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PHYSICS OF NANO-DEVICES

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Optical Properties of Materials with Negative Refraction: Perfect Lenses and Cloaking

Presented by:

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Optical Properties of Materials with Negative Refraction: Perfect Lenses and Cloaking

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ICTP 06

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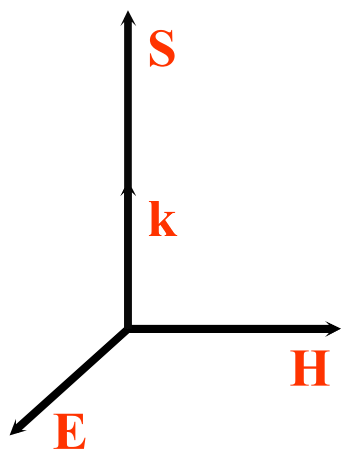
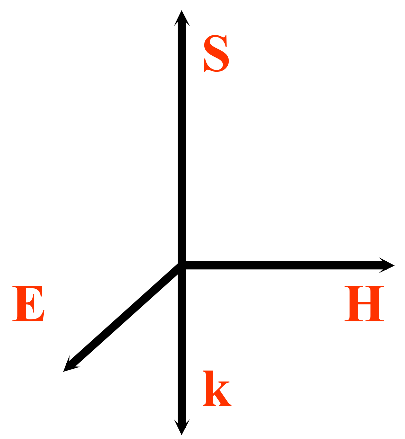
In his seminal work Veselago[†] has introduced the concept of the Left Handed Materials (LHM's). In his defenition the LHM's are materials where in some frequency range both electric permittivity ε and magnetic permeability μ are negative.



Usual (right-handed) material ($\varepsilon > 0$, $\mu > 0$)



Left-Handed Material

† V. G. Veselago, Sov. Phys.-Solid State **8**, 2854 (1967); Sov. Phys. Uspekhi **10**, 509 (1968).

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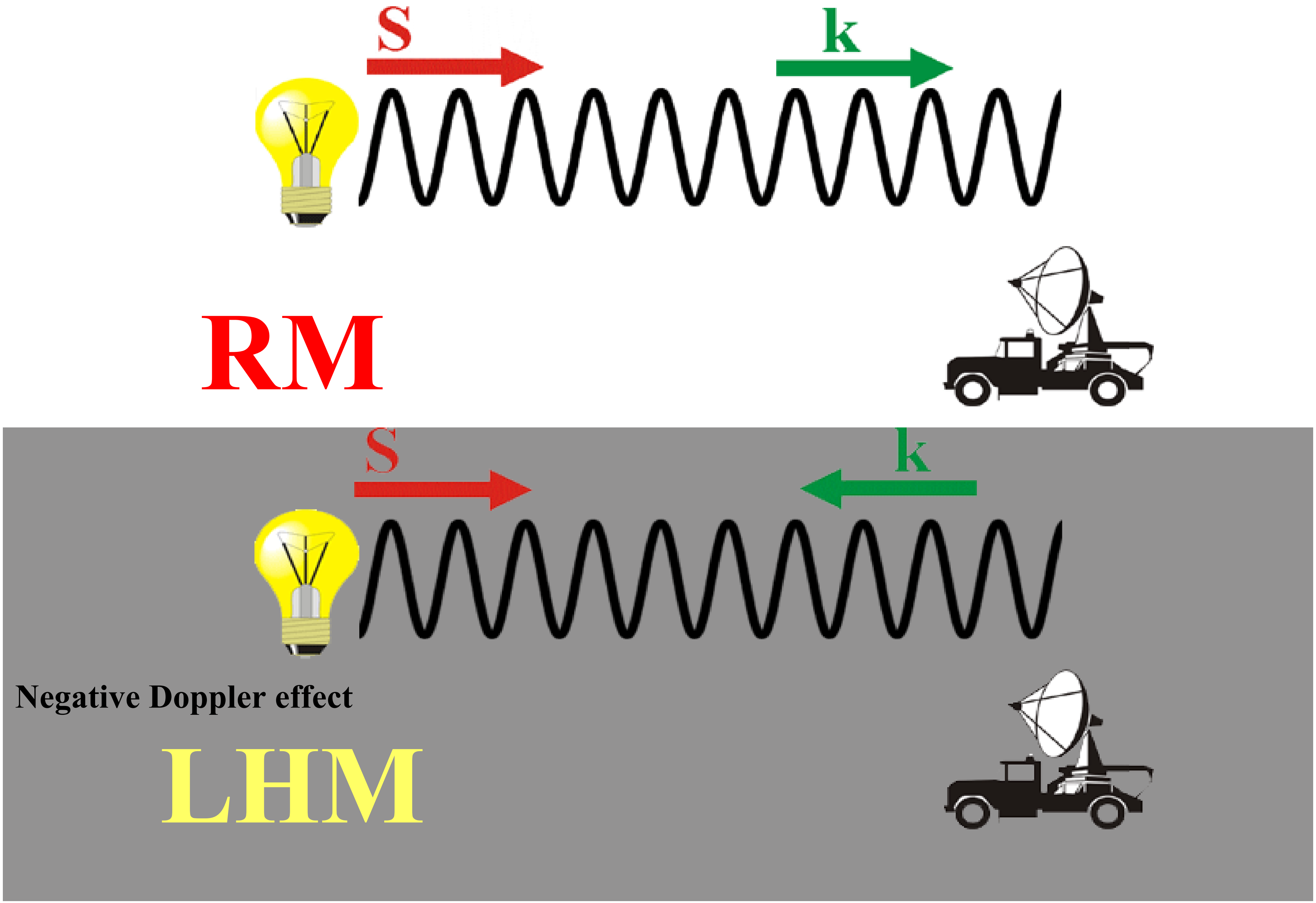


S
k
RM
S
k
Negative Doppler effect
LHM

Nowadays the most common definition of these materials is “Negative refractive index materials”

I think it is misleading: Refractive index n does not enter into Maxwell’s equations. A usual definition is

◆=c|k|/n, so n>0. One can change it as ◆= -c|k|/n, so that n<0. But this can be done for any material and leads to changes in some other equations of electrodynamics.

Theorem 1: ***The product n'n" is positive in the RM and negative in the LHM***

$$n^2 = \varepsilon\mu$$

$$(n' + in'')^2 = (\varepsilon' + i\varepsilon'')(\mu' + i\mu'')$$

Im[…]

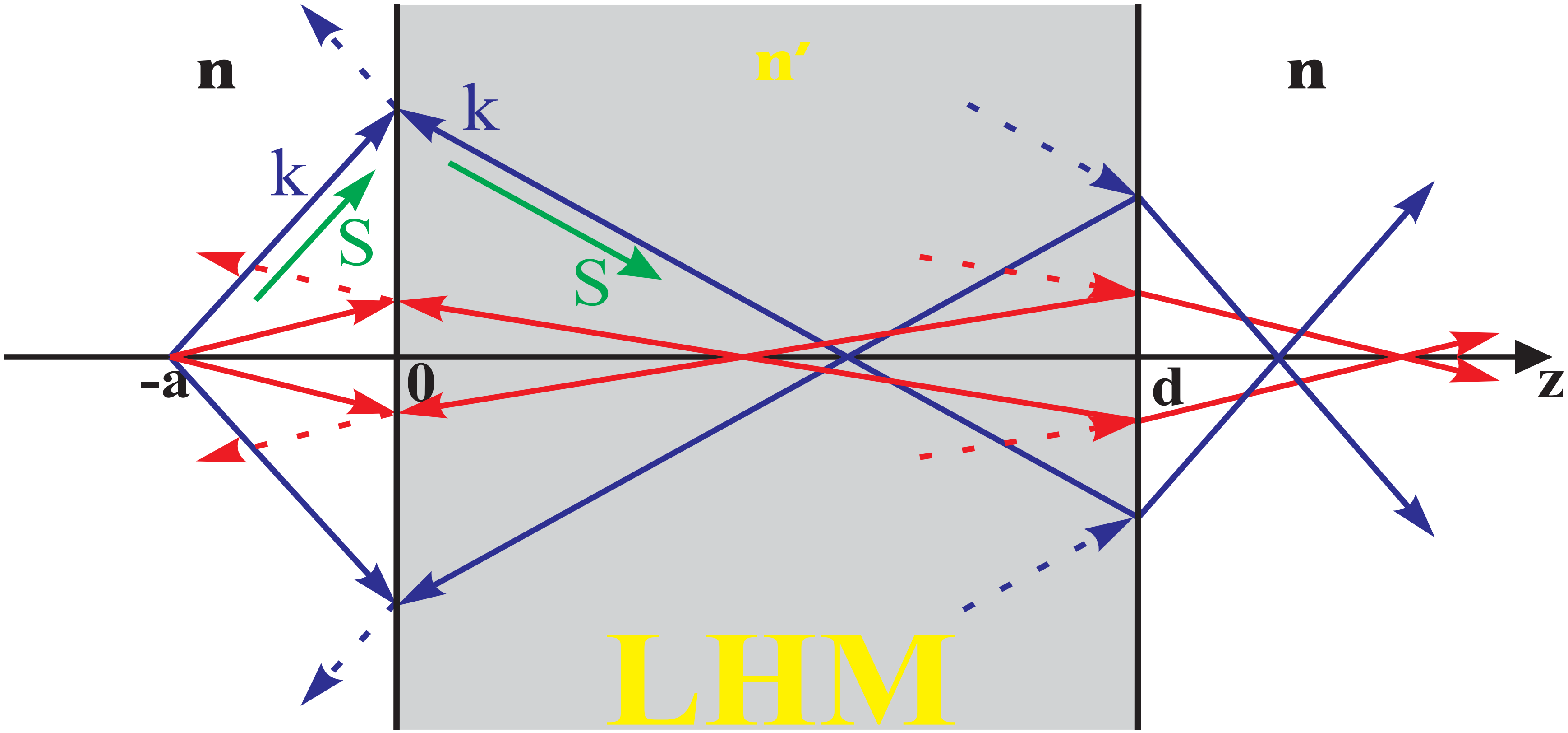
$$\varepsilon'\mu'' + \mu'\varepsilon'' = 2n'n''$$

$$\varepsilon'' > 0 \qquad \mu'' > 0$$

$n'n'' < 0$ For the LHM

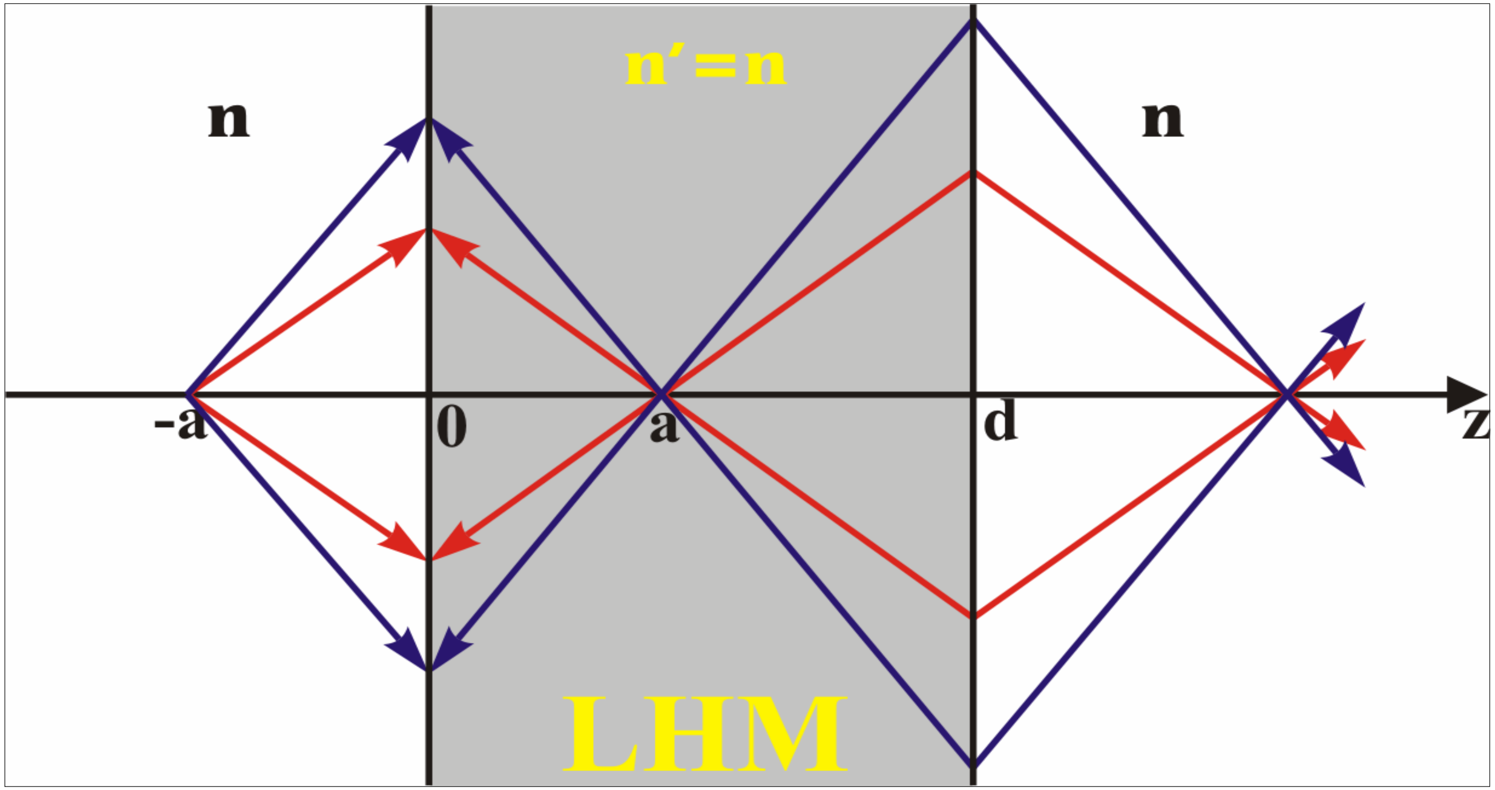
$n'n'' > 0$ For the RM

A. L. Pokrovsky, A. L. Efros, Solid St. Com. 124,283, 2002



Reflection and refraction of light outgoing from a point source at z=-a and passing through the slab of the LHM at 0<z<d. Refraction of light is described by the anomalous Snell's law. The wave vectors of the reflected waves are shown by dashed lines near interfaces only. The slab is surrounded by the usual material (ε>0, μ>0).

The Veselago lens



For this lens $\varepsilon=-\varepsilon'$ and $\mu=-\mu'$, then $n' = n$ and $i = -r$.

The reflected wave is completely absent.

All rays have foci at points $z=a$ and $z=2d-a$.

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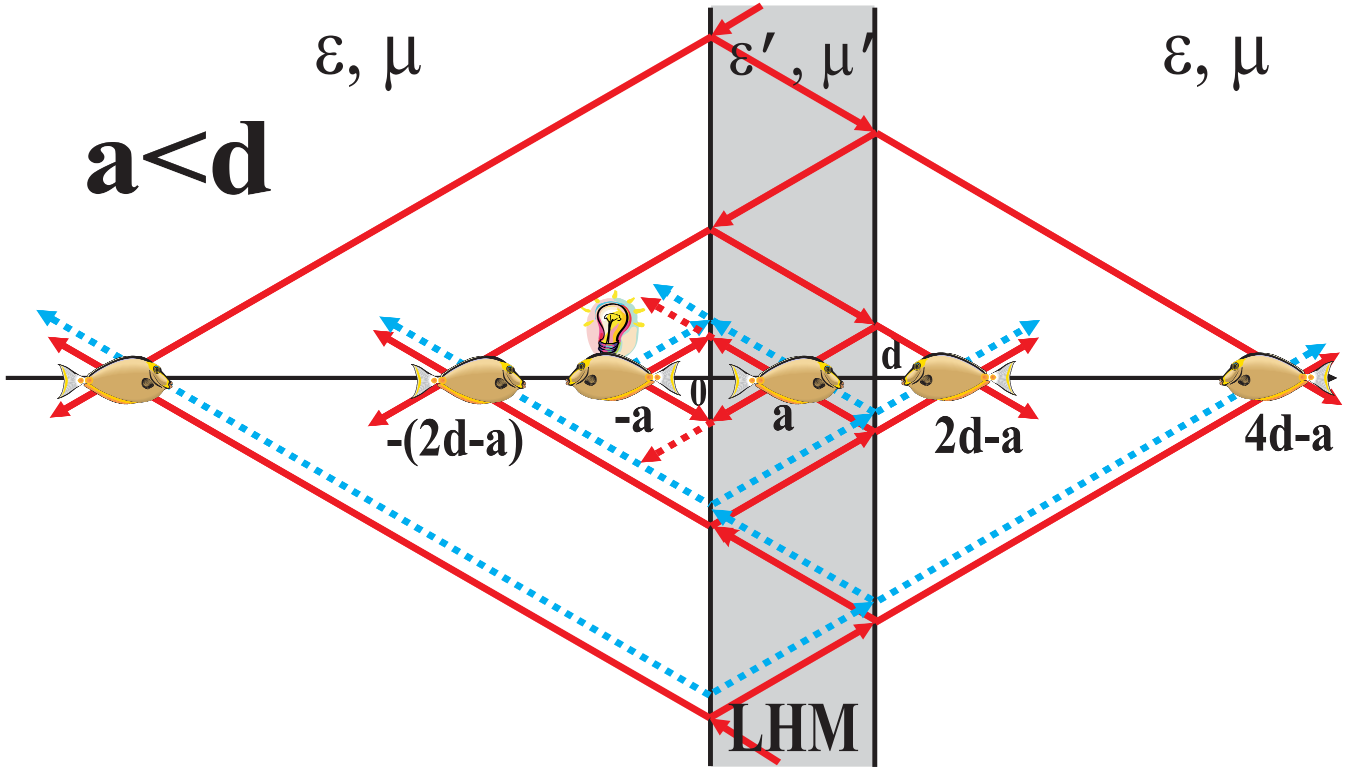
$\varepsilon\mu = \varepsilon'\mu'$, $\varepsilon \neq -\varepsilon'$, $\mu \neq -\mu'$



A. L. Pokrovsky and A. L. Efros, Applied Optics (2003).

Is it a perfect lens in terms of wave optics as well?

J. B. Pendry:

Negative Refraction Makes a Perfect Lens,

Phys. Rev. Letters **85**, 3966 (2000)

Perfect lens?????

“A suspicious object like that , it was clearly full of Dark Magic –” Rowling, Harry Potter, book 2

A. L. Pokrovsky and A. L. Efros: ***Diffraction in Left-Handed Materials and Theory of Veselago Lens***, cond-mat/0202078 (2002);Proceedings of ETOPIM, Physica B (2003)

N. Garcia and M. Nieto-Vesperinas: ***Left-Handed Materials Do Not Make a Perfect Lens***, Phys. Rev. Letters **88**, 207403 (2002)

F. D. M. Haldane: ***Electromagnetic Surface Modes at Interfaces with Negative Refractive Index make a "Not-Quite-Perfect" Lens***, cond-mat/0206420 (2002)

Pendry's idea of "perfect lens": Ampification of evanescent wave

Source magnetic field:

$$H_s = iH_0 H_0^{(1)}(\rho k_0)$$

$$k_0 = \omega n / c$$

$$H_0^{(1)} = J_0 + iN_0$$

$$H_s = H_p + H_{ev}$$

$$H_p = i\frac{H_0}{\pi}\int_{-k_0}^{k_0} \frac{\exp i(ky + x\sqrt{k_0^2 - k^2})}{\sqrt{k_0^2 - k^2}} dk$$

$$H_{ev} = \frac{H_0}{\pi}\int_{|k|>k_0} \frac{\exp(iky - x\sqrt{k^2 - k_0^2})}{\sqrt{k^2 - k_0^2}} dk$$

Pendry considered one EW

In the RHM:

$$H_{ev}(k) = \frac{\exp(iky - x\sqrt{k^2 - k_0^2})}{\sqrt{k^2 - k_0^2}}$$

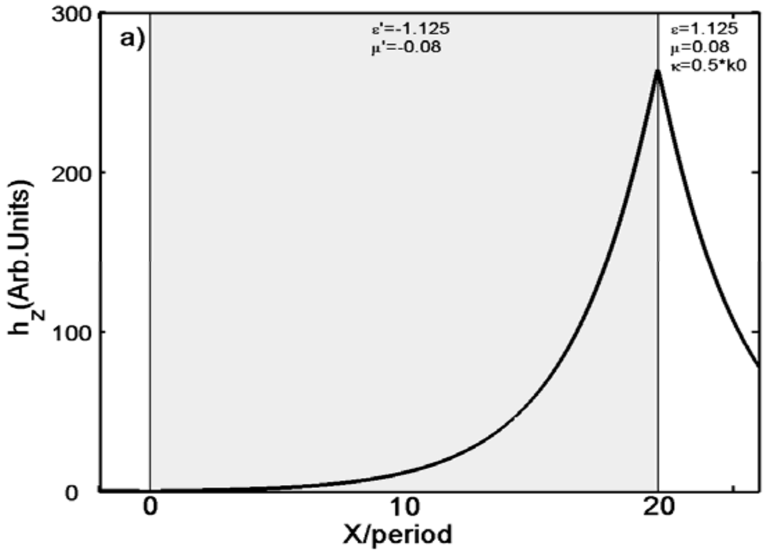
The boundary condition in RH-LH interface is continuity of

$$E_y = (1/\varepsilon)dH_z/dx \rightarrow \sqrt{k^2 - k_0^2}/\varepsilon$$

In the LHM:

$$H_{ev}(k) = \frac{\exp(iky + x\sqrt{k^2 - k_0^2})}{\sqrt{k^2 - k_0^2}}$$

$$\int_{-\infty}^{\infty} |H_{ev}|^2 dk = \infty$$



Photonic crystal as a Left-Handed Material

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Theorem : ***The group velocity in an isotropic medium is positive in the regular medium (RM) and negative in the LHM.***

$$\omega^2 n^2 = c^2 k^2$$

$$\frac{\partial \omega}{\partial \vec{k}} = \frac{2c^2 \vec{k}}{d[\omega^2 n^2]/d\omega}$$

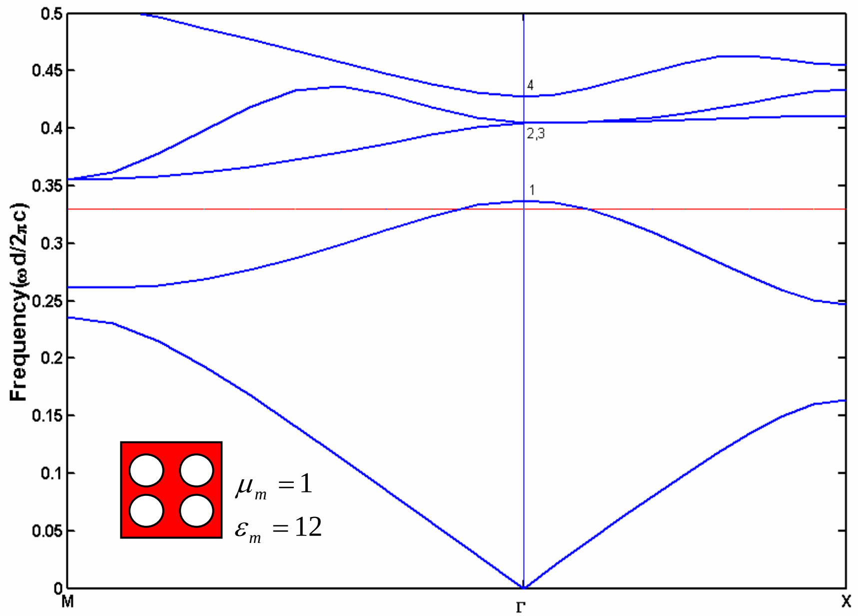
$$\bar{U} = \frac{1}{16\mu\omega}\frac{d[\omega^2 n^2]}{d\omega}|E|^2 > 0$$

$$\frac{d[\omega^2 n^2]}{d\omega} < 0$$ For the LHM

$$\frac{d[\omega^2 n^2]}{d\omega} > 0$$ For the RM

A. L. Pokrovsky, A. L. Efros, Solid St. Com. 124,283, 2002

Dielectric Photonic Crystal is a LHM



P-polarization

$$\bar{U} = \frac{1}{16\mu_{zz}\omega}\frac{d[\omega^2 n^2]}{d\omega}|E|^2 > 0, \qquad n^2 = \mu_{zz}\varepsilon_\perp$$

In what follows: $\mu=\mu_{zz}$, $\varepsilon=\varepsilon_\perp$

$$\vec{v}_g = \frac{2c^2\vec{k}}{d[\omega^2 n^2]/d\omega}$$

$$\bar{U} = \frac{1}{8\mu\omega}\frac{c^2k^2}{\vec{v}_g\cdot\vec{k}}|E|^2 > 0$$

If $v_g \cdot k < 0$, $\mu<0$, $\varepsilon<0$

A. L. Pokrovsky, A. L. Efros, Solid St. Com. 129,643, 2004

Results for ε and μ

numerical

$$\varepsilon = -1.12$$

$$\mu = -0.08$$

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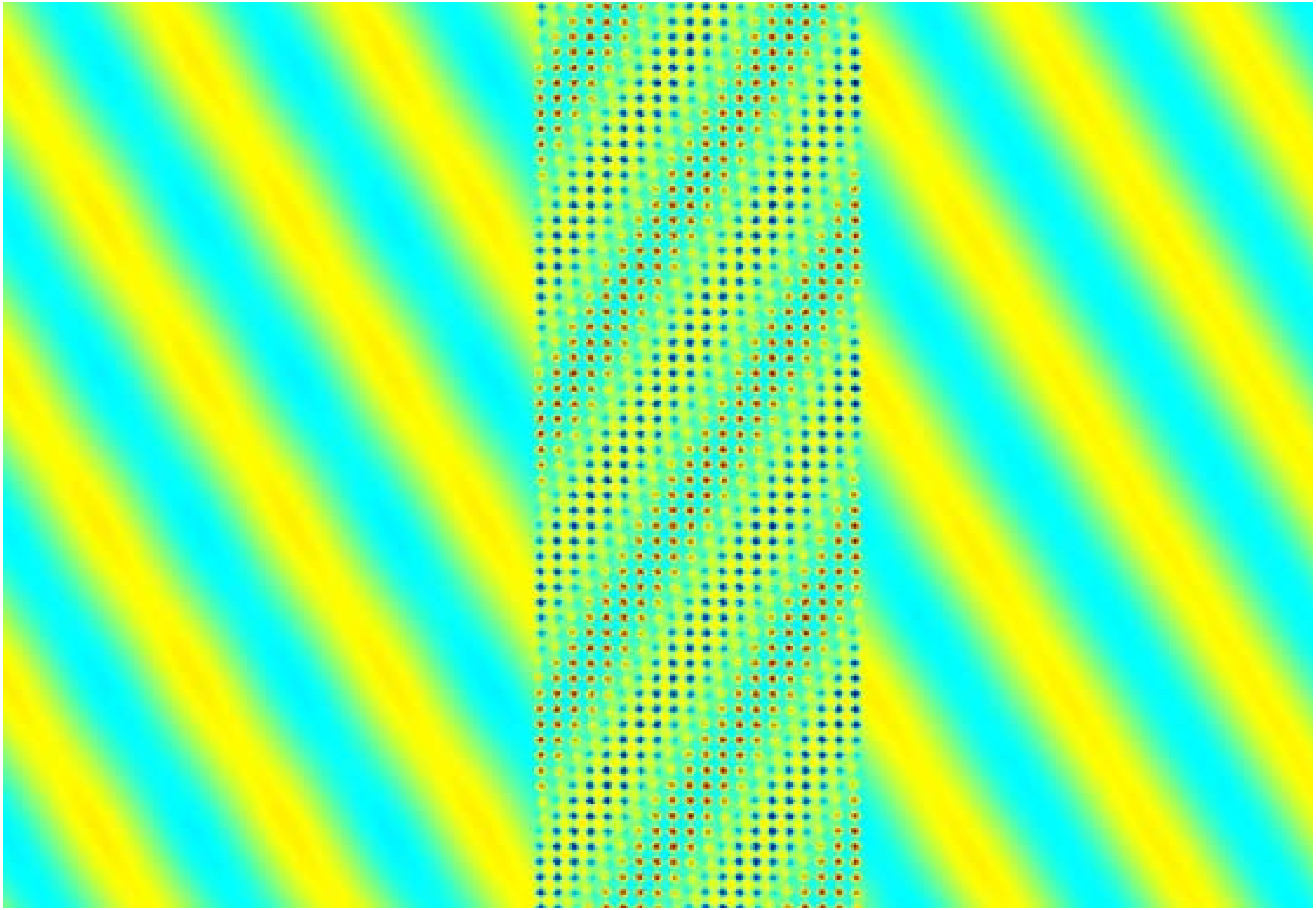
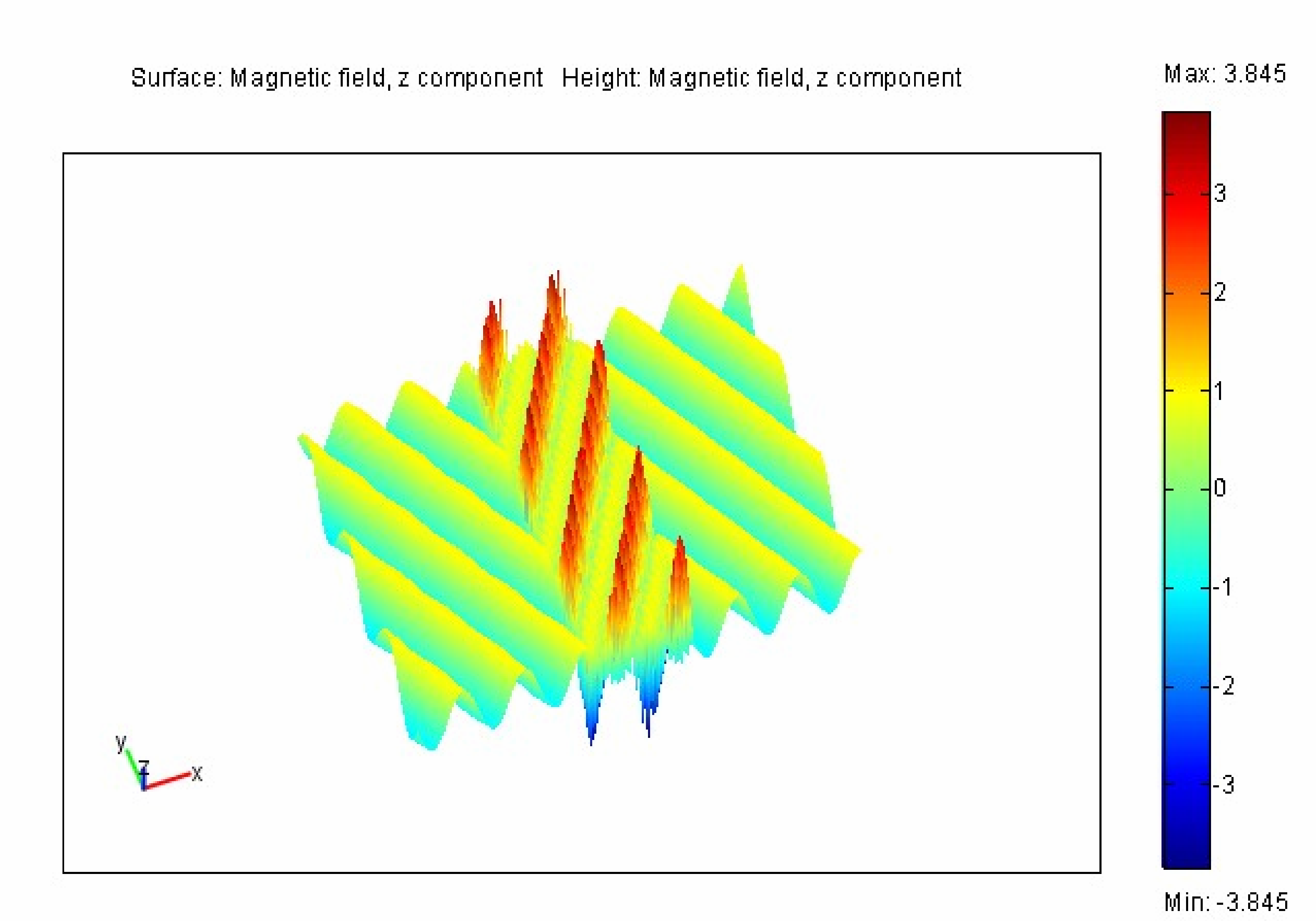


Negative refraction of a beam of light in dielectric photonic crystals



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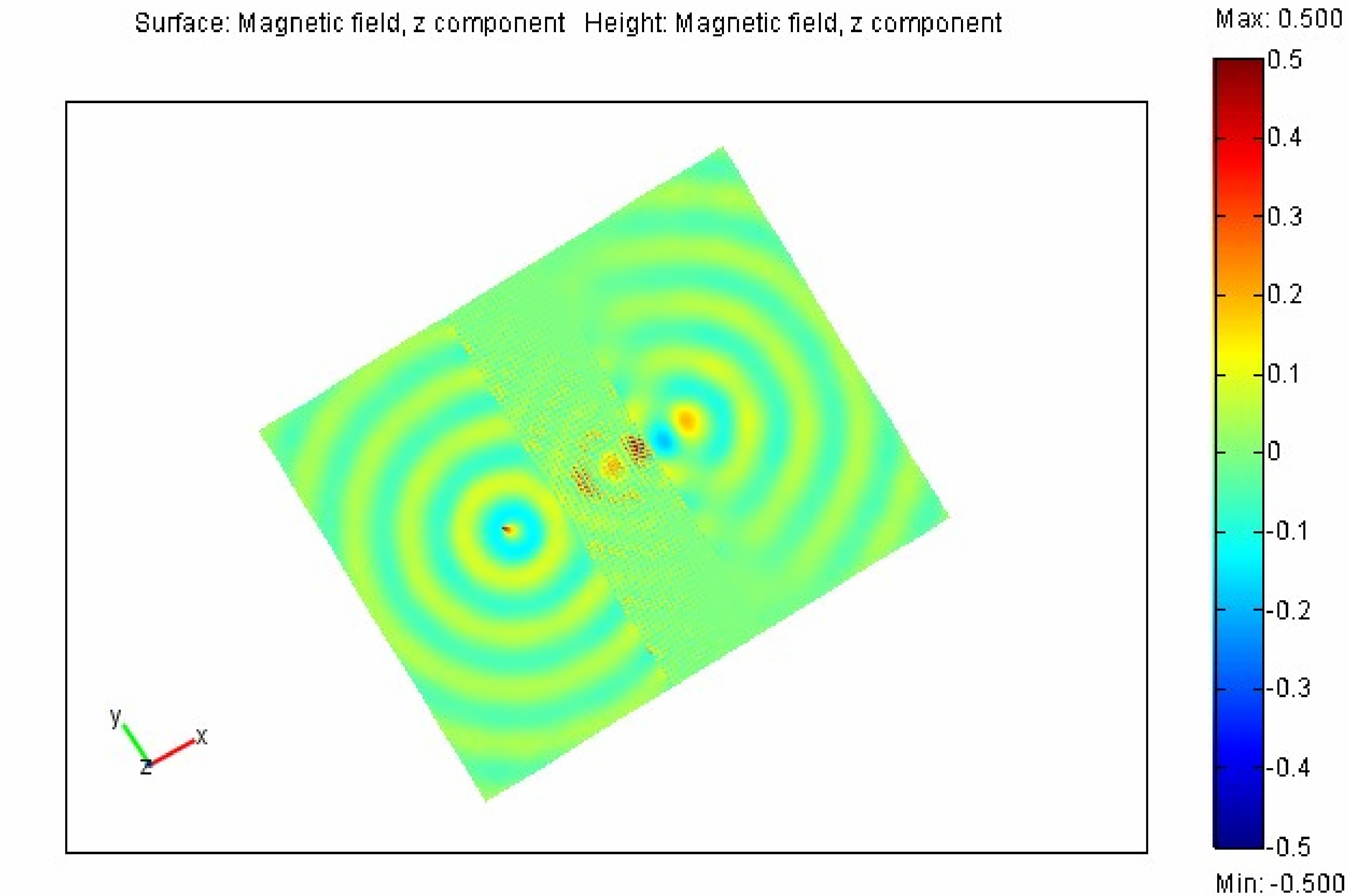
Veselago lens



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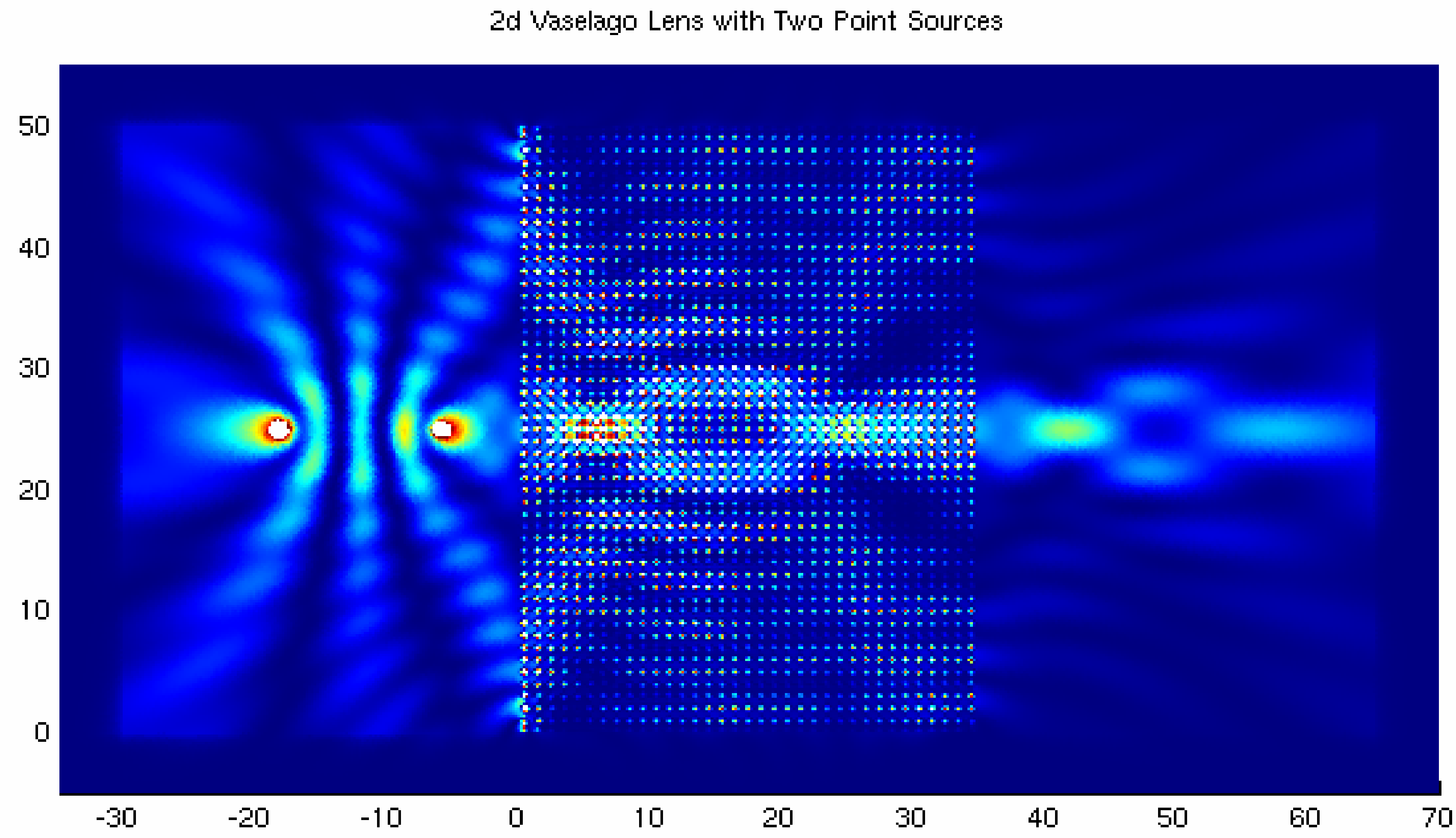
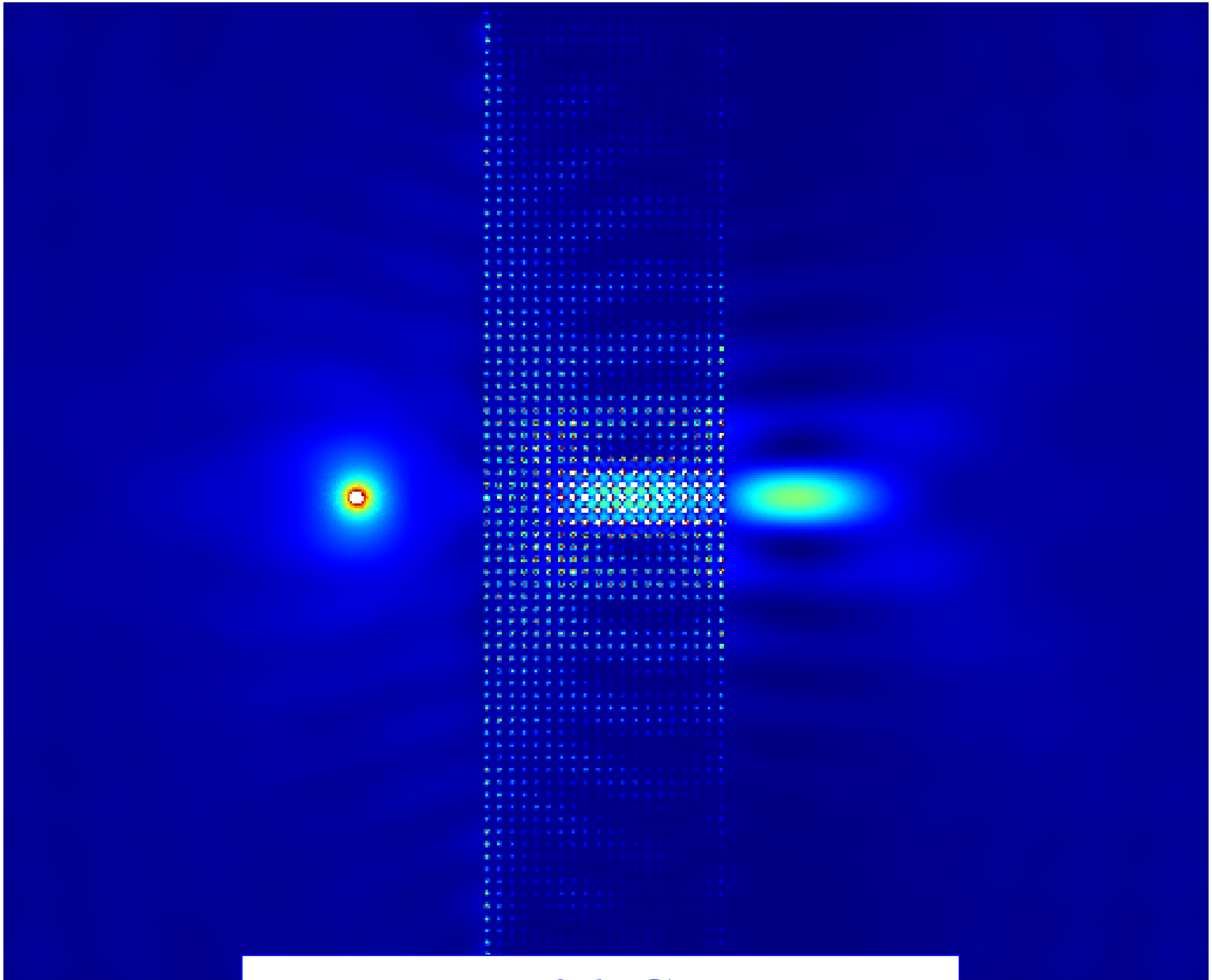
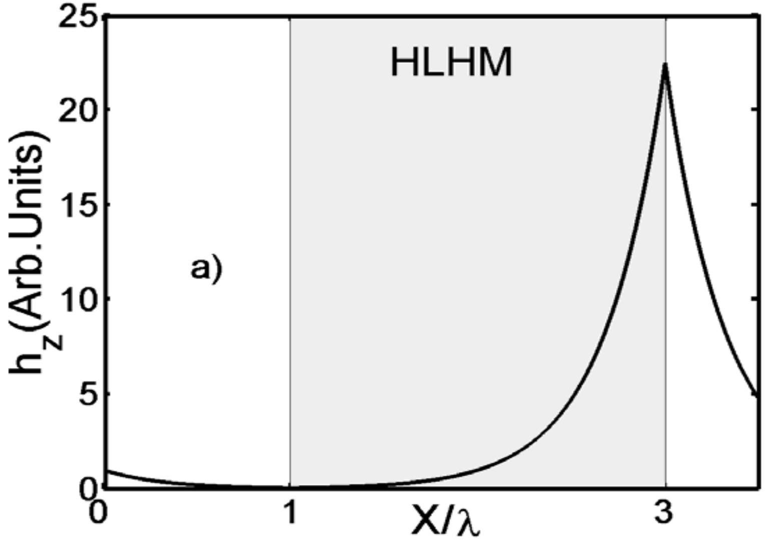

2d Vaselago Lens with Two Point Sources
50
40
30
20
10
0
-30
-20
-10
0
10
20
30
40
50
60
70



FTM-06, Greece

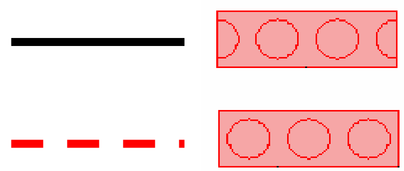


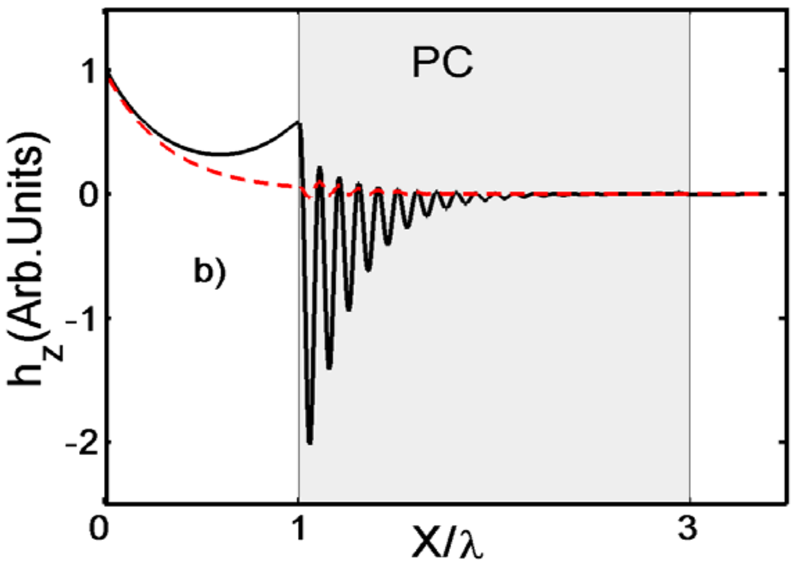
$$k_y = (\sqrt{5}/2)k_0$$

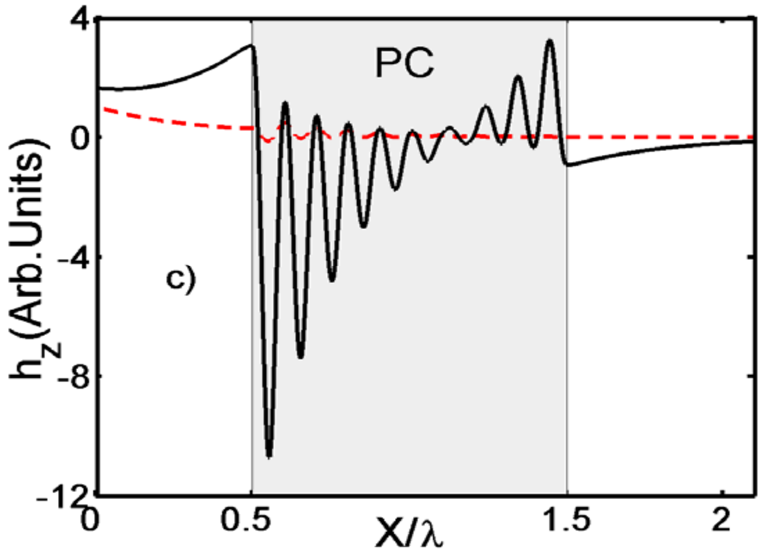
$$\kappa = 0.5k_0$$

$$k_0 = n\omega / c$$

$$h_z = \exp(ik_y y), dh_z / dx = -\kappa h_z$$







Explanation

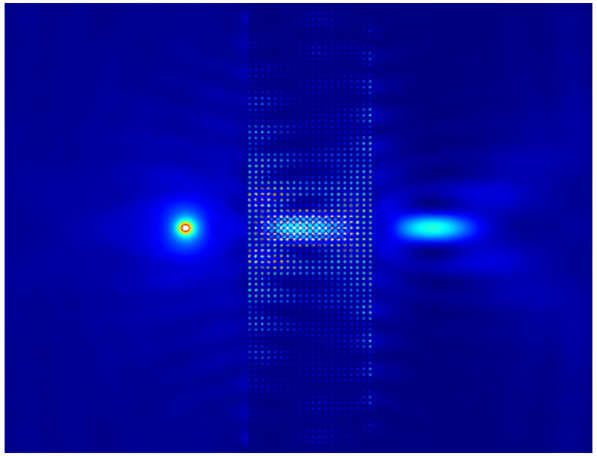
In general PC is described by $\varepsilon(\omega,k)$. Near Γ-point $\varepsilon(\omega,|k|)$ and $\omega=\omega(|k|)$. Thus we can come to $\varepsilon(\omega)$ and $\mu(\omega)$. However this $\varepsilon(\omega)$ and $\mu(\omega)$ are rather the property of the mode than the property of the material.

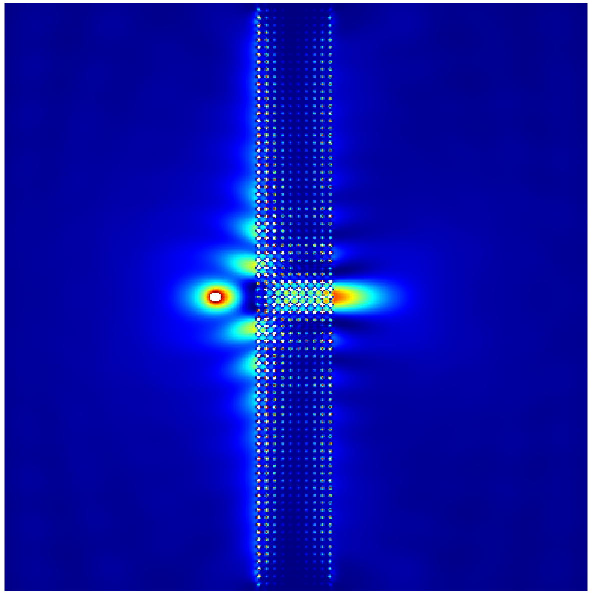
Important question is now whether or not the evanescent modes have the same $\varepsilon(\omega)$ and $\mu(\omega)$ at a given frequency as the propagating ones and whether they can be described by any k-independent parameters. The general answer to these questions should be negative, because the fields of EW has a form

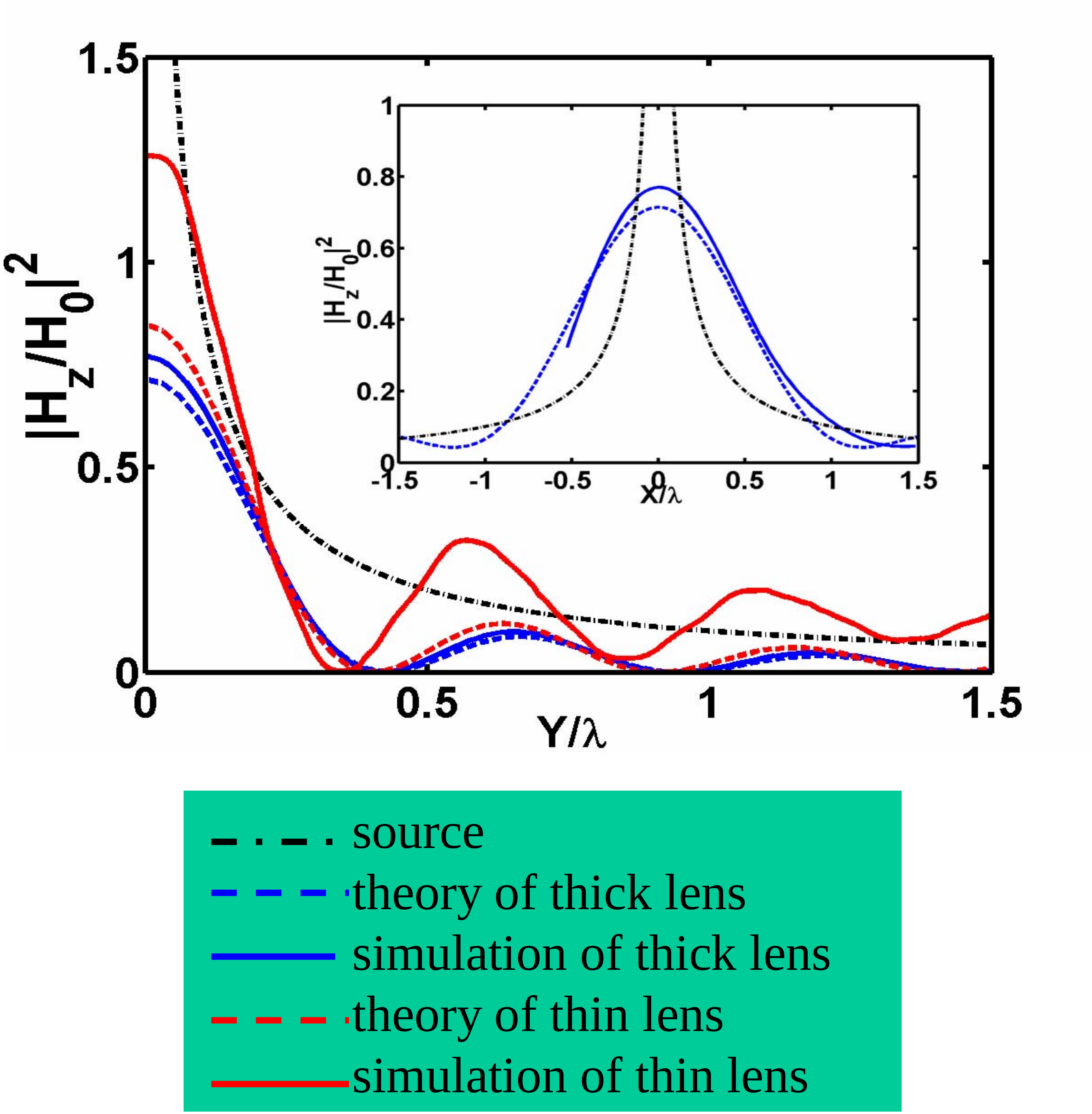
$$H_z = \exp(ik_y y - \kappa x)$$

and their dispersion law is anisotropic even in a vacuum:

$$\omega^2 = c^2 (k_y^2 - \kappa^2)$$



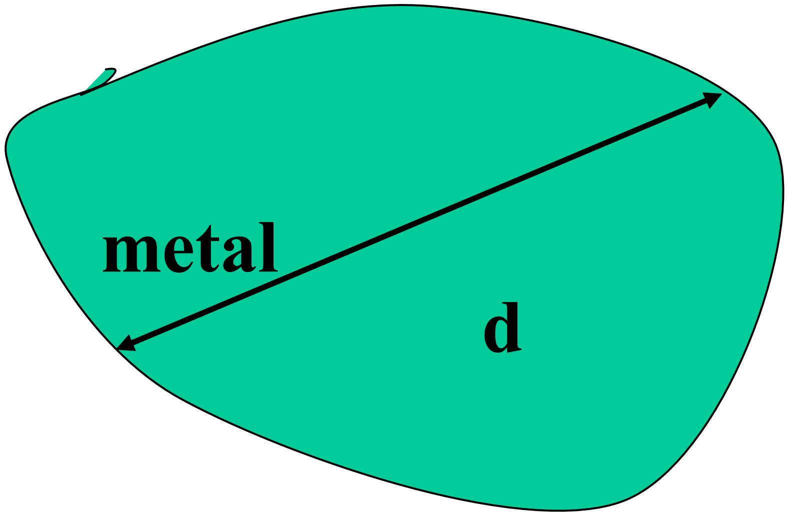




$|H_z/H_0|^2$
Y/λ
0
0.5
1
1.5
$|H_z/H_0|^2$
X/λ
-1.5
-1
-0.5
0
0.5
1
1.5
0.2
0.4
0.6
0.8
source
theory of thick lens
simulation of thick lens
theory of thin lens
simulation of thin lens

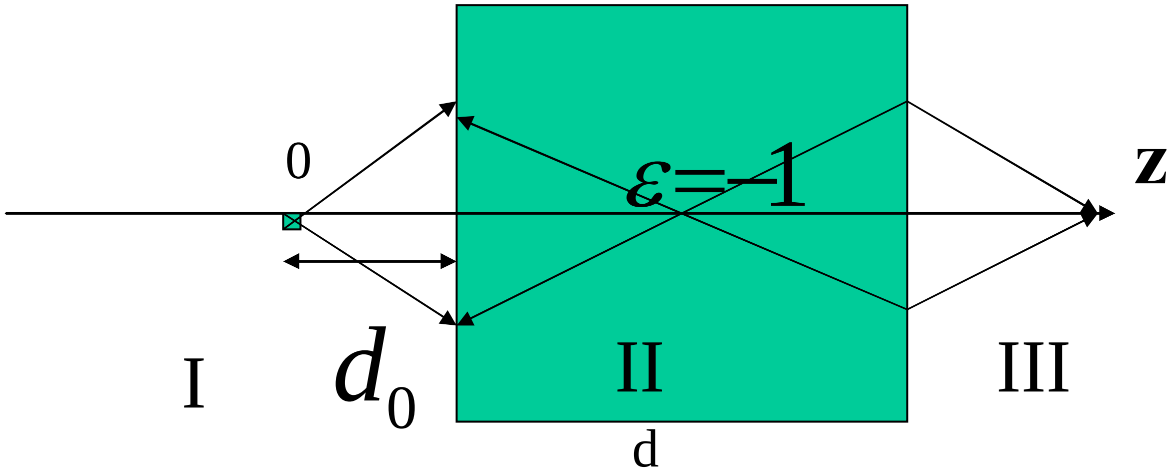
Quasistatic lens

$$\lambda >> d.$$



$$\nabla \varepsilon(\vec{r}) \nabla V(\vec{r}) = 0$$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$



$$\frac{\partial V_I}{\partial z} = -\frac{\partial V_{II}}{\partial z}$$

$$\frac{\partial V_{II}}{\partial z} = -\frac{\partial V_{III}}{\partial z}$$

$$V(x,y,0) = \sum_{k_x,k_y} C(k_x,k_y)\exp(ik_x x + ik_y y)$$

$$V(x,y,z) = V(x,y,0)\,f(z)$$

$$f_I(z) = \exp(-kz);\; f_{II}(z) = \exp[k(z-2d_0)];$$

$$f_{III} = \exp[-k(z-2d_0)] \qquad k = \sqrt{k_x^2 + k_y^2}$$

The fields in the region II lose square integrability so this is not a solution.

Note, that harmonic function in free space does not have not only a singularity but even a smeared maximum. Thus the chances of the “perfect lens” in the

Quasistatic case are much smaller that in Veselago’s case.

Many people consider quasistatic lensing assuming that

$\varepsilon = -1 + i\delta,$ where ≏<<1. Then one can speak about “focus”, which is actually a saddle point with a size of the order of $2\pi d / |\ln \delta|$

See Podolsky et al Appl. Phys. Lett 87,231113,2005, Milton et al Proc.R. Soc. A 461, 3999,2005 and many others.

There are experiments using mostly Ag films that demonstrate image of the order of the film thickness (about 40 nm)

(See Fang et al Science, 308, 534,2005 and some others)

Meanwhile it has been shown (Efros, Milton)

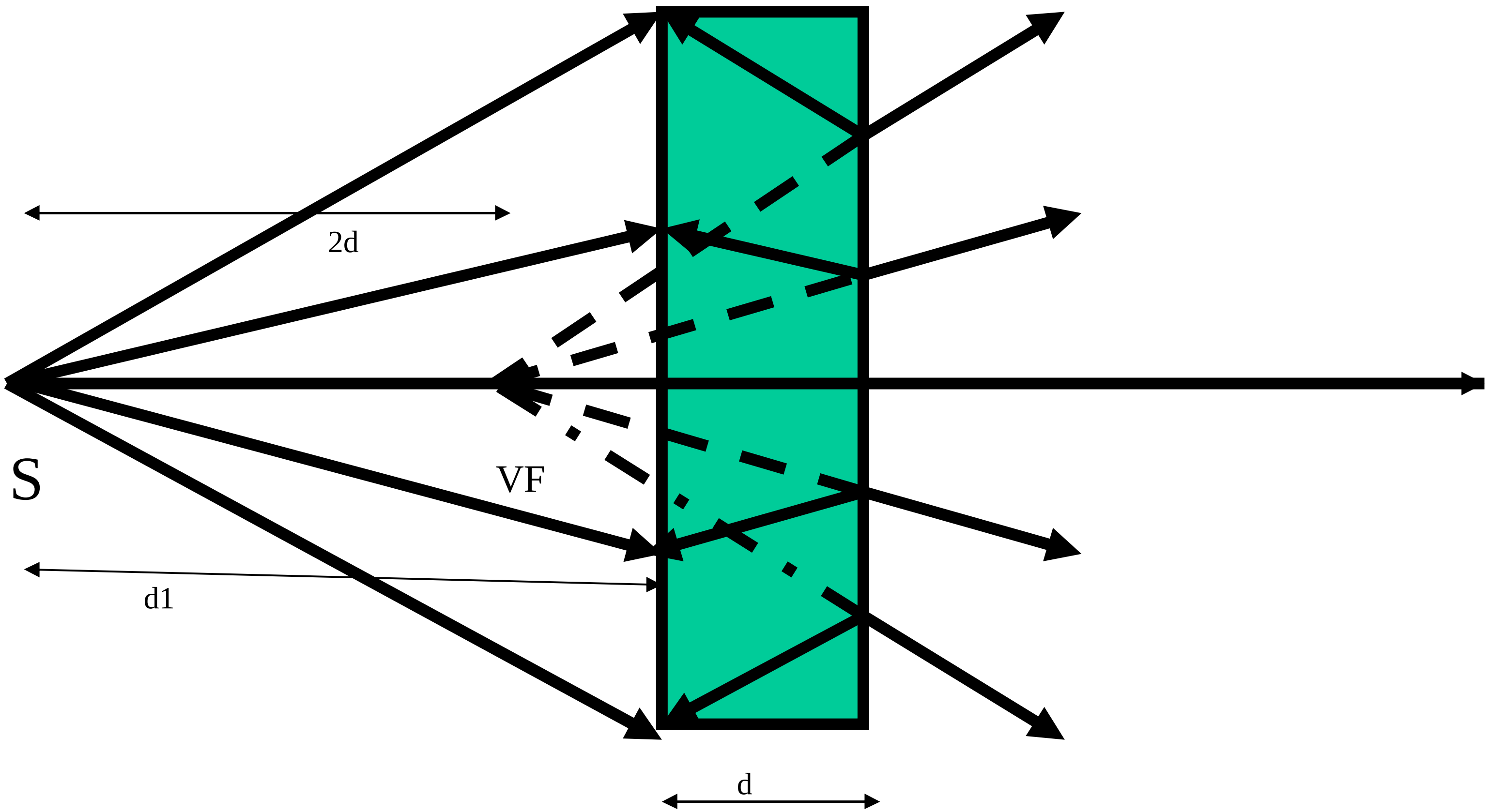
That if the source is closer to the lens than d/2,

Total absorption

δE^2 **Tends to infinity as**

$$\delta = \operatorname{Im} \varepsilon \to 0$$

Example of “perfect lens” with virtual focus:



S is the source at a distance d1from a lens, d1>d. Observer at any point to the right of the lens sees object to be of the same size and shape as it is in the source. The amplitude of the field decreases with the distance from the VF. That is true for quasistatics and wave regime, for 2 and 3 dimensions. Efros, unpublished.

Finally there is a universal limit for any lens based upon metal:

$$k < \omega / v_f$$

where v_f **is Fermi velocity.**

For Ag it gives maximum resolution about 10nm (See Larkin and Stokman,

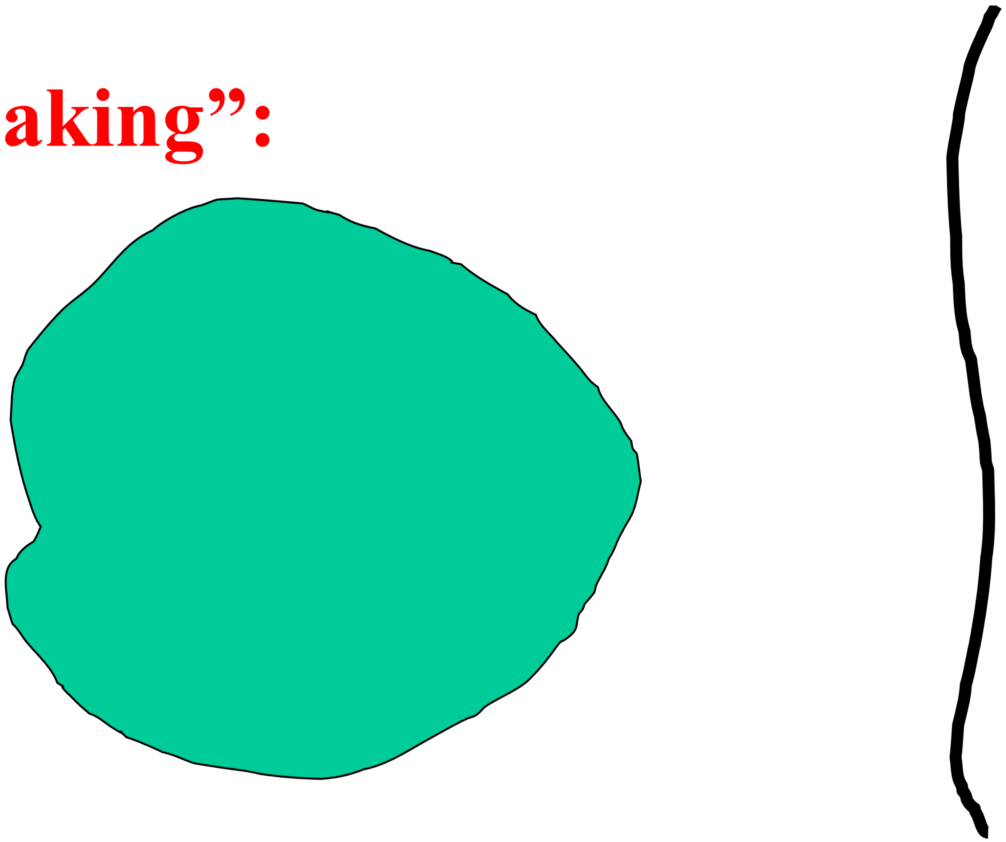
Nano Letters, 5, 339, 2005).

CLOAKING

**"Useful things…
your father used it
mainly for sneaking
of to the kitchen to
steal food."**

Rowling, Harry Potter, book 1

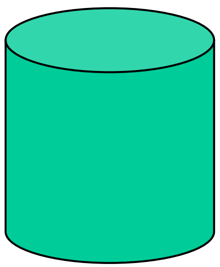
What is "cloaking":

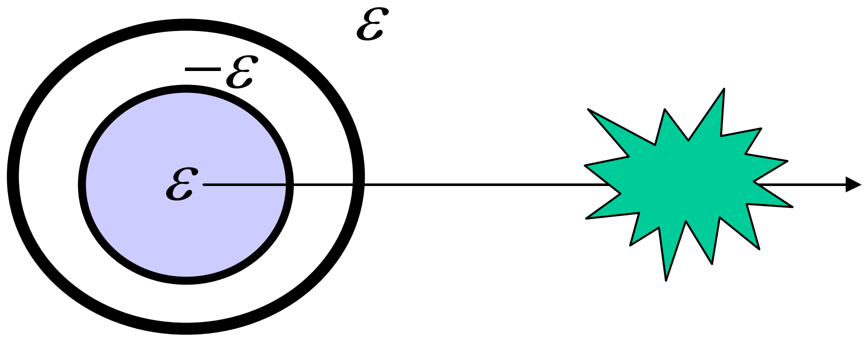


Body is cloaked if an arbitrary external electromagnetic field outside this body is the same as it is without the body.

Cloaking is obviously impossible if the body absorbs the energy

Nicorovici, McPhedran, Milton 1994





Two dimensional problem : Cylinder is invisible in for arbitrary sources.

Milton, Nicorovici (2006): Generalization for a Veselago lens. Cloaked is also an arbitrary collection of polarizable dipole near the cylinder.

J.B. Pendry, scienceexpress 25 May 2006

A new method is proposed how to cloak any object doing corresponding coordinate transformation and choosing proper

$\varepsilon(r)$ $\mu(r)$

The method is based upon three dimensional mapping of a proper region.

BBC NEWS May 25, 2006.

“Plan for cloaking device unveiled   Cloaking devices are a staple of science fiction stories. Researchers in the US and Britain have unveiled their blueprints for building a cloaking device.

So far, cloaking has been confined to science fiction; in Star Trek it is used to render spacecraft invisible.

Professor Sir John Pendry says a simple demonstration model that could work for radar might be possible within 18 months' time. “

Conclusion: What is next?

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$B = \mu H \qquad D = \varepsilon E$$

$\varepsilon<0, \mu<0$

$$S = E \times H$$

$$k \times E = -\omega |\mu| H$$

$$k \times H = \omega |\varepsilon| E$$

$$c^2 = \frac{1}{\varepsilon\mu} > 0$$

FTM-06, Greece

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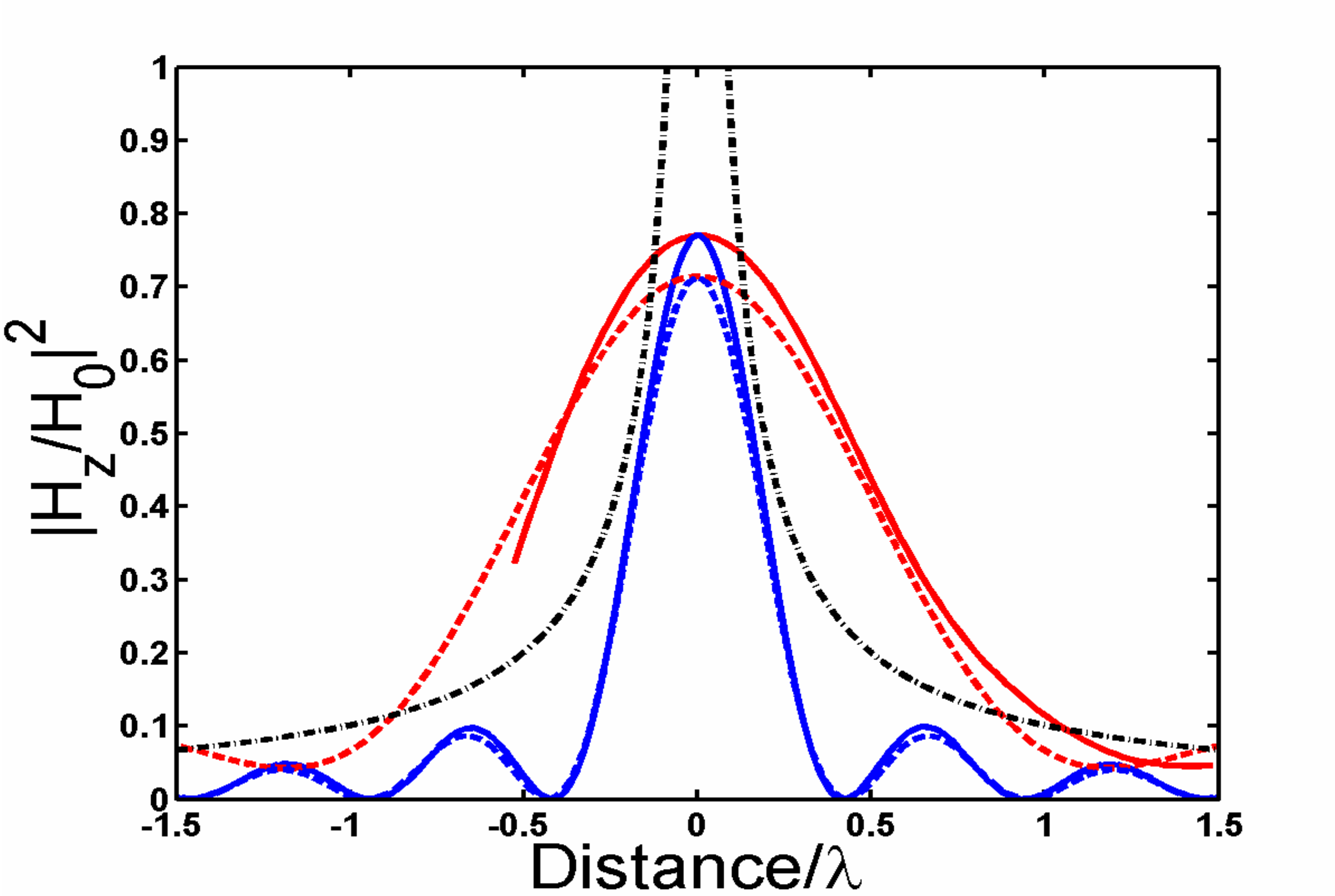
Intensity distribution near the focus for thick lens

(20 periods)



Calculation of μ and ε

$$B_z = \iint h_z(\boldsymbol{r})\,dxdy$$

$$(\boldsymbol{k} \times \boldsymbol{E})_z = \frac{\omega}{c} B_z$$

$$\frac{1}{\mu} = 1 - \frac{4\pi M_z}{B_z}$$

$$\left\langle \frac{\partial \boldsymbol{p}}{\partial t} \right\rangle = \frac{\partial \boldsymbol{P}}{\partial t} + c\nabla \times \boldsymbol{M}$$

$$\varepsilon_{xx} = 1 + 4\pi \frac{P_x}{E_x}$$

Calculation of magnetization

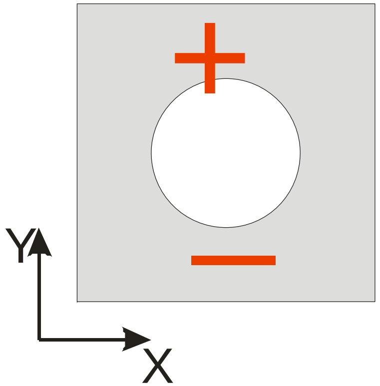
$$\omega^2 = \omega_1^2 - \frac{\kappa_0^2}{\omega_2^2 - \omega_1^2} c^2 k^2$$

$$h_z(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left(u_1 - \frac{i\kappa_0 k_x c^2}{\omega_2^2 - \omega_1^2} u_2 - \frac{i\kappa_0 k_y c^2}{\omega_2^2 - \omega_1^2} u_3 - \frac{\kappa_0 \kappa_1 k^2 c^4}{(\omega_2^2 - \omega_1^2)(\omega_4^2 - \omega_1^2)} u_4 \right)$$

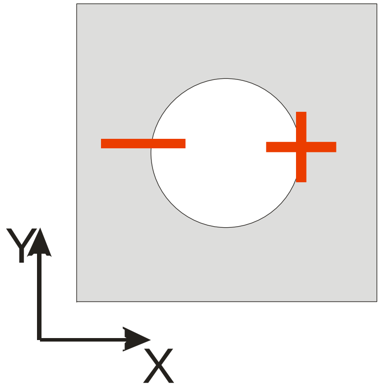
$$\boldsymbol{M} = \int \boldsymbol{r} \times \frac{\partial \boldsymbol{p}}{\partial t} ds$$

$$\boldsymbol{p} = \boldsymbol{e}(\boldsymbol{r}) \frac{\varepsilon(\boldsymbol{r}) - 1}{4\pi}$$

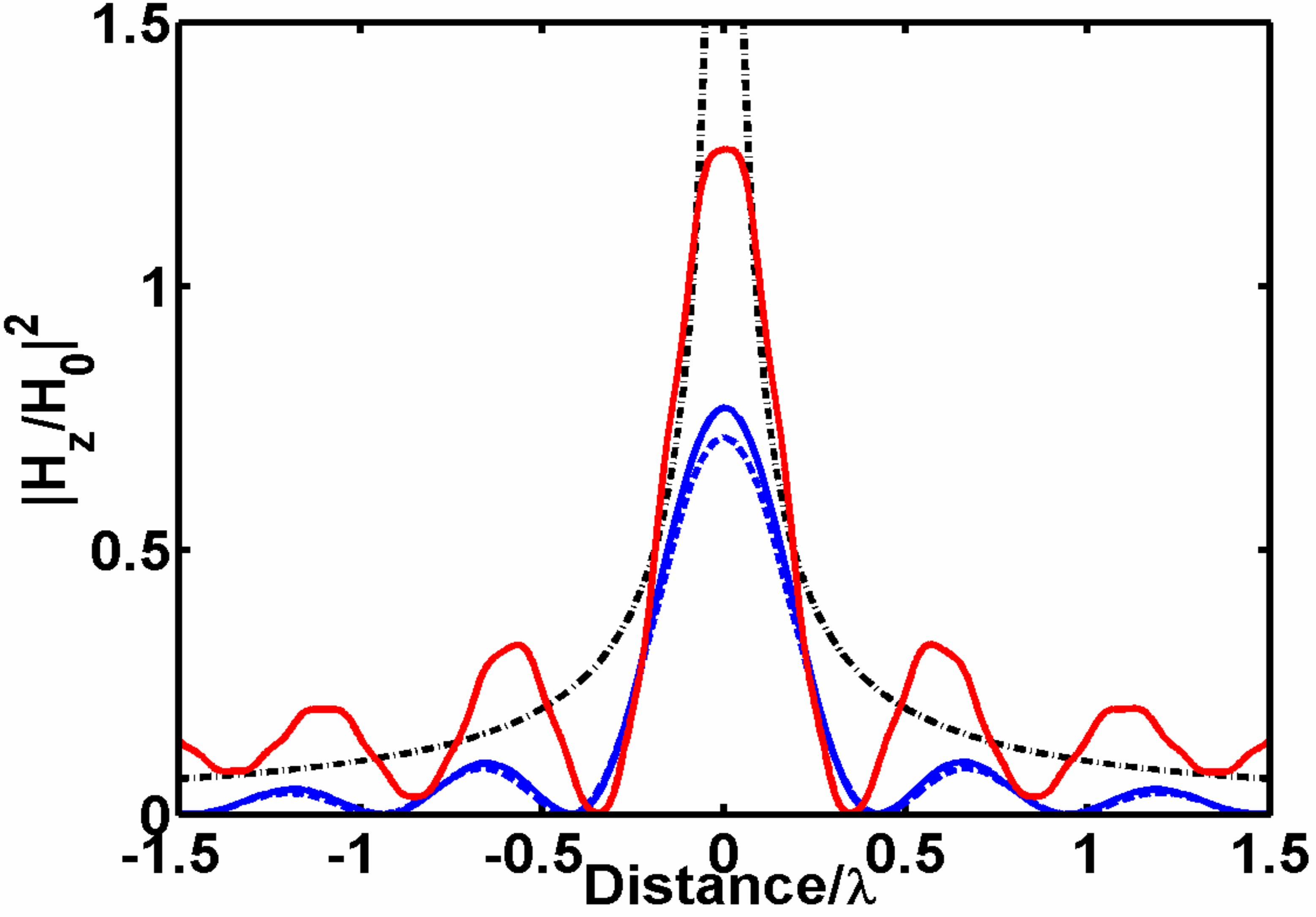
e_x



e_y



Intensity distribution for thin lens (10periods))



Calculation of magnetization

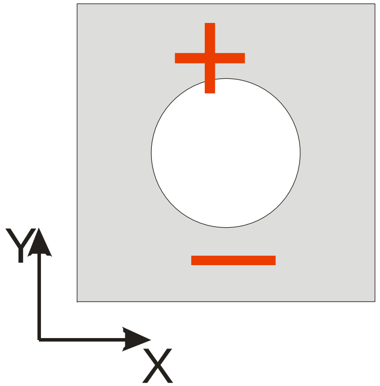
$$\omega^2 = \omega_1^2 - \frac{\kappa_0^2}{\omega_2^2 - \omega_1^2} c^2 k^2$$

$$h_z(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left(u_1 - \frac{i\kappa_0 k_x c^2}{\omega_2^2 - \omega_1^2} u_2 - \frac{i\kappa_0 k_y c^2}{\omega_2^2 - \omega_1^2} u_3 - \frac{\kappa_0 \kappa_1 k^2 c^4}{(\omega_2^2 - \omega_1^2)(\omega_4^2 - \omega_1^2)} u_4 \right)$$

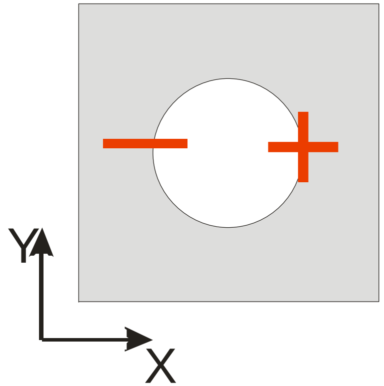
$$\boldsymbol{M} = \int \boldsymbol{r} \times \frac{\partial \boldsymbol{p}}{\partial t} ds$$

$$\boldsymbol{p} = \boldsymbol{e}(\boldsymbol{r}) \frac{\varepsilon(\boldsymbol{r}) - 1}{4\pi}$$

e_x



e_y



Intensity distribution near the focus of a thick lens

$$H_p = i\frac{H_0}{\pi}\int_{-k_0}^{k_0} \frac{\exp i(ky' + x'\sqrt{k_0^2 - k^2})}{\sqrt{k_0^2 - k^2}} dk$$

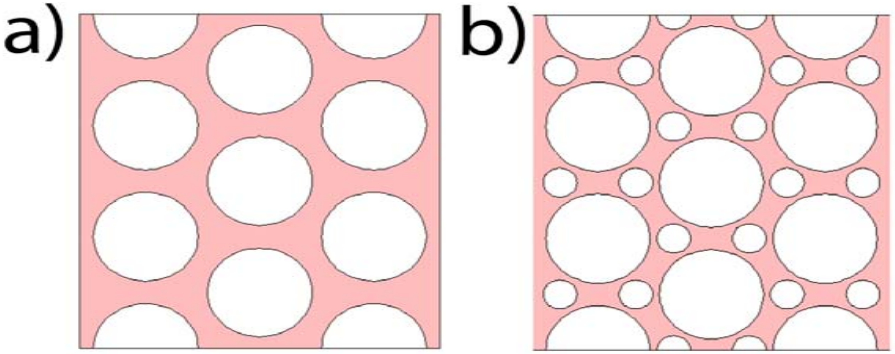
$$H_p(x' = 0) = H_0 J_0(y' k_0)$$

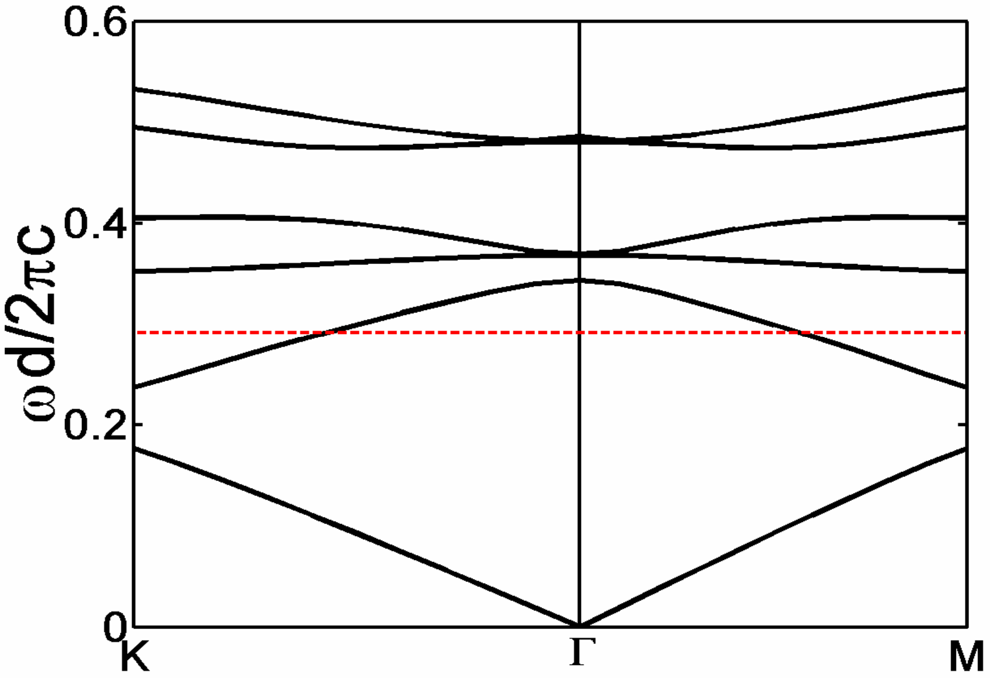
To take into account finite length h in y-direction

$$k_0 \to k_0 (h/2a)\sqrt{1 + (h/2a)^2}$$

$$h/2a = 4$$

X. Wang et al. Opt. Express **12**, 2919 (2004)





$$\omega^2 = \omega^2_1 - \alpha c^2 k^2 + \beta k^4 + O(k_x^3 k_y^3)$$

$$\omega^2 = \frac{c^2 k^2}{n^2}$$

$$n^2 = \frac{1}{\alpha}\left(-1 + \frac{\omega^2_1}{\omega^2}\right)$$

$$\mathrm{r} = 0.74;\ \varepsilon' = -5.67;\ \mu' = -0.18;\ n^2 = \varepsilon'\mu' = 1$$

$$\vec{\mathcal{M}} = \frac{1}{2c}\int \vec{r} \times \frac{\partial \vec{p}}{\partial t} ds$$

$$\vec{p} = \frac{\varepsilon_m(\vec{r}) - 1}{4\pi}\vec{e}(\vec{r})$$

$$\vec{E} = <\vec{e}(\vec{r})>$$

$$\vec{B} = c\vec{k} \times \vec{E} / \omega$$

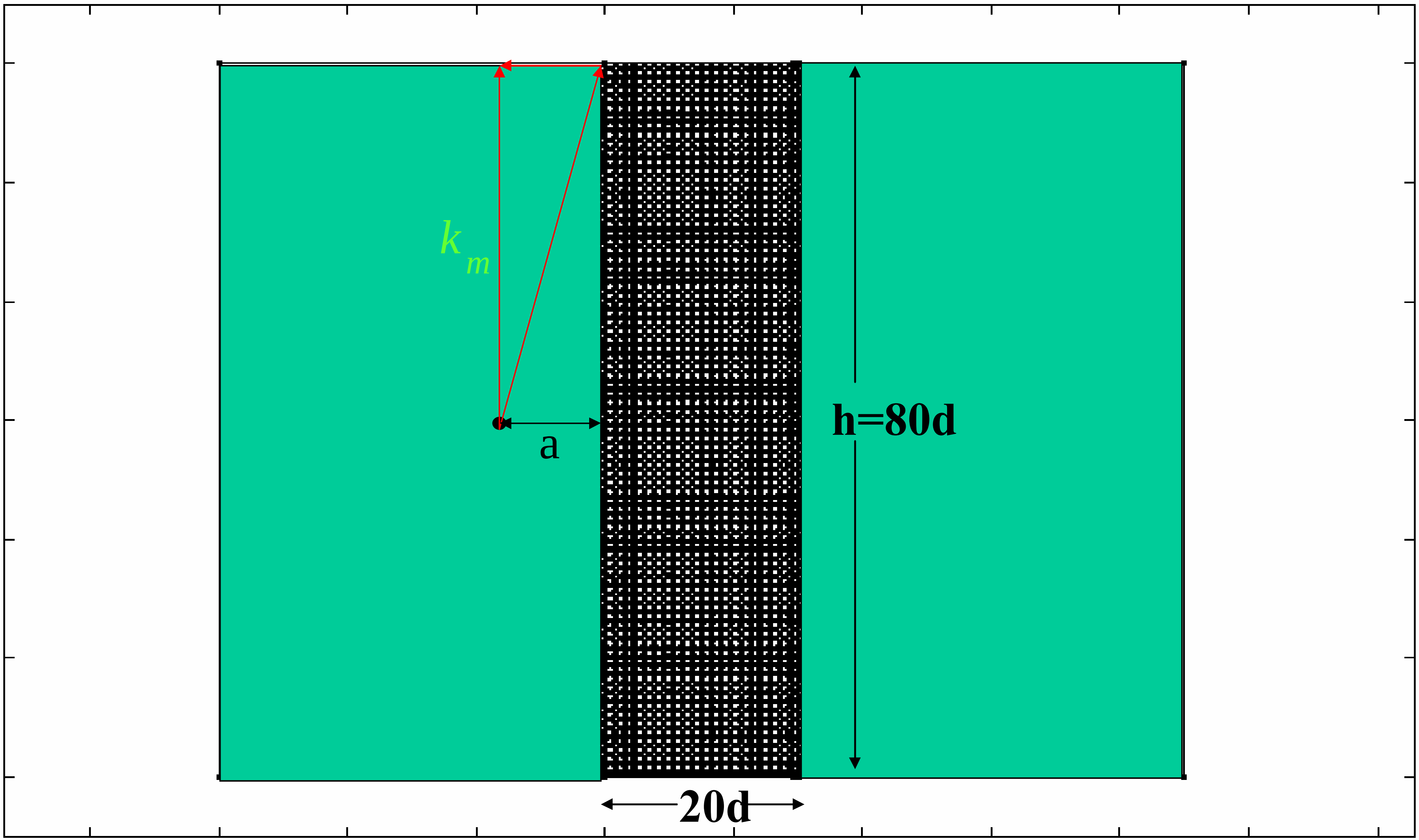
$$\vec{M} = \frac{\vec{\mathcal{M}}}{S_{cell}}$$

$$\vec{B} = \vec{H} + 4\pi\vec{M}$$

$$\vec{B} = \mu' \vec{H}$$

$$\mu' = 1 - \frac{4\pi M}{B}$$

$$\mu' = -0.2$$



k_m
a
h=80d
20d

$$\frac{\omega d}{2\pi c} = 0.3$$

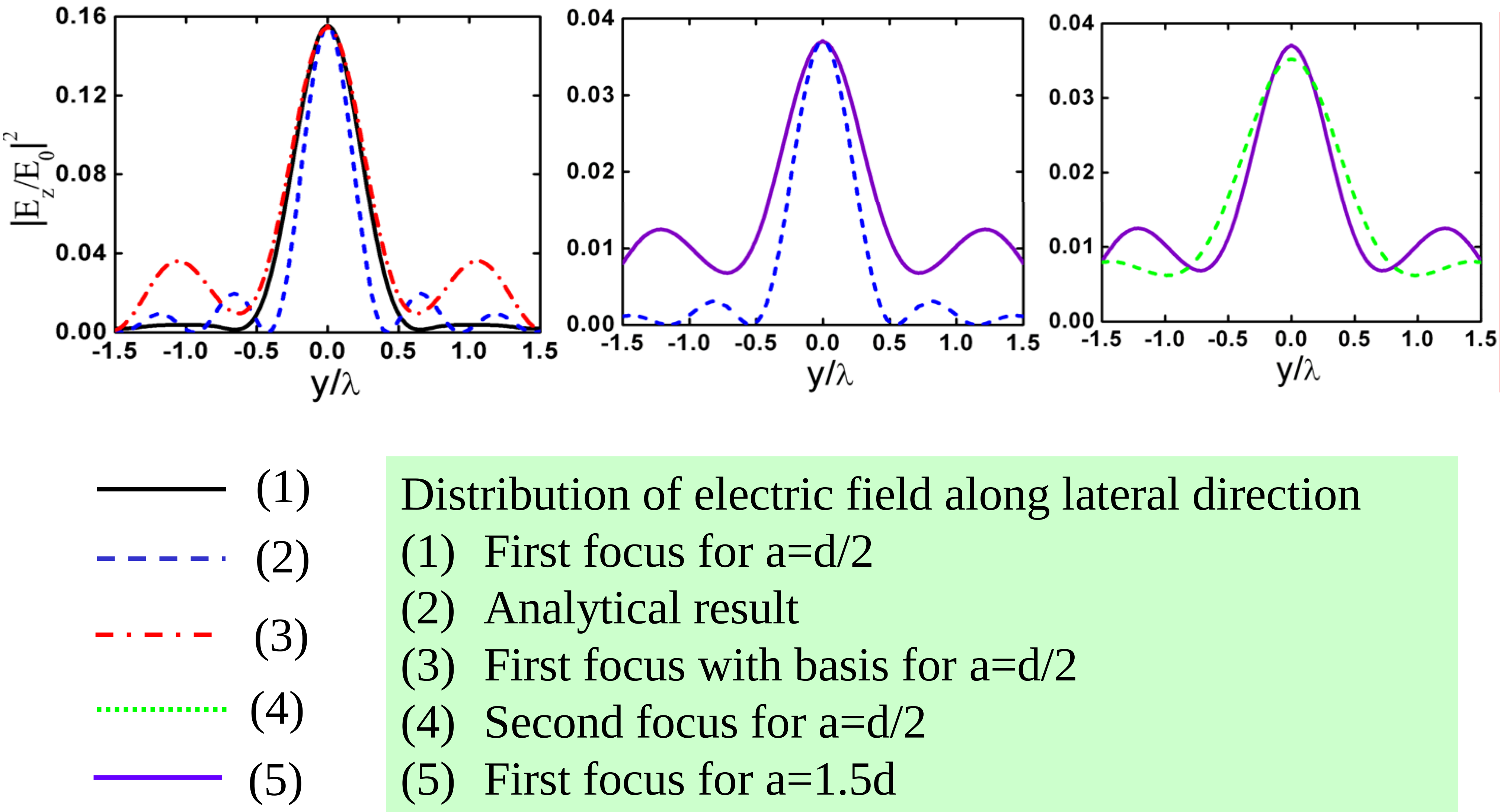
$$\frac{d}{\lambda} = 0.3$$

$$\lambda = 1.5\mu m$$

$$d = 0.45\mu m$$

CONCLUSION

1. **We proposed to make the LHM from the dielectric photonic crystals.**
2. **We consider important fundamental questions connected with propagation of both propagating and evanescent waves in the PC's. We have found that due to special dispersion EW's are not described by q-independent dielectric constant. As a result, there is no amplification of EW's due to polaritons.**
3. **We have found an analytical expression for the image provided by the Veselago lens in the far field regime.**



Distribution of electric field along lateral direction

(1) First focus for a=d/2

(2) Analytical result

(3) First focus with basis for a=d/2

(4) Second focus for a=d/2

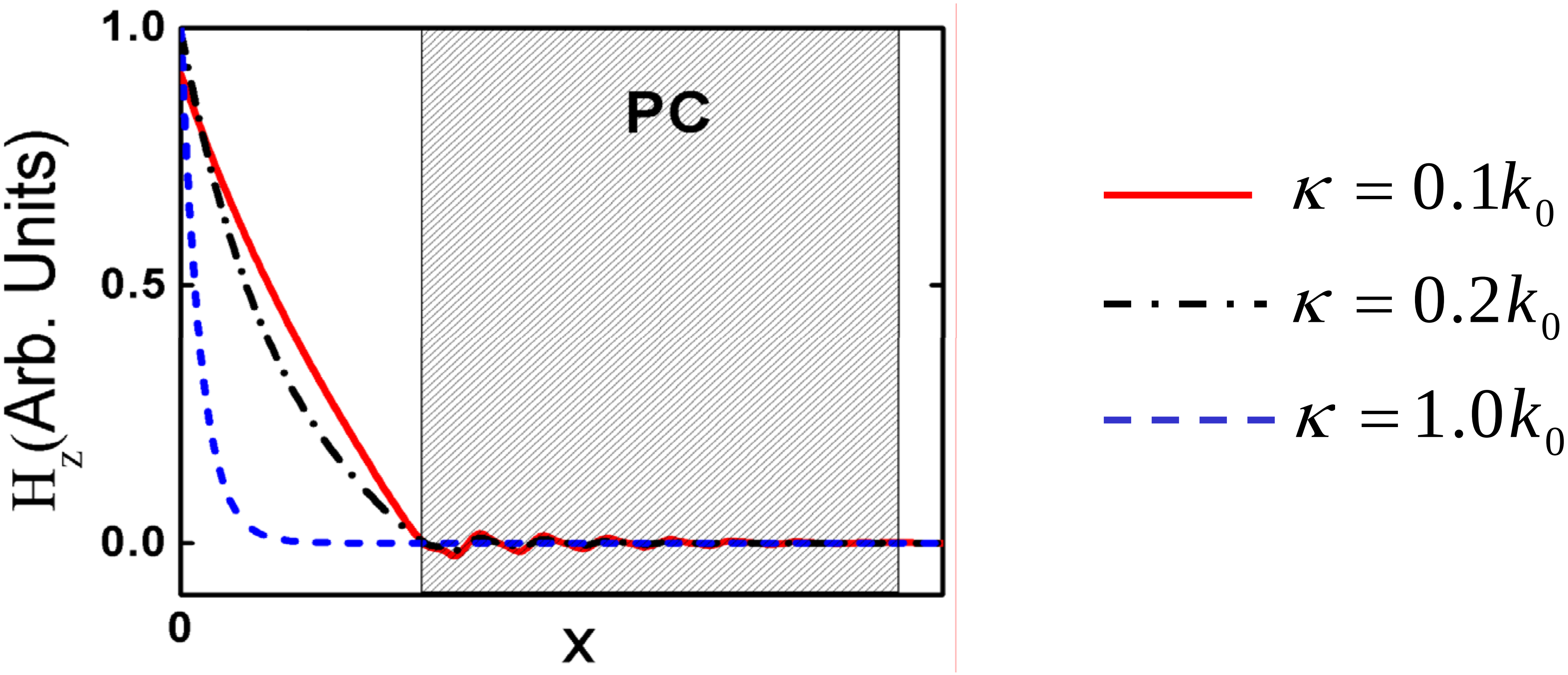
(5) First focus for a=1.5d

Conclusion

1. **We have shown that in some frequency range 2D-photonic crystal with a triangular lattice is a left handed material.**
2. **At the frequency, where refractive index n=1, we found**

$$\varepsilon = -5.67, \mu = -0.18$$

3. **We consider the image of the lens that is a slab of the PC in the air. Because of a mismatch of the impedances there is a strong reflection at the interface so that the lens has multiple foci.**
4. **The evanescent waves inside the PC rather decay than increase and any superlensing is completely absent though the foci are sharp.**
5. **Our analytical result for intensity distribution near the foci is in a good agreement with the computational result.**
6. **The lens can be constructed in the infrared range using the silicon technology.**

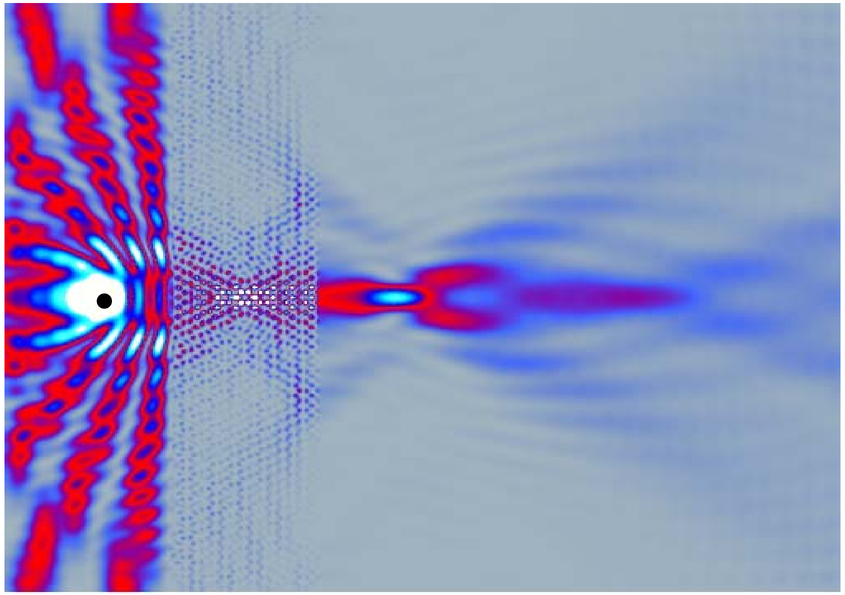


$$k_y^2 - \kappa^2 = k_0^2$$

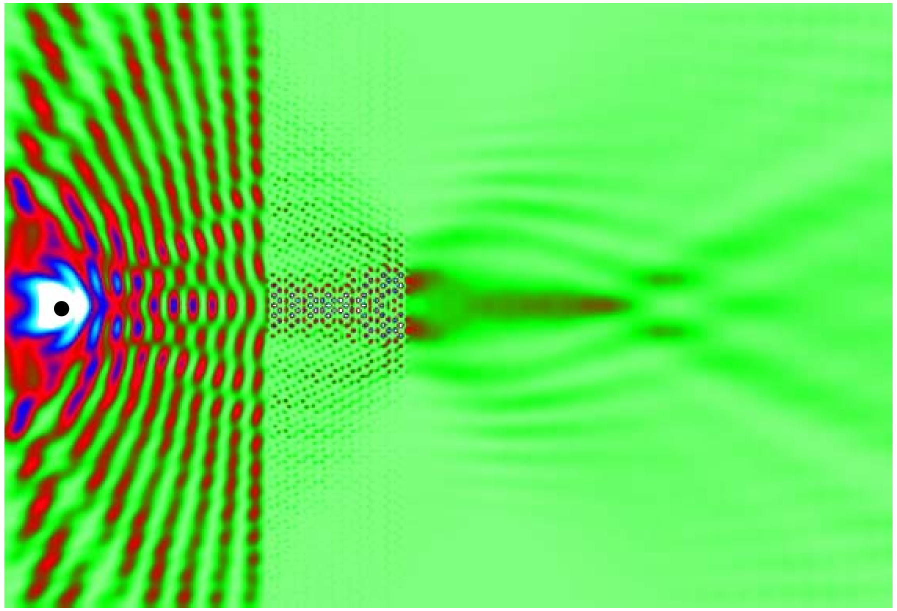
$$k_0 = \omega / c$$

$$\frac{\omega d}{2\pi c} = 0.3$$

a<d **d<a<2d**



Two foci



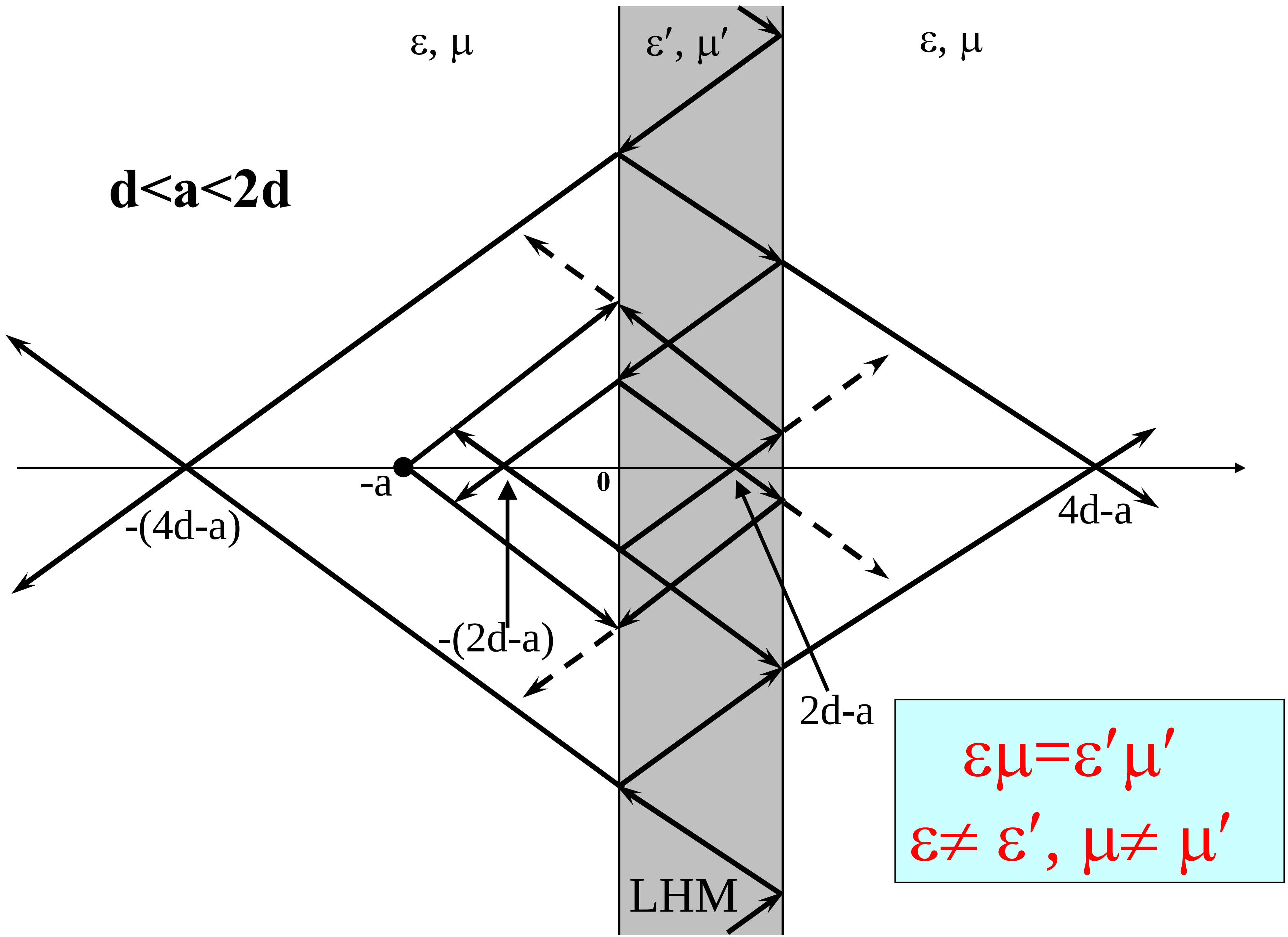
One focus

$$I_1 = (1 - r^2)^2 I_0$$

$$I_2 = (1 - r^2)^2 r^4 I_0$$

$$I_2 = (1 - r^2)^2 r^4 I_0$$

$$r = \frac{|\varepsilon'| - \varepsilon}{|\varepsilon'| + \varepsilon}$$



ε, μ
ε′, μ′
ε, μ
d<a<2d
-a
0
-(4d-a)
4d-a
-(2d-a)
2d-a
LHM
εμ=ε′μ′
ε≠ ε′, μ≠ μ′

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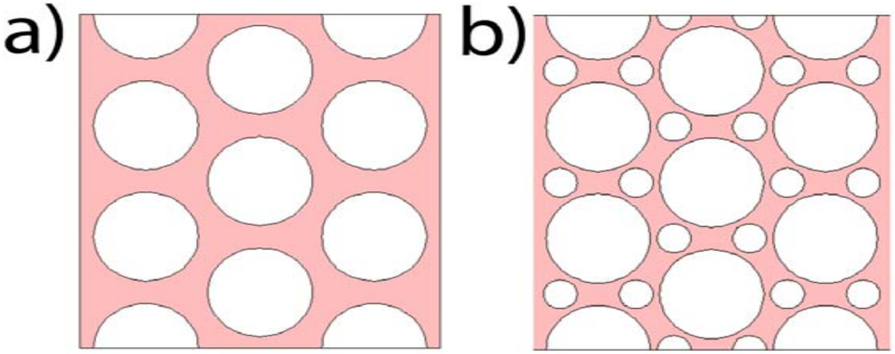

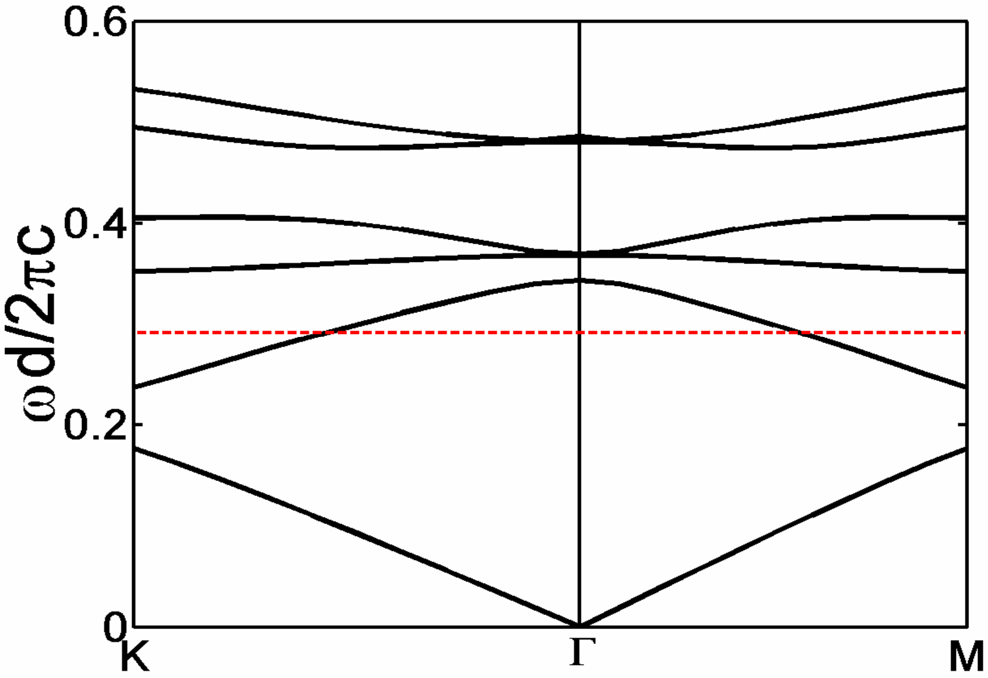

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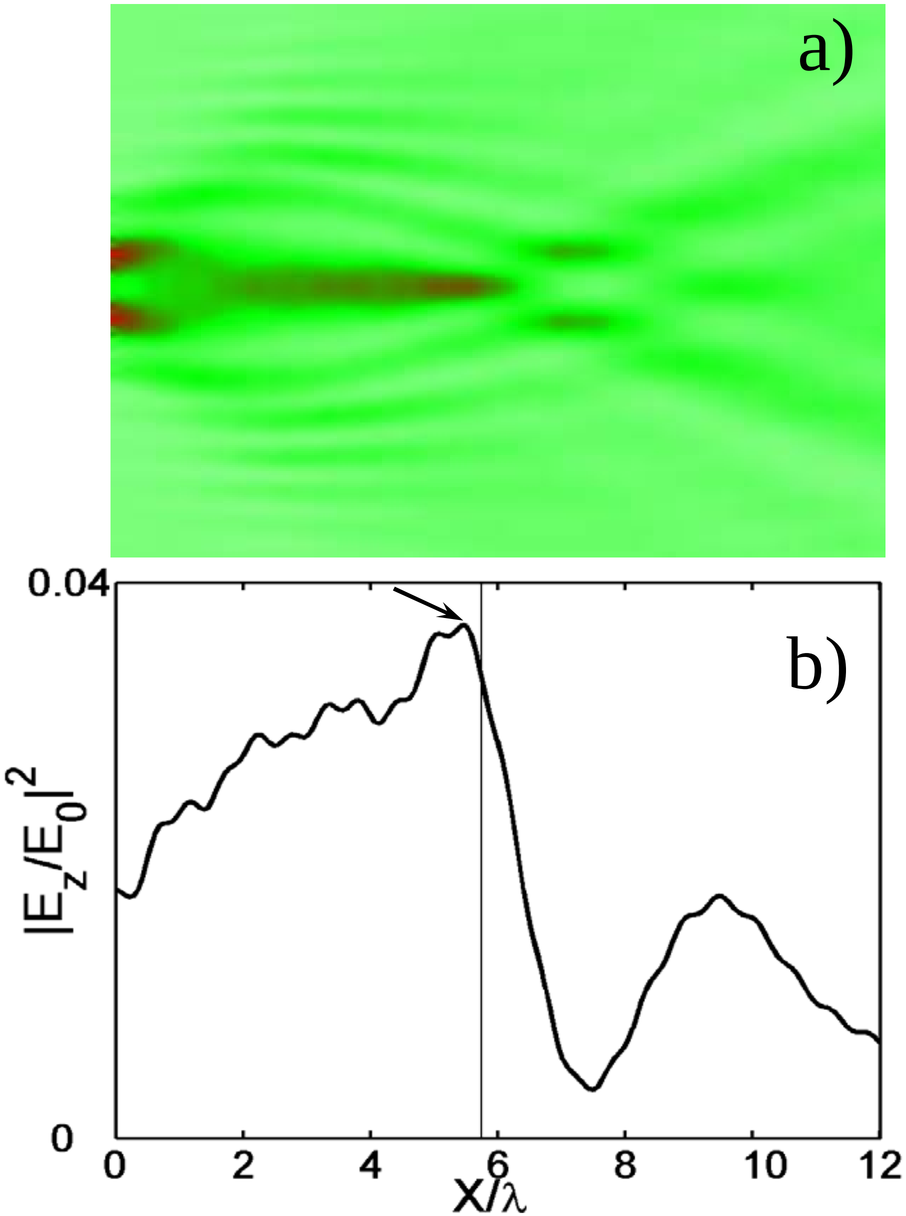
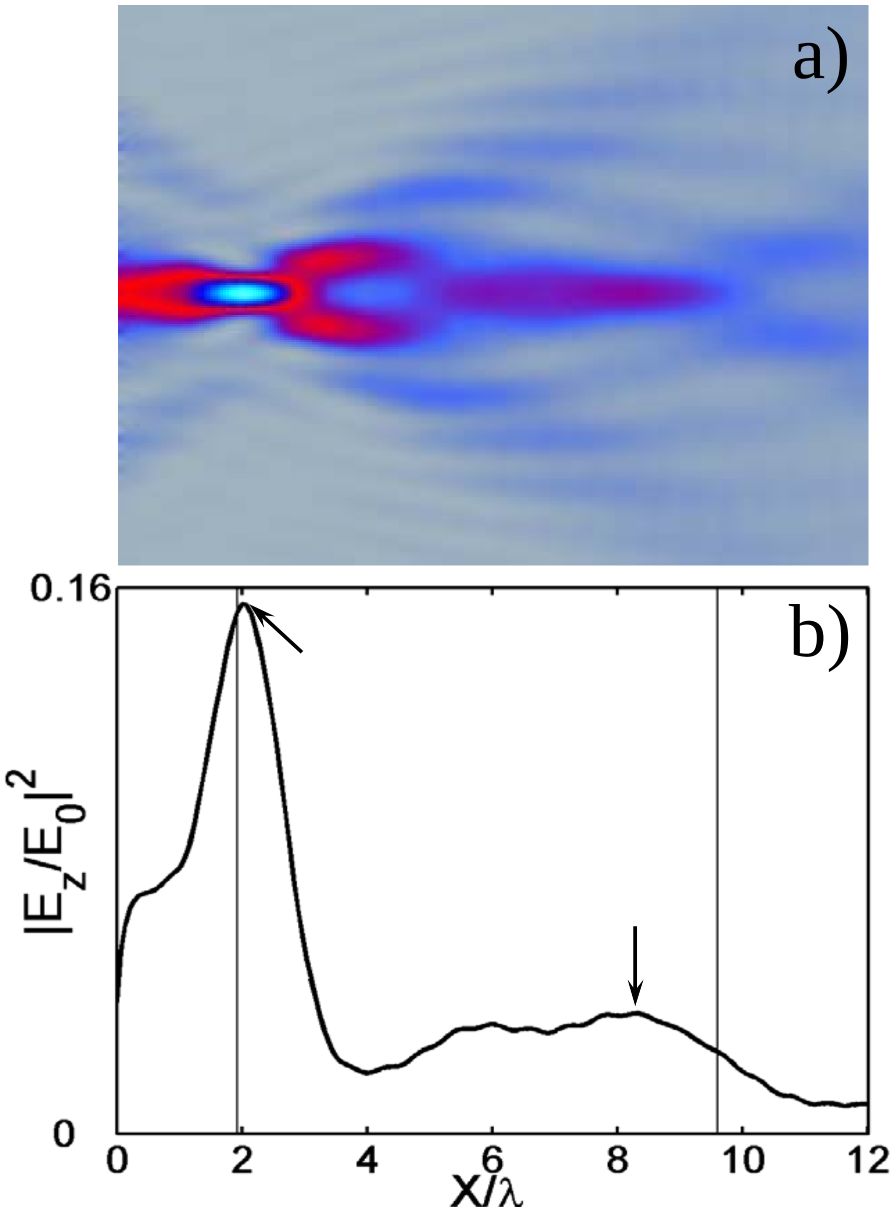
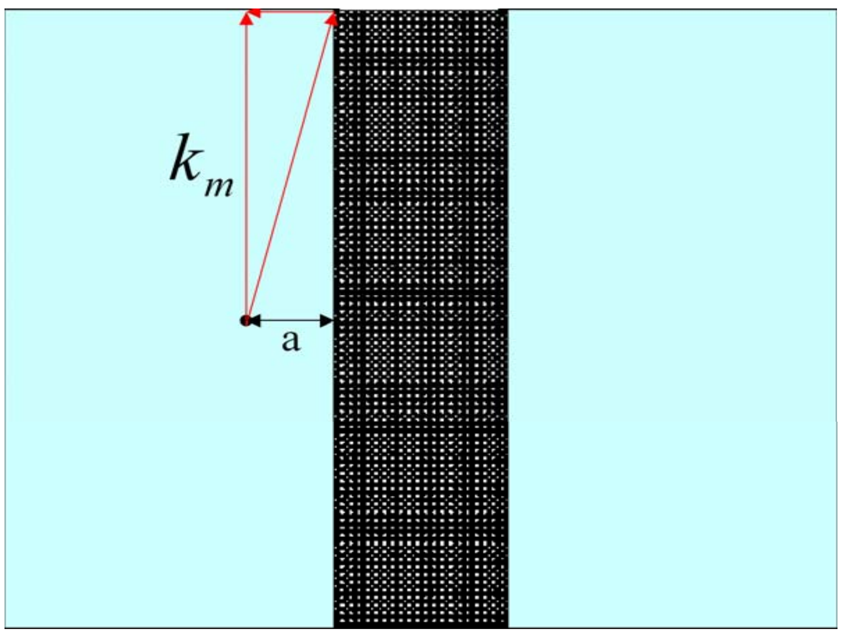

a)
b)
0.04
0
0
2
4
6
8
10
12
$|E_z/E_0|^2$
x/λ

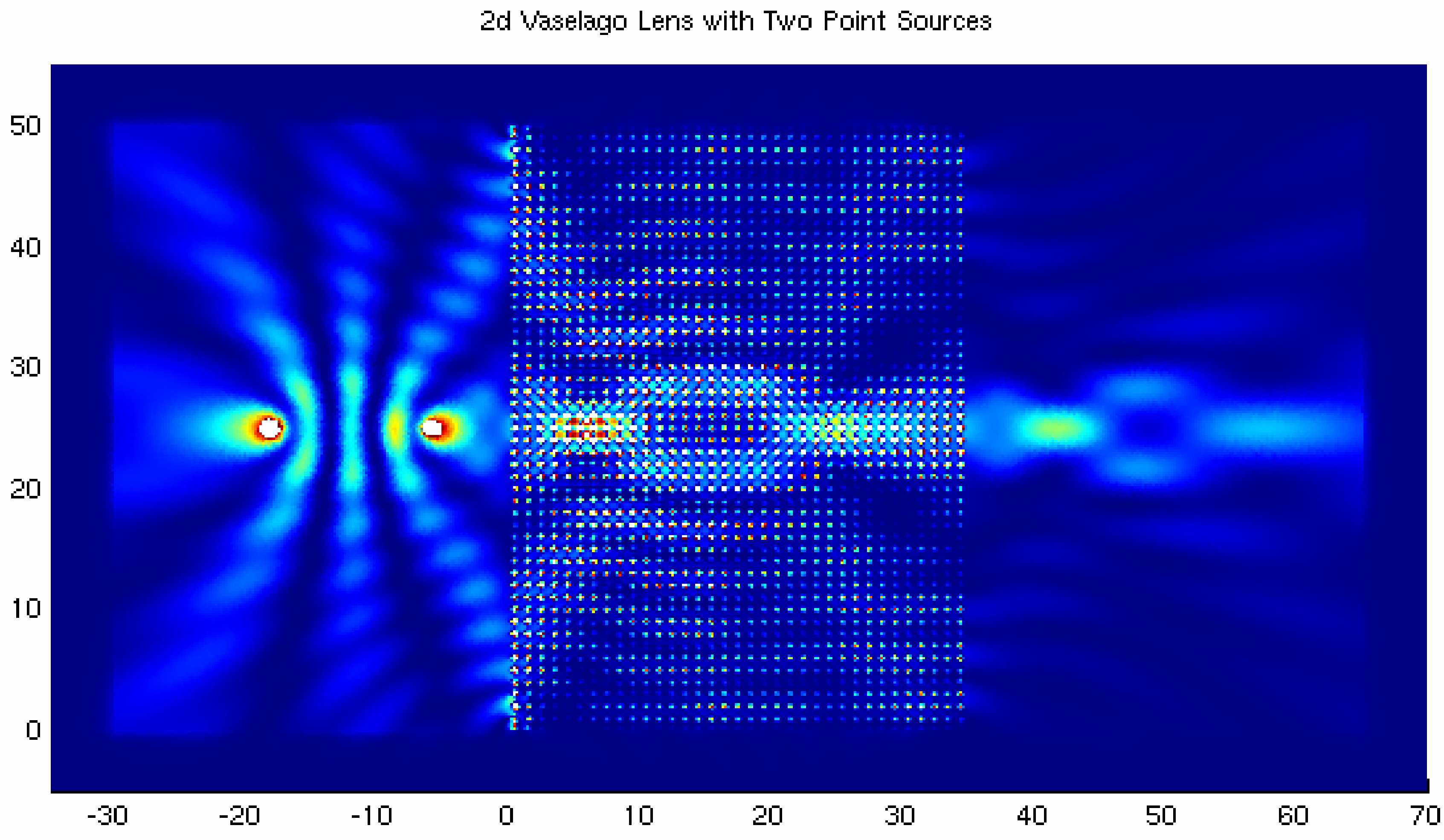

a)
b)
0.16
0
0
2
4
6
8
10
12
$|E_z/E_0|^2$
x/λ

$$E_s = iE_0 H_0^{(1)}(\rho k_0) e^{-i\omega t}$$

$$H_0^{(1)} = J_0 + iN_0$$

$$E(x', y) = \frac{iE_0}{\pi} \int_{-k_m}^{k_m} \frac{\exp i(ky + x'\sqrt{k_0^2 - k^2} - \omega t)}{\sqrt{k_0^2 - k^2}} dk$$




2d Vaselago Lens with Two Point Sources
50
40
30
20
10
0
-30
-20
-10
0
10
20
30
40
50
60
70