



The Abdus Salam
International Centre for Theoretical Physics



SMR 1760 - 25

COLLEGE ON

PHYSICS OF NANO-DEVICES

10 - 21 July 2006

Quantum transport of chiral electrons in graphene II

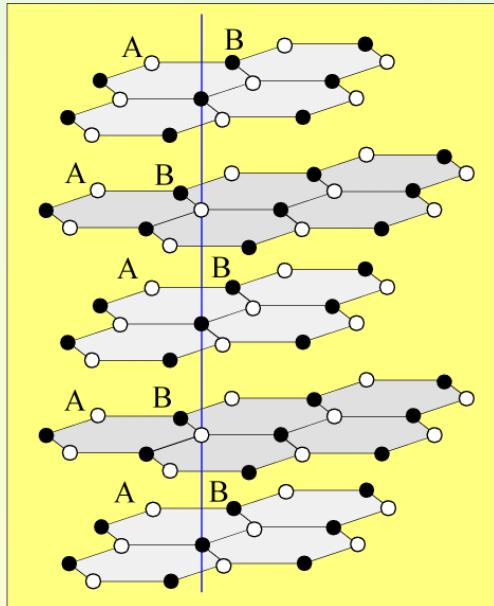
Presented by:

Vladimir Falko

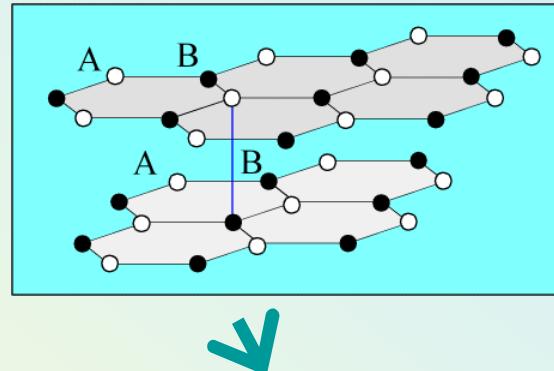
Lancaster University, U.K.

Lecture 2: observable effects

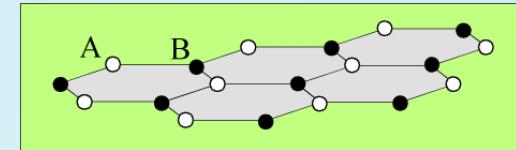
Graphite



Bilayer



Monolayer



1. Tight-binding-model for the Dirac-type electron spectrum in graphene and lattice symmetry.
2. Landau levels of chiral electrons in graphene, Quantum Hall effect in monolayers (graphene) and bilayers.
3. Chirality of carriers in graphene and Berry's phase $J\pi$, quantum transport properties of chiral 2D electrons.

Summary from Lecture 1

chiral electrons in graphene monolayer and bilayer

$$\hat{H}_1 = \zeta v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix}$$

valley

$$\hat{H}_2 = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} + \zeta v_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix}$$

dominant at a high magnetic field
and in high-density structures

$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=+1}$$

‘trigonal warping’:

$$\begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix}_{\zeta=-1}$$

monolayer

$$H_1 = \zeta v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

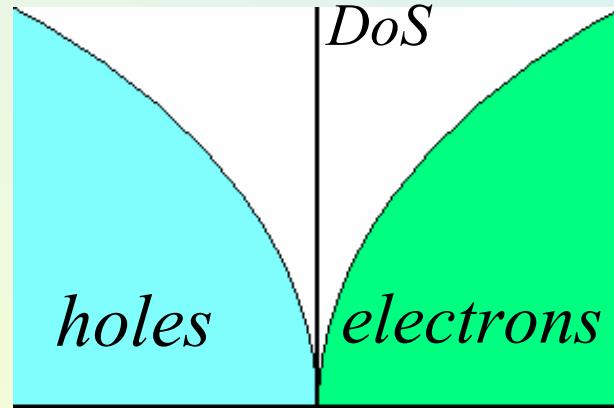
$$\vec{p} = -i\hbar\nabla - \frac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$
$$\pi = p_x + ip_y; \quad \pi^+ = p_x - ip_y$$

$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=+1}$$
$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=-1}$$

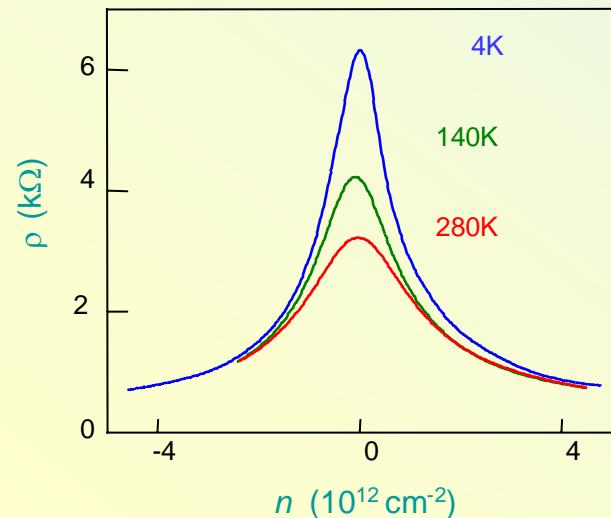
bilayer

$$H_2 = \frac{1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

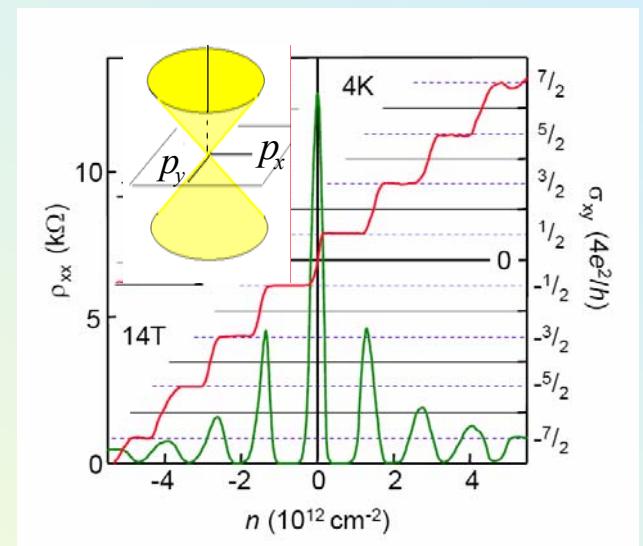
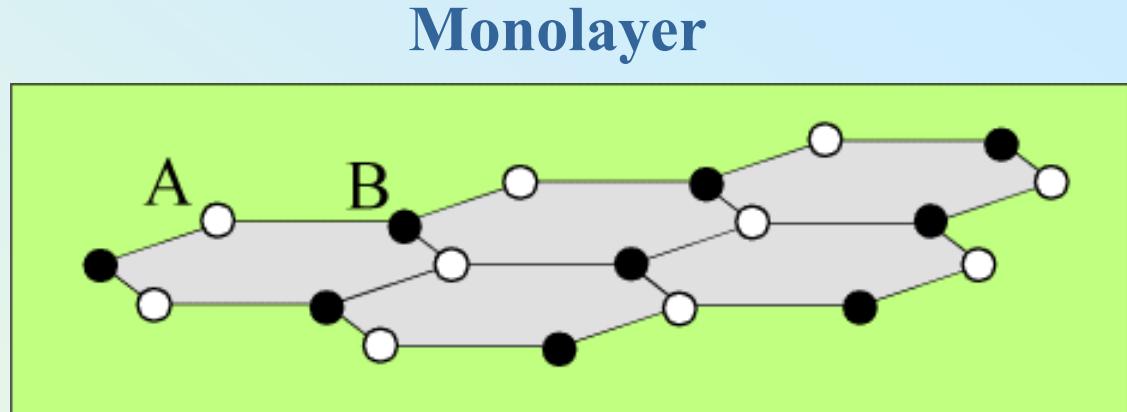
$$\begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix}_{\zeta=+1}$$
$$\begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix}_{\zeta=-1}$$



$$n_{carriers} \propto V_{gate}$$



K. Novoselov et al., Science 306, 666 (2004)

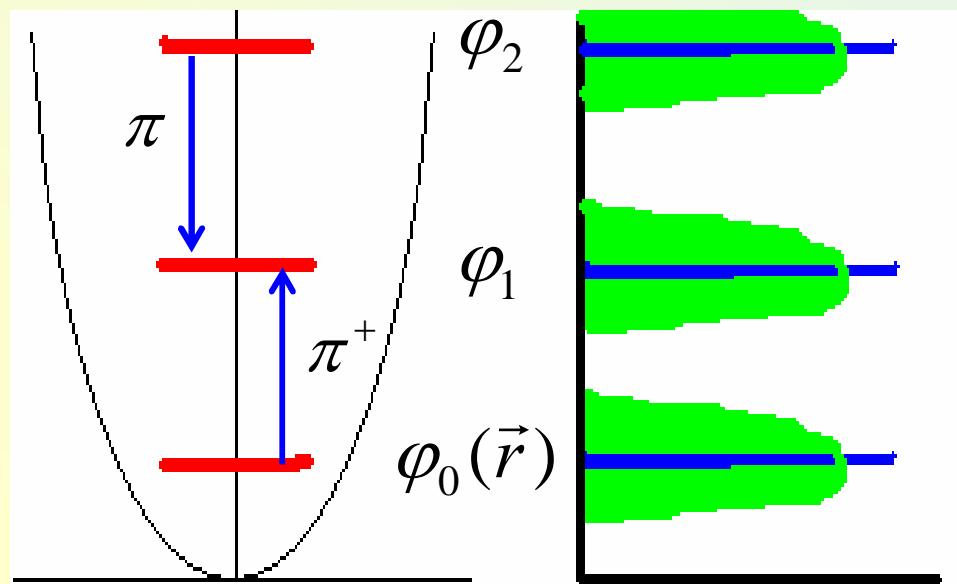


K. Novoselov et al., Nature 438, 197 (2005)
Y. Zhang et al., Nature 438, 201 (2005)

2D Landau levels

semiconductor
QW / heterostructure
(GaAs/AlGaAs)

$$H = \frac{\vec{p}^2}{2m} = \frac{\pi\pi^+ + \pi^+\pi}{4m} \Rightarrow (n + \frac{1}{2})\hbar\omega_c \leftarrow \text{energies / wave functions}$$



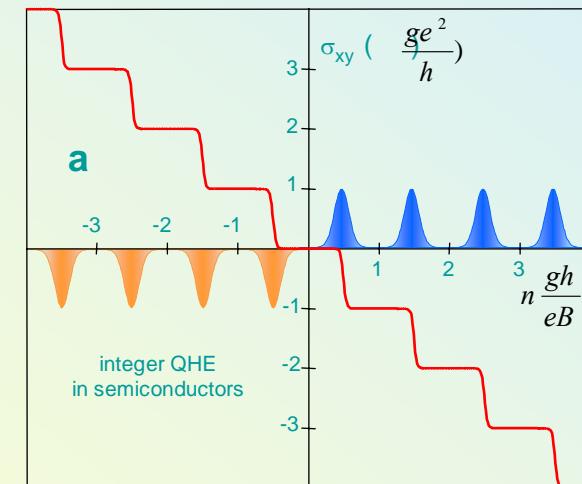
$$\vec{p} = -i\hbar\nabla - \frac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \quad \pi^+ = p_x - ip_y$$

$$\pi\varphi_0 = 0$$

$$\varphi_{n+1} = \frac{\lambda_B}{\sqrt{n+1}} \pi^+ \varphi_n$$

↑
energies / wave functions



Monolayer:

$$H = v\xi \begin{pmatrix} 0 & \pi^+ \\ \pi^- & 0 \end{pmatrix}$$

Bilayer:

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

In a perpendicular magnetic field B :

$\pi \rightarrow$ lowering operator
 $\pi^+ \rightarrow$ raising operator

} of magnetic oscillator eigenstates ϕ_n

We are able to determine the spectrum of discrete Landau levels

States at zero energy are determined by

$$\text{monolayer: } \pi\phi_0 = 0$$

$$\text{bilayer: } \pi^2\phi_0 = \pi^2\phi_1 = 0$$

2D Landau levels of chiral electrons

$J=1$ monolayer
 $J=2$ bilayer

$$\pi^J \varphi_0 = \dots = \pi^J \varphi_{J-1} = 0$$

also, two-fold real
spin degeneracy

$$g \begin{pmatrix} 0 & (\pi^+)^J \\ \pi^J & 0 \end{pmatrix} \psi = \varepsilon \psi$$

$$\begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} \varphi_{J-1} \\ 0 \end{pmatrix} \Rightarrow \varepsilon = 0$$

4J-degenerate zero-energy
Landau level

valley
index

$$\begin{pmatrix} 0 & (\pi^+)^J & & \\ \pi^J & 0 & & \\ & & 0 & (\pi^+)^J \\ & & (-\pi)^J & 0 \end{pmatrix} \begin{pmatrix} A & + \\ \tilde{B} & + \\ \tilde{B} & - \\ A & - \end{pmatrix}$$

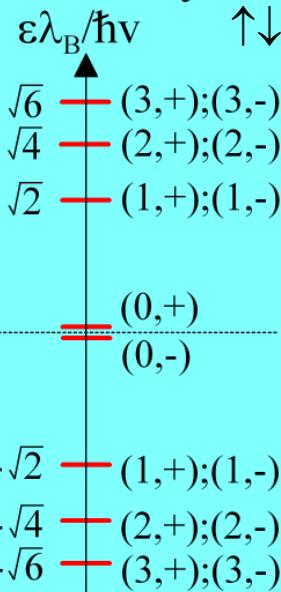
monolayer:

energy scale $\hbar v/\lambda_B$
where $\lambda_B = \sqrt{\frac{\hbar}{eB}}$

state at zero energy:

$$\pi\phi_0 = 0$$

monolayer



J.McClure, Phys. Rev. 104, 666 (1956)

F.Haldane, Phys.Rev.Lett. 61, 2015 (1988)

Y.Zheng, T.Ando

Phys. Rev. B 65, 245420 (2002)

$$\varepsilon_n^\pm = \pm \frac{\hbar v}{\lambda_B} \sqrt{n}$$

bilayer:

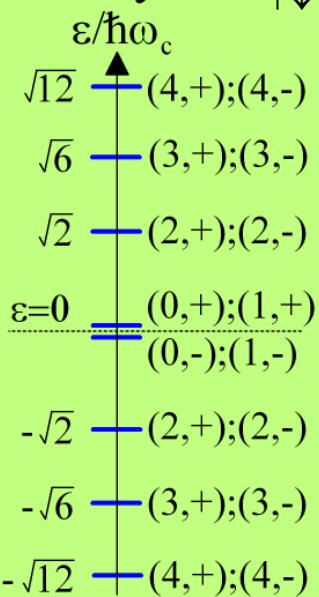
energy scale $\hbar\omega_c$
where $\omega_c = \frac{eB}{m}$

states at zero energy:

$$\pi^2\phi_0 = 0$$

$$\pi^2\phi_1 = 0$$

bilayer

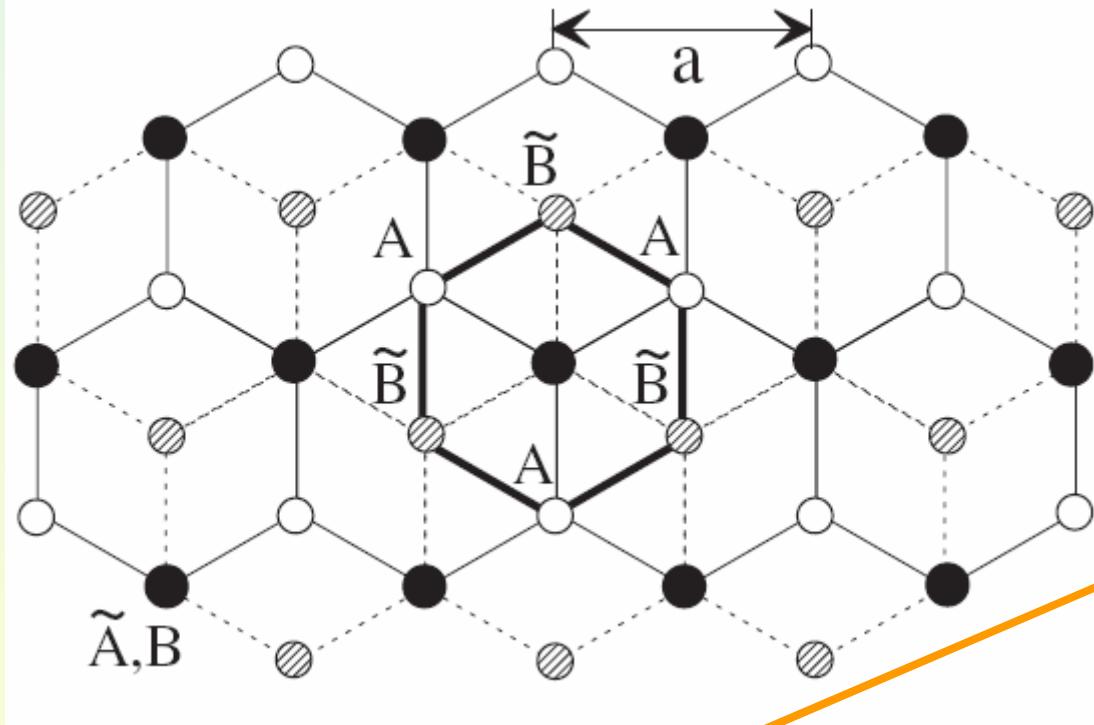


$$\varepsilon_n^\pm = \pm \hbar\omega_c \sqrt{n(n-1)}$$

E.McCann, V.Fal'ko

Phys. Rev. Lett. 96, 086805 (2006)

**8-fold degenerate
zero-energy Landau level**



$$\pi = p_x + i p_y$$

$$-\frac{v^2}{\gamma_1} \begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix}$$

Hops between A and \tilde{B} via $\tilde{A}B$

$$\hat{H}_2 = -\frac{1}{2m} \left[\sigma_x (p_x^2 - p_y^2) + \sigma_y (p_x p_y + p_y p_x) \right]$$

$$+ v_3 (\sigma_x p_x - \sigma_y p_y)$$

$$\mathcal{SV}_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix}$$

Direct inter-layer hops between A and \tilde{B} , $\frac{\nu_3}{\nu} \sim 0.1$

$$-\frac{1}{2m} \begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix}$$

weak magnetic field

$$\lambda_B^{-1} \sim p < m v_3$$

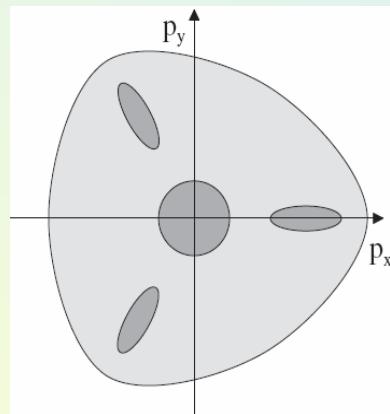
strong magnetic field

$$\lambda_B^{-1} \sim p \gg m v_3$$

$$\begin{aligned}\pi &= p_x + i p_y, \\ \mathbf{p} &= -i\hbar\nabla - e\mathbf{A} \\ [\pi, \pi^\dagger] &= 2\hbar e B.\end{aligned}$$

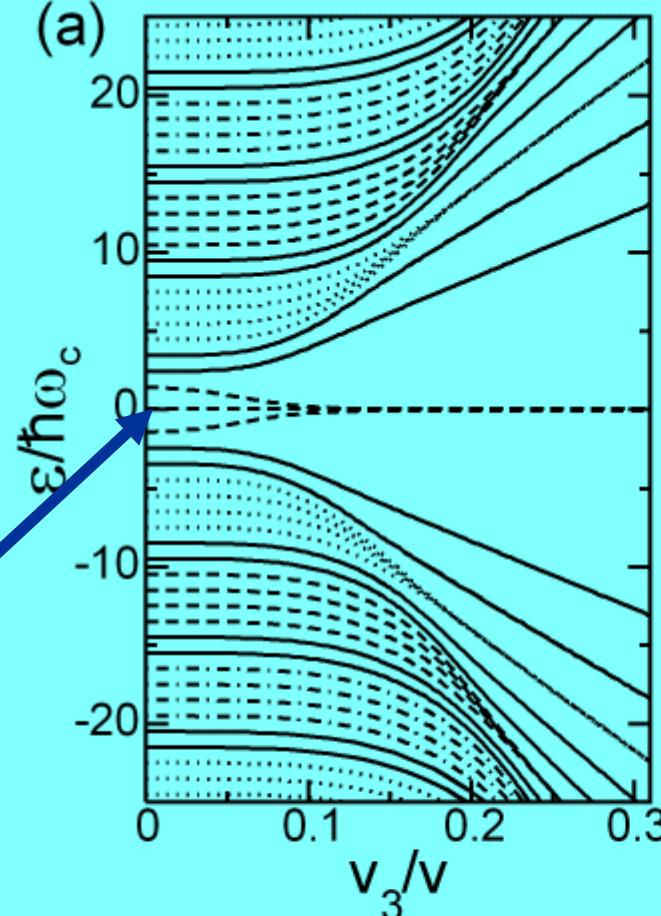
**Landau
wavefunctions**

$$e^{iky} \phi_n(x) \quad \pi^\dagger \phi_n = i(\hbar/\lambda_B) \sqrt{2(n+1)} \phi_{n+1}$$

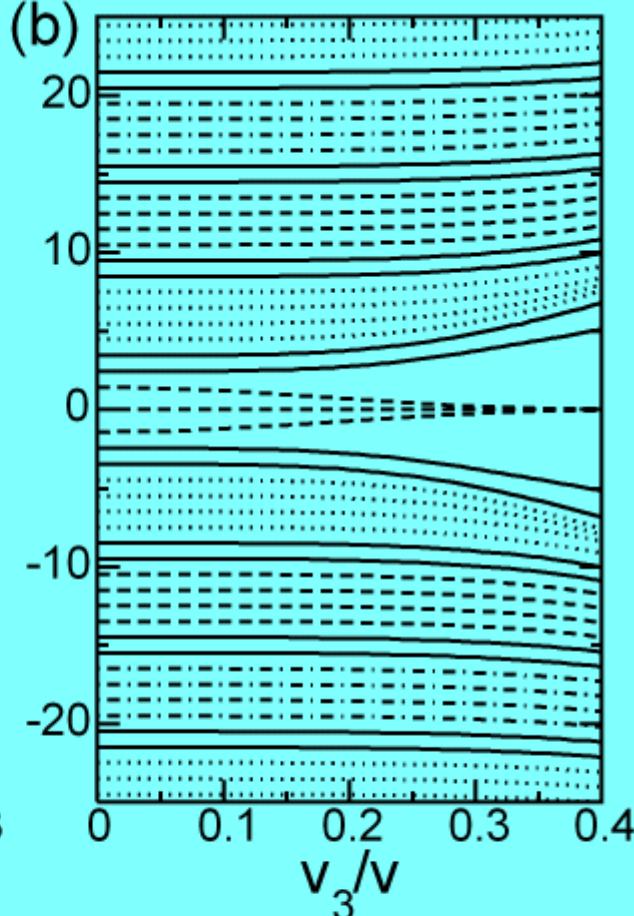


8-fold degenerate
zero-energy
Landau level

(a)



(b)



$$B = 0.1 \text{ T}$$

$$\hbar\omega_c = 0.216 \text{ meV}$$

$$\lambda_B = 0.0812 \mu\text{m}$$

$$B = 1 \text{ T}$$

$$\hbar\omega_c = 2.16 \text{ meV}$$

$$\lambda_B = 0.0257 \mu\text{m}$$

monolayer:

$$H = v \xi \begin{pmatrix} 0 & \pi^+ \\ \pi^- & 0 \end{pmatrix}$$

$$\varepsilon_n^\pm = \pm \frac{\hbar v}{\lambda_B} \sqrt{n}$$

state at zero energy:

$$\pi \phi_0 = 0$$

monolayer
 $\varepsilon \lambda_B / \hbar v$ $\uparrow \downarrow$

$$\sqrt{6} \text{ --- (3,+);(3,-)}$$

$$\sqrt{4} \text{ --- (2,+);(2,-)}$$

$$\sqrt{2} \text{ --- (1,+);(1,-)}$$

$$(0,+)$$

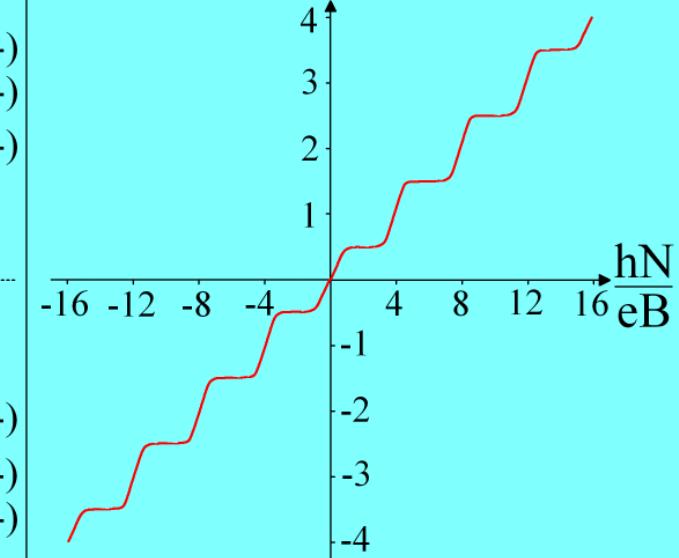
$$(0,-)$$

$$-\sqrt{2} \text{ --- (1,+);(1,-)}$$

$$-\sqrt{4} \text{ --- (2,+);(2,-)}$$

$$-\sqrt{6} \text{ --- (3,+);(3,-)}$$

$\sigma_{xy} (-4e^2/h)$



bilayer:

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

$$\varepsilon_n^\pm = \pm \hbar \omega_c \sqrt{n(n-1)}$$

states at zero energy:

$$\pi^2 \phi_0 = 0$$

$$\pi^2 \phi_1 = 0$$

bilayer $\uparrow \downarrow$

$$\sqrt{12} \text{ --- (4,+);(4,-)}$$

$$\sqrt{6} \text{ --- (3,+);(3,-)}$$

$$\sqrt{2} \text{ --- (2,+);(2,-)}$$

$$\varepsilon=0 \text{ --- (0,+);(1,+)}$$

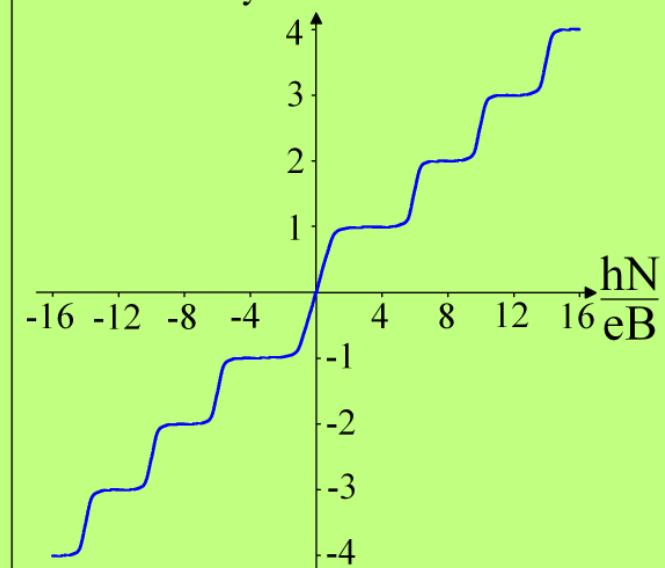
$$\text{--- (0,-);(1,-)}$$

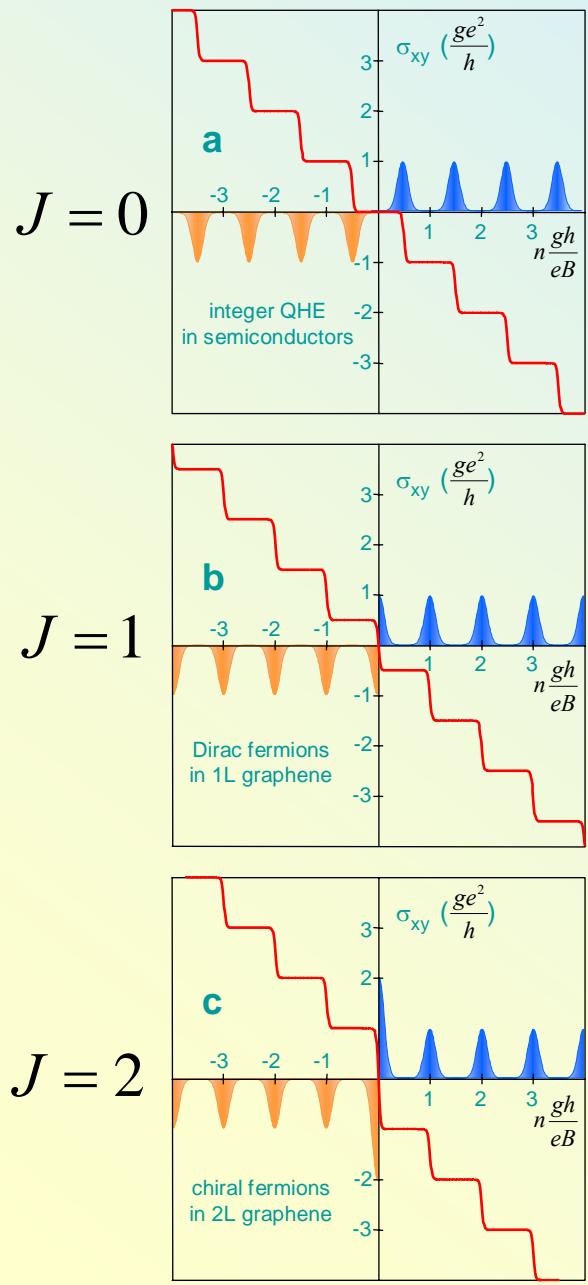
$$-\sqrt{2} \text{ --- (2,+);(2,-)}$$

$$-\sqrt{6} \text{ --- (3,+);(3,-)}$$

$$-\sqrt{12} \text{ --- (4,+);(4,-)}$$

$\sigma_{xy} (-4e^2/h)$





conventional
semiconductor

Monolayer

Bilayer

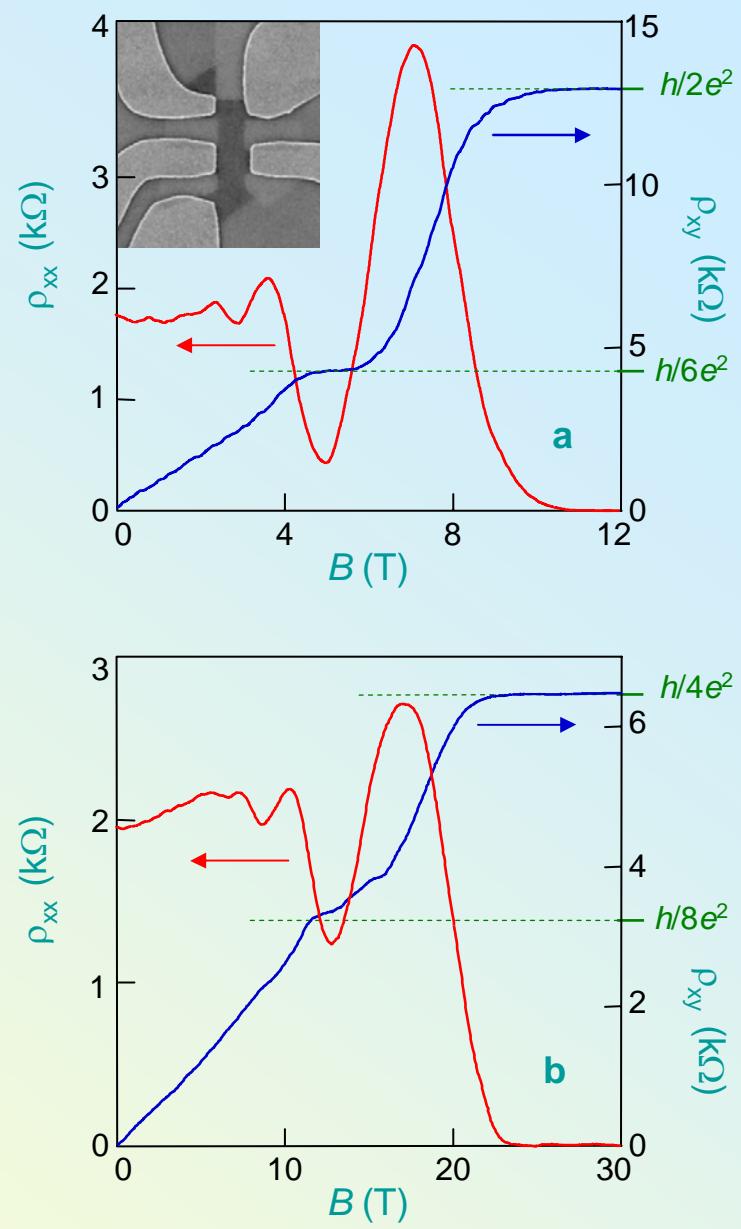
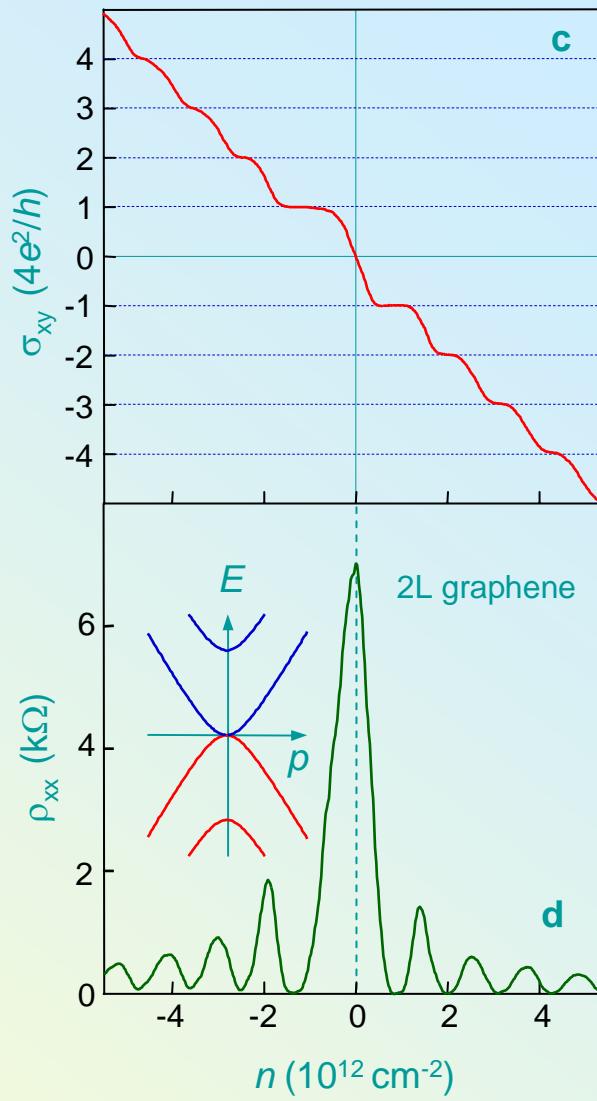
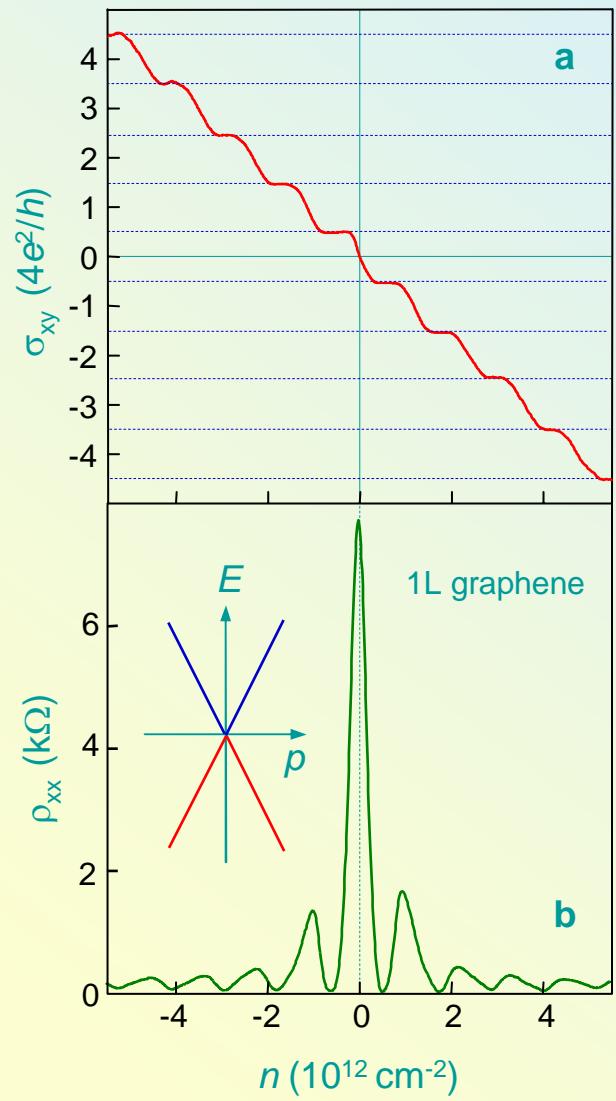


Figure 1

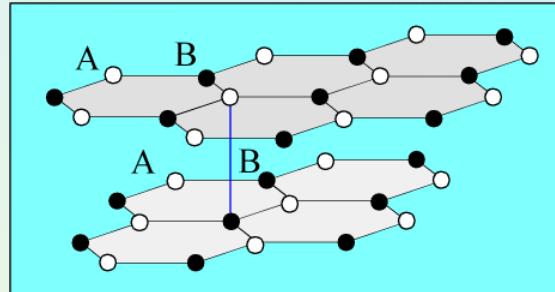


Unconventional quantum Hall effect and Berry's phase of 2π in bilayer graphene

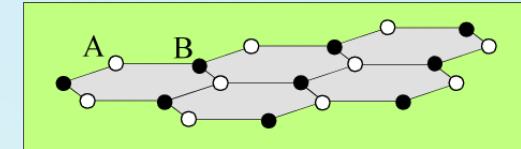
K.Novoselov, E.McCann, S.Morozov, V.Fal'ko, M.Katsnelson, U.Zeitler, D.Jiang, F.Schedin, A.Geim
 Nature Physics 2, 177 (2006)

Content

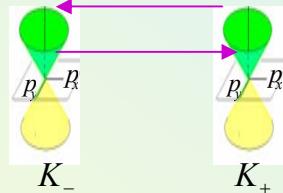
Bilayer



Monolayer



1. Tight-binding-model for the Dirac-type electron spectrum in graphene and lattice symmetry.
2. Landau levels of chiral electrons in graphene, Quantum Hall effect in monolayers (graphene) and bilayers.
3. Chirality of carriers in graphene, suppression of backscattering, p-n junction in graphene.



$$\hat{H}_1 = \begin{pmatrix} v\vec{\sigma} \cdot \vec{p} & 0 \\ 0 & -v\vec{\sigma} \cdot \vec{p} \end{pmatrix} + \hat{V}(\vec{r})$$

↑
Disorder:

**4x4 matrix in the
isospin-valley space**

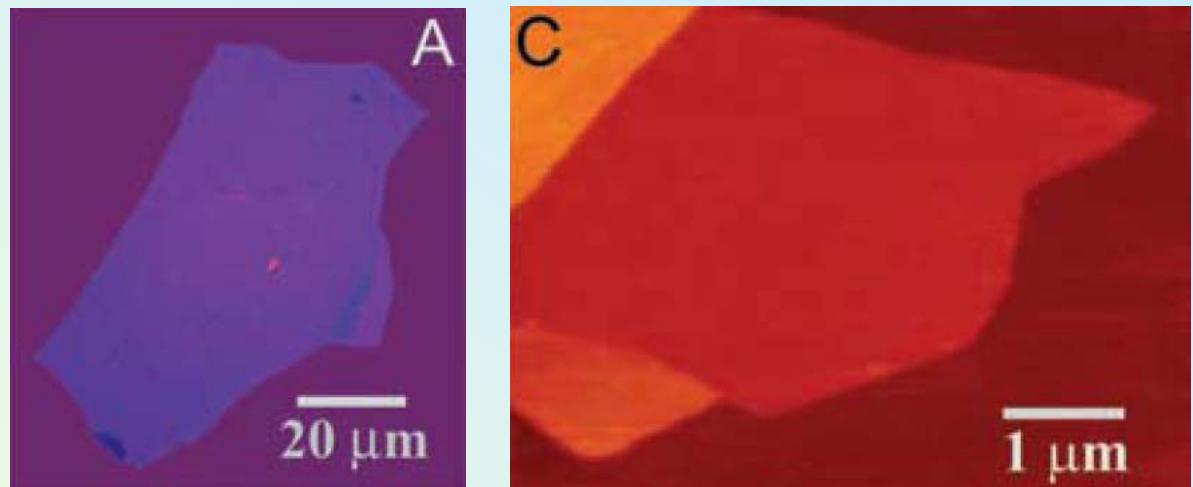
$$\hat{V} = \begin{pmatrix} Iu + u_x\sigma_x + u_y\sigma_y + u_z\sigma_z & wI + w_x\sigma_x + w_y\sigma_y \\ w^*I + w_x^*\sigma_x + w_y^*\sigma_y & Iu + u_x\sigma_x + u_y\sigma_y - u_z\sigma_z \end{pmatrix}$$

atomic-range distortion of the
lattice breaking A-B symmetry

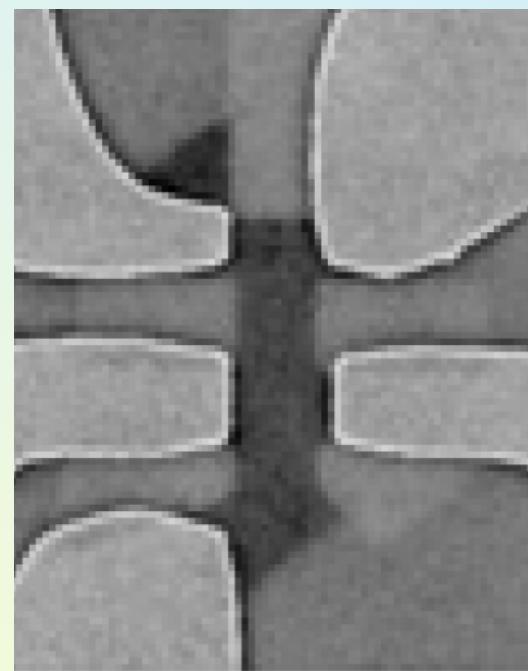
Intervalley-scattering disorder

Ultra-thin graphitic films: from flakes to micro-devices

K. Novoselov et al.,
Science 306, 666 (2004)



- K. Novoselov et al., Nature 438, 197 (2005)
J. Bunch et al., Nano. Lett. 5, 287 (2005)
Y. Zhang et al., Phys. Rev. Lett. 94, 176803 (2005)
Y. Zhang et al., Nature 438, 201 (2005)
K. Novoselov et al., Nature Physics 2, 177 (2006)



**Charged impurities in the substrate (or on its surface)
are the dominant disorder: $V=u(r)I$**

$$D = \frac{1}{2} v^2 \tau_{tr}$$

$$g = e^2 \gamma_F D = \frac{1}{2} e^2 v^2 \gamma_F \tau_{tr} \sim \frac{e^2 v^2}{\langle u_{p_F}^2 \rangle} \sim \frac{e^2}{h} \frac{n_e}{n_{imp}} \left(1 + \frac{hv}{e^2}\right)^2$$

$$\tau_{tr}^{-1} \propto \gamma_F \int d\theta (1 - \cos \theta) |u_{\vec{p}-\vec{p}'}|^2$$

$\theta = \vec{p} \wedge \vec{p}'$

$$u_q \sim \frac{e^2}{q + \kappa_{scr}}, \quad q \sim p_F$$

$$\kappa_{scr} \sim e^2 \gamma_F \sim \frac{e^2}{h\nu} \frac{p_F}{\hbar}$$

**Experiment shows
a linear relation between
conductivity, g
and carrier density, n_e**

K. Novoselov et al., Science 306, 666 (2004)

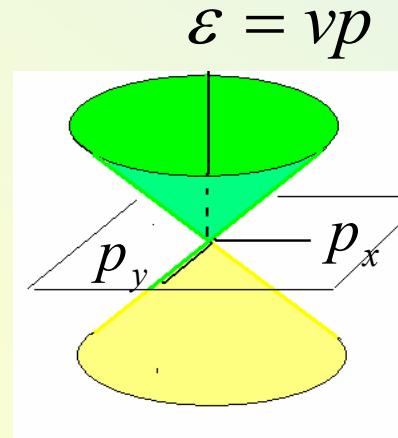
K.Nomura, A.MacDonald,
cond-mat/0604113

$$H_1 = v \begin{pmatrix} 0 & \pi^+ \\ \pi^- & 0 \end{pmatrix} = v \vec{\sigma} \cdot \vec{p} = vp \ \vec{\sigma} \cdot \vec{n}$$

$$\pi = p_x + ip_y = pe^{i\varphi}$$

$$\pi^+ = p_x - ip_y = pe^{-i\varphi}$$

Chiral electrons: isospin direction is linked to the electron momentum.

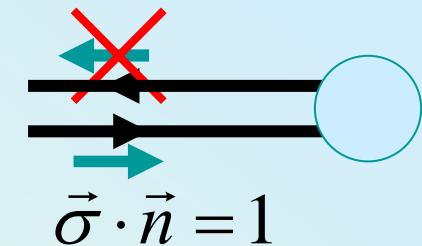


for conduction band electrons,

$$\vec{\sigma} \cdot \vec{n} = 1 \quad \vec{p}$$

$$\vec{\sigma} \cdot \vec{n} = -1 \quad \vec{p}$$

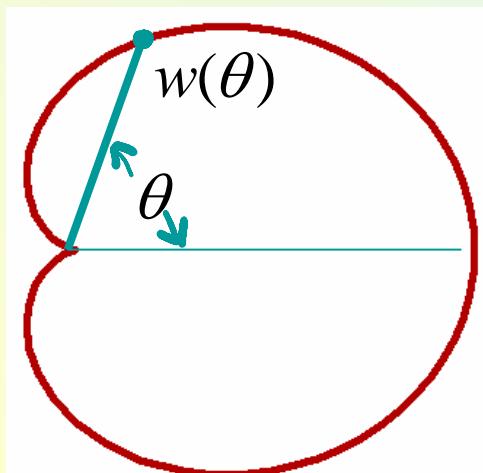
valence band ('holes')



Due to the isospin conservation, A-B symmetric electrostatic potential cannot scatter a chiral fermion in a backward direction.

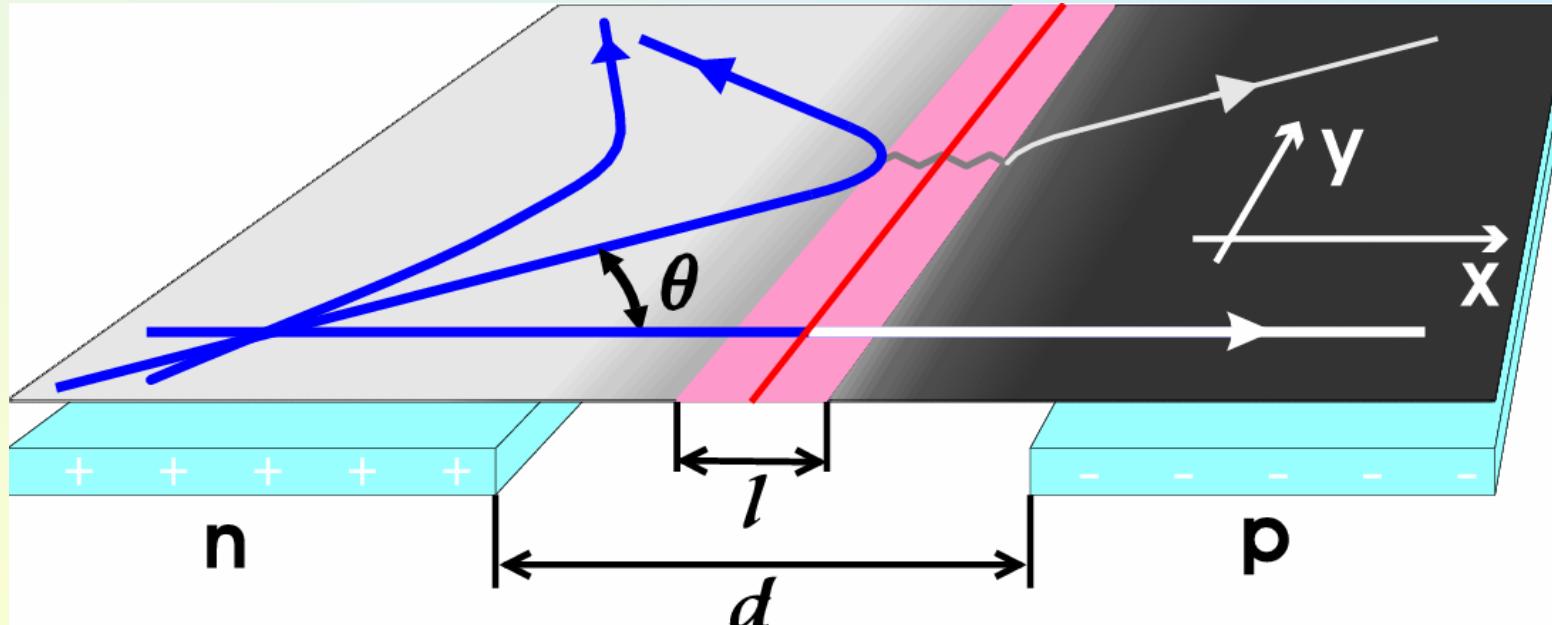
$$\hat{V}(\vec{r}) = Iu\delta(\vec{r})$$

$$w(\theta) \sim \cos^2 \frac{\theta}{2}$$



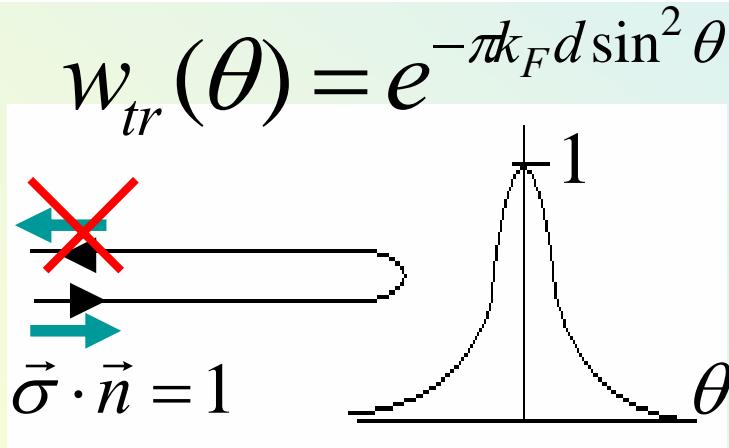
T.Ando, T.Nakanishi, R.Saito,
J. Phys. Soc. Japan 67, 2857 (1998)

Suppressed backscattering of chiral quasiparticles from a potential step leads to a selective transmission properties of an $n-p$ junction

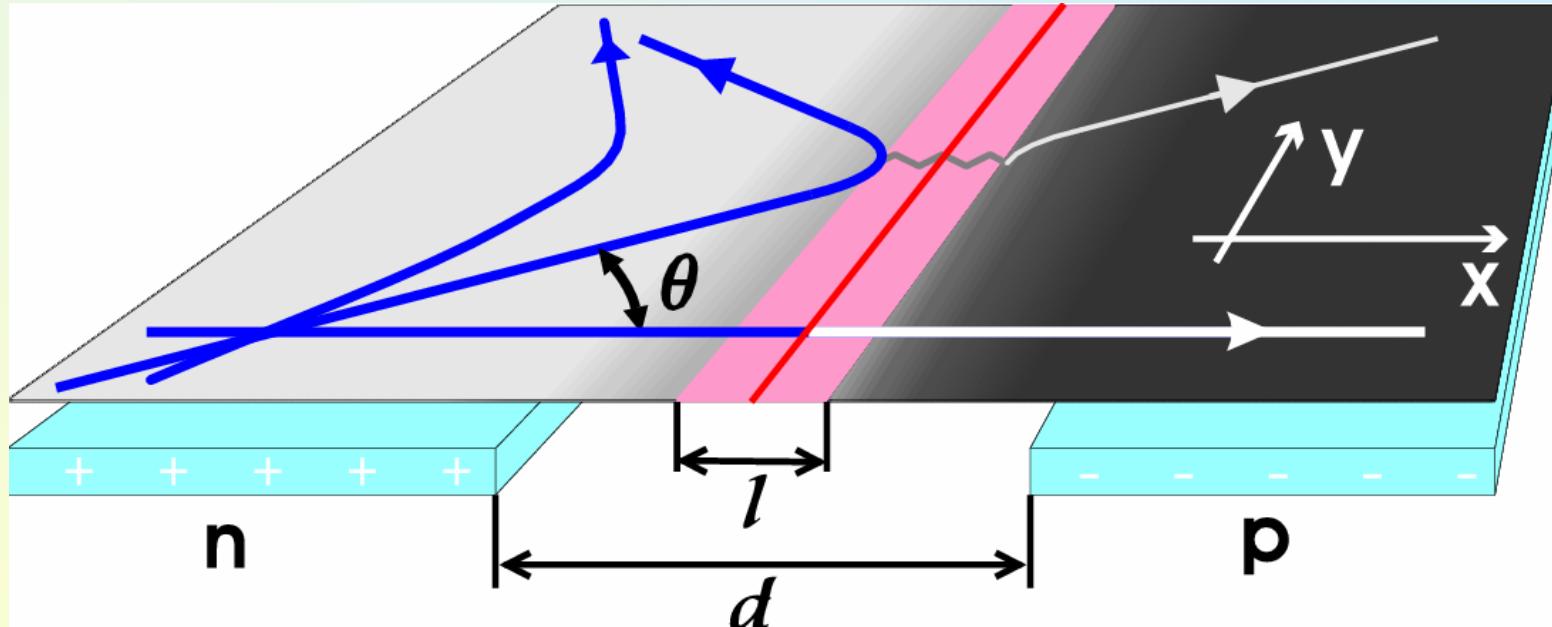


Due to the isospin conservation,
electrostatic potential cannot
scatter a chiral fermion in a
backward direction.

V.Cheianov, V.Fal'ko, cond-mat/0603624



Suppressed backscattering of chiral quasiparticles from a potential step leads to a selective transmission properties of an *n-p* junction

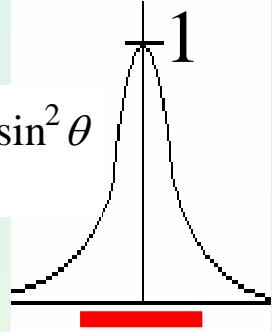


Due to selective transmission of electrons with a small incidence angle, an *n-p* junction in graphene should display a finite conductance per unit length and a characteristic Fano factor

$$g_{np} = \frac{2e^2}{\pi\hbar} \sqrt{\frac{k_F}{d}}$$

$$\langle I \cdot I \rangle = \left(1 - \sqrt{\frac{1}{2}}\right) eI$$

$$w_{tr}(\theta) = e^{-\pi k_F d \sin^2 \theta}$$



selective transmission in an *n-p* junction

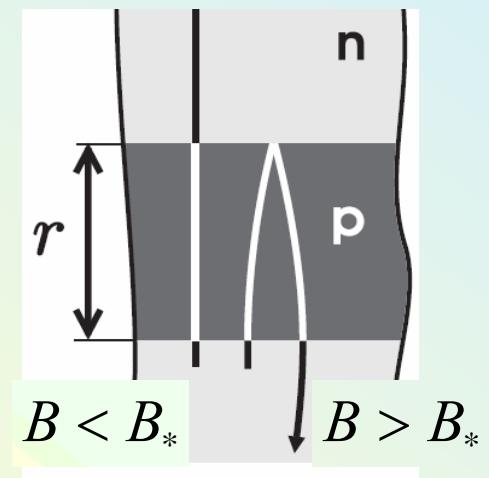
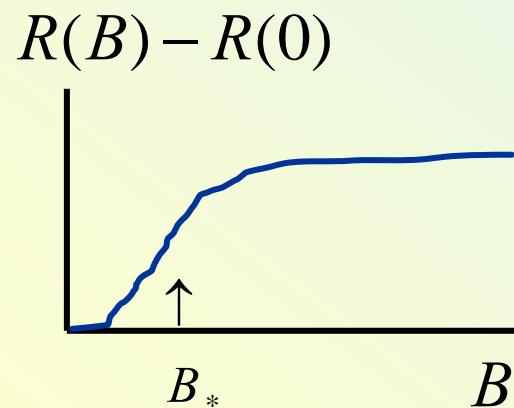
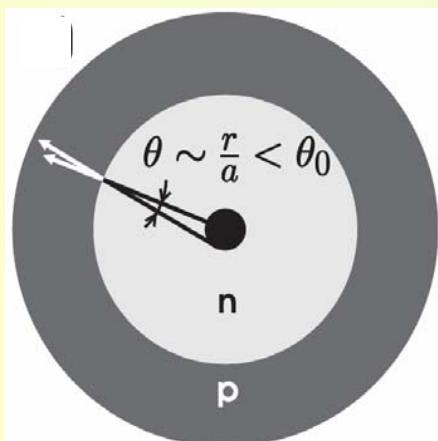
$$|\theta| < \theta_0 = \sqrt{\pi k_F d}$$

$$\theta' \approx \frac{r}{r_c(B_*)} = \theta_0$$

weak-field magnetoresistance of ballistic *n-p* junctions

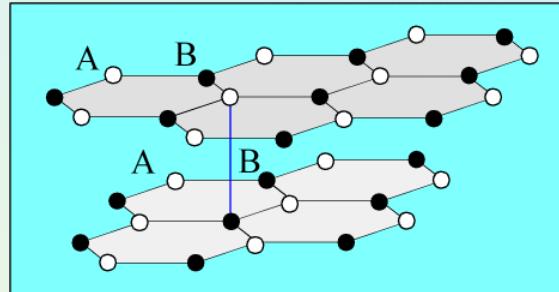
$$R(B) = R_{cont} + \frac{2\pi a}{g_{np}} f\left(\frac{B}{B_*}\right)$$

$$g_{npn} = \frac{g_{np}}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dz}{e^{z^2} + e^{(z+B/B_*)^2} - 1}$$

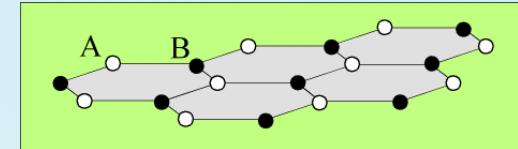


Content

Bilayer



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3. Chirality of carriers, Berry's phase, and quantum transport properties of disordered graphene.

General case:

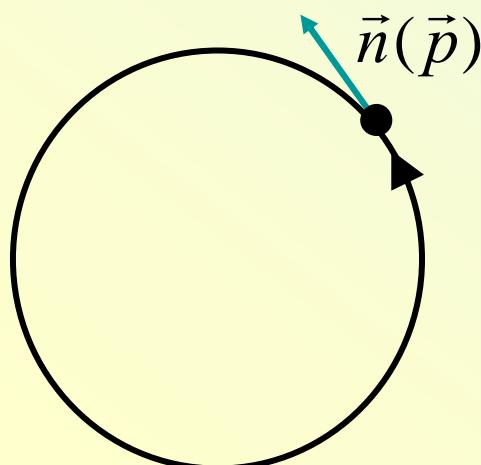
$$H = g \begin{pmatrix} 0 & (\pi^+)^J \\ \pi^J & 0 \end{pmatrix}$$

$$\pi = p_x + i p_y = p e^{i\varphi} \quad \pi^+ = p_x - i p_y = p e^{-i\varphi}$$

$$H = g |p|^J \begin{pmatrix} 0 & e^{-iJ\varphi} \\ e^{iJ\varphi} & 0 \end{pmatrix} = g |p|^J (\sigma_x \cos J\varphi + \sigma_y \sin J\varphi)$$

$$H = g |p|^J (\boldsymbol{\sigma} \cdot \mathbf{n})$$

$$\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$$
$$\mathbf{n} = (\cos J\varphi, \sin J\varphi)$$



$$\psi \rightarrow e^{J2\pi \frac{i}{2}\sigma_3} \psi = e^{iJ\pi} \psi$$

Berry phase $J\pi$
(for a monolayer π
for a bilayer 2π)

$$H_1 = \nu \begin{pmatrix} 0 & \pi^+ \\ \pi^- & 0 \end{pmatrix}$$

Berry phase π
suppressed backscattering
weak anti-localisation ?

H. Suzuura, T. Ando, Phys. Rev. Lett. 89, 266603 (2002)
D. Khveshchenko, cond-mat/0602398

Berry phase romantics

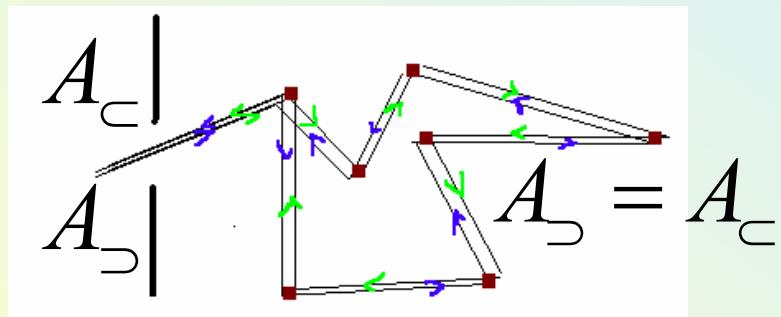


$$H_2 = \frac{1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^- & 0 \end{pmatrix}$$

Berry phase 2π
weak localisation ?

Weak localisation vs anti-localisation

$$w \sim |A_{\subset} + A_{\supset}|^2 = |A_{\subset}|^2 + |A_{\supset}|^2 + [A_{\subset}^* A_{\supset} + A_{\subset} A_{\supset}^*]$$



$$A_{\subset}^* A_{\supset} = |A_{\subset}|^2 > 0$$

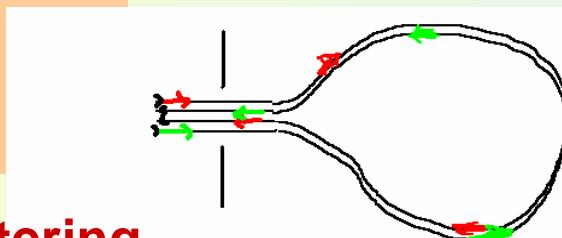
WL = enhanced backscattering in time-reversal symmetric systems

$$e^{i\phi_{\supset}} = e^{i\phi_{\subset}}$$

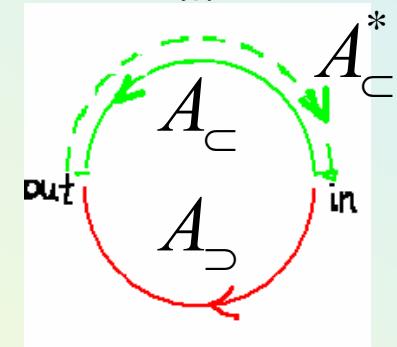
chiral electrons with $\vec{\sigma} \cdot \vec{p} = 1$

$$\begin{aligned} A_{\subset} A_{\supset}^* &= e^{-i2\pi(\sigma_z/2)} |A_{\subset}|^2 \\ &= -|A_{\subset}|^2 < 0 \end{aligned}$$

WAL = suppressed backscattering

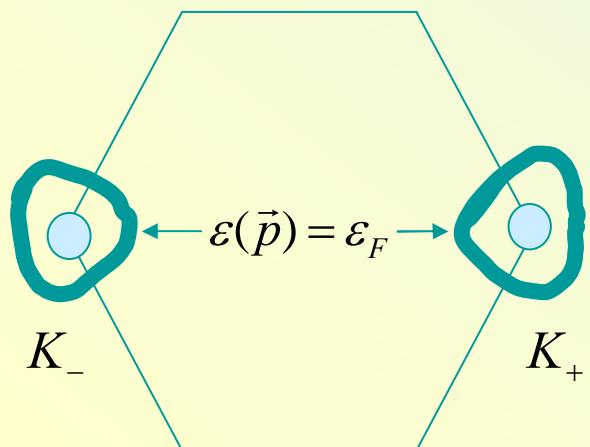


$$\psi_{out} = e^{-i\phi(\sigma_z/2)} \psi_{in}$$



$$\hat{H}_1 = \zeta \nu \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + \hat{V}(\vec{r})$$

valley



$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=+1}$$

$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=-1}$$

‘trigonal warping’:
symmetry of wave vector K is lower
than the hexagonal symmetry

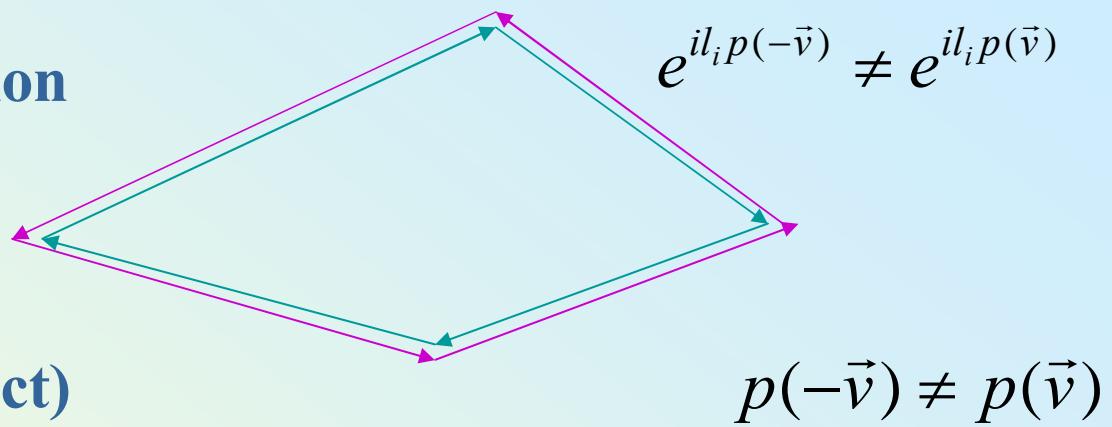
in one valley,
 $\epsilon(-\vec{p}) \neq \epsilon(\vec{p})$

for oppositely propagating electrons,
 $p(-\vec{v}) \neq p(\vec{v})$

**Weak anti-localisation
is suppressed by the
asymmetry of dispersion
in each valley**

$$e^{i\varphi_{\supset}} \neq e^{i\varphi_{\subset}}$$

(no Berry phase π effect)



$$\delta g_1 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

**can be
suppressed
only by
decoherence**

$$\epsilon_F \tau \gg 1$$

**may be
suppressed
by the
intervalley
scattering**

$$\tau_i$$

~~$+ C_{K_+ K_+} + C_{K_- K_-}$~~
**killed by
trigonal warping
reflecting
the asymmetry
 $\epsilon(-\vec{p}) \neq \epsilon(\vec{p})$**

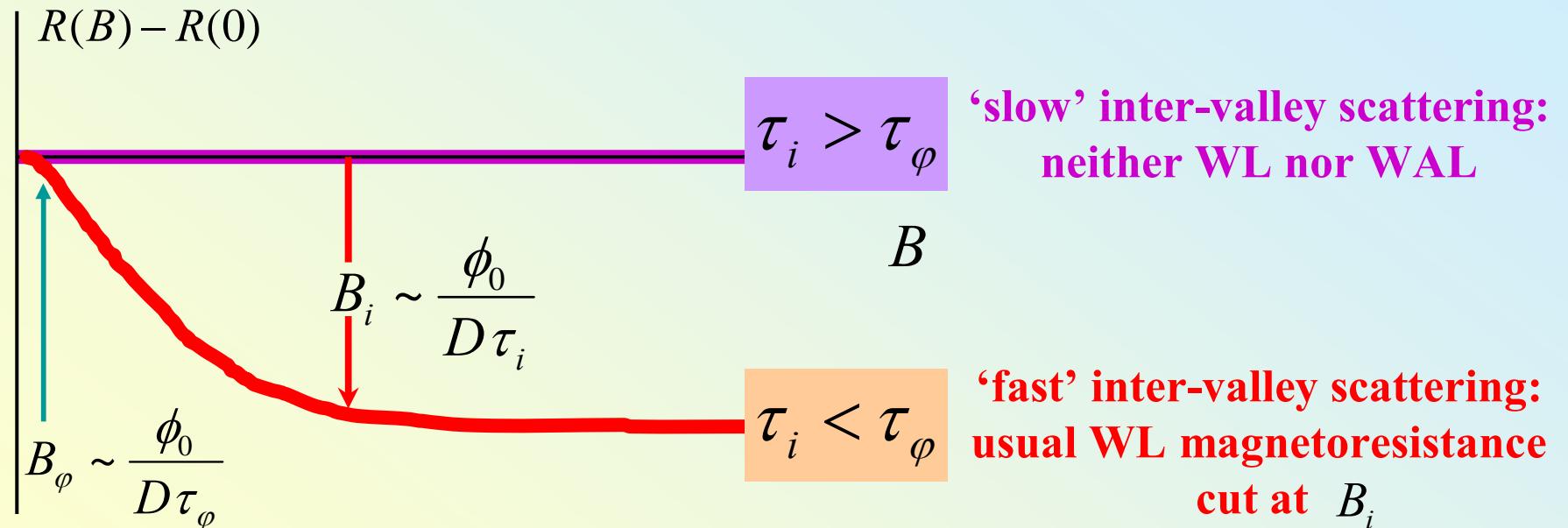
in each valley

Weak localisation magnetoresistance

$$\varepsilon_F \tau \gg 1$$

$$\delta g_1 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

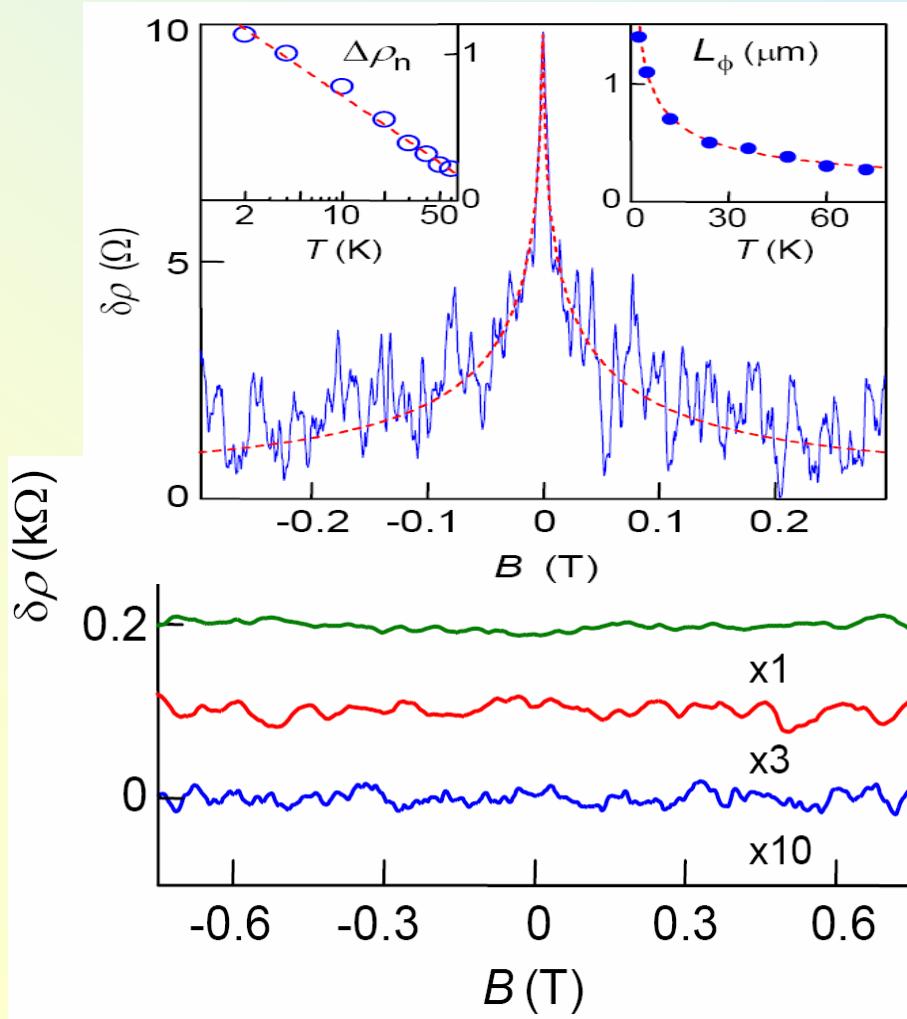
E. McCann, K. Kechedzhi,
 V. Falko, H. Suzuura, T. Ando,
 B. Altshuler
 cond-mat/0604015



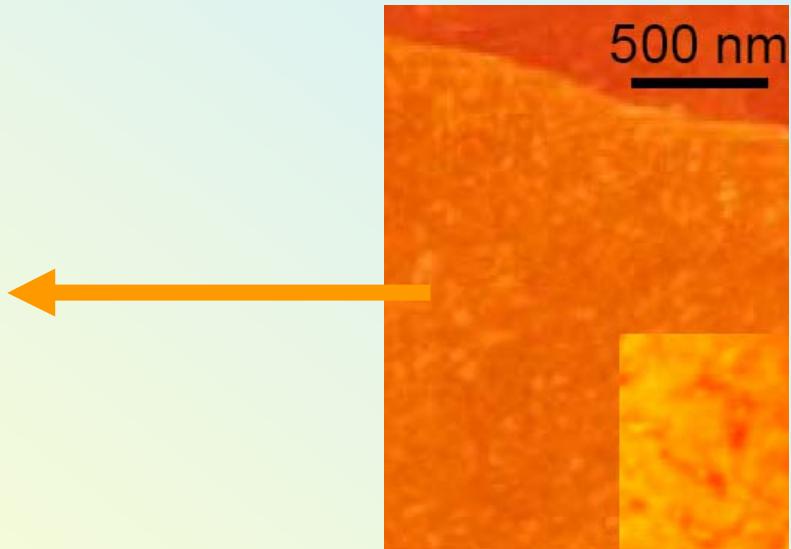
$$\Delta\rho \sim \frac{e^2 \rho^2}{\pi h} \left(F\left(\frac{B}{B_\varphi}\right) - F\left(\frac{B}{B_\varphi + 2B_i}\right) \right)$$

$$F(z) = \ln z + \psi\left(\frac{1}{2} + z^{-1}\right)$$

Weak localisation magnetoresistance in graphene



S.V. Morozov et al, cond-mat/0603826
(Manchester group)



$$H_1 = \zeta \mathcal{V} \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} + \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + \hat{V}(\vec{r})$$

valley

$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix}_{\zeta=+1}$$

$$\begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix}_{\zeta=-1}$$

‘trigonal warping’:
symmetry of wave vector K is lower
than the hexagonal symmetry

$$H_2 = \frac{1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} + \mathcal{V}_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix} + \hat{V}(\vec{r})$$

Weak localisation

$$\delta g_1 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

can be suppressed only by decoherence

$$\delta g_2 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

may be suppressed by the intervalley scattering τ_i due to atomically sharp scatterers or edges

$\epsilon_F \tau \gg 1$ High electron (hole) density and remote Coulomb scatterers

~~$$+ C_{K_+ K_+} + C_{K_- K_-}$$~~

killed by trigonal warping reflecting the asymmetry

$$\epsilon(-\vec{p}) \neq \epsilon(\vec{p})$$

in each valley

~~$$- C_{K_+ K_+} - C_{K_- K_-}$$~~

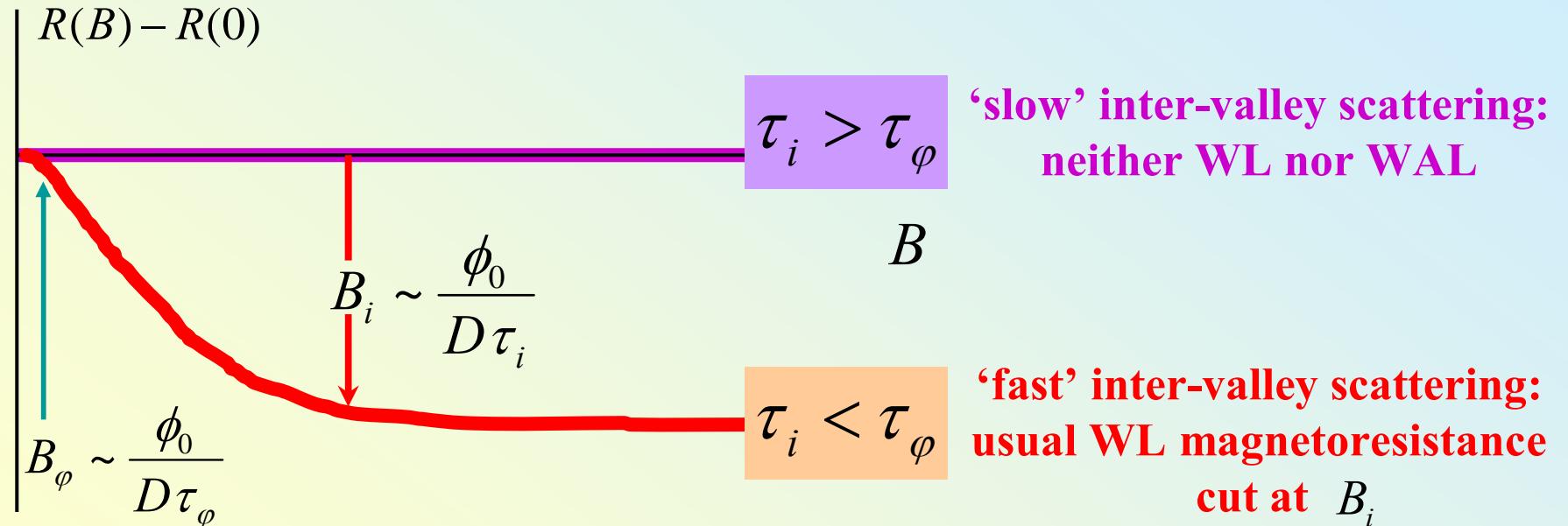
Berry phase 2π

Weak localisation magnetoresistance

$$\varepsilon_F \tau \gg 1$$

$$\delta g_1 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

E. McCann, K.Kechedzhi,
V.Fal'ko, H.Suzuura, T.Ando,
B.Altshuler
[cond-mat/0604015](https://arxiv.org/abs/cond-mat/0604015)



$$\delta g_2 = - C_{\text{valley-symm}} + C_{\text{valley-antisymm}}$$

K.Kechedzhi, E. McCann,
V.Fal'ko, B.Altshuler, 2006

Summary

Graphene – a new type of 2D electron systems with chiral quasiparticles displaying Berry's phase $J\pi$ and $4J$ -times degenerate ‘zero’ energy Landau levels manifested in the QHE behavior.

Chirality of electrons in graphene suppresses backscattering from Coulomb centres and determines selective transmission in $n-p$ junction.

Graphene (monolayers) and bilayers have a rich diagram of weak localisation - antilocalisation regimes

Many more interesting questions

Conductivity of un-doped structures.

e-e interaction effects in monolayers and bilayers.

Correlations in QHE liquid with highly degenerate LL's, FQHE.

Pierls-type transition stimulated by a magnetic field.

Graphene-based nanostructures (electrostatic QDs are not possible).

Hybrid junctions with superconductors and ferromagnets.