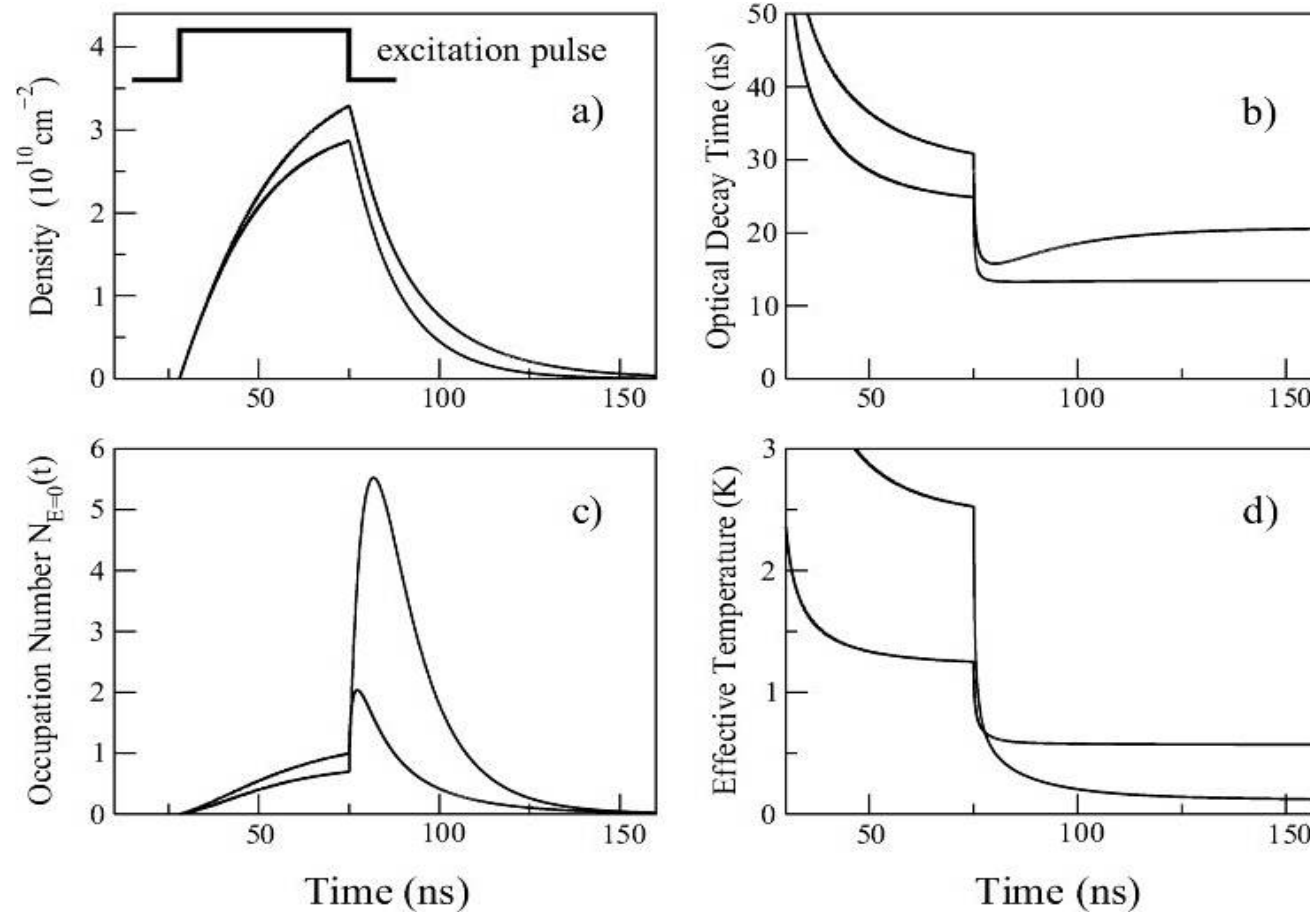


## PL dynamics in the presence of $H_{\perp}$

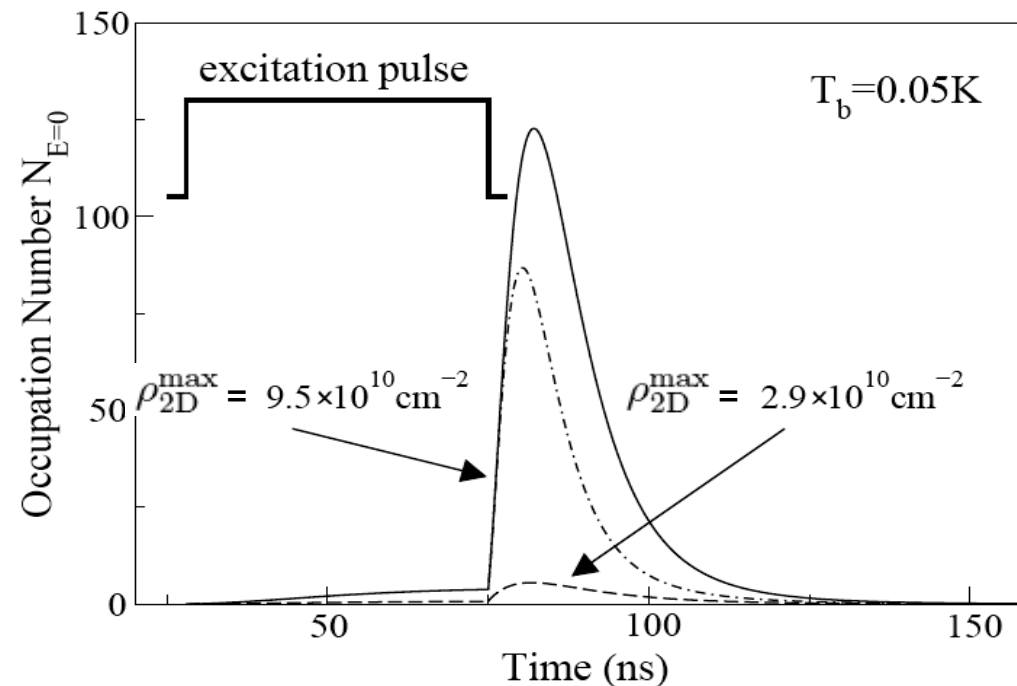
→  $H_{\perp} = 0$  and  $g = 4$  (solid lines);

→  $H_{\perp} = 14$  T,  $g = 1$  and  $M_x \simeq 7.1 M_x(H_{\perp}=0)$  (dashed lines)

(Yu. Lozovik and A. Ruvinskii, 1997).



# Occupation dynamics of the ground-state mode



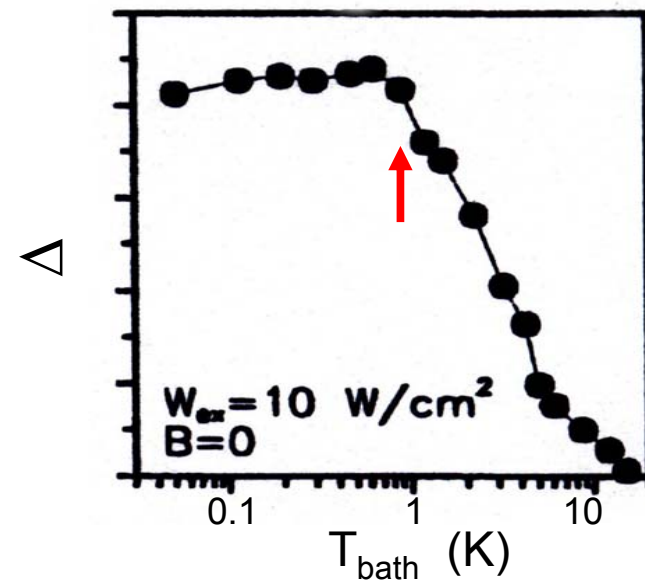
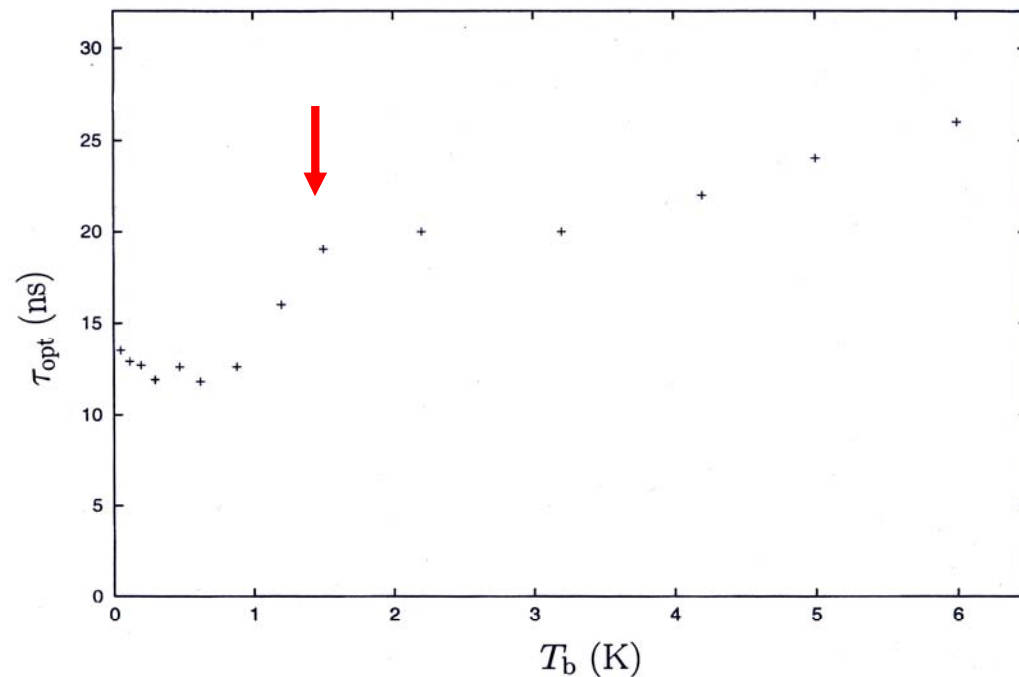
- Two contradictions in “modelling - experiment”

$$(10^9 \text{ cm}^{-2} \leq \rho_{2D}^{\max} \leq 3 \times 10^{10} \text{ cm}^{-2}):$$

- strongly nonlinear increase of  $1/\tau_{\text{rise}}$  with  $\rho_{2D}^{\max}$  (experiment)  
against  $1/\tau_{\text{rise}}$  nearly independent of  $\rho_{2D}^{\max}$  (theory);
- no PL-jump for  $\rho_{2D}^{\max} \simeq 10^9 \text{ cm}^{-2}$  (experiment)  
against a well-developed PL-jump with  $\delta_{\text{PL}} \simeq 2$  (theory).

## Narrowing effect due to the dipole-dipole interaction of indirect excitons

- building up of the narrowing effect with increasing  $\rho_{2D}$  ;
- phonon-assisted hopping against a well-defined  $\mathbf{k}_{\parallel}$  ;
- for  $\rho_{2D} \leq 10^9 \text{ cm}^{-2}$  the disorder effects are well-developed  
(S Baranovskii, E Runge and R Zimmermann) .



[L V Butov et al. (2001)]

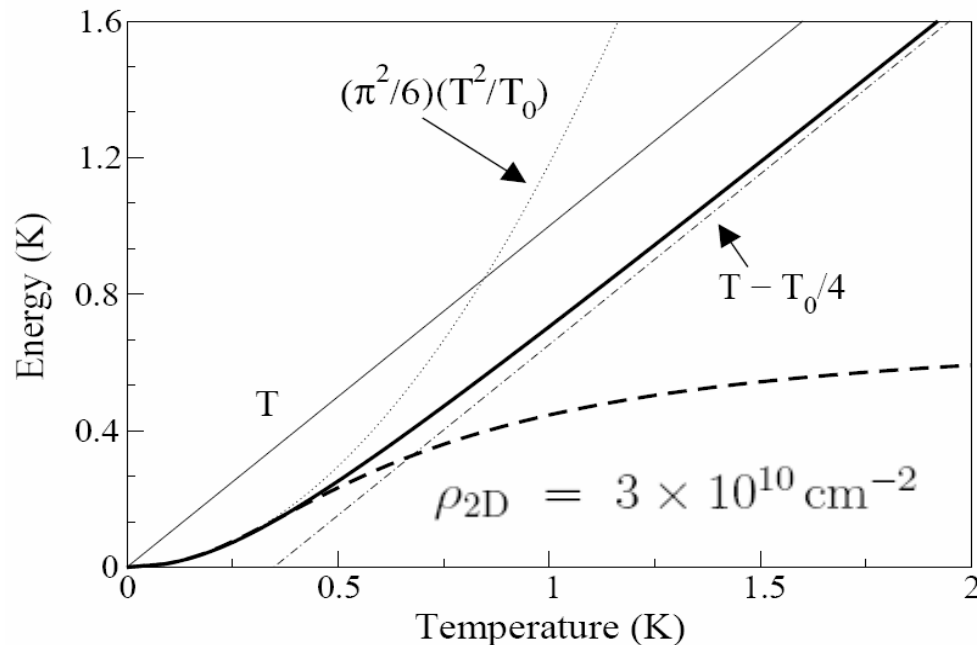
## Recombination heating or cooling of indirect excitons

$$E_{\text{kin}}^{(2d)} = \frac{T^2}{T_0} I_1(T, T_0) \quad \text{and} \quad E_{\text{opt}}^{(2d)} = E_\gamma \frac{\tau_{\text{opt}}}{\tau_{\text{E}}};$$

- $k_B T$  is much larger than  $k_B T_0$  and  $E_\gamma$ :

$$E_{\text{kin}}^{(2d)} = k_B T \left( 1 - \frac{1}{4} \frac{T_0}{T} \right) \simeq k_B T \quad \text{and} \quad E_{\text{opt}}^{(2d)} = \frac{3}{5} E_\gamma .$$

→ A net heating effect .



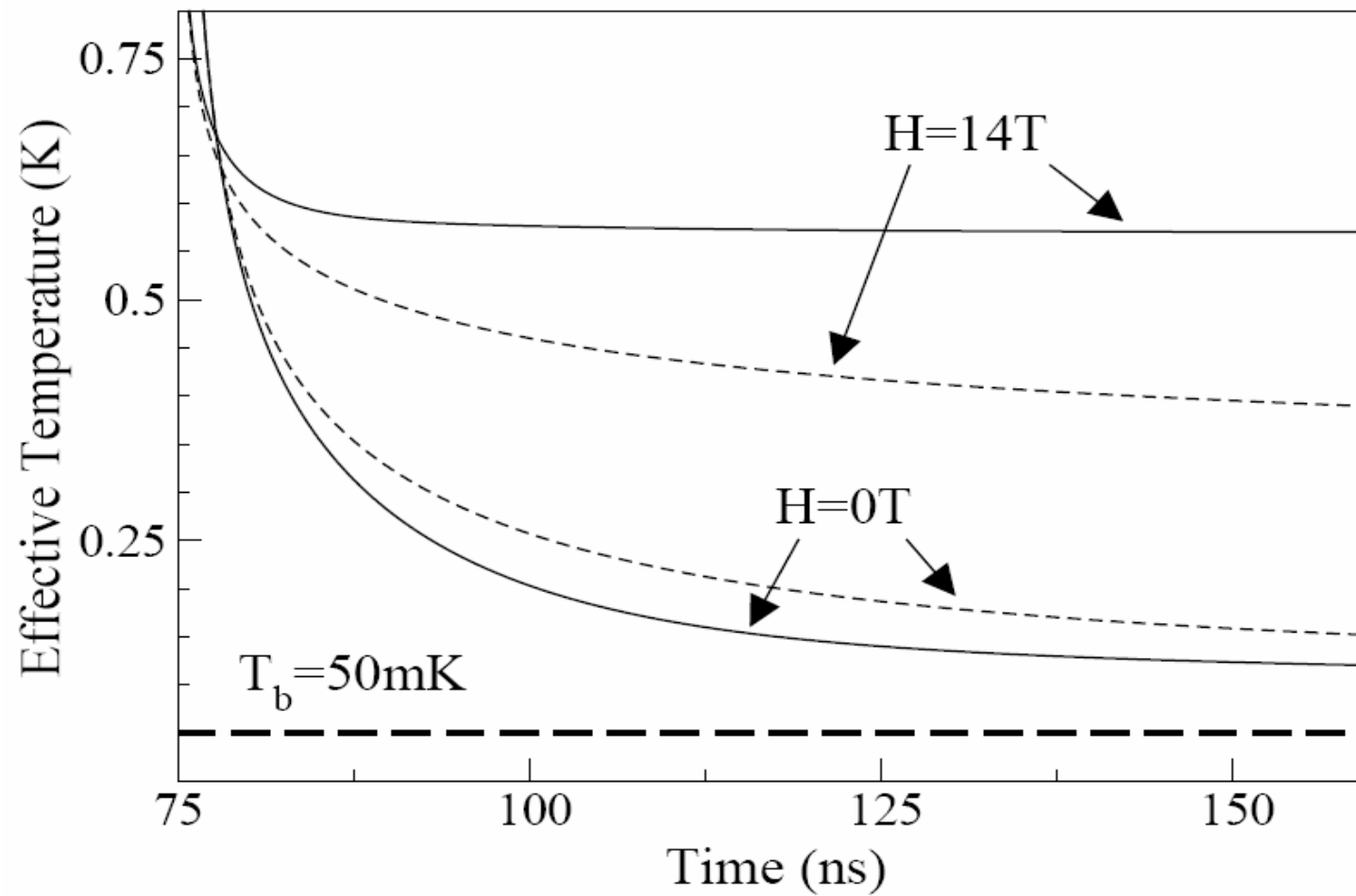
- $k_B T$  is much smaller than  $k_B T_0$  and  $E_\gamma$ :

$$E_{\text{kin}}^{(2d)} \simeq E_{\text{opt}}^{(2d)} \simeq \frac{\pi^2}{6} k_B \frac{T^2}{T_0} .$$

→ A net cooling effect .

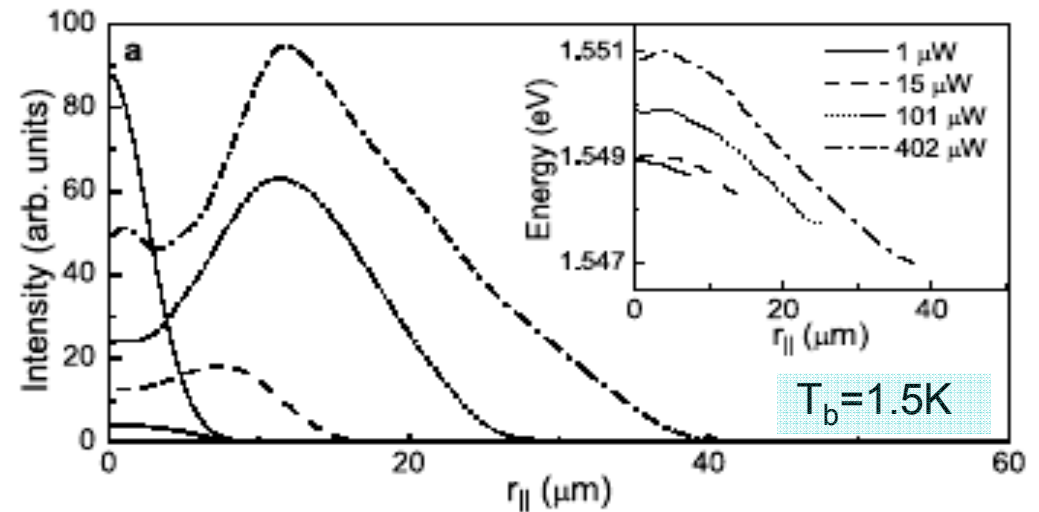
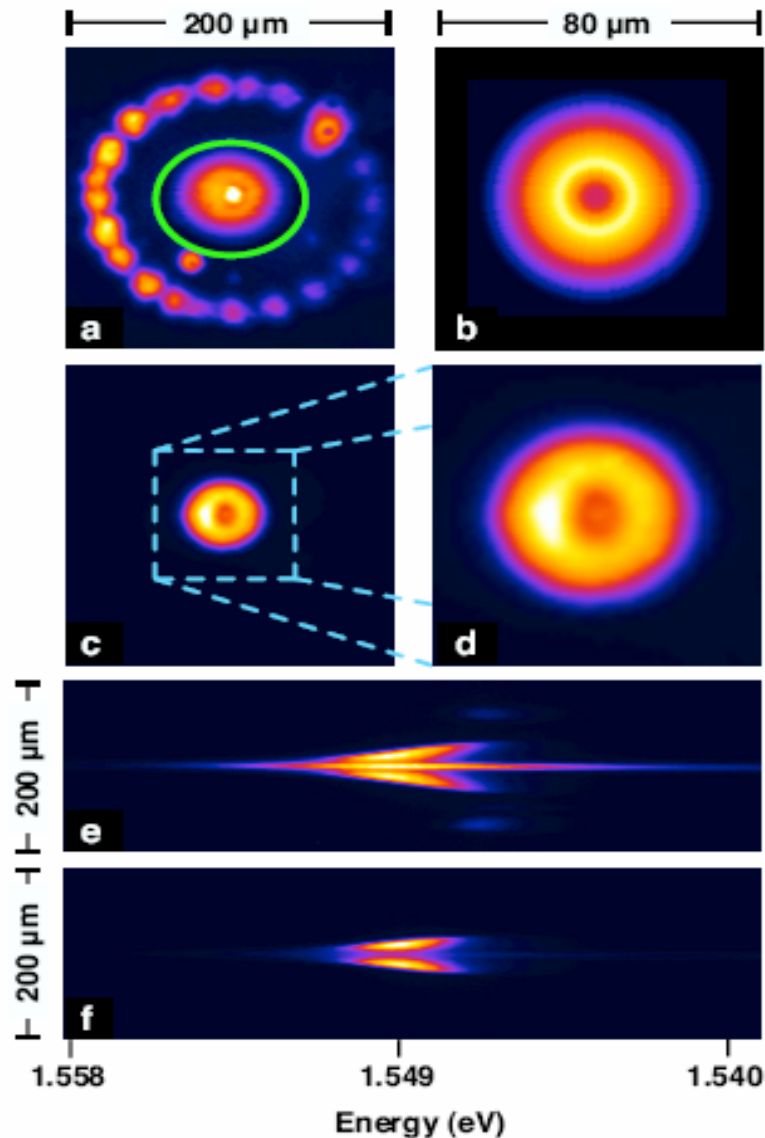


## Heating/cooling of indirect excitons due to the optical evaporation



# Origin of the inner ring in PL patterns of indirect excitons

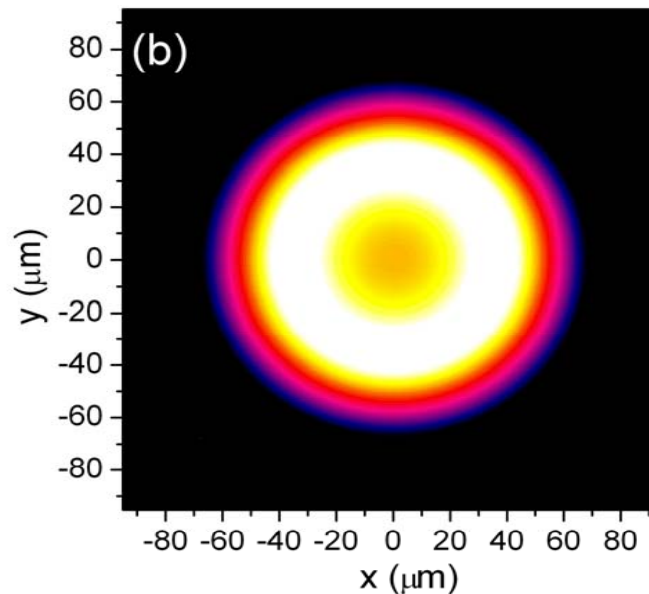
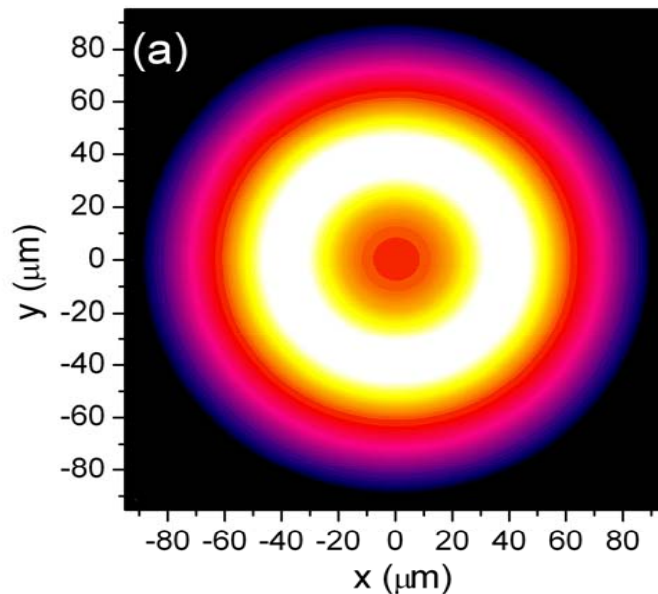
[A L Ivanov, L E Smallwood, A T Hammack, S Yang, L V Butov and A C Gossard, EPL (2006)]



- The first, inner PL ring is akin to the PL jump.
- Spatial pinning of the PL signal: importance of the disorder effects.

# Spatial pinning of the PL signal

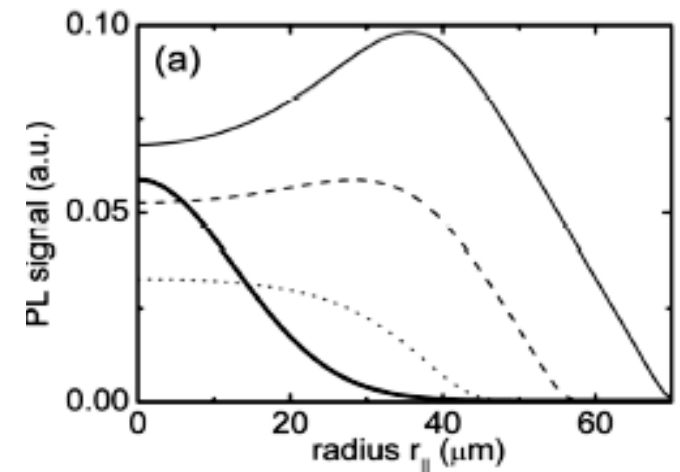
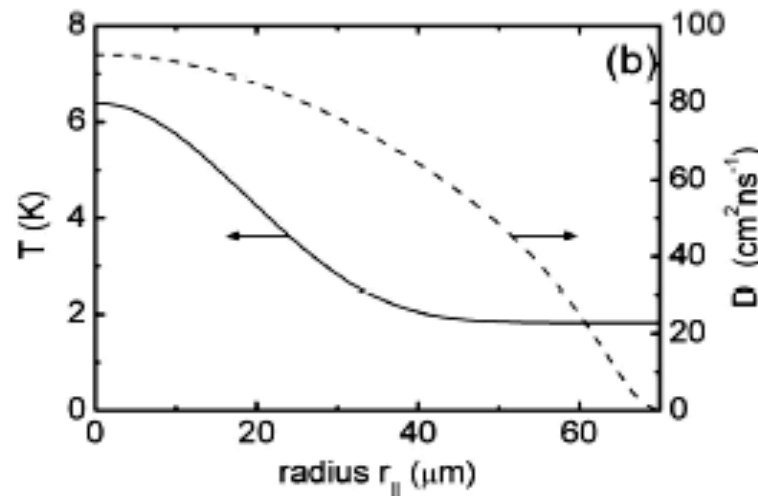
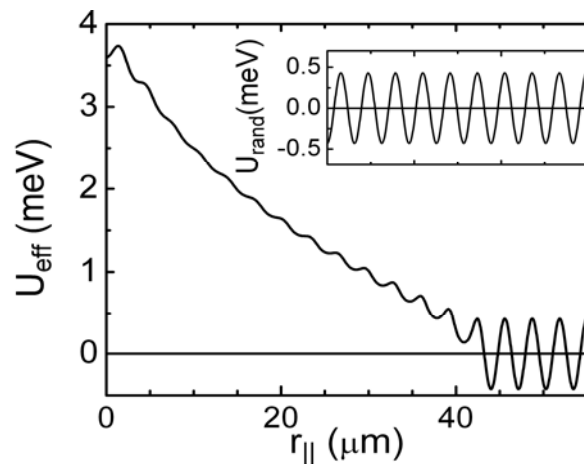
$$I_{\text{PL}} \propto \exp \left[ -(\Gamma_{\text{opt}}/D_{\text{x}}^{(2\text{D})})^{1/2} r_{\parallel} \right] ;$$



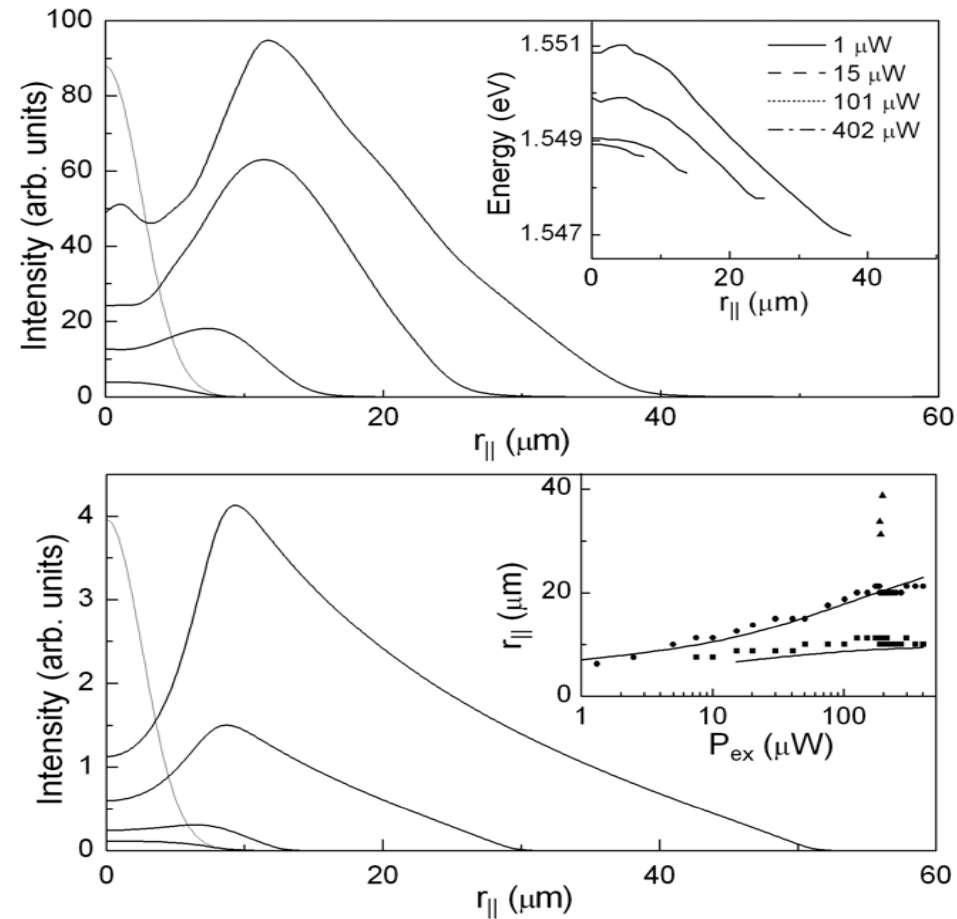
- Thermionic model:

$$\tilde{D}_{\text{x}}^{(2\text{D})} = D_{\text{x}}^{(2\text{D})} \exp \left[ -\frac{U_{\text{rand}}(\mathbf{r}_{\parallel})}{k_{\text{B}}T + u_0\rho_{2\text{D}}^{(0)}} \right] .$$

$$D_{\text{x}}^{(2\text{D})} = 100 \text{ cm}^2/\text{s}$$



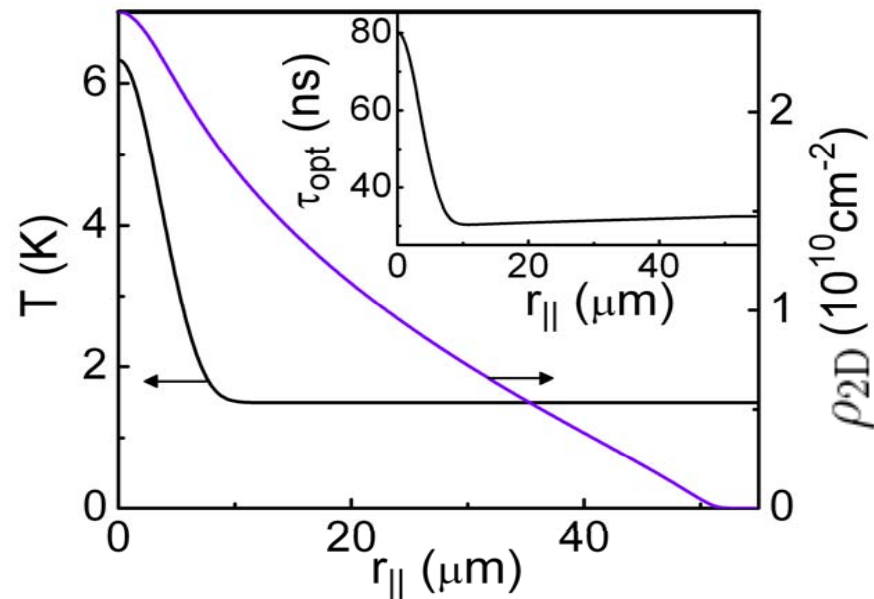
# Modelling of the inner ring



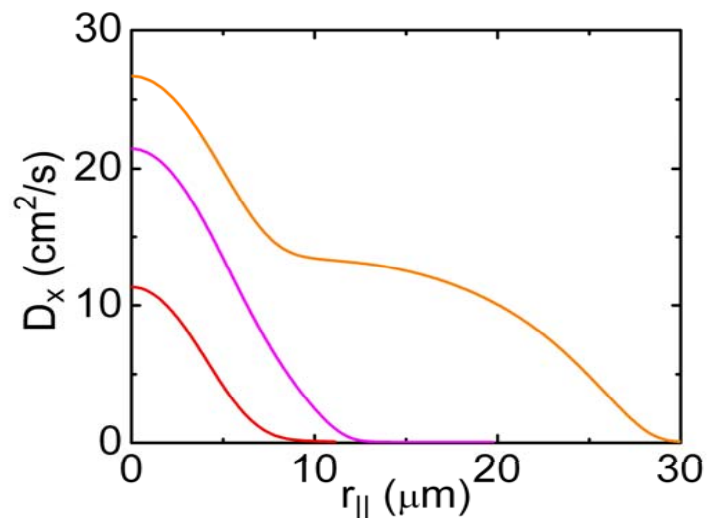
Disorder potential:

$$U_0/2 = 0.45 \text{ meV.}$$

## Effective temperature and radiative lifetime against radius



## Diffusion coefficient against radius



$$D_x^{(2D)} = \frac{D_{x-x}^{(2D)} D_{x-\text{imp}}^{(2D)}}{D_{x-x}^{(2D)} + D_{x-\text{imp}}^{(2D)}};$$

→  $D_{x-x}^{(2D)} = C_{x-x}(T/T_0)$  – self-diffusion coefficient ,

→  $D_{x-\text{imp}}^{(2D)}$  – diffusion due to scattering by imperfections .

[consistent with Z Voros, R Balili, D Snoke et al., PRL (2005) ]

## In-plane drift and diffusion velocities

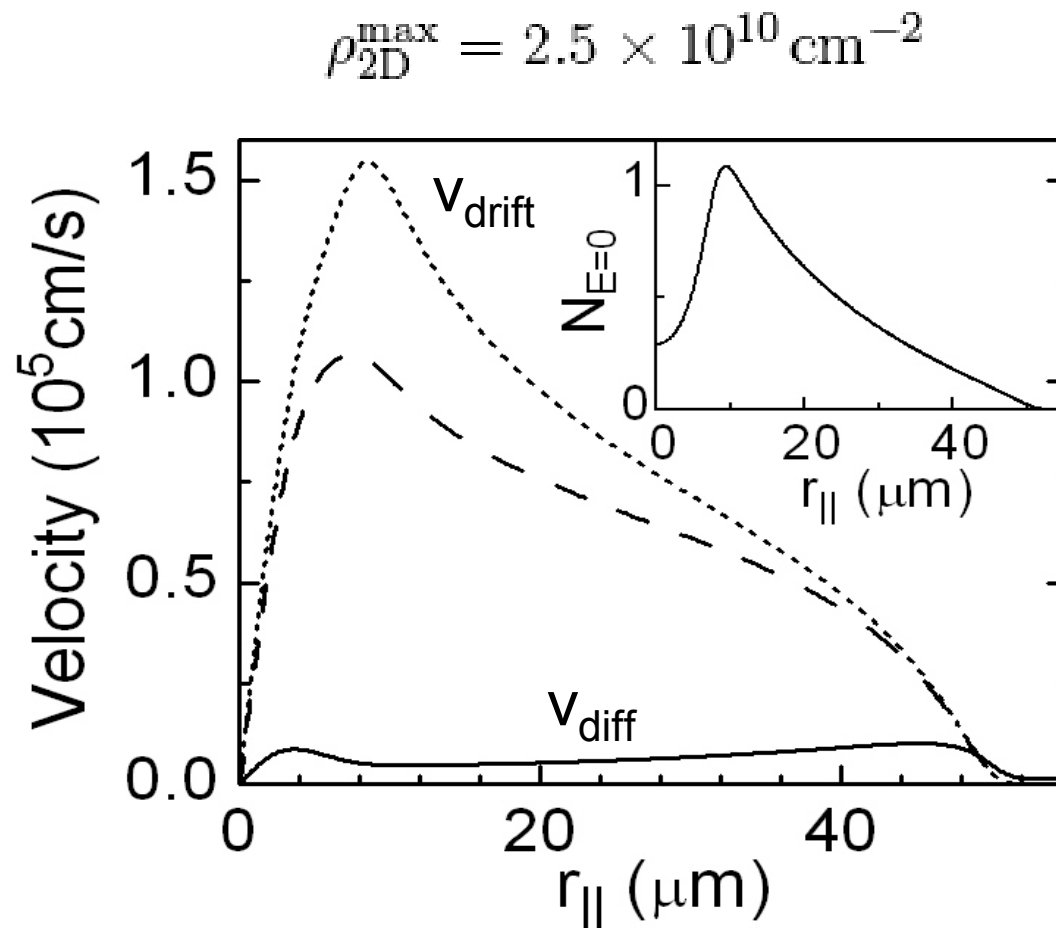
- Drift and diffusion velocities:

$$v_{\text{drift}} = -D_{\text{x}}^{(2\text{D})} \frac{(e^{T/T_0} - 1)}{k_{\text{B}}T_0} u_0 \nabla \rho_{2\text{D}};$$

$$v_{\text{diff}} = -D_{\text{x}}^{(2\text{D})} \frac{\nabla \rho_{2\text{D}}}{\rho_{2\text{D}}}.$$

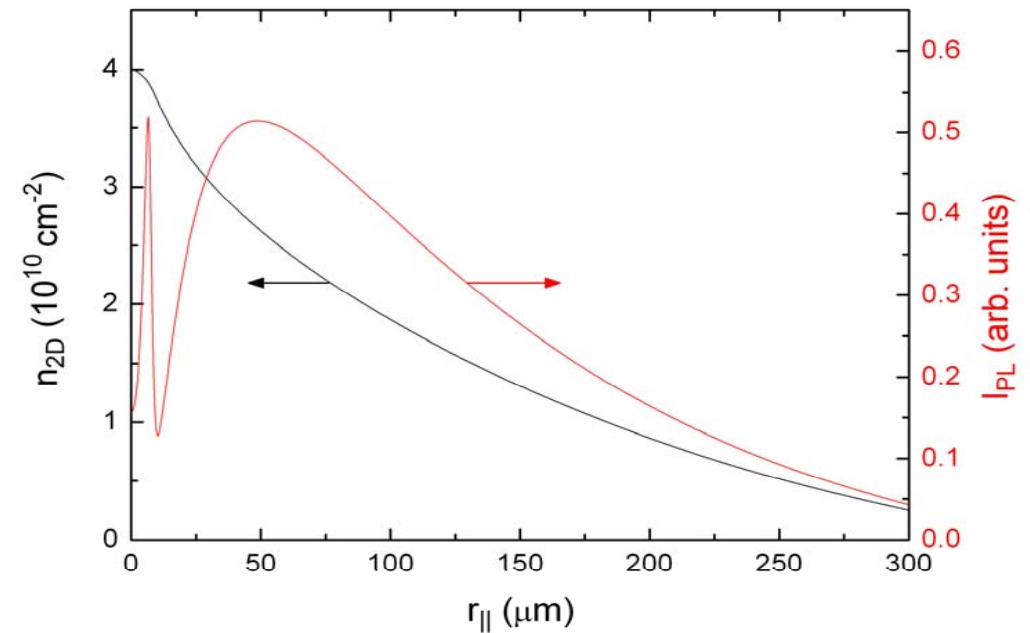
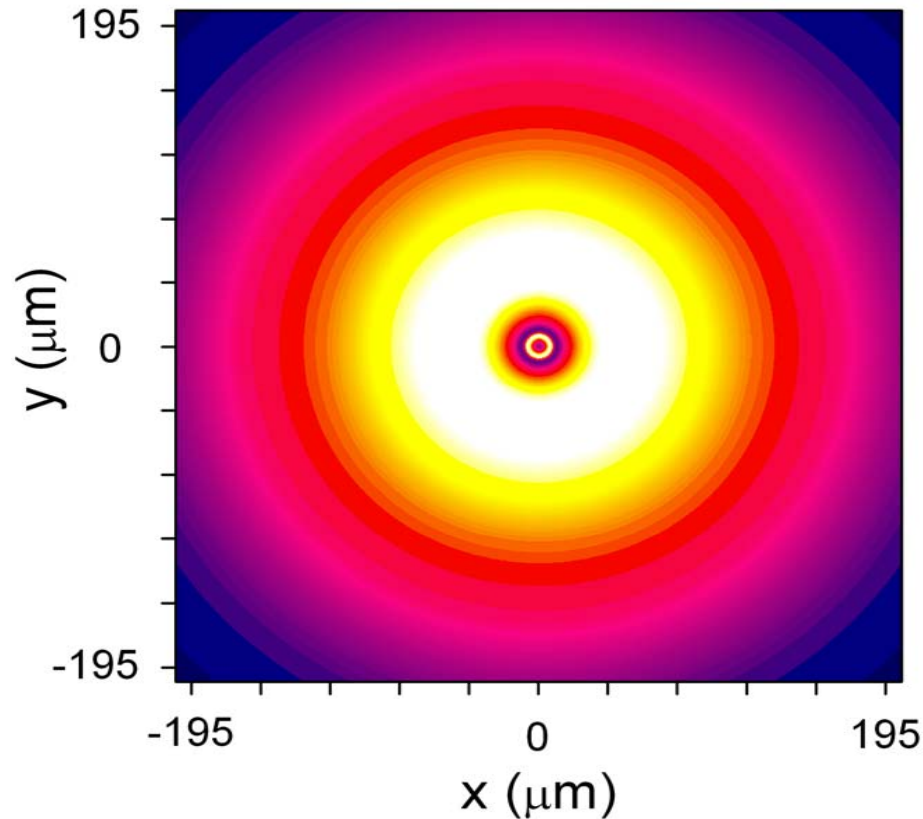
- The mean-field energy gradient:

$$u_0 |\nabla \rho_{2\text{D}}(r_{\parallel} \simeq r_{\parallel}^{\text{rg}})| \simeq 1.6 \text{ eV/cm}.$$



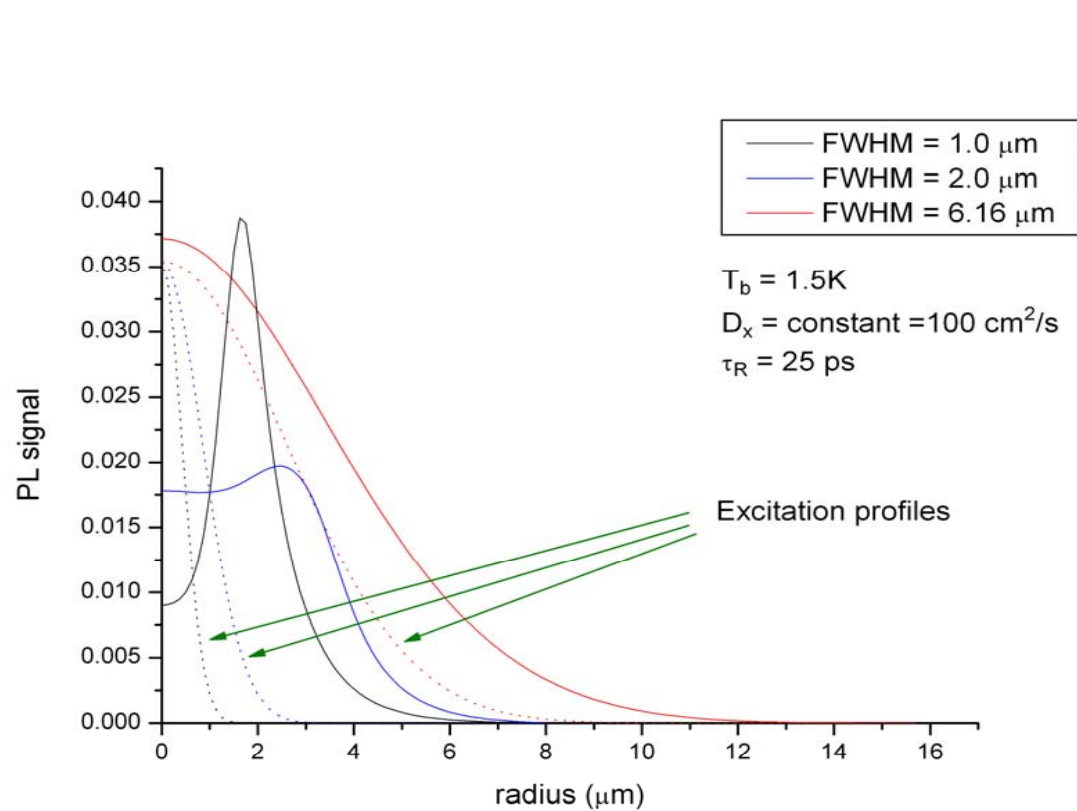
# Second inner ring in spatially-resolved photoluminescence

[L Smallwood, PhD, 2006]

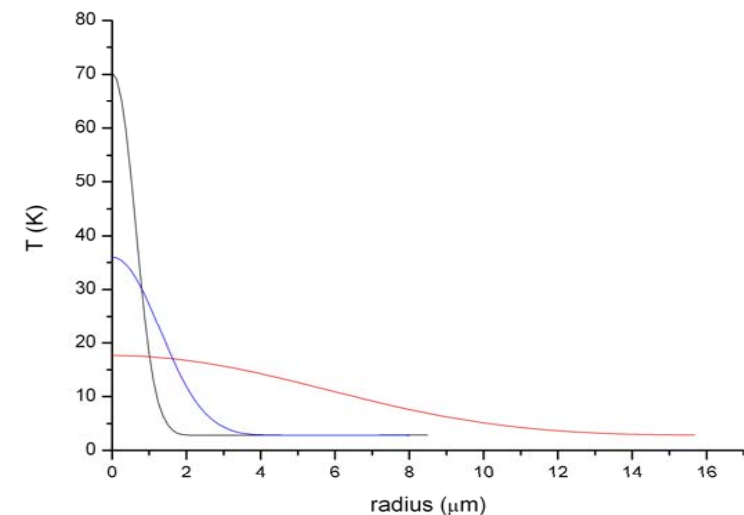
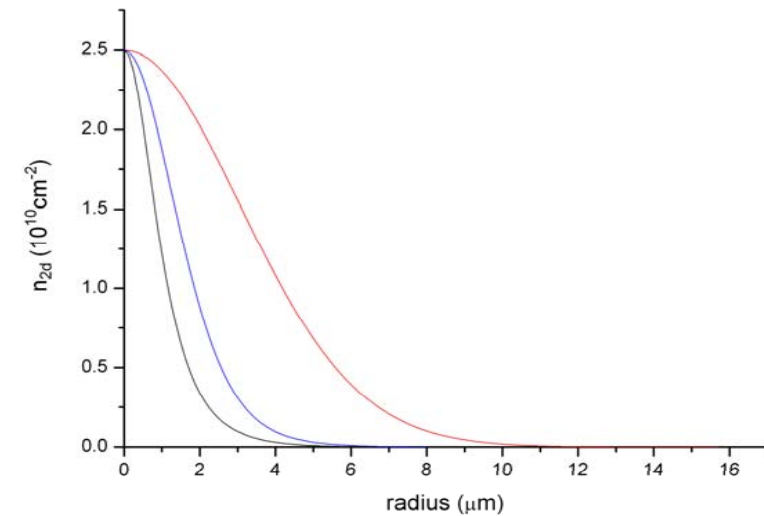


$D_x = 250 \text{ cm}^2/\text{s}$ ,  $\text{FWHM} = 6.16 \text{ microns}$ ,  
 $T_b = 1.5\text{K}$ ,  $U_0 = 0.9 \text{ meV}$ ,  $\tau_R = 13 \text{ ns}$ .

# Inner ring in spatially-resolved photoluminescence from direct excitons in a single quantum well



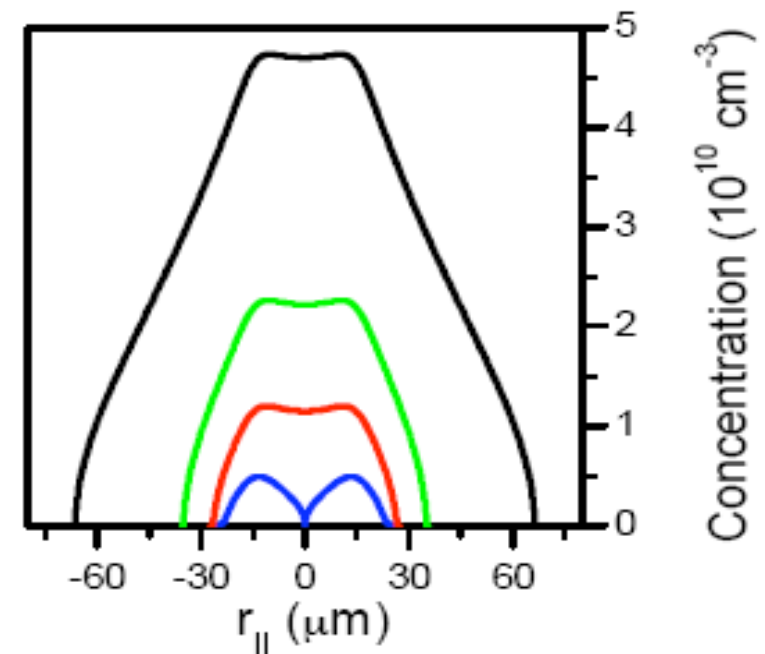
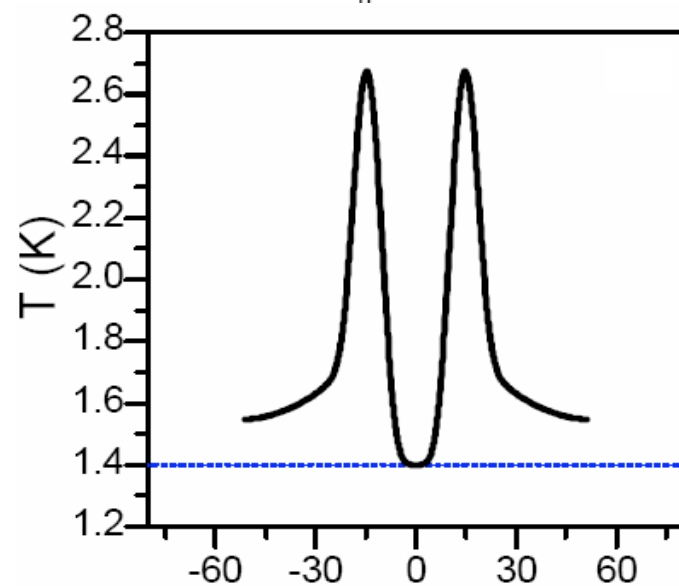
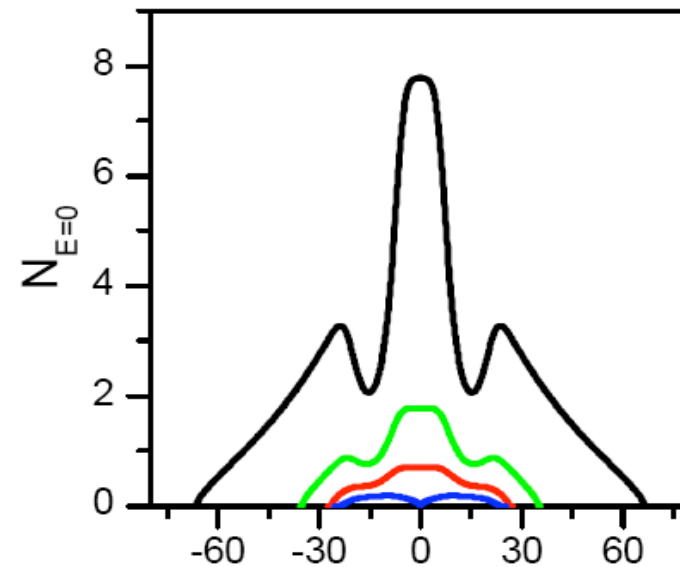
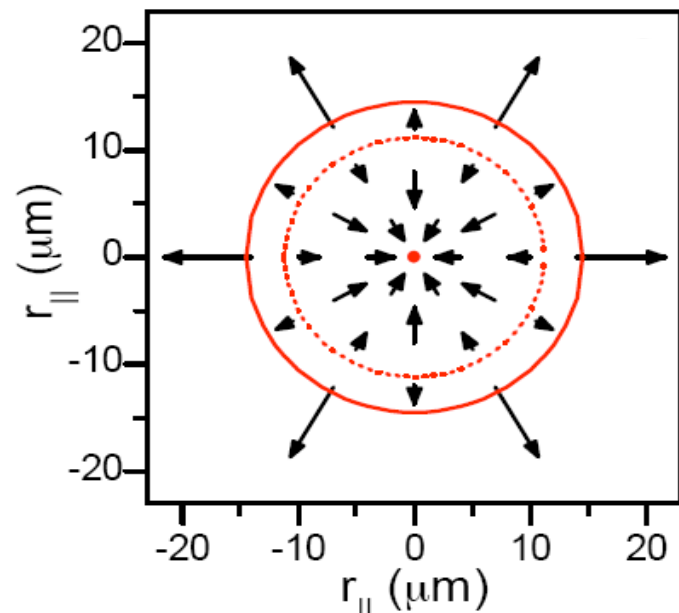
No mean-field energy and no drift





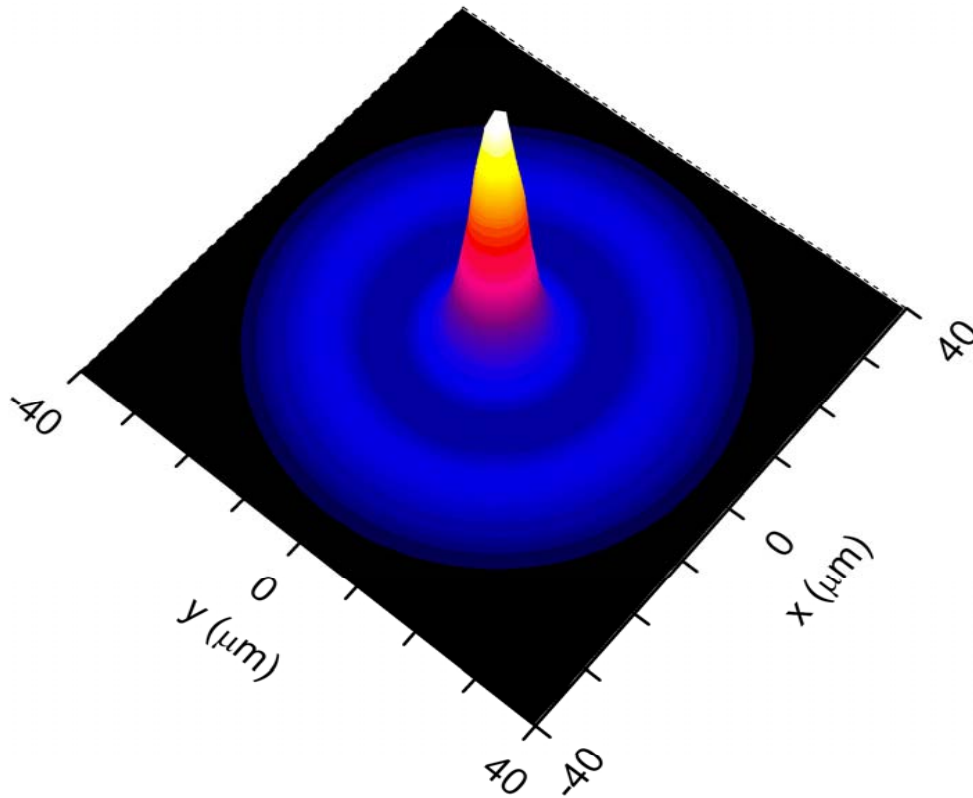
# Optically-induced traps for indirect excitons

[A T Hammack, M Griswold, L V Butov, L E Smallwood, A L Ivanov and A C Gossard, PRL (2006)]



# Ultra-cold indirect excitons

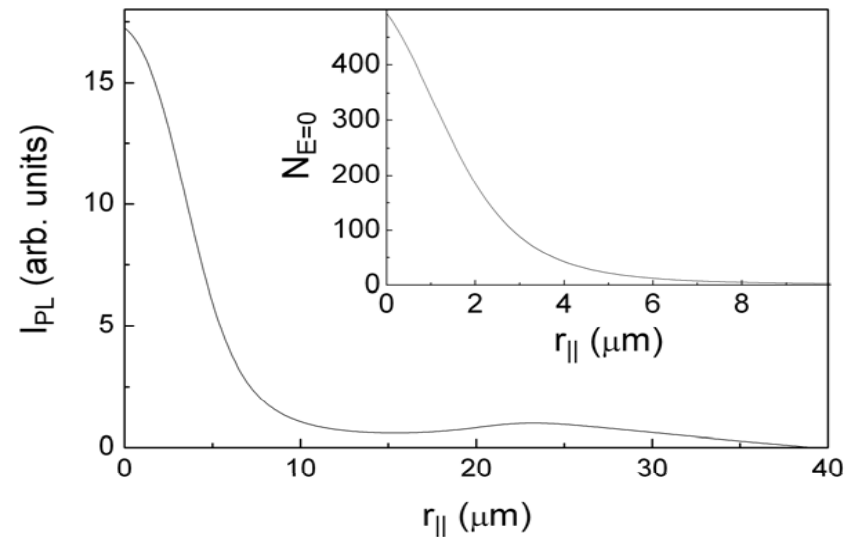
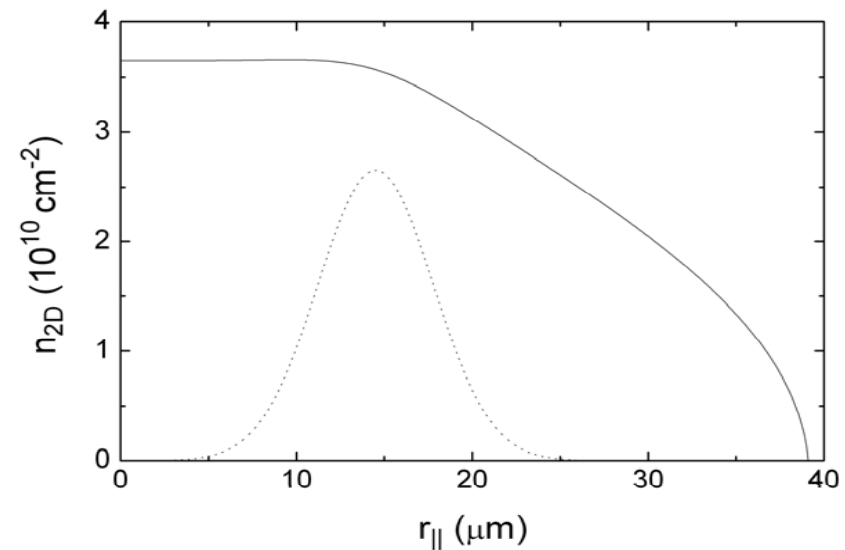
[L Smallwood, PhD, 2006]



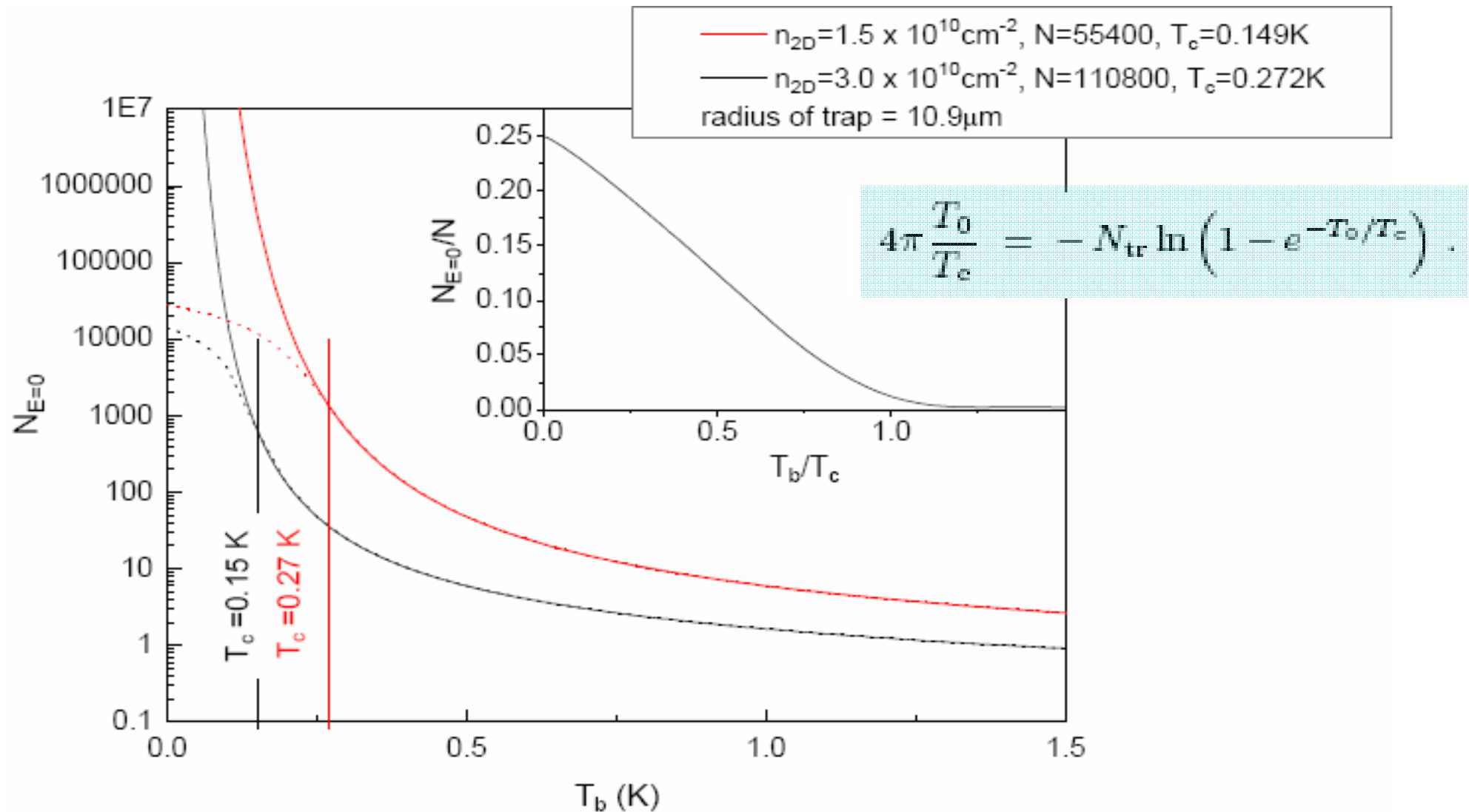
$T_b = 0.4\text{K},$

FWHM = 7.7 microns,

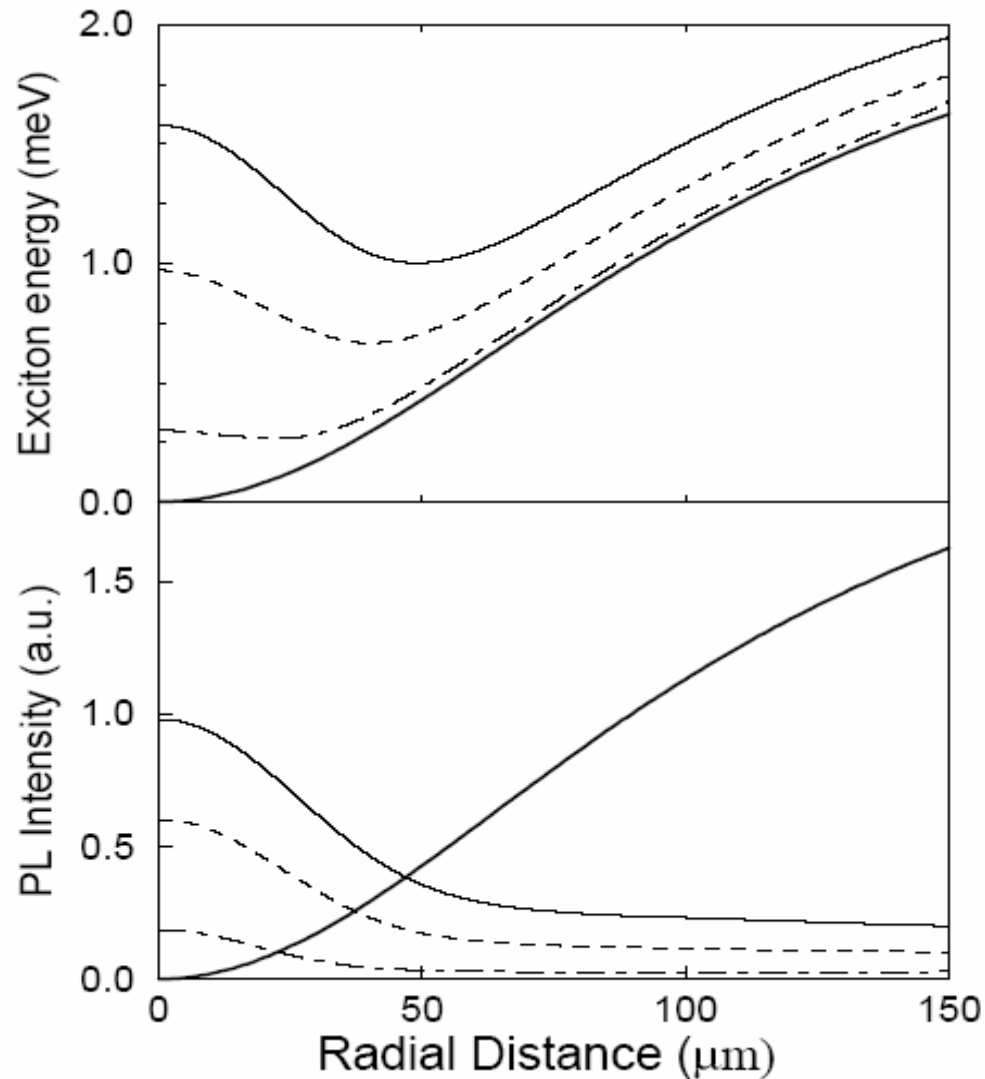
$U_0 = 1.8 \text{ meV}, \tau_R = 21 \text{ ns}.$



# Towards BEC of indirect excitons



# Photoluminescence of indirect excitons in an in-plane trap



- An in-plane trap:

$$U_{\text{trap}}(r_{\parallel}) = \frac{k r_{\parallel}^2}{2(1 + r_{\parallel}^2/r_0^2)},$$

$$\rightarrow r_0 = 110 \mu\text{m} \text{ and } k = 0.4 \mu\text{eV}/\mu\text{m}^2$$

$$\text{so that } U_{\text{trap}}^{\text{max}} = 2.5 \text{ meV};$$

$$\rightarrow \rho_{2\text{D}}^{\text{max}} \simeq 10^{10} \text{ cm}^{-2}.$$

## Hydrodynamics of statistically-degenerate indirect excitons

$$v \frac{\partial \rho_{2D}}{\partial r_{\parallel}} + \rho_{2D} \frac{\partial v}{\partial r_{\parallel}} + \frac{1}{r_{\parallel}} \rho_{2D} v + \frac{\rho_{2D}}{\tau_{\text{opt}}} = \Lambda(r_{\parallel}),$$

$$\begin{aligned} (M_x v^2 + T I_0 + \rho_{2D} B) \frac{\partial \rho_{2D}}{\partial r_{\parallel}} + 2 M_x \rho_{2D} v \frac{\partial v}{\partial r_{\parallel}} + \rho_{2D} \left( 2 \frac{T}{T_0} I_1 - I_0 \right) \frac{\partial T}{\partial r_{\parallel}} \\ + M_x \rho_{2D} \frac{v^2}{r_{\parallel}} = - \rho_{2D} \frac{\partial U_{\text{QW}}}{\partial r_{\parallel}}, \end{aligned}$$

$$\begin{aligned} \left( v \frac{M_x v^2}{2} + 2 v T I_0 + B \rho_{2D} v \right) \frac{\partial \rho_{2D}}{\partial r_{\parallel}} + \left( \frac{3}{2} M_x v^2 + 2 E_{\text{kin}} \right) \rho_{2D} \frac{\partial v}{\partial r_{\parallel}} \\ + 2 v \left( 2 \frac{T}{T_0} I_1 - I_0 \right) \rho_{2D} \frac{\partial T}{\partial r_{\parallel}} + \frac{1}{r_{\parallel}} v \rho_{2D} \left( \frac{M_x v^2}{2} + 2 E_{\text{kin}} \right) \\ = - \rho_{2D} v \frac{\partial U_{\text{QW}}}{\partial r_{\parallel}} + \Lambda(r_{\parallel}) \Omega_0. \end{aligned}$$



## Conclusions

- In-plane diffusion of statistically-degenerate indirect excitons:
  - a) Quantum diffusion equation and generalized Einstein relationship.
  - b) Mean-field energy gives rise to effective screening of QW disorder.
  - c) Large drift velocities,  $v_{\text{drift}} \geq 10^5$  cm/s.
- Relaxation thermodynamics of indirect excitons:
  - a) Heating of indirect excitons by the laser pulse ( $T$  can be much larger  $T_b$ ).
  - b) Recombination heating or cooling of indirect excitons.
  - c) Heating due to conversion of the MF energy to the internal one.
  - d) It is extremely difficult, but still possible, to get  $T < 1\text{K}$  and  $N_{E=0} > 10$ .
- Modelling of the inner ring in the PL patterns of indirect excitons:
  - a) The ring is due to cooling of excitons in their propagation from the laser spot.
  - b) Diffusion coefficient  $D_x = 10\text{-}30$  cm<sup>2</sup>/s and amplitude of disorder  $U_0/2 = 0.5$  meV.
- Synergetics of indirect excitons:

Three coupled nonlinear equations for  $\rho_{2D}$ ,  $T$  and  $\Gamma_{\text{opt}}$ .