



The Abdus Salam  
International Centre for Theoretical Physics



SMR 1760 - 7

**COLLEGE ON  
PHYSICS OF NANO-DEVICES**

**10 - 21 July 2006**

*Introduction to the Physics of  
Semiconductor Quantum Dots  
The physics of nano-electronics*

Presented by:

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Massachusetts Institute of Technology, USA

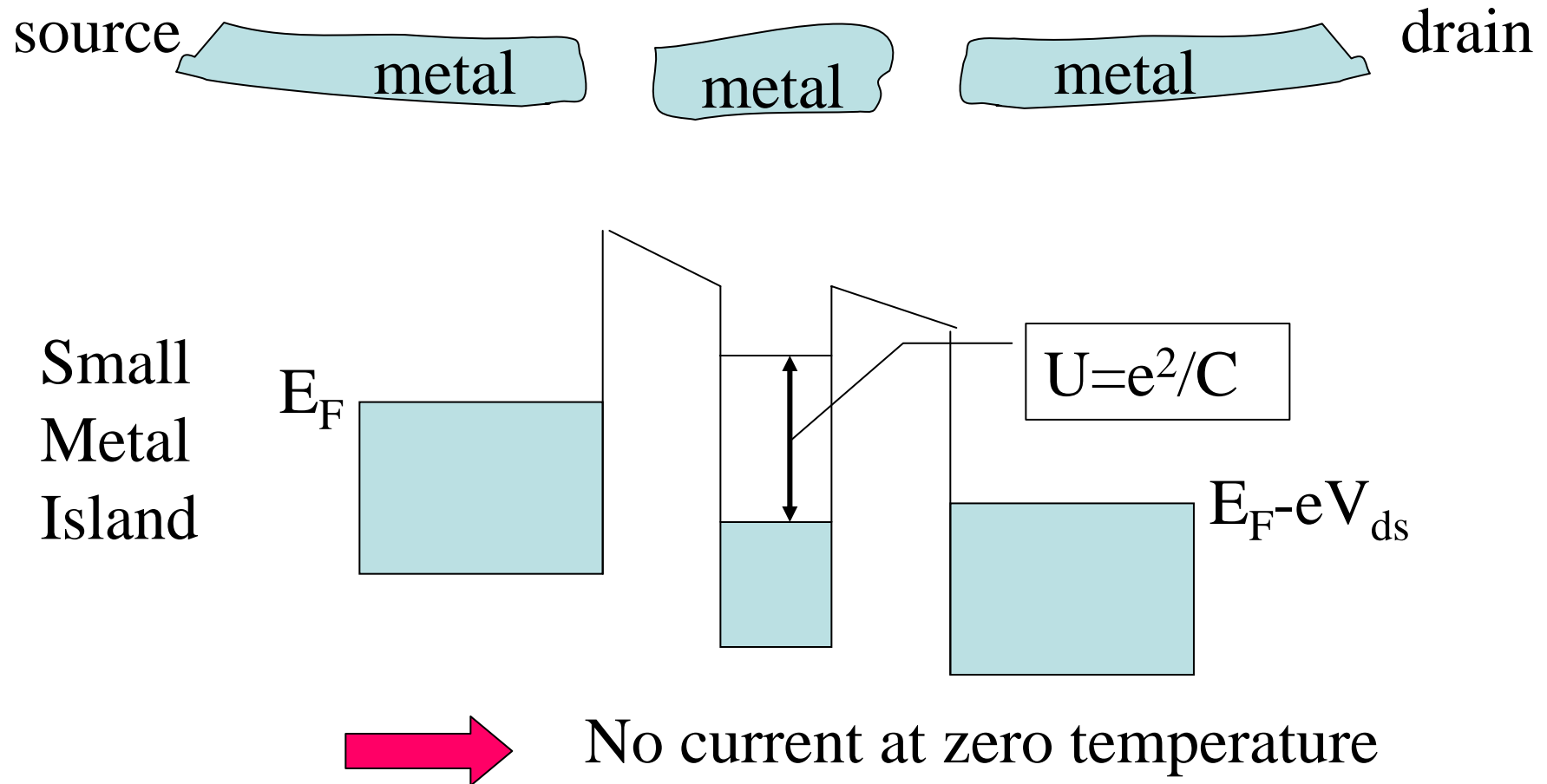
# Introduction to the Physics of Semiconductor Quantum Dots

M. A. Kastner, MIT  
Trieste 2006

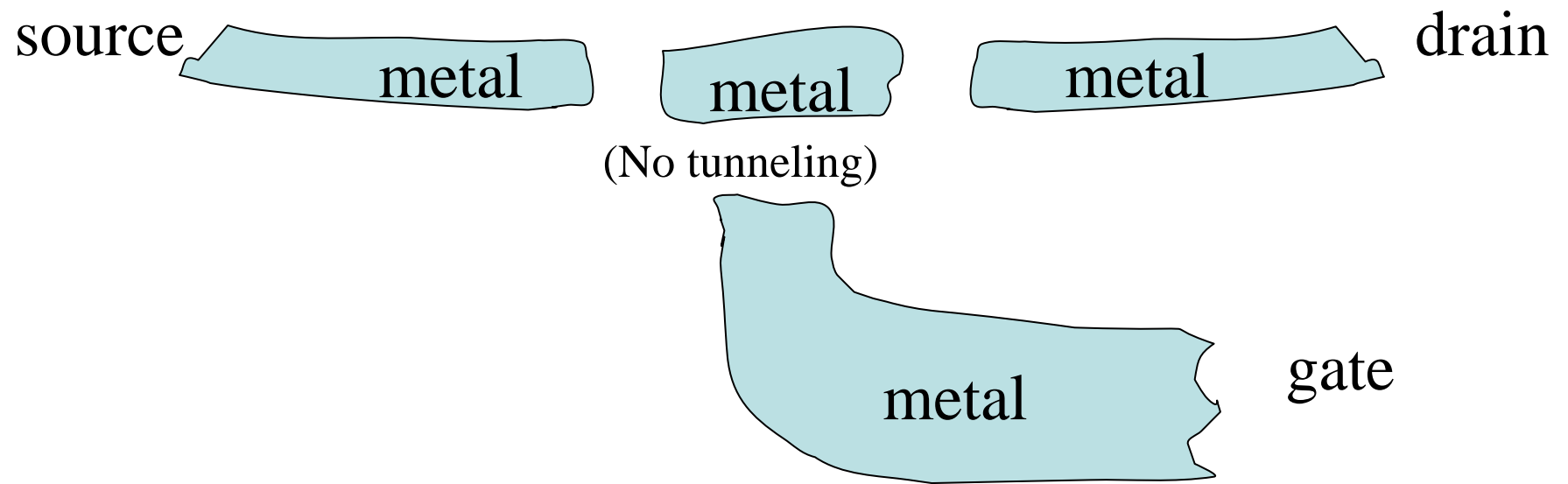
# Outline: Measuring Energy Scales

- Coulomb blockade energy  $U$  (Metal Single Electron Transistor)
- Energy level spacing  $\Delta\varepsilon$  (Semiconductor SET)
- Coupling to the leads  $\Gamma$

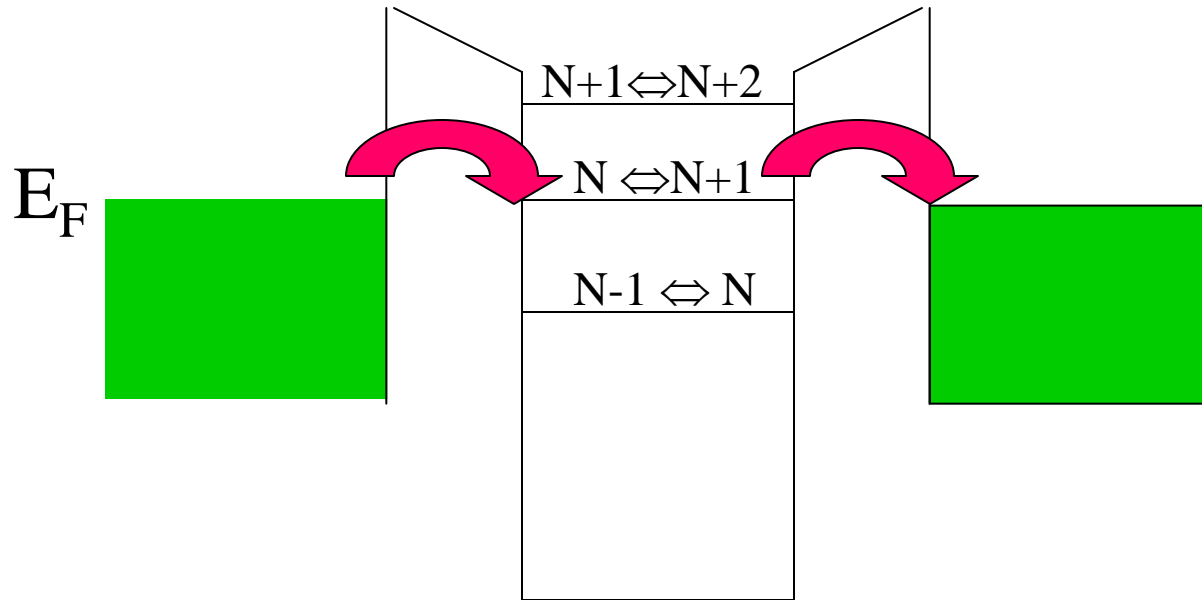
# Two Barriers with Small Island



# Schematic of Metal SET



# Sequential Charging



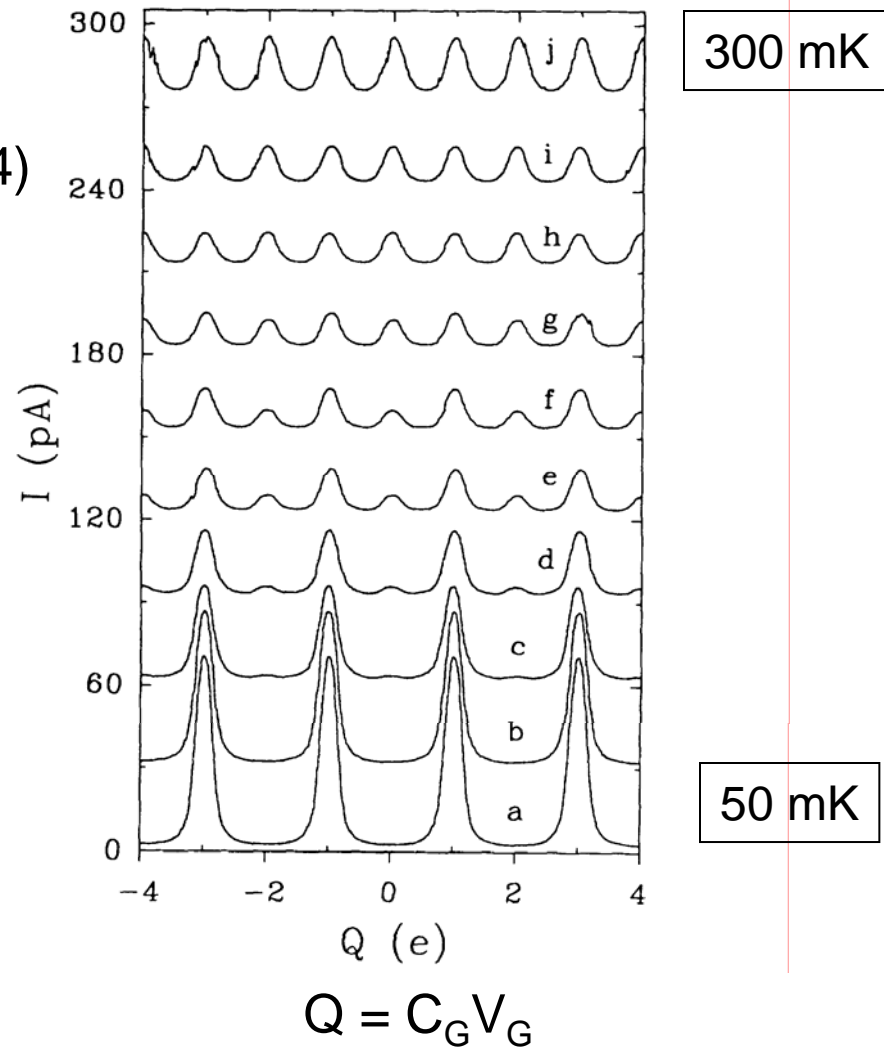
At low  $T$  and with very small  $V_{ds}$  get one sharp peak for each electron added.

# Current vs. Gate Voltage

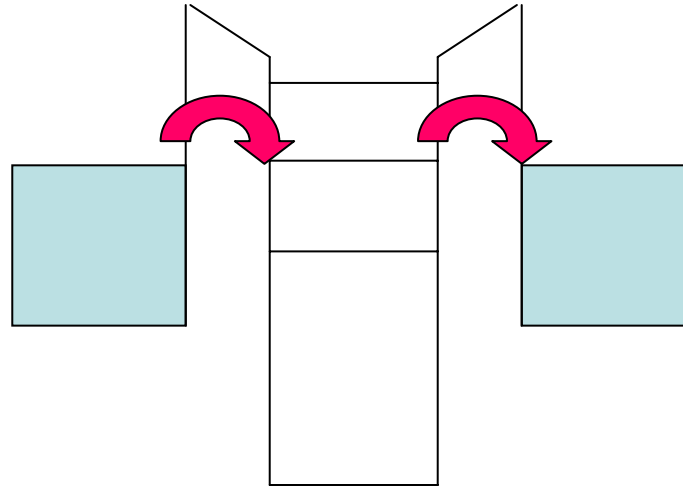
Aluminum SET

A. Amar et al. PRL **72**, 3234 (1994)

Note: Because of superconductivity, below  $T_C$  one gets a peak every time Cooper pair is added.



# Condition for Charge Quantization



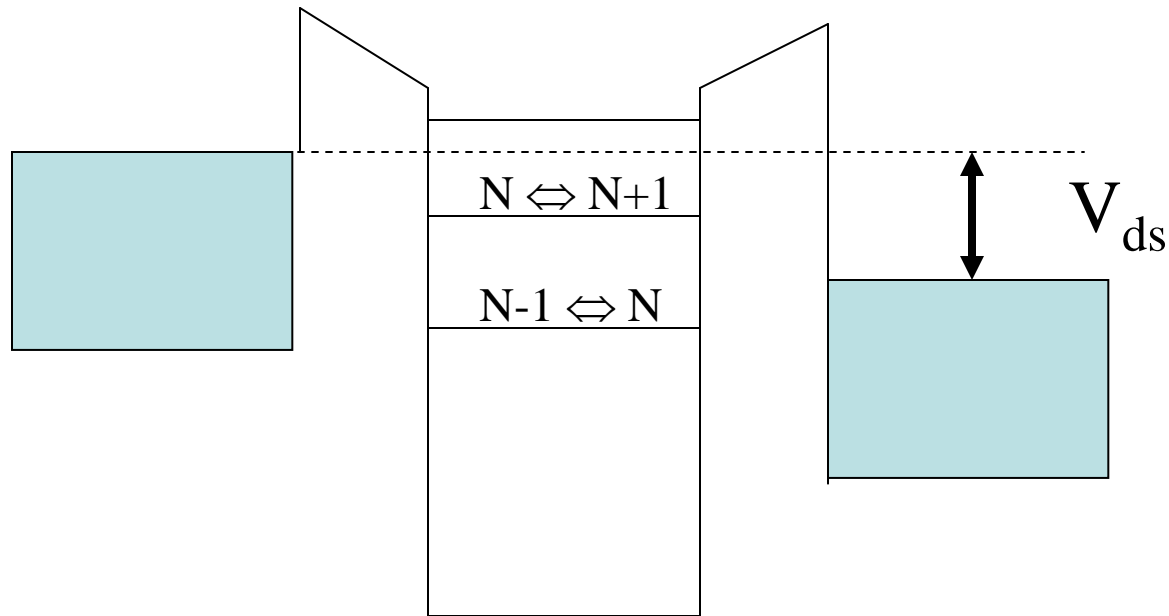
An extra electron stays on the island for time  $RC$ .  
This time must be long enough that the uncertainty  
in its energy is less than  $U$ .

$$U > \hbar/RC, \text{ but } U = e^2/C$$

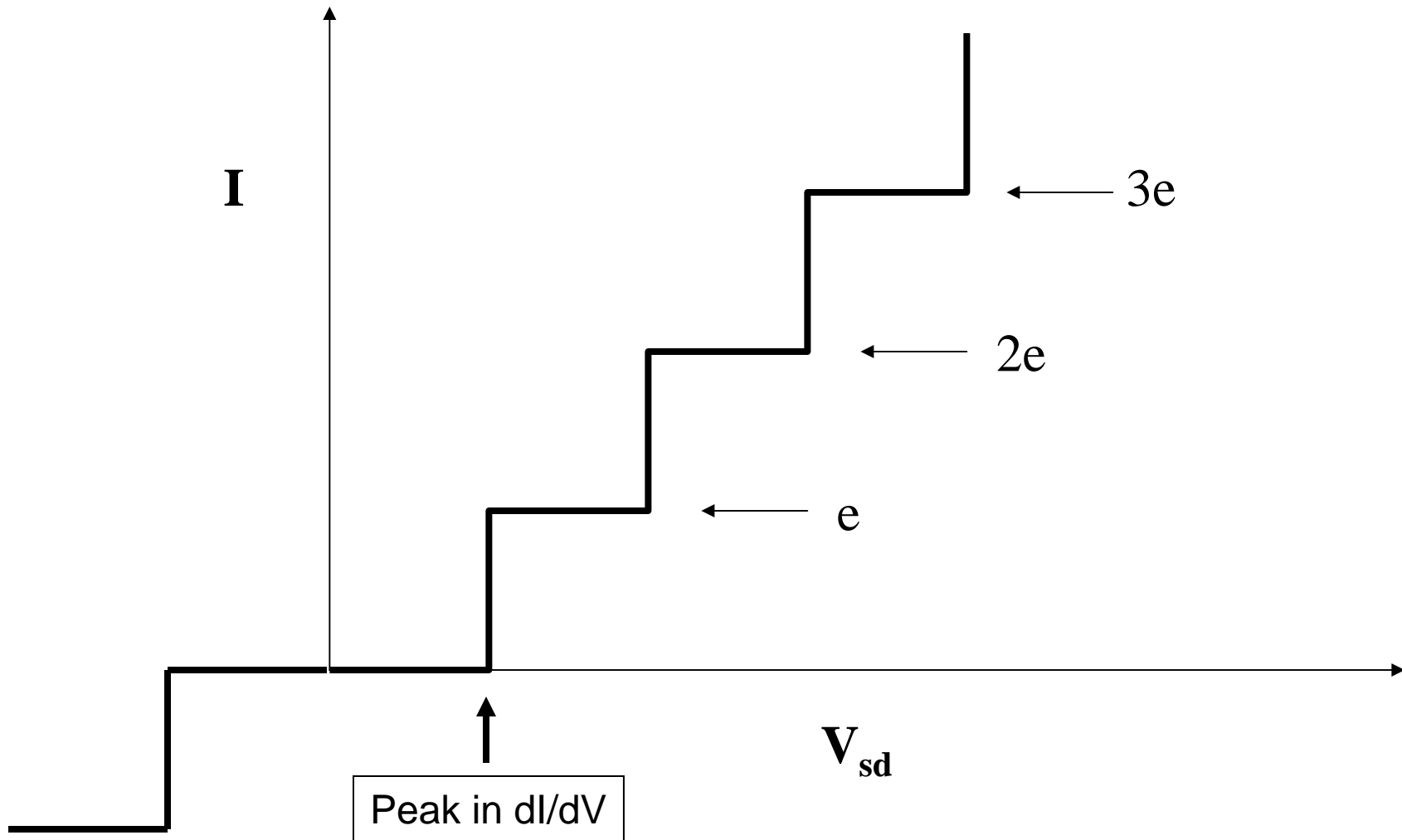
$$R > \hbar/e^2 \text{ or } G < e^2/\hbar$$



# Adding Charge by Source-Drain



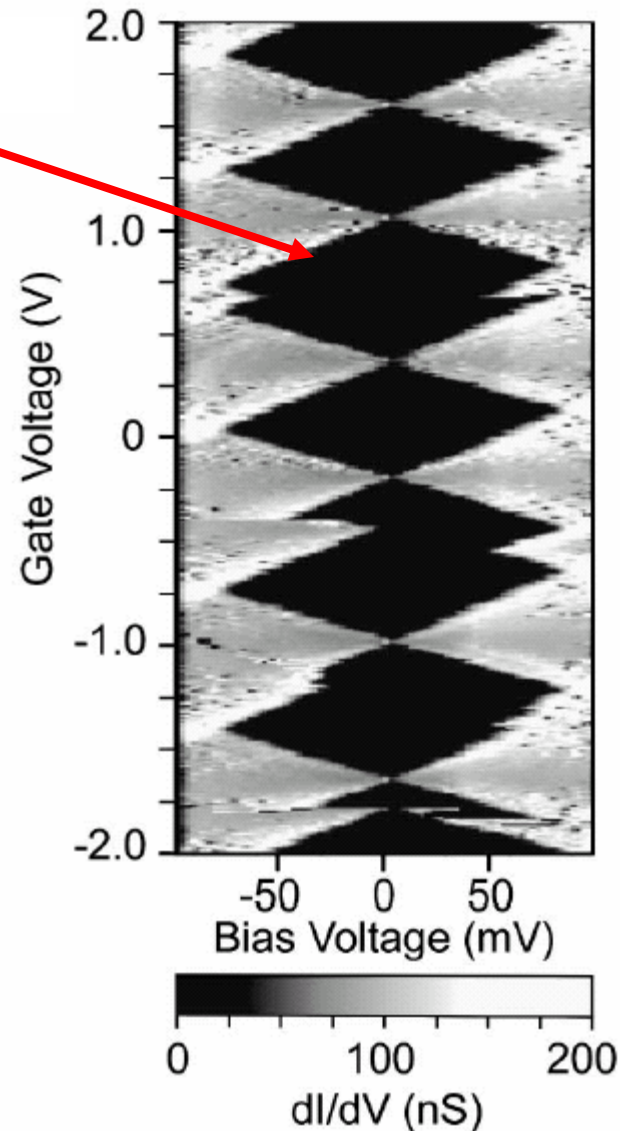
# Coulomb Staircase



# Coulomb Diamonds

First step  
in  
Coulomb  
staircase

Slopes of  
diamonds  
give  
capacitance  
ratios

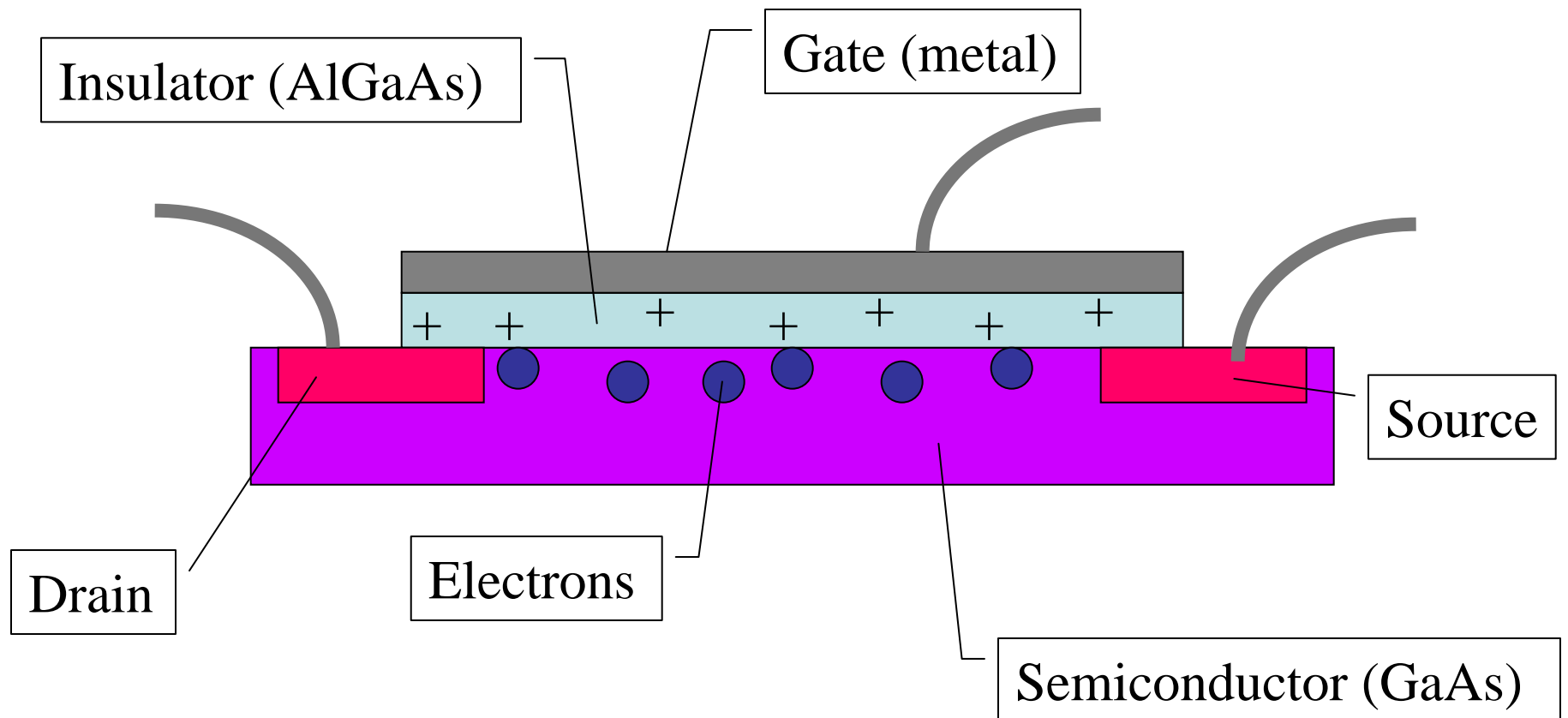


SET made with  
nano-particle,  
Bolotin et al. APL  
**84**, 3154 (2004)

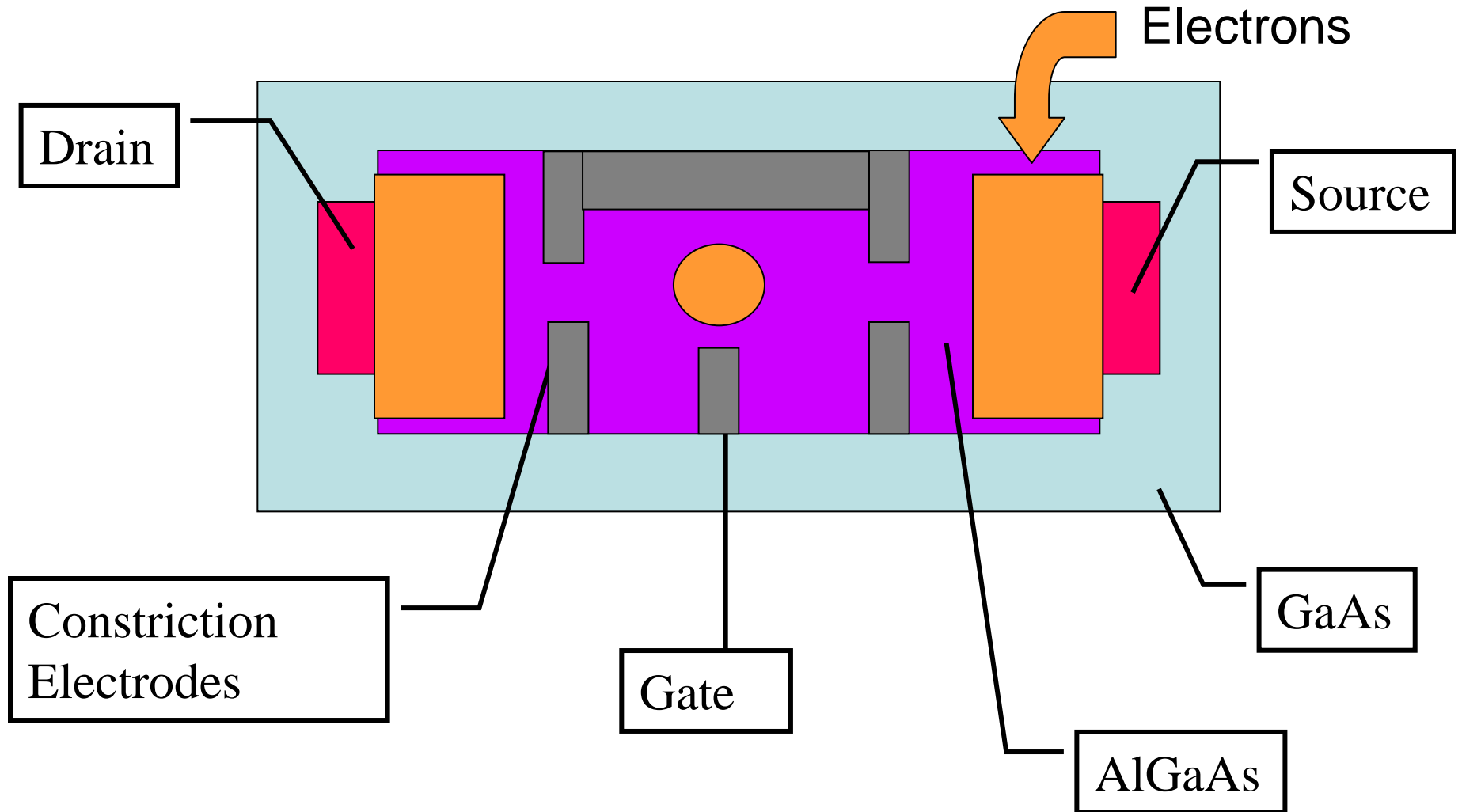
Note: switching from  
nearby charges

# Making Semiconductor SETs

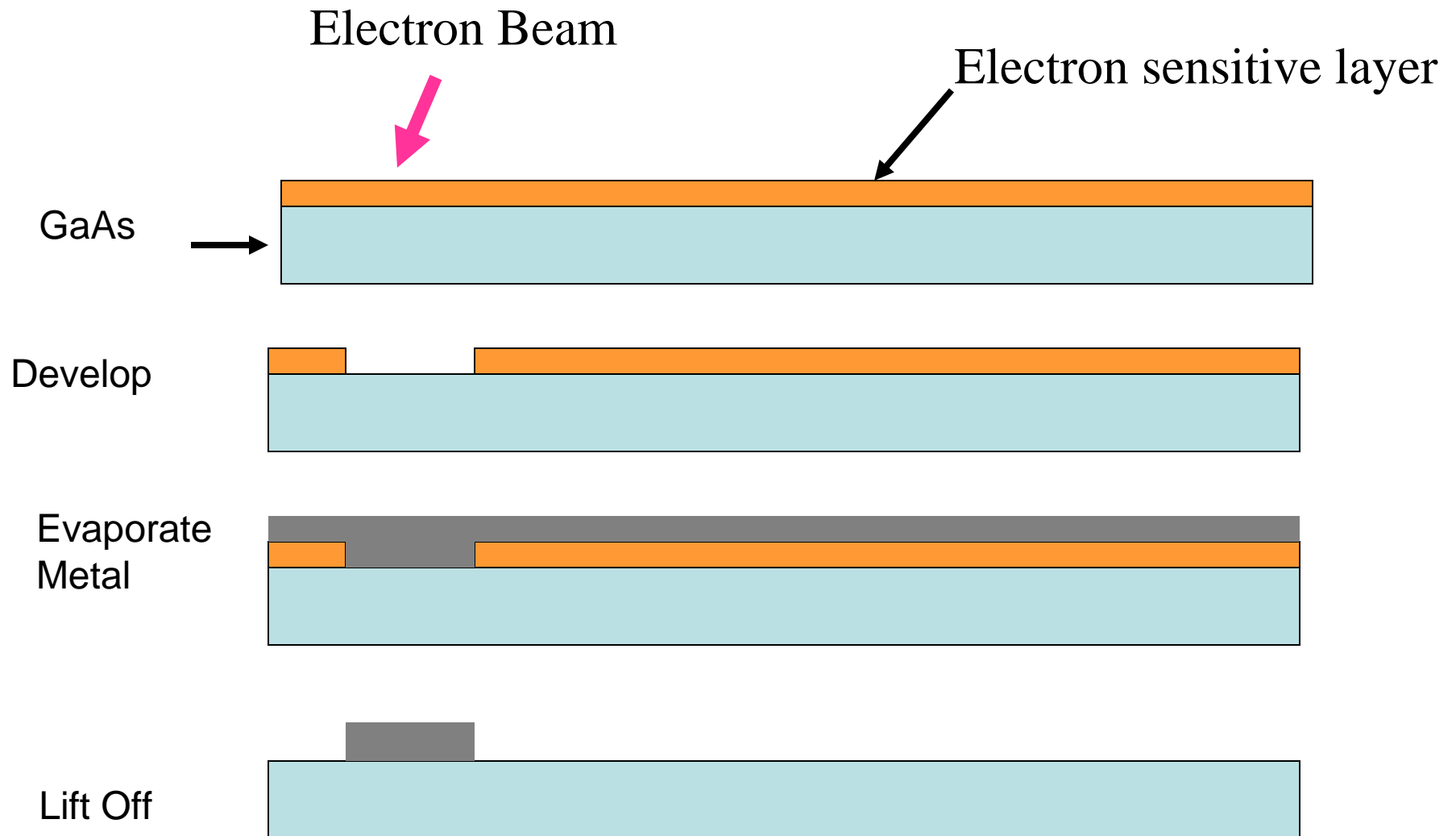
## GaAs Field Effect Transistor



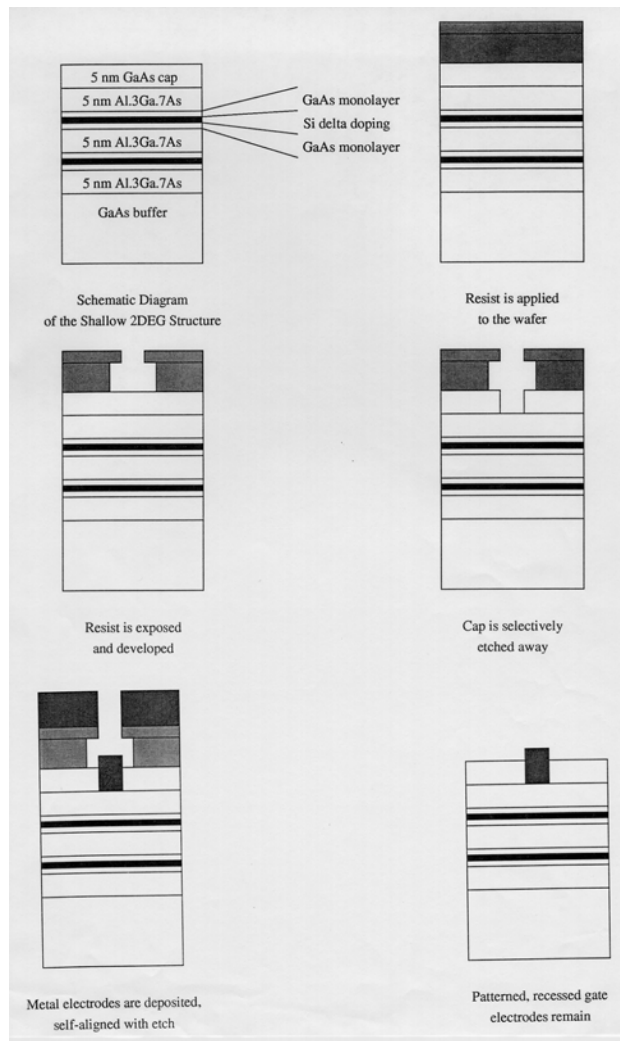
# Schematic GaAs SET



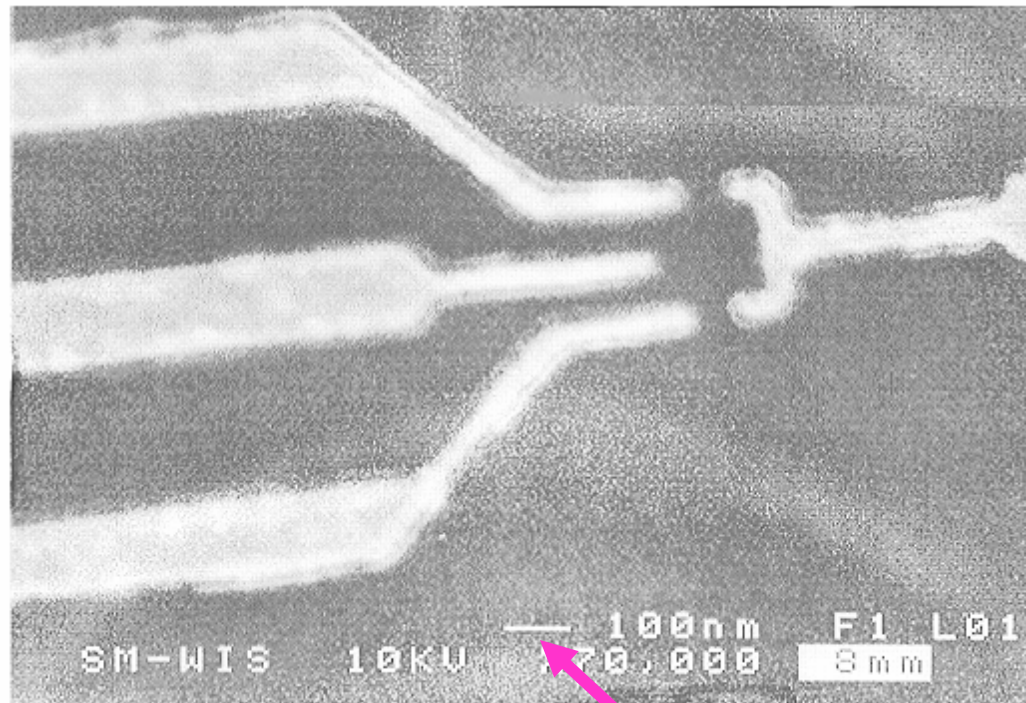
# Electron Beam Lithography



# Actual Process



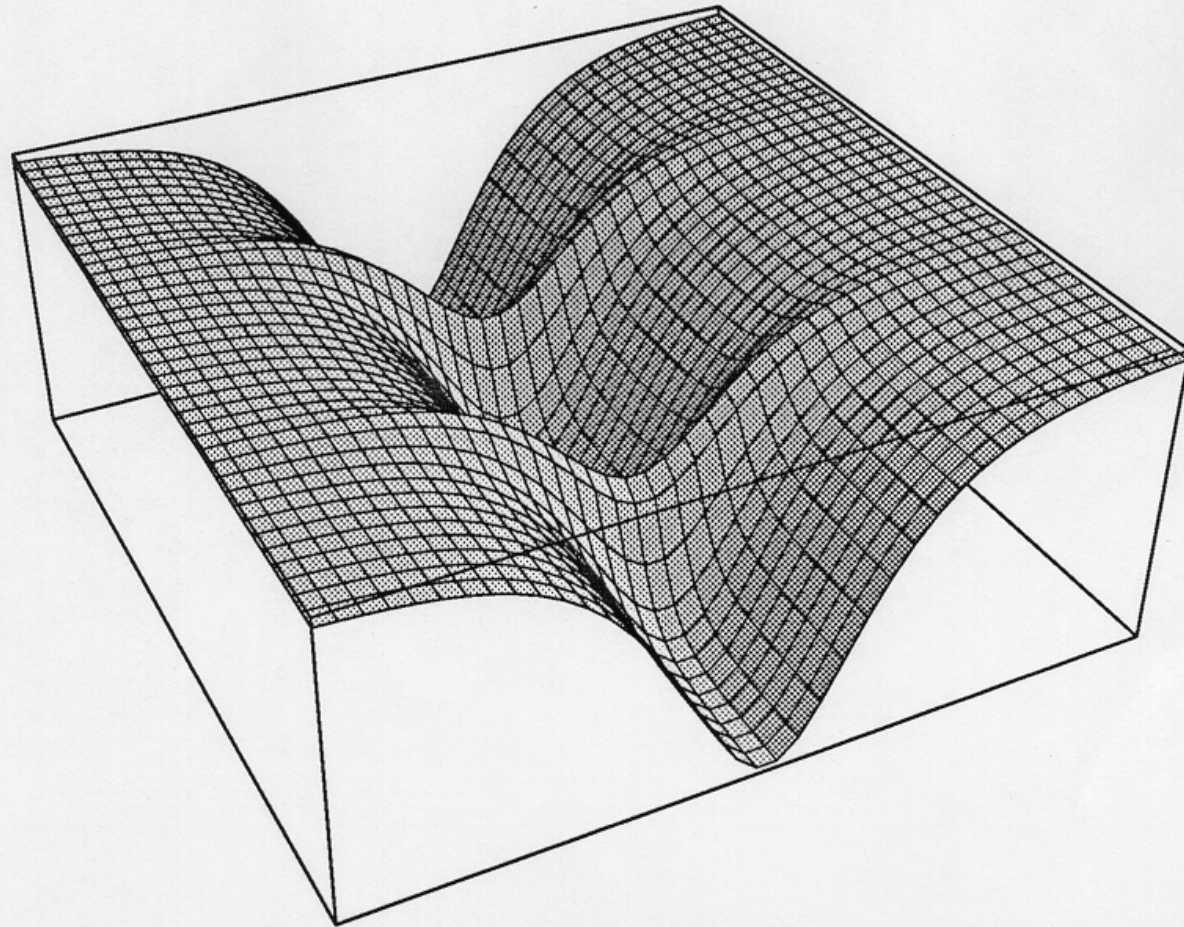
# GaAs SET



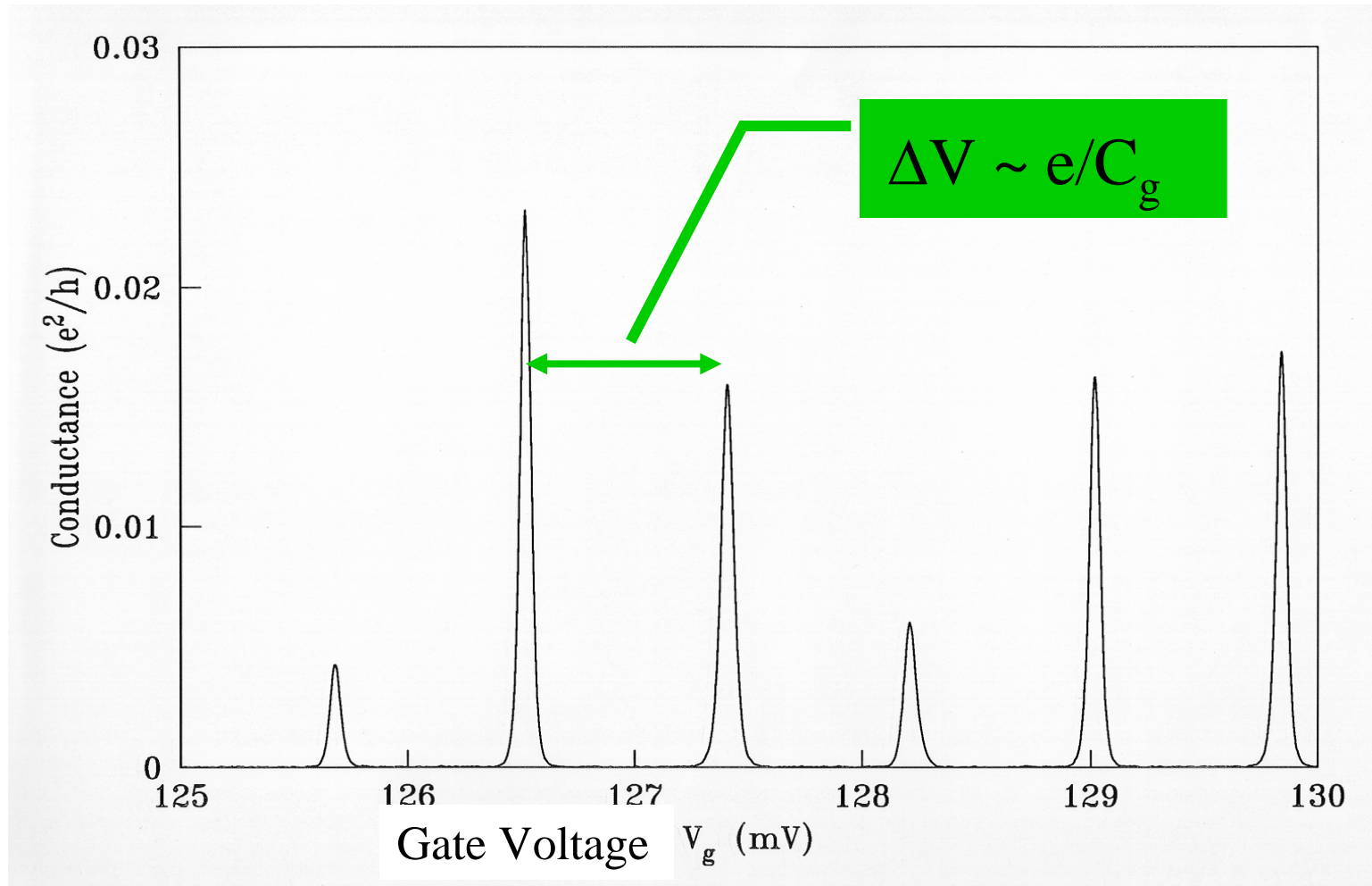
100nm = 1000Å



# Schematic Potential in SET

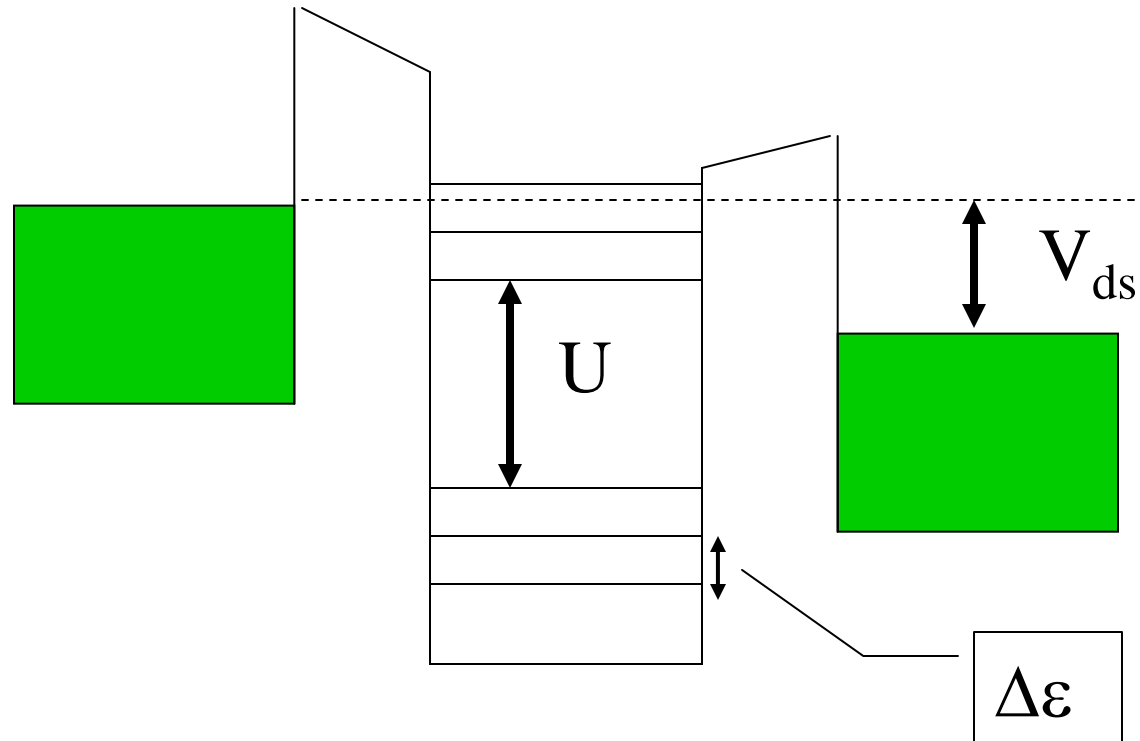


# Coulomb Charging Peaks



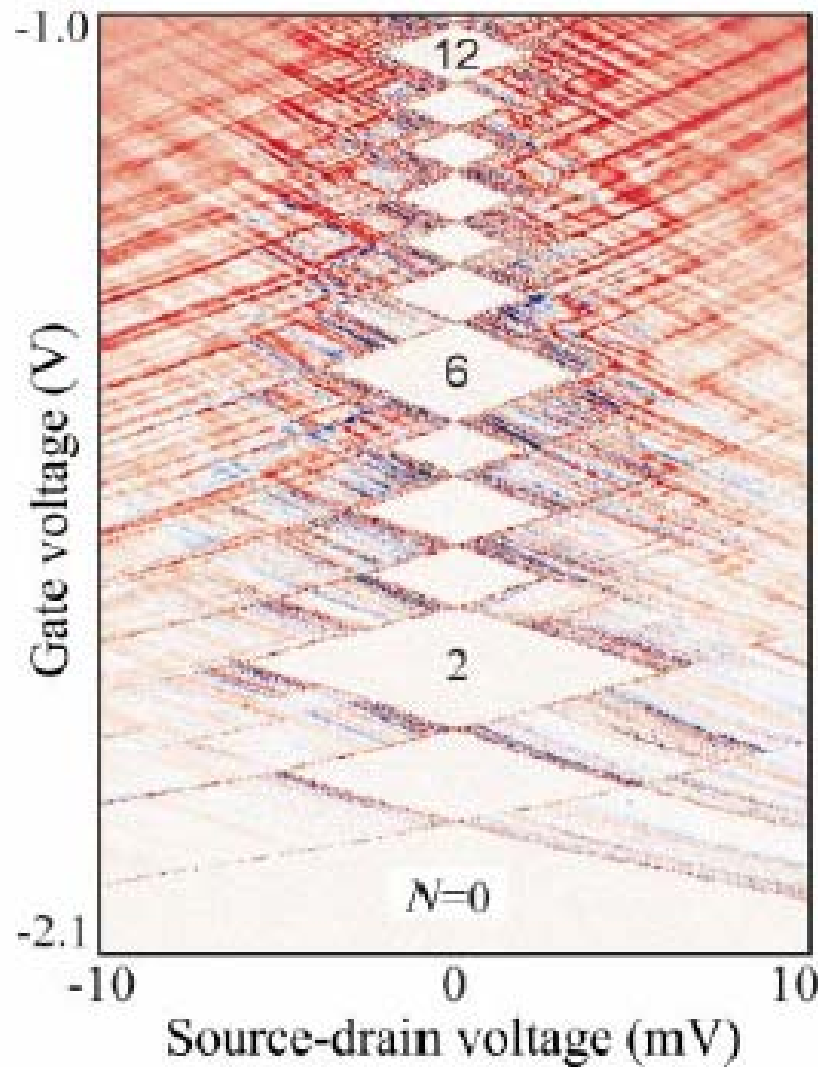
Note: Variation of peak height and spacing reflects individual levels.

# Quantized Energy Levels



There is a peak in  $dI/dV_{sd}$  for every energy level. Although these have been detected in metal SET's it is hard because density of states is so large.

# Excited State Spectroscopy



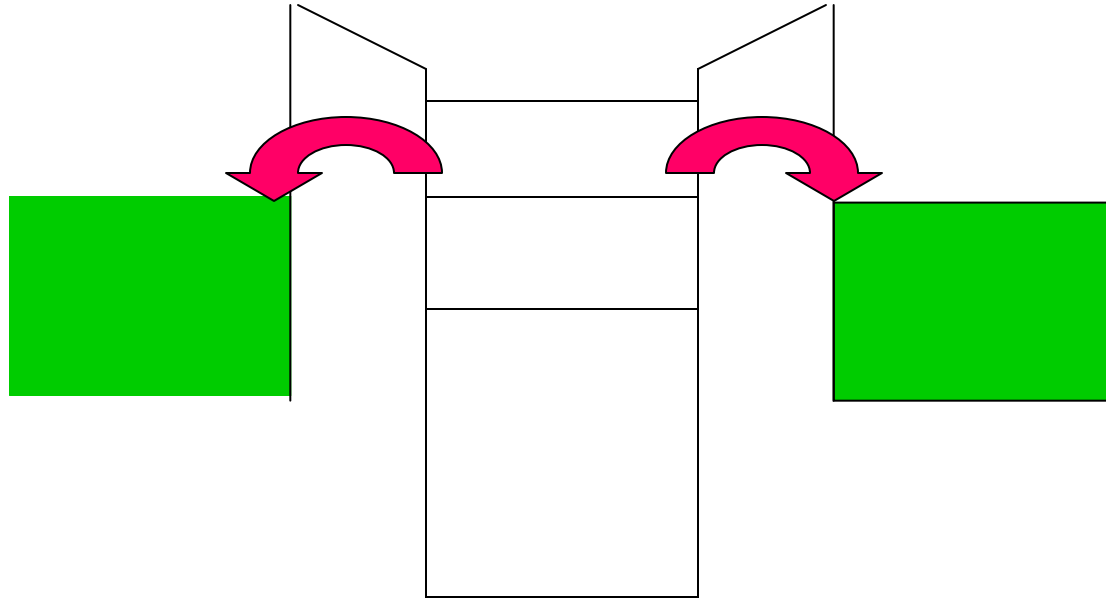
$dI/dV_{sd}$  has peak when level crosses  $E_F$

Very small dot  $\Rightarrow$  peaks no longer periodic along  $V_{sd} = 0$

Electron interactions are more complicated than just  $U$  and involve exchange.

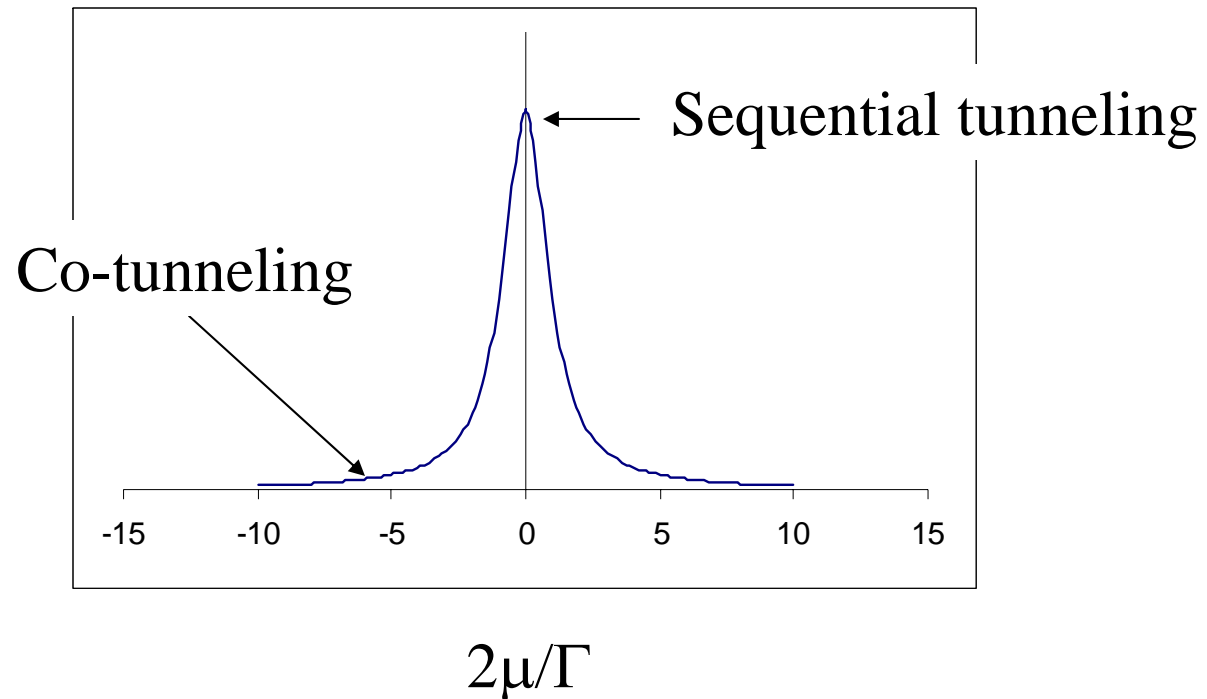
Kouwenhoven et al  
Science **278**, 1788, 1997

# Lifetime Broadening



Probability of electron remaining in a level on the dot decays as  $\exp(-t/\tau)$ , so the level broadens into a Lorentzian with energy width  $\Gamma = \hbar/\tau$

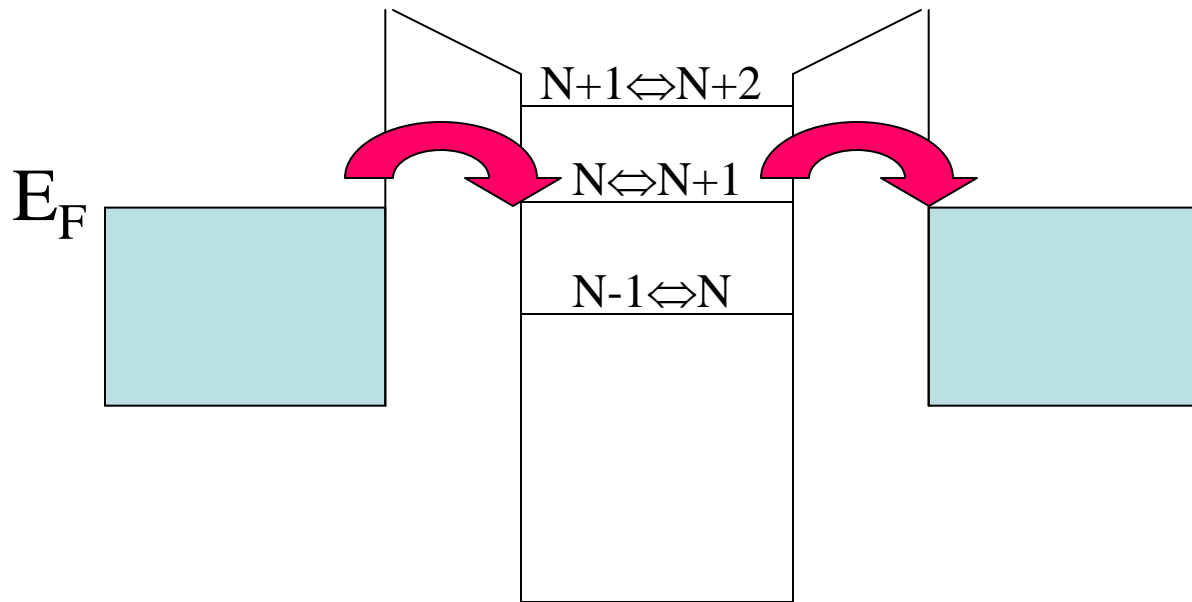
# Lorentzian Line Shape of Peaks vs Gate Voltage



The chemical potential  $\mu$  is proportional to the gate voltage.

The full width at half maximum is  $\Gamma$ .  $\tau = \hbar\Gamma^{-1}$  is the time for the electron to tunnel off.

# Temperature



$$I \sim \int T(E) [f(E) - f(E - eV_{sd})] dE \quad f(E) = \frac{1}{\exp[(E - E_F)/kT] + 1}$$

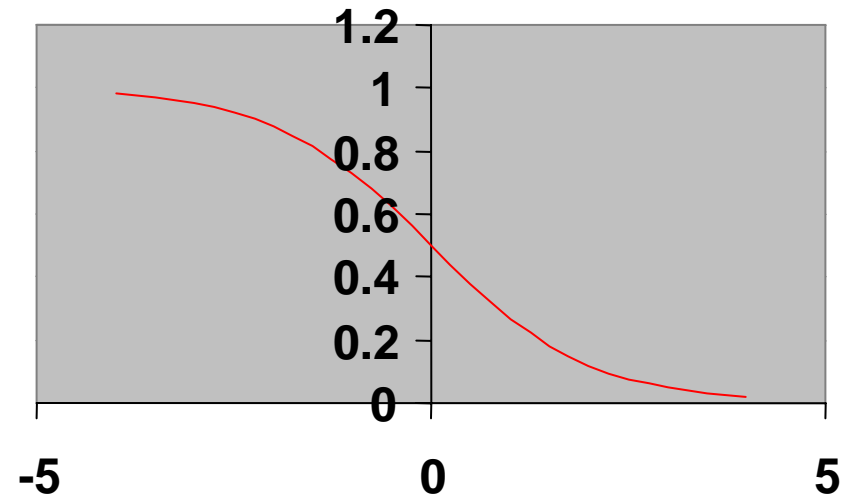
For resonant tunneling near zero bias, i.e.  
 $eV_{sd} < kT$ , if  $\Gamma$  is very small,  $T(E) = \delta(E)$ ,  $I = eV_{sd} df/dE$

# Thermal Broadening

Fermi-Dirac  
Distribution

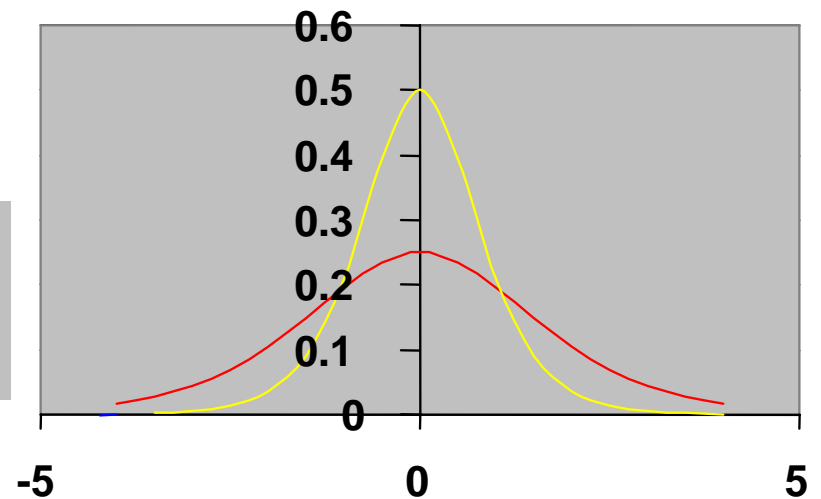
Thermal broadening  
gives width =  $3.5kT$   
Height  $\sim 1/T$

$f(E)$



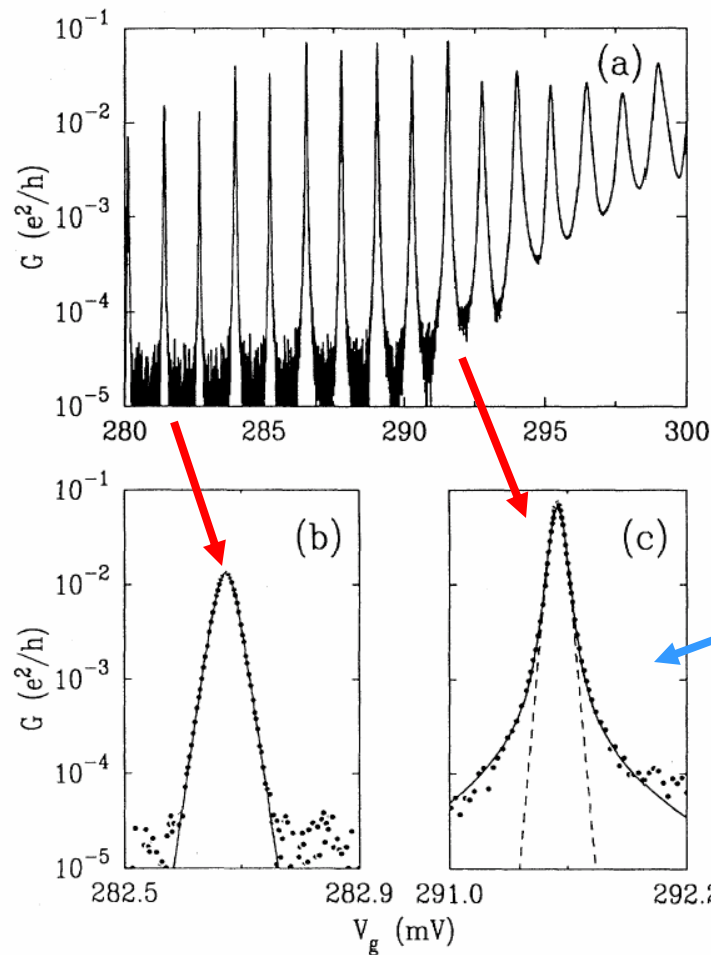
$df/dE$

$kT = 0.5$   
 $kT = 1$





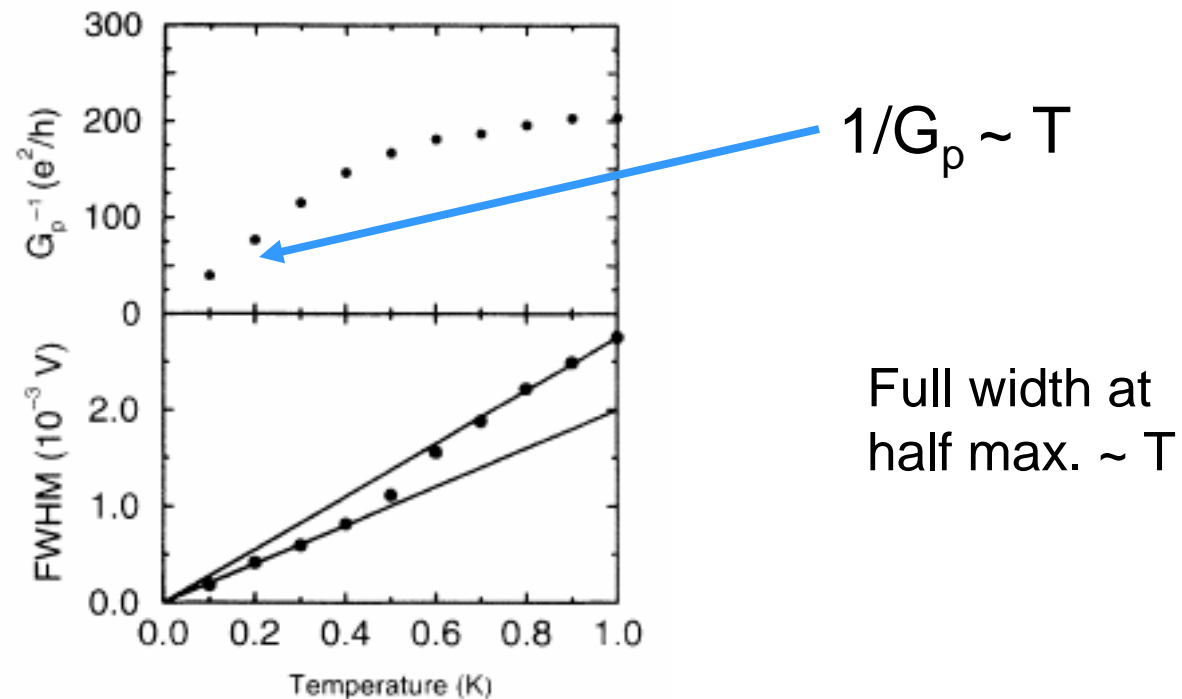
# Thermal and Intrinsic Broadening



Foxman et al. Phys. Rev  
B 47, 10020 (1992)

Dashed line from  
Fermi alone, solid  
includes Lorentzian

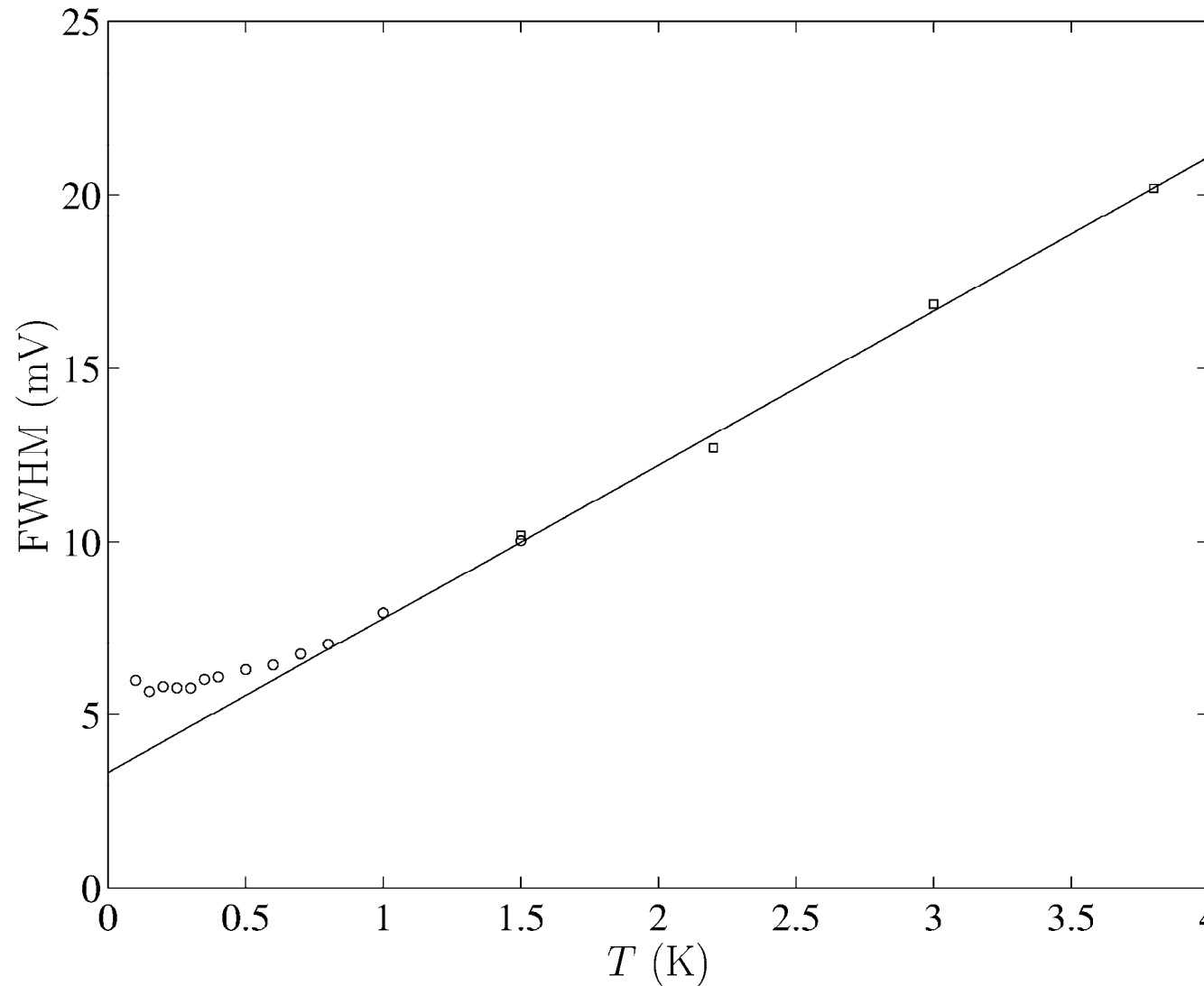
# Absolute Thermometer



When  $kT > \Delta\varepsilon$  the peak conductance becomes constant and the width changes slope slightly.

For thermometer application see Pekola et al. PRL **73**, 2903 (1994)

## Determining $\Gamma$ from peak width

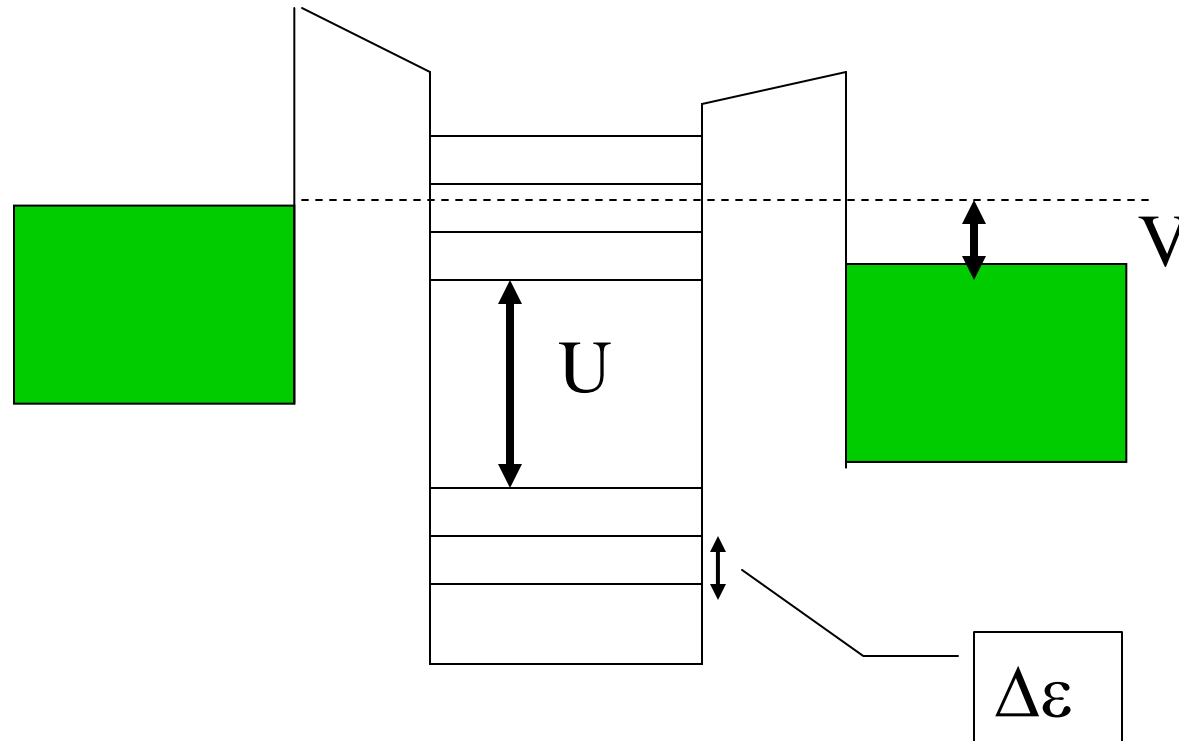


Slope gives  
conversion  
of voltage to  
energy.

$T=0$  intercept  
gives  $\Gamma$

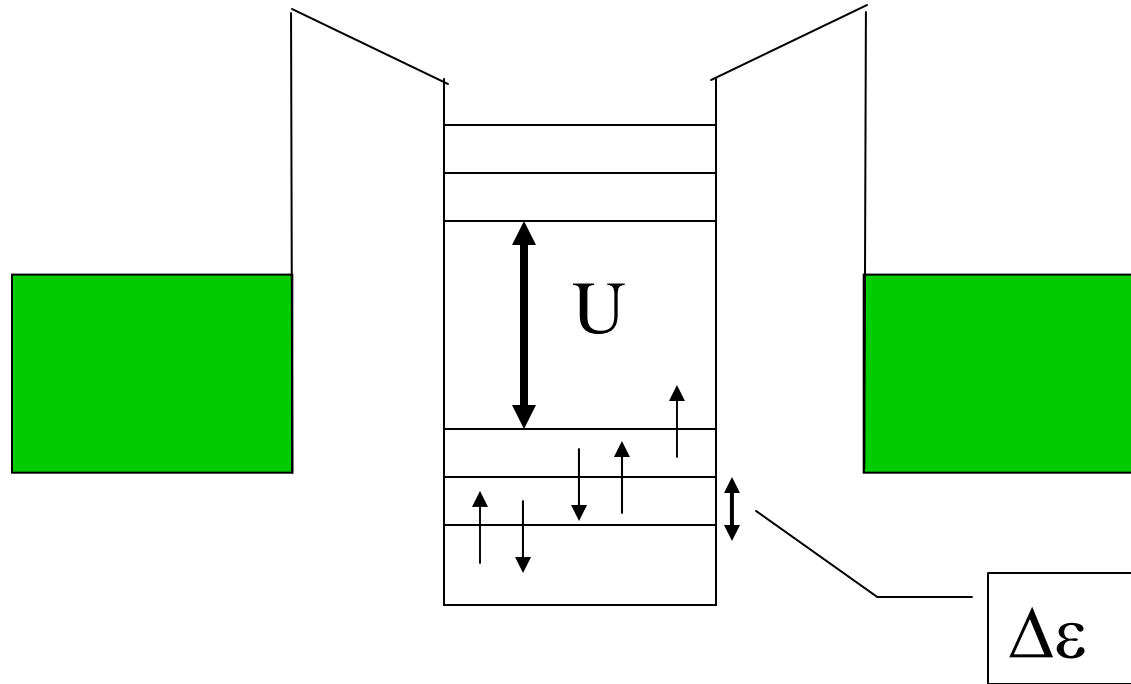
Note: in this  
case  
 $\Gamma/h \sim 50\text{GHz}$

# Condition for Charge Quantization is Condition for Level Separation



Above Coulomb gap, the current is  $I = Ne/\tau$ ,  $\tau = h\Gamma^{-1}$  and  $N = eV/\Delta\epsilon$   
 $G = I/V = (e^2/h)(\Gamma / \Delta\epsilon)$   
 $G < e^2/h \Rightarrow \Gamma > \Delta\epsilon$

# Constant Interaction Model

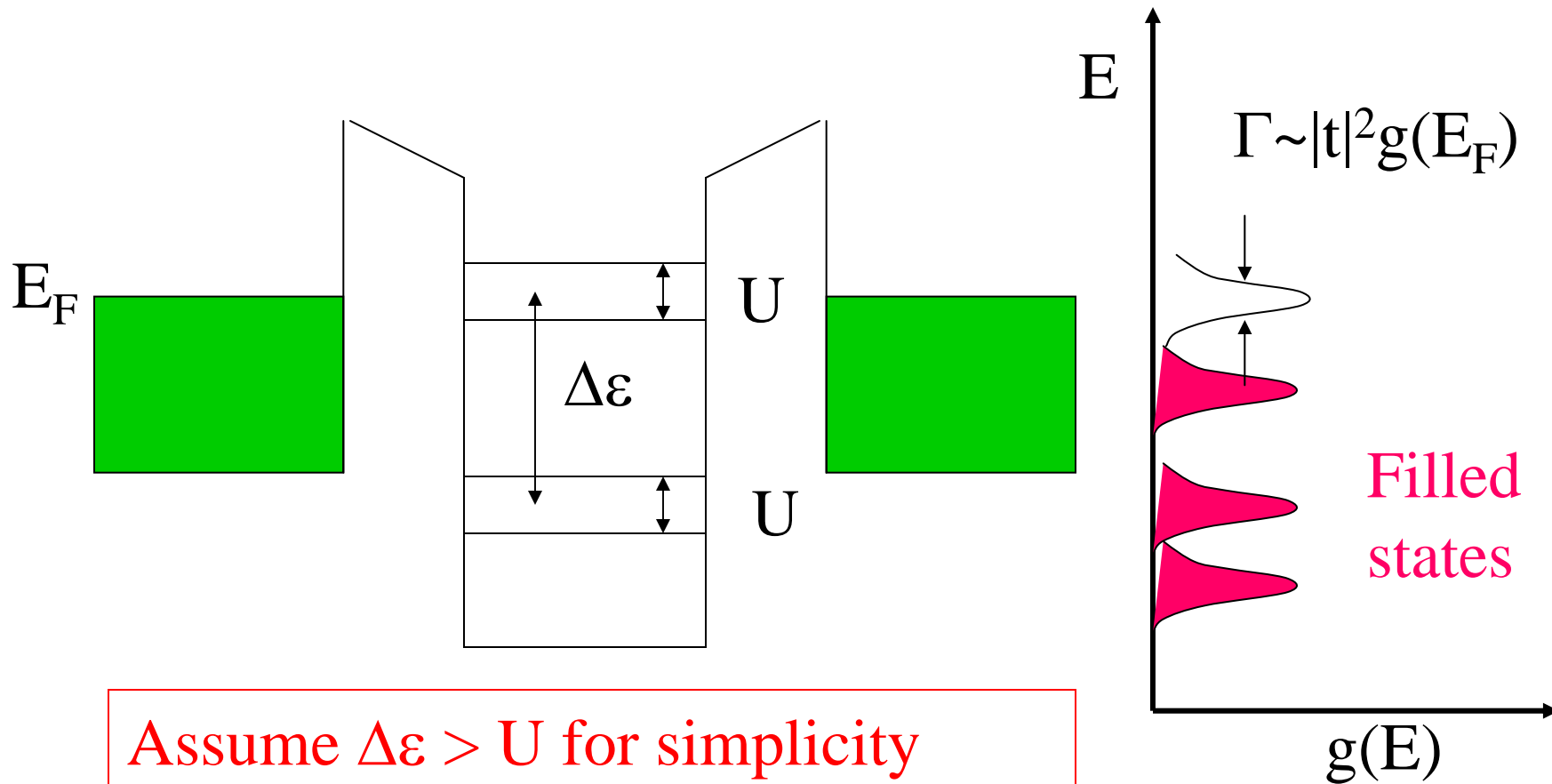


Ignore interactions among electrons on artificial atom.

States fill two at a time.

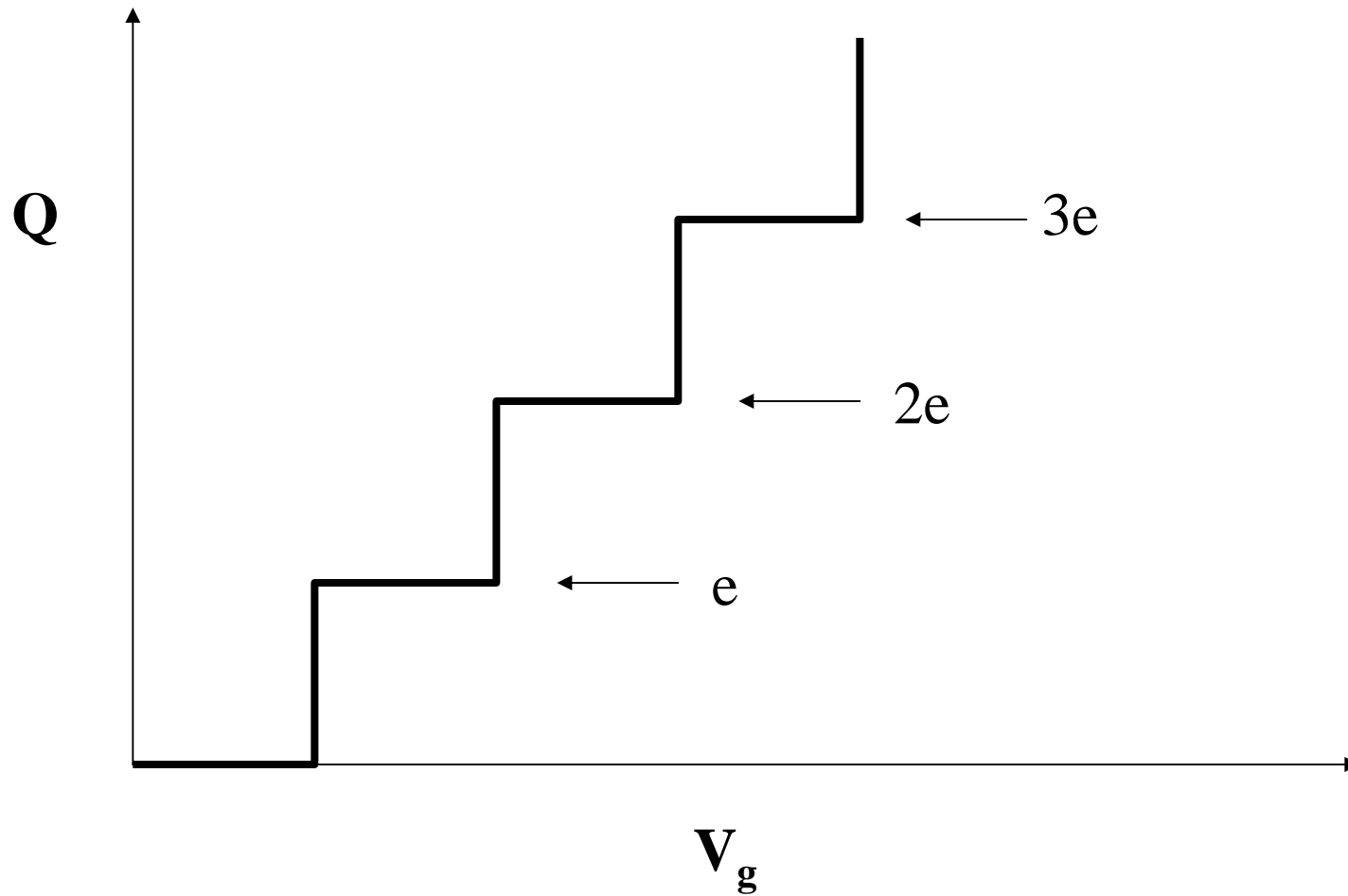
Actually more complicated, but it is a useful starting point.

# Energy Scales in SET

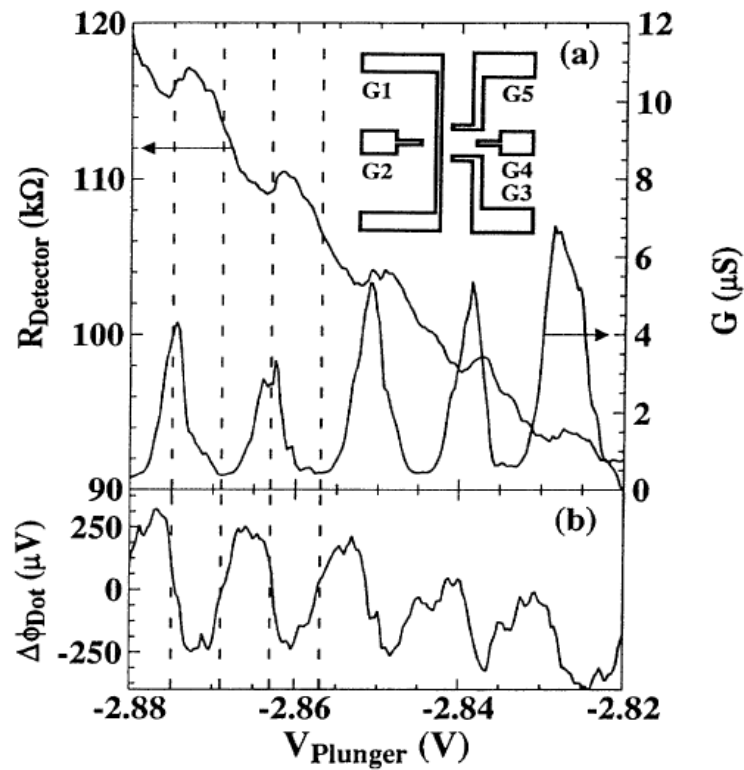


Assume  $\Delta\epsilon > U$  for simplicity  
 $t$  is the hopping matrix element  
between dot and leads

# Charge Quantization



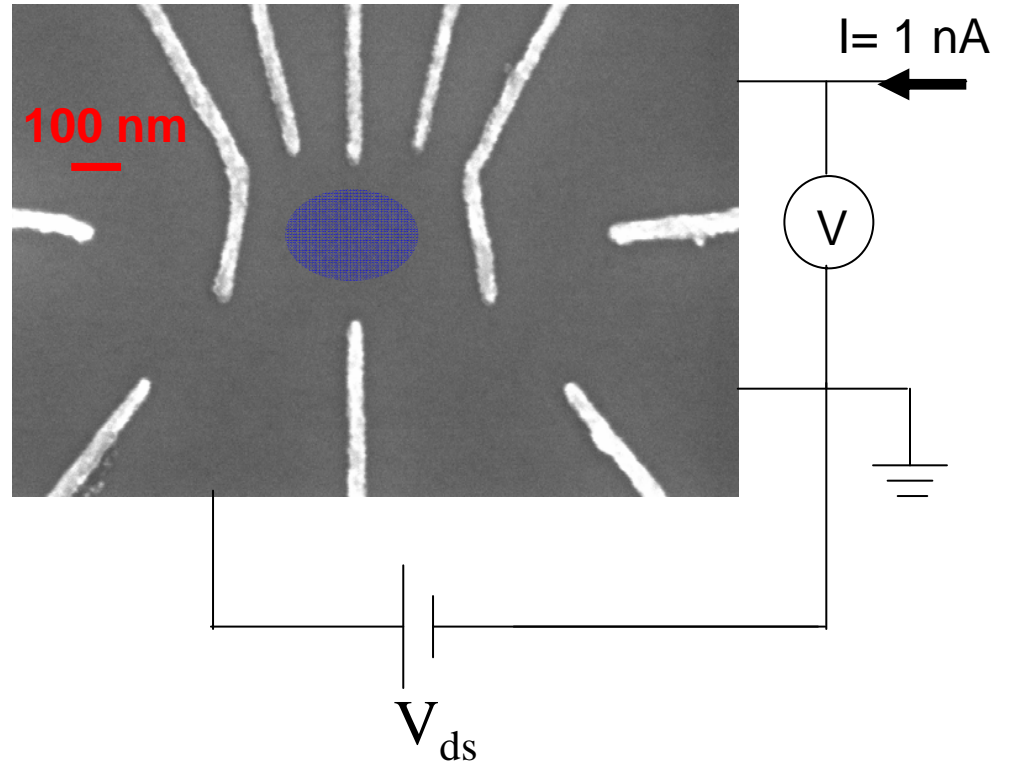
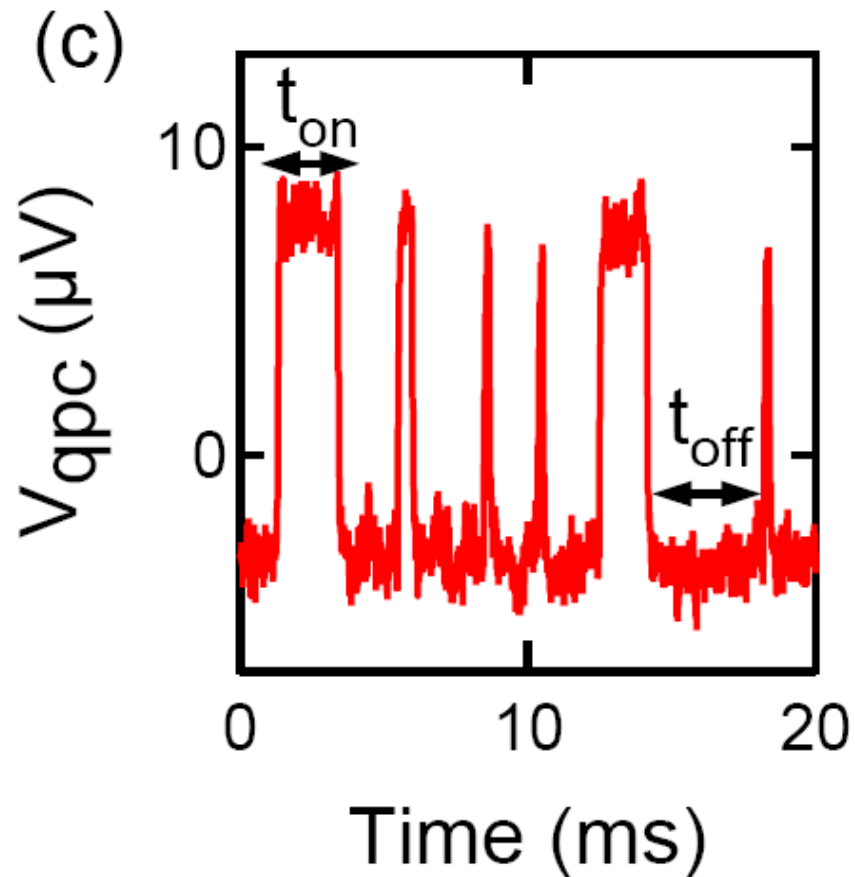
# Measuring Charge



Field et al PRL **70** 1311 (1993)

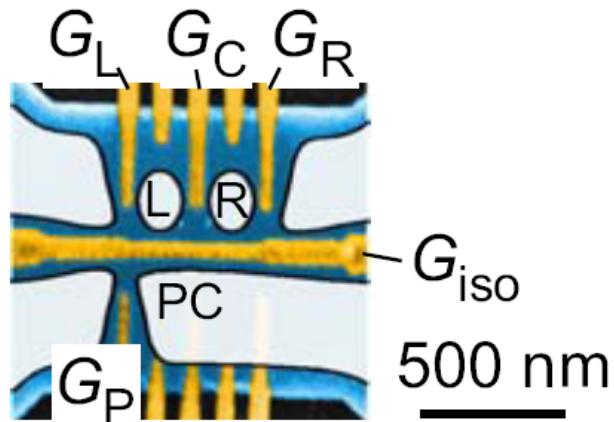


# Measuring Very Small $\Gamma$

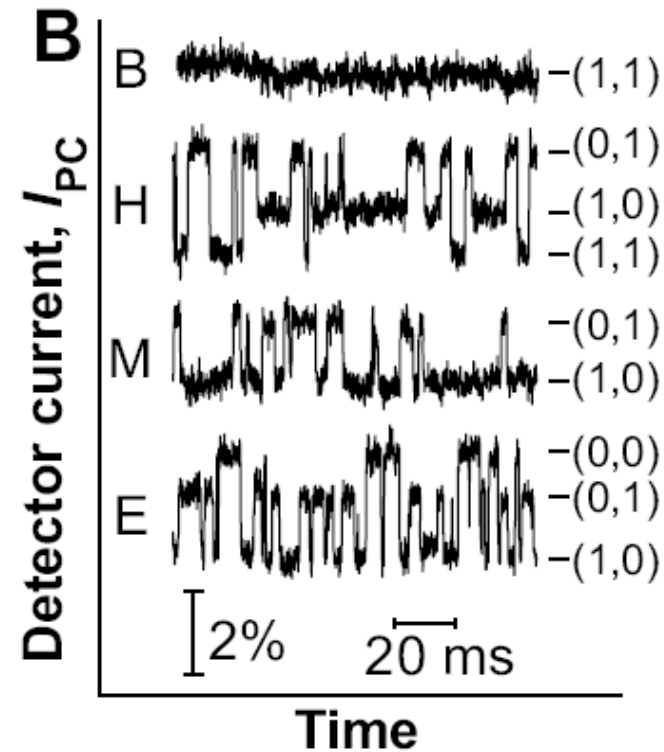
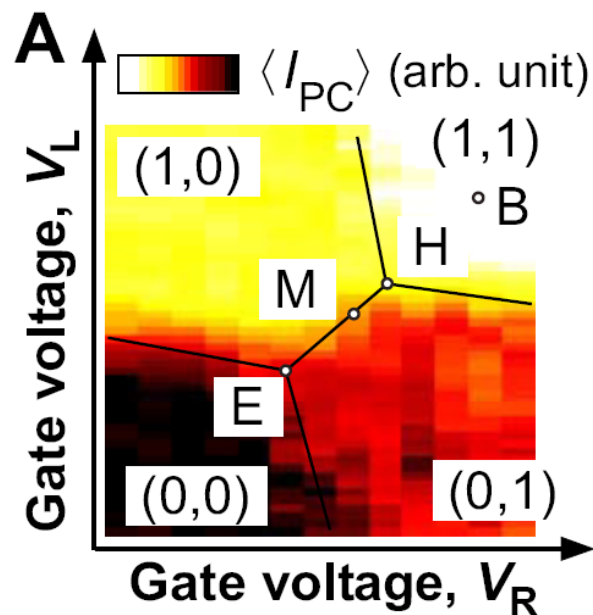


$\Gamma/h$  can be measured  
from 10-1000 Hz, compared to  
10-50 GHz from peak shapes

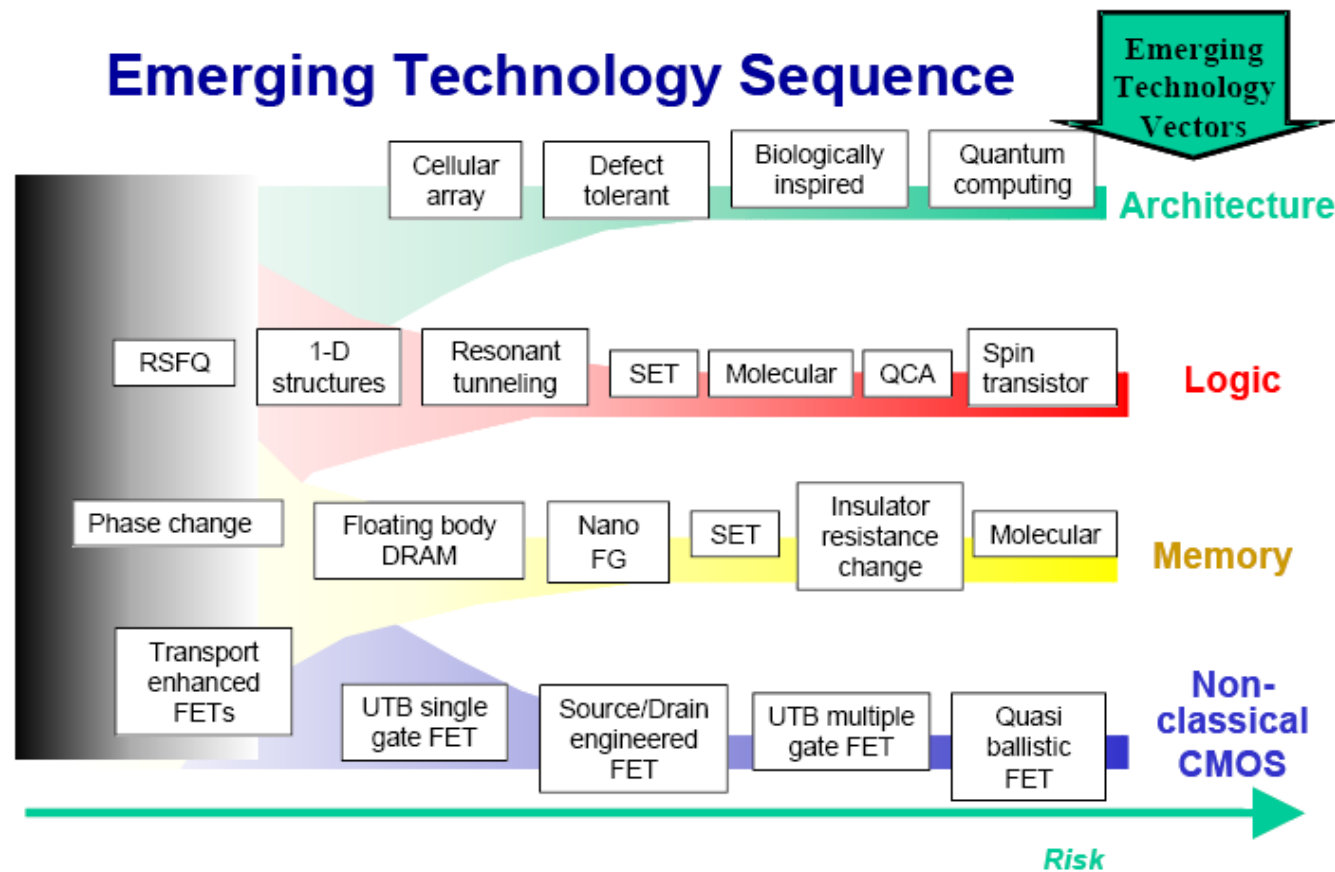
# Measuring Electron Statistics



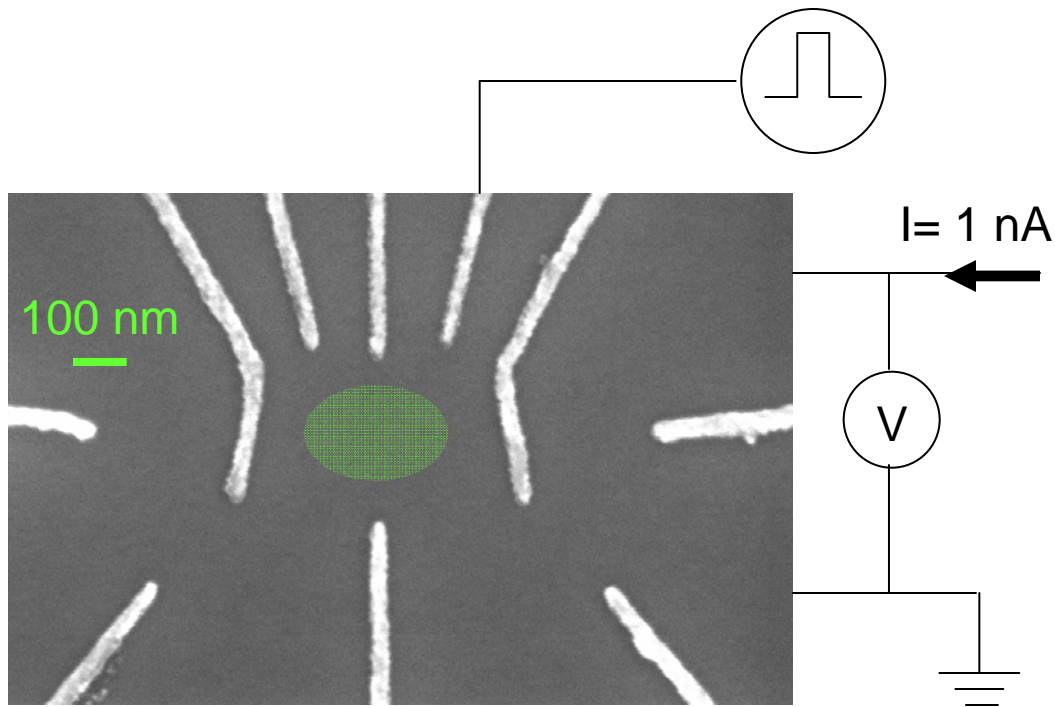
Fujisawa et al Science 312, 1634 (2006)



# International Semiconductor Roadmap 2003



# Measuring Spin Orbit Relaxation of Single Spin



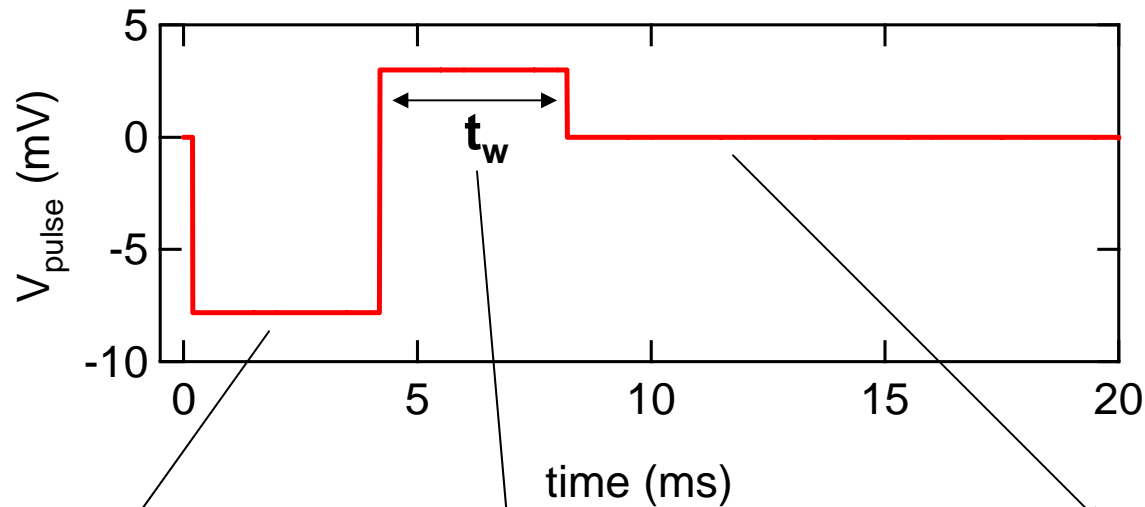
1. Load one electron
2. Adjust chemical potential so only the higher energy spin state can be ionized

# Pulse Sequence

Fujisawa, *et al.*, Physical B **298**, 573 (2001)

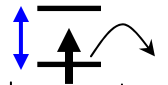
Hanson, *et al.*, PRL **91**, 196802 (2003)

Elzerman, *et al.*, Nature **430**, 431 (2004)



**Ionize**

$g\mu_B B$

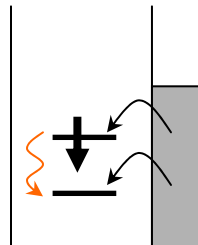


$$P_i = \varepsilon_i$$

$$P_g = 1 - \varepsilon_i$$

$$P_e = e^{-g\mu_B B/kT} P_g$$

**Charge and Relax**

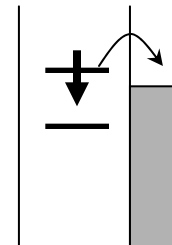


$$P_i(t_w)$$

$$P_g(t_w)$$

$$P_e(t_w)$$

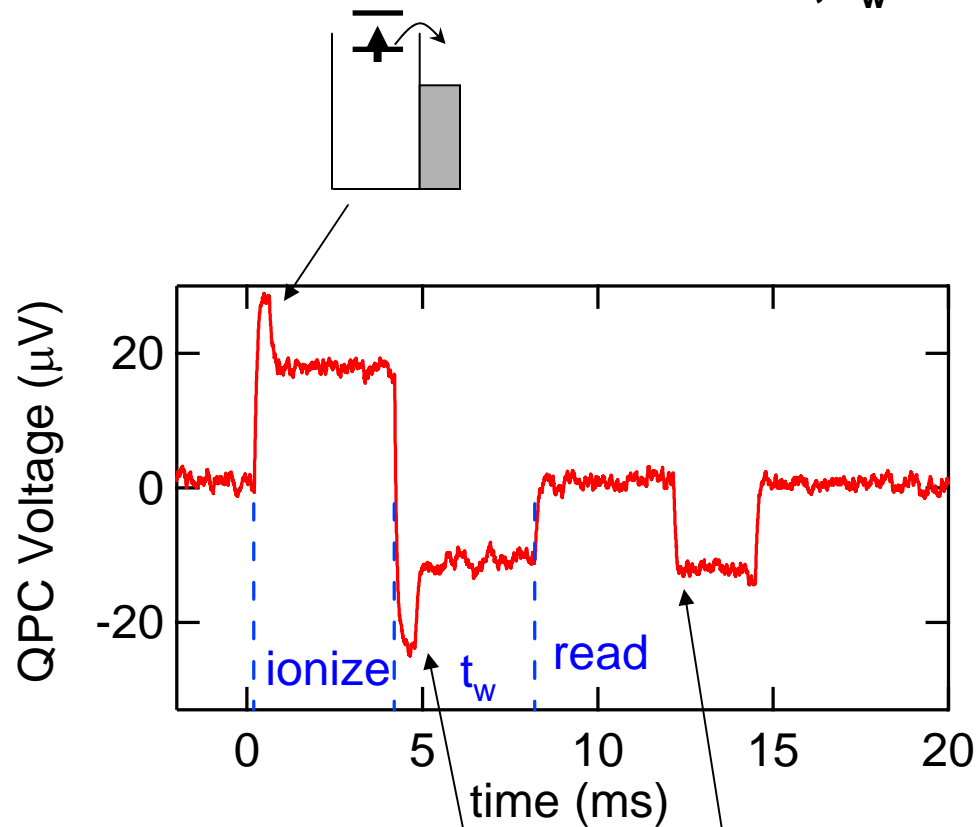
**Real-time Readout**



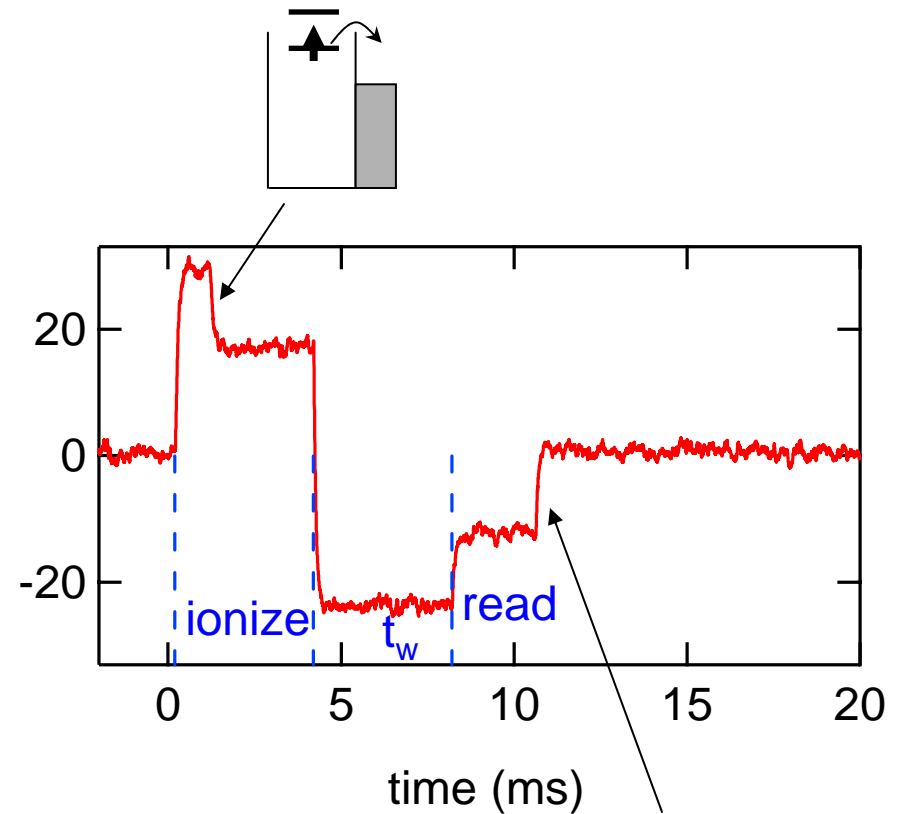
$$P_e(t_w) \text{ gives } W \equiv 1/T_1$$

# Real-Time Readout

$B = 2.5 \text{ T}$ ,  $t_w = 4 \text{ ms}$ ,  $N_{\text{pulses}} = 14300$



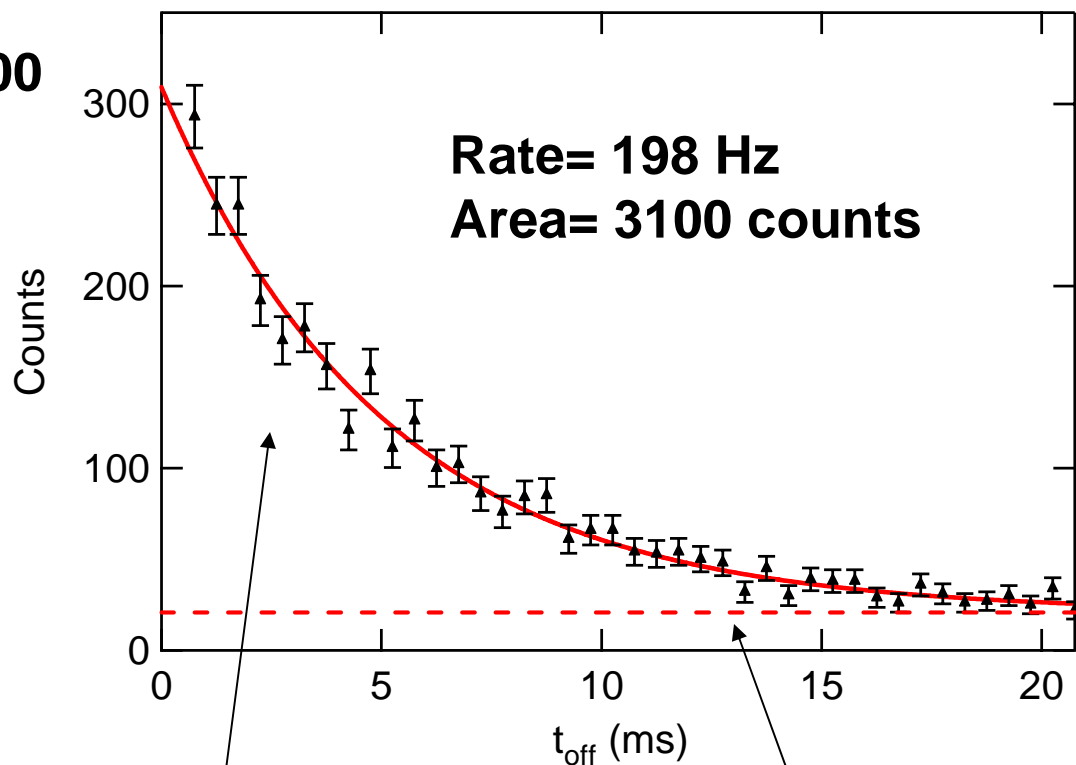
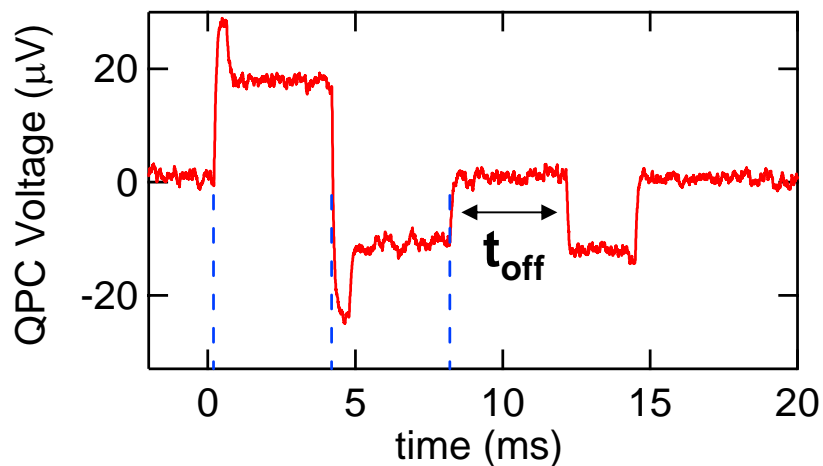
**Electron tunnels  
from excited state**



**Electron tunnels  
into empty dot**

# Measuring $P_e(t_w)$

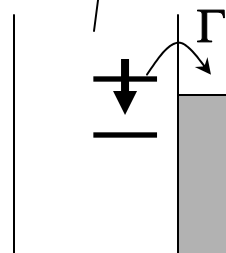
**B= 2.5 T,  $t_w = 4$  ms,  $N_{\text{pulses}} = 14300$**



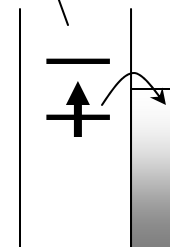
**For  $\Gamma \gg W$ :**

$$\begin{aligned} \text{Rate} &= \Gamma \\ \text{Area} &= N_e(t_w) \end{aligned}$$

$$P_e(t_w) = N_e(t_w) / N_{\text{pulses}}$$



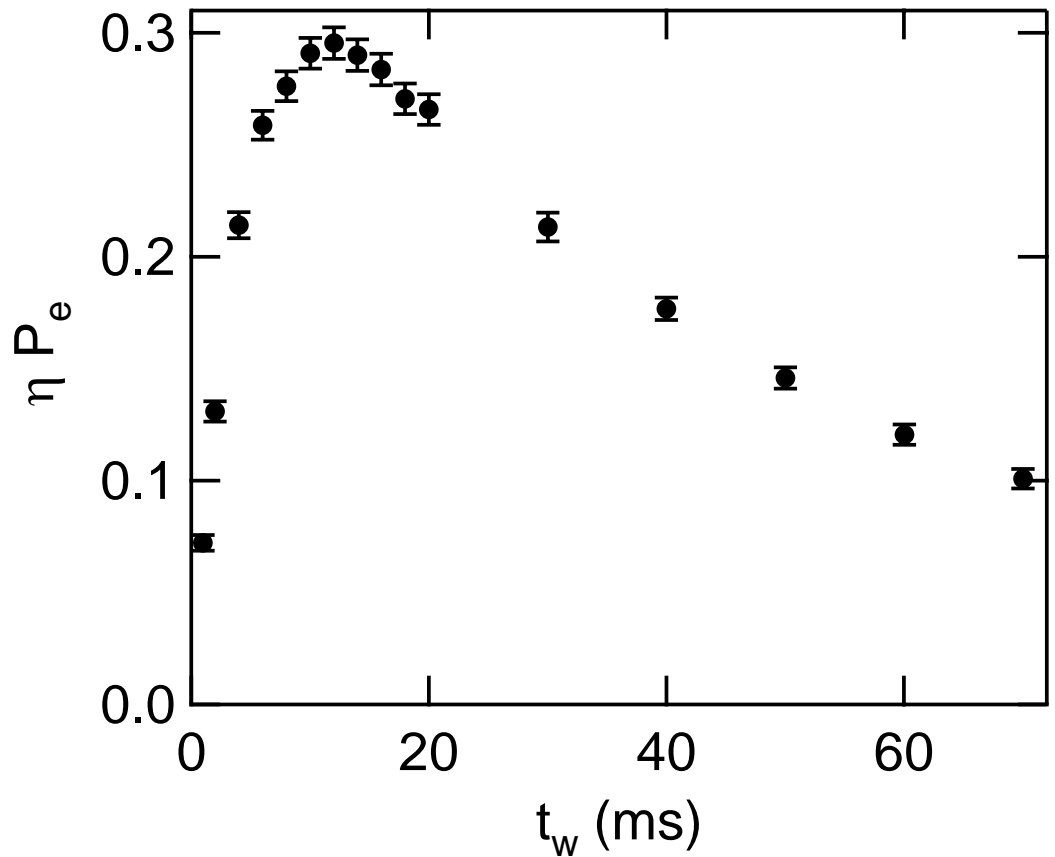
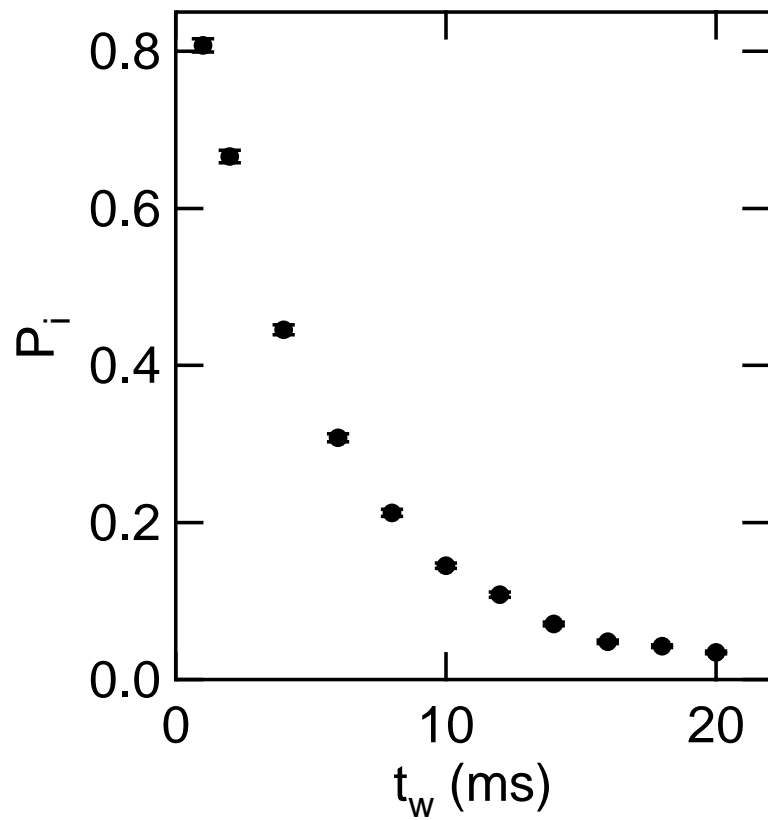
**Electron tunnels  
from excited state**



**Thermally activated  
tunneling from  
ground state**

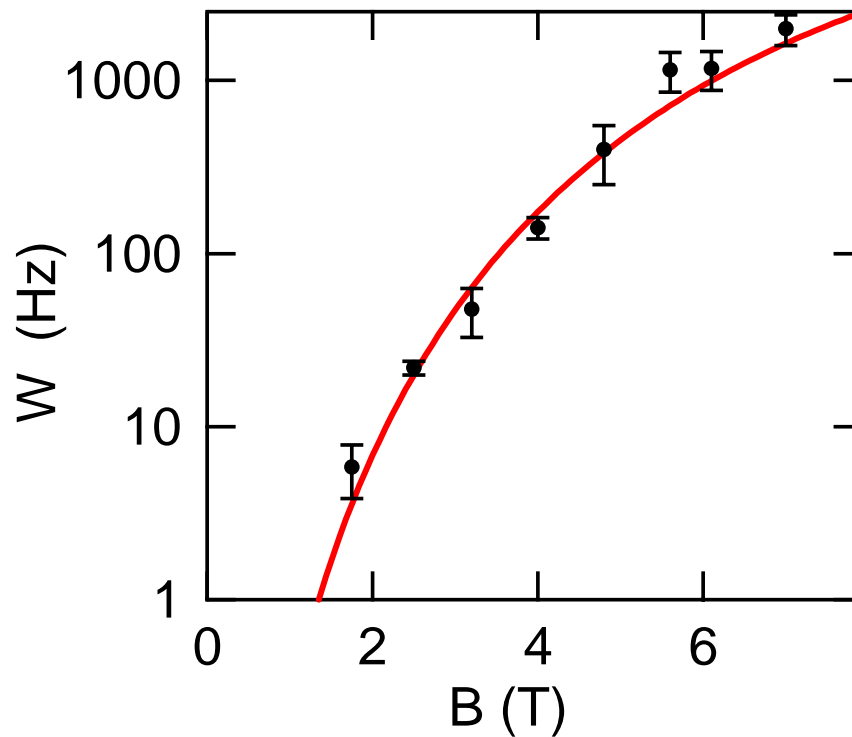
# $P_i(t_w)$ and $P_e(t_w)$

**B= 2.5 T**





# Theory vs Experiment



Theory:  
Golovach, *et al.*  
PRL, **93** 016601  
(2004)