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COLLEGE ON

PHYSICS OF NANO-DEVICES

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Introduction in Bosonization III

Presented by:

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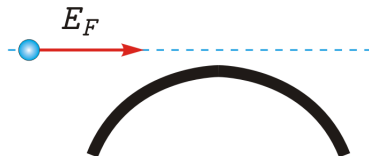
Lancaster University, U.K.

Outline
Two Terminal Conductance of a Clean Quantum Wire
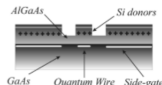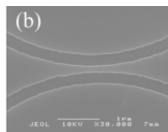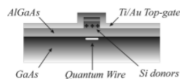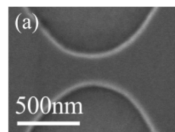Conductance of the Luttinger Liquid With an Impurity

Introduction in Bosonization III

Vadim Cheianov

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1. TWO TERMINAL CONDUCTANCE OF A CLEAN QUANTUM WIRE

2. CONDUCTANCE OF THE LUTTINGER LIQUID WITH AN IMPURITY

WHAT ARE THE TRANSMISSION PROPERTIES?

Landauer-Büttiker formula for two terminal conductance

$$\frac{dI}{dV} = -\frac{2e}{h} \int dE \mathcal{T}(E) \frac{d}{dE} n_F(E)$$

assumes that electrons are non-interacting. In fact, in the point contact region the characteristic energy of Coulomb interaction can be higher than the Fermi energy.

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Two Terminal Conductance of a Clean Quantum Wire

FORMULATION OF THE PROBLEM

- The system is clean (no impurity backscattering)
- The contacts are adiabatic (no contact backscattering)
- The electron-electron interactions are strong in the wire region

Q: What is the two-terminal DC conductance?
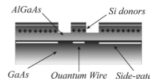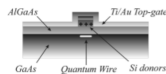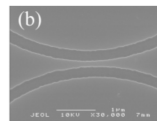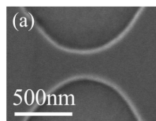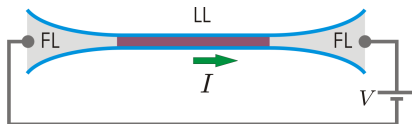
THE MODEL: INHOMOGENEOUS LUTTINGER LIQUID

The system can be modelled by a Luttinger Liquid with adiabatically varying Luttinger parameter:

- In the leads interactions are weak, $K = 1$
- In the wire $K < 1$
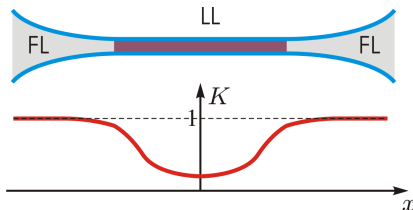
[D. L. Maslov and M. Stone, Phys. Rev. B, 52, 5539 (1995)]

THE MODEL ACTION

$$S_i = \frac{1}{2\pi} \int_0^\beta d\tau \int \frac{dx}{K(x)} \left[\frac{1}{v} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + v \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

THE KUBO FORMULA

The Kubo formula relates the electric current to the equilibrium current-current correlation function.

The DC response to electric field $E(x)$ is given by

$$I(x) = \lim_{\omega \to 0} \int_{-L}^{L} \sigma_\omega(x, x') E(x') dx'$$

where

$$\sigma_\omega(x, x') = \frac{ie^2}{\hbar\omega} \int_0^\beta d\tau \langle Tj(x, \tau)j(x', 0)\rangle^{-i\omega_n\tau} \Big|_{\omega_n \to i\omega - \epsilon}$$

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The Current-Current Correlation Function

Use the current conservation law $\partial_x \rho + \partial_t j = 0$ to derive the bosonized representation of current:

$$\rho(x) = \frac{1}{\pi} \partial_x \phi \quad \Rightarrow \quad j(x) = -\frac{1}{\pi} \partial_t \phi = -i \frac{1}{\pi} \partial_\tau \phi$$

The current-current correlation function is expressed in terms of the correllator of ϕ fields

$$\sigma_\omega(x, x') = \frac{e^2}{\hbar} \frac{i\omega_n^2}{\pi^2 \omega} G(x, x'; i\omega_n) \bigg|_{\omega_n \to i\omega - 0}$$

$$G(x, x'; i\omega_n) = \int_0^\beta d\tau \langle T \phi(x, \tau) \phi(x', 0) \rangle e^{-i\omega_n \tau}$$

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Boson Propagator

The theory has a Gaussian action

$$S_i = \frac{1}{2\pi} \int_0^\beta d\tau \int \frac{dx}{K(x)} \left[\frac{1}{v} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + v \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

Therefore the propagator coincides with Green's function of the equation of motion:

$$\left[\frac{\omega_n^2}{vK} - \partial_x \frac{v}{K} \partial_x \right] G(x, x', i\omega_n) = \pi \delta(x - x')$$

It can be seen that if $K(x) \to 1, \quad x \to \pm\infty$

$$G(x, x', \omega - i0) \to \frac{i\pi}{2\omega}, \quad \omega \to 0$$

THE TWO TERMINAL CONDUCTANCE

The conductivity tensor is given by

$$\sigma_\omega(x, x') = -\frac{e^2}{\hbar}\frac{i\omega}{\pi^2}G(x, x'; \omega - i0) = \frac{e^2}{h}, \quad \omega \to 0$$

So the current is

$$I(x) = \int_{-L}^{L} \frac{e^2}{h}E(x')dx' = \frac{e^2}{h}U$$

This result can be generalized to the n-channel case.

CONCLUSION:

The DC conductance of a clean quantum wire is not renormalized by electron-electron interactions and has a universal value ne^2/h as long as the Luttinger Liquid description is appropriate.

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IMPURITY IN THE LUTTINGER LIQUID

Like in the non-interacting case, an impurity induces

- Forward scattering which shifts the phase of right (left) moving particle
- Backscattering, in which left-moving particle becomes right-moving and vice versa

In backscattering processes a large momentum $2k_F$ is transferred to the impurity.

We shall concentrate on the regime $T > \hbar v / L$
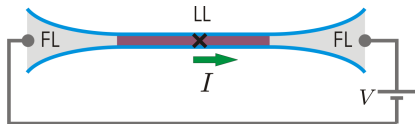
BOSONIZATION OF IMPURITY HAMILTONIAN

For simplicity we shall consider the following Hamiltonian

$$H_{\mathrm{imp}} = u_F[\rho_L(0) + \rho_R(0)] + u_B[\psi_R^\dagger \psi_L(0) + \psi_L^\dagger \psi_R(0)]$$

The bosonization rules for
operators entering impurity
scattering

ψ_R	$e^{i(\theta - \phi)}$
ψ_L	$e^{i(\theta + \phi)}$
$\rho_L + \rho_R$	$\frac{1}{\pi}\partial_x \phi$

BOSONIZATION OF THE THE IMPURITY HAMILTONIAN

$$H_{\mathrm{imp}} = \int dx \delta(x)[\gamma_F \partial_x \phi + \gamma_B \cos(2\phi)]$$

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Eliminating the Forward Scattering

The action with the forward scattering term:

$$S = \frac{1}{2\pi K} \int_0^\beta d\tau \int dx \left[\frac{1}{v} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + v \left(\frac{\partial \phi}{\partial x} \right)^2 \right] + \int_0^\beta d\tau \gamma_F \partial_x \phi(0)$$

Forward scattering can be absorbed into field redefiniton

$$\phi \rightarrow \phi + \frac{\pi K \gamma_F}{v} \text{sign}(x)$$

THE BOUNDARY SINE-GORDON THEORY

Backscattering cannot be eliminated from the action. After eliminating the forward scattering the action becomes

$$S_{\text{BSG}} = S_0 + \int_0^\beta d\tau \int dx \delta(x) \gamma_B \cos(2\phi)$$

where S_0 is the Luttinger action

$$S_0 = \frac{1}{2\pi K} \int_0^\beta d\tau \int dx \left[\frac{1}{v} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + v \left(\frac{\partial \phi}{\partial x} \right)^2 \right]$$

The theory with the action S_{BSG} is called the boundary Sine-Gordon theory. It is not free. However it is integrable.

SCALING OF THE BOUNDARY TERM.

The Luttinger model Hamiltonian is an infrared fixed point of the RG flow. Let us perform the Lyapunov analysis of the Boundary SG term. First, we investigate the scaling of field $\cos(\phi)$ at the fixed point:
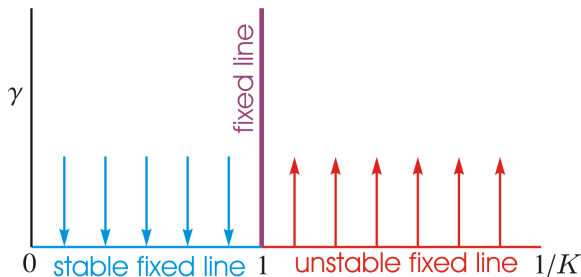
$$\langle T \cos 2\phi(\mathbf{x}) \cos 2\phi(0) \rangle = \frac{1}{(x^2 + v^2\tau^2)^K} \quad \Rightarrow \quad h_{\cos(2\phi)} = K$$

Note, that in Hamiltonian $\cos(2\phi)$ is multiplied by a δ function , which has a scaling dimension 1, so the scaling dimension of the boundary term is $h_B = 1 + K$

> ### THE RG EQUATION
>
> $$\lambda \frac{d}{d\lambda}\gamma_B = (2 - h_B)\gamma_B = (1 - K)\gamma_B$$

THE RG FLOW

$$\lambda \frac{d}{d\lambda} \gamma_B = (1-K)\gamma_B$$

For repulsive interactions $K < 1$ the boundary term is relevant and the Luttinger liquid becomes an unstable fixed point.

CONDUCTANCE FOR WEAK BACKSCATTERING

Suppose the DC conductance (in units e^2/h) of the system at temperature T is $g(T, \gamma)$. Rescaling of the (x, τ) plane: $(x, \tau) \to (\lambda x, \lambda \tau)$ and $T \to T/\lambda$ and $\gamma_B \to \tilde{\gamma}_B$ should leave the physical conductance invariant

$$\lambda \frac{d}{d\lambda} g(T/\lambda, \tilde{\gamma}) = 0 \quad \Rightarrow \quad \left(T \frac{\partial}{\partial T} - \beta(\gamma) \frac{\partial}{\partial \gamma} \right) g(T, \gamma) = 0$$

For small γ_B the conductance should be given by

$$g = 1 - c\gamma^2 \quad \Rightarrow \quad g(T) - 1 \propto -T^{2K-2}$$

The conductance drops with decreasing temperature and presumably at $T = 0$ the system becomes insulating.
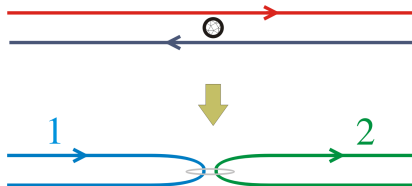
HYPOTHETIC STABLE FIXED POINT

Hypothesis: the stable fixed point describes two decoupled semi-infinite Luttinger liquids with Dirichlet boundary conditions

$$j_{1,2}(0, \tau) = \partial_\tau \phi_{1,2}(0, \tau) = 0$$

This boundary condition is equivalent gluing ψ_L and ψ_R at $x = 0$:

$$\psi_L(0) = \psi_R(0) = e^{i\theta(0)}$$

PERTURBATIONS OF THE FIXED POINT

The leading-order perturbation, which exchanges particles between two semi-infinite liquids is the tunnelling Hamiltonian

$$H_{\mathrm{t}} = \gamma_t \psi_1^\dagger(0)\psi_2(0) + h.c., \qquad \psi_a(0) = e^{i\theta_a(0)}$$

where $a = 1, 2$. The scaling dimension of ψ_a is found from the correlation function:

$$\langle Te^{-i\theta_a(\tau)}e^{i\theta_a(0)}\rangle \sim \frac{1}{|\tau|^{2/K}} \quad \Rightarrow \quad h_t = 1 + K^{-1}$$

$$\lambda\frac{d}{d\lambda}\gamma_t = (1 - K^{-1})\gamma_t \qquad \text{– irrelevant for } K < 1$$

EXERCISE

Investigate the operator content of the theory described by semi-infinite Luttinger liquid and prove the stability of the fixed point discussed above.

TEMPERATURE FOR STRONG BACKSCATTERING

Suppose the DC conductance $g(T, \gamma_t)$ satisfies

$$\left(T \frac{\partial}{\partial T} - \beta(\gamma_t) \frac{\partial}{\partial \gamma_t} \right) g(T, \gamma_t) = 0, \qquad \beta(\gamma_t) = (1 - K^{-1})\gamma_t$$
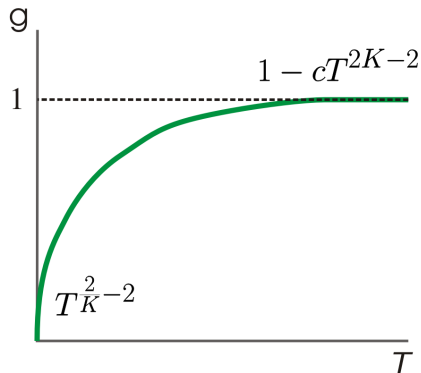
For small γ_t the conductance should be given by

$$g = c\gamma_t^2 \quad \Rightarrow \quad g(T) \propto T^{\frac{2}{K} - 2}$$

The conductance drops with decreasing temperature and at $T = 0$ the is an insulator.

TEMPERATURE FOR STRONG BACKSCATTERING

This picture summarizes the analysis. There are analytical results for intermediate temperatures, but they do not contribute too much to the physical picture.

IMPURITY AT $1 - K \ll 1$

In this special case one can interpret the blocking of transport as Bragg reflection on Friedel oscillations. The sharp temperature dependence of transmission is due to the smearing of Friedel oscillations at $x \sim v/T$.

$u(x)$

x
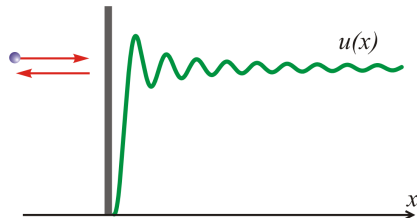
The scaling equation for the Transmission amplitude t can be calculated by means of perturbative RG

$$\lambda \frac{dt}{d\lambda} = -(1 - K)t(1 - |t|^2)$$

SUMMARY

- We considered two related problems, where the bosonization can be used for analyzing the transport properties of a system
- We demonstrated that electron-electron interactions (however strong) do not renormalize the conductance of a clean one-dimensional quantum wire as long as it can be considered as a Luttinger liquid
- We found that introducing impurities in Luttinger liquids makes them unstable in the RG sense
- We found a new universality class of interacting systems - semi-infinite Luttinger liquid, which is the infrared limit of interacting system with an impurity.
- Using scaling arguments we predicted the temperature dependence of the conductance of a system with an impurity. The conductance has distinct features, which cannot be found in Landauer-Büttiker picture

LITERATURE

Luttinger Model

A.O. Gogolin, A.A. Nersesyan, and A.M. Tsvelik, *Bosonization and strongly correlated systems*, Cambridge University Press, Cambridge, England, 1998.

T. Giamarchi, *Quantum physics in one dimension*, Clarendon, Oxford, England, 2004.

LITERATURE

Renormalization Group

R. Shankar, *Renormalization Group Approach to Interacting Fermions*, cond-mat/9307009

H.J. Maris, L.P. Kadanoff, American Journal of Physics, **46**, 652-657 (1978)

LITERATURE

The Impurity Problem

M. P. A. Fisher and L. I. Glazman, cond-mat/9610037

A. Furusaki, cond-mat/0409016